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ABSTRACT

When latent class parameters are estimated, maximum likelihood and Pearson chi-square statistics can be derived for assessing the fit of the model to the data. This study used simulated data to compare these two statistics, and is based on mixtures of latent binomial distributions, using data generated from five dichotomous manifest variables. Data were generated for a two-class unconstrained model and a three-class constrained model. Data were also generated under three independent variables--sample size and two parameters that define latent class models (conditional response probability and latent class proportion). Parameters were estimated for the generation models and a series of subsumed and subsuming models. Maximum likelihood and Pearson chi-square statistics were derived for each estimation. Distributions of fit statistics were produced by 1,000 replications. For each distribution of statistics, the overall fit to the appropriate chi-square distribution was assessed. In addition, the mean, variance, and tail weights were examined. The distributions of the statistics varied according to the parameter values of the models and the type of models estimated. When the estimated model accurately reflected the data, the Pearson statistic was generally distributed as a chi-square for both large and small samples, while the maximum likelihood statistic was distributed as a chi-square only at the large sample size. Fourteen tables of supporting statistics are included. (Author/TJH)

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Comparison of Maximum Likelihood and Pearson Chi-Square Statistics for Assessing Latent Class Models

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ABSTRACT

When latent class parameters are estimated, maximum likelihood and Pearson chi-square statistics can be derived for assessing the fit of the model to the data. This study uses simulated data to compare these two statistics.

Data were generated for a 2-class unconstrained model and a 3-class constrained model. Data were also generated under three independent variables. Those variables are sample size and two parameters that define latent class models--conditional response probability and latent class proportion.

Parameters were estimated for the generation models and a series of subsumed and subsuming models. Maximum likelihood and Pearson chi-square statistics were derived for each estimation. Distributions of fit statistics were produced by 1000 replications. For each distribution of statistics, the overall fit to the appropriate chi-square distribution is assessed. In addition, the mean, variance, and tail weights are examined.

The distributions of the statistics vary according to the parameter values of the models and the type of models estimated. When the estimated model is an accurate reflection of the data, the Pearson statistic is generally distributed as chi-square for both large and small samples, while the maximum likelihood statistic is distributed as chi-square only at the large sample size.

COMPARISON OF MAXIMUM LIKELIHOOD AND PEARSON

CHI-SQUARE STATISTICS FOR LATENT CLASS MODELS

Latent class modeling is a probabilistic modeling strategy that can be used for establishing relations between response patterns on manifest variables and latent categorical variables. This probabilistic approach allows for the establishment of statistical criteria for testing the fit of theoretical models to the observed data (Bergan, 1982; Clogg, 1977; Dayton & Macready, 1976; Duncan & Sloane, 1982; Goodman, 1974 & 1979; Macready & Dayton, 1977 & 1980).

Dichotomous manifest variables, which are applicable to a wide range of content areas, are frequently used with latent class models. Equation 1 defines a general latent class model that may be used with dichotomously-scored variables.

$$P(\underline{r}) = \sum_{j=1}^{J} \left\{ \theta_j \prod_{i=1}^{I} \alpha_{ij}^{r_i} (1 - \alpha_{ij})^{1-r_i} \right\} \text{(1)} \qquad (1)$$

where:

$\underline{r}$ = vector of r_i = {0,1} manifest responses for variables $i = 1,\ldots,I$;

θ_j = probability of membership in the j^{th} latent class;

α_{ij} = probability of a level one (i.e., positive or correct) response to the i^{th} manifest variable given that the assessed element (e.g., respondent) is a member of the j^{th} latent class.

There are some basic assumptions underlying this latent class model. First, it is assumed that the latent classesare is mutually-exclusive and exhaustive. Second, it is assumed that local independence (see Goodman, 1974) is present within latent classes. This means that the manifest variables conditional on latent class membership are independent and, thus, $P(\underline{r} \mid LC=j) = \prod_{i=1}^{I} \alpha_{ij}^{r_i} (1 - \alpha_{ij})^{1-r_i}$. Third, it is assumed that the response options to each manifest variable i are mutually-exclusive and exhaustive.

The likelihood function for the estimation of latent class model parameters is:

$$\lambda = \prod_{l=1}^{L} P (r_l) \qquad (2) \qquad\qquad (2)$$

where l is the respondent index.

A maximum likelihood fit statistic may be derived, which is based on the likelihood function in Equation 2. This statistic is:

$$G^2 = -2 \ln \lambda \qquad (3) \qquad\qquad (3)$$

Bartlett (1937) showed that as N increases, the distribution of G^2 approaches a χ^2 distribution with degrees of freedom:

$$df = 2I - 1 - (J - 1) - IJ \qquad (4) \qquad\qquad (4)$$

where:

I = number of dichotomous manifest variables;

J = number of latent classes.

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A Pearson fit statistic may also be derived. This statistic is:

$$\chi^2 = \sum_{l=1} (f_l - F_l)^2 \; / \; F_l \quad (5) \qquad\qquad (5)$$

where:

f_l = observed frequency for response pattern l;

F_l = expected frequency for response pattern l, based on the estimated model.

It is also distributed asymptotically as χ^2 with degrees of freedom specified in Equation 4.

In addition to the absolute fit of a selected model, it is also possible to test in the same manner the absolute fits of one or more constrained versions of that model. There are several ways in which a model may be restricted. Three basic methods were discussed by Dayton & Macready (1988). These three methods involve the imposition of linear, linear logistic, and other functional constraints. Recent developments in latent class modeling have incorporated outside variables within the model which allow for interesting new formulations of constrained models (Clogg & Goodman, 1984, 1985 & 1986; Formann, 1982, 1984, 1985; Dayton & Macready, 1988; Macready & Dayton, 1986).

The application of linear constraints is the most widely reported method that is used for restricting models (Bergan, 1982; Clogg 1981; Dayton & Macready, 1976; Macready & Dayton, 1980). There are two special cases of linear constraints, which have frequently been considered in the application of latent class modeling. The first case involves fixing parameters at user-specified values. The second case involves equating, in which the values of two or more parameters are set equal to one another.

Although the value of the maximized likelihood for a given model is greater than or equal to that of any model it subsumes (i.e., a model that is obtained by placing one or more constraints on the parameters of the initial model), sometimes a more constrained model will provide fit that approaches that obtained under the more general subsuming model. In addition, the constrained model provides an advantage of increased parsimony. This increased parsimony provides a gain in degrees of freedom, resulting from the reduced number of independent parameters to be estimated.

The degrees of freedom for the constrained model are expressed in terms of Equation 4 as:

$$df_C = df_S + K \qquad (6) \qquad (6)$$

where:

df_S = degrees of freedom for the subsuming model;

K = the number of non-redundant constraints placed on the parameters in the more general model in defining the subsumed model.

The final selection of a preferred model should be based upon a balance between fit and parsimony. Fit requires that a model is able to effectively explain or replicate the data. Parsimony requires that a model is as simple (i.e., constrained) as possible. Thus a preferred model is one which is as simple as possible yet still provides acceptable fit to the data.

Method

This study is based on mixtures of latent binomial distributions, using data generated from five dichotomous manifest variables. The number of response patterns for those variables is $2^5 = 32$. A total of sixteen data sets were generated, representing two generation models under eight data conditions resulting from all combinations of three independent variables at two levels each.

The IMSL (1980) subroutine, GGUBS, was used to generate random numbers in a uniform distribution, ranging from zero to one. These random numbers were used to produce the raw data for the study. The frequencies for the response patterns were produced by comparing the generated numbers to each of the 32 cumulative probability intervals. This was repeated for 1000 replications for each of the sixteen data generation conditions that are considered.

Two latent class models were specified for the data generation. The first criterion for the selection of those models was their applicability to a wide range of research in the areas of education, psychology, and other social sciences. The second criterion was that they were as unconstrained as was feasible.

The first generation model is a general unconstrained 2-class model defined by Equation 1. The second model is a 3-class constrained model with equality restraints imposed on the conditional probabilities. An unconstrained 3-class model was originally considered for this second generation model. However, it was rejected because, although it is in general identified, a comparable 4-class unconstrained model which would have been appropriate for comparison would is not identified for I=5.

The first independent variable is sample size, which has been recognized as an important factor affecting the distribution of the logarithm of the likelihood ratio. The values selected are based, in part, on the results of Hayek (1978), as well as on practical considerations. The first level of this variable is specified as N=160, which results in an average cell frequency of 5 for each of the 32 response patterns. The second level is specified as N=960, which results in an average cell frequency of 30 per response pattern.

The remaining two independent variables vary the levels of the two types of parameters used in defining latent class models. This includes the magnitude of the disparity of α_{ij} across latent classes and the proportion of elements from each latent class.

The first level of the disparity of α_{ij}, which is "small" disparity, places the response probabilities near the center of the parameter space. The second level, which is "large" disparity, places them nearer the boundaries of the space. The specific values of α_{ij} for the 2-class generation model are designated in Table 1. This type of model has been proposed for use in the assessment of mastery (see Macready & Dayton, 1980), as well as a wide variety of other uses (see Bergan, 1982 and Clogg, 1981).

(Insert Table 1 about here)

Note from Table 1 that there is no overlap in the ranges of α_{ij} values across latent classes, and there is equal variability of α_{ij} values within each latent class.

The specific values of α_{ij} for the 3-class constrained generation model are listed in Table 2. The restraints imposed on the second latent class specify that $\alpha_{i2} = \alpha_{i3}$ for $i = 1,2$ and $\alpha_{i2} = \alpha_{i1}$ for $i = 3,4,5$. Constraints of this type are applicable when the three latent classes represent progressive states of acquisition (i.e., non-acquisition, partial acquisition, and acquisition), as suggested by Dayton & Macready (1976).

(Insert Table 2 about here)

The last manipulated variable considered in this study is the relative proportion of elements from each latent class. The first level of latent mixture considered is equal latent proportions, where $\theta_1 = \theta_2 = .5$ for the 2-class generation model and $\theta_1 = \theta_2 = \theta_3 = .333$ for the 3-class generation model. The second level considered is an unequal latent class mixture, where $\theta_1 = .8$ and $\theta_2 = .2$ for the 2-class generation model and $\theta_1 = .6$, $\theta_2 = .3$, and $\theta_3 = .1$ for the 3-class generation model. The first level of this variable places the parameters near the center of the parameter space, while the second level places them nearer the boundaries of that space.

Parameter Estimation

Maximum likelihood estimation was used to estimate the model parameters. Clogg's (1977) FORTRAN program, which is based on the Iterative Proportional Fitting algorithm (Goodman, 1979), was adapted and used as a subroutine for this purpose.

It may be noted that the Newton-Raphson procedure could alternatively have been used in parameter estimation. This is the algorithm which is used in the general purpose computer program written by Formann (1984). An advantage of this alternative approach is that convergence is usually reached in a fewer number of iterations. However, the time in which each iteration is completed is usually somewhat longer.

Parameters were estimated for the two generation models and for a series of subsumed and subsuming models. The two criteria for the selection of the models were their applicability for research and their relation to the generation models. The data generated from the 2-class model were used to estimate parameters for that model, as well as for an unconstrained 1-class model, an unconstrained 3-class model, a constrained 3-class model, and a constrained 4-class model. Similarly, the data generated from the 3-class model were used to estimate parameters for that model, as well as for an unconstrained 1-class model, an unconstrained 2-class model, and a constrained 4-class model.

The constrained 3-class models considered under both levels of data generation are the same model. The constrained 4-class model is specified with the same constraints on the first three latent classes as those imposed for the constrained 3-class model. In addition, constraints on the fourth latent class are: $\alpha_{i4} = \alpha_{i1}$ for $i = 1,2$ and $\alpha_{i4} = \alpha_{i3}$ for $i = 3,4,5$. This model corresponds to an acquisition model in which item subsets {1,2} and {3,4,5} are at the same latent level of acquisition, namely, "masters" or "non-masters" (see Macready, 1982).

Two separate fit statistics were derived for each model estimation. Those are G^2, defined in Equation 3, and Pearson χ^2, defined in Equation 5.

Analysis

The distributions of fit statistics in this study were analyzed according to their overall fit to χ^2 distributions with the appropriate degrees of freedom. Each distribution of 1000 observed statistics was divided into 100 intervals, based on a central χ^2 distribution. Each of these intervals, therefore, had an expected cell frequency of 10. A Pearson χ^2 statistic with 99 degrees of freedom was calculated to assess the fit of the observed G^2 and Pearson χ^2 fit statistics to a central χ^2 distribution.

In addition to overall fit, each of the distributions is examined in terms of its tail weights. The tail weights for the distributions are compared to the distribution means and to the overall fit statistics.

Results

The goodness of fit for each model estimation is examined first. These are reported in Tables 3 through 6.

(Insert Table 3 about here)

The fit of the observed statistics to a χ^2 distribution is presented in Table 3 for the generation models. All of the observed Pearson χ^2 statistics are distributed as χ^2, while the G^2 statistics are consistently distributed as χ^2 only at N=960. Although there are some exceptions, the average fit is better for Pearson χ^2 than for G^2 at both sample sizes. The fit is also better, on the average, for both statistics at N=960 than at N=160. At both sample sizes, G^2 consistently demonstrates better fit when there is small disparity in the conditional probabilities. Pearson χ^2 does not show any pattern of performance in relation to this variable.

The fit statistics are also examined for the estimation of the subsuming models that are overfitted to the data. One estimation for an unconstrained model is reported in Table 4, and three estimations for constrained models are reported in Table 5.

(Insert Table 4 about here)

Table 4 contains the data generated under the 2-class unconstrained model and estimated under a 3-class unconstrained model. With the exception of Pearson χ^2 at N=160, the average fit statistics for this estimation are not distributed as χ^2. The average fit for both statistics becomes worse as sample size increases, which contradicts the pattern of fit for the generation models.

(Insert Table 5 about here)

The results of the estimation under a series of constrained subsuming models are listed in Table 5. In general, the estimation of the 3-class constrained model for the data generated under the 2-class model demonstrates better fit than the estimation of the 4-class constrained model for the data from either of the generation models. However, there is a pattern of fit across these three subsuming models that parallels the generation models. On the average, Pearson χ^2 fits better than G^2 and the fit for both statistics is better at N=960. In addition, G^2 consistently shows better fit when the disparity is small for the conditional probabilities.

(Insert Table 6 about here)

The fit statistics are examined last for the 1-class and 2-class unconstrained nonfitting subsumed models. These are reported in Table 6. The fit to a χ^2 distribution is consistently better for the estimation of the 2-class model from data generated under the 3-class model than for the estimation of the 1-class model from data generated under both models. For the estimation of the 2-class model, the pattern of fit is the same for both Pearson χ^2 and G^2. They are distributed as χ^2 only when N=160 and there is small disparity in the conditional probabilities. Although neither statistic for the estimation of the 1-class model is distributed as χ^2, the fit is consistently better for both statistics for N=160 and small disparity in the conditional probabilities. In addition, the deviation from χ^2 is not as great for any of the subsumed models when latent class membership is unequal.

In addition to testing goodness of fit for each of the model estimations, the mean and variance are examined for each distribution. These are reported in Tables 7 through 10.

(Insert Table 7 about here)

Table 7 contains the means and variances for the estimation of the generation models. The means of the distributions exhibit the same pattern as the goodness of fit, but the pattern for the variances is not consistent. For the distributions of G^2 statistics at N=160 that deviate from χ^2, the means are all larger than their expected values. The means for G^2 at N=960 are consistently smaller and closer to their expected values than at N=160. The means for Pearson χ^2 are consistently smaller than the means for G^2 and are, on the average, closer to the expected values for both sample sizes. When the disparity in the conditional probabilities is small, the means for G^2 are consistently smaller and closer to their expected values than when disparity is large.

(Insert Table 8 about here)

The means and variances for the estimation of the unconstrained subsuming model are reported in Table 8. The direction of deviation for this estimation is opposite that observed for the generation models. In this case, the larger means of the distributions for G^2 and N=160 are closer to their expected value than the smaller means observed for Pearson χ^2 and N=960. The pattern for the means is paralleled by the pattern for the variances for this estimation.

(Insert Table 9 about here)

The means and variances are reported in Table 9 for the three estimations of the constrained subsuming models. The pattern, including the direction of deviation, for the means of the distributions for these estimations is consistent with that observed for the generation models. In general, the means are smaller and closer to their expected values for N=960 and for Pearson χ^2 at both sample sizes. The means for G^2 at N=160, which are the largest and farthest from their expected values, are not as deviant when there is small disparity in the conditional probabilities.

(Insert Table 10 about here)

Table 10 contains the means and variances for the estimation of the 1-class and 2-class subsumed models. The pattern for the means and variances is consistent with the pattern of fit for these estimations. In general, the means and variances for both statistics are smaller and closer to their expected values when N=160, there is small disparity in the conditional probabilities, and latent class membership is unequal. For the 1-class model when the disparity in the conditional probabilities is large, the deviation for Pearson χ^2 is greater than for G^2.

In addition to the overall fit and first two moments, the tail weights of the distributions of fit statistics are examined. It is in the tail of a distribution that a decision is made regarding the acceptance or rejection of a model for a given set of data. Tables 11 through 14 contain the proportion of rejections at the critical values of .01, .05, and .10.

(Insert Table 11 about here)

The tail weights for the generation models are listed in Table 11. Pearson χ^2 produces an appropriate proportion of rejections, on the average, for both sample sizes. G^2 generally produces a larger proportion of rejections than Pearson χ^2, with the largest discrepancy observed at N=160. These heavier tail weights for the distributions of G^2 are consistent with their larger means reported in Table 8. The improvement of G^2 for small disparity in the conditional probabilities is not as consistent in the tails as it is in the goodness of fit. On the average, there is improvement for both sample sizes at .10, improvement only for the small samples at .05, and no improvement at .01.

(Insert Table 12 about here)

The tail weights for the estimation of the unconstrained subsuming model are listed in Table 12. The pattern of the tail weights for this estimation parallels the patterns for the means and variances. The heaviest tail weights, which are also closest to their expected values, are observed for G^2 at N=160. Pearson χ^2 and N=960 produce an inappropriately small proportion of rejections.

(Insert Table 13 about here)

Table 13 contains the tail weights for the estimation of the constrained subsuming models. These results are consistent with the tail weights from the estimation of the generation models. The average tail weights for Pearson χ^2, which are the same for both sample sizes, are smaller and closer to their expected values than for G^2. G^2 produces as inappropriately large proportion of rejections, particularly at N=160. The effect of small disparity in the conditional probabilities on G^2 is inconsistent. It is nonexistent at .01, moderate at .05, and larger at .10.

(Insert Table 14 about here)

The tail weights for the estimation of the subsumed models are listed in Table 14. The tails for these estimations are generally larger than expected, resulting in an inappropriately large proportion of rejections. The tail weights that are smallest and closest to their expected values are associated with N=160, small disparity in the conditional probabilities, and unequal latent class membership. This is consistent with the pattern of fit and the observed means of these distributions. The largest effect on the tail weights is the disparity of the conditional probabilities. The large disparity condition produces tail weights of 1.00 for both statistics, indicating that all 1000 statistics in each distribution fall beyond P<.001.

Discussion

The nature of the estimated model influences the direction of deviation for those statistics that are not distributed as χ^2. The direction is consistent for the generation models, the constrained subsuming models, and the subsumed models. In each of these estimations, the distributions that deviate from χ^2 contain statistics that are larger than their expected values. However, the distributions that deviate from χ^2 for the estimation of the unconstrained subsuming model contain statistics that are smaller than their expected values.

Sample size is an important variable in determining the distribution of the fit statistics. The larger sample size produces statistics that are smaller in value for the estimation of the generation and subsuming models and larger in value for the estimation of the subsumed models. For the estimation of the generation models and constrained subsuming models, the smaller statistics result in a slightly better fit for Pearson χ^2 and a much better fit for G^2. However, for the estimation of the unconstrained subsuming model, the smaller statistics result in a slightly worse fit on the average for G^2 and a much worse fit for Pearson χ^2. For the subsumed models, the larger statistics result in a worse fit for both statistics.

The disparity of the conditional probabilities also affects the distribution of the fit statistics. Small disparity produces a better fit for G^2 for the estimation of the generation models and constrained subsuming models. It also produces a better fit for both statistics for the estimation of the subsumed models. However, it has no consistent effect for the estimation of the unconstrained subsuming model.

The equality of latent class membership only affects the estimation of the subsumed models. For this estimation, unequal membership produces a better fit for both statistics.

When the model that is estimated is an accurate reflection of the data, the Pearson χ^2 fit statistic is distributed as χ^2. On the average, this statistic results in an appropriate proportion of rejections for even a relatively small sample size. However, G^2 for the same estimation is distributed as χ^2 only for the larger sample size. At the smaller sample size, this statistic results in a larger average proportion of rejections than

Pearson χ^2. At the larger sample size, G^2 results in the same average proportion of rejections as Pearson χ^2 at P=.01. However, it results in a larger proportion of rejections at p-values of .05 and .10.

The true model is not normally known when the data are analyzed. However, using I=5 with the types of models examined in this study, some generalizations are possible. If either fit statistic has a very large p-value in relation to the critical value that is established, it is likely that the estimated model underfits the data and a more complex model would be appropriate for testing. If either fit statistic has a very small p-value in relation to the critical value, it is possible that another less complex model would be appropriate. If the fit statistic has a p-value that is relatively close to the critical value, caution should be exercised and other factors should be considered in making a decision on whether to accept or reject the model. These other factors include sample size, disparity of the conditional response probabilities, equality of latent class membership, and the specific fit statistic used. In particular, G^2 tends to have heavier tails than Pearson χ^2 and is more sensitive to the disparity of the conditional probabilities and to sample size when the estimated model is an accurate reflection of the data.

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TABLE 1

CONDITIONAL PROBABILITIES: 2-CLASS UNCONSTRAINED GENERATION MODEL

	Small Disparity		Large Disparity	
Var. #	Latent Class 1	Latent Class 2	Latent Class 1	Latent Class 2
1	$\alpha_{11}=.49$	$\alpha_{12}=.79$	$\alpha_{11}=.29$	$\alpha_{12}=.99$
2	$\alpha_{21}=.47$	$\alpha_{22}=.77$	$\alpha_{21}=.27$	$\alpha_{22}=.97$
3	$\alpha_{31}=.45$	$\alpha_{32}=.75$	$\alpha_{31}=.25$	$\alpha_{32}=.95$
4	$\alpha_{41}=.43$	$\alpha_{42}=.73$	$\alpha_{41}=.23$	$\alpha_{42}=.93$
5	$\alpha_{51}=.41$	$\alpha_{52}=.71$	$\alpha_{51}=.21$	$\alpha_{52}=.91$

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TABLE 2

CONDITIONAL PROBABILITIES: 3-CLASS CONSTRAINED GENERATION MODEL

	Small Disparity			Large Disparity		
Var. #	Latent Class 1	Latent Class 2	Latent Class 3	Latent Class 1	Latent Class 2	Latent Class 3
1	$\alpha_{11}=.49$	$\alpha_{12}=.79$	$\alpha_{13}=.79$	$\alpha_{11}=.29$	$\alpha_{12}=.99$	$\alpha_{13}=.99$
2	$\alpha_{21}=.47$	$\alpha_{22}=.77$	$\alpha_{23}=.77$	$\alpha_{21}=.27$	$\alpha_{22}=.97$	$\alpha_{23}=.97$
3	$\alpha_{31}=.45$	$\alpha_{32}=.45$	$\alpha_{33}=.75$	$\alpha_{31}=.25$	$\alpha_{32}=.25$	$\alpha_{33}=.95$
4	$\alpha_{41}=.43$	$\alpha_{42}=.43$	$\alpha_{43}=.73$	$\alpha_{41}=.23$	$\alpha_{42}=.23$	$\alpha_{43}=.93$
5	$\alpha_{51}=.41$	$\alpha_{52}=.41$	$\alpha_{53}=.71$	$\alpha_{51}=.21$	$\alpha_{52}=.21$	$\alpha_{53}=.91$

TABLE 3

GOODNESS OF FIT FOR DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: 2-CLASS UNCONSTRAINED AND 3-CLASS CONSTRAINED GENERATION (FITTING) MODELS

# Classes	Mixture	Disparity	N	L-ratio	Pearson
2	Equal	Small	160	117.0	118.4
2	Equal	Small	960	103.4	105.8
2	Equal	Large	160	292.4*	117.6
2	Equal	Large	960	109.8	105.4
2	Unequal	Small	160	94.2	114.2
2	Unequal	Small	960	110.6	116.6
2	Unequal	Large	160	383.0*	130.8
2	Unequal	Large	960	117.0	73.2
2	Combined	Combined	160	221.6*	120.2
2	Combined	Combined	960	110.2	100.2
3	Equal	Small	160	166.2*	104.2
3	Equal	Small	960	84.6	92.2
3	Equal	Large	160	253.4*	126.0
3	Equal	Large	960	117.6	88.2
3	Unequal	Small	160	151.0*	91.2
3	Unequal	Small	960	118.6	110.2
3	Unequal	Large	160	429.4*	143.4
3	Unequal	Large	960	130.8	108.2
3	Combined	Combined	160	250.0*	116.2
3	Combined	Combined	960	112.9	99.7
Combined	Combined	Combined	160	235.8*	118.2
Combined	Combined	Combined	960	111.6	100.0

* P <.001

TABLE 4

GOODNESS OF FIT FOR DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: ESTIMATION OF 3-CLASS UNCONSTRAINED SUBSUMING (OVERFITTED) MODEL FROM DATA GENERATED UNDER 2-CLASS UNCONSTRAINED MODEL

Mixture	Disparity	N	L-ratio	Pearson
Equal	Small	160	87.0	148.4*
Equal	Small	960	203.2*	223.0*
Equal	Large	160	194.2*	169.6*
Equal	Large	960	132.4	191.8*
Unequal	Small	160	123.8	115.2
Unequal	Small	960	163.6*	167.6*
Unequal	Large	160	218.6*	117.0
Unequal	Large	960	150.4*	226.0*
Combined	Combined	160	155.9*	137.6
Combined	Combined	960	162.4*	202.1*

* P <.001

TABLE 5

GOODNESS OF FIT FOR DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: ESTIMATION OF 3-CLASS AND 4-CLASS CONSTRAINED SUBSUMING (OVERFITTED) MODELS

Gen. Model	Est. Model	Mixture	Disparity	N	L-ratio	Pearson
2-Class	3-Class	Equal	Small	160	144.6	107.6
2-Class	3-Class	Equal	Small	960	109.4	113.6
2-Class	3-Class	Equal	Large	160	375.0*	138.8
2-Class	3-Class	Equal	Large	960	130.4	115.4
2-Class	3-Class	Unequal	Small	160	157.6*	101.8
2-Class	3-Class	Unequal	Small	960	92.6	91.4
2-Class	3-Class	Unequal	Large	160	510.6*	143.0
2-Class	3-Class	Unequal	Large	960	130.4	79.0
2-Class	3-Class	Combined	Combined	160	297.0*	122.8
2-Class	3-Class	Combined	Combined	960	115.7	99.8
2-Class	4-Class	Equal	Small	160	225.6*	121.0
2-Class	4-Class	Equal	Small	960	148.0	139.4
2-Class	4-Class	Equal	Large	160	514.6*	166.6*
2-Class	4-Class	Equal	Large	960	150.0*	129.8
2-Class	4-Class	Unequal	Small	160	241.6*	124.2
2-Class	4-Class	Unequal	Small	960	114.0	140.6
2-Class	4-Class	Unequal	Large	160	731.6*	208.4*
2-Class	4-Class	Unequal	Large	960	136.2	134.4
2-Class	4-Class	Combined	Combined	160	428.4*	155.0*
2-Class	4-Class	Combined	Combined	960	137.0	136.0
3-Class	4-Class	Equal	Small	160	204.4*	115.2
3-Class	4-Class	Equal	Small	960	104.4	99.0
3-Class	4-Class	Equal	Large	160	373.6*	132.4
3-Class	4-Class	Equal	Large	960	187.8*	119.0
3-Class	4-Class	Unequal	Small	160	192.6*	114.4
3-Class	4-Class	Unequal	Small	960	122.4	106.0
3-Class	4-Class	Unequal	Large	160	602.2*	220.0*
3-Class	4-Class	Unequal	Large	960	193.6*	127.6
3-Class	4-Class	Combined	Combined	160	343.2*	145.5*
3-Class	4-Class	Combined	Combined	960	152.0*	112.9
Combined	Combined	Combined	Combined	160	356.2*	141.1
Combined	Combined	Combined	Combined	960	134.9	116.2

* $P < .001$

TABLE 6

GOODNESS OF FIT FOR DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: ESTIMATION OF 1-CLASS AND 2-CLASS UNCONSTRAINED SUBSUMED (NONFITTING) MODELS

Gen. Model	Est Model	Mixture	Disparity	N	L-ratio	Pearson
3-Class	2-Class	Equal	Small	160	146.2	116.8
3-Class	2-Class	Equal	Small	960	473.6*	465.8*
3-Class	2-Class	Equal	Large	160	76023.2*	68209.2*
3-Class	2-Class	Equal	Large	960	99000.0*	99000.0*
3-Class	2-Class	Unequal	Small	160	100.4	113.4
3-Class	2-Class	Unequal	Small	960	183.0*	156.2*
3-Class	2-Class	Unequal	Large	160	47977.8*	43923.0*
3-Class	2-Class	Unequal	Large	960	99000.0*	99000.0*
3-Class	2-Class	Combined	Combined	160	31061.9*	28090.6*
3-Class	2-Class	Combined	Combined	960	49664.2*	49655.5*
2-Class	1-Class	Equal	Small	160	8989.2*	11415.8*
2-Class	1-Class	Equal	Small	960	99000.0*	99000.0*
2-Class	1-Class	Equal	Large	160	99000.0*	99000.0*
2-Class	1-Class	Equal	Large	960	99000.0*	99000.0*
2-Class	1-Class	Unequal	Small	160	1621.9*	1111.4*
2-Class	1-Class	Unequal	Small	960	66849.0*	73581.2*
2-Class	1-Class	Unequal	Large	160	99000.0*	99000.0*
2-Class	1-Class	Unequal	Large	960	99000.0*	99000.0*
2-Class	1-Class	Combined	Combined	160	52152.8*	52631.8*
2-Class	1-Class	Combined	Combined	960	90962.2*	92645.3*
3-Class	1-Class	Equal	Small	160	2112.2*	1676.2*
3-Class	1-Class	Equal	Small	960	75144.2*	79835.8*
3-Class	1-Class	Equal	Large	160	99000.0*	99000.0*
3-Class	1-Class	Equal	Large	960	99000.0*	99000.0*
3-Class	1-Class	Unequal	Small	160	542.0*	271.4*
3-Class	1-Class	Unequal	Small	960	9600.8*	11038.6*
3-Class	1-Class	Unequal	Large	160	98800.2*	98800.2*
3-Class	1-Class	Unequal	Large	960	99000.0*	99000.0*
3-Class	1-Class	Combined	Combined	160	50113.6*	49937.0*
3-Class	1-Class	Combined	Combined	960	70686.2*	72218.6*
Combined	Combined	Combined	Combined	160	44442.8*	43553.1*
Combined	Combined	Combined	Combined	960	70437.5*	71506.5*

* $P < .001$

TABLE 7

MEAN AND VARIANCE OF DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: 2-CLASS UNCONSTRAINED AND 3-CLASS CONSTRAINED GENERATION (FITTING) MODELS

# Classes	Mixture	Disparity	N	Mean L-ratio	Mean Pearson	Variance L-ratio	Variance Pearson
				EXPECTED: 20.00		EXPECTED: 40.00	
2	Equal	Small	160	20.98	19.37	47.34	36.76
2	Equal	Small	960	20.02	19.83	43.66	42.19
2	Equal	Large	160	22.65	20.07	36.10	41.73
2	Equal	Large	960	20.50	19.92	41.76	39.34
2	Unequal	Small	160	20.95	19.61	46.55	36.45
2	Unequal	Small	960	19.98	19.82	38.66	37.44
2	Unequal	Large	160	23.38	20.99	39.71	41.28
2	Unequal	Large	960	20.44	20.01	40.66	38.51
2	Combined	Combined	160	21.99	20.01	42.42	39.06
2	Combined	Combined	960	20.24	19.90	41.18	39.37
				EXPECTED: 19.00		EXPECTED: 38.00	
3	Equal	Small	160	20.53	18.77	42.40	32.78
3	Equal	Small	960	19.04	18.86	39.34	37.93
3	Equal	Large	160	21.16	19.57	32.57	44.13
3	Equal	Large	960	20.15	19.22	42.89	39.27
3	Unequal	Small	160	20.37	18.91	40.85	32.59
3	Unequal	Small	960	19.26	19.11	37.66	36.64
3	Unequal	Large	160	22.42	20.36	38.10	40.51
3	Unequal	Large	960	19.96	19.41	37.60	35.26
3	Combined	Combined	160	21.12	19.40	38.48	37.50
3	Combined	Combined	960	19.60	19.15	39.37	37.28

TABLE 8

MEAN AND VARIANCE OF DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: ESTIMATION OF 3-CLASS UNCONSTRAINED SUBSUMING (OVERFITTED) MODEL FROM DATA GENERATED UNDER 2-CLASS UNCONSTRAINED MODEL

Mixture	Disparity	N	Mean L-ratio	Mean Pearson	Variance L-ratio	Variance Pearson
			EXPECTED: 14.00		EXPECTED: 28.00	
Equal	Small	160	14.05	12.84	31.59	23.73
Equal	Small	960	12.33	12.21	23.41	22.47
Equal	Large	160	14.91	12.89	20.92	19.17
Equal	Large	960	12.76	12.24	22.85	19.69
Unequal	Small	160	14.33	13.36	31.27	24.02
Unequal	Small	960	12.60	12.52	22.84	22.29
Unequal	Large	160	15.51	13.53	24.36	21.75
Unequal	Large	960	12.70	12.31	22.84	20.30
Combined	Combined	160	14.70	13.16	27.04	22.17
Combined	Combined	960	12.60	12.32	22.98	21.19

TABLE 9

MEAN AND VARIANCE OF DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: ESTIMATION OF 3-CLASS AND 4-CLASS CONSTRAINED SUBSUMING (OVERFITTED) MODELS

G. Mod.	E. Mod.	Mixture	Disparity	N	Mean L-ratio	Mean Pearson	Variance L-ratio	Variance Pearson
					EXPECTED: 19.00		EXPECTED: 38.00	
2-Cl.	3-Cl.	Equal	Small	160	20.45	18.87	45.62	35.22
2-Cl.	3-Cl.	Equal	Small	960	19.54	19.36	43.41	41.99
2-Cl.	3-Cl.	Equal	Large	160	22.14	19.51	34.67	38.78
2-Cl.	3-Cl.	Equal	Large	960	19.98	19.38	41.08	38.11
2-Cl.	3-Cl.	Unequal	Small	160	20.60	19.27	46.82	36.67
2-Cl.	3-Cl.	Unequal	Small	960	19.47	19.32	36.93	35.75
2-Cl.	3-Cl.	Unequal	Large	160	22.89	20.47	38.16	39.38
2-Cl.	3-Cl.	Unequal	Large	960	19.98	19.52	39.96	37.49
2-Cl.	3-Cl.	Combined	Combined	160	21.52	19.53	41.32	37.51
2-Cl.	3-Cl.	Combined	Combined	960	19.74	19.40	40.34	38.34
					EXPECTED: 18.00		EXPECTED: 36.00	
2-Cl.	4-Cl.	Equal	Small	160	20.18	18.62	44.86	34.76
2-Cl.	4-Cl.	Equal	Small	960	19.26	19.08	42.97	41.51
2-Cl.	4-Cl.	Equal	Large	160	21.85	19.17	34.21	37.88
2-Cl.	4-Cl.	Equal	Large	960	19.50	18.87	40.29	36.99
2-Cl.	4-Cl.	Unequal	Small	160	20.34	19.03	46.32	36.36
2-Cl.	4-Cl.	Unequal	Small	960	19.24	19.08	36.16	34.96
2-Cl.	4-Cl.	Unequal	Large	160	22.63	20.11	37.79	38.67
2-Cl.	4-Cl.	Unequal	Large	960	19.57	19.09	40.00	37.53
2-Cl.	4-Cl.	Combined	Combined	160	21.25	19.23	40.80	36.92
2-Cl.	4-Cl.	Combined	Combined	960	19.39	19.03	39.86	37.75
3-Cl.	4-Cl.	Equal	Small	160	20.10	18.30	42.18	32.24
3-Cl.	4-Cl.	Equal	Small	960	18.54	18.35	38.90	37.57
3-Cl.	4-Cl.	Equal	Large	160	20.83	19.01	31.53	40.78
3-Cl.	4-Cl.	Equal	Large	960	19.80	18.84	41.88	37.76
3-Cl.	4-Cl.	Unequal	Small	160	20.01	18.53	40.72	32.16
3-Cl.	4-Cl.	Unequal	Small	960	18.68	18.54	36.37	35.41
3-Cl.	4-Cl.	Unequal	Large	160	22.15	19.89	37.84	39.53
3-Cl.	4-Cl.	Unequal	Large	960	19.60	18.99	37.17	34.27
3-Cl.	4-Cl.	Combined	Combined	160	20.77	18.93	38.07	36.18
3-Cl.	4-Cl.	Combined	Combined	960	19.16	18.68	38.58	36.25

TABLE 10

MEAN AND VARIANCE OF DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: ESTIMATION OF 1-CLASS AND 2-CLASS UNCONSTRAINED SUBSUMED (NONFITTING) MODELS

					Mean		Variance	
G.Mod.	E.Mod.	Mixture	Disparity	N	L-ratio	Pearson	L-ratio	Pearson
					EXPECTED:	20.00	EXPECTED:	40.00
3-Cl.	2-Cl.	Equal	[illegible]	160	21.37	19.71	45.02	36.54
3-Cl.	2-Cl.	Equal	[illegible]	960	[illegible]	23.60	57.45	57.23
3-Cl.	2-Cl.	Equal	[illegible]	160	51.06	49.71	138.13	162.76
3-Cl.	2-Cl.	Equal	[illegible]	960	192.29	188.67	647.63	591.02
3-Cl.	2-Cl.	Unequal	[illegible]	[illegible]	20.85	19.39	41.85	33.31
3-Cl.	2-Cl.	Unequal	Small	960	21.86	21.73	45.89	45.04
3-Cl.	2-Cl.	Unequal	Large	160	44.24	42.60	133.75	126.05
3-Cl.	2-Cl.	Unequal	Large	960	158.97	167.43	721.88	544.96
3-Cl.	2-Cl.	Combined	Combined	160	34.38	32.85	89.69	89.66
3-Cl.	2-Cl.	Combined	Combined	960	99.19	100.36	368.24	309.56
					EXPECTED:	26.00	EXPECTED:	56.00
2-Cl.	1-Cl.	Equal	Small	160	40.00	41.00	122.18	155.67
2-Cl.	1-Cl.	Equal	Small	960	99.64	112.47	365.66	582.22
2-Cl.	1-Cl.	Equal	Large	160	298.68	502.81	1009.04	7577.71
2-Cl.	1-Cl.	Equal	Large	960	1653.28	2884.88	5539.93	41136.48
2-Cl.	1-Cl.	Unequal	Small	160	33.16	32.21	89.52	88.34
2-Cl.	1-Cl.	Unequal	Small	960	58.32	62.55	184.28	249.90
2-Cl.	1-Cl.	Unequal	Large	160	172.93	455.00	926.54	11163.24
2-Cl.	1-Cl.	Unequal	Large	960	895.40	2628.31	5590.45	66897.17
2-Cl.	1-Cl.	Combined	Combined	160	136.19	257.76	536.82	4746.24
2-Cl.	1-Cl.	Combined	Combined	960	676.66	1422.05	2920.08	27216.44
3-Cl.	1-Cl.	Equal	Small	160	34.30	33.40	86.41	89.34
3-Cl.	1-Cl.	Equal	Small	960	60.68	64.15	190.45	241.46
3-Cl.	1-Cl.	Equal	Large	160	193.06	289.27	707.64	3166.93
3-Cl.	1-Cl.	Equal	Large	960	1006.99	1586.46	3700.49	16342.46
3-Cl.	1-Cl.	Unequal	Small	160	30.50	28.86	67.31	60.03
3-Cl.	1-Cl.	Unequal	Small	960	40.73	41.23	119.13	129.70
3-Cl.	1-Cl.	Unequal	Large	160	103.58	163.56	423.34	2508.56
3-Cl.	1-Cl.	Unequal	Large	960	482.00	866.08	2261.72	14461.77
3-Cl.	1-Cl.	Combined	Combined	160	90.36	128.77	321.18	1456.22
3-Cl.	1-Cl.	Combined	Combined	960	397.60	639.48	1567.95	7793.85

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TABLE 11

TAIL WEIGHTS FOR DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS:
2-CLASS UNCONSTRAINED AND 3-CLASS CONSTRAINED GENERATION (FITTING) MODELS

				L-ratio			Pearson		
		Expected values:		.01	.05	.10	.01	.05	.10
#Classes	Mixture	Disparity	N						
2	Equal	Small	160	.02	.08	.14	.01	.04	.08
2	Equal	Small	960	.02	.05	.10	.01	.05	.09
2	Equal	Large	160	.01	.09	.16	.02	.06	.10
2	Equal	Large	960	.01	.06	.12	.01	.05	.11
2	Unequal	Small	160	.02	.07	.13	.01	.03	.09
2	Unequal	Small	960	.01	.06	.10	.01	.06	.10
2	Unequal	Large	160	.02	.10	.22	.01	.06	.12
2	Unequal	Large	960	.01	.06	.11	.01	.05	.10
2	Combined	Combined	160	.02	.08	.16	.01	.04	.10
2	Combined	Combined	960	.01	.06	.11	.01	.05	.10
3	Equal	Small	160	.02	.08	.16	.01	.04	.08
3	Equal	Small	960	.01	.06	.10	.01	.05	.10
3	Equal	Large	160	.02	.07	.14	.02	.06	.11
3	Equal	Large	960	.02	.07	.15	.02	.06	.10
3	Unequal	Small	160	.02	.08	.15	.01	.03	.08
3	Unequal	Small	960	.01	.05	.10	.01	.05	.10
3	Unequal	Large	160	.02	.12	.23	.02	.08	.14
3	Unequal	Large	960	.01	.06	.12	.01	.05	.10
3	Combined	Combined	160	.02	.09	.17	.02	.05	.10
3	Combined	Combined	960	.01	.06	.12	.01	.06	.10
Combined	Combined	Combined	160	.02	.09	.17	.01	.05	.10
Combined	Combined	Combined	960	.01	.06	.11	.01	.05	.10

TABLE 12

TAIL WEIGHTS FOR DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: ESTIMATION OF 3-CLASS UNCONSTRAINED SUBSUMING (OVERFITTED) MODEL FROM DATA GENERATED UNDER 2-CLASS UNCONSTRAINED MODEL

			L-ratio			Pearson		
	Expected values:		.01	.05	.10	.01	.05	.10
Mixture	Disparity	N						
Equal	Small	160	.01	.06	.11	.01	.03	.07
Equal	Small	960	.01	.02	.05	.00	.03	.05
Equal	Large	160	.00	.04	.09	.00	.02	.05
Equal	Large	960	.00	.03	.05	.00	.01	.04
Unequal	Small	160	.01	.05	.11	.01	.03	.06
Unequal	Small	960	.00	.03	.06	.00	.02	.06
Unequal	Large	160	.01	.06	.13	.00	.03	.06
Unequal	Large	960	.00	.03	.06	.00	.02	.05
Combined	Combined	160	.01	.05	.11	.00	.03	.06
Combined	Combined	960	.00	.03	.06	.00	.02	.05

TABLE 13

TAIL WEIGHTS FOR DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: ESTIMATION OF 3-CLASS AND 4-CLASS CONSTRAINED SUBSUMING (OVERFITTED) MODELS

					L-ratio			Pearson		
			Expected values:		.01	.05	.10	.01	.05	.10
G. Mod.	E. Mod.	Mixture	Disparity	N						
2-Cl.	3-Cl.	Equal	Small	160	.02	.08	.15	.01	.04	.09
2-Cl.	3-Cl.	Equal	Small	960	.02	.06	.11	.02	.05	.10
2-Cl.	3-Cl.	Equal	Large	160	.02	.10	.18	.02	.07	.11
2-Cl.	3-Cl.	Equal	Large	960	.02	.07	.13	.01	.06	.12
2-Cl.	3-Cl.	Unequal	Small	160	.02	.09	.15	.01	.05	.10
2-Cl.	3-Cl.	Unequal	Small	960	.01	.05	.10	.01	.06	.10
2-Cl.	3-Cl.	Unequal	Large	160	.02	.12	.25	.02	.08	.14
2-Cl.	3-Cl.	Unequal	Large	960	.01	.06	.12	.01	.06	.11
2-Cl.	3-Cl.	Combined	Combined	160	.02	.10	.18	.02	.06	.11
2-Cl.	3-Cl.	Combined	Combined	960	.02	.06	.12	.01	.06	.11
2-Cl.	4-Cl.	Equal	Small	160	.03	.10	.18	.01	.06	.11
2-Cl.	4-Cl.	Equal	Small	960	.03	.07	.13	.02	.07	.12
2-Cl.	4-Cl.	Equal	Large	160	.02	.12	.21	.02	.08	.12
2-Cl.	4-Cl.	Equal	Large	960	.02	.08	.15	.01	.07	.13
2-Cl.	4-Cl.	Unequal	Small	160	.03	.11	.19	.02	.06	.12
2-Cl.	4-Cl.	Unequal	Small	960	.02	.07	.13	.01	.07	.12
2-Cl.	4-Cl.	Unequal	Large	160	.03	.16	.28	.02	.09	.17
2-Cl.	4-Cl.	Unequal	Large	960	.02	.08	.14	.02	.07	.13
2-Cl.	4-Cl.	Combined	Combined	160	.03	.12	.22	.02	.07	.13
2-Cl.	4-Cl.	Combined	Combined	960	.02	.08	.14	.02	.07	.12
3-Cl.	4-Cl.	Equal	Small	160	.02	.10	.18	.01	.05	.10
3-Cl.	4-Cl.	Equal	Small	960	.02	.06	.12	.02	.06	.11
3-Cl.	4-Cl.	Equal	Large	160	.02	.08	.15	.02	.06	.11
3-Cl.	4-Cl.	Equal	Large	960	.02	.09	.18	.02	.06	.12
3-Cl.	4-Cl.	Unequal	Small	160	.02	.10	.17	.01	.05	.10
3-Cl.	4-Cl.	Unequal	Small	960	.01	.06	.11	.01	.06	.11
3-Cl.	4-Cl.	Unequal	Large	160	.03	.15	.27	.02	.09	.16
3-Cl.	4-Cl.	Unequal	Large	960	.02	.08	.16	.01	.05	.13
3-Cl.	4-Cl.	Combined	Combined	160	.02	.11	.19	.02	.06	.12
3-Cl.	4-Cl.	Combined	Combined	960	.02	.07	.14	.02	.06	.12
Combined	Combined	Combined	Combined	160	.02	.11	.20	.02	.06	.12
Combined	Combined	Combined	Combined	960	.02	.07	.13	.02	.06	.12

TABLE 14

TAIL WEIGHTS FOR DISTRIBUTIONS OF LIKELIHOOD-RATIO AND PEARSON STATISTICS: ESTIMATION OF 1-CLASS AND 2-CLASS UNCONSTRAINED SUBSUMED (NONFITTING) MODELS

					L-ratio			Pearson		
			Expected values:		.01	.05	.10	.01	.05	.10
G.Mod.	E.Mod.	Mixture	Disparity	N						
3-Cl.	2-Cl.	Equal	Small	160	.02	.08	.15	.01	.04	.08
3-Cl.	2-Cl.	Equal	Small	960	.05	.16	.24	.05	.15	.24
3-Cl.	2-Cl.	Equal	Large	160	.88	.96	.98	.83	.95	.97
3-Cl.	2-Cl.	Equal	Large	960	1.00	1.00	1.00	1.00	1.00	1.00
3-Cl.	2-Cl.	Unequal	Small	160	.01	.06	.13	.00	.03	.07
3-Cl.	2-Cl.	Unequal	Small	960	.02	.09	.16	.02	.08	.16
3-Cl.	2-Cl.	Unequal	Large	160	.69	.87	.93	.66	.85	.91
3-Cl.	2-Cl.	Unequal	Large	960	1.00	1.00	1.00	1.00	1.00	1.00
3-Cl.	2-Cl.	Combined	Combined	160	.40	.49	.55	.38	.47	.51
3-Cl.	2-Cl.	Combined	Combined	960	.52	.56	.60	.52	.56	.60
2-Cl.	1-Cl.	Equal	Small	160	.28	.50	.62	.33	.52	.61
2-Cl.	1-Cl.	Equal	Small	960	1.00	1.00	1.00	1.00	1.00	1.00
2-Cl.	1-Cl.	Equal	Large	160	1.00	1.00	1.00	1.00	1.00	1.00
2-Cl.	1-Cl.	Equal	Large	960	1.00	1.00	1.00	1.00	1.00	1.00
2-Cl.	1-Cl.	Unequal	Small	160	.11	.25	.36	.08	.22	.32
2-Cl.	1-Cl.	Unequal	Small	960	.82	.94	.97	.86	.95	.97
2-Cl.	1-Cl.	Unequal	Large	160	1.00	1.00	1.00	1.00	1.00	1.00
2-Cl.	1-Cl.	Unequal	Large	960	1.00	1.00	1.00	1.00	1.00	1.00
2-Cl.	1-Cl.	Combined	Combined	160	.60	.69	.74	.60	.68	.73
2-Cl.	1-Cl.	Combined	Combined	960	.96	.98	.99	.96	.99	.99
3-Cl.	1-Cl.	Equal	Small	160	.12	.28	.42	.10	.27	.37
3-Cl.	1-Cl.	Equal	Small	960	.87	.96	.98	.90	.97	.98
3-Cl.	1-Cl.	Equal	Large	160	1.00	1.00	1.00	1.00	1.00	1.00
3-Cl.	1-Cl.	Equal	Large	960	1.00	1.00	1.00	1.00	1.00	1.00
3-Cl.	1-Cl.	Unequal	Small	160	.04	.16	.25	.02	.10	.17
3-Cl.	1-Cl.	Unequal	Small	960	.29	.53	.66	.32	.54	.67
3-Cl.	1-Cl.	Unequal	Large	160	1.00	1.00	1.00	1.00	1.00	1.00
3-Cl.	1-Cl.	Unequal	Large	960	1.00	1.00	1.00	1.00	1.00	1.00
3-Cl.	1-Cl.	Combined	Combined	160	.54	.61	.67	.53	.59	.64
3-Cl.	1-Cl.	Combined	Combined	960	.79	.87	.91	.80	.88	.91
Comb.	Comb.	Combined	Combined	160	.51	.60	.65	.50	.58	.62
Comb.	Comb.	Combined	Combined	960	.75	.81	.83	.76	.81	.84