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ABSTRACT

This publication contains seven papers by the staff of the University of Exeter School of Education and by invited outside contributors. The focus is on the changing mathematics curriculum, instructional practices, and research findings. The papers are: (1) "Mathematics Education for the 21st Century: It's Time for a Revolution!" (David Burghes); (2) "Using Interactive Video in Secondary Mathematics" (Dudley Kennett); (3) "Mathematics--An Alternative Approach" (David Hobbs); (4) "Classroom Processes and Mathematical Discussions: A Cautionary Note" (Charles Desforges); (5) "Investigations: Where To Now? or Problem-Posing and the Nature of Mathematics" (Stephen Lerman); (6) "Classroom Based Assessment" (Susan Pirie); (7) "The Use of the History of Mathematics in Teaching" (Derek Stander). (MNS)

TEACHING and LEARNING MATHEMATICS

Part 1

PERSPECTIVES 33

Series Editor
Peter Preece

Issue Editor
Paul Ernest

Perspectives is a series of
occasional publications on
current educational topics

Issues contain papers by
the staff of the University of
Exeter School of Education and
by invited outside contributors

Forthcoming topics include:

Teaching and Learning Mathematics: Part 2
Racial Prejudice
Gender and Education
Implementing GCSE

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PERSPECTIVES 33

TEACHING AND LEARNING MATHEMATICS PART 1

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TEACHING AND LEARNING MATHEMATICS PART 2

EDITOR'S INTRODUCTION

British mathematics teaching is currently in a state of ferment and change. The last five years have seen the publication of a number of major official documents recommending changes in the teaching of mathematics, most notably Cockcroft (1982) and HMI (1985); or signalling changes in the assessment of school mathematics (DES, 1985 and 1987).

At the same time as these central government initiatives, the 1980s have seen the greatest ever impact of technology upon the teaching of mathematics. Electronic calculators are universally available, microcomputers are increasingly central to the mathematics curriculum and new technologies, such as interactive video, are beginning to make their own impact.

A third strand of development has also been taking place. Research is beginning to deliver knowledge of the processes of learning and teaching mathematics. Detailed knowledge of the outcomes of the mathematics curriculum is now available, for the first time, in publications such as Hart (1981) and APU (1986).

Theories which explain the learning of mathematics, such as Constructivism, are beginning to emerge.

One outcome of these forces is a new round of curriculum developments in mathematics, including major projects at King's College, London (Nuffield Secondary Mathematics) and Exeter (Alternative Mathematics).

<u>Perspectives</u> 33 and 34 reflect the changes that are happening in mathematics education in Great Britain. A number of the papers focus on innovations and developments in the teaching of mathematics and give an insight into what mathematics teaching will be like in the 1990s. Some of the papers provide state-of-the-art research perspectives on the teaching and learning of mathematics, by researchers of national and international renown. Unusually, these two issues bring together expert researchers both from within the mathematics education community, and from the broader educational research community.

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The range of contributions, from those of practitioners to those of pioneering researchers, shows that for all the difficulties of economic recession and cutback, there has never been a more exciting time to be involved with mathematics education than the present.

Paul Ernest

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MATHEMATICS EDUCATION FOR THE 21ST CENTURY:

IT'S TIME FOR A REVOLUTION!

1. Initiatives in Mathematics Education

There is so much activity in mathematics education post Cockcroft that it is difficult to know where to begin. We have:

* 300 or so Maths missionaries spreading the gospel throughout the land

* D.E.S. priority for mathematics in-service courses throughout the country

* New maths education centres at Universities

* Mathematics education research projects (e.g. slow learners, graded tests, investigations, etc. etc.)

* Countless new books, G.C.S.E. schemes, packs, software etc.

* New forms of assessment - eventually including grade related criteria

- but will all this activity make any difference to mathematics education? There are two important questions to ask:

(i) Will we produce a more numerate and adaptable generation?

(ii) Will we produce more school leavers who have enjoyed their maths at school?

Although most maths educators seem convinced that they have seen the light (e.g. investigations, practical work, discussion and problem solving), I think it is time to question this new faith. I do not do this in order to put the clock back or to undermine confidence, but in order to provoke a **real revolution** in the way we teach (and pupils learn) mathematics.

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2. The Faith

Almost everyone reacted favourably to the Cockcroft report - it was particularly welcomed by those of us in the mathematica education community who for so long have been trying to show our prospective teachers that there is more to a mathematics education than simply getting sums right. Yet mathematics at school and at University has continued up to now to promote this view of mathematics. A well trained and hard working parrot could get a first class degree in mathematics at many Universities - provided it could write rather than talk!

The Cockcroft Report ('Mathematics Counts', published by the D.E.S. in 1982) has gained an international acclaim - amongst many recommendations it advocates that:

Discussion, Practical Work, Investigations and Problem Solving

should be part of a mathematical diet as well as the usual theory and practice. So the faithful, up and down the country, now think that they have the answer. But just imagine bottom set fourth year late on a Friday afternoon - will it really motivate them to find out how many squares you can make on an n x n pegboard? Indeed, it probably will motivate them, if it is taught by a teacher with complete conviction and who has the personality to put it across in a meaningful way. But what about the average harrassed, hard-pressed teachers - how are they going to cope?

It is not that I am against introducing such teaching methods into mathematics - I do very positively encourage it, and with good teachers, and quality in-service work, I am convinced that these changes should take place. If nothing else, it makes teaching mathematics a far more unpredictable occupation. What I am not so convinced about is whether having made these changes (which will be both costly to implement, and difficult for many teachers to cope with), we will have had much effect on giving positive answers to my questions (i) and (ii) above. In fact, if the changes are made in a haphazard way with insufficient backing (and despite all the money that has been pumped into maths education, there is already evidence of this with the new G.C.S.E. courses), we might well have worsened the situation.

It is very easy to criticise, but clearly not so easy to suggest alternative strategies, but I will at least attempt to put forward some tentative ideas. My suggestions (unlike Cockcroft's) might well be unpopular with the maths education community, but they are an honest attempt to suggest changes - no, more than changes - **a revolution**, which might prove more successful in the long run than our current attempts at change.

3. An Alternative Faith: Motivation

Pupils will work at mathematics (or any other subject) if they are motivated to do so - so how do we provide the motivation for mathematics? It will depend on such factors as:

- (i) the personality and talent of the teacher,
- (ii) the school environment, and class associates,
- (iii) the home environment,
- (iv) job prospects (or lack of),
- (v) the curriculum,
- (vi) assessment,

and many other things. I put the teacher first as I do believe that they are the most crucial motivator - just think back to your own time at school. Although clearly children have particular talents for various topics, a good teacher can make all the difference between a pupil losing interest (which is exceedingly difficult to recover in mathematics) or working conscientiously, enjoying the subject. So my faith starts with the teacher - if somehow we could take the most talented dozen or so maths teachers in the country and make thousands of clones, many of our problems in maths education would be solved. But, and this is where I am on very shaky ground, all my experience shows that good maths teachers are born with the talent, and I'm not yet convinced that training does much good. Indeed, I liken this to my experience as a potential Middlesex cricketer! At the age of 14, I was 'spotted' by my school after a spectacular 5 wicket fast bowling spell in a House game (which was the first time I had bowled in a competition match) - during the winter, I was sent to the M.C.C. for training. I know that the coaches were doing their best, but I spent three further years representing the school at cricket, and I never took more than two wickets in one innings again! . My inelegant style and jumping action had been replaced by a nice, clean, smooth action but my bowling was doomed for evermore. Gone was the enthusiasm and confidence - replaced by a professionalism that was not needed! Are we in danger of doing the same when we train maths teachers?

Although the school, home and job prospects come next on my list, they are really outside the scope of this article and so I look next at the curriculum and its assessment. Firstly the curriculum - here perhaps we can make some progress, as it is clear to me that certain topics are more motivating than others. In Appendix 1 we show an algorithm for finding the day of the week for dates this century - you can use this for classes from age 11 up to 100 - it always motivates - not because pup') are keen to know why it works, but because **they want to know the answer** - they want, for example, to know which day of the week they were born on, and they enjoy actually performing the calculation to get the answer - the computer could do it all for you, but that actually takes the fun out of the problem. So a key seems to be in finding activities in which pupils need to use mathematics to find out results that **they** want to find out. It's probably not possible to rewrite the whole maths curriculum in such a way that pupils are always motivated in this way, but we all know that too often pupils are performing calculations or learning techniques that have absolutely no relevance for them at all.

So my faith begins with **motivation** through

(i) the ability of good teachers
(ii) the relevance of the curriculum.

Of course, so far this faith is compatible with the post Cockcroft faith, but my interpretation will start to diverge from others.

4. Relevance of the Curriculum

I will start by assuming that for most people, there is little or nothing in G.C.S.E. Mathematics beyond List 1 (see Appendix 2) that is directly relevant .or future life. For those continuing onto A-level and into H.E., there is a natural progression upwards, but for the vast majority of children who complete their mathematical instruction at age 16, there is precious little in secondary mathematics that we can in all honesty defend as being directly useful for future life. That is not to say that there are not good reasons for the inclusion of many topics but direct relevance or usefulness is certainly not the only reason!

Topics of direct relevance, such as:

Basic Numeracy
Percentages
Weighing and Measuring
Money Transactions
Reading Timetables
Betting Odds

do not, at present, constitute a large part of the mathematics curriculum and, indeed, many are dealt with at primary rather than secondary level.

So what should we fill up the maths curriculum with - one suggestion is a very simple one - we don't fill it up at all! Why do we continue to protect mathematics as a subject that needs 2/3 hours every week for 5 years in secondary schools! Our usual meaning of mathematics probably needs such time - and probably much more for many pupils. But why bother at all? Why put our children through this misery week after week - surely we can think of something better and more positive to do with the time. Could we accept the thesis that children do not have to succeed at **academic** mathematics to be worthwhile citizens? What we must aim for is to make sure that they are numerate (and this often means making sure that they do not get worse at sums when they leave secondary school than when they entered!) and that they are offered something mathematical which is enjoyable, creative and stimulating, whilst enabling them to reach their individual mathematical potential.

Another suggestion is to teach mathematics through applications and contextual situations. This is the theme of the Alternative Mathematics Project, which is a modular approach to teaching G.C.S.E. Mathematics. Here, at least, pupils will see the relevance of mathematics. The modules are in:

* Business and Commerce
* Design
* Science and Technology
* Environment
* Leisure and Recreation

So, in some way, we have taken mathematics off the curriculum. Full details of this project are given by David Hobbs in this issue of Perspectives.

5. Challenge in Mathematics

I have already outlined the theme of motivation through relevance - but relevance is not enough. Challenge is also needed, but the appropriate challenge for each individual. Appendix 3 gives an example of a worksheet on the classical 'Travelling Salesman Problem'. It is structured so that all students can make progress - they can all find a possible solution, and the challenge is to find the best possible solution. In Appendix 4, there is another example of a problem, which has a steady build up through to the final problem (which is not as easy as it looks!). It is clear to me that challenge is, like relevance, an important ingredient that provides the motivation needed for effective and enjoyable teaching and learning of mathematics.

6. New Technology

Elsewhere in this issue of Perspectives, you will learn more about new technology, and its possible impact for teaching and learning of mathematics. At this stage, it is very easy to become obsessed by Interactive Video (its potential is understandably enormous - its acceptability as a teaching medium still remains to be proved!), but we must not forget the potential of earlier technologies (audio-tapes, video and computer software). We should also note that we already have hand-held calculators which can give a graphic display for functions, and it will not be long before we have small calculators capable of formal algebra and calculus; i.e. given a function, say x^2, they will, at the press of a button, give the answer '$x^3/3 + c$'. The existence of such programs (currently available for main-frame computers, although packages are now being developed for microcomputers) should initiate a debate over the content of school (and university) mathematics curricula - indeed, it gives strength to the argument that it really doesn't matter what mathematical content we teach since it can all be done by computers now - it's the process skills that are important. These are skills such as:

* understanding problems
* implementing strategies for solution
* interpreting data
* transferring skills to new problems
* tackling unseen problems
* critical judgement
* working positively in a group

It is this type of criterion which is far more important to later life than the usual stress on yet more and more abstract content.

Finally though, I would add that I would not like to see all of mathematics go out of the window. Just doing mathematics can be an enjoyable activity for many people - it should be encouraged, since there are so many beautiful and elegant aspects to mathematics, for example:

4-colour theorem for maps
Perfect numbers
Fibonacci Sequence
Golden Ratio
Mobius Strips

Unfortunately, topics of this sort do not occur on conventional syllabuses.

6. Final Remarks

I have tried to outline possible developments in the teaching and learning of mathematics over the next decade. I think though that it should be stressed that any historical perspective gives little cause for comfort - the Cockcroft Report is just the latest of a long series of reports into what is wrong with the teaching of mathematics that have been published over the past century. We have always had problems. Maybe the heart of the problem is that mathematics (as we conventionally see it) is a difficult subject, and there is no chance of any dramatic improvement (unless we move the goalposts!)

Surely though the mathematics education community must recognise that currently we do much harm to many pupils at school - my rough estimate would be that more than half the population leave school with a distaste for the subject. They become the adults of tomorrow who cannot cope with even simple sums. Mathematics is a subject that I am convinced can be **enjoyed** by everyone.

Although enjoyment in mathematics is my ultimate goal, it should also be stressed that the nation does need more able generations, who are adaptable, flexible and able to use their mathematical abilities. We do need a revolution in the way we teach mathematics. Maybe I'm alone in my thinking, but I cannot be convinced that the presently planned changes in mathematics teaching will produce anything but chaos. If we are going to have chaos, then let's go the whole way and have a revolution - we might ultimately achieve more!

David Burghes

Appendix 1

Birthday Algorithm

Step 1

Find $S = Y + D + \left[\frac{Y-1}{4}\right]$,

where Y = Year

D = Day of Year (Jan 1st = 1, ..., Jan 31st = 31, Feb 1st = 32, ...)

$\left[\frac{Y-1}{4}\right]$ means ignore the remainder

Step 2

Divide 5 by 7 and note the remainder

Step 3

The day of the week is given by the key

Remainder	Day
0	Friday
1	Saturday
2	Sunday
3	Monday
4	Tuesday
5	Wednesday
6	Thursday

Example : For the birthday of the Princess of Wales, we have the birth date July 1st, 1961.

Step 1

Y = 1961

D = 31 + 28 + 31 + 30 + 31 + 30 = 1 = 182

$$\left[\frac{Y-1}{4}\right] = \left[\frac{1960}{4}\right] = 490$$

So S = 1961 + 182 + 490 = <u>2633</u>

Step 2

S divided by 7 gives 376 and remainder 1

Step 3

Using table, 1 corresponds to SATURDAY

So the Princess of Wales was born on a Saturday.

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Whole numbers: odd, even, prime, square. Factors, multiples, idea of square root. Directed numbers in practical situations. Vulgar and decimal fractions and percentages; equivalences between these forms in simple cases; conversion from vulgar to decimal fractions with the help of a calculator.

Estimation.

Approximation to obtain reasonable answers.

The four rules applied to whole numbers and decimal fractions.

Language and notation of simple vulgar fractions in appropriate contexts, including addition and subtraction of vulgar (and mixed) fractions with simple denominators.

Elementary ideas and notation of ratio. Percentage of a sum of money. Scales, including map scales. Elementary ideas and applications of direct and inverse proportion. Common measures of rate.

Efficient use of an electronic calculator; application of appropriate checks of accuracy.

Measures of weight, length, area, volume and capacity in current units. Time: 24 hour and 12 hour clock. Money, including the use of foreign currencies.

Personal and household finance, including hire purchase, interest, taxation, discount, loans, wages and salaries. Profit and loss, VAT. Reading of clocks and dials. Use of tables and charts. Mathematical language used in the media. Simple change of units including foreign currency. Average speed.

Cartesian coordinates. Interpretation and use of graphs in practical situations including travel graphs and conversion graphs. Drawing graphs from given data.

The use of letters for generalised numbers. Substitution of numbers for words and letters in formulae.

The geometrical terms: point, line, parallel, bearing, right angle, acute and obtuse angles, perpendicular, similarity.

Measurement of lines and angles. Angles at a point. Enlargement.

Vocabulary of triangles, quadrilaterals and circles: properties of these figures directly related to their symmetries. Angle properties of triangles and quadrilaterals.

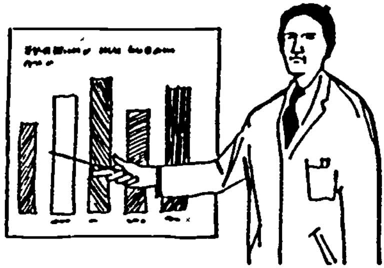
Simple solid figures.

Use of drawing instruments. Reading and making of scale drawings. Perimeter and area of rectangle and triangle. Circumference of circle. Volume of cuboid.

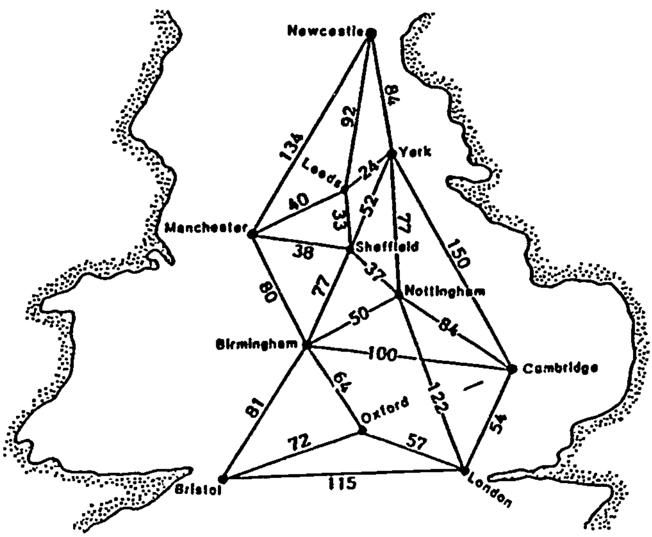
Collection, classification and tabulation of statistical data. Reading, interpreting and drawing simple inferences from tables and statistical diagrams. Construction of bar charts and pictograms. Measures of average and the purposes for which they are used.

Probability involving only one event.

TRAVELLING



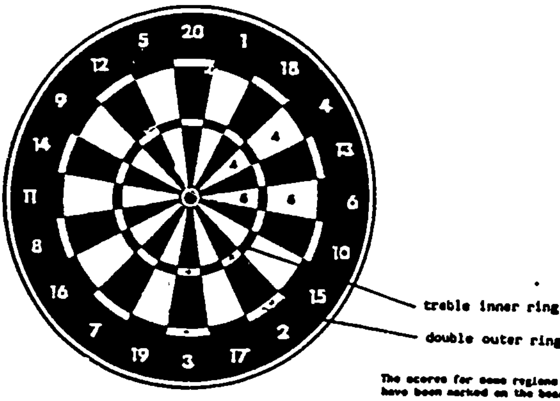
A computer salesman, who lives in London, wants to visit all the cities shown on the map.



1. Find a route for him.
2. What is its total length?
3. Can you find a route of length less than 750 miles?
4. How long is the shortest route you can find?
5. If he no longer has to visit Sheffield, what is his shortest route?

An Edward Arnold Master

DARTS - 1



The scores for some regions have been marked on the board.

The picture shows the usual darts board.

If the dart lands

(i) in the outer ring, you score twice the number shown (we call these doubles)

(ii) in the next narrow ring, you score three times the number (we call these trebles)

(iii) in the centre, outer ring you score 25

(iv) in the centre, inner ring (called the bull) you score 50

You usually throw three darts in each turn, from a line nine feet from the board (called the ockey). In most games you play to score 501, and you must finish with a double or bull.

DARTS - 2

PROBLEMS

1. Make a list of all the integers from 1 to 60. Indicate which of these numbers can be thrown with a single dart.
   Which of these numbers are doubles?
   Which of these numbers are trebles?

2. How many numbers between 1 and 60 cannot be thrown with one dart?

3. What is the most that you can score with 3 darts?

4. You need 57 to finish and it is your turn. With your first dart you score 8. How many ways are there of finishing, using the next two darts?
   (Remember – you must finish with a double or bull.)

5. Find the least number of throws needed to score 501 (ending with a double or bull).

6. Can you finish in three darts, if you need to score

   (a) 61 (b) 72 (c) 89 (d) 111 (e) 132
   (f) 145 (g) 166 (h) 170 (i) 177?

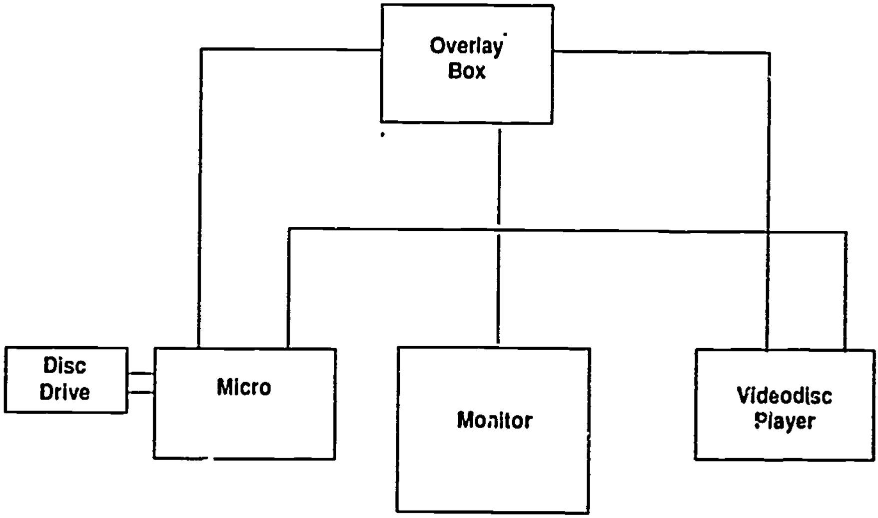
7. Experts aim for treble 20 (often scoring 20). If, however, you just miss 20, you would land in 1 or 5.
   Suppose you are not an expert dart thrower. Which sector should you aim for in order to score as much as possible?

8. What is the smallest number (not including 1) that you cannot score with at most three darts in order to finish?

USING INTERACTIVE VIDEO IN SECONDARY MATHEMATICS

For many years now computer software has been developed for use in education and training. We have recognised the strengths of this technology and looked for ways of overcoming its weaknesses. A particular weakness has been the quality and control of visual displays and sounds. Laser technology now enables us to move forward. It gives us the opportunity to have moving video images, computer text and graphics, all under the control of a low cost microcomputer. Central to this development is the video-disc; each side of which can hold up to 54,000 frames or 36 minutes of sound tracks. This can all be controlled by computer, has fast random access, and very high quality of sound and vision. It is known as Interactive Video (IV).

A typical workstation is illustrated in Fig.1. It is made up from a videodisc player, monitor, microcomputer, disk drives and an overlay box. The overlay box combines the video outputs of the microcomputer and videodisc player for display on the monitor. A complete system can cost from £5,000 to as little as £3,000, and there are clear signs that £2,000 will be a typical price in the near future. At such prices users in education may be interested in using IV, but they need to have material to use on these systems. Since 1982 and until recently software for IV has been developed almost exclusively for Industry and Commerce. Training departments have found it to be an effective and cost efficient method of training, and in marketing it is used for point of sale and other promotions. It is quite clear that managers and accountants believe that this is worthwhile. They are investing in software and hardware, and expect a return on their money. In education there is a problem. Investment in the development of curriculum materials is usually on a small scale, and our objectives are less clear than those of the trainer. One step to help overcome this was taken in 1985 when the D.T.I. set up the Interactive Video in Schools Project (IVIS).



<u>The Layout of an Interactive Video Workstation</u>

Figure 1.

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IVIS is a research and development project which aims to answer the question 'How can IV be used in the classroom?' It consists of a central co-ordinating team, eight project teams each producing and trialling an IV package, and an evaluation team. The eight project teams and their subject areas are:

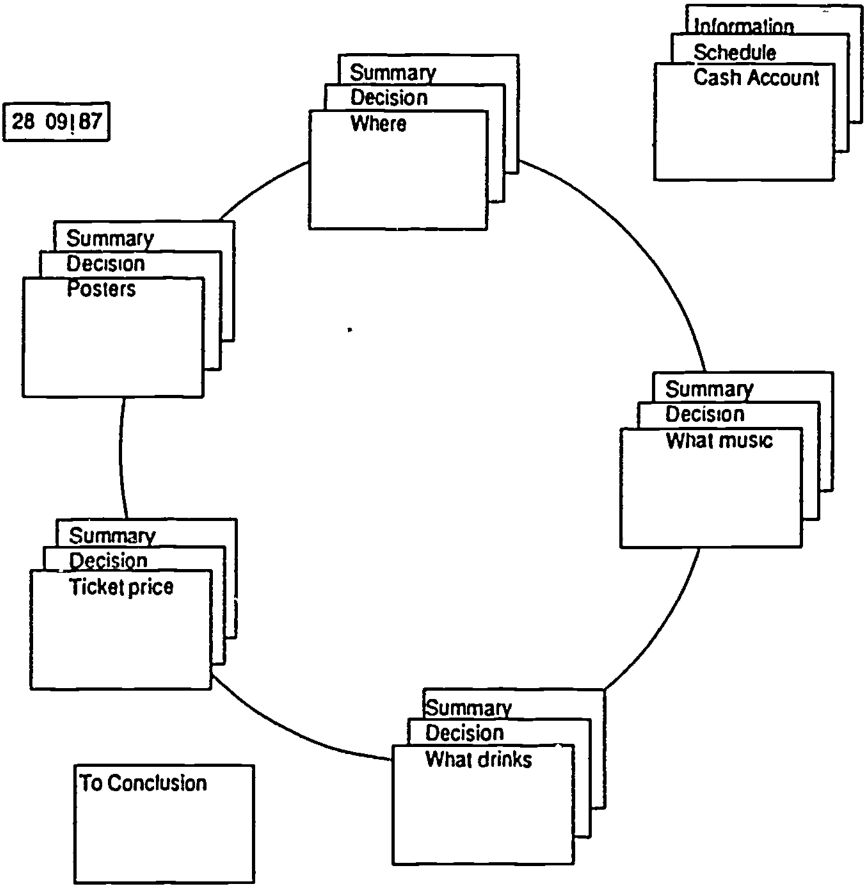
Mathematics : University of Exeter
Environmental Studies : Moray House College of Education
Modern Languages : Shropshire LEA
Geography : Loughborough University
Design : Leicestershire LEA
Primary Science : Bulmershe College
Teacher Training : Bishop Grosseteste College
Social Skills : Centre for Learning Resources, N.I.

The evaluation is being carried out by the Centre for Applied Research in Education, University of East Anglia, and the research report will be published in April 1988.

As part of IVIS the Centre for Innovation in Mathematics Teaching at the School of Education, University of Exeter, working with Blackrod Interactive Services Ltd., has developed an interactive video package for use in the secondary mathematics classroom. The title of the package is 'School Disco'.

'School Disco' brings the mathematics of the real world into the classroom by placing students in an environment where they can see how the decision making processes of a straightforward business situation require some basic mathematics for their efficient solution. When asked to plan and organise a school disco, they are faced with the problems of deciding where to hold the event, who to hire to play the music, what refreshments to provide, the price of the tickets, and what publicity is needed. The consequences of their decisions are then seen when they view the success, or otherwise, of the event they have planned.

The curriculum content of the package includes: basic arithmetic, statistics, graphs, modelling and optimisation. It is aimed at students of average ability in the 14-16 age range.



Main Map

Figure 2.

On starting the system, pupils are asked to type in the date and their names; after this they see an opening sequence which sets the scene. A headteacher advances them a float with which they start their enterprise, and from there they are moved to the 'Main Map', a diagram (Fig.2) which gives an overview of the system and from which they can choose the activity they wish to do first. There is no prescribed strategy, and the system includes many random events. The values of the variables change, and it is unlikely that anyone using it more than once will be faced with the same combination of situations.

As they move from one activity to another, the pupils have to collect, record and summarise information. Sometimes this involves straightforward arithmetic and recording in tabular form, on other occasions graphs have to be drawn and interpreted. Help is always available, it is in three forms: help with the system, help with the game, and help with the mathematics. The latter is presented automatically if needed. There are also guides to the additional activities, or follow-up materials, which include supplementary problems and investigations.

The design of 'School Disco' enables a teacher to use it in several different ways. At one level it can be presented to pupils as a game to be played fairly rapidly making naive decisions, but nevertheless reaching a satisfactory end point. This can be carried out by individuals or small groups of pupils. At another level, a teacher might use it as the basis for a class lesson in which a single decision point is studied in detail. For example, the choice of ticket price for the disco. This decision is critical to the success of the venture, and the mathematics involved includes the consideration of a price-demand graph, which then leads to a price-revenue graph, and from there to the idea of a maximum possible value for the revenue for a given choice of venue and music. After several such sessions, looking at various decision points, the activity can again be based on individual or small group work, but this time the emphasis would be on making optimum decisions using the available mathematics.

We can attempt to classify IV packages by drawing upon the typology of computer based learning activities. Much of the IV in training corresponds to Computer Aided Instruction, in which the system transmits information and tests the learning. Sometimes this is seen as drill and practice and sometimes it is considered as an intelligent programmed learning machine. Some would argue that as exposition is still the main teaching style for large parts of mathematics courses, and the Rule-Example-Practice paradigm is frequently used throughout the mathematics classroom, then IV has a role to play. It could 'do' certain parts of the course. In the United States a commercial company, Systems Impact Inc., are marketing such materials in a series called 'Core Concepts in Action'. A quotation from their sales literature gives an impression of the approach.

> Core concepts incorporates the principles of effective instruction; models proper techniques for presentation; focuses on important 'building block' concepts; uses motivating graphics which appeal to all age levels; and offers extensive 'branching' capabilities for remediation.

The series includes the titles 'Mastering Fractions', 'Mastering Decimals and Percents', 'Mastering Ratios' and 'An Introduction to Algebra'. This approach of exposition is used in parts of School Disco. For example the Help modules are straightforward descriptions of the technique required for the current problem.

Computer Based simulations are a second category of Computer Aided Learning. In these the learner can explore a model of a system while the computer acts as an umpire in a decision making game. School Disco fits clearly into this category. The same can be said of the work on secondary mathematics of the Interactive Learning Group at the University of Newcastle. They too have chosen a game or simulation for their package, which develops the ideas of chance and probability for secondary school pupils.

A third category is software which provides an aid to conjectural learning. The best known example is the computer language Logo. Using Logo, a learner is supplied with an environment in which ideas and concepts can be explored. As yet there is no IV equivalent, but it is a prospect we can look forward to. Soon, it will be possible to use the high quality images and sounds of IV to create a new generation of mathematical microworlds. The closest point reached so far in this development is the 'generic' type of package, in which the user is provided with a videodisc and software consisting of a wide range of images stored as a database. These can then be explored in an open manner, with suitable software in the form of an authoring language used to produce sequences of scenes and images as required. One example of this type is the IVIS Geography disk by Loughborough University. At Exeter we are exploring the possibility of producing similarly structured materials specifically for the use of mathematics teachers.

As yet we have no clear idea of the full impact of IV in the classroom. The IVIS trialling period is just starting. In education many have high hopes, but others may be rejecting the technology already. When looking at a package in isolation it is important to remember that it is effectiveness in the classroom that is important. It is easy to be swept along by the excitement of the images, or be impressed by the technology, when it should be the outcome in terms of student learning that really counts. Alternatively, one aspect of a complex package may conflict with a personal view or approach, and the whole package or IV itself may be rejected too hastily because of this.

There can be no doubt that as the technology advances, attempts will be made to use it in education. Some will see it as the answer to their problems - teacher shortages or an untrained workforce. Others will use it just because it is new. Whatever happens the work of the IVIS project will be seen as one more step along a path of increasingly sophisticated and effective devices for teaching and training. We will still need teachers, but the resources available to them and the techniques they use are changing.

Dudley Kennett

MATHEMATICS - AN ALTERNATIVE APPROACH

In 'Aspects of Secondary Education in England' (DES, 1979) attention was drawn to two aspects of mathematics teaching. Firstly, the style: most lessons were of a stereotyped nature - the teacher presented a topic on the blackboard, worked through an example, and then the pupils did practice exercises. Secondly, the content: the mathematics seldom related to anything else, nor was it inherently challenging.

Both of these points were followed up by the Cockcroft report 'Mathematics Counts' (DES, 1982). The celebrated paragraph 243 suggested that mathematics teaching at all levels should include opportunities for:

* exposition by the teacher;
* discussion between teacher and pupils and between pupils themselves;
* appropriate practical work;
* consolidation and practice of fundamental skills and routines;
* problem solving, including the application of mathematics to everyday situations;
* investigational work.

Later (paragraph 462) an extract from a submission is quoted:

> "Mathematics lessons in secondary schools are very often not about anything. You collect like terms or learn the laws of indices, with no perception of why anyone needs to do such things. There is excessive preoccupation with a sequence of skills and quite inadequate opportunity to see the skills emerging from the solution of problems."

The 'GCSE National Criteria for Mathematics' (DES, 1985) broke new ground by including objectives which could not be fully tested through written examinations:

> 3.16 respond orally to questions about mathematics, discuss mathematical ideas and carry out mental calculations;

3.17 carry out practical and investigational work, and undertake extended pieces of work.

The National Criteria also state that between 20% and 50% of the overall assessment has to be given to course work.

With regard to content the National Criteria are less adventurous. Dull lists of mathematical topics are provided for the least able group (List 1) and the middle ability group (List 2). For example, in List 2, we have

Basic arithmetic processes expressed algebraically.
Directed numbers.
Use of brackets and extraction of common factors.
Positive and negative integral indices.
Simple linear equations in one unknown.
Congruence.

A syllabus for the most able group is not provided in the National Criteria, the GCSE boards being allowed freedom to decide their own (although the contents of List 2 must be included).

The GCSE boards have responded by producing helpful material and guidelines for coursework, but in most cases their syllabuses are Lists 1 and 2 of the National Criteria with a few words of explanation included.

Mathematics in Context

Most people believe, often naively, that mathematics is 'useful', by which they mean that it is of use in everyday life and in many jobs. This is a view which parents, employers and politicians have propagated and which has an influence on children. In the classroom teachers often use it as a carrot, although much of what is done - algebraic manipulation, congruence, etc. - seems far removed from being useful. The response to 'What is the point of this?' is often that the use will be perceived at a later stage, the argument being that we will do the mathematics now and you will apply it later. However, in practice, the applications seldom emerge and for most pupils this later stage never comes.

An alternative approach, which is by no means new, is to recognise that much of mathematics has been developed in response to problems arising in the world around and to start with these contexts. Thus the everyday experience of sending a letter can be the starting point for a variety of activities. A discussion about how a letter gets from Manchester to London in a few hours can lead to points such as:

- Where are local post boxes sited? (Are there regulations about distances between post boxes? In view of housing development, should the local boxes be resited?)
- How are collection and delivery routes planned? (Elementary ideas of operational research)
- How are post codes allocated? (Leading to discussion about other coding systems - ISBN, bar-codes)
- How is mail sorted using post codes?
- How efficient are first and second class posts?

Also, at a low level, it can involve practical work, first estimating weights of letters and parcels and then weighing them, and it can involve skills of obtaining information from tables in Post Office leaflets. The problem of designing a useful book of stamps to cost £1, say, can appeal at all levels.

A contextual approach is equally valid for able children. Interpretation of the rules for maximum dimensions of parcels soon leads to higher-level mathematics. Again, an advertisement such as the one shown in Figure 1 raises the question of how the APR (annual percentage rate) has been calculated.

If you make the minimum deposit of 20% the rate is 4 9% (APR 9.5%). If you deposit 50% or more the rate is even less. Only 2.5% (APR 4 8%).

Cash price	£5452.86
9.5% APR Initial Payment (Minimum 20%)	£1090.57
36 Monthly Payments of (Starting 1 month after contract)	£138.99
Charge for Credit	£641.35
Total Credit Price	£6094.21

Figure 1. A sample advertisement and what it entails

This immediately leads to some interesting mathematics requiring the summation of a geometic sequence and the solution of an equation of degree 37. The use of a microcomputer together with trial and error techniques makes this accessible to able pupils. Such an approach uses the motivation of exploring contexts to which pupils can relate and beginning where pupils are in their conception of what mathematics is about.

The Alternative Mathematics Project

A curriculum development project to explore this contextual approach, Alternative Mathematics, has been set up at the Centre for Innovation in Mathematics Teaching, Exeter University, funded initially by the Department of Trade and Industry. The aim is to produce material which can be used as a GCSE course for secondary years 4 and 5 and also as a resource in further education courses such as CPVE. Influenced by TVEI thinking, it is planned to organise the course on a modular basis in five blocks:

* Business and commerce
* Environment
* Design
* Science and technology
* Leisure and recreation

Each block will contain material for one term's work in a GCSE mathematics course and will be available at the three levels referred to in the National Criteria for Mathematics (see Figure 2).

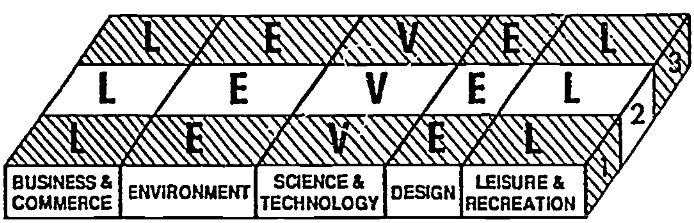


Figure 2. The structure of Alternative Mathematics

The main feature of the course is that it will develop mathematical concepts and techniques from real contexts. Practical work, applications, investigations and problem solving will arise naturally from these contexts. Thus the realisation of the GCSE objectives 3.16 and 3.17 will be an essential part of the pupil's work and not a 'Friday afternoon' activity. Fifty per cent of the overall assessment will be given to course work. Using appropriate contexts the mathematical content of the National Criteria lists will be covered. There will be a need to consolidate and practise, and material for that purpose will be provided but it should be seen as supplementary and not as the main component. The course will aim to show that mathematics is about something and that it relates to other school subjects. Collaborative teaching will be encouraged, thus making the material suitable for use in the curriculum structures being developed through TVEI and CPVE.

Form of the Materials

Recent improvements in reprographic technology have made it easier for teachers to produce their own material and many teachers are now looking for a more flexible resource than a single textbook. In particular, desk-top publishing opens up possibilities for the production of material by a curriculum development project.

The trial version of Alternative Mathematics is being produced as photocopyable master sheets. Some topics have been written as booklets involving a progression of ideas with each page or double page being complete in itself. Thus teachers who wish can put the pages together to make a structured booklet. Other teachers might prefer to use the pages singly. Either way there is the flexibility to insert other material or to replace pages. Some topics lend themselves to an unstructured format and the material is presented as one-off sheets giving activities for groups or individuals. Thus whilst providing a course which is, as far as possible, complete, there is flexibility for teachers to personalise it.

A further advantage of providing photocopyable master sheets is that up-dating is easier than with a textbook. This is particularly important where contextual material is being used.-

prices of cars, interest rates, etc. can soon become out of date. Looking further ahead, the establishment of electronic databases such as NERIS will enable even more rapid up-dating to take place.

It could be argued that in order to encourage the styles of teaching recommended in paragraph 243 of the Cockcroft report it would be better to produce material for teachers rather than for pupils - written pupil material can inhibit discussion and practical work. Although such an approach might be welcomed by adventurous, independent teachers the evidence from the past suggests that such material does not get used by most teachers. For example, the Schools Council 'Mathematics for the Majority' (1970-1974) project produced books for teachers, but their influence was slight, and a subsequent project was then set up to produce pupil material. An intermediate approach, which is being explored, is to put the discussion ideas in the pupils' material, not for the pupils to work at alone, but for use as starting points by the teacher, as is illustrated in two sample pages (Figures 3 and 4). Figure 3 is a sample page from the Level 1 module on Business and Commerce. Figure 4 is from the Level 3 module in the same subject area. The difficulty is to keep written material fairly open, without closing it down, yet to give sufficient help for teachers and pupils to work on. For example, in a draft unit on Time (for the Science and Technology block) the intention is to incorporate practical work which involves making simple devices to measure time. An open approach is to pose the problem, "You are shipwrecked on a desert island. Invent a method for measuring time." In a draft version a stimulus page has been included consisting of pictures showing various historical methods (sand clock, water clock, candles, pendulum, etc.). Should there be further pages giving constructional details of such devices? Again, the problem of determining the day of the week for any given date arises. Should it be posed and then left entirely for discussion, should a structured approach be developed or should a procedure be given? To what extent should the amount of guidance depend on the ability of the pupils? These are the sorts of questions the project team is having to answer. For example, in addition to the topic 'booklets' the material contains back-up sheets providing consolidation and practice together with 'gap-filling' to allow for the different courses pupils will have followed in previous years.

Managing Money

At Sarah's school, people from the Modland Bank come in once a week.
Sarah opens an account.
There is a special account for students.
Interest is paid at 5% per year.

E1 Sarah puts £100 into her account.
How much would she have at the end of a year?
How much would she have at the end of two years?

E2 Sarah takes out £30 at the end of the first year.
How much would she have at the end of the second year?

Mr Money has £100 in his current account.
He withdraws £40, £30, £19, £11 and keeps a record like this –

TAKEN OUT	AMOUNT LEFT
£40	£60
£30	£30
£19	£11
£11	£0
£100	£101

He adds up this column and finds it comes to £101

He claims the bank owes him £1 – Do you agree?

THINGS TO FIND OUT

* What are current accounts?
* What are deposit accounts?
* What do banks do with the money?
* Find out the names of the main High Street banks.
* Find out about their 'special offers for young savers'.

Did you know?

The word bank comes from an Italian word, banca, which means bench.
Many years ago merchants did all their business sitting on wooden benches in the market place.
When a merchant couldn't pay his debts, his bench was broken up. Our word bankrupt comes from the Italian words for bench broken.

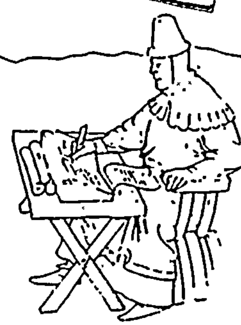


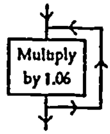
Figure 3.

Jane's money grows like this:

Amount at the beginning of this year = Amount at the beginning of last year x 1.06

There is a repetitive process here:

multiply by 1.06
multiply by 1.06
multiply by 1.06
etc.




```
10 INPUT "Amount "A
20 INPUT "Percentage "P
30 INPUT "Number of years "N
40 C=0
50 REPEAT
60   C=C+1
70   PRINT C,A
80   A=A*(1+P/100)
90 UNTIL C=N+1
```

When the interest rate is P% per year the multiplying factor is

$$1 + \frac{P}{100}$$

This multiplying factor can be seen in line 80 of the computer program.

Here is a printout for Jane's investment:

```
Amount 200
Percentage 6
Number of years 3
      1        200
      2        212
      3        224.72
      4        238.2032
```

Beginning of Year 4, i.e. 3 years of growth.

A5 Try the program with the examples on page 2.

A6 One hundred years ago, Jane's great-great-great grandfather invested £1 at 5%. What is it worth now? More realistically, try it with a variable investment rate, for example, 3% for the first 20 years, 4% for the next 20 years, etc.

Follow-up activities:

*Extend the program by adding some more words. Improve the lay-out in line 70 - try replacing A by INT(100*A + 0.5)/100*

What does this do?

Use the program to find how long it takes to double your money at (i) 6%, (ii) 7%, (iii) 8%. Try some other percentages.

Can you find a rough rule for the doubling time, given the percentage? What about tripling times?

Follow up 'Things to find out' on page 2 by writing a report showing how £100, say, would grow when invested for 10 years in various schemes.

Recommend a 'best buy' for an 8-year old with £100 to invest.

Figure 4.

Teachers will be encouraged to use a variety of different styles by the range of supporting materials: It is planned to produce a newspaper containing articles of current interest, advertisements, data (finance, sport, etc.), puzzles, etc., presented in a lively style. It can be dipped into when appropriate by the teacher and used for stimulus activities. An advantage of a newspaper is that it can be up-dated cheaply without becoming fossilized like a textbook.

The use of microcomputers will be encouraged by provision of software consisting of simulations, investigations, for use by the teacher from 'out front' and by individual children, data files, and a version of some of the back-up material. Audio-tapes, containing conversations in context, radio-style programmes, simulated discussions, etc., will be provided to exploit an approach used in other school subjects but not often in mathematics. Video has great potential for showing contexts outside the classroom. Material relevant to mathematics is already available through television and film libraries. Further videos will be produced to accompany the course. Interactive Video brings together the facilities of video discs and micro-computers, thus giving instant access to pictures and allowing decision taking with feedback. A disc on the theme of running a School Disco has been produced which will be a valuable resource in the Busiess and Commerce block.

Alternative Mathematics aims, therefore, to tackle the two issues of content and style identified in the reports referred to in the introduction. It is hoped that by providing motivation the course will enable students to enjoy their mathematics at school, reaching their mathematical potential and gaining confidence in their ability to use and apply mathematics in a variety of situations.

David Hobbs

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CLASSROOM PROCESSES AND MATHEMATICAL DISCUSSIONS: A CAUTIONARY NOTE

Introduction

Classroom discussion features prominently in the modern rhetoric of mathematics education. As part of a programme to take children beyond competence in basic skills and towards confidence and success in applications work, discussion, both between teacher and children and amongst children independently of their teachers, is considered to play a valuable part. Through discussion, it seems children should learn to articulate their points of view, to listen to others, to learn to ask appropriate questions, to learn how to recognise and respond to mathematically relevant challenges and in these ways to develop their mathematical conceptions and their applications.

Surprisingly, given the prominence, the empirical basis and the theoretical rationale on which this view is based are rarely spelled out. The value of discussion, it seems is taken to be self evident. This is possibly part of the modern vogue in which children are seen as 'social constructionists' building their understanding of the world by reflecting on their social experiences which take a predominantly linguistic form. Or perhaps it is just plain common sense. Whatever its provenance, the merits of discussion in mathematics are widely extolled. Cockcroft (1982) suggested conversations to be a key element in the mathematics curriculum. Some enthusiasts go further and consider that such exchanges should be at the heart of children's classroom mathematics experience (Easley and Easley, 1983).

The Easley's rest their enthusiasm in part on a constructivist philosophy of cognitive development and in part on their interpretation of four months' observations of the technique in practice in a Japanese elementary school. The Easleys report that the teachers at the Kitameo School seemed to place their highest priority on teaching children how to study mathematics largely through group work. Their normal technique was to ask the children to work individually on a challenging problem and then have them discuss their answers and reasons with other members of a mixed ability group to try to achieve a consensus of opinion.

The quality of these exchanges is illustrated by the Easleys in the following example in which the children were discussing the illustration in Figure 1.

Figure 1 Elementary Addition Task

Child 1: You can't add three and zero. There is no answer.

Teacher: Do you mean the answer is zero?

Child 1: Yes.

Teacher: Who agrees?

Child 2: You can't add three and zero so your answer is three.

Teacher: You can't add or there is nothing to add? Which is it?

Child 2: There is nothing to add.

Child 3: There are three here, even if there are none here, there are three.

What is impressive about this exchange is that it is unimpressive. Given the Easleys' enthusiasm and their desire to proselytise, this excerpt from their observations might be taken to be exemplary. Yet it contains no pupil-pupil exchanges. Nor is it at all clear what the children are making of the various contributions. The Easleys' example makes poor advertising copy.

With material like this it is perhaps no surprise that the use of discussions in mathematics has yet to make much of an appearance in primary schools. H.M.I. persistently complain about a preponderance of pencil ' paper work in maths. Their observations have been supported by independent studies. Bennett et al. (1984) recorded no instances of discussion in more than 300 mathematics tasks observed in over two terms in 32 primary classes. Recent research by me and Anne Cockburn confirm that mathematical discussions are at a premium (Desforges and Cockburn, 1987). The same research, however, suggests that this has less to do with the advertising than with the product: discussion causes a large number of problems for teachers and is, apparently, not well adapted to classroom conditions. In the rest of this paper I illustrate and expand on these problems and consider their implications for the development of primary mathematics teaching.

The Research

A great deal of work has been done on understanding children's mathematical thinking, on identifying and sequencing educational objectives and on designing attractive teaching materials. The work has generated plenty of advice to teachers. Most of this advice does not seem to be practised in most classrooms.

The research reported here was part of an attempt to understand teachers' classroom behaviour (Desforges and Cockburn, 1987). It was based on the view that since teachers in our experience seem to know, understand and accept the precepts of good mathematics teaching and yet do not practise them, and that since most teachers are industrious and have the best interests of children at heart, there were probably some good reasons why traditional practices are resistant to exhortation. We argued that since familiar forms of curriculum development have had li tle impact on mathematics teaching it was necessary, before further attempts at change were made, to understand the forces that constrain teachers' actions.

To this end we recruited a sample of seven primary school teachers who had good reputations. Preliminary interviews established that the teachers held elaborate views of children's learning and recognised the virtues of the broad range of teaching methods described in the Cockcroft report. They subscribed to the view that teaching methods should make the most of young children's well developed thinking capacities.

The teachers were each observed teaching mathematics over a period of three weeks. A sample of their teaching was video-recorded. The recordings were used to stimulate the teachers' recall of their thinking during teaching. The teachers were invited to stop the video whenever they felt a significant event had occurred or a decision had been made. The discussions about the tapes were held on the same day as the lesson was recorded. They were tape recorded and subsequently analysed to identify the teachers' experience of classroom interactions. The following comments apply only to the teachers' accounts of their thinking during discussion. These sessions accounted for less than 10% of the sessions videod.

Classroom Processes

The teachers reported a large number of tensions experienced whilst conducting discussions. Some of these are illustrated in the following example. Here the teacher set out to get a group of six year olds to discuss different ways of making ten. The extract below is a record of one minute of her session.

Mrs D: Make a stick of ten cubes that are all the same colour. (Children do so) Right. Check you've got ten. Now. Show me with your cubes a number story that adds up to ten.

(The children break up their sticks and mutters of 'one add nine', 'two add eight' can be heard. Ross breaks his into two fives, looks round, rebuilds his ten and breaks it into one and nine.)

Mrs D: No Ross. You were right. I just said 'show me a number story'.

(Ross reproduces two sticks of five.)

Mrs D: Now. What have you got, Ross?

Ross: Five add five.

Mrs D: What have you got Hazel?

Hazel: One add nine.

Mrs D: What have you got Kai?

Kai: Three add six equals ten. (Kai is holding up a stick of six and a stick of four.)

Mrs D: Nooo...have another look.

Kai: Four add six.

Robert: Two add seven.

Mrs D: (Waving away Freddie from another group.) You count again. What do you think they are Darren?

Darren: Three add eight.

Before examining the teacher's experience of this brief exchange it is worth noting that she had thirty five children in a very small room. She was working with one of three groups. Her comments on the action were as follows:

> 'I saw Ross do something unusual in making five add five. As soon as someone said 'one add nine' he (Ross) started to put his back. I was also trying to see what everyone else had done and who was looking at whose. Louise, Hazel and Lindsey are never too sure. I saw Hazel had it right so I called on her to give her a bit of confidence. I saw Kai had it right. But I was not surprised when he said it wrong. Robert was the same. He and Kai both knew the larger number but had probably guessed the smaller number instead of counting it. The work is totally inappropriate for Darren. He should not be in this group but I have to keep an eye on him. I waved Freddie away without a second thought - he knows the rules.

In this one brief instant of teaching Mrs D. found herself sustaining the general pace of the action, monitoring individual responses, choosing one child to boost confidence, another to check understanding and yet another to attract attention. All the while she found it necessary to interpret their particular responses in the light of her knowledge of their typical behaviour. The wrong answer of one child was judged to be worth checking immediately whilst that of another was judged best left for later discussion.

In situations similar to those illustrated above the teachers perceived the demand for decisions to be incessant and exhausting. They admitted that on occasions they went into 'automatic pilot' and held periods of routine questioning in order to, 'have a bit of a rest' or, 'to give my brain a breather'. The challenge was not only in the sheer amount of information they had to process. It lay also in the frequency of difficult decisions which they felt had to be made. Some of these are illustrated in the following excerpts in which Mrs G. set out to explore and discuss three dimensional shapes. She had previously had a lot of examples of these in her mathematics area. The children had played with and talked about them. As a preliminary to further exploration, Mrs G. decided to check the children's grasp of the notion of sphere. A globe had been passed round, its spherical properties emphasised and its technical name was mentioned several times.

> Mrs G: Sphere. That's right. Now. Can anyone else tell me anything that is a sphere? Adam? (who has his hand up).
>
> Adam: Square.
>
> At this point Mrs G. recalled, 'I was ever so surprised he said that. I nearly fell off the chair. He is so keen and so shy. I thought, 'how on earth am I going to cope with this without putting him down?'

She decided to ignore his answer and to get Adam to feel the globe and describe it as a sphere. She then turned her attention to another child. Seconds later, in giving a further example of sphere, Ben offered 'half the world'. Mrs G. later noted:

> I thought 'oh dear!' I cannot get into all that now. If he he said it later when we had got the main idea established I could have developed it using plasticene. As it was,

lots of the children were just beginning to get the idea and I thought it would be too confusing. I suspected Ben would have benefited from some extension but I decided to pass over his suggestion for the sake of the rest of them.'

There followed a period of acceptable responses for sphere shaped objects and then Samantha suggested 'a circle'. Mrs G. thought:

'Help!' This is all getting to be more difficult than I had anticipated. I was not prepared to have the group sit and wait whilst we cut a circle out to make the concrete contrast. They would have lost interest.

In this brief space of time, Mrs G. had perceived that she had had to resolve a number of difficult problems and had done so by making rather messy compromises. She had seen conflicts between a concern for individual and group needs, between intellectual challenge and children's personalities and between the management of materials and of time. Mrs G's experience was typical of the teachers' reactions to the few exploratory sessions observed. Once the teacher engaged in relatively free discussion, the pupils' responses became unpredictable. Levels of uncertainty in the teachers' decisions rose dramatically. Their reflections revealed a fine awareness of their children - not only as mathematicians but as humans with sensitivities as well as academic strengths and weaknesses. Also revealed was an awareness of the social context and the need to maintain the pace of interactions to sustain group interest. What the teachers reflections did not reveal is an awareness of the increasing mismatch between their thinking and their behaviour. For the less routine and familiar was the children's behaviour the more fixated became the teachers' actions.

The teachers frequently rushed to interpret children's responses and to correct errors. They did not pause to check or test their own interpretations let alone the child's responses. Under the press of events, the teachers tended to close options down, become repetitive, allow repetitive responses from the children and generally reduce the proceedings to a routine.

In these respects the teachers were behaving like most humans under potential information overload. Information processing techniques are brought to bear to make as much information processing as possible into a routine so that attention can be focussed on some central issu- This process of routinisation may be understood by reference to learning to drive a car. Initially every manipulation requires total attention. Gradually one learns to stear and change gear at the same time. Ultimately one manages to drive through traffic whilst maintaining conversations. This is accomplished by making as much as possible of the performance into a routine. Unfortunately making matters routine is the complete antithesis of a searching discussion. The implication is that erudite mathematical discussions may be beyond the limits of human information processing in the press of classroom events.

Teachers' Implicit Theories

The press of events however might not be the sole reason for the teachers' responses under these circumstances. From the range of events happening at any moment the teachers make their selections. We may presume that their choices are based on pre-occupations that the teachers bring with them to the situation. In this research the teachers did not spell out their reasons for selecting what they focussed their attention on. We are therefore left to speculate as to what ideas constrain the teachers' selective attention.

One powerful idea seemed to be that in their teachers' eyes, children are extremely vulnerable both intellectually and socially. Teachers frequently mentioned the need to sustain children's confidence. Of course as a general principle this is unimpeachable. But at the level of practice, at the first sign of a hesitant glance, the teacher- tended to jump in and help. The price of confidence can be dependence. It is a price often paid in the maelstrom of classroom exchanges.

The teachers also evidently had the idea that children can be very easily misled into false notions. The teachers recognised the fertility of ideas proferred by their children but passed them over for the sake of some of the members in the group who were judged to be not ready to deal with them.

The third major theme which appeared to focus the teachers' attention was the need to get on with the curriculum in recognition of the scarcity of time. The teachers observed that they had a commitment to their colleagues to cover the school mathematics programme which generally took the form of a commercial scheme. In this context, discussions always seemed to be pushed along at a brisk pace by the teacher.

Of course it is easy to point out that each of these considerations completely defeats the object of sharing notions through discussions. A discussion necessarily involves the challenge of ideas and could hardly be sustained towards this end if the chairperson feels that the preservation of confidence carries a higher priority. Equally, discussions present opportunities for sharing and testing ideas amongst participants with a broad range of experience. There would be little point in discussing ideas only with those who were 'ready'. And whilst teachers have only a finite amount of time at their disposal, they do have considerable autonomy in the manner of its use. Discussions with a guillotine hanging over the chairperson's head are unlikely to be searching.

Whilst these arguments may have considerable logical merit in showing that the ideas which appeared to preoccupy the teachers sounded the death knell for discussion in principle (and, as it turned out, in practice) they are likely to have as much impact in a classroom as readings from Clauswitz would have on a soldier in no-man's land. For the fact is that the teacher's selection mechanisms are all highly adaptive to classroom life. Dispirited or confused children spell management disaster. An uncovered syllabus spells official and parental disapproval. Whatever the logic of the teachers' preoccupations as they go through the motions of conducting mathematical discussions, the reality is they must manage and be seen to manage their classrooms and to cover the syllabus. Their preoccupations are highly adaptive to meeting these demands under the circumstances in which they work.

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Summary and Conclusions

Discussions are generally considered to be an important part of a child's mathematical experience and particularly so in respect of the development of skills associated with reflection and application. Despite the exhortation of decades, fruitful discussions are rarely seen in primary classrooms. This is so even in the classrooms of teachers who recognise and endorse their value. When discussions are seen they are mainly teacher dominated, brief and quickly routinised.

It has been suggested that the sheer information load a teacher must process in order to manage her class and conduct a discussion makes the successful appearance of such work an unlikely prospect. Whilst there have been reported sightings of this rare event, its existence remains to be confirmed.

What are the implications of this argument? Certainly there are few implications for the teacher. It would be futile to write out advice to the effect that teachers should try harder. Nor will teachers be convinced by demonstrations of discussions given on a one-off basis by a lavishly prepared and equipped virtuoso and involving half a dozen children. Even when these extravaganzas are impressive as performances they say little about what the children learned.

The implications of this paper arise in the main for those who dictate how classrooms are resourced both intellectually and materially. If discussions are to be standard practice then quite clearly the time will have to be made to conduct them in an undistracted fashion. This entails reducing drastically the breadth of the curriculum that teachers feel obliged to rush over. Secondly it entails having more adults in classrooms to ease the teachers' management concerns. These are the very minimum requirements.

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<u>INVESTIGATIONS: WHERE TO NOW?</u>

<u>or PROBLEM-POSING AND THE NATURE OF MATHEMATICS</u>

Recently, I observed a lesson in which an experienced mathematics teacher was asked, by his Head of Department, to 'do an investigation'. He chose one from a mathematical journal, and prepared it carefully beforehand, including writing out the completed solution on a piece of paper, which he held in his hand during the lesson. At the start, he introduced the problem to the pupils, who clearly had never worked in this way before, and set them the task. He then proceeded to go around the class, offering advice such as "no, no that way, it won't lead anywhere, try this", and "that's right, keep on that way and you'll get the right answer" or "don't give up, look here's the answer, on my paper". It wasn't long before a pupil at the back put up her hand, and as if speaking for the whole class, asked the teacher to "tell us how to do it now please sir". The teacher managed to resist the request, and the pupils worked on. However, at the end of the lesson, the teacher commented to me that there didn't seem to be much in the "business of investigations that was any different to normal mathematics lessons".

Clearly, there had been no opportunity for the teacher to discuss, or examine, what an investigation might be, how it might differ from 'normal' mathematics, how to conduct such a lesson, or indeed any other aspects of problem-solving and investigations. Thus, inevitably, the investigative work of the class became just like any other lesson in mathematics, except that in this case the teacher seemed to the pupils to be behaving somewhat perversely, refusing to tell them how to do the problem before it was set or even during the lesson. It may also indicate, though this was just one school, how little contact there is between the work going on in most schools and the research, or new developments, taking place in the mathematical associations, in departments of education or amongst some teachers.

Our knowledge and experience of problem-solving and investigations has grown considerably in the last few years. Recently, some articles have appeared that attempt to look at the whole nature of this area of teaching mathematics, and what part it plays in mathematics education, and it is [illegible] that experiences like the above reveal the necessity of such fundamental considerations.

In this article, I intend to dig deeply into these issues, and to attempt to propose a theoretical framework that may help to structure our thinking and research in this area. I will examine first how problem-solving might relate to different conceptions of the nature of mathematics, and the issue of 'process v content', and then support and develop the notion of e.g. Stephen Brown (1984), and others, of a shift of discussion from 'problem-solving' to 'problem posing'. Finally, I will suggest an alternative direction for development in the teaching of mathematics that differs radically from the direction in which we seem to be heading at the moment.

'Process v Content' or 'the Chicken and the Egg'

The idea that in mathematics education we should be concerned with enabling pupils to acquire the skills and techniques that we identify as doing mathematics, or thinking mathematically, has been aired and discussed for some time now. We always insist, though, that we must have content as well as a focus on processes. Since 'content' has traditionally been the only conscious focus of school mathematics, we seem to resort to specifying what we are going to teach, and leave the teaching of processes, or the development of problem-solving skills in pupils, in general vague terms, and as general guiding principles. The National Criteria (DES, 1985) exhibits this tendency. It is after all what we are good at, discussing whether calculus should be in the 'O' level syllabus, or linear programming, or geometry proofs.

Now, there is no doubt that concentrating attention on processes is very problematic. For instance, we do not have any clear idea of how to teach 'generalising', or 'reflecting', for example, nor do we know whether these processes are carried over from one problem to the next. And then, how does one assess understanding of 'conjecturing'? We seem to believe that we know how to assess understanding of content, and that is usually by a suitable examination question.

In fact, the major obstacle to be overcome to enable a aerious consideration of a process-orientated mathematics curriculum, I suggeat, may be our own reluctance, as teachers of mathematics, whose own mathematical education was so totally concerned with content, to think in terms of the processes of doing mathematics. As a personal illustration of this, I remember quite vividly my reaction, several yeara ago, to being offered the post of reaearch mathematician in a team of scientiata, constructing a mathematical model of the pollution of an important lake. I could not recall having studied 'Mathematical Modela of Lakea' at university, and in a atate of mild panic I turned the job down. I could not see myself as being able to do mathematics in any creative way, merely as being fairly able at remembering and reproducing what I had been taught.'

My atarting point is that there are many problems with 'content' as a focus, problems that challenge the teaching of mathematica in such significant ways, that a fundamental reconaideration is necessary. These problems include the following.

As teachera of mathematics, we know only too well that just a very small minority of pupila will be able to go on from such a school mathematics syllabus to be able to use the skills, or knowledge acquired, in any aituation other than school mathematics, i.e. not in other school subjects, in work situations or other adult-life needs. If there ia any minimum competency demanded of school mathematics, it is aurely the adaptability of knowledge and skills to changing problems, situations and needs in adult life and in employment (where there is any - an issue I will return to later). This is not a content issue. It is of course far from minimal in terms of the demands on mathematics education to 'provide' auch competency!

A aecond problem with 'content' is that if we are certain that the mathematical knowledge we have is true, absolutely, there is aome justification in the idea of a content-focussed mathematics curriculum. The alternative epiatemological position, that mathematical knowledge is fallible, has aignificant implications for mathematics education. First let us briefly examine the issue of absoluteness and fallibility in the nature of mathematical knowledge. (For a more complete development of this, and the consequences for the teaching of mathematics, see Lerman 1986).

Since the 1830s we have been aware that Euclidean Geometry is simply a geometry, one of the possible ways of describing the physical structure of the universe, and that the decision of which geometry to use is thus not known **a priori**. The independence of the Continuum Hypothesis has done the same for arithmetic. Russell demonstrated the possible consequences of 'impredicative definitions' in mathematics, but the mathematics we use is full of them. There might be any number of barbers who both shave themselves, and do not shave themselves! Lakatos has shown that the image we have of mathematics as proceeding by infallible deductions, from intuitively obvious assumptions, to certain conclusions, is not correct. Mathematics, instead, develops by the re-transmission of falsity. Even the notion of truth is, it may be maintained, a relative one in mathematics, to be defined at the start of any discussion involving its use. (For a development of this thesis, see Grabiner 1974.) We cannot expect any correspondence between the predictions of a mathematical theory and the objective facts of the outside world to confirm the truth of the theory, or determine which of rival theories is the correct one. What constitutes such a critical experiment, how the experiment is set up, and how the event is interpreted are all theory-laden decisions. The <u>certainty</u> we have, is limited to the tautological, that is for example, if $3x + 7 = 2x + 11$ then $x = 4$ and even this, in theory at least, may lead to a contradiction. All this, I suggest, ought to lead us, as teachers of mathematics, to question the way we teach. If we, the possessors of this esoteric and certain knowledge, whose job it is to convey, or reveal, this certainty to pupils, are deluding ourselves, and that what characterises mathematical knowledge is not its absoluteness, but the particular ways of dealing with certain aspects of the world and society around us, then our status in relation to knowledge, and hence to pupils in the school situation, must change.

A third problem is what constitutes indispensible, irreducible, absolutely-basic content? We cannot get beyond the Four Rules without confronting controversial issues such as the teaching of long multiplication, long division, directed numbers, number bases etc., let alone the rest! We use all kinds of rationales for such decisions, such as statistically determined levels of competency for particular ages and topics (Hart et al., 1981). As soon as one begins to question what must be included in a school mathematics syllabus, one falls back on what content will provide the skills that we all broadly identify as necessary in adult life, working or otherwise. As suggested above, whether content leads to skills for the great majority is doubtful.

Awarene of the central significance of context and meaning in mathematics education, for instance as described recently by Cobb (1986), throws into question most of our mathematics teaching. We all know of instances of children with considerable skills outside of the classroom that we would consider mathematical, who are unable to exhibit those abilities, and that confidence, inside the classroom. The 'constructivist' view of children's learning, focusses on what the child perceives in any situation, including what we as teachers present, and suggests that this necessarily has to be our starting point for understanding, or interpreting what the child knows. It is inadequate to persevere with the idea that what matters is how we present the mathematical knowledge, how we explain it, and that doing this better will directly facilitate the child's learning. It seems to me that we have no hope of engaging in these issues as long as we hold on to 'content' as our focus.

If, then, focussing primarily on 'content' is hard to justify, inadequate and unsuccessful, we should free ourselves from the constraints of our pre-conceptions, and examine the alternative. In the next section I will look at what might be the consequences of reversing the order, and thinking of 'process' as coming before 'content'.

Mathematics through Problem-Posing

The assumptions, then, for this part of the discussion, are as follows. Since the context and the meaning, and hence the engagement of the child, is an individual response, and not necessarily consequent upon the stimulus of the teacher, then offering the child an open situation, in which the child is encouraged to pose questions for her/himself, is the only way of enabling the child to advance conceptually. In a sense, the child is doing this in any case, when s/he is learning, that is providing the meaning and context that is thus 'meaningful' to that child. The problem is that most of what we do in the mathematics classroom is 'meaningless' to most children, and therefore does not get learnt.

Cobb (1986) describes the situation as follows:

> "Self-generated mathematics is essentially individualistic. It is constructed either by a single child or a small group of children as they attempt to achieve particular goals. It is, in a sense, anarchistic mathematics. In contrast, academic mathematics embodies solutions to problems that arose in the history of the culture. Consequently, the young child has to learn to play the academic mathematics game when he or she is introduced to standard formalisms, typically in first grade. Unless the child intuitively realizes that standard formalisms are an agreed-upon means of expressing and communicating mathematical thought they can only be construed as arbitrary dictates of an authority. Academic mathematics is then totalitarian mathematics."

Mathematical knowledge de-reified, seen as a social intervention, its truths, notions of proof etc., relative to time and place, has to be seen as integrally involved with the doing of mathematics, and indeed cannot be separated from it. Mathematics is identified by the particular ways of thinking, conjecturing, searching for informal and formal contradictions, etc., not by the specific 'content'.

I will give here an example of a problem that may best be termed a 'situation', in that asking the question is left up to the student. This example should serve as an illustration for the discussion that follows.

In a recent seminar with a group of post-graduate students, who were non-mathematicians, I presented the following investigation, taken from SMILE Investigations (1981):

"Consider triangles with integer sides. There are 3 triangles with perimeter 12 units. Investigate." (See Fig.1)

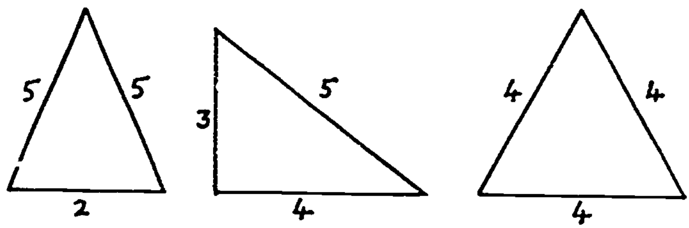

Figure 1 Diagram Accompanying Investigation (from SMILE, 1981)

A number of groups were disconcerted, saying that they had no idea what to do, since there was no question being asked. Other groups worked on areas of the triangles, perimeters of the triangles, perimeters of rectangles etc. All the students found this to be quite different from the other investigations they had tried before, and very challenging. Brown (1984) describes a number of other examples like this one, where students de-pose or re-frame the question as stated, and generate their own problems.

It can be suggested that there are more radical consequences in changing from a 'content' to a 'process' focus. There are political implications of the notion of problem-posing, as is suggested by Cobb's use of the terms 'anarchistic' and 'totalitarian'. Freire (1972), in writing about his literacy work with the oppressed people of Brazil and elsewhere in South America, describes two rival conceptions of education. The traditional view of education is the 'banking concept', whereby pupils are seen as initially empty depositories, and the role of the teacher is to make the deposits. Thus the actions available to pupils are storing, filing, retrieving etc. In this way, though, pupils are cut off from creativity, transformation, action and hence knowledge. The alternative view of education, Freire describes as the 'problem-posing' concept. By this view, knowledge is seen as coming about through the interaction of the individual with the world. 'Problem-posing' education responds to the essential features of the conscious person, intentionality and meta-cognition. Freire's discussion of opposing concepts of education is integrally tied with oppression and freedom.

We are working in education, at a time when our students may be faced with a lifetime of unemployment, and uncertainty; with threats to the ecology of the planet; an increase in disparity of wealth between rich and poor in society and between nations, and even the threat of total annihilation. Traditionally, these have not been issues that have been thought of as having any relationship to mathematics. We have always rested safe in the knowledge that mathematics is value-free, non-political, objective and infallible. Freire's analysis, together with the doubts about such conceptions of the nature of mathematics and children's learning described above, suggest that this is not the case. Quite the contrary, we have a particular responsibility, since so many political, moral and other issues are decided using 'mathematical' techniques, to enable pupils to examine situations, make conjectures, pose problems, make deductions, draw conclusions, reflect on results etc. These situations can be as in the investigation above, or the Fibonacci sequence (see Brown, 1984), information about expenditure on arms by the United States and the USSR, expenditure on education for different racial groups in South Africa, etc. There is plenty of 'content'. The latter two situations would call on statistical and graphical techniques, for example. The difference is that engagement in the problems posed by the pupils puts mathematical knowledge in a different, and in my view appropriate position. It is seen as a library of accumulated experience, and just as any library is useless to someone who cannot read, so too this library is useless unless people have access to it. When a problem is generated which reveals the need for some of this knowledge, be it multiplying decimals, standard index form, complex numbers, or catastrophe theory, if the individual recognises firstly that such help is needed, and secondly that it is available, the context, relevance and meaning of mathematical knowledge is established.

Thus, it is being proposed here, enabling students to examine situations and to pose problems for themselves, reflects the fallibilist or relativist view of mathematical knowledge, reflects the constructivist perspective of children's learning, and places a powerful tool in the hands of people to examine what is happening to their lives, and provide them with the possibility of changing it.

Conclusion

Course work, extended pieces of work and investigations are becoming a compulsory part of the teaching of mathematics, through their inclusion in GCSE mathematics. Far from these elements fundamentally changing the teaching of mathematics, we are in danger of losing what open-endedness exists in the mathematics classroom to examination-led criteria for assessment of children's work. Far from focussing on situations that relate to the context and meaning that children bring into the classroom, we are in danger of extending and reinforcing our judgements of childrens' 'abilities' in mathematics, based on school-mathematics. (The use of quotation marks around the word 'abilities' is intended to devalue the notion. Given our growing awareness of some of the issues discussed here, we have to be highly suspicious of our usual use of the term, I suggest.)

Perhaps the National Criteria should consist of processes of mathematical thinking, with some illustrations of the types of content areas that may be called upon by particular examples of situations that can be presented to pupils. Perhaps we should forget assessment for some years, until we have developed new ways of working in the classroom, and give every child a 'pass' in the meantime. Perhaps we should move towards a totally school-based assessment, as with English. Perhaps the criterion of success in a mathematics course should be the ability to take a newspaper, be it the 'Sun' or the 'Times', or a piece of government legislation, and reveal some of the underlying assumptions and methods of deduction that have been used to reach conclusions and determine policies and attitudes that at the moment dominate our lives, and against which we feel ourselves to be powerless.

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CLASSROOM BASED ASSESSMENT

Uppermost in many mathematics teachers' minds at present is their worry about course work assessment. This paper aims to take an overview of current practice related to assessment and then consider ways of integrating the demands of schemes such as the GCSE into classroom teaching.

One of the concomitants of anxiety is often an inability to separate out important features from irrelevancies and to see clearly to the heart of the problem. An outside perspective can often help to clarify a situation which seems intractable from within. It is therefore necessary to look closely at assessment in general before focussing on teacher based assessment. It is pertinent at this point to consider more precisely the language to be used. 'Marking' and 'assessing' are not synonymous. They are, however, used interchangeably by many people, making it difficult to discriminate between teachers' actions and intentions. It is particularly important with the advent of attempts to assess practical and oral work that the distinction be made clear. It may sound tautologous to state that marking requires a mark to be written on a piece of work. This is usually a number, letter, tick or cross, and can be interpreted in many ways. What is crucial is that although the mark may well be the result of an assessment of the worth of the work, it may equally well be nothing more than a recognition of the work's existence, its length, or, in mathematics particularly, whether it is right or wrong. Assessment, on the other hand, implies that a considered judgement has been made on some aspect of the work. This may be communicated by a mark, but discussion or alteration to a teaching program are also possible outcomes.

Why Assess Work?

It is worth pausing a while with this question since teachers frequently fall into a pattern of marking pupils' work simply because it is the thing to do. This is not a reason to be ignored. Inexperienced teachers can feel that unless they are seen to be doing a certain amount of marking their senior colleagues may judge them to be lazy or, worse, uncaring about their pupils' progress. However, unless the teacher has some other explicit reason for assessing pupil-work the effort will be little more than a time-wasting charade. Time, it is important to remember, is a commodity which teachers do not have in abundance. Basically reasons for assessing can fall into three categories: for the benefit of the pupil, for the benefit of the teacher, to satisfy the perceived needs of others. The requirements of the third category are rather different from those of the first two and will therefore be considered first.

'Everything at school is assessed', 'the system expects it', are possibly accurate but nonetheless disturbing responses. They posit a view of schooling as 'doing, remembering and testing', rather than 'developing pupils' understanding and mental growth'. It is certainly true that many parents see a marked piece of work as evidence that the teacher is doing his job properly: concerning himself with the work of their own particular children. The converse impression, that the lack of regularly marked work implies an indifference to the progress of the pupils, may also be held by parents in the absence of information to the contrary. Assessment procedures should of course be appropriately selected by teachers, not dictated by parents' needs, but parents have inalienable rights to be interested in the education of their sons and daughters, and teachers should take it upon themselves to ensure that information about their forms of assessment is available. In its absence parents have no option but to fall back on their memories of the meanings of marks when they were at school and these may be highly irrelevant today. Other assessment demands are imposed by the desirability of grouping pupils according to their ability whether to achieve streamed or true mixed-ability classes, by the need to inform other teachers about particular pupils' progress and, more dubiously, by the schools' wish to keep records on pupils. From time to time it may be necessary to concentrate on assessment for these purposes, but the majority of the time the over-riding factors should be the value to the pupil and the teacher.

Assessing and marking children's work is one way of reporting to them how they achieved at a particular task. Many pupils expect the school system to provide them with feedback in such a form although written comments or a personal discussion with the teacher might be a much more appropriate and effective method. Marks can be used for motivation - 'I did better than last time' or 'Rani had a higher mark then me' - although such competitiveness may have doubtful validity. ᵥils are tempted to focus on the mark rather than gain understanding from reviewing their work. Marking can also offer encouragement - 'I got it right' - or prèssure - 'I'd better do it 'cos I have to hand it in'. Despite pupil expectations, one of the major reasons for the day to day assessment of work in an educational setting must be to diagnose each child's weaknesses and strengths and then to devise some apposite action. This affects both teacher and learner, as a means of communication has to be found which apprises the learner of her difficulties and offers ways of working at the problem. If this is an aim, marking is rarely of va' ɔ.

It is possible, even desirable, that from time to time a teacher's reasons for assessing work take on a more egoistical hue. To find out what the class can do, to find out what it cannot do, to evaluate a piece of teaching are all legitimate reasons for assessment for which marking has no relevance whatsoever and considered teacher action is the appropriate response. In order to use time wisely and economically it is essential to address the question of 'why assess?' with seriousness and honesty. The later part of this paper looks at realistic ways to react when 'why' has been established.

What to Assess?

Even for the traditional piece of written mathematical homework this should not be a trivial question. Accuracy, results, method of working, evidence of thinking, mathematical ideas assimilated, transfer of understanding, demonstration of skills and techniques, personal effort, all these are responses which should trigger different approaches and outcomes of assessment. Coursework of an extended, practical or investigational nature poses additional criteria to be considered. Much of this type of work involves pupils in activity, discussion and rough working. Following this they are usually asked to produce a written account of some form. The value of this latter activity is discussed later.

Should observed activity be assessed? What about overheard or teacher-led discussion? Even when considering the written work there are different areas to consider. For exploratory and problem solving approaches to the learning of mathematics to be effective for pupils it is critical that the image of mathematics as 'a neat piece of written algebra which is either right or wrong' be eroded both in the minds of the pupils and of their teachers. The real search for mathematical truths involves conjectures and predictions, checking and refining, errors and reappraisal and much rough working. To encourage this view of mathematics a distinction needs to be made, overtly, between recording and writing-up - both valuable activities but with different purposes. Recording is personal. It is to record passing thoughts, hunches and ideas. It is to try out calculations, illuminating diagrams and possible lines of attack. It is for making mistakes and building on the insight thus obtained. It is not about neat writing, erasing unfruitful attempts, ruled margins, right answers and external coherence. Writing-up, however, is for communication to others. It may not be necessary to explain the false starts, unprofitable avenues and inaccuracies that occured during the course of the work. The pure mathematician may present a few lines of elegant proof as the result of years of search and struggle. It may, on the other hand, be of value to expose these events to the reader so that others may be made aware of fruitful and frustrating paths of exploration, and may themselves build on the author's trains of thought. This is the writer's decision. What is necessary is a clear explanation of the problem and comprehensible account of the activity and its outcome, and this communication skill probably needs to be explicitly taught. What then should be assessed? Clearly one may not assess the recording, although the recordings might be consulted to throw light on what mathematical thinking was taking place. It must be born in mind that only positive assessment can be made thus; absence of written evidence of thinking is not evidence of lack of thought. The final (or concurrent) writing-up can be judged from two stances. Is it a clear, informative account of the mathematical activity? Alternatively, what is the level of mathematical activity as revealed by the write-up? It is this latter aspect which is most commonly the basis of assessment and if this is to continue to be so, pupils should be made aware of the processes which need to be explicitly articulated in their writing. The hardest aspect of all to assess is that which should be the most valued: has an increase in mathematical understanding taken place?

The aim in the preceding paragraphs has been to raise to a level of consciousness the decisions which ought to be considered before selecting an appropriate method of assessment; only then can an answer be given to the question, 'How to assess work?'

Formative or Summative Outcome?

Assessments can be usefully divided into two categories: formative assessment and summative assessment. The former is a judgement made with the express intention that action will result, usually in the form of communication between teacher and pupil, and the subsequent offer of relevant follow-up tasks. The latter implies a judgement passed on mathematical ability at a particular moment and assumed to be a comment on the current position of understanding reached. Two illustrations of these categories are, respectively, considering a pupil's homework with a view to planning the next lesson and marking end-of-year examinations. Newly added to this last category is the assessment by teachers of coursework for GCSE. If this is not to become the time-consuming burden which many teachers fear, it is necessary to look closely at existing assessment and marking and prune away self-imposed, unnecessary or inefficient tasks.

Formative and summative intentions require very different assessment procedures. Consider first ordinary classwork and homework; answer the questions 'why?' and 'what?'. If the responses concern markbook records, parents' evenings, a picture of the class as a whole, where pupils are at the end of a topic, then a quick, briefly recordable, summative scheme is needed. Common currency of meaning among colleagues may also be desirable. Two suggestions are offered here for serious consideration. The first is to rank order the pieces of work as they are read without heart-searching over precise, individual positions and accepting that only accuracy within, say, five places, is expected. Then number the pile from 1 to n. This achieves a quick overview of pupils' relative understanding or ability on a particular piece of work. Initially this method requires discipline and strength of purpose and conviction that precise rank ordering is not necessary, but the rewards in time saved can be considerable. It may well be undesirable to give these marks to the pupils who will probably try to attach too much importance to their exact positions, but bear in mind that feedback to pupils was not the reason for the assessment on this occasion. An alternative and possibly easier method, is, having read a piece of work to, allocate it to one of four groups; A, B, C, and D. Do not agonise over A-- and B++ because the purpose is to produce a quick, rough descriptive mark. The groups are unlikely to be of equal size, but will give a view of the spread within the class on that specific topic. If the purpose of the homework was practice of a particular technique why mark the work outside the classroom? A tick acknowledges the receipt of the work and pupils marking the work in class takes but a few minutes.

Formative assessment clearly needs more attention to the detail of individual pieces of work. Time spent on this task aims to facilitate the learning of mathematics. A single mark has little meaning and comments, whether written or discussed with a pupil, are more appropriate. If the purpose is to inform future teaching, then perhaps it is unnecessary to communicate with individual pupils. The feedback can be in the form of a general class statement.

All the foregoing suggestions have been made relative to traditional, written work. How does practical, investigative and group work, given public credibility by the Cockcroft Report (1982), affect the scene? Some teachers have been working in this way with their pupils for many years. It is however a sad, but nonetheless true, indictement of British educational society that 'if it is not seen to be assessed, it is not seen to be valued', and many teachers have, with arguably the best interests of their pupils at heart, refrained from 'wasting time' which could be more profitably spent concentrating on the examination syllabus. Enjoyment will never be an item assessed for external purposes! Course work however now is, and suddenly teachers are being asked to work in unfamiliar ways and at the same time to summatively assess their pupils' performance in these new areas of learning. What is crucial at this moment is that teachers retain their confidence in their own professional capabilities. They know their pupils' real mathematical abilities far better than any external examiner can ever do when basing a judgement, which may dramatically effect a candidate's future, on a couple of written, timed papers. Teachers must stake their professional integrity on their ability to summatively assess their own pupils fairly and accurately. If pupils, of any age, are indeed to approach mathematical learning successfully by new routes they will, in addition, need guidance through formative assessment. They will need continual feedback, encouragement and assistance.

What Does Assessing Coursework Involve?

Again the question 'what is to be assessed?' must first be asked, the answers discriminated one from another and the pupils made aware of what is expected of them. In a practical task, is the final artifact or the 'doing' to be judged? Is planning and research important or are only actual physical skills to be assessed on this particular occasion? Is the aim of the practical work to enhance mathematical learning in general, or to develop specific skills and techniques? The aim behind any project must affect the light in which it will be judged. How far should choice of suitable materials for a given task be left to the individual pupil, particularly if this radically affects the possible outcome of a task? This last question opens up the whole minefield of help which is at the root of teachers' fears as to their ability to make universally valid assessments; how much guidance may pupils be given? Careful consideration of what is being assessed will help to resolve this delemma. Consider, for example, the task to build a scale model of the classroom. If the final model is the focus of assessment, then discussion of the best scale to use is quite in order, since the judgement will be passed on the outcome of following this advice. If, however, the underlying mathematics is at issue, then tips on the best glue and cardboard to use for the construction are totally permissable since the beauty of the final model will enhance a pupil's feelings of achievement without prejudicing the assessment of the work. A third possible reason for this practical task might be to test measuring skills, in which case neither help on choice of scale nor model building hints will affect the area under consideration. In such an extended piece of work the teacher may wish to look at several different aspects of activity and a form of continual assessment is apposite; a combination of summative and formative methods should be used. In the above illustrative example, the ability to measure acurately and use appropriate tools could be judged, and then the measurements discussed with the pupil so that inaccuracies here do not affect the final model. The use of scale could then be considered for suitability and accuracy of calculation, and any errors fed back to the pupil in order that subsequent assessment of model building skills should not be jeopardised. This kind of approach is certainly in line with the common examination practice of following through and awarding marks for written work based on an earlier incorrect calculation.

Other forms of newly assessable coursework include investigations and problem solving. One of the aims behind the inclusion of this type of open working in the experience of school mathematics is to increase pupils' enjoyment and remove the artificial view many have that to answer any problem in a mathematical lesson they need consult only the necessary and sufficient information given in the question; irrelevant detail will not be offered and assumptions or predictive approximations will not be required. Formal assessment of open working must be approached with caution, particularly when it is not topic content but use of mathematical processes which is being examined. One of the advantages of investigative work is that a pupil may explore paths of personal interest, thus making mathematical activity relevant to the individual's current abilities and inclinations. The dangers for assessment are that pupils may not create for themselves the opportunity to display their full capabilities. Should pupils be expected to choose an activity which will demonstrate as many skills as possible, as in some Home Economics examinations, or is this the responsibility of the teacher? Is one of the areas of assessment to be the ability to predict a mathematically rich avenue of enquiry? It is in this field that assessment by a teacher who knows the pupil is essential. Over a span of time and breadth of experiences, professional judgement can be made which does not depend on a pupil attempting to reveal, in the exploration of one problem, all her capabilities.

How Can Coursework be Assessed?

This question can now be addressed from the two angles of day-to-day classroom working and formal, final accreditation. Unless the sole focus of attention is to be the end product, be it model, write-up or correct answer, much of the assessment must take place in the classroom with the teacher eavesdropping on the pupils talking, engaging them in discussion about what they are doing, answering a question with a question and really comprehending the reply. Pupils need encouraging to talk about mathematics before being expected to write about it. There need be no attempt to evaluate each pupil's contribution in each lesson. A more realistic strategy is to approach the class with an intention to focus on four or five individual pupils and assess their working on some pre-arranged aspect. It can indeed be valuable to inform the pupils on what you will be focussing as this is then seen as an important aspect of their mathematics. At the end of the lesson a few sentences jotted down help to produce any more formal record which might later be needed. A more structured and probably easier method of classroom assessment is to use a check list of areas to consider and a record card for each pupil which is filled in from time to time. One example of this, which has been used with success, is described in detail in Pirie (1987). It offers a way of recording which enables the teacher to be both systematic and to respond to unforseen insights when they occur.

A written account of a piece of mathematical activity is a new concept in most classrooms, yet if pupils are to form some record of their progress in open working situations it is a valuable adjunct to their learning. It will not, however, come easily to most children because it does not lie within their expectations of the mathematics lesson. Written mathematics for most pupils involves numbers and letters, but not words. Many pupils when asked to write down the explanation they had just given orally would join with Simon, an articulate, bright, motivated 11 year old, in saying "But I dont know how to write it in maths". It is wise, therefore, to sometimes have, as the aim of a particular mathematical task, the assessment of the write-up, not for the thought and understanding revealed, but for its effectiveness as a means of revealing any learning which might have taken place. This is not to say that every project must be written-up. Class discussion, peer interaction, or practical demonstration may frequently all be better ways of concluding an activity. Ideas and expressions of understanding can be lost in the struggle to provide a written account. Talking allows modification of thoughts as they are articulated and is a dynamic, personally involving process within which errors can be perceived and acted upon. Writing can distance the thinker from the thought producing a static, final record. However, nationwide credibility of qualifications is of importance in our society and the write-up is highly valued for its very permanence and assumed irrefutable evidence of ability. Since this is the present situation, pupils should be specifically and overtly taught the skills of representation and communication which will enable their work to be justly assessed.

What about G.C.S.E.?

One thing is very clear: if pupils are not to spend a large part of the fourth and fifth years producing work from which they learn nothing, the tasks for the GCSE must be both summatively and then formatively assessed. In the late 1970s, when N and F levels were being considered as an alternative scheme to 'A' level, the Association of Teachers of Mathematics produced a booklet (ATM, 1978) describing various methods of assessing sixth form investigative and extended work. Much of their thinking could be used to form a basis for formal assessment of coursework at 16+. Pirie (1987) offers three alternative ways of approaching this task, all of which have the merit of relative simplicity and do not try to force investigational work into a mould dictated by an examination mark scheme. The new examination boards have each produced schemes indicating markedly different attitues to course-work assessment, ranging from the Northern Examining Association whose material contains statements such as "Centres are responsible for deciding the nature of the coursework", "teachers are free to devise their own assignments" to the Midland Examining Group which specifies the number, length and topic area of the assignments, which must then be assessed according to a very tight numerical scheme. There are teachers who will welcome the freedom to assess coursework as they consider appropriate; there are teachers who will feel more confident working within a strictly prescribed mark scheme. These latter must be wary lest the coursework be too tightly controlled, reducing what should be creative work to the status and form close to a series of written, timed examinations. If teachers are to assess their own pupils' work then they should be offered guidance, but nothing more. How can work produced to fit a rigid scheme be called 'open'? Such schemes, too, can become over-bearingly time-consuming if marks of 1 and 2 must be accumulated to produce a final grade of, say, B.

Teachers must be trusted to make qualitative statements based on their knowledge of a pupil and her work. After all, University tutors have for years been able to say with unchallenged confidence 'that is a 1st class piece of work; that is barely worth a IIii'. Now, as possibly never before, the classroom teacher is in a position to influence national assessment methods and demonstrate a professional competence.

The final word in the debate on how coursework should be assessed lies at present with the teacher, and it is an opportunity which should not be missed. Public assessment can be used as a means of enhancing the school child's classroom experience in the field of mathematics; if teachers so choose, it need no longer be an end in itself.

Susan Pirie

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THE USE OF THE HISTORY OF MATHEMATICS IN TEACHING

Life without some knowledge of mathematics would be very difficult in the latter end of the 20th Century. In some cases mathematics is essential for continuing one's profession or to understand one's personal affairs. Many young people are put off mathematics either by bad teaching or by being exposed to too much mathematics at an early stage in their education. The opinion has been expressed by some college lecturers and teachers that the content of the new GCSE mathematics syllabus is even now too large.

Motivation and interest may be promoted in mathematics by showing the practical value of learning the subject. Experimental and discovery work can also be employed to raise the pupil's interest. The History of Mathematics can be used to provide some enrichment of the subject and present a view that mathematics is connected with the development of our culture. Simons (1923) writing in the Mathematics Teacher suggested that using history and recreations was a way of vitalising the teaching of mathematics. This opinion was supported by Hassler (1929) writing in the same journal. Hadamard (1954) in an essay says that Henri Poincaré advocated an historical approach to mathematics teaching. Barzum (1947) and Wiltshire (1930) express similar opinions to those of Simons and Hassler. The American mathematical historian Jones (1957) strongly encourages the use of the History of Mathematics as a teaching tool. From a pupil's point of view an historical approach would hopefully increase motivation by making the subject more human and by showing that the greatest mathematicians were in fact human beings with human frailties who worked very hard to achieve their results. The pupil's confidence increases with the realisation that the great did not write their answers down immediately. An historical approach to mathematics teaching can also help the pupil realise that mathematics is not a once and for all discovery, but is constantly changing. There are links between mathematics and other subjects which can easily be shown to exist by studying the historical development of the subject. It is possible that an historical approach encourages competence as there is greater exposure to mathematics and mathematical ideas. For some pupils a study of the History of Mathematics will show that good notation and setting out will help in producing not only the right answer but also many discoveries.

There appear to have been very few previous investigations of any depth into the use of the History of Mathematics in the classroom. The only two found by the author were made by D.R.Green (1974) and L.F. Rogers (1976). In a survey conducted by the author no adverse opinion was found for the use of the historical development of topics as a tool in mathematics teaching.

The Use Being Made of the History of Mathematics in Education

In order to find out the use being made of the History of Mathematics in education a number of primary schools were visited, a questionnaire was sent to the Mathematics Advisers of all the L.E.As in England and a second questionnaire was sent to the mathematics departments of all the universities on the British mainland. From the information gathered about the teaching of mathematics in these establishments is was found that little use was being made of the History of Mathematics.

Before this investigation was started it was assumed that in the primary school sector of state education great use would be made of the History of Mathematics to enrich mathematical studies as a means of providing such project materials for pupils. The reality is, however, quite different.

In the secondary sector it is important to differentiate between History of Mathematics courses and the History of Mathematics being used as enrichment or a teaching tool. There are very few History of Mathematics courses being taught. Green (1974) reported less than 1/20 of 1 per cent of the total examination entry in mathematics being made up of History of Mathematics course candidates. The L.E.A. mathematics advisers in their replies to the questionnaire indicated that there was little use being made of the History of Mathematics in secondary schools. They suggested that they knew of a few teachers who told their pupils of the achievements of some named mathematicians but in general there was no indication of departmental policy to use the historical development of mathematics as a teaching tool.

In the universities there appears to be an underlying desire to make greater use of the History of Mathematics in the teaching of mathematics. A few universities offer History of Mathematics courses. A lack of courses means a lack of potential lecturers. This in turn leads to a lack of courses - a cycle! It was obvious considering the evidence from lecturers who replied to questionnaires or who were interviewed during the investigation that there is in universities considerable potential for making use of the History of Mathematics. One problem, however, appears to be that university lecturers have to include a high mathematical content in their courses and they have little opportunity either to prepare material which they consider suitable or to present this material when they have prepared it.

It was thought reasonable as part of this investigation to compare the use of the History of Mathematics in Britain with its use in other countries. Thirty-one other countries were selected according to their secondary school enrolment ratio. The London Embassies of the thirty-one countries were contacted with requests for information about the teaching of mathematics in their countries with particular reference to the History of Mathematics. From the replies received it was evident that in the Scandinavian countries, Denmark and Norway, considerable interest is taken in the History of Mathemati s. The History of Mathematics is much used, investigated and taught as courses in the United States of America. It is apparent that it is the United States which is leading the field in this aspect of mathematics. Little evidence was found that any other country used the History of Mathematics.

Are Attitudes Towards Mathematics Changed by Introducing the History of Mathematics to the Mathematics Classroom?

Two experiments were carried out to study changes in attitude towards mathematics when history was used in the teaching of mathematics. These experiments involved pupils from secondary schools and student primary school teachers in training. In these experiments an attitude questionnaire taken from Shaw and Marvin (1967) consisting of twenty items using the Likert scaling procedure was used. This attitude questionnaire was designed to measure changes in attitude when the History of Mathematics was used in teaching mathematics.

In the first experiment the mathematics and history of the Euler relationship ($V - E + F = 2$) for convex polyhedra was used. The experiment was carried out with 22 girls in a private school and 41 boys and girls in a comprehensive school. In each school the pupils were divided into two groups. One group was presented with a teaching package containing the mathematical material connected with the Euler relation. This group was regarded as a control group. The second group was presented with the same mathematical material but with additional material which was concerned with the discovery and proof of the Euler relationship. Each group took two weeks to study these materials. An analysis of the results obtained from the two schools showed that there was no change in attitude within or between the history group and the control group in the private school. Similarly there was no significant change in attitude in either of the groups in the comprehensive school, and again there was no significant difference between the groups.

This experiment clearly showed that a limited short term inclusion of some History of Mathematics material had no effect on pupil attitudes towards mathematics. Although it is well known that attitudes can be changed, any change might take some time to effect. This is possibly the case when history is introduced into mathematics teaching.

The second experiment involved potential primary shool teachers. The entry of students to a University School of Education was divided into two groups. One of the groups was given a weekly historical enrichment sheet about the mathematics they had been studying. This was done for one term. The second group or control group was given no enrichment material. A Likert type attitude questionnaire was given to all the students before the experiment was begun. The same questionnaire was again given at the end of the experiment. An analysis of the results showed that using this kind of enrichment material made no difference to the attitudes of these future teachers. This might be because of the limited time devoted to this experiment. It is worth noting here that when these future teachers were interviewed the majority stated that they enjoyed reading about the mathematicians connected with the mathematics they were studying but that they would be reluctant to spend time finding out about the historical development for themselves.

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History of Mathematics Resources

Resource materials for the History of Mathematics can be found in books and mathematical journals. Films and videos, sound tape sequences, filmstrips, single slides and wall charts are also available.

The author has identified 493 books about the History of Mathematics with 19% of these books b ing in print in 1986. A book was included if it met two criteria; it had to be written in English and the major part of its content had to be about the History of Mathematics. The four most popular topics written about were Numbers, Euclidean Geometry, Greek Mathematics and Astronomy. The five most popular mathematicians were Newton, Galileo, Einstein, Babbage and Kepler. The majority of the books identified are excellent reading for mathematics teachers but regrettably there are few that would be suitable for pupils up to fifth form level in the secondary school.

Articles about the History of Mathematics appear infrequently in mathematical journals, but when they do they are generally of very high quality and are often difficult to read because of the high level of the mathematics being discussed.

Only six films/videos about the History of Mathematics have been identified by the author. One of these was a very useful cartoon called 'Donald In Mathmagic Land' which would provide an enjoyable background to geometry and trigonometry teaching. Four of the others were about the history of measurement and the sixth was a film used to introduce the Open University Mathematics Foundation course.

All the sound tape sequences identified as being currently available are about measurement. There are a number of filmstrips and single slides available particularly from the Bodleian Library Oxford. (The Bodleian Library will send a detailed catalogue on request.)

I.B.M. produce a free frieze poster which is about twelve feet long and gives a large amount of detail about mathematicians and mathematical events from 1100 AD to the present day. This chart is supplied free to schools and is admirably suitable for classroom display. Posters about the Möbius Band, The Euler Solids and Archimedes are available from the Mathematical Association.

Conclusions

The value of mathematics and the need to learn mathematics is apparent. For learning, there generally has to be a teacher. Any teacher has a great opportunity to stimulate interest in his subject. A way that would seem to make mathematics interesting to students and laymen is to approach it in the spirit of enrichment and enjoyment. The application of mathematics to practical problems, however, is a very serious matter indeed. But no student is motivated to learn advanced group theory, for example, by telling him that he will find it beautiful and stimulating, or even useful, if he becomes a particle physicist. If the teacher uses his time with drilling his students in routine operations, he kills their interest, hampers their intellectual development, and misuses his opportunity. If he challenges the curiosity of his students by setting them stimulating problems which demonstrate possible applications and enriching his teaching with the many fascinating aspects of the historical development which shows mathematics as an ever evolving human endeavour, he may induce in them a taste for independent thinking and a liking of mathematics.

The limited experimental evidence available tends to suggest that there is little improvement in attitudes towards mathematics when historical material is included in teaching over a short period of time. To evaluate the effect on attitudes when the historical development of mathematics is employed in teaching would require longer experiments with more pupil involvement. This in turn would necessitate a greater participation on the part of teachers. But teachers are largely untrained in the use or substance of the History of Mathematics. Again we are in a cycle. Surely there is a need for one or two Schools of Education not only to teach History of Mathematics courses but also to illustrate to students ways of using history in the mathematics classroom. This would result in a number of mathematics teachers being conversant with the development of their subject. It would then be possible to carry out attitude experiments from a wider base. There might then be a clearer indication of the value of using the historical development of mathematics.

Lack of resources for the pupil certainly inhibits the use of historical material in mathematics classrooms. There is a large volume of historical resource material available for the interested teacher provided he has available to him a 'friendly' library or Teachers Centre.

An indexed listing of all the books, articles, films/videos, filmstrips/slides, posters and charts and portraits of mathematicians identified by the author about the History of Mathematics is available from the Centre for Innovation in Mathematics Teaching, at the School of Education, University of Exeter.

Derek Stander

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