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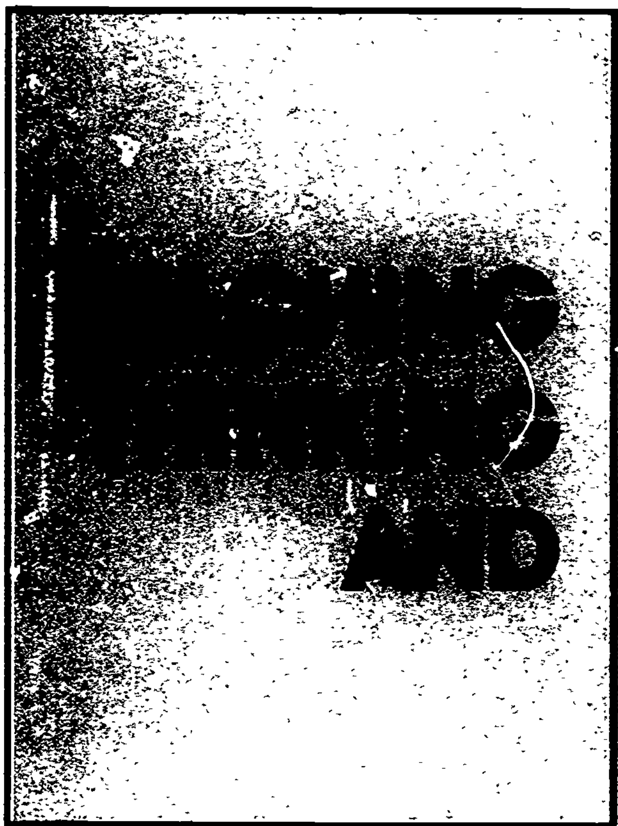
ABSTRACT

This bibliography presents 21 references and abstracts for improving mathematics instruction in two ways. The first aim is to serve as a guide in locating higher-order thinking skills material in mathematics; the second is to encourage the integration of higher-order thinking skills techniques into the mathematics curriculum. Entries are included that address instruction at the elementary, middle, and high school levels. Also addressed are instructional issues related to problem solving, cognitive science, computer-assisted instruction, and metacognition. (YP)



PROBLEM SOLVING

A HIGHER ORDER THINKING SKILLS RESOURCE FOR MATHEMATICS

A SOUTHEASTERN EDUCATIONAL IMPROVEMENT LABORATORY RESOURCE

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TEACHING THINKING AND PROBLEM SOLVING

AN ANNOTATED BIBLIOGRAPHY ON THINKING SKILLS IN MATHEMATICS

By
Anthony Boswell
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A Cross-Laboratory Project on Teaching Thinking

ANNOTATED BIBLIOGRAPHY ON THINKING SKILLS INSTRUCTION IN MATHEMATICS

This bibliography seeks to improve mathematics instruction in two ways. The first aim is to serve as a guide in locating higher-order thinking skills material in mathematics; the second is to encourage the integration of higher-order thinking skills techniques into the mathematics curriculum. Entries are included that address instruction at the elementary, middle, and high school levels. Also addressed are instructional issues related to problem solving, cognitive science, computer-assisted instruction, and metacognition. This annotation was compiled by the SEIL Graduate Assistants in Mathematics, Anthony Boswell and Boyd Coan, under the direction of Frederick Smith. Many thanks to Betty Howie, Joan Taylor, and Barbara Meeks for proofreading and assistance with typing the document. Special thanks is extended to Beau Fly Jones, North Central Regional Educational Laboratory and Barbara Presseisen, Research for Better Schools for the entries they submitted for this document.

1. Baroody, A.J. (1987). Children's mathematical thinking: A developmental framework for preschool, primary, and special education teachers. Urbana-Champaign, IL: University of Illinois.

Educators have long needed a resource capable of giving them a clear insight into the real nature of mathematical learning. This book provides a practical psychological framework for understanding children's mathematical development and the ways in which that development can go wrong, thereby giving teachers a basis for making informed decisions concerning mathematics education. Suggestions for remediating learning difficulties and overcoming math anxiety are presented in this important contribution to mathematics education for young children in preschool through third grade and for students with special needs.

2. Bransford, J., Sherwood, R., Vye, N., & Rieser, J. (1986). Teaching thinking and problem solving. American Psychologist, 41(10), 1078-1089.

The authors focus on two general research approaches. The first derives from studies of individuals who are experts in particular domains and emphasizes the role of domain-specific knowledge. A second approach emphasizes general strategic and metacognitive knowledge. The authors conclude that many existing programs that are designed to teach thinking and problem solving involve an emphasis on general skills and strategies, in contrast to domain-specific knowledge. They argue that these programs can be strengthened by focusing more explicitly on domain knowledge, especially when students are helped to understand how different ways of learning new knowledge can affect their abilities to solve relevant problems. Of particular interest to the practitioner is the IDEAL approach to problem solving, which has five components: Identify, Define, Explore, Act, and Look and Learn. The ability to solve learning problems often requires a number of passes through this IDEAL cycle.

3. Burton, L. (1980). The teaching of mathematics to young children using a problem solving approach. Educational Studies in Mathematics, 11, 43-58.

This is a report on the research project, "The Skills and Procedures of Mathematical Problem Solving in Pupils of 9-13 Years," conducted at the Polytechnic of the South Bank, London. The need for this project, both from a mathematical and from a pedagogical point of view, is explored. Two main tasks were attempted: The first was the construction of an inventory of mathematical problem-solving skills and procedures; the second was to design and test a structured teaching programme.

4. Capper, J. (1986). Computers in mathematics learning: Research review and instructional implications. The Research Into Practice Digest, 1(4).

To assist educators in making effective use of their computers, this digest reviews useful research with practical classroom applications. This issue provides a synthesis of research on the use of computers to teach mathematics and addresses questions such as:
(1) Should programming be used to teach mathematics?
(2) What are intelligent tutoring systems and how can they help students be more reflective and thoughtful problem solvers?
(3) Can graphing problems help reduce girls' math anxiety?
Research is reviewed on programs such as the Geometry Tutor, Algebraland, The Geometric Supposer, Green Globs, Algebra Arcade, and microcomputer-based laboratories. The software is reviewed from the perspective of the cognitive science research dealing with how people develop their thinking and problem-solving abilities.

5. Carpenter, T.P. (1985). Learning to add and subtract: An exercise in problem solving. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives (pp. 17-40). Hillsdale, NJ: Lawrence Erlbaum Associates.

A central premise of this paper is that the solutions of problems involving a routine application of a single arithmetic operation, particularly solutions of young children, do in fact involve real problem-solving behavior. A related premise is that research on children's solutions of simple arithmetic word problems can provide insights into the development of more complex problem-solving abilities. Finding an appropriate representation for a mathematical problem so that it can be solved may involve more than mere translation into mathematical form and more than using an approach based strictly on syntax and/or key words. Many successful problem solvers

attend to the semantics of the problem. For this reason, the author offers a refinement of some schemata for classifying addition and subtraction word problems. This particular analysis proposes four broad classes of addition and subtraction problems: Change, Combine, Compare, and Equalize.

6. Confrey, J. (1987). Mathematics learning and teaching. In V. Richardson-Koehler (Ed.), C.M. Clark (Math Ed.), Educators' handbook: A research perspective (pp. 3-25). New York: Longman.

In this chapter, a variety of studies are reviewed, beginning with a survey of the large existing national data base on mathematics education. Then, the author focuses on what students learn from instruction and describes different forms of mathematics instruction, including problem solving. Sociological issues arising from mathematics instruction are described. The article concludes with a discussion of the research on two current areas of interest: levels of thinking in microcomputers and geometry.

7. Davis, R. B. (1984). Learning mathematics: The cognitive science approach to mathematics education. Norwood, NJ: Ablex Publishing.

There has been a history of controversy in mathematics between the drill-and-practice orientation that emphasizes rote memorization of mathematical formulas and procedures and the approach based on comprehending and creatively using mathematics skills. This book seeks to explain and diffuse this dispute by taking a broad view of the cognitive-science approach to the teaching and learning of mathematics. It is primarily concerned with providing a deeper understanding of the thought processes involved in mathematical thinking, including what goes on inside children's heads as they learn mathematics and do mathematical problem solving. Among the areas considered are: the cognitive science approach to mathematics education; deficiency in typical school curricula; the nature of representations; retrieval, construction and mapping; and basic concepts used to facilitate the discussion of human information processing as it relates to mathematical problem solving.

8. Ekenstam, A.A., & Greger, K. (1983). Some aspects of children's ability to solve mathematical problems. Educational Studies in Mathematics, 14, 369-384.

This survey presents the essentials of a study concerning problem-solving ability in children ages 12-13. It forms part of a large project concerning the impact of calculators and computers in school mathematics and the consequences for certain abilities. In brief, the contents of the survey are: (1) definition of problem-solving ability, (2) test construction, (3) interviews, and (4) supplementary investigations.

9. Laing, R.A. (1985). Activities: Extending problem solving skills. Mathematics Teacher, 78(1), 36-44.

Three problem-solving skills, guess, test, and simplification, are introduced and reinforced through a variety of motivating problem situations. Solutions of sample problems and extensive directions for the teacher encourage the use of Polya's four-step model of problem solving.

10. Leinhardt, G. (1986). Expertise in mathematics teaching. Educational Leadership, 43(7), 28-33.

Research on how expert teachers structure lessons and manage content may suggest how others can communicate mathematics more skillfully and how supervisors can assist them in doing so. Conducted over a period of six years, the study of the arithmetic teaching of seven "expert" elementary school teachers suggests that content knowledge is critical. Supervisors may assist this content development in mathematics by holding miniseminars on topics such as: regrouping, equivalent fractions, multidigit multiplication and division, and word problems. Although expert teachers do many of the same things well, they do not necessarily do them in the same way. With close attention to this individuality, supervisors may also assist teachers by locating where in the teaching repertoire support is most needed and providing this support.

11. LeMoyne, G., & Tremblay, C. (1986). Addition and multiplication: Problem solving and interpretation and relevant data. Educational Studies in Mathematics, 17(2), 97-123.

This study involved 48 students ages 9 to 12. The experimental group of 29 students was divided into smaller groups; each of the sub-groups then performed learning exercises designed to improve skills in analyzing and processing expressions frequently included in addition and multiplication problems. The study concludes that the exercises helped significantly to improve problem-solving performance.

12. Lindquist, M.M. (in press). Strategic teaching in mathematics. In B.F. Jones, A.S. Palincsar, D.S. Ogle, & E.G. Carr, Strategic teaching and learning: Cognitive instruction in content areas. Alexandria, VA: Association for Supervision and Curriculum Development.

This chapter does not solve the dilemma of whether or not thinking is taught in mathematics. Rather, it considers the application of research on cognition and cognitive instruction to mathematics instruction. It is noted that many of the same assumptions articulated about learning in general are present in the research and folklore of mathematics education. For example, the parallels between language arts and mathematics is evident in at least six basic themes. However, some differences in the way those involved in

mathematics education view the organizational patterns are pointed out. Two examples illustrate the use of the author's Planning Guide for Instruction in Mathematics. The first example is an algorithm for subtracting three-digit numbers which focuses on the teaching of procedural knowledge to enhance thinking; the second uses the Rhombus to illustrate how to teach concepts. Some adaptations in the strategies and planning guides for the more or less proficient student may become necessary, but it is suggested that if the tenets of cognitive instruction are followed, all students will benefit.

13. Mayer, R.E. Mathematics. (1986). In R.F. Dillon & R.J. Sternberg, Cognition and instruction (pp. 127-153). Orlando, FL: Academic Press.

    The relationship between the mathematics curriculum and the psychology of human learning and cognition is explored in this chapter, and an overview of the mathematics curriculum and the psychology of mathematics learning is provided. Also presented are five representative areas of the mathematics curriculum (counting, arithmetic computation, arithmetic application, algebraic computation, and algebra application). For each area, examples of research are given to show how cognitive research and the mathematics curriculum are related. This chapter is based on the premise that cognitive psychology has implications for organizing and teaching the mathematics curriculum. In a parallel manner, cognitive psychologists can benefit from the challenges posed by the need to provide mathematics instruction to children.

14. Montague, M., & Bos, C.S. (1986). The effect of cognitive strategy training on verbal math problem-solving performance of learning-disabled adolescents. Journal of Learning Disabilities, 19(1), 26-33.

    Six adolescents with learning disabilities participated in an eight-step cognitive strategy designed to enable students to read, understand, carry out, and check verbal math problems encountered in the general math secondary curriculum. Visual analysis of the data indicates that the strategy was an effective intervention for this sample of students who had deficits in verbal math problem solving.

15. Nicely, R. F. (1985). Higher-order thinking skills in mathematics textbooks. Educational Leadership, 42(7), 26-30.

    Student acquisition of problem-solving and higher-order thinking skills has long been a goal of schools in general and of mathematics educators specifically. Thus, the tools used to teach these skills must be equal to the challenge of this goal. This article discusses the fact that mathematics textbooks are not doing enough to actively involve students in the development, practice, and acquisition of higher-order thinking skills. The author offers an analysis scheme for evaluating textbooks and for classifying printed instructional materials according to the type of content, level of cognitive activity, stage of mastery, and mode of response.

16. Resnick, L.B., & Omanson, S.F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), Advances in instructional psychology (pp. 41-ff). Hillsdale, NJ: Lawrence Erlbaum Associates.

The authors are concerned with: the nature of understanding in procedural domains, whether and how the understanding of mathematical principles enhances performance skill, and the ways in which both understanding and procedural competence are learned. Arithmetic is a convenient arena in which to work, in that it is a domain in which procedures are codified and directly taught. In this study, the domain is place value; its procedural expression is multidigit subtraction. The initial theoretical analysis considers the nature of errors in subtraction and their implications and outlines a set of principles that provides the mathematical justification for many subtraction algorithms. In addition, several sets of empirical data are examined. Some data provide rough evidence of the extent to which elementary school children know the principles, while the description of a set of instructional experiments helps to establish some understanding of procedural skill. This analysis is compatible with earlier findings, but is not identical with them.

17. Russell, R.L., & Ginsburg, H.P. (1984). Cognitive analysis of children's mathematics difficulties. Cognition and Instruction, 1(2), 217-244.

The aim of this study is to investigate the informal and formal mathematical knowledge of children suffering from "mathematics difficulty" (MD). Children in the study group were individually presented with a large number of tasks designed to measure key mathematical concepts and skills. Major findings of the study suggest that:
(1) MD children are not seriously deficient in key informal math concepts and skills.
(2) MD children's calculational errors often result from common error strategies.

(3) MD children display severe difficulty in recalling common addition facts, but, in the area of problem solving, MD children are in many respects similar to normal, younger peers.

18. Schoenfeld, A.H. (1979). Can heuristics be taught? In J. Lochhead and J. Clement, Cognitive process instruction: Research on teaching thinking skills (pp. 315-338). Philadelphia: The Franklin Institute Press.

This article addresses the question, "Can students be taught general strategies that truly enhance their abilities to solve mathematical problems?" Schoenfeld presents the rationale for heuristics and notes some questions about their effectiveness in the teaching of problem solving. These questions are discussed, and the course he used to implement the heuristics is described. Tips for the effective use of heuristics, as well as obstacles that prevent successful use, are also mentioned.

19. Schoenfeld, A.H. (1985). Metacognitive and epistemological issues in mathematical understanding. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives (pp. 361-379). Hillsdale, N.J.: Lawrence Erlbaum Associates.

Aspects of mathematical understanding that extend beyond the mastery of routine facts and procedures are explored in this paper. The author deals with three aspects of such understanding, summarized in the following three assertions:
(1) Metacognitive skills and a mathematical epistemology are essential to mathematical competence.
(2) Most students do not develop very many metacognitive skills or a mathematical epistemology because math instruction deals with the understanding of basic facts and the memorization of mathematical procedures.
(3) It is possible, although it takes a lot of time and effort, to develop these skills in students.
Schoenfeld discusses the nature of metacognition and how development of this level of thought can be successfully incorporated into a curriculum for students. He also believes that there are issues that mathematics education can address beyond the scope of cognitive science, such as classroom realities and how they must be shaped to facilitate a successful learning environment in the future.

20. Shaklee, B.D., & Amos, N.G. (1985, November). The effectiveness of teaching creative problem-solving techniques to enhance the problem-solving ability of kindergarten students. Paper presented at the Annual Meeting of the Mid-South Educational Research Conference, Biloxi, MS. (ERIC Document Reproduction Service No. ED 264 292)

The purpose of this research was to determine the effectiveness of using a version of Creative Problem Solving (CPS) to increase problem solving activities of kindergarten students, and to determine the practical significance of using CPS as a method for increasing problem-solving skills of kindergarten students. The statistical analysis of the data showed no significant differences between the control and experimental groups on problem-solving ability after the six-week interval. However, the results did indicate an educationally significant impact on problem-solving acquisition when using creative problem-solving techniques with kindergarten children.

21. Sowder, L. (1986). The looking back step in problem solving. Mathematics Teacher, 79(7), 511-513.

The often neglected strategy of looking back is discussed, and an example is given. The author sees this step as one that can give students a glimpse at the creation of conjectures. It can also give students a small taste of designing mathematical problems, rather than just absorbing the already polished procedures. Looking back can develop the outlook that how an answer is obtained is more important than the answers. The what-if-not procedure is described in which attributes of a situation are systematically varied to create new situations and questions. A special case of this procedure, "Can the problem be generalized," is applied to the example used in the article.