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ABSTRACT

The present study was designed to test how well the measures of the cognitive constructs of M-space, cognitive style, and short-term storage space (STSS) could predict the amount of practice needed to induce successful problem solving behaviors, the ability to transfer behaviors to different contexts, and the readiness to abandon unsuccessful behaviors. Thirty-four non-science majors enrolled in a physical science course participated in the study. Those 23 students who did not use formal reasoning schemes to solve ratio problems on a pretest took part in the training phase designed to help them to induce the ratio scheme. The training consisted of balance beam and probability problems at five levels of difficulty corresponding to five levels of development of the ratio scheme. Most of the students induced strategies that permitted them to solve problems at the formal level during the training phase. The STSS could be a predictor for various aspects in the development of complex problem solving strategies. Factor analysis showed the Figural Intersection Test and the Group Embedded Figures Test as one factor and the Ratio Span and the Backward-digit Span Test as another factor. Some implications for science education research were discussed. (YP)

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**Factors in the Development of Reasoning in Two Problem Contexts**

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### Factors in the Development of Reasoning in Two Problem Contexts

The term "neo-Piagetian" refers to modern developmental theories linking advances in theories of human information processing and Piaget's stage theory. Within such theories separate but related theoretical constructs have been proposed by Pascual-Leone (1970) and Case (1985).

Pascual-Leone proposed a developmental construct, *M*, which is measured by the maximum number of discrete chunks of information an individual can attend to at any one time. This construct, often referred to as *M*-space or *M*-capacity, has been shown to correlate significantly with developmental tasks in various domains (Pascual-Leone, 1970; Pulos, Stage, & Karplus, 1982; Karplus, Pulos, & Stage, 1983; de Ribeaupierre & Pascual-Leone, 1979). However, *M*-space does not seem to be the sole determinant of performance on such tasks; that is, the functional, i.e. actually available, *M*-space may be much less than the structural, i.e. maximum, *M*-capacity.

One factor that may bring about such a decrease in processing capacity is an individual's susceptibility to misleading information, a fact which Pascual-Leone accounted for by including the construct of cognitive style in terms of field-dependence-independence. Cognitive style has been reported to correlate significantly with general problem solving performance (Adi & Pulos, 1989; Pulos, Stage, & Karplus, 1980; Karplus, Karplus, & Wollman, 1974; Hill & Redden, 1984; Niaz & Lawson, 1985), with speed and accuracy (Rowe, 1985), with the readiness to change problem solving behaviors (Adi & Pulos, 1980), and with the re-use of inadequate behaviors (Tourniere & Pulos, 1985).

In recent years, the *M*-operator theory has been superseded by theories with a more differentiated view of the processes in short-term memory (Brainerd &

Kingma, 1985; Baddeley, 1982; Hitch, 1984; Case, 1985). According to these theories, short-term memory is divided into a short-term storage space (STSS) and a processing or operating space (OS). A number of studies showed high correlations of STSS with learning potential (Case, 1985: six studies; Liu, 1981; Daneman & Case, 1981; Brainerd, 1981), reading comprehension, and performance on the SAT (Daneman & Carpenter, 1980; Daneman, 1982; Hess & Radtke, 1981).

The present study was designed to test how well the measures of the cognitive constructs of M-space, cognitive style, and short-term storage space could predict the amount of practice needed to induce successful problem solving behaviors, the ability to transfer behaviors to different contexts, and the readiness to abandon unsuccessful behaviors.

#### DESIGN

To test the predictive powers of the neo-Piagetian theories of Pascual-Leone (1970) and Case (1985) four instruments were used. The Figural Intersection Test (Burtis & Pascual-Leone, 1978; Bereiter & Scardamalia, 1978) and the Backward-digit Span Test (Zimmerman & Woo-Sam, 1973) were measures of the M construct; the Group Embedded Figures Test (Witkin, Oltman, Raskin, & Karp, 1971) measured cognitive style; and the Ratio Span Test (Case, 1985) estimated an individual's short-term storage space.

Thirty-four non-science majors enrolled in a physical science course participated in the study. An eight-item test which contained the recipe Problem (Thornton & Fuller, 1981), Mr. Tall and Mr. Short (Karplus & Peterson, 1970), and six juice-mixing problems (Noelting, 1980), was used to divide the subjects into formal (two or more correct answers) and non-formal

reasoners (less than two correct answers). An item was scored as correct only if both the answer and the accompanying reason were correct.

Those 23 students who did not use formal reasoning schemes to solve ratio problems on a pretest took part in the training phase designed to help them to induce the ratio scheme. The training consisted of balance beam and probability problems at five levels of difficulty corresponding to five levels of development of the ratio scheme (Noelting, 1980; Case, 1985). The problems were presented by means of a computer which also provided feedback regarding the subjects' answers. All sessions were tape-recorded while the subjects were "thinking aloud" (Ericson & Simon, 1980). After correctly answering consecutive five problems, a subject proceeded to the next level of problem difficulty. Most students attended two thirty minute sessions weekly until they completed the treatment or until they attended a maximum of twelve sessions.

#### Dependent Variables

Amount of Practice (TRIALS). The number of problem-solving schemes which can be co-activated simultaneously is a function of an individual's short-term storage resources (Case, 1985). The rate at which subordinate problem-solving schemes can be integrated into more complex ones, thus, also becomes a function of the available short-term storage resources. Although mathematical learning models which include short-term memory (or other individual characteristics) are in general non-linear (Aldridge, 1983; Anderson, 1983), there is evidence that linear learning models may be sufficient to describe the relationship between amount of practice and achievement. In the present study, the number of trials subjects needed until they mastered a level of

difficulty constituted the dependent variable.

Strategy Selection (CHOICE). Individuals learning a rule from specific examples, in general, will maintain their hypothesis regarding the nature of the rule as long as it leads to correct results. If the feedback is negative, the rule will be modified so that it is consistent with past examples (Anderson, 1985). Such a strategy requires memory for past items, and thus favors subjects with larger STSS measures. Past research has also shown correlations between field-dependence and strategy selection (Adi & Pulos, 1980; Tourniere & Pulos, 1985). In the present study, the independent variables of field-dependence and short-term storage space were correlated with CHOICE, the fraction of problems for which subjects maintained a reasoning strategy after positive feedback and changed a strategy after negative feedback.

Using guidelines from Ericson & Simon (1980), the transcripts were coded in terms of the reasoning strategies used. An initial list of strategies was identified during a pilot study and deemed satisfactory by both authors.

Transfer Ability (TRANSFER). The dichotomous variable representing transfer (TRANSFER) was coded "one" when a subjects transferred their reasoning level from the training phase to the posttest which contained novel, but structurally equivalent problems. TRANSFER was coded "zero" when a subject did not use reasoning at a lower level than during the training phase.

Monitoring the Learning/Problem Solving Process (MONITOR). The frequency of monitoring statements such as 'I must be going backwards', 'I must be doing something wrong', or 'If this doesn't work then you can't add them up [weight and distance on balance problems]' constituted the dependent variable MONITOR.

### Treatment

Before the treatment students were given training in the thinking aloud procedure so they could verbalize their thoughts in an ongoing manner. The treatment consisted of probability and balance beam problems at five levels (Noelting, 1980) which were presented by a microcomputer. Table I presents the structure of the problems at the different levels. The subjects were asked to reason out loud before giving the answer to a problem. Most students attended two to three 30 minute sessions per week until they completed the treatment or for a maximum of twelve sessions.

Probability Problems. The subjects were shown two boxes with white and pink squares (Figure 1). The task was to find the box where there was a greater likelihood of pulling a white square in a blindfolded experiment.

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Insert Figure 1 about here

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Balance Beam Problems. There were three types of balance beam problems (Figure 2). The subjects had to predict either a weight (screen 1) or a distance (screen 2), with all other weights and distances given; or the subjects faced a situation where they had to predict the movement of the balance beam after it was released (screen 3).

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Insert Figure 2 about here

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### RESULTS

The intent of this study was to investigate the predictive power of

constructs from two neo-Piagetian theories regarding selected variables in the development of reasoning in two domains, probability and balance beam problems. In addition, the results of this study also seem to raise some questions about the widely used measures to assess neo-Piagetian constructs. Accordingly, this results section is divided into three sections. The first section reports on the predictive power of the measures of M-space and field-dependence (Pascual-Leone, 1970) and short-term storage space (Case, 1985) in the domain of probability problems. Then, the predictive power of the measure of short-term storage space on aspects in the development of reasoning on the balance beam is discussed. Finally, the results of a factor analysis on the independent variables are presented.

#### Probability Problems

In this training program, 17 of the 23 subjects mastered level five problems, i.e., they used strategies at level IIIB of the Piagetian classification. Figure 3 shows the distribution of the number of trials at

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Insert Figure 3 about here

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each level of problem difficulty. It is evident from Figure 3 that the distribution of the number of trials until a workable algorithm was induced expands with the difficulty level. At the same time, it should be noted that the performance of the subjects at the different levels was not consistent. A subject may have taken one or two trials at level 4 problems but 10 trials at level 5, while another subject showed a reverse tendency.

Pascual-Leone's Constructs of M-space and Field-dependence. Statistical

analyses showed that neither M-space nor cognitive style, alone or in combination, predicted the amount of practice (TRIALS) needed to induce successful behaviors in either domain (Table II & III). Also, there existed

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Insert Table II about here

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no significant correlations between cognitive style and either the ability to transfer (TRANSFER) or the tendency to avoid unsuccessful behaviors (CHOICE; Table III).

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Insert Table III about here

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Case's Construct of Short-term Storage Space (STSS). Ratio span (RATIO), the measure of STSS, proved to be a successful predictor for the number of hypotheses generated until students induced the product-moment rule on the balance beam,  $r(17) = -.40, p < .05$ . RATIO also showed high correlations with the ability to transfer,  $r(22) = .46, p < .05$ , and the tendency to avoid unsuccessful behaviors  $r(23) = .60, p < .01$  (Table III).

#### Balance Beam Problems

The major finding in this domain was that the majority of the 23 subjects (18) induced the product-moment rule (PMR) rather than the ratio rule as predicted by Piagetian theories. The mean number of trials until these 18 subjects used PMR consistently was  $\bar{X} = 14.5, SD = 11.5$ , and a range from 1 to 51 trials. Two of the remaining subjects induced the ratio rule at level 3 after 49 and 149 hypotheses, one induced the ratio rule after 52 trials.

Ratio span was the only neo-Piagetian variable with a significant correlation to the amount of practice students needed to induce the product-moment rule,  $r = -.40$  (Table IV). Significant correlations existed between

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Insert Table IV about here

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the measure of short-term storage space, *RATIO*, and the dependent variables of *CHOICE* (the tendency to avoid unsuccessful behaviors),  $r = .69$ ,  $p = .001$ , and *MONITOR* (the tendency to monitor one's problem-solving activity),  $r = .59$ ,  $p = .01$  (Table IV). The correlations between the foregoing variables and the amount of practice are negative and lie between  $r = -.39$  and  $r = -.63$ .

Ratio span was also a good predictor for the strategy an individual used to do balance problems, i.e. ratio or product-moment rule. For this analysis, data from all subjects, including the formal students, were used. The ratio span means of the two strategy groups, 4.04 and 2.86, respectively, were significantly different,  $t(15) = 4.15$ ,  $p < .001$ .

#### Analysis of the Independent Variables

The correlation matrix of the four independent variables used in this study shows relationships similar to those reported from studies with children (Case & Globerson, 1974; Collis & Romberg, 1979). Table II reveals high correlations between the measures of M-space, *FIT* and *BACK*,  $r = .51$ , and between *FIT* and field-dependence (*GEFT*),  $r = .61$ . The measure of short-term storage space, *RATIO*, correlated significantly with *BACK*,  $r = .49$ . All other correlations were not significant.

This matrix was analyzed by means of a principal components factor analysis

followed by an oblique rotation. Two distinct factors emerged with the measures of M-space and field-dependence loading on the same factor with loadings of .94 and .79, respectively, while RATIO and BACK showed significant loadings, .88 and .82, respectively, on the other factor (Table V). The correlation between the two factors was  $r = .27$ .

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Insert Table V about here

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#### DISCUSSION

The results of this study provide answers to the following three questions. (1) Does a program which is constructed such that the normal stages of the development are recapitulated help students to induce strategies suitable to solve problems at the formal level? (2) Do measures of the neo-Piagetian constructs of M-space, field-dependence, and short-term storage space predict the amount of practice needed until successful strategies (algorithms) are induced as well as other aspects during the development of reasoning strategies? (3) Are the widely used measures of the neo-Piagetian constructs independent of each other.

This study answered the first question affirmatively. In both the domains of probability and balance beam, the majority of students (17 and 19 respectively) induced strategies that permitted them to solve problems at the formal level of reasoning. The subjects achieved this without any other intervention than simple feedback whether their answer was correct or incorrect. These results support the contention of mastery learning advocates (Block, 1971; Bloom, 1971) that if students received quality instruction and

the learning time required there would be little or no relationship between aptitude and achievement.

The answer to the second major question, the suitability of the neo-Piagetian constructs of M-space, field-dependence, and short-term storage space (STSS) as predictors for various aspects in the development of complex problem-solving strategies (algorithms) is negative for the measures of the first two variables, but largely positive for STSS.

The results of this study are consistent with previous research using the same constructs in several respects. First, the inter-correlations between the measures of the constructs of M-space, cognitive style, and STSS seems to be stable across different samples (Niaz & Lawson, 1985; Lawson, 1985; Lawson, 1987; Case, 1985; Case & Globerson, 1974; Collis & Romberg, 1979). Second, the finding that M-space and cognitive style alone or in combination are not significant predictors when developmental level has been controlled corroborates the results of other groups (Niaz & Lawson, 1985; Blake, 1978; Lawson, 1985).

The high correlations between ratio span and the dependent variables of amount of practice on balance problems, the tendency to avoid unsuccessful behaviors, the ability to transfer behaviors to new contexts, the tendency to monitor problem solving behavior, and the tendency to look for patterns is in agreement with the findings of other studies. These had indicated high correlations with learning potential (Case, 1985: six studies; Liu, 1981; Daneman & Case, 1981; Brainerd, 1981) as well as reading comprehension and performance on the SAT (Daneman & Carpenter, 1980; Daneman, 1982; Hess & Radtke, 1981).

The third question, which concerned the widely used measures of the neo-Piagetian constructs used in this study, arose during the study when the high correlations between the Group Embedded Figures Test (field-dependence), Figural Intersection Test, and Backward-digit Span Test (both M-space) became evident. A factor analysis was used to find an answer to this question.

The factor analysis of the correlation matrix of the cognitive variables suggests that the Figural Intersection Test and the Group Embedded Figures Test, which are used to assess two different constructs, are in fact assessing the same underlying ability, the storing of visual images. The Ratio Span and Backward-digit Span Tests load on a second factor that can be interpreted, due to the nature of the tasks used, as a verbal component of short-term memory. These results are in agreement with the findings of other teams (Case & Globerson, 1974; Collis & Romberg, 1979) and support the current theories of short-term memory (Baddeley, 1983; Hitch, 1984) which incorporate visuo-spatial and verbal components in their short-term memory constructs. The concurrent use of the Embedded Figures and Figural Intersection Tests has to be cautioned.

The significance of this study for science educators lies in the recognition of short-term storage space as a powerful construct. The presented results seem to indicate that individuals with sufficient resources in short-term storage can coordinate a variety of monitoring or pattern seeking behaviors in addition to the scheme(s) necessary to solve the problem at hand. These monitoring and pattern seeking behaviors lead to a faster integration of subordinate schemes into superordinate schemes at a higher level (Figure 4).

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Insert Figure 4 about here

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The power of the construct of STSS also lies in the fact that it is useful to describe the development of reasoning as well as the process of problem-solving in individuals beyond the age to which developmental theories are traditionally applied. In a subsequent study, physics problems were analyzed in terms of their STSS demand. Computer-generated homework programs based on this analysis led to significant improvements in the problem solving ability of college students. A tutoring program is in the planning stage which will provide students with specific props to overcome STSS limitations.

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TABLE I

Structure of problems and Levels of Cognitive  
Development According to Piaget and Case (Balance Beam)

Level	Structure (w1:w2)cmp(d2:d1)	Sample Problem	Levels	
			Piaget	Case
1	w1=n*w2, d2=n*d1 w2, d1 = 1	(1:2)cmp(2:4)	IIB	4.1a
2	w1=(m/n)w2, d2=(m/n)d1, (w2, d1) > 1	(9:6)cmp(6:4)	IIB	4.1b
3	w2=1, d2/d1=1+1/n	(2:1)cmp(3:2)	IIIA1	4.2
4	w1/w2=1+1/n, d2/d1=1+1/m	(4:3)cmp(7,6)	IIIA2	4.3a
5	w1/w2=p+m/n, d2/d1=q+r/s m,n,r,s > 1	(5:7)cmp(3:5)	IIB	4.3b

w1, w2, d1, d2, m, n, p, q, r, s = integer

w = weight, d = distance

Table I continued

## Problem Structure and Developmental Levels (Probability)

Level	Structure	Sample	Levels	
	(a:b)::(c:d)	Problem	Piaget	Case
1	$a=b, c=d$	(4:4)::(3:3)	IIA	4.0
2	$a=nb, c=nd$			
	(b or d = 1)	(1:2)::(2:4)	IIB	4.1a
3	$a=(m/n)b, c=(m/n)d$			
	(b, d = 1)	(9:6)::(6:4)	IIB	4.1b
4	$b=1, c/d = 1+1/n$	(2:1)::(3:2)	IIIA	4.2
5	any ratio not belonging to levels 1 - 4	(5:7)::(3:5)	IIIB	4.3

a,b,c,d, m,n = integer

Table II  
 Relationship Between the Number of Problems and M-space while  
 Statistically Partialling out the Effect of  
 Field Dependence (Probability)

Level	N	Model				
		Full		Reduced	Change	
		R2	F	R2	R2	F
1	20	.092	1.01	.092	.000	0.01
2	20	.079	0.86	.078	.001	0.03
3	20	.050	0.53	.025	.025	0.52
4	20	.134	0.25	.024	.110	2.54
5	14	.020	0.14	.000	.020	0.28

Table III

Grand Correlation Table of all Independent and Dependent Measures<sup>1</sup> (N = 23)  
(Probability Problems)

Variable	2	3	4	5	6	7	8	9	10	11
1. FIT	62 <sup>5</sup>	76 <sup>6</sup>	18	-24	-21	-22	-33	10	24	09
2. BACK		40	51 <sup>4</sup>	-21	-45 <sup>2</sup>	07	-22	19	31	19
3. GEFT			10	-30	-28	-16	-16	02	05	00
4. RATIO				-16	-27	-20	-34	-33	46 <sup>2</sup>	60 <sup>3</sup>
5. TRIALS Level 1					16	14	-05	-02	08	-26
6. TRIALS Level 2						40	54 <sup>4</sup>	21	-32	-48 <sup>4</sup>
7. TRIALS Level 3							75 <sup>6</sup>	62 <sup>4</sup>	-37	-69 <sup>6</sup>
8. TRIALS Level 4								37	-49 <sup>4</sup>	-65 <sup>6</sup>
9. TRIALS Level 5 (N = 17)									12	-67 <sup>5</sup>
10. TRANSFER										50 <sup>4</sup>
11. CHOICE										

<sup>1</sup> All decimals omitted

<sup>2</sup>  $p < .05$  one-tailed for directional hypotheses

<sup>3</sup>  $p < .01$  one-tailed for directional hypotheses

<sup>4</sup>  $p < .05$  two-tailed

<sup>5</sup>  $p < .01$  two-tailed

<sup>6</sup>  $p < .001$  two-tailed

TABLE IV  
Result of Principal Components Factor Analysis  
with following Oblique Rotation

Variable	Factor	
	I	II
FIT	.94	.27
GEFT	.79	-.09
BACK	.05	.82
RATIO	-.14	.88

$$r_{I,II} = .27$$

TABLE V  
Grand Correlation Matrix Dependent and  
Independent Variables (Balance Beam)

Variable	1	2	3
1. STSS			
2. CHOICE	.69 <sup>3</sup>		
3. MONITOR	.59 <sup>2</sup>	.56 <sup>2</sup>	
4. TRIALS	-.40 <sup>1</sup>	-.39	-.63 <sup>3</sup>

<sup>1</sup>  $p < .05$

<sup>2</sup>  $p < .01$

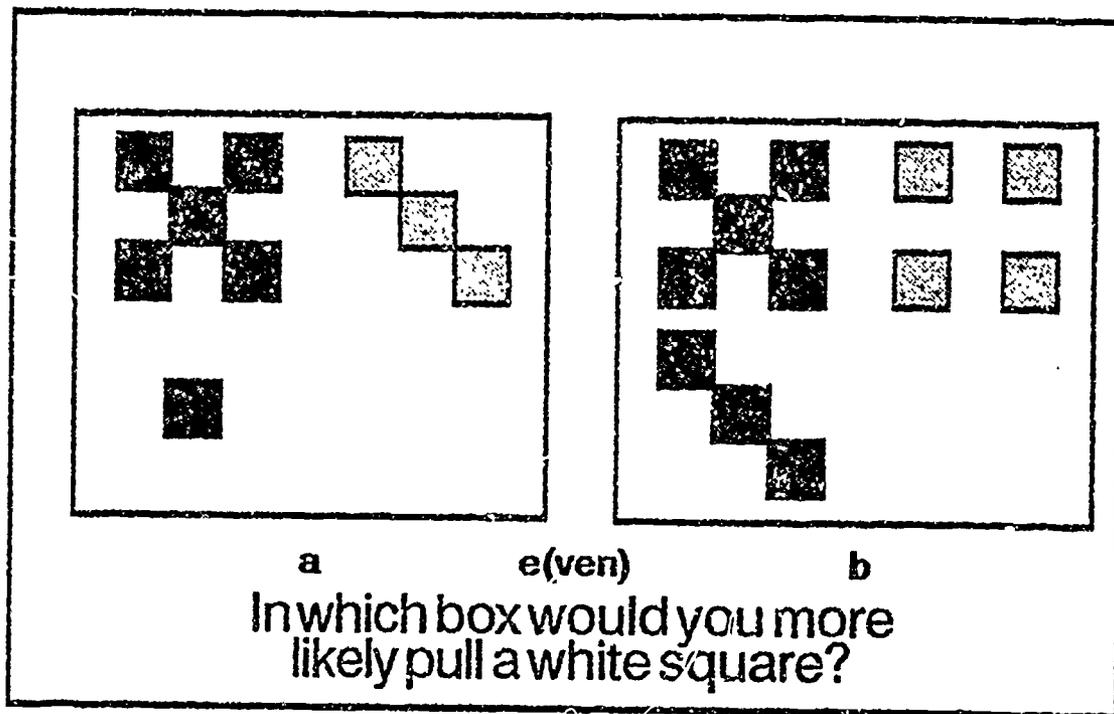
<sup>3</sup>  $p < .001$

FIGURE 1:

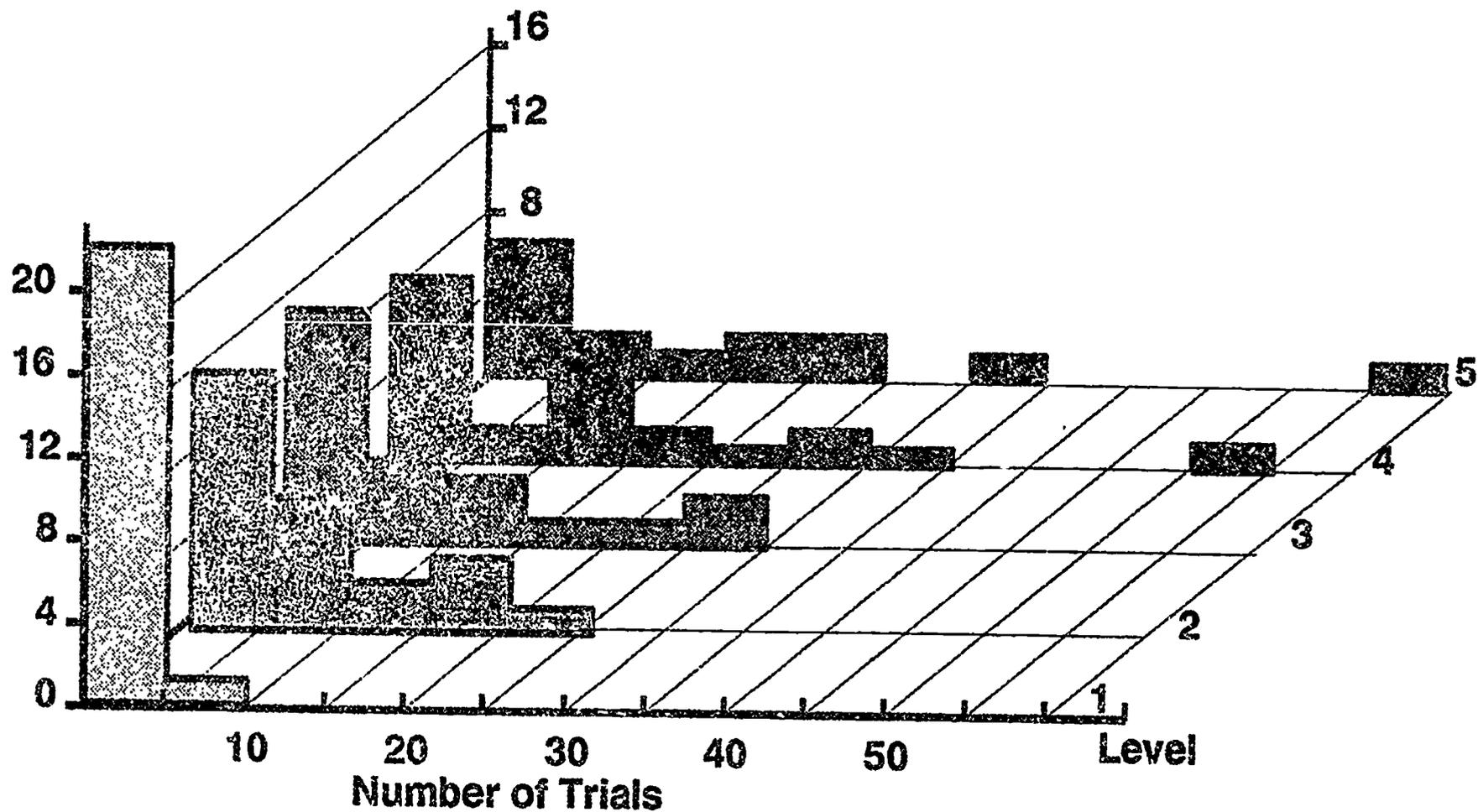
Which weight goes to the arrow to make a balanced beam?

Move the left weight to make a balanced beam.  
Move left: → Move right: ←

Indicate whether the beam will balance (b), move right down (r), or move left down (l).



**Figure 2. Sample screen display for probability problems.**



**Figure 3. Number of trials until a workable strategy was induced at each of the five levels of problem difficulty.**

# Figure

## Model for problem-solving and development

