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ABSTRACT

This book combines suggestions for the teaching of problem solving with activities and carefully discussed non-routine problems which students should find interesting as they gain valuable experience in problem solving. The over 300 activities and problems have been gleaned from a variety of sources and have been classroom tested by practicing teachers. Topics included are an explanation of problem solving and heuristics, the pedagogy of problem solving, strategy games, and non-routine problems. An extensive number of problems and strategy game boards are given. A 50-item bibliography of problem solving resources is included. (Author/MVL)

# PROBLEM SOLVING

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## A HANDBOOK FOR ELEMENTARY SCHOOL TEACHERS

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# Preface

During the past decade, problem solving has become a major focus of the school mathematics curriculum. As we enter the era of technology, it is more important than ever that our students learn how to successfully resolve a problem situation.

This book is designed to help you, the elementary school teacher—whether you are a novice or experienced—to teach *problem solving*. Ever since mathematics has been considered a school subject, the teaching of problem solving has been an enigma to mathematics teachers at all levels, whose frustrating efforts to teach students to become better problem solvers seem to have had little effect. The teaching of problem solving must begin when the child first enters school. Even the youngest of children face problems daily.

This book combines suggestions for the teaching of problem solving with activities and carefully discussed non-routine problems which your students will find interesting as they gain valuable experience in problem solving. The activities and problems have been gleaned from a variety of sources and have been classroom-tested by practicing teachers. We believe that this is the first time such an expansive set of problems has appeared in a single resource, specifically designed for the elementary school.

Problem solving is now considered to be a basic skill of mathematics education. However, we suggest that it is more than a single skill; rather, it is a group of discrete skills. Thus, in the chapter on pedagogy, the subskills of problem solving are

enumerated and then integrated into a teachable process. The chapter is highlighted by a flowchart that guides students through this vital process. Although there are many publications that deal with the problem-solving process, we believe that this is the first one that focuses on these subskills.

We are confident that this book will prove to be a valuable asset in your efforts to teach problem solving.

S.K. and J.R.

## CHAPTER ONE

# An Introduction to Problem Solving

Chapter One

## WHAT IS A PROBLEM?

A major difficulty in discussing problem solving seems to be a lack of any clear-cut agreement as to what constitutes a "problem." A problem is a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent path to obtaining the solution. The key to this definition is the phrase "no apparent path." As children pursue their mathematical training, what were problems at an early age become exercises and are eventually reduced to mere questions. We distinguish between these three commonly used terms as follows:

(a) question: a situation that can be resolved by recall from memory.
(b) exercise: a situation that involves drill and practice to reinforce a previously learned skill or algorithm.
(c) problem: a situation that requires thought and a synthesis of previously learned knowledge to resolve.

In addition, a problem must be perceived as such by the student, regardless of the reason, in order to be considered a problem by him or her. If the student refuses to accept the challenge, then at that time it is not a problem for that student. Thus, a problem must satisfy the following three criteria, illustrated in Figure 1–1:

1. *Acceptance*: The individual accepts the problem. There is a personal involvement, which may be due to any of a variety of reasons, including internal motivation, external motivation (peer, parent, and/or teacher pressure), or simply the desire to experience the enjoyment of solving a problem.
2. *Blockage*: The individual's initial attempts at solution are fruitless. His or her habitual responses and patterns of attack do not work.
3. *Exploration*: The personal involvement identified in (1) forces the individual to explore new methods of attack.

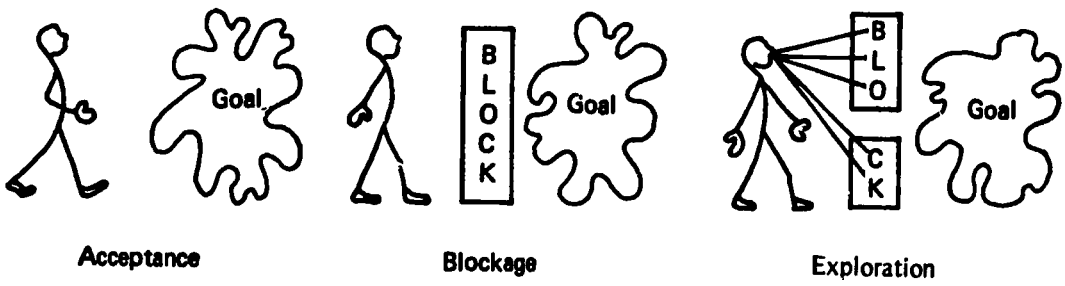



Figure 1–1

The existence of a problem implies that the individual is confronted by something he or she does not recognize, and to which he or she cannot merely apply a model. A situation will no longer be considered a problem once it can be easily solved by algorithms that have been previously learned.

### A word about textbook problems

Although most mathematics textbooks contain sections labeled "word problems," many of these "problems" should not really be considered as problems. In many cases, a model solution has already been presented in class by the teacher. The student merely applies this model to the series of similar exercises in order to solve them. Essentially the student is practicing an algorithm, a technique that applies to a single class of "problems" and that guarantees success if mechanical errors are avoided. Few of these so-called problems require higher-order thought by the students. Yet the first time a student sees these "word problems" they could be problems to him or her, if presented in a non-algorithmic fashion. In many cases, the very placement of these exercises prevents them from being real problems, since they either follow an algorithmic development designed specifically for their solution, or are headed by such statements as "Word Problems: Practice in Division by 4."

We consider these word problems to be "exercises" or "routine problems." This is not to say that we advocate removing them from the textbook. They do serve a purpose, and for this purpose they should be retained. They provide exposure to problem situations, practice in the use of the algorithm, and drill in the associated mathematical processes. However, a teacher should not think that students who have been solving these exercises through use of a carefully developed model or algorithm have been exposed to problem solving.

## WHAT IS PROBLEM SOLVING?

Problem solving is a process. It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. The process begins with the initial confrontation and concludes when an answer has been obtained and considered with regard to the initial conditions. The student must synthesize what he or she has learned, and apply it to the new and different situation.

Some educators assume that expertise in problem solving develops incidentally as one solves many problems. While this may be true in part, we feel that problem solving must be considered as a distinct body of knowledge and the process should be taught as such.

The goal of school mathematics can be divided into several parts, two of which are (1) attaining information and facts, and (2) acquiring the ability to use information and facts. This ability to use information and facts is an essential part of the problem-solving process. In effect, problem solving requires analysis and synthesis.

## WHY TEACH PROBLEM SOLVING?

In dealing with the issue of why we should teach problem solving, we must first consider the larger question: Why teach mathematics? Mathematics is fundamental to everyday life. All of our students will face problems, quantitative or otherwise, every day of their lives. Rarely, if ever, can these problems be resolved by merely referring to an arithmetic fact or a previously learned algorithm. The words "Add me!" or "Multiply me!" never appear in a store window. *Problem solving provides the link between facts and algorithms and the real life problem situations we all face.* For most people, mathematics *is* problem solving!

In spite of the obvious relationship between mathematics of the classroom and the quantitative situations in life, we know that children of all ages see little connection between what happens in school and what happens in real life. An emphasis on problem solving in the classroom can lessen the gap between the real world and the classroom world and thus set a positive mood in the classroom

In many mathematics classes, students do not see any connections among the various ideas taught during the year. Most regard each topic as a separate entity. Problem solving shows the interconnections among mathematical ideas. Problems are never solved in a vacuum, but are related in some way to something seen before or to something learned earlier. Thus, good problems can be used to review past mathematical ideas, as well as to sow seeds for ideas to be presented at a future time.

Problem solving is more exciting, more challenging, and more interesting to children than barren exercises. If we examine student performance in the classroom, we recognize the obvious fact that success leads to persistence and continuation of a task; failure leads

to avoidance. It is this continuance that we constantly strive for in mathematics. The greater the involvement, the better the end product. Thus, a carefully selected sequence of problem-solving activities that yield success will stimulate students, leading them to a more positive attitude toward mathematics in general and problem solving in particular.

Finally, problem solving permits students to learn and to practice heuristic thinking. A careful selection of problems is a major vehicle by which we provide a "sharpening" of problem-solving skills and strategies so necessary in real life.

## WHEN DO WE TEACH PROBLEM SOLVING?

Problem solving is a skill everyone uses all their lives. The initial teaching and learning of the problem-solving process must begin as soon as the child enters school, and continue throughout his or her entire school experience. The elementary school teacher has the responsibility for beginning this instruction and thus laying the foundation for the child's future problem-solving experiences.

Since the process of problem solving is a teachable skill, when do we teach it? What does it replace? Where does it fit into the day-to-day schedule?

Experiences in problem solving are always at hand. All other activities are subordinate. Thus, the teaching of problem solving should be continuous. Discussion of problems, proposed solutions, methods of attacking problems, etc., should be considered at all times. Think how poorly students would perform in other skill areas, such as fractions, if they were taught these skills in one or two weeks of concentrated work and then the skills were never used again.

Naturally there will be times when studies of algorithmic skills and drill and practice sessions will be called for. We insist that students be able to add, subtract, multiply, and divide. Problem solving is not a substitute for these computational skills. However, these times will permit the delay necessary for the incubation period required by many problems, which need time to "set." By allowing time between formal problem-solving sessions, you permit students to become familiar with the problem-solving process slowly and over a longer period of time. This is important, since the emphasis is on the process and not merely on obtaining an answer. The development of the process takes time!

## WHAT MAKES A GOOD PROBLEM SOLVER?

Although we cannot easily determine what it is that makes some students good problem solvers, there are certain common characteristics exhibited by good problem solvers. For instance, good problem solvers know the *anatomy of a problem*. They know that a problem contains facts, a question, and a setting. They also know that most problems (with the exception of some word problems in textbooks) contain distractors, which they can recognize and eliminate.

Good problem solvers have a *desire* to solve problems. Problems interest them; they offer a challenge. Much like climbers of Mt. Everest, problem solvers like to solve problems because they exist!

Problem solvers are extremely *perseverant* when solving problems. They are not easily discouraged when incorrect or when a particular approach leads to a dead end. They go back and try new approaches again and again. They refuse to quit!!

If one method of attacking a problem fails to yield a satisfactory solution, successful problem solvers try another. They usually have a variety of methods of attack at their disposal and they will often try the opposite of what they have been doing in the hope that new information will occur to them. They will ask themselves many "What if..." questions, changing conditions within the problem as they proceed.

Good problem solvers show an ability to skip some of the steps in the solution process. They make connections quickly, notice irrelevant detail, and often require only a few examples to generalize. They may show a lack of concern about neatness while developing their solution process.

Above all, good problem solvers are *not afraid to guess!* They will make "educated guesses" at answers, and then attempt to verify these guesses. They will gradually refine their guesses on the basis of what previous guesses show them, until they find a satisfactory answer. They rarely guess wildly, but use their own intuition to make carefully thought-through guesses.

We would suggest that good problem solvers are students who hold conversations with themselves. They know what questions to ask themselves, and what to do with the answers they receive as they think through the problem.

## WHAT MAKES A GOOD PROBLEM?

Problem solving is the basic skill of mathematics education. It is the primary reason for teaching mathematics. Fundamental to the teach-

ing of problem solving is the development and the utilization of "good" problems.

What constitutes a good problem? Good problems can be found in virtually every aspect of daily living as well as in traditional mathematical settings. And problems need not always be word problems in order to be good problems.

**PROBLEM** Which doesn't belong?

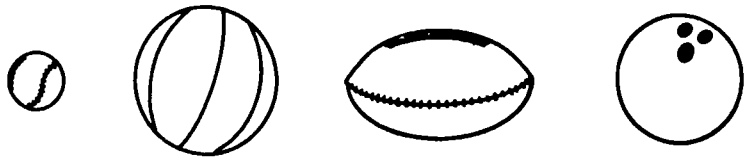


Figure 1–2

**Discussion** This non-verbal problem requires the student to determine the characteristics common to three objects, but not to the fourth. In this case, the baseball, basketball, and bowling ball are all round; the football is oval. Thus, the football does not belong.

Some children might also recognize that three begin with the letter "b"; again, the football does not belong.

Notice that some students may arrive at their answer using a different reason. In fact, some may arrive at a different answer entirely. This is an important fact: *Answers can vary!!* In life, there are times when several answers can serve or be acceptable. The same should be true in our classroom problems.

In the example, some students may decide that the bowling ball does not belong, since it is made from a synthetic (non-leather) material, is the only one without any stitching or lacing, and is the only one with holes in it. A discussion of all of these is vital to the teaching of problem solving.

The problems that follow have been chosen to illustrate specific ideas. They may not all be suitable for your classroom. However, you should modify the problems wherever possible to suit your particular classroom situation.

**1. The solution to the problem involves the understanding of a mathematical concept or the use of a mathematical skill.**

Many problems may appear, on the surface, to be non-mathematical in context, yet the solution to the problem involves basic mathematical principles. Perhaps a pattern can be found that the students

recognize. Or some application of a skill may quickly resolve the problem. In any case, there should be some basic mathematical skill and/or concept embedded in the problem and its solution.

**PROBLEM** John is taller than Mary. Mary is taller than Peter. Who is the shortest of the three children?

**Discussion** The solution of this problem depends on an understanding of the order principle and the property of transitivity. Some students may have to act out the problem in order to solve it, by choosing three classmates that fit the given conditions.

**PROBLEM** George weighs 36 pounds, Luisa weighs 46 pounds, and Julia weighs 39 pounds. Arrange them in order of their weight.

**Discussion** The solution to this problem depends on an understanding of order. However, number concepts have also been introduced. Notice that two answers are possible:

George—Julia—Luisa
or
Luisa—Julia—George

**PROBLEM** There are 27 children in line to go through the Haunted House. Each car carries exactly 3 children. Jorge and Paula are numbers 16 and 17 in line. Will they ride in the same car?

**Discussion** To solve this problem, we can make a table:

| *Student Number* | *Car Number* |
|---|---|
| 1,2,3 | 1 |
| 4,5,6 | 2 |
| 7,8,9 | 3 |
| 10,11,12 | 4 |
| 13,14,15 | 5 |
| 16,17,18 | 6 |

Yes, they will both ride in car number six.

A table is not necessary if the child understands the concept of division and interpreting remainders. Divide 3 into 16 and then into 17. In each case, we get 5 and a remainder. Thus 5 cars go before, and children 16 and 17 are in the next car, number six.

**PROBLEM** The floor of the monkey house at the local zoo is in the form of a square, 6 feet by 6 feet, and covered with Astroturf. Next to this is the gorilla house. The floor is also a square, but its sides are each 12 feet. How much more Astroturf is used to cover the floor of the gorilla house?

**Discussion** The floor of the monkey house requires $6 \times 6$ or 36 square feet of Astroturf. The floor of the gorilla house is 12 feet by 12 feet or 144 square feet. The gorilla house requires $144 - 36$ or 108 square feet of additional Astroturf.

This problem depends on the concept of area as it relates to the square. Notice that this is a multi-stage problem, requiring the student first to find the area of each floor and then to subtract. (Notice, too, that this problem lays the foundation for the later study of the relationship between changes in the dimensions of a figure and the change in area that results).

**PROBLEM** The new school has exactly 1,000 lockers and exactly 1,000 students. On the first day of school, the students meet outside the building and agree on the following plan: The first student will enter the school and open all of the lockers. The second student will then enter the school and close every locker with an even number (2,4,6,8, . . . ). The third student will then "reverse" every third locker. That is, if the locker is closed, he will open it; if the locker is open, he will close it. The fourth student will reverse every fourth locker, and so on until all 1,000 students in turn have entered the building and reversed the proper lockers. Which lockers will finally remain open?

**Discussion** It seems rather futile to attempt this experiment with 1,000 lockers, so let's take a look at 20 lockers and 20 students, and try to find a pattern.

In our smaller illustration in Figure 1–3, the lockers with numbers 1, 4, 9, and 16 remain open (O), while all others are closed (C). Thus, we conclude that those lockers with numbers that are perfect squares will remain open when the process has been completed by all 1,000 students. Notice that a locker "change" corresponds to a divisor of the locker number. An odd number of "changes" is required to leave a locker open. Which kinds of numbers have an odd number of divisors? Only the perfect squares!

In summary, this problem has embedded in it several basic mathematical concepts, namely factors, divisors, composites, and perfect squares.

This problem also lends itself to an experiment, by having students act it out. Twenty students holding cards, each numbered

| Locker # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Student 1 | O | O | O | O | O | O | O | O | O | O | O | O | O | O | O | O | O | O | O | O |
| 2 | ▲ | C | O | C | O | C | O | C | O | C | O | C | O | C | O | C | O | C | O | C |
| 3 | | | C | C | O | O | O | C | C | C | O | O | O | C | C | C | O | O | O | C |
| 4 | | | | O | O | O | O | O | C | C | O | C | O | C | C | O | O | O | O | O |
| 5 | | | | ▲ | C | O | O | O | C | O | O | C | O | C | O | O | O | O | O | C |
| 6 | | | | | | C | O | O | C | O | O | O | O | C | O | O | O | C | O | C |
| 7 | | | | | | | C | O | C | O | O | O | O | O | O | O | O | C | O | C |
| 8 | | | | | | | | C | C | O | O | O | | O | O | O | O | C | O | C |
| 9 | | | | | | | | | O | O | O | O | O | O | O | C | O | O | O | C |
| 10 | | | | | | | | | ▲ | C | O | O | O | O | O | C | O | O | O | O |
| 11 | | | | | | | | | | | C | O | O | O | O | C | O | O | O | O |
| 12 | | | | | | | | | | | | C | O | O | O | C | O | O | O | O |
| 13 | | | | | | | | | | | | | C | O | O | C | O | O | O | O |
| 14 | | | | | | | | | | | | | | C | O | C | O | O | O | O |
| 15 | | | | | | | | | | | | | | | C | C | O | O | O | O |
| 16 | | | | | | | | | | | | | | | | O | O | O | O | O |
| 17 | | | | | | | | | | | | | | | | ▲ | C | O | O | O |
| 18 | | | | | | | | | | | | | | | | | | C | O | O |
| 19 | | | | | | | | | | | | | | | | | | | C | O |
| 20 | | | | | | | | | | | | | | | | | | | | C |

Figure 1–3

from 1 to 20, represent the lockers. Having them turn facing forward (open) or facing backward (closed) as they are "reversed" enables them to demonstrate the action described in the problem.

**2. The problem should be capable of extension.**

A problem is not necessarily finished when a satisfactory answer has been found. The solution should suggest variations on parts of the original problem. The problem might be changed from a two-dimensional, plane geometry problem to a three-dimensional situation. Circles become spheres; rectangles become "boxes."

The problem should lend itself to extension by means of "What if..." questions. *What if* we hold one variable constant and let another change? *What if* the shapes of the given figures vary? *What if* the dimensions change? *What if* we increase or decrease the numbers in the problem?

**PROBLEM** A party of 18 people goes to a restaurant for dinner. The restaurant has tables that seat 4 or 6 people. Show how the maître d' can seat the party.

**Discussion** They can be seated in two different ways:

(a) 3 tables of 6
(b) 1 table of 6 and 3 tables of 4

The problem can now be extended:

What if two more people join the group? Now how might they be seated? The answer is still only two ways, but they are different:

(a) 5 tables of 4
(b) 2 tables of 6 and 2 tables of 4

What if one table that seats 8 people were available? What if the party consisted of 21 people?

**PROBLEM** The Greens are having a party. The first time the doorbell rings 1 guest enters. On the second ring, 3 guests enter. On the third ring, 5 guests enter, and so on. That is, on each successive ring, the entering group of guests is 2 larger than the preceding group. How many guests will enter on the eighth ring?

**Discussion** Let's make a table and search for a pattern:

| *Ring* | *People Enter* |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |
| 5 | 9 |
| 6 | 11 |
| 7 | 13 |
| 8 | 15 |

The table reveals that, on the eighth ring, 15 people will enter.
Several extensions can now be considered:

(a) How many people enter on the twentieth ring?
[Perhaps the students will notice that the number of people who enter is twice the "ring" number minus 1 ($2n-1$). They can use this "formula" to solve the extension without the table.]
(b) How many people are in the room after the eighth ring? After the fifteenth ring?
(c) On what ring do 21 people enter?
(d) On what ring do 28 people enter?

**PROBLEM** A set of children's blocks comes in two shapes: triangles and circles. Each shape comes in red, yellow, and blue. The blocks are thick or thin. How many blocks are in a set?

**Discussion** The solution to this problem utilizes the Fundamental Counting Principle. Thus, there are

$$2 \text{ (shape)} \times 3 \text{ (color)} \times 2 \text{ (thickness)} = 12 \text{ blocks.}$$

We can extend this problem as follows:

(a) How many red pieces are in the set?
(b) How many triangles are in the set?
(c) What if we introduce a third shape, rectangles?
(d) What if a fourth color, green, is added?

**3. The problem lends itself to a variety of solution techniques.**

Most problems can be solved by more than one method. A single problem can often be acted out; can be reduced to a simple arithmetic, algebraic, or geometric relationship; can be resolved with a drawing; or can be resolved by an application of logical reasoning. It is of greater value to the development of the problem-solving process to solve a single problem in four different ways than to solve four problems each in one way.

**PROBLEM** A farmer has some pigs and some chickens. He finds that together they have 70 heads and 200 legs. How many pigs and how many chickens does he have?

**Discussion 1** A series of successive approximations together with a table to record the data will enable students to solve the problem:

| CHICKENS | | PIGS | | TOTAL | | |
|---|---|---|---|---|---|---|
| Number of heads | Number of legs | Number of heads | Number of legs | Number of heads | Number of legs | |
| 70 | 140 | 0 | 0 | 70 | 140 | (Not enough legs) |
| 50 | 100 | 20 | 80 | 70 | 180 | (Still not enough legs) |
| 40 | 80 | 30 | 120 | 70 | 200 | |

Figure 1–4

**Discussion 2** We can reduce the problem's complexity by dividing through by 10. Thus, we now have 7 heads and 20 legs.

(When the answer is obtained, we must remember to multiply by 10.) Draw 7 circles to represent the 7 animals (7 heads tells us there are 7 animals) and attach two legs to each. This accounts for 14 legs.

Figure 1–5

We must now distribute the remaining 6 legs. Affix two additional legs to each of the first three animals.

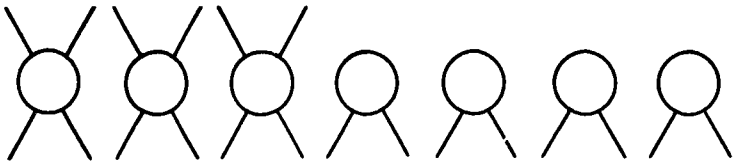


Figure 1–6

This shows three "pigs" and 4 "chickens." The final answer, then, is 30 pigs and 40 chickens.

**Discussion 3** Use the idea of a one-to-one correspondence. All chickens stand on one leg, all pigs stand on hind legs. Thus, the farmer will see 70 heads and 100 legs. The extra 30 legs must belong to the pigs, since the chickens have one leg per head. Thus, there are 30 pigs and 40 chickens.

**PROBLEM** Dana had 8 cookies. Her mother gave her 10 more. Then Dana gave 6 cookies to her sister. How many cookies does Dana have left?

**Discussion 1** This two-stage problem can be done by following the arithmetic computation as it appears in the problem. Thus, the answer can be found by

$$8+10-6=12$$

Dana has 12 cookies left.

**Discussion 2** An alternate approach would be to act it out. Use cookies,

bottle caps, or some other manipulative, and have the children follow the action.

**PROBLEM** Larry, Pam, Mae, and Nick are going on a scavenger hunt. They will form teams of two people each. How many different teams can they form?

**Discussion 1** Four students can act out the problem.

**Discussion 2** We can simulate this experiment by arranging slips of paper on which are written the children's names. Thus, the slips of paper represent the children.

**Discussion 3** The problem can be solved by making an organized list that pairs the children:

| | | | |
|---|---|---|---|
| Larry–Pam | Pam–Larry | Mae–Larry | Nick–Larry |
| Larry–Mae | Pam–Mae | Mae–Pam | Nick–Mae |
| Larry–Nick | Pam–Nick | Mae–Nick | Nick–Pam |

Now have the children strike out the duplicates, noting that Larry–Pam and Pam–Larry are the same pair, and so on:

| | | | |
|---|---|---|---|
| Larry–Pam | ~~Pam–Larry~~ | ~~Mae–Larry~~ | ~~Nick–Larry~~ |
| Larry–Mae | Pam–Mae | ~~Mae–Pam~~ | ~~Nick–Mae~~ |
| Larry–Nick | Pam–Nick | Mae–Nick | ~~Nick–Pam~~ |

(This problem could set the stage for a later discussion of permutations and combinations.)

**4. The problem should be interesting and appealing to the child.**

A child's world differs significantly from that of an adult. Problems that would normally interest adults, such as some real-life applications, may be of no interest to children. Therefore, the problem setting and the action must center on the child's world. Teachers must be familiar with the child's world in order to select those problems that are both instructive and appealing.

**PROBLEM** Janice has fewer than 10 baseball cards. If she puts them into piles of three, she has none left over. But when she puts them into piles of four, there is one left over. How many baseball cards does Janice have?

**Discussion** Children enjoy collecting. Baseball is a sport that interests most of them. Thus, it is likely that a problem such as this one will appeal to most children.

The first two facts tell us that Janice has 3, 6, or 9 cards.

The final fact tells us that the answer must be 9.

Notice that this problem personifies a good problem in that it exhibits all four characteristics of a good problem. First of all, the mathematical content centers on the concept of division with remainders. Second, the problem can be extended by increasing the number of cards in the collection and/or by changing the number of cards in each arrangement. Third, it lends itself to more than one method of solution. That is, it can actually be acted out by using cards or other manipulatives. It can also be done abstractly with paper and pencil, or it can be done mentally by guessing at an answer and then testing the guess. Finally, as we have already stated, collecting baseball cards is within the realm of the child's world and is usually of interest to most children.

## WHAT MAKES A GOOD TEACHER OF PROBLEM SOLVING?

All we have said about problems, problem solving, and problem solvers depends on the teacher for implementation and fruition. Without an interested, energetic, enthusiastic, and involved guide or model, nothing positive will take place.

Success in problem solving requires a positive teacher attitude toward the problem-solving process itself. This means that teachers must prepare carefully for problem solving and be aware of the opportunities for problem solving that present themselves in everyday classroom situations. You may have to modify a particular problem to ensure its pedagogical value—its scope may have to be reduced, or the problem restated in terms of the students' experiences. Knowing your students helps you make these choices. Problems should be solved in class carefully, with the teacher allowing for and encouraging a wide variety of approaches, ideas, questions, solutions, and discussions. Teachers must be confident in class and must exhibit the same enthusiasm for the problem-solving process that they wish to instill in their students.

Some teachers dislike problem solving because they have not had enough successful experiences in this area. Practice will provide these experiences. Teachers who encourage their students to solve problems, who make the students think, and who ask carefully worded questions (rather than merely giving answers) will provide their students with a rich problem-solving experience.

## CHAPTER TWO

# A Workable Set of Heuristics

## WHAT ARE HEURISTICS?

Problem solving is a process—a process that starts when the initial encounter with the problem is made and ends when the obtained answer is reviewed in light of the given information. Children must learn this *process* if they are to deal successfully with the problems they will meet in school as well as in other walks of life. The *process* consists of a series of tasks and thought processes that are loosely linked together to form what is called a set of heuristics or a heuristic pattern. They are a set of suggestions and questions that a person must follow and ask himself in order to resolve his dilemma.

Heuristics should not be confused with algorithms. Algorithms are schemas that are applied to a single class of problems. In computer language, they are programs that can be called up to solve specific problems or classes of problems for which they were developed. For each problem or class of problems, there is a specific algorithm. If one chooses and properly applies the appropriate algorithms and makes no arithmetic or mechanical errors, the answer that is obtained will be correct. In contrast, heuristics are general and are applicable to all classes of problems. They provide the direction needed by all people to approach, understand, and obtain answers to problems that confront them.

There is no single set of heuristics for problem solving. Several people have put forth workable models, and whether the student follows the one put forth by Polya or the one that appears in this book is not important; what is important is that our students learn some set of carefully developed heuristics and that they develop the habit of applying these heuristics in all problem-solving situations.

It is apparent that simply providing students with a set of heuristics to follow would be of little value. There is quite a difference between understanding the process on an intellectual plane (recognizing and describing it) and being able actually to apply the process. Thus, we must do more than merely hand the heuristics to the students; rather, instruction must focus on each stage of the process that the problem solver goes through while considering a problem. It is the process, not the answer, that is problem solving.

## A SET OF HEURISTICS TO USE

Over the years, several heuristic plans have been developed to assist students in problem solving. For the most part, all of these are quite similar. We now put forth a set of heuristics that has proven to be successful with students and teachers at all levels of instruction:

1. Read
2. Explore
3. Select a strategy
4. Solve
5. Look back and extend

These represent a continuum of thought that every person should use when confronted by a problem-solving situation. As a continuum, these are not discrete. In fact, "Read" and "Explore" could easily be considered at the same time under a single heading such as "Think." Also, at the same time the problem solver is exploring, he or she is considering what strategy to select.

## 1. Read

Read, of course, means much more than merely reading the words. A problem has an anatomy. It consists of four parts: a setting, a question, some facts, and some distractors. During the read step of the process, the student must identify each of these four parts.

1a. Describe the setting and visualize the action.
1b. Restate the problem in your own words.
1c. What is being asked?
1d. What information is given?
1e. What are the key facts?
1f. Is there extra information?

**PROBLEM** Lucy and Carla leave school at 3:00 P.M. and start toward home. Their homes are on the same street, but lie in opposite directions from the school. Lucy lives 3 miles from the school, while Carla lives 2 miles from the school. How far apart are their homes?

**Discussion** Can you visualize the action? Can you describe what is taking place? What does 3:00 P.M. have to do with the problem? The key word here is "opposite."

**PROBLEM** Lucy and Carla leave school at 3:00 P.M. and start toward home. Their homes are on the same street, and lie in the same direction from the school. Lucy lives 3 miles from the school, while Carla lives 2 miles from the school. How far apart are their homes?

**Discussion** Can you visualize the action? Can you describe what is taking place? What makes this problem different from the preceding one? What key word or words make the difference?

**PROBLEM** Find the difference in the number of apples in a 5-pound bag that contains 27 apples and a basket that contains 2 dozen apples.

**Discussion** Notice that the 5-pound bag is extra information. There is no question mark in this problem. What is the question?

**PROBLEM** Jeff weighs 160 pounds. His sister Nancy weights 108 pounds. Scott weights 26 pounds more than Nancy. What is the average weight of all three people?

**Discussion** Here, the important words are "more than" and "average." Words such as "more than," "less than," "subtracted from," etc. are often overlooked by students.

**PROBLEM** Mary is 12 years old and her brother George is 5 years older. How old is George?

**Discussion** Here the key fact is that George is 5 years *older* than Mary. We will refer to words such as "older" as *directional words*, since they *direct* the problem solver along the solution path.

## 2. Explore

Explore is an activity that most experienced problem solvers do without conscious thought. It is the analysis and synthesis of the information contained in the problem, which has been revealed during the *read* stage. It is in this stage of the process that possible paths are mentally examined (hence the name "explore").

2a. Organize the information.
2b. Is there enough information?
2c. Is there too much information?
2d. Draw a diagram or construct a model.
2e. Make a chart or a table.

**PROBLEM** At the ballpark, pizza costs 95¢ a slice, soft drinks cost 75¢, and hot dogs cost $1.25. Gladys bought a hot dog and a soft drink. How much change did she receive?

**Discussion** This problem contains excess information and, at the same time, has insufficient data. The cost of the pizza is excess, while the answer cannot be found because the amount of money given to the cashier is not known.

**PROBLEM** A log is to be cut into five equal pieces. How many times must the woodsman saw through the log?

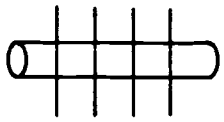


Figure 2–1

**Discussion** Students should draw a diagram as shown in Figure 2–1. The drawing reveals that the number of cuts is one less than the actual number of pieces required. Thus, the woodsman must saw through the log four times.

**PROBLEM** Mike has 8 hamsters. Together they eat 7 carrots each week. How many carrots will the hamsters eat in one year (52 weeks)?

**Discussion** This problem contains excess information. The number of hamsters (8) is not needed to solve the problem. Yet it serves as a distractor to many students who multiply $8 \times 7 \times 52$ and get 2912 as their answer, rather than 364. Notice that the word "together" is the directional word.

**PROBLEM** Danny is giving his comic book collection to his friends. He has 400 comics to give away. He gives Miriam half of his comic books. Then he gives Susan half of what he has left. Then he gives Bobby half of what he now has left. Finally, he gives Peter half of what he has left. how many comic books did Danny give to each friend?

**Discussion** One path to the answer would be to organize the information by means of a table:

| Miriam | Susan | Bobby | Peter |
|---|---|---|---|
| 200 | 100 | 50 | 25 |

**PROBLEM** Antelope Hill, Buffalo Corner, Coyote Canyon, and Desperado Gulch lie along a straight road in the order named. The distance from Antelope Hill to Desperado Gulch is 100 miles. The distance from Buffalo Corner to Coyote Canyon is 30 miles. The distance from Buffalo Corner to Desperado Gulch is 60 miles. How far is it from Antelope Hill to Buffalo Corner?

**Discussion** Although the problem sounds cumbersome and complicated, it can be simplified by the use of a drawing as shown in Figure 2–2.

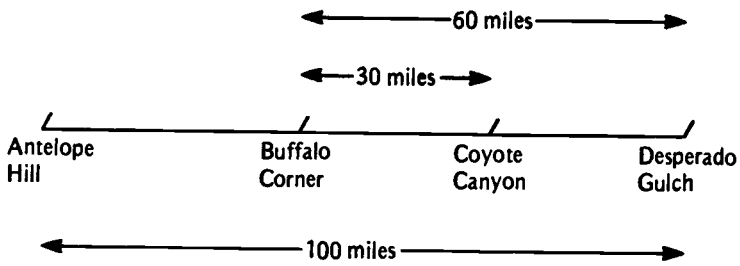



Figure 2–2

The key fact here is the phrase "in the order named." Notice that this makes the distance from Buffalo Corner to Coyote Canyon excess information.

### 3. Select a strategy

As a result of the exploration stage, the problem solver now selects the path that seems most appropriate. Below are eight identified strategies that are used most often, either independently or combined in some manner. Different people might approach a particular problem in different ways. A single problem can probably be solved by applying several combinations of these strategies. No one strategy is superior to any other; however, some strategies may offer a more elegant path to the answer than others.

3a. Pattern recognition
3b. Working backward
3c. Guess and test
3d. Simulation or experimentation
3e. Reduction/solve a simpler problem
3f. Organized listing/exhaustive listing
3g. Logical deduction
3h. Divide and conquer

**PROBLEM** Find the next few terms in the sequence 2, 4, 6, ....

**Discussion** The most obvious pattern is to use the sequence of even numbers. Thus, the next few terms might be 8, 10, 12, ....
However, some persons might think of 2, 4, 6, 10, 16, 26, ..., where each term (beginning with the third term) is the sum of the two previous terms.
Some may decide that the sequence only contains five terms, and is symmetric. Thus, this solution would be 2, 4, 6, 4, 2.

Some cheerleaders might even say that the next term is "one dollar." Can you discover their pattern?

**PROBLEM** Find the next term in the sequence Ann, Brad, Carol, ...

**Discussion** Not all problems are numerical. This sequence involves names. Note that this sequence contains three variables: alternating of gender, leading letters, and the number of letters in each name. Thus, the next terms might be Daniel and Eleanor. (How far can *you* carry the sequence?)

**PROBLEM** Barbara is giving her baseball card collection away. First she gives half of the collection to her sister Suzy. Then she gives half of what is left to Mike. She then gives the remaining 20 cards to David. How many cards did Barbara start with?

**Discussion** We can solve this problem by working backward and using logic. David received 20 cards. This represented the half remaining after Mike received one-half. Thus, just prior to Mike receiving his share, Barbara must have had 40 cards. These 40 cards represent the half left after Suzy received her half. Thus, Barbara must have started with 80 cards.

**PROBLEM** How many different ways can you add four even whole numbers and get 10 as the sum?

**Discussion** Students should remember that "sum" implies addition. They should notice that they are to add four numbers, none of which can be an odd number. This problem is an excellent illustration of the use of the guess-and-test strategy.

Students will try different sets of four numbers to see if they add up to 10. Keep guessing and checking until all the ways have been found. Keeping the results in an organized list will help. Notice that $4+2+2+2$ is the same as $2+2+2+4$ is the same as $2+4+2+2$, and so on. These all count as one way.

**PROBLEM** How many board erasers can you line up on the chalk tray of the chalkboard in your classroom?

**Discussion** The problem can be done by an experiment. Actually line up a series of erasers along the chalk tray and count how many there are. A more sophisticated solution would be to measure the length of the chalk tray, the length of one eraser, and then divide.

**PROBLEM** How many thirds are there in three quarters?

**Discussion** Replace the "thirds" by 2 and the "three quarters" by 8. We now have a simpler problem:

How many 2s are there in 8?

Solving this simpler problem will indicate the method or approach to use to solve the original problem.

**PROBLEM** How many ways can Jeff make change for 50¢ without using pennies?

**Discussion** An organized list enables us to solve this problem:

| *Quarters* | *Dimes* | *Nickels* |
|---|---|---|
| 2 | 0 | 0 |
| 1 | 2 | 1 |
| 1 | 1 | 3 |
| 1 | 0 | 5 |
| 0 | 5 | 0 |
| 0 | 4 | 2 |
| 0 | 3 | 4 |
| 0 | 2 | 6 |
| 0 | 1 | 8 |
| 0 | 0 | 10 |

There are ten different ways to make change.

Notice that in addition to being an organized list, this list is exhaustive. That is, *all* the possibilities have been listed. This solution is also an example of a simulation. We have simulated making change with a table.

**PROBLEM** Albert weighs 50 pounds. Together, Bennet and Carlos weigh 100 pounds. If Carlos weighs more than Bennet, arrange the three boys from heaviest to lightest.

**Discussion** If Bennet and Carlos weighed the same, each would weigh exactly 50 pounds. But Carlos weighs more than Bennet, therefore more than 50 pounds. And so, Bennet weighs less than 50 pounds.

**PROBLEM** "Babe" Richardson has hit 7 doubles, 5 triples, 6 home runs, and 19 one-base hits. How many total bases has he hit?

**Discussion** Divide the problem into its component parts and solve each separately. The answer is then obtained by adding the results of the four parts.

$$19 \times 1 \text{ base} = 19 \text{ total bases}$$

$$7 \times 2 \text{ bases} = 14 \text{ total bases}$$

$5 \times 3$ bases = 15 total bases

$6 \times 4$ bases = 24 total bases

TOTAL BASES = 72

**4. Solve**

Once the problem has been understood and a strategy has been selected, the student should perform the mathematics necessary to arrive at an answer. In most cases in the elementary grades, this mathematics consists of basic computational skills with whole numbers, decimals and fractions, some metric properties of geometry, and some elementary logic.

4a. Use computational skills.
4b. Use geometric skills.
4c. Use elementary logic.

**PROBLEM** Dr. Leka looked at 7 small plants under her microscope. Each plant had 4 leaves. How many leaves did she see?

**Discussion** The problem requires that the students understand the setting and the action. Multiplication is the required operation.

**PROBLEM** Gladys, Jeanette, Jesse, and Steve went fishing. Gladys caught 16 fish, Jeanette caught 13 fish, Jesse caught 17 fish, and Steve caught 14 fish. How many more fish did Jesse and Steve catch than Gladys and Jeanette?

**Discussion** The data in the problem can best be organized with a simple table:

| | |
|---|---|
| Gladys | 16 |
| Jeanette | 13 |
| Jesse | 17 |
| Steve | 14 |

Now we solve the problem by adding and then subtracting.

Jesse and Steve caught $17 + 14 = 31$

Gladys and Jeanette caught $16 + 13 = 29$

$$\begin{array}{r} 31 \\ -29 \\ \hline 2 \end{array}$$

**PROBLEM** At the amusement park, 18 people are waiting to go on a ride.
A square car seats 4 people while a circular car seats 6 people. How can the people be seated in the cars?

**Discussion** Students should use guess-and-test strategy, keeping track of their guesses in a table. The problem involves a knowledge of the 4 and 6 tables of related facts.

| Cars With 6 | Cars With 4 | Total Number |
|---|---|---|
| 3 | 0 | 18 |
| 2 | 1 | — |
| 1 | 3 | 18 |
| 0 | 4 | — |

They can ride in 3 circular cars or in 1 circular car and 3 square cars.

**PROBLEM** Mary and Mike are each fencing in a garden in the shape of a rectangle with an integral number of units on each side. They each used 16 meters of fencing, yet Mary's garden contains 1 square meter more than Mike's. What were the dimensions of their gardens?

**Discussion** The student must know about the area and perimeter of a rectangle. If the perimeter is 16, then the sum of one length and one width (the semiperimeter) would be 8. Several rectangles can now be drawn meeting the given condition:

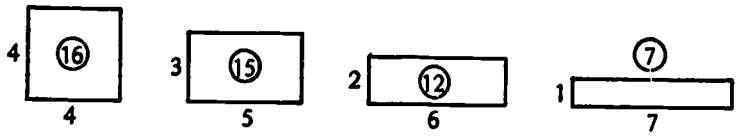



Figure 2–3

Thus, the drawing reveals that Mary's garden was really a square ($4 \times 4$) while Mike's was a rectangle $5 \times 3$.

### 5. Look back and extend

The "answer" is *not* the "solution"! The solution is the process by which the answer is obtained. Therefore, once the answer has been arrived at, there is more to be done. This stage of the process consists of verifying the answer, checking the arithmetic, mentally recording the procedures that were followed, and then looking for extensions.

5a. Verify your answer.
5b. Look for interesting variations on the original problem.
5c. Ask "What if..." questions.
5d. Discuss the solution.

**PROBLEM** Find the length of 1 school desk if the sum of the lengths of 4 such desks is 20 feet.

**Discussion** Notice that in this problem, the word "sum" does not ensure that the problem will be solved by addition.

$$20 \div 4 = 5$$

The answer appears to be 5 feet. Are the units correct? Does the method appear to yield a correct answer? Does the answer "make sense"? Why or why not?
What if the sum of the lengths of the 4 desks had been 22 feet?
Now how long would each desk be?
If the sum of the lengths of the 4 desks had been 18 feet, would each desk be longer or shorter than in the original problem? Why?

**PROBLEM** How many different ways can you add four even whole numbers and get 10 as a sum?

**Discussion** In a previous section, we discussed this problem. However, it lends itself to an interesting variation and extension. *What if* we use the set of integers in place of the whole numbers? Now, the number of answers becomes infinite. Why is this so? This should be discussed carefully.

**PROBLEM** Ila threw 3 darts at the dartboard shown in Figure 2–4. All 3 hit the board, What was her maximum score?

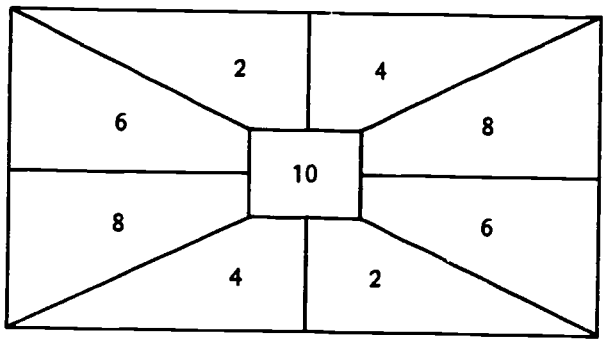



Figure 2–4

**Discussion** Although the given problem merely requires an understanding of the word "maximum," the problem lends itself to several interesting extensions and variations. For example:

(a) How might Ila score 20 points with her 3 darts?
(b) What if Ila's first dart missed the board? What would have been her maximum score now? Her minimum score?
(c) Suppose she hit the board with 3 darts, each hitting a different number. now what might her score have been?
(d) How might Ila have scored 15 points with exactly 4 darts?

**PROBLEM** A menu for a fast-food restaurant is shown in Figure 2–5. How much did Miguel pay for a lunch of one hot dog, one slice of pizza, and one glass of milk?

| MENU | |
|---|---|
| Hot Dog | .85 |
| Pizza (Slice) | .80 |
| Grilled Cheese Sandwich | 1.00 |
| Chicken Nuggets | 1.45 |
| Soft Drinks | |
| Small | .40 |
| Medium | .55 |
| Large | .65 |
| Milk | .45 |
| Desserts | .75 |

Figure 2–5

**Discussion** The problem involves taking the appropriate data from the

menu and adding the prices together. However, once again, the extensions of the problem are more interesting.

(a) Miguel bought 2 hot dogs and a third item. If he spent $2.25, what was the third item Miguel bought?
(b.) How might Miguel have spent exactly $2.05 for lunch?

The usual practice in many textbook word problems is to find the answer, check it, and then go on to the next problem. However, much more can be achieved toward the development of problem-solving ability if the conditions of the problem are altered and the resulting effect on the answer is examined. This provides the student with a much deeper insight into what has taken place in the problem-solving process.

## APPLYING THE HEURISTICS

Now that each step of the heuristic process has been presented, discussed, and illustrated, let's apply the model to several problems. As the solutions are developed, be certain that you are aware of the thought processes being utilized in each step. Remember, problem solving is a *process;* the answer is merely the final outcome.

**PROBLEM** Twelve couples are seated at dinner in a restaurant. The couples are seated at a series of small square tables that can seat one person on each side. The tables are placed end to end so as to form one large long table. How many of these small tables are needed to seat them?

**Discussion** 1. Read

*Describe the setting and visualize the action. Restate the problem. What is being asked? What information is given? What are the key facts?*

The key facts in the problem are "square tables," "twelve couples," "placed end to end," and "seat one person on each side." We are asked to find the number of tables required. Notice that 12 couples translates to 24 people.

2. Explore

*Make a drawing.*

Let the drawing show three tables. Mark an × where each person can sit. Notice that the end tables each seat three people while the inner tables each seat two people:

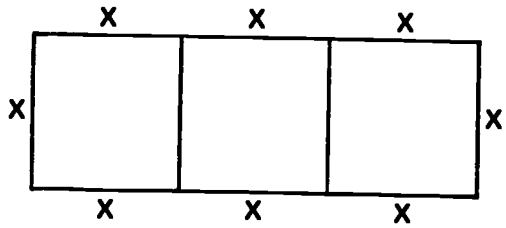



Figure 2–6

The exploration reveals one way in which we can resolve the problem. We could continue drawing tables and placing X's on the available seats until we reach 24.

3. Select a strategy

*Reduction. Make a table. Look for a pattern.*

Let's begin with one table. The drawing in Figure 2–7 shows that we can seat 4 people. Now try two tables; we can seat 6 people.

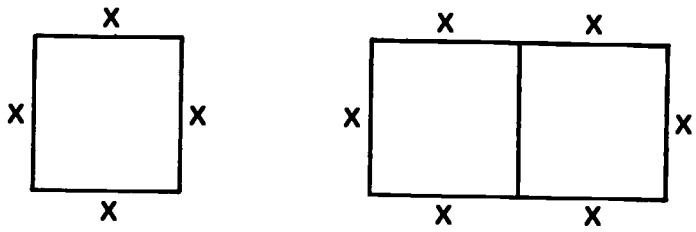



Figure 2–7

Let's record the data in a table:

| | |
|---|---|
| Number of Tables | 1 2 3 ... |
| Number of Guests | 4 6 8 ... |

There seems to be a pattern here; as we add a table, the number of people who can be seated is increased by two. Thus, if we had four tables, we could seat 10 guests. We can now continue the table until we reach 24 people.

4. Solve

*Carry through your strategy.*

The answer is now obtained by continuing the table until we reach the required number of guests.

| Number of Tables | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Number of Guests | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |

It will take 11 tables to seat the 24 people.

5. Look back and extend

*Verify your answer. Look for variations. Ask "What if..." questions. Describe the solution.*

You can verify your answer by solving the problem in a different way. Let's use logic. No matter how many tables are in the row, the end tables each seat 3 people, or a total of 6 people. Each of the other tables can only seat 2 people. Thus, we subtract 6 from 24 (the number seated at the two end tables from the total number of people), leaving 18. These people require 9 tables. Thus, we will need 11 tables. Several "What if..." questions might be asked:

(a) What if two people could be seated at each side of the tables?
(b) What if the tables were placed to form a large square (either open or closed)?
(c) What if there were an odd number of people waiting to be seated at the restaurant?

**PROBLEM** Mrs. Brewster's bicycle store had 25 bicycles and tricycles for rent. She had 7 more bicycles than tricycles. How many of each kind did she have?

**Discussion** 1. Read

The key facts are "25 bicycles and tricycles" and "7 more bicycles than tricycles." We wish to find how many of each kind she has for rent.

2. Explore

The problem tells us that the sum of the two numbers must be 25. We could find two numbers whose sum is 25, and whose difference is 7.

3. Select a strategy

Guess and test! Along with guess and test, we must keep a record of the guesses.

4. Solve

*First guess*: 15 bicycles
10 tricycles

These do add up to 25, but there are not 7 more bicycles than tricycles.

*Second guess*: 17 bicycles
8 tricycles

These also add up to 25, but there are too many bicycles.

*Third guess*: 16 bicycles
9 tricycles

Correct! These do add up to 25 and differ by 7. The answer is 16 bicycles and 9 tricycles.

5. Look back and extend

(a) Could there be 6 more bicycles than tricycles? Why or why not?
(b) What if Mrs. Brewster had 40 bicycles and tricycles, but had 8 more bicycles? How many of each would she now have?
(c) Mrs. Brewster rented all of her 16 bicycles at $8 a day, and 3 of her tricycles at $5 each per day. How much money did she collect that day?

**PROBLEM** Mrs. Glatzer is redecorating her home. She wants to cover the floors in three rooms with tiles that are one foot square. The rooms, all rectangular in shape, measure, in feet, $6 \times 10$, $12 \times 14$, and $11 \times 15$. How many tiles does she need for all three rooms?

**Discussion** 1. Read

There are 3 rooms to be tiled. They are each rectangular in shape. They measure 6′ × 10′, 12′ × 14′, and 11′ × 15′. The tiles are 1-foot squares. We are asked to find the number of tiles.

2. Explore

A tile that is 1-foot square measures 1 foot by 1 foot. Its area is 1 square foot. The total number of squares needed is the sum of the tiles needed for each room. Since the measures of each room are in whole numbers, the tiles will fit exactly.

3. Select a strategy

Divide and conquer. Find the area of each room and then add.

4. Solve

Room 1: 6 × 10 or 60 square feet
Room 2: 12 × 14 or 168 square feet
Room 3: 11 × 15 or 165 square feet
Total: 393 square feet

She will need 393 tiles.

5. Look back and extend

Did you answer the question? Is your answer a number of tiles or an area measure (in square units)?
What if each tile measured 6 inches on a side?
What if the dimensions of the rooms were not whole numbers?

## CHAPTER THREE

# The Pedagogy of Problem Solving

Problem solving is a process. Thus, we must develop a set of heuristics to follow and then be certain to use it! Whether we use the heuristics developed in Chapter 2, the four-step heuristics of Polya, or some other set of seven, eight, or even more steps is not important. What is important is that the students learn a heuristic model, develop an organized set of "questions" to ask themselves, and that they constantly refer to the questions when they confront a problem situation.

What can the teacher do to help the students in developing their own heuristic process and to assist them in becoming good problem solvers? In this chapter, we will present ideas and activities that the teacher can use in the classroom.

**1. Create an atmosphere of success.**

The old adage "Nothing succeeds like success" holds true in the mathematics classroom. If the students are successful in the introductory problems they encounter, they will be more willing to attempt more difficult problems. Choose the problems carefully. Begin with relatively simple problems so as to ensure a reasonable degree of success. If students are successful, they are likely to be "turned on" to problem solving, whereas repeated failure or constant frustration can have a devastating effect on motivation, attitude, and the desire to continue. However, remember that success must be truly earned, not just "given."

The breadth and depth of knowledge required, as well as the sequence of problems chosen, should be kept in mind as major criteria for developing suitable problem-solving situations. Consider the possibility of a series of brief, quickly done problems, each one leading to a more difficult problem, until the original problem-solving task has been completed. This procedure of breaking up a problem into a series of short steps will help those students with short attention spans to enjoy success and to become interested in progress toward the solution of problems.

**PROBLEM** In a class, half of the students are boys. Six boys are present. One-fourth of the boys are present. How many students (both boys and girls) are there in the entire class when all the students are present?

**Discussion** Introduction of this problem might be followed by a series of brief questions, such as:

1. How many boys are present?
2. If this is one-fourth of the boys on roll, how many boys are on roll?

3. What part of the entire class is boys?
4. What part of the entire class is girls?

(Notice that these last two questions are related. Students should realize that one-half of the class being boys implies that the other one-half of the class must be girls.)

Answering these short questions could easily lead the students to the solution of the original problem, namely 48 students.

**PROBLEM** Danny and Nancy left their home at 9:00 in the morning to do some shopping. They got to the mall at 9:30 and spent the next hour in the record shop. They then went next door and had brunch. After 45 minutes in the restaurant, they left the mall and started home. They arrived home in 30 minutes. At what time did they arrive home?

**Discussion** The problem can be simplified by asking questions such as:

1. At what time did they leave home?
2. At what time did they leave the record shop?
3. At what time did they leave the mall for home?
4. How long did the trip home take?

Success in problem solving means more than obtaining the correct answer. When the students become absorbed in a problem and make a sustained attempt at solution, they should be made to realize that this is also success.

**2. Encourage your students to solve problems.**

In order for students to become good problem solvers, they must be constantly exposed to and involved in problem solving. If a student refuses even to attempt to solve a problem, there can be no problem-solving activity taking place. In teaching someone to swim, the "theory" can go just so far; eventually, the real ability to swim must come from actually swimming. It is the same way in problem solving. Students must solve problems! The teacher should try to find problems that are of interest to the students. Listen to them as they talk; they will often tell you about the things in which they are interested. (Problems derived from television and sports always generate enthusiasm among students.)

**PROBLEM** Each lion in the lion house at the local zoo eats 30 pounds

of meat each day. How much meat does each lion eat in a week?

**Discussion** The problem should interest children, since animals have a natural appeal. Most children are not aware of the food consumption of a lion. The problem also requires the child to draw on the fact that there are seven days in a week.

**PROBLEM** Superman and Lothar are mortal enemies. Superman is about to enter Lothar's secret hideaway. Inside, Lothar has 5 "zap" guns, each of which can fire 3 kryptonite bullets at Superman. How many times can Lothar fire a kryptonite bullet at Superman?

**Discussion** Notice the similarity to the basic textbook problem, "Jane wants to buy 3 oranges at 5¢ each. How much money does she need to buy the oranges?" However, the Superman setting is much more appealing to the children.

**ACTIVITY** Take several exercises from the students' textbook. Encourage the students to change the setting of the problem to one that is more interesting to them. Emphasize that the problem must contain the same data as the original.

**Discussion** One effect of this activity will be to force the students to decide what the problem is really all about. At the same time they will be engaged in creating problems similar to the original, but more interesting to them.

Another way to encourage your students to solve problems is to stop occasionally in class in order to analyze what is being done and why the particular processes were undertaken in a particular manner. Focus the students' attention on the larger issue of a general strategy as well as on the specific details of the particular problem at hand. If difficulties arise, make yourself available to *help* students; *do not* solve the problems for them.

**ACTIVITY** Present students with problems that do not contain specific numbers. Ask them to discuss what operations are called for.

*Example:* Jeff wants to buy a pen, a notebook, and an eraser for school. How can he be sure that he has sufficient money?

*Example*: You know the age of an elephant in years and months. How many months old is the elephant?

*Example*: Harry is placing some stamps he just bought into his album. He put the same number of stamps in each row. How many rows did he use?

Try to solve problems in more than one way. Doing this will increase the number of alternative approaches available to the student the next time he or she faces a problem situation.

**PROBLEM** Grace is making 5 bracelets out of beads. Each bracelet contains 3 rows of beads. Each row contains 18 beads. How many beads does she need for the 5 bracelets?

**Discussion** One way to solve this problem is to make a drawing and actually count the number of beads. Some children may count the number of beads in one bracelet and then multiply by 5.
Another way would be to determine that there would be $5 \times 3$ or 15 rows of beads. Since each row contains 18 beads, we multiply $15 \times 18$ to find the answer.
Still another way is to determine how many beads are needed for one bracelet ($3 \times 18 = 54$) and multiply that answer by 5.

**PROBLEM** Grace worked on her bracelets for 30 minutes every morning and 1 hour every evening. How long did she work on the bracelets in 1 week?

**Discussion** One way to solve this problem is to determine how long she works each day, and multiply by 7. Another approach is to determine how long she worked in the 7 mornings, how long she worked in the 7 evenings, and then add. (This problem is an excellent illustration of the distributive principle of multiplication over addition.)

### 3. Teach students how to read the problem.

As we have said before, every problem has a basic anatomy. This anatomy consists of four parts: a setting, facts, a question, and distractors. (In some introductory problems, distractors should be omitted.) Since most problems that students are asked to solve in school are presented to them in written form, proper reading habits are essential. Students must be able to read with understanding.

**ACTIVITY** It is important to alert your students to the fact that not all mathematics is read from left to right. The eye views simple expressions such as

$$\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

in a variety of different directions, as shown by the following "arrow diagram":

$$\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

Prepare a transparency containing several different mathematical expressions. Have students draw correct arrow diagrams for each one.

Fundamental to any problem is an understanding of the setting. If the setting is unfamiliar to the students, it will be impossible for them to solve the problem. Some time should be devoted to merely having the students relate what is taking place in a problem setting. Leading questions can be asked to help.

**PROBLEM** Bobby and Susan are picking strawberries. Bobby picked 6 pints while Susan picked 4 pints. How many pints did they pick together?

**Discussion** Although this problem seems simple to an adult, it is not necessarily simple to youngsters. Ask the children to describe what is going on in the problem in their own words. Questions may be asked to help extract this information. For example:

(a) How many people are involved?
(b) What are they doing?
(c) Where is the action taking place?

(Some students might answer this last question by saying that it takes place at the fruit stand. This is also correct.)

Many activities should be used to help students sharpen their ability to read critically and carefully for meaning. One such technique is to have the students underline or circle words that they consider to be critical facts in a problem. Discuss these words with the class. Have students indicate why they consider these particular words to be critical.

**PROBLEM** Scott has two dogs. Charcoal weighs 40 pounds, while Koko weighs half as much. How much does Koko weigh?

**Discussion** In this problem, the critical facts are "Charcoal weighs *40 pounds,*" and "Koko weighs *half as much.*"

**ACTIVITY** Write a problem on a slip of paper. Have one student read the problem silently, put it away, and then relate the problem in his or her own words to the rest of the class. In this way, students often reveal whether they have found the facts that are really important to the solution of the problem or whether they have missed the point entirely.

**ACTIVITY** Show a problem on a transparency on the overhead projector. After a short period of time, turn the projector off and have the class restate the problem in their own words.

**ACTIVITY** One way to encourage practice in reading mathematics problems slowly and for understanding is to mimeograph a page from a mathematics textbook, cut the page into pieces much like those of a jigsaw puzzle, and have the students put the page together again.

Since many words have a special meaning in the mathematics classroom that is different from the regular, everyday meaning, the class should discuss a list of such words together with their various meanings. A beneficial project is to have the students compile a "dictionary" in which each word is defined both in mathematical and in other contexts.

**ACTIVITY** Discuss the different meanings of the following words:

| | | |
|---|---|---|
| times | volume | prime |
| difference | foot | order |
| pound | face | figure |
| count | chord | |

After the students have gained an understanding of a problem and can relate it in their own words, it is necessary for them to be able to identify the question to be answered.

**ACTIVITY** Give students a set of problems and have them circle or underline the sentence that tells them what they must find. Note that this "question" may be in interrogative form or stated in declarative form.
*Example*: Andrea went to the hobby shop and spent 59¢. How much change will she receive from a $1 bill?
*Example*: Michael has just planted a new garden. He wants to put a rope around it to keep people from walking on it. Find the amount of rope he will need if the garden is in the shape of a square 10 feet on each side.

**ACTIVITY** Every problem must have a question in order to be a problem. What's the Question is an activity that requires the student to supply a reasonable question based on a given situation. Asking students to do this forces them to analyze the situation and understand what is given. This is a needed skill for problem solving.
*Example:* Ralph has 6 oranges, Marlisse has 9 oranges, and George has 15 oranges.
Make this a problem by supplying the question.

*Example:* Howard and Donna drove to Georgia from their home, a distance of 832 miles. On the first day they drove 388 miles and bought 38 gallons of gasoline.
Make this a problem by supplying the question.

The information necessary to solve a problem sometimes appears in verbal form. Other times, it may appear in picture form. Give the students activities similar to the ones that follow to help determine which facts in a problem are important.

**ACTIVITY** Read the following paragraph. Then answer the questions.

Mr. and Mrs. Rogers and their three children went to the movies to see "Gulliver's Travels." Tickets were $5 for adults and $3 for children. The show lasted 2½ hours. They left the theater at 4:00 p.m.

1. What was the name of the movie they saw?
2. How much do adult tickets cost?
3. How much do children's tickets cost?
4. How long did the show last?
5. How many members of the Rogers family went to the movies?
6. At what time did the show end?
7. At what time did the show begin?

**ACTIVITY** Look at the toys in Figure 3–1. Then answer the questions.

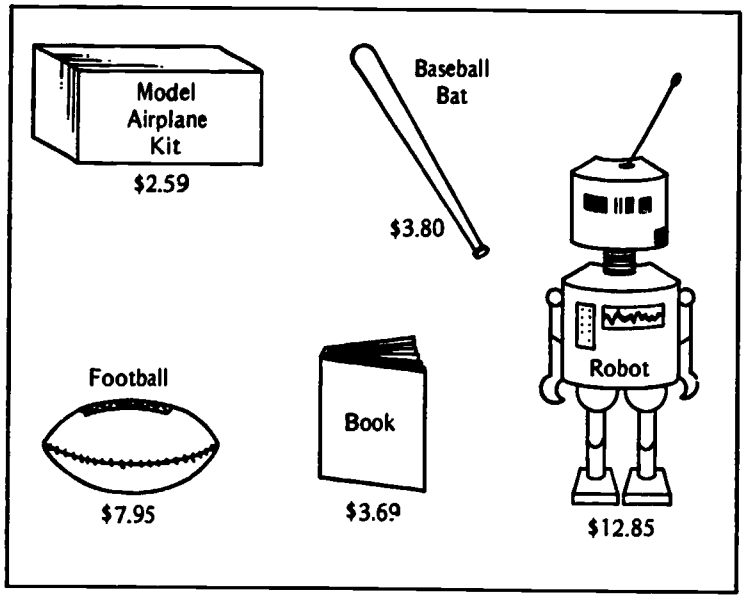



Figure 3–1

1. Which toy costs $2.59?
2. How much does the football cost?
3. Which toy is the most expensive?
4. Which toy is the least expensive?
5. Which toys cost less than $5.00?
6. Which toys cost more than $6.00?
7. Miriam has $10 to spend. Which toys can she buy?

Most of the questions in these two activities can be answered directly from the statements in the text or the facts in the drawing. However, some of the questions may require either some computation or use of inference. To many of us who are more experienced than the students, these inferences may be immediate. It is most important that the students learn to differentiate between what is given directly and what can be extrapolated from the given information.

**ACTIVITY** Read the following paragraph. Then answer the questions.

In the Watkins family, there are 4 children. Cynthia is 6 years old. Her twin sister, Andrea, is 3 years older than her brother, Matthew. Their brother James is the oldest, and is 3 times as old as Matthew.

1. How many children are in the Watkins family?
2. How old is Cynthia?
3. Who is the oldest child?
4. How old is Andrea?
5. How old is Matthew?
6. How old is James?

Reading a problem also means being able to discriminate between necessary and unnecessary information. In many cases, information is put into a problem to serve as a distractor. In other cases, necessary data may have been omitted. We strongly recommend that students be given activities that will enable them to distinguish between necessary and superfluous data, as well as to determine when there are insufficient data to solve the problem. Students should be asked to supply the necessary facts when they are missing from a problem.

**ACTIVITY** Give the students a problem in written form. Include one piece of extra information. Tell the students to cross out what they think is unnecessary. Have them read what remains. Can they now solve the problem? Repeat the activity, but add a second distractor to the problem. Have them now cross out both pieces of extra data.
*Example*: Summer is 3 months long. Winter is 3 months long. How many months of the year are not summer?

*Discussion*: The students should cross out the sentence "Winter is 3 months long." Notice that this problem requires the student to know that there are 12 months in a year.

*Example*: The Janeway School was having a cake sale. Each cake sold for $3.50. Louise sold 8 cakes. Martin sold 7 cakes. Karen sold 4 cakes. How many cakes did Louise and Karen sell together?
*Discussion*: The students should cross out the price of the cakes ($3.50 each) and the number of cakes that Martin sold.

**ACTIVITY** Give the students a problem in written form. Omit one piece of necessary information. Have one student identify what is missing. Then have a second student supply a *reasonable* fact so that the problem can be solved. Now have the class solve the problem.
*Example:* Barbara's family stayed at the seashore four days longer than Roger's family. How long did Barbara's family stay at the seashore?
*Discussion*: The students should recognize that the answer depends on the length of time that Roger's family stayed at the seashore. Be certain that the number of days they supply is reasonable.

**ACTIVITY** Prepare a collection of problems on 3″ × 5″ cards. Some of the problems should have excess information. Others should have missing data. Students must decide which, and supply the data needed to solve the problem if the facts are missing.

**ACTIVITY** Problem Reader–Problem Solver is an activity that helps to sharpen students' ability to read and comprehend a problem quickly and accurately. It is particularly effective in helping students ascertain the important elements of a problem. The students in the class are divided into teams of four. One pair of students on each team is designated the Problem Readers, while the other pair is designated the Problem Solvers. The Problem Solvers close their eyes while a problem is displayed via the overhead projector for about 30 seconds. (The time will depend on the ability of the students and the difficulty level of the problem itself.) During this time, the Problem Readers may take any notes or make any drawings that they deem necessary. The problem is then taken off the overhead, and the Problem Readers present the problem (as they saw it) to their partners who must solve the problem. The Problem Readers and the Problem Solvers then reverse roles and play the game again.

**ACTIVITY** Divide the class up into three or four teams consisting of five to eight students on each team. Tell the students that they are going on a mathematical scavenger hunt. Each team must find problems in their textbooks as asked for on the list of items that they will receive. Present each team with the same list of questions, similar to the following:

1. Find a problem that has a setting in a supermarket.
2. Find a problem that deals with sports.
3. Find a problem where the answer is an amount of money.
4. Find a problem where some of the information is given in a picture.
5. Find a problem where the answer is in hours.
6. Find a problem that contains too much information.
7. Find a problem that contains insufficient information.
8. Find a problem where subtraction is used to find the answer.

Activities such as these will help your students to become better readers of problems.

**4. Involve your students in the problem.**

It is important that we involve the elementary school student in the problem-solving process, both physically and mentally. Create problems that require action, and let the students be the actors. Involve the students in activities such as surveys and shopping expeditions. Have them record their own data. Let them experiment!

**PROBLEM** How far can you walk in one minute?
Who is the fastest walker in your class?

**PROBLEM** How many M&M candies are in one pound?
How many of these are yellow?

**PROBLEM** How many students in your class have the letter "R" in their first name?

**PROBLEM** How many times can you bounce a basketball in one minute?

Notice that these problems allow the children actually to get into the problem. They become a part of the story. Out come stopwatches, packages of candy, basketballs, and so on. The entire class becomes actively engaged in the problem-solving process.

**PROBLEM** Three boys stood on a scale and put a nickel in the slot. The scale showed 205 pounds as their total weight. One

boy stepped off the scale. It then showed 140 pounds. The second boy stepped off the scale and it then showed 85 pounds. Find the weight of all three boys.

**Discussion** In class, you could ask three boys actually to act out this problem. When all three boys are "on the scale," show a sign that reads 205 pounds. Have one boy "step off." Now show a sign that reads 140 pounds. Continue the action.

Using manipulable materials in problem solving is another way to enable your students to become active participants. Students must work directly with the materials to solve problems; they cannot just sit back and be spectators. The materials can easily be stored in shoeboxes or in large envelopes, along with a series of activity cards posing the problems.

**PROBLEM** How many 3-rod trains can you make that are equal in length to one orange rod?

**Discussion** Notice that this is the same problem as "How many different ways can you select three natural numbers such that their sum is 10?"
Yet it involves the use of concrete materials.

**PROBLEM** On a geoboard, use rubber bands to construct several figures whose area is 16 square units. How many can you find?

**PROBLEM** Figure 3–2 contains pictures of three figures that were

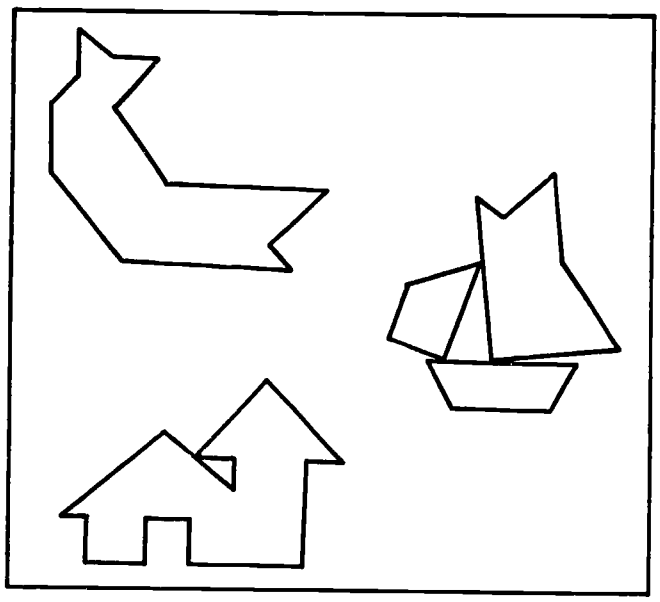


Figure 3–2

made with the seven tangram pieces. Try to make each one using all seven pieces. Record your results on paper.

**5. Require your students to create their own problems.**

Nothing helps children to become better problem solvers than having them make up their own problems. In order to create a problem, the students must know the ingredients. They must relate setting, facts, questions, and distractors. We know that children create excellent problems. In fact, their problems are usually more relevant (and often more complex) than the ones typically found in textbooks.

However, in order to generate problems, students need something to write about. Presenting a suitable stimulus will aid pupils in this endeavor.

**ACTIVITY** Show pictures that have been taken from old magazines, old catalogues, old textbooks, etc. Have the students make up a story problem to fit each picture.

**Discussion** This activity helps students learn to decide on numerical data that makes sense, for they must use realistic numbers in their problem designing. At the same time, this activity helps students to integrate mathematical problems with their other subjects, such as social studies, language arts, and science.

**ACTIVITY** Ask your students to write a "menu problem." That is, given the following menu, write a problem about it:

| | |
|---|---|
| Hot dog | .95 |
| Hamburger | 1.25 |
| Pizza (slice) | .85 |
| Tuna sandwich | 1.25 |
| Grilled cheese sandwich | 1.05 |
| Apple | .25 |
| Banana | 2 for .45 |
| Milk (white) | .30 |
| Milk (chocolate) | .35 |
| Candy bar | 2 for .65 |

**Discussion** At first, many students will probably write a problem that merely lists an order of two or more items and ask for the cost. A higher level of problem may be given by some students in which the amount of change from a large bill is asked for. A third-level problem might involve stating the total amount spent and asking for what could have

been purchased. (Students may be surprised when multiple answers appear.) Adding the tax may complicate the problem even further.

**ACTIVITY** Provide students with a set of answers to problems, such as 15 books, $7.32, 8 miles, 12½ gallons, etc. Have them create problems for which each of these is the correct answer.

**ACTIVITY** Have the students create problems that involve each of the computational operations. For example, make up a story problem that uses the arithmetic sentence $4+8=12$ in its solution.

Select several of the students to present their problems to the class. Have the entire class solve the problems. Sharing problems that have been written by other students should be an integral part of your classroom procedure. The fact that the problems have been designed by classmates usually heightens interest in solving them. These problems may simply be variations of other problems that students have seen, or they may be entirely original creations.

As the students gain experience in creating their own problems, the problems will become more sophisticated. There will be some with insufficient information and some with excess information. This is highly desirable because the problem-solving process is what should be stressed. Problems that appear in textbooks often emphasize one particular skill or operation. On the other hand, these student-generated problems frequently involve extraneous data and possibly more than one operation.

**PROBLEM** How many slices of pizza will four children eat?

**PROBLEM** How many apples will Mrs. Rose have in 14 boxes of apples?

**PROBLEM** Jeff ran a 50-meter race in 15 seconds. Larry beat Jeff to the finish line by running 50 meters in 12 seconds. Who won the race?

**PROBLEM** Jan has 84 sea shells in her collection. She decided to give five shells to each of three friends. How many shells did she give away?

**PROBLEM** An ostrich egg weighs 48 ounces. How many duck eggs would it take to weigh this much?

**PROBLEM** Pat gives three free nutcrackers for each 30 you order. How many free nutcrackers would you receive if you order nutcrackers every month for 24 months?

Working these student-generated problems will involve a considerable amount of discussion. This will be time well spent! Remember, the answer is not the most important aspect of problem solving. The answer is only the vehicle to the process.

**6. Have your students work together in pairs or in small groups.**

Team problem solving and group brainstorming are viable techniques in the business world. Rarely does any one person solve major problems alone. While the final decision does fall on one person, the group input helps in the problem-solving process. A student's inability to help in the group process can be a direct hindrance to decision making.

The groundwork for brainstorming can be laid in the elementary school grades. The classroom teacher must provide guidance and practice in the particular skills involved in sharing ideas. The teacher should encourage *all* students to contribute to the discussion. Keep in mind that in brainstorming:

1. There should be no evaluation of any kind.
2. Everyone is encouraged to allow his or her imagination to run rampant.
3. Students are encouraged to put forth as many ideas as possible.
4. Everyone is encouraged to build on the ideas or to modify the ideas of others.

The important task is to work together toward solving the problem.

The interaction between the students will help them learn to modify one another's thinking and to clarify their own. They will also learn to express their thoughts more clearly by the use of precise language, especially mathematical terminology. Students will find it difficult to communicate with others unless they use language that every member of the group can agree on.

The group may want to focus on a problem-solving situation as a series of motion picture frames. The sequential nature of this image helps students decide what comes first, what comes second, and so on. With contributions from various members of the group, the class can develop a sequence of activities to solve the problem.

ACTIVITY Divide your class into groups of five or six students. Each group should be provided with a pair of totally unrelated words, such as:

(a) pencil–apple
(b) elephant–lollipop

(c) sunglasses–flashlight
(d) radio–cereal

Notice that the words in each pair do not appear to have anything in common. It is the group's task to connect the two words in a problem. Be creative, use your imagination.

**ACTIVITY** "Notice" is a quiz that is administered to students in groups. The entire group must arrive at a single true or false decision for each statement. Here are some sample statements:

(a) The Statue of Liberty uses her right hand to hold the torch. (True)
(b) A record on a turntable will turn clockwise. (True)
(c) Page 82 of a book is a right-hand page. (False)
(d) Most pencils have eight sides. (False)
(e) Q is the only letter that is missing on a telephone. (False)

**ACTIVITY** Students are divided into groups of five or six. Each group is given the task of building the highest possible "tower" out of the available materials in a fixed time period. The group must decide who will be responsible for the various tasks required to build the tower. Materials might consist of the following: small boxes (cereal boxes, for example), construction paper, adhesive tape, rolls from inside paper towels, pieces of Styrofoam, metal scraps, pipe cleaners, etc. The only restriction is that the tower may not be attached in any way to the walls or ceiling for support.

Problems in which students have to list all the possible outcomes lend themselves well to the group discussion method. It is the group's decision when all the possible outcomes have been listed.

**PROBLEM** Your group has been given $1,000 to spend in a 24-hour period. How would you spend the money? (You may buy only one of each item selected.)

**PROBLEM** Your group contains five members. How many different pairs of students can you form from the students in your group?

While we do not advocate that students solve all problems in groups, the group process is a good method for allowing students to develop a respect for one another's abilities and to learn to look for many possibilities in solving problems. It also permits the children to share and refine ideas.

**7. Encourage the use of drawings.**

One goal of school mathematics is to bring children to the abstract level of reasoning. Simulation is an intermediate step between the concrete and abstract stages. Simulating an activity by means of materials and drawings is the stage that naturally follows experimentation. Students must be taught to use paper and pencil drawings to simulate action. These drawings need not be exact in their representation, but they must be neat, accurate, and carefully drawn. Students should draw carefully labeled diagrams. Perpendiculars should look as if they form 90° angles; equal lengths should be drawn approximately equal. Directions should be carefully indicated.

This means, too, that the teacher must serve as a model for the students. When drawing diagrams at the board, make them carefully, but without the use of tools that differ from those the students might use. Although rulers should not be used, a straightedge might be allowed. Practice in making freehand drawings is essential, since few teachers are so artistically inclined that they can draw well the first few times they attempt freehand drawings.

**ACTIVITY** Distribute a series of problem situations that can be described by a drawing. Have each student make a drawing to illustrate the action. Discuss the various drawings they have made.

*Example:* Paula walked 5 blocks due north from her house. She then walked 4 blocks due east, and then 5 blocks due south. How many blocks is she from her home by the shortest path?

The drawing in Figure 3–3 illustrates the problem setting and reveals that the figure formed is a rectangle.

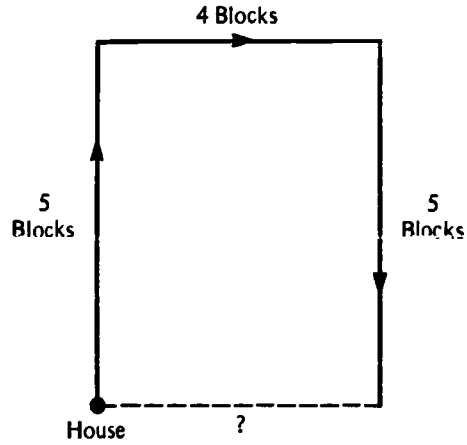



Figure 3–3

**ACTIVITY** Distribute several drawings that illustrate problem situations. Have your students make up a problem for which the given sketch is appropriate. Discuss these problems with the entire class.

*Example*: Although the problems that the students will create may vary, here is one possible problem suggested by the drawing (Figure 3-4):

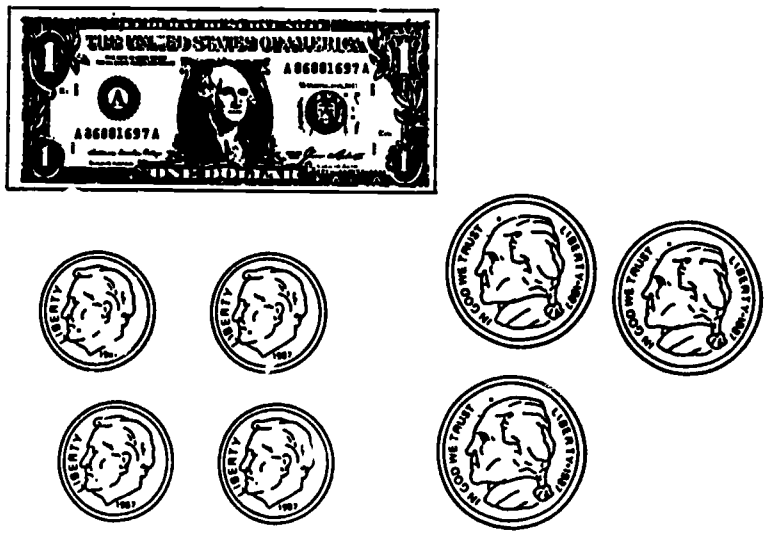


Figure 3–4

At the end of the day, Joshua put all of his money on the table. He had a $1 bill, 4 dimes, and 3 nickels. How much money did he have?

*Example*:

5 miles ——————————>

3 miles ————————>

One problem that a student might create from this drawing is:

One movie theater is 5 miles from Gregory's house. The second movie theater is 3 miles from his house in the same direction. How far apart are the two theaters?

**PROBLEM** A snail is at the bottom of a fish tank that is 16 inches deep. Each day the snail climbs up 3 inches. Each night it slips back 2 inches. Does the snail get out of the tank in two weeks?

**Discussion** Although the problem sounds complex, a drawing of the situation in Figure 3-5 reveals the method for obtaining an answer.

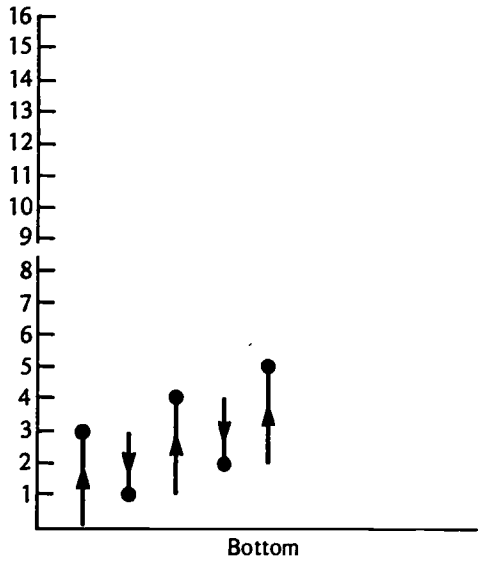



Figure 3–5

**8. Suggest alternatives when the present approach has apparently yielded all possible information.**

It often happens that a chosen strategy fails to provide an answer. Good problem solvers do not get discouraged! Instead, they seek an alternative path. Many students, however, may continue the same approach even though it does not yield an answer. This is a predisposing condition or mind-set that usually leads to the same end over and over again, by blocking out any kind of variable behavior. This mind-set must be changed and some other approach undertaken if the student has not successfully resolved the problem. Many students give up or follow the same path again and again. It is at this point that some teachers err; they often direct the students through the most efficient path to the solution, rather than allowing further exploration. The teacher should guide the additional exploration by pointing out facts and inferences that might have been overlooked.

**ACTIVITY** Present a problem to the class. Have students work in groups and attempt to solve the problem in as many different ways as possible. Have each group present its solutions to the entire class. See which group can find the greatest number of different solutions.

**PROBLEM** How many squares are there on a 3-inch by 3-inch checkerboard?

**Discussion** Many students will quickly answer "Nine!" (See Figure 3–6.) Even if they are told that this is not the final answer, they often respond by counting the nine 1-inch squares that are most obvious in the figure. At this point, you should point out that the entire board is also a square, and then ask if there are squares of any other sizes on the checkerboard?

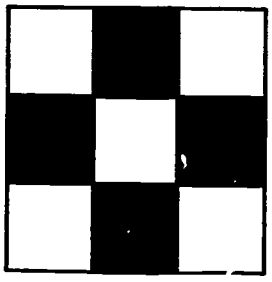


Figure 3–6

**PROBLEM** Janina has a piece of wood that is 32 inches long. She cuts off an 11-inch piece. How many 3-inch pieces can she make from the wood she has left?

**Discussion** Students will divide 32 by 3 and give an answer of $10\frac{2}{3}$ pieces. Others will make a drawing to represent the 32-inch piece of wood and mark off 3-inch pieces. It may be necessary for you to point out that the problem will not work unless the 11-inch piece is *first* removed from the 32 inch piece of wood.

**PROBLEM** A man left 17 horses to be divided among his three children. The oldest was to receive half of the horses, the youngest was to receive one-ninth of the horses, and the middle child was to receive $\frac{1}{3}$ of the horses. How were the 17 horses divided up?

**Discussion** This is a classic puzzle that has been used to baffle students for many years. Seventeen does not contain 2, 3, and 9 as factors. Students will probably not find the answer. They will not notice that $\frac{1}{2}+\frac{1}{3}+\frac{1}{9}\neq 1$(the whole). In fact, the three fractions add up to $\frac{17}{18}$. After the students have been baffled for a while, "lend" them one additional horse. Suggest that they consider the situation where 18 horses are used. Now, one-half of 18 = 9, one-third of 18 = 6, one-

ninth of 18=2, and 9+6+2=17 (and you now take that eighteenth horse back!).

This next example is a very interesting problem, but a successful solution depends on geometric facts that your students may not yet possess.

**PROBLEM** In an office, there are two square windows. Each window is 4 feet high, yet one window has an area twice that of the other window. Explain how this could take place.

**Discussion** The usual way to envision a square window is with the sides parallel to and perpendicular to the floor. However, if we consider a window that has been rotated through 45°, as in Figure 3–7, we can readily see the explanation.

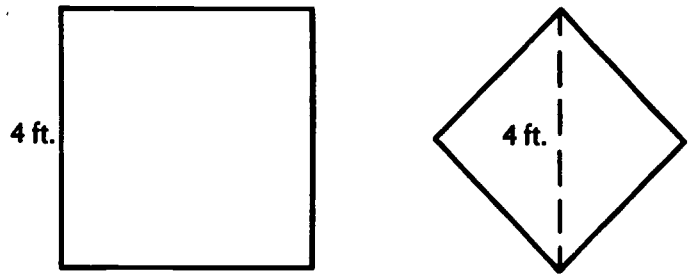



Figure 3–7

When students are stuck, you might suggest that they look back at other problems they have solved in the past that were similar to the problem under consideration. This might lead to some ideas of what to do. Even a suggestion as to what might be done at a particular point is sometimes in order. Thus, you could suggest to students that "it might be a good idea if they":

1. looked for a similar problem whose solution they know
2. made a guess and checked it
3. tried a simpler version of the problem
4. made a table
5. drew a diagram
6. used a physical model
7. used a calculator
8. worked backward
9. looked for a pattern
10. divided the problem into several parts and solved each
11. used logical thinking.

Even encouraging students simply to pause and reflect carefully on the problem is a good technique to try when students are totally stymied.

**9. Raise creative, constructive questions.**

Teachers often commit a basic error. In the interest of time, they often take the students directly to the answer by showing them a finished solution. (This does not help the students become problem solvers.) Instead, you should ask questions that will provide the students with guidance and direction, and will also allow for a wide range of responses. Give them time to think before they respond to your questions. Research indicates that the average teacher allows less than 3 seconds for students to respond to a question. Problem solving is a complex process. You must allow time for reflection. Don't rush your questions.

In trying to guide your students through a solution, use open-ended questions frequently. Implied questions such as "Count the number of...," "Find all...,"or "How many..." are all non-threatening questions that lead to successful student responses. In addition, as we have stated before, the use of "What if..." questions is important to their full understanding of the problem.

**PROBLEM** How many triangles can you find in Figure 3–8?

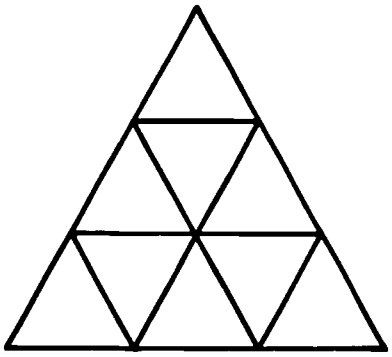


Figure 3–8

**Discussion** Since the question asks "How many triangles can *you* find ...," it cannot be answered incorrectly, even if the student can only find a minimal number of triangles.

Throughout the problem-solving process, let your questions cause the students to reflect back on the problem. Too often we tend to turn away from a problem that has been "solved" (i.e., for which an answer has been found) in order to move on to the next problem. Thus, we miss a chance to glean extra values from our energy. Examine the solution carefully; ask questions about key points. Ask many "What if..." questions. Ask students "What new question does this suggest?" or "How else might I ask this question?"

Other questions might include:

1. Do you recognize any patterns?
2. What is another way to approach the problem?
3. What kind of problem that you've seen before does this problem remind you of?
4. What would happen if...
   - the conditions of the problem were changed to...?
   - the converse was asked?
   - we imposed additional conditions?
5. What further exploration of this problem can you suggest?

**PROBLEM** Gladys uses 9 shells to make a game. She wrote the numeral 1 on the first shell. The second shell has the numeral 4 on it, and the third shell has a numeral 7. If Gladys continues in this way, what numeral will be on the last shell?

**Discussion** You might ask some of the following questions in order to stimulate the class discussion:

(a) Do you recognize a pattern? What is it?
(b) What if Gladys only had 4 shells?
(c) Can you draw a picture of all 9 shells?
(d) What numeral would you put on the fourth shell?

When asking any questions of students, be careful that:

1. You do not change or alter the question while the students are considering it.
2. You give the students ample time before repeating the question.
3. You do not answer your own question, even when you are certain that the students have finished their responses. Perhaps then an additional hint or comment might lead them in the right direction.

**10. Emphasize creativity of thought and imagination.**

In a positive classroom atmosphere, students can be as free-thinking as they wish. You should not penalize "way out" answers if they show some thought on the part of the students. Again, keep in mind that it is the problem-solving *process* that is important!

**ACTIVITY** A student's response to the question "How can I divide 25 pieces of candy among three people?" was "I'll take 23 pieces and give 1 to each of my friends." Discuss the answer.

**ACTIVITY** Karen drove from New York to Philadelphia in 3 hours, a total distance of 120 miles. How fast did she drive? Discuss this problem.

The two activities suggested above will often yield answers that are quite different from what was expected. Students can interpret what is given and what is asked for in a variety of ways. Each way may be different in meaning, yet quite appropriate in thought. In all cases, you must lead the class in a thorough discussion of *why* these various interpretations took place.

Systematic trial and error and careful, selective guessing are both creative techniques to use. (How often have teachers been heard to say to a student, "Do you *know*, or are you just guessing?") Guessing, or careful trial-and-error reasoning, should be practiced and encouraged. It is important to be a good guesser.

**PROBLEM** I am thinking of two 2-digit numbers. They have the same digits, only reversed. The difference between the numbers is 54, while the sum of the digits of each number is 10. Find the two numbers.

**Discussion** While students in the upper grades might complete this problem by solving a pair of linear equations in two unknowns simultaneously, creative use of guess and test makes this problem suitable for a problem-solving activity at lower levels as well. The teacher could suggest the following steps:

1. List all the 2-digit numbers whose digit sum is 10:

   19, 28, 37, 46, 55, 64, 73, 82, 91

2. Subtract pairs which have the same digits:

$$91-19=72$$
$$82-28=54$$
$$73-37=36$$
$$64-46=18$$

At this point, you might consider asking the students how they know that 82 and 28 are the *only* pair of 2-digit numbers that satisfy the original problem.

**PROBLEM** Peggy and Barbara each want to buy a turkey for Thanksgiving dinner. Peggy needs a larger turkey, since she has a bigger family. The butcher has two turkeys left. He tells the women that "Together they weigh a total of 20 pounds, and the smaller one costs 48¢ a pound, while the larger one costs 46¢ a pound." Together they spend $9.34 for the two turkeys. How much does each turkey weigh?

**Discussion** Let's solve the problem with some imagination, assisted by systematic trial and error. Suppose we assume that the smaller turkey weighs 8 pounds. Then the larger turkey must weigh 20 − 8 or 12 pounds. The cost for an 8-pound turkey would be $3.84, while the cost for the larger turkey would be $5.52. Thus, their total cost would then be $9.36, which is wrong. Our next try should use a smaller weight for the smaller bird. In one or two more tries, we should arrive at the answer of 7 pounds and 13 pounds, respectively.

Puzzle problems that involve practice in arranging and rearranging numbers are helpful in developing student skills at organized guessing.

**PROBLEM** Which disk in Figure 3–9 would you move to another box so that all three boxes would then have a sum of 15? Show your move.

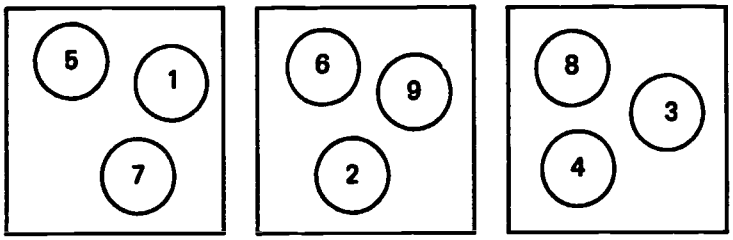



Figure 3–9

**PROBLEM** Place the numbers 1, 2, 3, 4, 5, and 6 in the spaces provided in Figure 3-10 so that each side of the triangle shows a sum of 10.

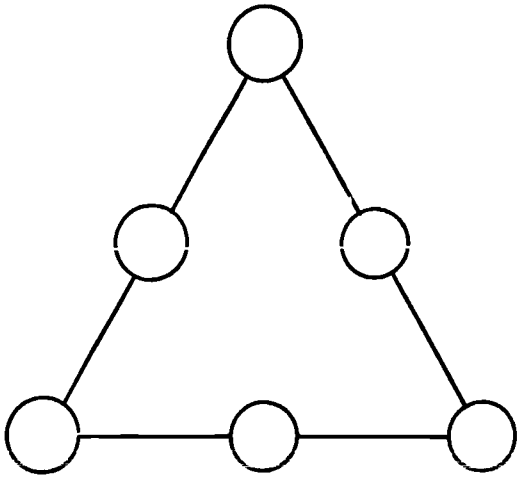


Figure 3–10

**PROBLEM** Suppose you have a standard 8×8 square checkerboard and 32 dominoes. Each domino covers exactly two squares on the board. Thus, you can cov the entire 64 squares with the 32 dominoes. Now, remove one square from each of two opposite corners on the board, and take away one domino How would you cover the remaining 62 squares with the 31 dominoes?

**Discussion** Copy the 64-square checkerboard shown in Figure 3–11 onto a sheet of paper, one for each student. Give the students 32 dominoes and let them actually cover the 64 squares. Now have them cut off the appropriate squares and throw away one domino. (See Figure 3–11.)

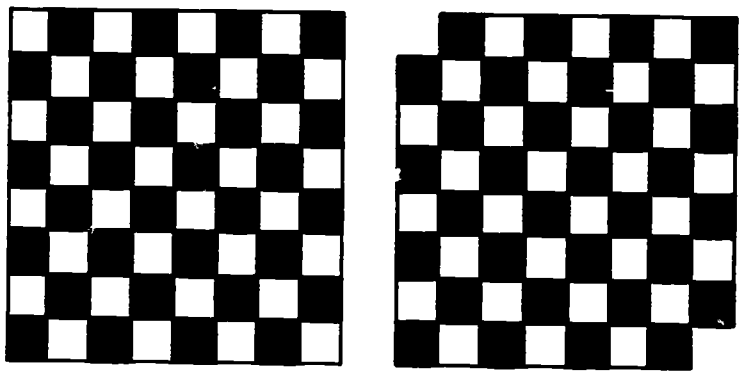


Figure 3–11

After they try to solve the problem by doing it, discuss the problems that the students have run into. After a while, ask about the colors of the squares involved. Lead them to see that a single domino covers one black and one white square. They should then note that they have cut off two white squares (or two black squares if they cut the opposite set of corners). Thus, they have left 32 black squares and 30 white squares. The dominoes cannot cover the checkerboard as described.

**PROBLEM** You are planning a class party for 20 children. You have $10 to spend. Popcorn costs 80¢ a bag, soda costs 40¢ a can, pretzels cost $1.10 a bag, and ice cream costs $2.00 a half-gallon. What would you buy?

**11. Emphasize estimation.**

With the increasing prominence of technology, particularly the hand-held calculator, estimation has become a very important topic

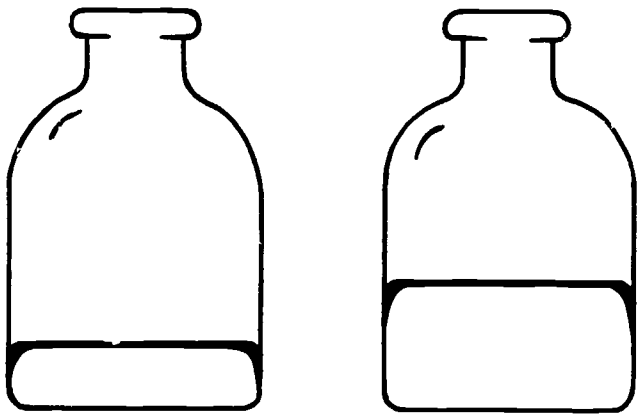
in school mathematics. It also plays an important role in problem solving. Students must know if their answers to problems are "in the right ballpark."

**ACTIVITY** Find a book with a large number of pages. Note the number of pages in the book. Place a bookmark anywhere in the book. Have the students look at the bookmark and guess the number of the page it is marking. Try this several times. Have students keep a record of how many times their guesses were within 10 pages of the bookmark.

**ACTIVITY** Ask the students to close their eyes and guess how long a minute is. Check them with a clock. See if they can come within 15 seconds, then within 10 seconds, then within 5 seconds. Have them record how close they come each time.

**ACTIVITY** Have the students look closely at the classroom. Have them estimate its length, its width, and its height. Check the student guesses by having them actually measure as much as they can, using a tape measure, yardstick, or other measuring devices.

Drawings can also be used to provide students with practice in estimation skills. Figures 3–12 and 3–13 will give the students an opportunity to estimate. In each case, they must establish a reference base.

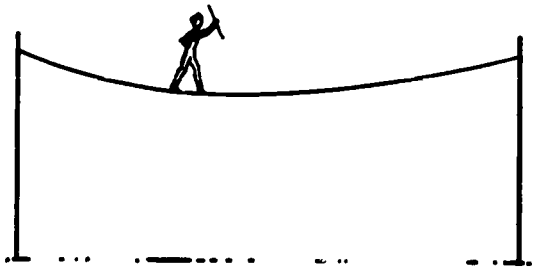


This jar contains 10 ounces of oil.

How many ounces of oil does this jar contain?

(Answer: Approximately 20 ounces)

Figure 3–12



The tightrope walker has walked 20 feet. How many more feet must he walk to reach the other side?

(Answer: Approximately 40 feet more)

Figure 3–13

ACTIVITY Prepare a sheet of "Is It Reasonable?" problems for the students. They are not to solve the problems. Rather, they are to decide if the answer given is reasonable. Here are some samples you might use.

1. A hurricane is moving northward at the rate of 19 miles per hour. The storm is 95 miles south of Galveston. Approximately how long will it take the hurricane to reach Galveston?
   *Answer*: It will take the storm 5 minutes to reach Galveston. (Not reasonable—it will take approximately 5 *hours*, not 5 minutes.)
2. A cog railroad makes 24 round trips each day up the side of a mountain. On Monday, a total of 1,427 people rode the cog railroad. About how many people rode on each trip?
   Answer: About 60 people rode on each trip. (Reasonable.)
3. During the baseball season, 35,112 fans went to a Red Sox–Yankees game one Sunday, while 27,982 fans saw the Phillies–Braves game that same day. About how many more fans were at the Red Sox–Yankees game?
   Answer: About 700 more fans were at the Red Sox–Yankees game. (Not reasonable—it should be about 7,000.)

Activities such as these will help your students improve their ability to estimate answers. Most of all, such practice will encourage them to "take a guesstimate." In itself, this is an important part of problem solving.

**12. Encourage your students to use a calculator.**

Over 200 million hand-held minicalculators have been sold in the United States. The majority of your students probably either own a calculator or have access to one owned by someone within their immediate household. You should not use a calculator to replace the computational algorithms that are being developed in these grades. However, you can provide conceptual experiences long before the student makes the generalization, and concepts can often be extended to the real world through the use of the calculator. All students can add, subtract, multiply, and divide with a calculator. They can work with problems that are interesting and significant, even though the computations may be beyond their paper-and-pencil capacity. The focus can now be on problem solving for problem solving's sake; strategies and processes can be emphasized, with less time devoted to the computation within the problem-solving context.

**PROBLEM** Place the digits 1, 2, 3, and 4 in the boxes shown in Figure 3–14 to form a product. Use a different digit in each box.

(a) How many different products can you make?
(b) What is the largest product you can make?
(c) What is the least product you can make?

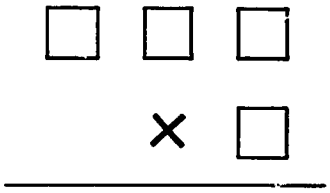


Figure 3–14

**Discussion** This problem affords the student an opportunity to examine the real meaning of place value as well as an understanding of the multiplication algorithm. The calculator simplifies the procedure and allows the student to concentrate on the findings.

**PROBLEM** Place the digits 5, 6, 7, and 8 in the boxes shown in Figure 3–15 to form a sum. Use a different digit in each box.

(a) How many different sums can you make?

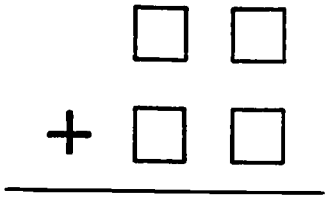


Figure 3–15

(b) What is the greatest sum you can make?
(c) What is the least sum you can make?

**Discussion** This problem should be examined in much the same way as the previous problem.

**PROBLEM** What is the largest whole number you can multiply by 6 and still have a product less than 79?

**Discussion** Students can readily multiply each of the numbers in turn, beginning with $1 \times 6$, $2 \times 6$, etc., until they find a product that is more than 78.

**PROBLEM** A magic square is a square array of numbers whose sum is the same horizontally, vertically, and diagonally. Complete the magic square begun below.

| | | |
|---|---|---|
| | | 76 |
| | | 33 |
| | 69 | 65 |

**PROBLEM** A gross of pencils is 144 pencils. If each pencil is 18 centimeters long, how long would the line be if all the pencils in a gross were put end to end?

**Discussion** Students should decide whether their answer will be a number of pencils or a number of centimeters. Once they have decided what to do, the operation may be done easily with a calculator. Suppose the students formed a 100-sided polygon with 100 pencils. What would its perimeter be? Suppose they form any 100-sided closed figure; what would its perimeter be?

**PROBLEM** Juanita likes to give her answers in large numbers. When she was asked her age, she replied that she has been alive a total of 7,358,400 minutes. How many years has Juanita been alive? (Consider a year as having 365 days.)

**Discussion** The students will have to change the minutes to hours, the hours to days, and the days to years. What operation(s) will these calculations involve? Suppose Juanita decided to give her age in seconds. For how many seconds has she been alive?

**PROBLEM** The 6 and 7 keys on my calculator do not work. Show how you might add 375 and 246 on the calculator.

**Discussion** There are many ways in which this problem can be answered. For example:

$$380-5+245+1$$
or
$$385-10+250-4$$

**13. Take advantage of computer programming.**

A rapidly increasing number of schools are including courses in computer literacy, awareness, and programming in their curriculum. The programming of a computer draws on many of the skills used in problem solving. When students are asked to write a program, they must analyze the task at hand, draw on their previous knowledge and experiences, and put together an organized plan of steps, operations, and commands that yield the correct results. This format of a program is very much like the heuristics pattern of problem solving. Several languages can be used, such as LOGO, BASIC, etc. We have used BASIC in the programs that follow.

**PROBLEM** Tell what the output will be for each of the following programs.

```
(a) 10   LET A=10
    20   LET B=5+8
    30   LET C=A * B
    40   PRINT C
    50   END
(b) 10   LET R=5-2
    20   LET S=16+4
    30   LET T=9 * R
    40   PRINT R+S
    50   END
```

**Discussion** An activity such as this one provides students with the opportunity to analyze a sequence of steps. This is a skill similar to that needed in analyzing problems. Notice that step 30 in (b) is excess data.

**PROBLEM** Put line numbers in the following program so that it will print out the perimeter of a rectangle.

```
LET  P=2 * L+2 * W
LET  L=14.6
START
END
PRINT "THE PERIMETER IS  "; P
LET  W=10.8
```

**PROBLEM** Write a program for finding the area of a rectangle given its length and width.

**Discussion** In order to do this, the student must know what a program is, what a rectangle is, and what is meant by length, width, and area. In other words, he or she must understand what is being asked and what is given. This is exactly the same as the first phase of the problem-solving process.

The student must now review his or her knowledge of areas and rectangles to find the proper formula. Now this information is synthesized and the program written. This carries the student through the "solve" phase.

```
5   HOME
10  REM AREA OF A RECTANGLE
20  PRINT "TYPE IN THE LENGTH IN INCHES"
30  INPUT L
35  PRINT
40  PRINT "TYPE IN THE WIDTH IN INCHES"
50  INPUT W
55  PRINT:PRINT
60  LET A=L * W
70  PRINT "THE AREA OF THE RECTANGLE IS  "; A ; "  SQUARE
    INCHES"
80  END
```

The student now runs the program. Does it work? (This is the "Review" phase.) If not, why not? If it does run correctly, how could it be modified to extend it to other geometric figures? (Extend)

**PROBLEM** Here is a program designed to find the average of three numbers. Some steps have been omitted. Supply the missing steps. Then run the program.

```
10  REM FINDING AVERAGES
20  INPUT A
30
40  INPUT C
50  LET S=A+B+C
60
70  PRINT "THE AVERAGE OF THE THREE NUMBERS IS "; M
80  END
```

**Discussion** This program is an exact parallel to the problem-solving situation in which there is missing data. Being able to determine what information is required to resolve a problem situation and to supply such information assures us that the problem solver has a thorough understanding of the problem situation.

**PROBLEM** Here is a program designed to tell you how much change you receive from a $10 bill when you make three purchases in a grocery store. Some steps have been omitted. Supply the missing steps. Then run the program.

```
10  REM CHANGE PROGRAM
20  PRINT "P IS WHAT YOU SPENT FOR POTATOES"
30  INPUT P
40  PRINT "B IS WHAT YOU SPENT FOR BANANAS"
50  INPUT B
60  PRINT "M IS WHAT YOU SPENT FOR MILK"
70  LET S=P+B+M
80  PRINT " YOUR CHANGE IS  "; C
90  END
```

**Discussion** This problem is similar to the previous one. However, it

has an additional complexity: the missing steps have not been identified:

```
65    INPUT M
75    LET C=10-S
```

*The programs in this section have been written for the Apple microcomputer.

**14. Have your students flow-ch ' their own problem-solving process.**

Students should be helped to develop their own set of heuristics, or problem-solving techniques. They should recognize the components of their strategy. Therefore, after students have gone through the problem-solving process several times, ask them to list and then to flow-chart the steps they have used. At first, the flow chart might be a simple one, like the one in Figure 3–16.

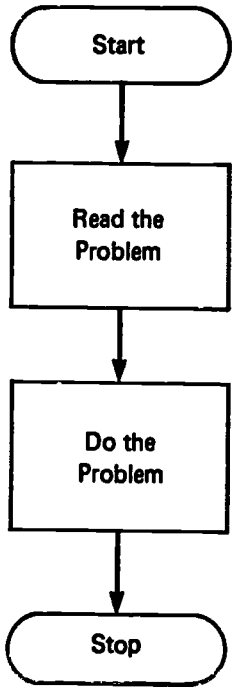



Figure 3–16

However, as the students develop and become more involved in problem solving, the flow chart should become more sophisticated like the one in Figure 3–17.

Notice that the students should begin to ask themselves ques-

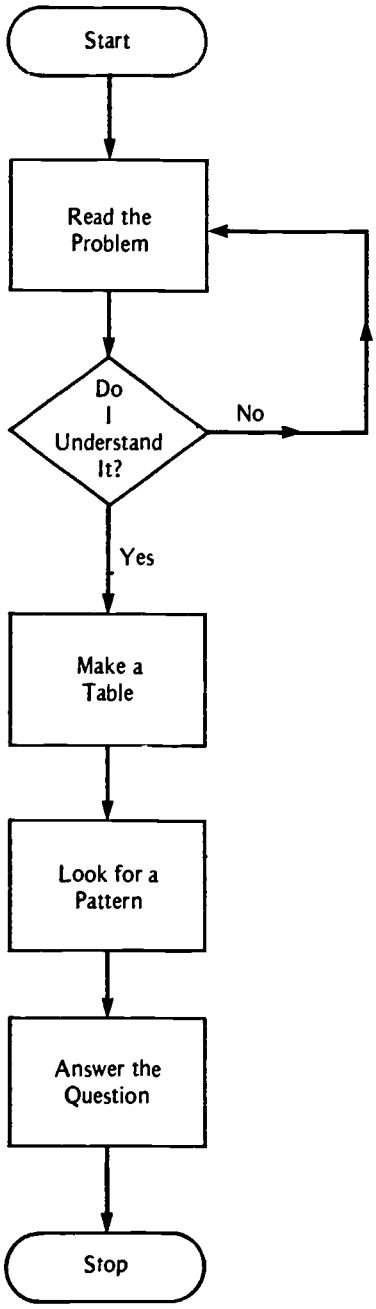



Figure 3–17

tions. This is extremely important in the problem-solving process, as we have stated before.

After more and more experiences with the problem-solving process, it is hoped that the students might evolve a problem-solving flow chart like the one in Figure 3–18.

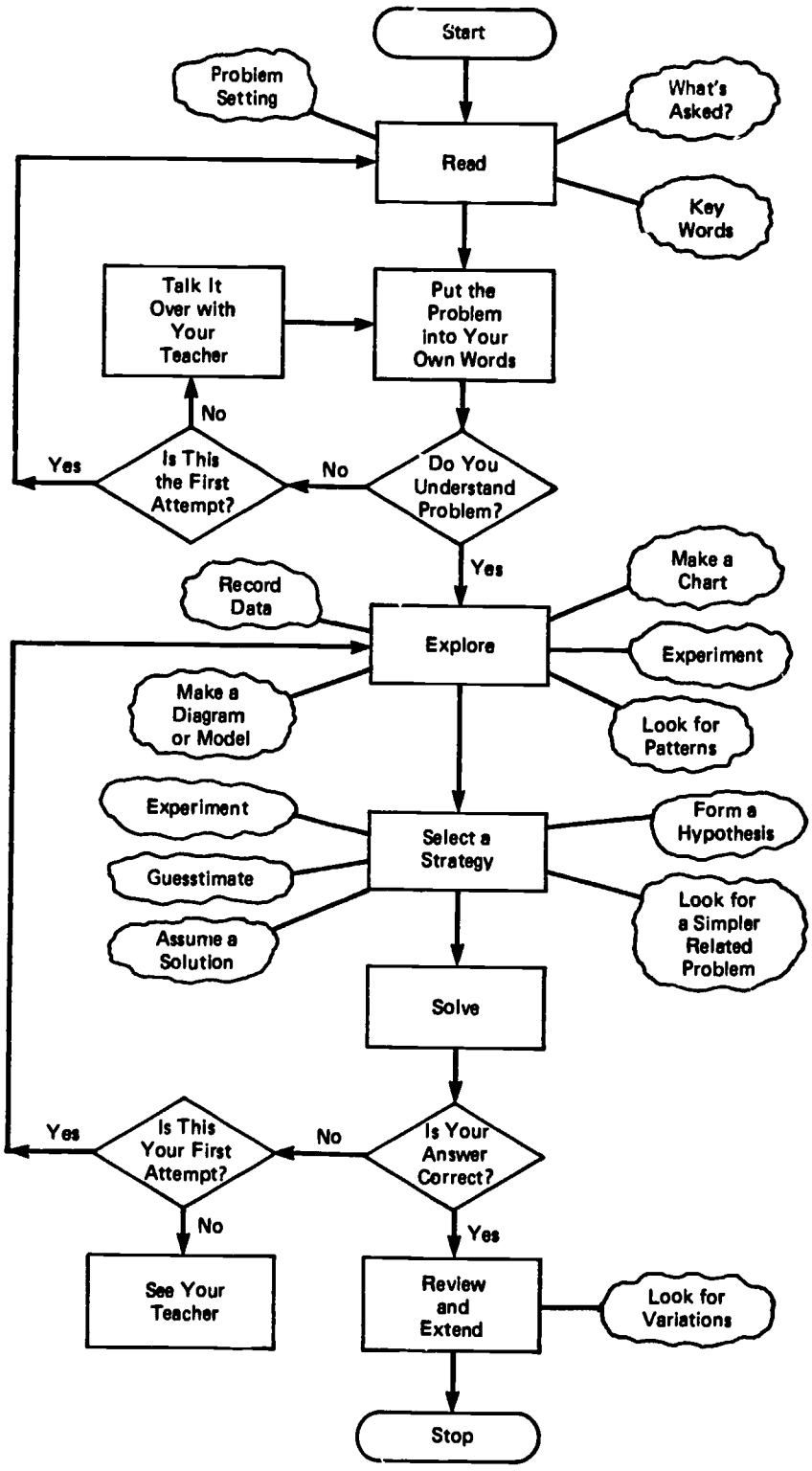

Start
Problem Setting
What's Asked?
Read
Key Words
Talk It Over with Your Teacher
Put the Problem into Your Own Words
No
Yes
Is This the First Attempt?
No
Do You Understand Problem?
Yes
Record Data
Make a Chart
Explore
Experiment
Make a Diagram or Model
Look for Patterns
Experiment
Select a Strategy
Form a Hypothesis
Guesstimate
Look for a Simpler Related Problem
Assume a Solution
Solve
Yes
Is This Your First Attempt?
No
Is Your Answer Correct?
No
Yes
See Your Teacher
Review and Extend
Look for Variations
Stop

Figure 3–18

The preparation of a flow chart for problem solving is an extremely valuable procedure for both the students and the teacher. It will help the students to organize their own thoughts better. (We feel that anyone who cannot flow-chart a process does not really understand that process.) At the same time, it will provide the teacher with a chance to examine the problem-solving process as the students perceive it. It is a visual example of what the students are thinking as they solve a problem.

**15. Use strategy games in class.**

Research has shown that children who have been guided in the playing of strategy games have improved their problem-solving ability. This section discusses the use of these strategy games as a vehicle for teaching problem solving. These are *not* games that necessarily involve arithmetic skills. Rather, to be considered a strategy game, the game must meet the following conditions:

1. The game must have a definite set of rules for the players.
2. The game must be played by at least two players, each of whom has a goal; these goals must be in conflict with each other.
3. The players must make "intelligent" choices of moves, based on whatever information is available at the time of the move.
4. Each player tries to stop the opponent from achieving his or her goal before doing so himself.

Notice that "luck" per se, or "chance," should play a minimal role in strategy gaming.

Games have a strong appeal for children and adults alike. In fact, most people enjoy games. Witness the many books of puzzles and games that are sold in bookstores and the puzzles and game sections that appear on napkins in restaurants or in magazines on the airlines. Currently, an entirely new wave of strategy games using the personal computer is appearing.

Children have been exposed to games and gaming all of their lives. They have learned what a game is, that games have rules to be followed, and that it is often possible to win at a particular game consistently by developing a strategy to follow. Most of your students are already familiar with some of the basic strategy games, such as tic-tac-toe, checkers, and chess. They already know some basic strategies for these games.

To students, games are real-world problem situations. They want to win! And they enjoy playing games. Remember that skills acquired under enjoyable conditions are usually retained for longer

periods of time than are skills acquired under stress or other adverse conditions.

When we develop a strategy for winning at a strategy game, we usually go through a series of steps that closely parallel those used in a heuristic system for solving problems.

## Strategy Gaming

1. Read the game rules. Understand the play of the game. What is a "move"? What pieces are used? What does the board look like? What is a "win"? When is a game over?
2. Correlate the rules with those from any related game. Is there a similar game whose strategy you know? Select several possible lines of play and follow them in an attempt to win the game.
3. Carry out your line of play. Can you counter your opponent's moves as the game proceeds?
4. Look back. If your strategy produced a win, will it work every time? Try alternative lines of play, alternative moves.

The similarity between this sequence and the one we suggested in Chapter 2 is marked indeed.

In order to use strategy games effectively with your students, you need a variety of games. These games can be found in many places. Best of all, games already known to the students can be varied by changing the rules, the pieces, or the game board. In these cases, students need not spend an inordinate amount of time learning all about a "new" game, but can immediately move on to developing a strategy for the play. For example, most of your students already know the game of tic-tac-toe. Under the usual rules, the player who first gets three of his or her own marks (usually Xs and Os) in a straight line, either vertically, horizontally, or diagonally, is the winner. A simple rule change might be that the first player who gets three of his or her own marks in a straight line is the loser. This creates an entirely new line of play and requires a different strategy.

When you use these games with your students, have them analyze and discuss their play. Have them record their moves in each game. You can help them in this analysis by asking key questions, such as:

1. Is it a good idea to go first?
2. Is it a good idea to play a defensive game (that is, to block your opponent)?
3. If you won, was it luck? Or, will your strategy produce a win again?

Have the students play each game several times. Have them refine their strategy each time. Discuss with the class those strategies that consistently lead to a "win."

You can find many examples of strategy games in a local toy store or by looking through the many game and puzzle books available in bookstores. A brief collection of strategy games has been suggested in Section A.

**16. Include problems that have more than one step.**

Students have difficulty with problems that involve more than one step in their solution. Thus, it is important that students recognize such problems when they encounter them in school or in daily situations.

*Example*: John bought a hamburger for $1.65 and a soft drink for 89¢. How much did he spend altogether?
This is an example of a one-step problem. The student merely adds $1.65 and .89 to obtain the answer, $2.54. However, we can make this into a two-step problem as follows:

*Example*: John bought a hamburger for $1.65 and a soft drink for 89¢. How much change does he receive from a $5 bill?
Just as before, the student must find out how much John spent altogether. This is sometimes referred to as a "Hidden Question." It is not asked directly, but its answer is needed in order to solve the final problem as it was asked.

ACTIVITY

Present your students with a sheet of problems that are all simple, one-step problems. Ask them to change each into a two-step problem by adding a statement and a new question. At the same time, the original question asked must be removed, although it remains, but as a Hidden Question. Some samples follow.

*Single step*: Admission to the dog show is $5 for adults. There were 20 adults at the show on Saturday. How much did these adults pay?

*Two step*: Admission to the dog show is $5 for adults. There were 20 adults at the show on Saturday. On Saturday, they took in $137 in admissions. How much money was collected from people other than adults?

*Single step*: Ramona has 5 fish tanks. If each tank uses 4 pounds of gravel, how much gravel does Ramona need?

*Two step*: Ramona has 5 fish tanks. If each tank uses 4 pounds of gravel, and she spent $3.60 for gravel, how much does gravel cost per pound?

**ACTIVITY** Give your students several problems with more than one step. In each case, have them identify the Hidden Question.

*Example*: Dr. L scored 33 points in last night's basketball game. He scored 13 field goals worth 2 points each, and the rest in foul shots worth 1 point each. How many foul shots did he make?
*Hidden Question*: How many points did Dr. L score in field goals?

*Example*: Jan bought five record albums at $7.95 each. The sales tax was 6%. How much was the tax on her albums?
*Hidden Question*: How much was the total cost of the five albums before tax?

You should be aware that there are two kinds of multi-stage problems. In the first kind, which we have been discussing, the answer to the first step is needed to answer the second step. In the second kind, each part is independent. The results of the various parts are then combined to yield the final answer.

*Example*: Maria's mother bought her three blouses that cost $9.50 each and two skirts that cost $15 each. How much did she spend?
*Discussion*: We must find the amount Maria's mother spent on the blouses and on the skirts. We can then combine these in any order. Notice that the parts of the problem are independent of each other. Thus, the strategy often used in solving these problems is usually referred to as "Divide and Conquer."

*Example*: Gary scored 6 field goals and 3 fouls in last night's third-grade basketball game. How many points did he score?
*Discussion:* Again, we divide the problem into its independent parts and solve each. The 6 field goals at 2 points each total 12 points. The 3 foul shots at 1 point each are 3 points. Now we combine these into a final total: $6+3=9$. Gary scored 9 points.

**17. Don't teach new mathematics while teaching problem solving.**

The development of the problem-solving process, not a review of mathematics skills, is the paramount reason for including problem solving in the curriculum. The traditional textbook exercises, labeled

"verbal problems," are primarily intended to practice a mathematical skill or algorithm. Such a practice can easily mask the problem-solving aspect of the activity. Try to keep the mathematics involved in your problems well within the students' level of ability. In some cases, find problems where the mathematics is a relatively minor part of the activity.

**PROBLEM** Janie is sorting her postcard collection. If she puts them into packages of four, she has none left over. When she puts them into packages of five, she again has none left over. If Janie has fewer than 40 cards in her collection, how many cards does she have?

**Discussion** One approach to this problem would be to make a package of 39 disks or bottle caps available to the students. Let them try different number combinations. Another method is to list all the multiples of 4 that are less than 40, and all the multiples of 5 that are less than 40. Note where the two sets intersect, at 20. A more sophisticated method is to note that conceptually the problem deals with the least common multiple (LCM) of 4 and 5. This LCM is 20.

**PROBLEM** Eight students are entered in a checker elimination tournament. Winners play winners until only one student is left. What is the total number of games that must be played?

**Discussion** This problem can be done by acting it out. Select 8 students and have them represent the tournament. The action can be simulated with an elimination drawing as shown in Figure 3–19.

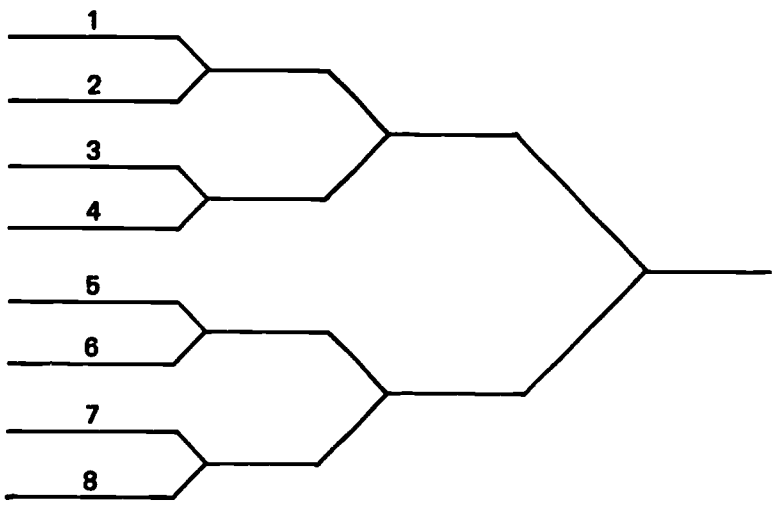



Figure 3–19

There is another more artistic solution to this problem, however. Since 1 student is eliminated in each game, and since we must eliminate 7 students, 7 games are needed to leave the final winner.

**PROBLEM** Strawberry Sam and Blueberry Barbara have a watermelon vine growing right on the border of their gardens. One night, Sam picked all 40 watermelons and put them on his porch. The next day, Barbara sneaked over and took 3 of the watermelons back to her porch. That night, Sam sneaked over and took 1 of the melons back. The next day, Barbara took 3 more melons; that night, Sam took 1 watermelon back. If this continues, when will Sam and Barbara have the same number of watermelons?

**Discussion** Some youngsters may suggest that the answer to the problem is "When they each have 20 melons!" This is correct, but it is not the answer we are seeking. The students can simulate the action by preparing a table similar to the following:

| | NIGHT | | DAY | |
|---|---|---|---|---|
| | Sam | Barbara | Sam | Barbara |
| 1 | 40 | 0 | 37 | 3 |
| 2 | 38 | 2 | 35 | 5 |
| 3 | 36 | 4 | 33 | 7 |
| 4 | 34 | 6 | 31 | 9 |

and so on until completion.

## SECTION A

# A Collection of Strategy Games

## TIC-TAC-TOE VARIATIONS

The basic game of tic-tac-toe is a good starting point for introducing strategy games to children. In many cases, your students will already be familiar with the rules of the game. Thus, it becomes a logical place to find a wide assortment of variations.

### 1. Tic-Tac-Toe

The game is played by two people on a nine-square array as shown in Figure A–1. Give each player five chips, markers, or tokens of a distinctive color. Or, assign X to one player and O to the other. The players alternate turns placing one of their markers in any empty cell. The winner is the person who gets three in a row, horizontally, vertically, or diagonally.

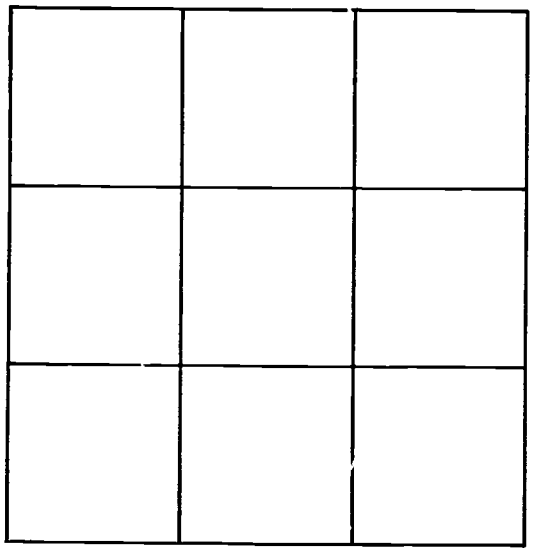


Figure A–1

### 2. Valley Tic-Tac-Toe

The game is played on an eight-cell board as shown in Figure A–2. Players take turns placing one of their own markers in any empty cell. The winner is the first person to get three of his or her own markers in a row, horizontally, vertically, or diagonally. There are six possible ways to win.

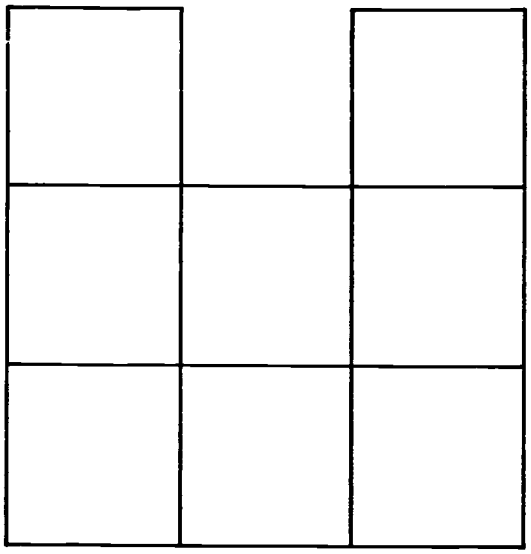


Figure A–2. Valley Tic-Tac-Toe Game Board

### 3. Mountain Tic-Tac-Toe

The game is played on a nine-cell board as shown in Figure A–3. Again, the rules of Tic-Tac-Toe are followed. The winner is the first person to get three of his or her markers in a row. There are seven ways to win.

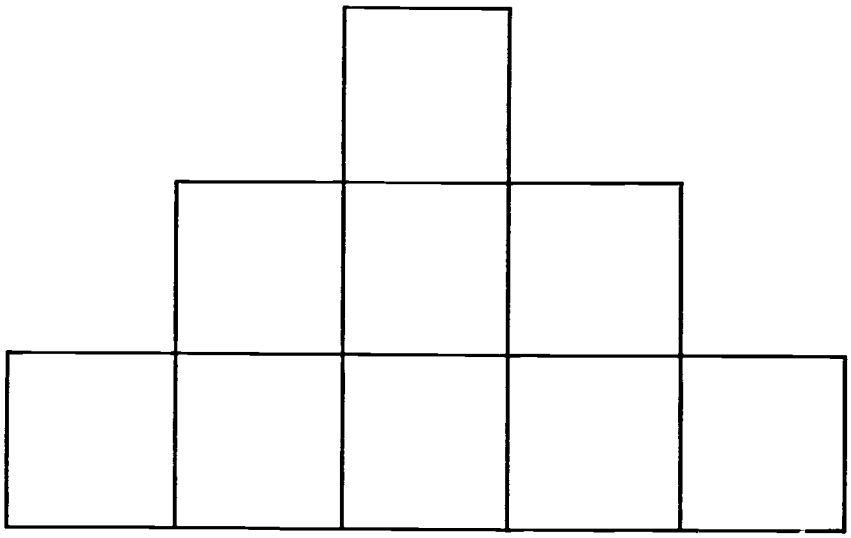


Figure A-3. A Mountain Tic-Tac-Toe Game Board

**4. Reverse Tic-Tac-Toe**

This game follows the rules of Tic-Tac-Toe described in game number 1. It simply changes the requirements for "a win." Players must take turns placing an O or an X on the basic, nine-cell game board, and try to *avoid* getting three markers in a row. If a person gets three of his or her markers in a row, the opponent has won the game. Notice that the idea of Reverse Tic-Tac-Toe can be used with Valley and Mountain Tic-Tac-Toe as well.

**5. Big 7 Tic-Tac-Toe**

This game is played on a playing surface consisting of 49 squares in a 7 × 7 array. Two players take turns placing either an O or an X into any open cell on the playing surface. Each places his or her own mark. The first player to get four marks in a row is the winner.

**6. Three-Person Tic-Tac-Toe**

Most versions of Tic-Tac-Toe are games between two players. In this version, however, three people play. Players use either an X, an O, or an I as a marker, and the game is played on a board that contains a 6 × 6 or 36-square array. Players put their own mark anywhere on the playing surface in turn. The first player to get three of his or her own markers in a row is the winner.

**7. Line Tic-Tac-Toe**

Fifteen dots are placed in a straight line. Two players alternate turns placing an X through any one dot anywhere on the line. The first player to mark off a dot so that there are three consecutive dots marked is the winner. The game can also be played as in Reverse Tic-Tac-Toe, so that the first player to mark the third consecutive X in the row is the *loser*.

**8. Dots-in-a-Row Tic-Tac-Toe**

The game is played on a surface as shown in Figure A–4. Players take turns crossing out as many dots as they desire, provided the dots are all along the same straight line. The player who crosses out the last dot is the winner.

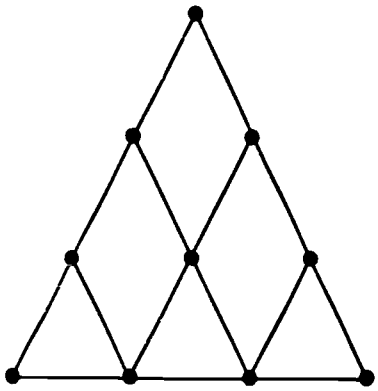


Figure A–4. Dots-in-a-Row Game Board

## 9. Tac-Tic-Toe

This game is played on a 4 × 4 square surface. Each player has four chips or markers of a single color. The starting position is shown in Figure A–5. Players take turns moving a single piece of their own color. A move consists of moving one piece onto a vacant square either horizontally or vertically, but not diagonally. There is no jumping or capturing in this game. No piece can be moved into an already occupied square, but must be moved into an open, adjacent square. The player who moves three of his or her own pieces into a row, either horizontally, vertically, *or diagonally*, with no intervening spaces or intervening squares occupied by an opponent's piece, is the winner.

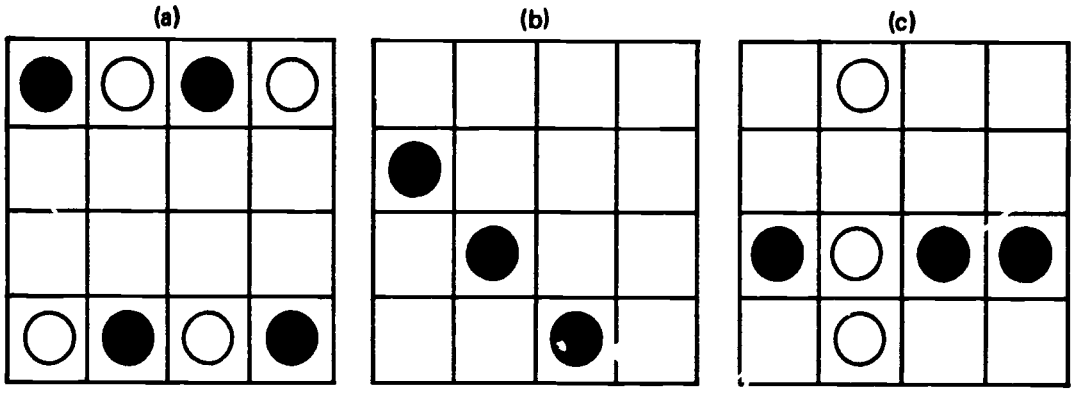



Figure A-5. (a) Starting Position for Tac-Tic-Toe. (b) A Win Position for Tac-Tic-Toe. (c) Nobody Wins in These Positions.

### 10. Triangular Tic-Tac-Toe

This game uses the basic rule—that is, the player scoring three of his or her marks in a straight line is the winner. The playing surface, however, has been changed into the triangular array shown in Figure A–6, rather than the usual square array.

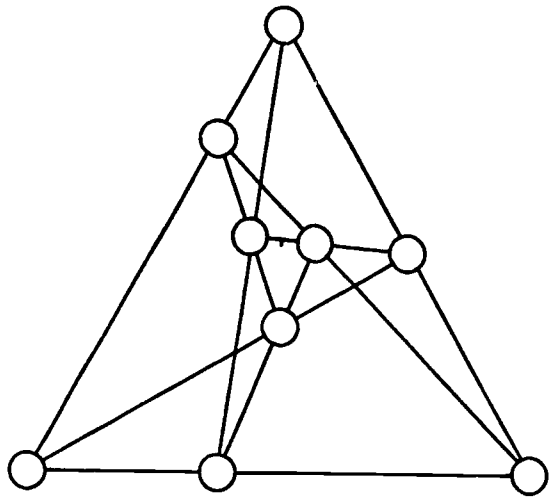


Figure A–6. Triangular Tic-Tac-Toe Board

## BLOCKING STRATEGY GAMES

### 11. Blockade

Playing pieces are placed on cells A and B in Figure A–7 for player number one, and on cells C and D for player number two. Players take turns moving one playing piece along lines on the playing surface into any vacant, adjacent circle. No jumps or captures are permitted. A player loses the game when he or she cannot move either of his or her two pieces in his or her turn.

### 12. Jest

This game is played on a 3 × 3 array of squares. Each player has three chips or markers of a single color. Starting position is as shown

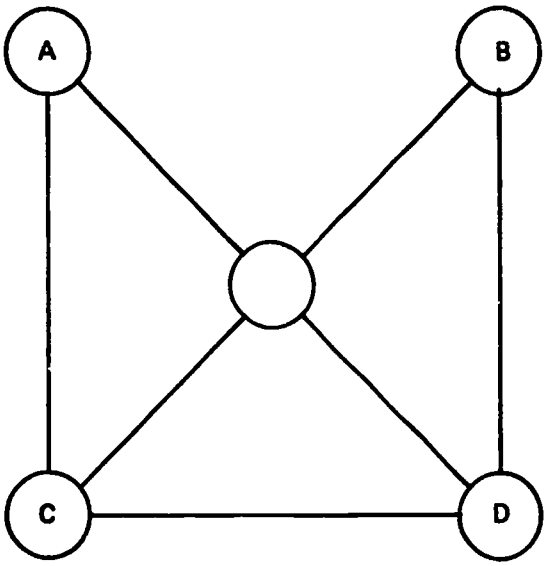



Figure A–7. Playing Surface for Blockade

in Figure A–8. Players take turns moving one of their own pieces. Each piece may be moved one square in any direction, forward, backward, horizontally, or diagonally. There is no jumping of pieces, nor may a piece be captured. A player is a winner when his or her pieces occupy the opponent's starting line.

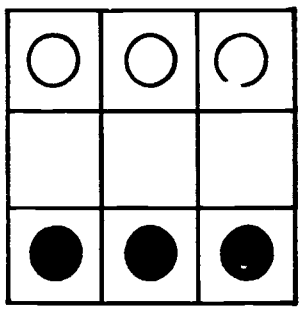


Figure A–8. Starting Position for Jest

## 13. Hex

The game of Hex is played on a diamond-shaped board made up of hexagons (see Figure A–9). The players take turns placing an X or an O in any hexagon on the board that is unoccupied. The winner is the first player to make an unbroken path from one side of the

board to the other. Blocking moves and other strategies should be developed as the game proceeds. The corner hexagons belong to either player.

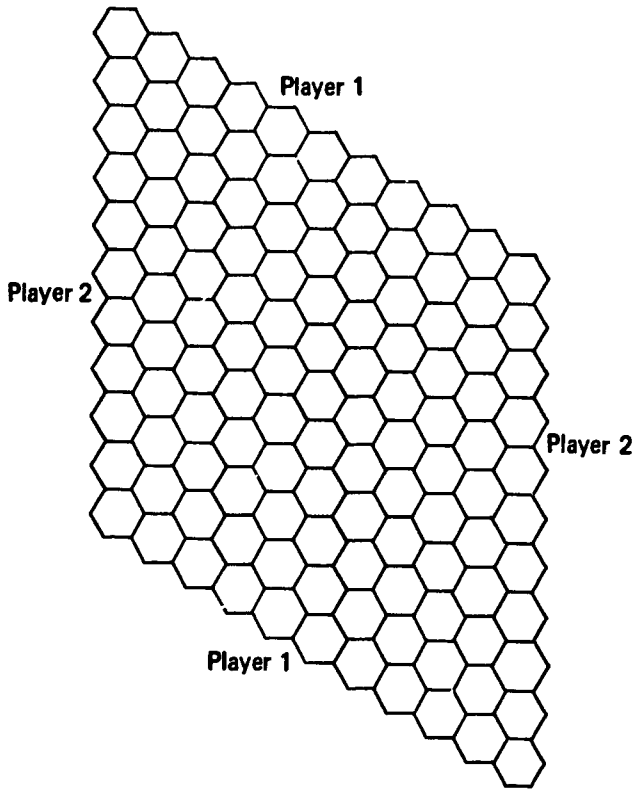



Figure A–9. Hex Board

## 14. The Four-Color Game

This game can be played by two, three, or four players. The game board is divided into various numbers of regions. One example is shown in Figure A–10. Each player, in turn, colors any region of his or her choice with an identifiable color. However, the regions adjacent to each other *may not* be of the same color. The first player who cannot make a move is the loser. Play continues until only one player is left. This player is the winner. Various colored chips or markers can be used instead of colored pencils if you wish.

## 15. Bi-Squares

This game is played on a playing surface that consists of 16 squares in one long, continuous row. Players take turns placing their mark

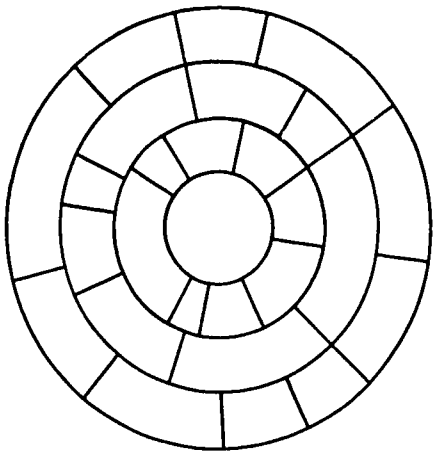


Figure A–10. A Gameboard for the Four-Color Game

(an X for player number one, and an O for player number two) into each of two adjacent, unoccupied squares anywhere on the board. The player who makes the last successful move on the board is the winner.

### 16. Domino Cover

This game is played on the standard 8 × 8 checkerboard, and uses a set of dominoes that will cover two adjacent squares, either horizontally or vertically. Players take turns placing a domino anywhere on the board, according to the following rules: (1) Player number one can only place his or her dominoes in a horizontal direction; (2) player number two can only place dominoes in a vertical direction. The loser is the player who cannot make a move by placing a domino in the correct position.

### 17. Connecting Dots

The game is played on an 8 × 8 array of 64 dots as shown in Figure A–11. Players alternate turns connecting any two vertically or horizontally adjacent dots (but not diagonally) with a straight pencil line. The player who draws the line that completes a one-by-one unit square places his or her initial inside the square and goes again. When the board is completely covered with initials, the player with the most squares having his or her initial is the winner.

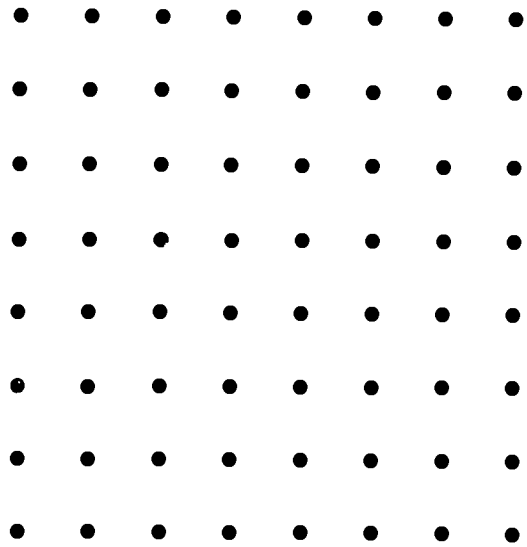


Figure A–11. A Game Board for Connecting Dots

## 18. Tromino Saturation

The game is played on a 5 × 5 square board. The playing pieces consist of the two basic tromino shapes shown in Figure A–12. Each tromino piece should exactly cover three squares on the playing board. Players alternate turns placing one of the pieces of either shape anywhere on the playing surface. The first player who cannot place a piece exactly covering three squares is the loser. (In order to allow each player a full choice of which piece to select on each play, prepare eight pieces of each shape.) If the size of the board is increased to a 6 × 6 board, prepare twelve pieces of each shape.

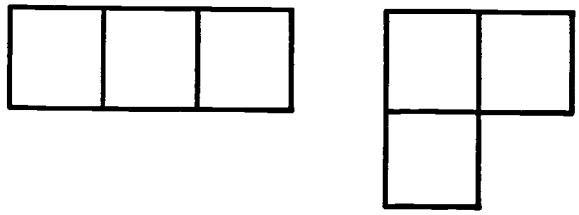


Figure A–12. The Two Basic Tromino Shapes for Saturation

## CAPTURE STRATEGY GAMES

### 19. Short Checkers

This game is played on a 6 × 6 square checkerboard, rather than on the traditional 8 × 8 square board. Each player uses six checkers of his or her own color, and places them in the starting position on the black squares in the first two rows. The game is played according to the same rules as the traditional game of checkers, having twelve checkers for each player.

### 20. Solitaire

This is a strategy game for one person. The playing surface consists of a board with fifteen circles, as shown in Figure A–13. Place chips or other counters on all of the cells except the darkened cell. The player must remove as many counters as possible by jumping counters over adjacent counters (along lines) into empty cells. The jumped counters are removed from the board. All counters but one can be removed in this manner. A winning game is one in which only one counter remains. A variation for experienced players is to try to make the one remaining counter end the game in the darkened cell.

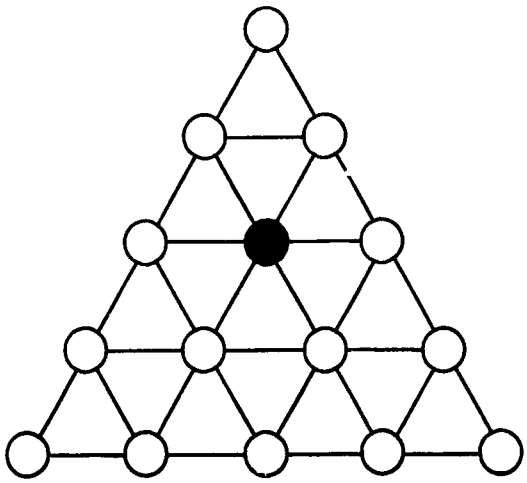


Figure A–13. A Solitaire Board

## 21. Fox and Geese

This game for two players is played on a surface with 33 cells, as shown in Figure A–14. The fox marker and the thirteen goose markers are placed as shown in the figure. The fox can move in any direction along a line—up, down, left, or right. The geese move one cell at a time along the lines, but may not move backward. The fox can capture a goose by making a short jump over a single goose along a line into the next cell, provided that the cell is vacant. The fox can make successive jumps on any one turn, provided vacant cells exist. The geese win if they can corner the fox so that he cannot move. The fox wins if he captures enough geese so that they cannot corner him.

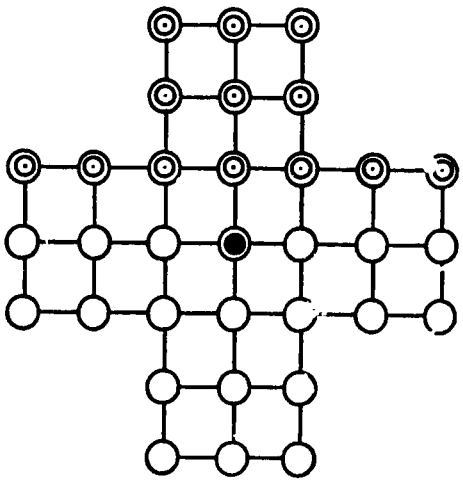


Figure A–14. Starting Position for Fox and Geese

## 22. Sprouts

Three dots are placed in a triangular array on a piece of paper. Players take turns drawing a line connecting any two dots, or connecting a dot to itself. After a line is drawn, a new dot is placed approximately midway between the two dots being connected, along the connecting line. No lines may cross, and no more than three lines may terminate in a single point. The last player to make a successful move is the winner. See Figure A–15. The new point, D, is shown along the line connecting point A to itself. The new point, E, is shown along the line connecting B to C. Notice that points D, E, and A each have two lines terminating.

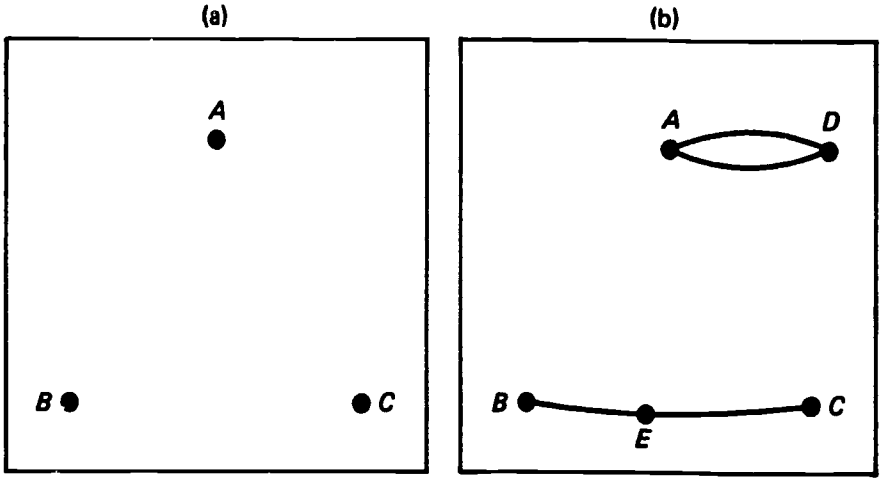



Figure A–15. (a) A Sprouts Board. (b) Typical Moves in Sprouts

### 23. Nim

The game is played with a set of eleven chips, bottle caps, or other markers. The markers are placed on the table between the two players. In turn, each player may pick up one, two, or three chips. The winner is the player who picks up the last chip. Note: The game may also be played so that the person who picks up the final chip is the loser.

### 24. Sum 15

The game is played on a nine-cell, 3 × 3 square board. Player number one uses the five digits 1, 3, 5, 7, and 9. Player number two uses the five digits 2, 4, 6, 8, and 0. The players decide who will go first. The players alternate turns writing one of their own digits in any empty square on the board. Each digit may be used only once in a game. The winner is the player who completes a row, either horizontally, vertically, or diagonally, with a sum of 15.

## SOME COMMERCIAL STRATEGY GAMES

### 25. Amoeba

This is a game in which players rotate individual pieces on the game board as they attempt to form amoeba-like shapes that match the

shapes on the cards in their hands. No two shapes are the same on the 54 cards in the deck. (Pressman Toys)

**26. Basis**

A strategy game in which players form numerals in different bases while preventing their opponents from doing the same. (Holt, Rinehart and Winston Co.)

**27. Battleship**

A game of strategy in which two players try to sink each other's ships, which are hidden from view. It is a good introduction to coordinates. (Creative Publications)

**28. Bee Line**

Players use strategy while attempting to make a "beeline" across the playing board. (SEE Corporation)

**29. Block 'N Score**

A strategy game for two players who work in binary notation. (Creative Publications)

**30. Equations**

A game designed to give students practice in abstract reasoning, to increase speed and accuracy in computing, and to teach some of the basic concepts of mathematics. The game can be varied to work in different bases for more advanced students. (Wff 'N Proof)

**31. Foo (Fundamental Order of Operations)**

A strategy game in which players try to combine seven cards into any multiple of 12. Extra cards are drawn and discarded until one player calls "Foo!" (Cuisenaire, Inc.)

**32. Helix**

Another three-dimensional tic-tac-toe game. Players place different-colored beads on a series of pins, trying to get four in a row. The pins are not only in straight lines, but also along arcs designated on the playing surface. (Creative Publications)

**33. Kalah**

A strategy game involving counting, skill, advanced planning, and logic. Chance is a minimal factor in this game. (Creative Publications)

**34. Mastermind**

A secret code of colored pegs is set up out of sight of one player. He or she then has ten chances to duplicate the colors and exact positions of the code pegs. Pure logic! (Cadaco, Invicta, Creative Publications)

**35. Numble**

A game similar to a crossword puzzle. Players place tiles with numerals from 0 to 9 on them to form addition, subtraction, multiplication, and division problems. (Math Media, Inc.)

**36. Othello**

A strategy game for two players that includes the moves and strategy of chess, checkers, and backgammon. Pieces change colors from player to player as the game progresses. (Gabriel Toys)

**37. Pressups**

A player must guide the direction of play so as to press down pegs of his or her own color. Traps must be set. The winner is the player who has more of his or her own color pegs depressed. (Invicta)

**38. Qubic**

Qubic expands tic-tac-toe into a four-level space game. Players win

by setting four markers in a straight line in one or several planes. (Parker Brothers)

**39. Racko**

By drawing from the pile, players attempt to replace cards in their racks so that the numbers read from high to low in numerical sequence. (Milton Bradley and Company)

**40. Rubik's Brain Game**

A game similar to Mastermind, but played with a Rubik's Cube. Players ask questions in order to determine the hidden $3 \times 3$ pattern of colors as they might appear on a Rubik's Cube. A game of logic and deduction. (Ideal Toys)

**41. Score Four**

Similar to Qubic and Helix. Players place wooden beads on metal pins and need to get four in a row to score. (Lakeside Toys)

**42. SOMA Cube**

An elegant cube with irregular sets of combinations of cubes. There are 1,105,920 mathematically different ways to come up with the 240 ways that the seven SOMA pieces fit together to form the original cube. (Parker Brothers)

**43. Triominos**

Triangular pieces replace the standard two-square dominoes in this game. Players must plan ahead to make matching numbers fit on all three sides of the piece that is being played. (Pressman Toys)

## SECTION B

# A Collection of Non-Routine Problems

The following set of problems has been chosen to provide practice in problem solving for your students. We have attempted to arrange the problems in increasing order of student maturity. However, the actual choice of problems for a particular student or group of students must be made by the classroom teacher. Only he or she is in a position to determine the appropriateness of a given problem for an individual child.

Notice, too, that the problems are all presented in written form. Obviously, many of the problems will have to be presented to the children orally, since the reading level may be beyond that of the class.

**PROBLEM 1** There are 3 girls and 4 boys waiting in line. How many children are waiting in line?

**Discussion** The solution to this problem depends on a conceptual understanding of addition. We can also utilize the strategy of experimentation by lining up 3 girls and 4 boys, and having a student count the number of children actually standing in the line.

**PROBLEM 2** What's next?

(a) 1, 2, 3, 4,_____.
(b) 2, 4, 6, _____.
(c)

**Discussion** Observation of patterns is a crucial skill in problem solving as well as in all mathematics. There may be more than one correct answer to (a) and (b), such as:

(a) *1, 2, 3, 4, 3, 2, 1.*
*1, 2, 3, 4, 5, 6, 7.*
(b) *2, 4, 6, 10, 16.*
*2, 4, 6, 8, 10.*

**PROBLEM 3** Amelia has 5 coins. Mike has 7 coins. Who has more coins? How many more?

**Discussion** Recognizing the correct operation is an important skill. As another illustration of experimentation, this problem can be acted out.

**PROBLEM 4** John is taller than Alex. Lucy is shorter than Alex. Arrange the three children in order of size with the shortest first.

**Discussion** Act it out! Select three children to represent John, Lucy, and Alex. Or, you can simulate the action with stick figures.

**PROBLEM 5** Andrea is taller than Michael. Danielle is taller than Michael. Arrange the three children in order of size with the shortest first.

**Discussion** There is not enough information to solve this problem. We *can* decide that Michael is the shortest person, but we cannot arrange the other two in order. Students should be taught that, in some cases, a problem will not contain sufficient information to determine an answer.

**PROBLEM 6** In our classroom, I have Spelling before Art. I have Math right after Art. Which class comes first?

**Discussion** Have the students draw a "time line." The answer requires a knowledge of the words "before" and "after" and their meanings.

**PROBLEM 7** Bianca jumped from the 4-foot line and landed on the 9-foot line. Joanne jumped from the 2-foot line and landed on the 6-foot line. Who had the longer jump?

**Discussion** Use a number line to determine the length of each jump. Or, depend on the students' understanding of subtraction and their knowledge of the basic subtraction facts.

**PROBLEM 8** How many ways can you get from A to B in Figure B–1?

**Discussion** There are 2 ways to go from A to C, and 2 ways to go from D to B. Thus, there are $2 \times 2$ or 4 ways to go from A to B. This is the fundamental counting principle; however, most students should actually trace the paths.

**PROBLEM 9** Nancy woke up at 8:00 A.M.
Danny woke up one hour after Nancy.
Jeff woke up two hours before Nancy.
At what time did Danny wake up?

**Discussion** This problem contains excess information. Careful reading is necessary. The question requires only the information about Nancy and Danny.

**PROBLEM 10** David woke up at 7:00 A.M.
Barbara woke up one hour after David.
Suzie woke up two hours before Barbara.
At what time did Suzie wake up?

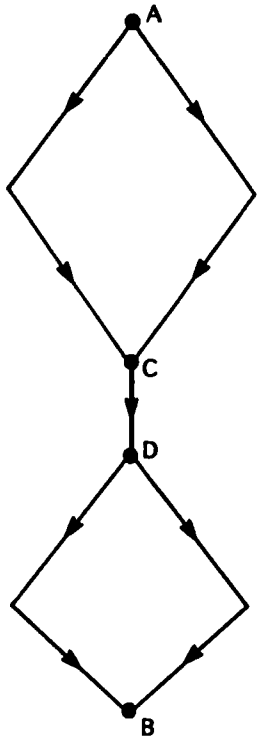



Figure B–1

**Discussion** Careful reading will reveal that *all* of the information in the problem is needed. David woke up at 7:00 A.M.; Barbara woke up one hour later—8:00 A.M. Suzie woke up two hours before Barbara—6:00 A.M.

**PROBLEM 11** How do you get 2 pints of water?

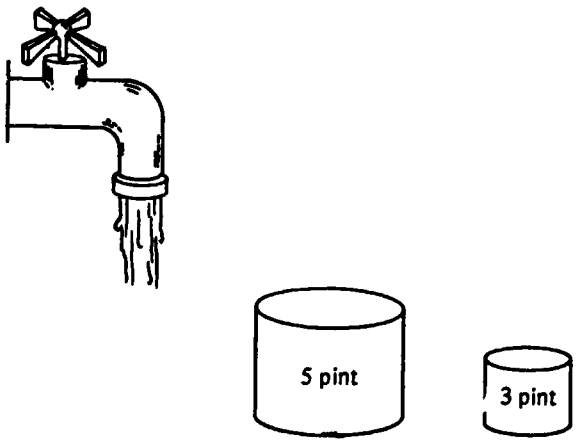
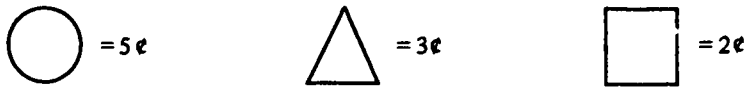



Figure B–2

**Discussion** This problem provides the child with early experience in guess and test as well as in logic. The procedure would be to fill the 5-pint bucket with water and pour it into the 3-pint bucket. There will be the required 2 pints of water remaining in the 5-pint bucket.

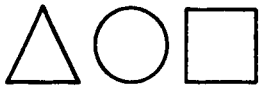
**PROBLEM 12**




Find the value of:

(a)

(b)



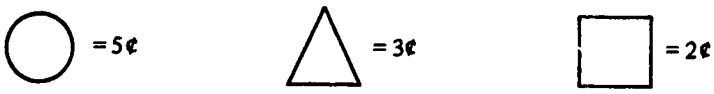
(c)

(d)

**Discussion** Symbolizing numerical values with geometric figures is analogous to algebraic representation. The answers are obtained by replacing each symbol with its given value. Thus,

(a) 5¢ + 5¢ = 10¢
(b) 3¢ + 3¢ = 6¢
(c) 3¢ + 5¢ + 2¢ = 10¢
(d) 3¢ + 3¢ + 5¢ + 2¢ + 2¢ = 15¢

**PROBLEM 13**



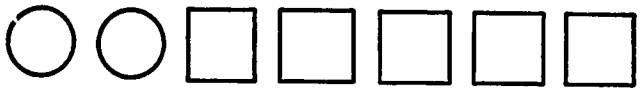

Make a picture worth 20¢.

**Discussion** Just as in the previous problem, symbolic representation is carefully stressed. However, this problem requires a higher level of sophistication, since it is more open-ended. Answers will vary; for example:

20¢ =

or

or



**PROBLEM 14** Erin saw 2 squirrels and 3 bluebirds in her yard. How many wings did she see in all?

**Discussion** The answer to this problem depends on an understanding of the number of wings on each bird (2). The problem has been complicated by the inclusion of excess information; i.e., the number of squirrels.

**PROBLEM 15** The Rockets scored 28 points. The Jets scored 7 points more. How many points did the Jets score?

**Discussion** This problem depends on the students' understanding the concept of "more" as being addition.

**PROBLEM 16** You can buy 1 marble for 1¢. Each extra marble costs 2¢. How much will you pay for 6 marbles? Finish the table.

| Number of Marbles | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Cost | 1¢ | 3¢ | 5¢ | 7¢ | | |

**Discussion** Students must not only be able to read a table, but should also be able to complete a table that has already been begun. As the number of marbles increases by 1, the cost increases by 2¢. Some more talented children might be able to generalize the situation—the cost of the marbles is twice the number of marbles minus 1.

**PROBLEM 17** Jim, Kim, and Lim had a race. Kim came in last. Lim did not win. Who won the race?

**Discussion** The construction of a time line along with logical thinking reveals the placement of the three contestants: Kim was third, Lim was second, and Jim won the race.

**PROBLEM 18** You want to buy each of the three stamps in order to mail the letters shown in Figure B–3. You have the coins that are shown in the figure. Which coins would you use to buy each stamp? You will use all of your coins.

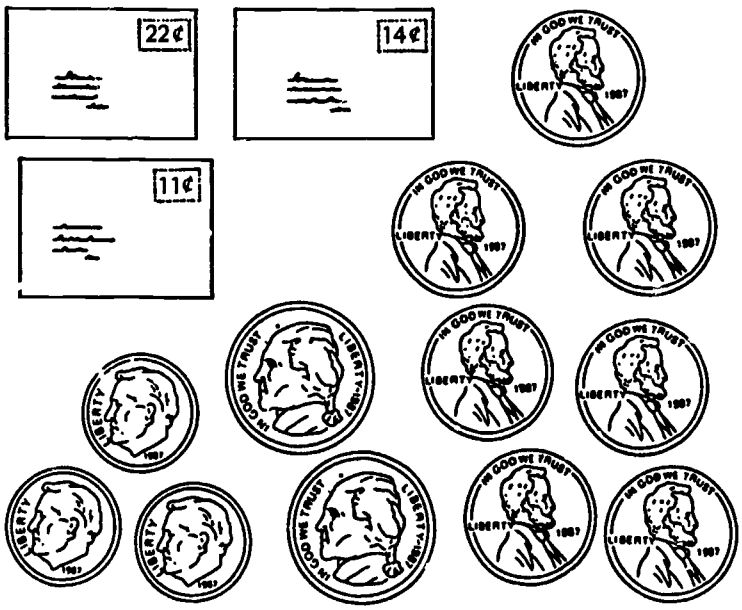



Figure B–3

**Discussion** Let's simulate the action. Provide the students with coins or cut-outs of coins. Allow them to guess and test until they arrive at the correct answer. The 22¢ stamp will be bought with two dimes and two pennies. The 14¢ stamp can be bought with one time and four pennies. The 11¢ stamp is then bought with the two 5¢ coins and one 1¢ coin.

**PROBLEM 19** Jill has 2 quarters and 2 dimes. A chocolate bar costs 20¢. What is the largest number of chocolate bars that Jill can buy?

**Discussion** First determine the amount of money that Jill has. Two quarters and two dimes equal 70¢. Have the students make a table to organize their work:

| Number of Bars | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Cost | 20¢ | 40¢ | 60¢ | 80¢ |

Jill can buy 3 chocolate bars.

**PROBLEM 20** How many days are there from May 5 through May 20?

**Discussion** The obvious approach for many students will be to subtract 5 from 20 and give the answer as 15 days. This is incorrect, since it does not include both end points. Drawing a calendar and actually counting will reveal the correct answer, 16 days.

**PROBLEM 21** Move only one block to another stack, and make the sum of the numbers in each stack be 12.

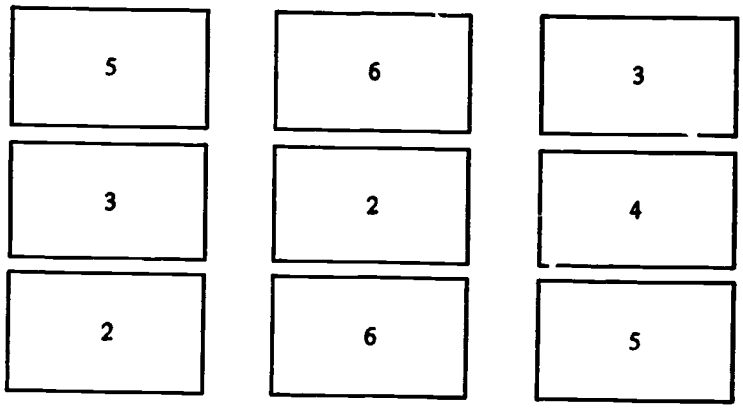



Figure B–4

**Discussion** Some students will solve this problem by observation. Others will add the numerals in each stack. Help them to solve the problem by an experiment. Provide a set of 9 cubes with the numbers written on them. Have students stack them as shown, and then actually shift them around. Or, provide a set of buttons, tokens, or sticks and have the children actually do the problem.

**PROBLEM 22** Kay's pencil is 7 incles long.
Ray's pencil is 2 inches shorter than Kay's.
May's pencil is 3 inches longer than Ray's.
Whose pencil is the longest.?

**Discussion** Use a number line as a measuring device. With three different colored pencils or crayons, represent each of the children's pencils in turn. Kay's pencil = 7 inches; Ray's pencil = 5 inches (2 inches shorter than Kay's); May's pencil = 8 inches (3 inches longer than Ray's). Some children may be able to omit the number line and deal with the problem abstractly.

**PROBLEM 23** July 4 is a Tuesday. Your birthday is on July 23rd. On what day of the week is your birthday?

**Discussion** A good way of attacking this problem is to sketch a calendar for the month. The answer is easily found this way. An alternative method without drawing the calendar is to use the fact that a week contains 7 days. Thus July 11, 18, and 25 are also Tuesdays. Counting back from July 25 to July 23 places the birthday on a Sunday.

**PROBLEM 24** I put my 10 checkers into two stacks. One stack has 4 more checkers than the other has. How many checkers are in each stack?

**Discussion** Act it out! Give each student 10 checkers or buttons. Have them guess at the size of each pile and test their guess with the checkers. They should find that there are 7 checkers in one stack and 3 checkers in the other.

**PROBLEM 25** Last week the Giants played the Dodgers. There were a total of 7 runs scored in the game. What could have been the final score?

**Discussion** Make a list. There are 8 possible scores:

| *Giants* | *Dodgers* |
|---|---|
| 7 | 0 |
| 6 | 1 |
| 5 | 2 |
| 4 | 3 |
| 3 | 4 |
| 2 | 5 |
| 1 | 6 |
| 0 | 7 |

**PROBLEM 26** There are 48 children in two clubs.
There are 15 boys in one club.
There are 10 boys in the other club.
How many girls are there in both clubs?

**Discussion** Careful reading is especially necessary to solve this problem. All the information is given in terms of the number of *boys*. Students must add 15 + 10 = 25 (the total number

of boys in both clubs) and then subtract from 48 to get the total number of girls, 23.

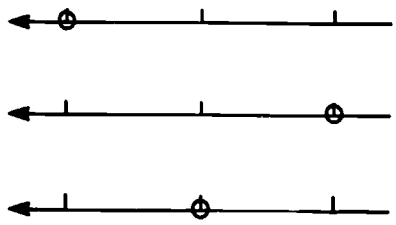
**PROBLEM 27** I have nine bills in my wallet. Five of them are $1 bills and the rest of them are $5 bills. How much money do I have in my wallet?

**Discussion** This problem is a multi-step problem and should be done one part at a time.

*Step 1*: 9 bills − 5 $1 bills = 4 bills
*Step 2*: 4 bills × $5 each = $20
*Step 3*: $20 + $5 = $25

**PROBLEM 28** In a line, there is a rabbit in front of two rabbits. There is a rabbit behind two rabbits. There is a rabbit between two rabbits. What is the smallest number of rabbits in the line?

**Discussion** Simulate the situation by making a drawing or series of drawings similar to the following:




The answer is 3 rabbits. Notice that the problem called for the *smallest* number of rabbits in the line.

**PROBLEM 29** A chicken can lay about 5 eggs each week. How many eggs can you expect 5 chickens to lay in 3 weeks?

**Discussion** Organize the work with a table:
1 chicken = 5 eggs in one week
5 chickens = 25 eggs in one week
5 chickens = 75 eggs in three weeks
You can expect 75 eggs from 5 chickens in 3 weeks

**PROBLEM 30** Here is a table showing the runs scored by two teams in three baseball games played against each other. If this scoring pattern continues, what will be the score of the 5th game that they play?

| Game | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Robins | 2 | 4 | 6 | | |
| Crows | 5 | 6 | 7 | | |

**Discussion** Look for the pattern that occurs in each row and continue the table. The Robins will win the 5th game by the score of 10 to 9. Notice that the 4th game will end in an 8–8 tie.

**PROBLEM 31** Which of the four numbers in the array doesn't belong?

| | |
|---|---|
| 23 | 20 |
| 25 | 15 |

**Discussion** This problem is very open-ended. Some students will decide that 23 doesn't belong, since it is the only number that does not contain 5 as a factor. Other students may decide that 15 doesn't belong, since it is the only number that does not have a tens' digit of 2. Others may eliminate the 20, since it is the only *even* number of the four given. All are correct!

**PROBLEM 32** Yew found 5 shells on the beach. She found 7 more shells on the dock. She gave away 3 of the shells to her brother. How many shells did Yew keep?

**Discussion** Some children will solve this problem with a number sentence, $5 + 7 - 3 = 9$. Others should act it out, using physical objects. Do it both ways in class.

**PROBLEM 33** Jimmy planted a tree on Tuesday. Jimmy is 7 years old, and the tree is 2 years old. How old will the tree be when Jimmy is 13 years old?

**Discussion** Some students may recognize that Jimmy will *always* be 5 years older than the tree. Thus, the tree will be 8 years old when Jimmy is 13. Other students may wish to make a table to reveal the answer:

| Jimmy's Age | 7 | 8 | 9 | ... | 13 |
|---|---|---|---|---|---|
| Tree's Age | 2 | 3 | 4 | ... | 8 |

**PROBLEM 34** Jeanne has \$8. Grace has \$6. Trish has \$7. Ann has \$4. Two of the girls put their money together and had a total of \$12. Who were the two girls?

**Discussion** Guess and test. Have students choose any two girls and find the total money. Continue until they guess Jeanne and Ann, \$8 + \$4 = \$12.

**PROBLEM 35** How far is it from Corcoran to Millville?

**Discussion** Make an arrow drawing along a number line. Students should see that the sum of the two given distances is the required distance. It is 252 miles from Corcoran to Millville.

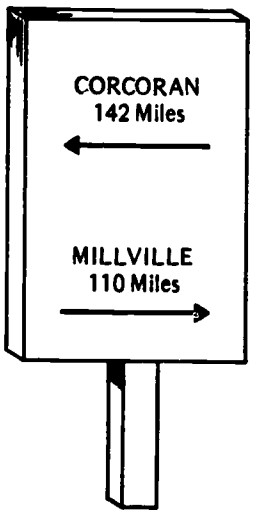



Figure B–5

**PROBLEM 36** How many 5¢ pieces of bubble gum can you buy if you have 27 pennies?

**Discussion** Have your students act it out. Give them 27 counters and have them arrange the counters in sets of 5. Each set buys one piece of bubble gum. The remainder of 2 is not enough to buy another piece of gum. An alternate solution is to make a table:

| Number of Pieces of Bubble gum | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Cost | 5¢ | 10¢ | 15¢ | 20¢ | 25¢ | 30¢ |

↑ too much

The more mathematically mature student may divide 27 by 5 and disregard the remainder (which would yield a fractional piece of bubble gum).

**PROBLEM 37** Ricardo's guppies had baby fish. He gave 6 of them to Marlene. He gave 5 of them to Sonja. If there were 18 baby fish to start, how many does Ricardo keep?

**Discussion** This is a multi-stage problem. Add the number of fish given away (6 + 5 = 11) and subtract this sum from the total number of fish to find the answer (18 − 11 = 7). Ricardo kept 7 fish.

**PROBLEM 38** Which box would you take off the balance scale to make it balance?

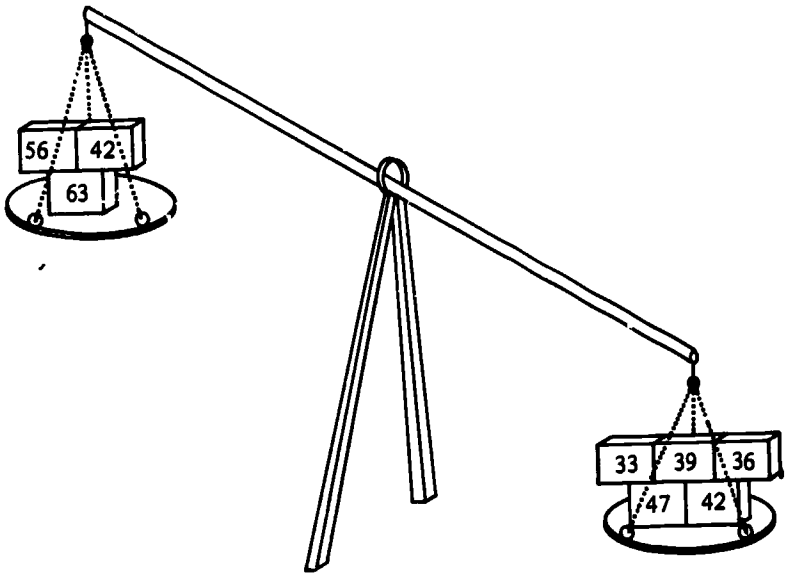



Figure B–6

**Discussion** In this problem, some of the information must be obtained from an examination of the picture. This is a technique that students must practice. This problem is an example of guess and test. Remove one number at a time from the side of the balance scale and add the remaining ones. Removing the 36 will make both sides total 161. Some students may find the sum of the boxes on each side (161 and 197, respectively). Then find the difference, 36, and remove it from the right side.

**PROBLEM 39** Peter, Paul, and Mary have 5 cookies. How many ways can they divide the cookies if each person must get at least one cookie?

**Discussion** Make a table:

| *Peter* | *Paul* | *Mary* |
|---|---|---|
| 1 | 3 | 1 |
| 1 | 2 | 2 |
| 1 | 1 | 3 |
| 2 | 1 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 1 |

There are a total of six ways it can be done.

**PROBLEM 40** The houses on Whitehall Street all have odd numbers. The first house is number 3. The second house is number 5. The third house is number 7, and so on. What is the number of the 10th house?

**Discussion** A list reveals the answer. Follow the pattern in each row of the list:

| House | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 |

The tenth house is number 21.

**PROBLEM 41** Find the number to fill the spots:

(a) $\begin{array}{r} 203 \\ +\ 47\square \\ \hline 675 \end{array}$ (b) $\begin{array}{r} 36\square \\ -\ \square 4\square \\ \hline 226 \end{array}$ (c) $\begin{array}{r} 25 \\ 2\square \\ +\ \square 7 \\ \hline 75 \end{array}$

**Discussion** This set of nonverbal problems requires the students to interpret the question. In the first case, the units digit of one of the addends is missing. In the second problem, two numerals of the subtrahend are missing. The third problem has two numerals missing in two different addends. To find the answers, the children must be familiar with the algorithms involved:

(a) $\begin{array}{r} 203 \\ +\ 472 \\ \hline 675 \end{array}$ (b) $\begin{array}{r} 368 \\ -\ 142 \\ \hline 226 \end{array}$ (c) $\begin{array}{r} 25 \\ 23 \\ +\ 27 \\ \hline 75 \end{array}$

**PROBLEM 42** The record store has a sale on tapes at \$4.00, \$5.00, \$6.00, and \$7.00 each. Lonnie bought two tapes and paid \$12.00. What price tapes did Lonnie buy?

**Discussion** The problem is solved by guess and test. However, more than one answer is possible: \$7.00 + \$5.00 or \$6.00 + \$6.00. Notice, too, that \$4.00 + \$4.00 + \$4.00 = \$12.00; however, the problem states that only *two* tapes were bought.

**PROBLEM 43** What's next for each of these?

(a) 53, 47, 41, 35, _____, _____.
(b) □, ○, △, □, ○, △, _____, _____.
(c) 1, 1, 2, 3, 5, 8, 13, _____, _____.
(d) 3, 12, 5, 9, 7, 6, 9, _____, _____.

**Discussion** Students should be asked to *verbally* describe the pattern rule in each series. In (a), each term is diminished by 6. The missing terms are 29 and 23. In (b), the sequence repeats every three terms. The next two terms would be the square and the circle. In (c), we have the well-known Fibonacci sequence, where each term after the first is found by adding the two previous terms. The next two terms would be 21 and 34. In (d), the students must realize that there are two embedded sequences. The even-numbered terms are decreasing by 3 (12, 9, 6) while the odd-numbered terms are increasing by 2 (3, 5, 7, 9). The missing terms are 11 and 3.

**PROBLEM 44** The number of my classroom is odd, and is between 20 and 30. It does not end in either a 7 or a 9. It is more than 23. What is my room number?

**Discussion** Make a list of the numbers described by the first two clues:

21, 23, 25, 27, 29

The next clue eliminates 27 and 29. The final clue tells us that the number is 25.

**PROBLEM 45** Every bike slot in the bicycle rack was filled. Donna's bike is in the middle. There are 6 bikes to the right of Donna's bike. How many bicycles are in the rack?

**Discussion** Make a drawing. Show Donna's bike in the center and show the 6 slots to the right of her bike. If Donna's bike is in the *middle*, then there are also 6 slots to the left of her bike. Thus, there are 6 + 6 + 1 (Donna's bike counts, too)

for a total of 13 bikes in the rack. An alternate procedure using logic tells us that, if Donna's bike is in the middle, there will be just as many bikes on either side. Thus, the 6 bike slots on the right side equal the 6 bike slots on the left side. Don't forget to count Donna's bike as well.

**PROBLEM 46** Samantha works in her father's gasoline station. Today she sold 4 new tires for each of 5 cars, and 2 new tires for each of 8 cars. How many tires did she sell today?

**Discussion** The most direct method would be to use $(4 \times 5) + (2 \times 8) = 20 + 16 = 36$ tires. However, if multiplication is not available to the students, we can use either tokens or chips (5 groups of 4 tokens and 8 groups of 2 tokens) and count. Other students may use a drawing that simulates the experiment.

**PROBLEM 47** Irv has 6 baseball cards. Bob has 4 baseball cards. Steve has 3 baseball cards. Sandra has 7 baseball cards. And Marcella has 9 baseball cards. Three of them put their cards together and had a total of 18 cards. Who put their cards together?

**Discussion** Guess and test. Try different combinations of three people until we get 18 cards. The answer is Irv (6), Steve (3), and Marcella (9).

**PROBLEM 48** Which of the numbers 4, 7, or 9 is my mystery number?

(a) It is more than 3.
(b) It is less than 8.
(c) It is more than 5.

**Discussion** Use the clues in turn to eliminate unwanted numbers. Notice that clue (a) is not needed.

**PROBLEM 49** Some children took 5 rides in a pony cart. Only 1 child went on the first ride, 3 children went on the second ride, and 5 children went on the third ride. Louis guessed that 9 children went on the fifth ride. Can you tell why Louis made that guess?

**Discussion** The children should realize that Louis found a pattern. This yields the sequence 1, 3, 5, 7, 9.

**PROBLEM 50** Mitch bought three different toys for his children. The gifts cost him $12. What did he buy?

| | | | |
|---|---|---|---|
| Football | $6.00 | Puppet | $2.00 |
| Soccer ball | $4.00 | Book | $5.00 |

**Discussion** Guess and test. The answer is a football, a soccer ball, and a puppet.

**PROBLEM 51** Arthur is making lunch. He makes sandwiches with white bread or rye bread. He uses either cheese, jelly, or lunch meat. How many different sandwiches can he make?

**Discussion** Make a list of all possible sandwiches.

| *White Bread* | *Rye Bread* |
|---|---|
| cheese | cheese |
| jelly | jelly |
| meat | meat |

He can make 6 different sandwiches. (Some students might decide to make a sandwich with one slice of rye bread and one slice of white bread. Thus, this student will have 9 different sandwiches on his or her list.)

**PROBLEM 52** After shopping, Stephanie had $3.00 left. She had spent $3.25 on a present for her mom, $4.25 for balloons for the party, and $5.00 for invitations. How much did she start with?

**Discussion** The students should work backward:

| | |
|---|---|
| $3.00 | money she had left at the end |
| + 5.00 | money spent for invitations |
| 8.00 | |
| + 4.25 | money spent for balloons |
| 12.25 | |
| + 3.25 | money spent for a present |
| $ 15.50 | amount she started with |

Have the students check their work by beginning with the $15.50 and carrying the action forward. Do they wind up with the $3.00 as the problem stated?

**PROBLEM 53** The Little League scores are on two facing pages of the local newspaper. The sum of the page numbers is 13. What are the page numbers?

**Discussion** Students must realize that the numbers on two facing pages of a newspaper are consecutive, with the lower number on the left side page. List all possible number pairs of successive integers and find the pair whose sum is 13 (pages 6 and 7). It is possible that some students may find the "average" page number (13 ÷ 2 = 6 ½), and then use the actual pages on either side of 6 ½, namely, 6 and 7.

**PROBLEM 54** Which two banks in Figure B–7 have a total of $8.25?

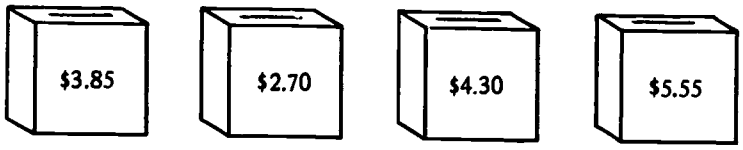



Figure B–7

**Discussion** Again, have students guess and test. Select any two banks at random and add the amounts shown in each. The correct answer is the banks that show $5.55 and $2.70.

**PROBLEM 55** Janet bought her goldfish on Thursday, July 10th. On what day of the week was the first day of the month?

**Discussion** Count backward from Thursday, July 10. Or, subtract 7 days placing July 3 on a Thursday as well. This reveals that the first of July was on a Tuesday. An alternative is to draw a calendar for the month of July.

**PROBLEM 56** Which of the following sums of money could you pay with exactly three coins? Tell how you would do it.

7¢ 16¢ 22¢ 56¢

**Discussion** Give the children several pennies, nickels, dimes, quarters, and half-dollars (or chips and tokens that are appropriately marked). Have them experiment until they find the proper combinations.

7¢ = 5¢ + 1¢ + 1¢
16¢ = 10¢ + 5¢ + 1¢
56¢ = 50¢ + 5¢ + 1¢

It is impossible to pay 22¢ with exactly three coins.

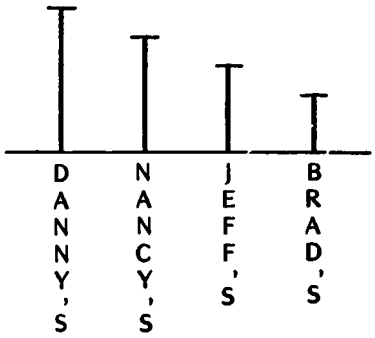
**PROBLEM 57** Mrs. Chen has lost the middle digit from her house number:

She knows that it is greater than the last number, and smaller than the first number. It is an even number. What is the missing number?

**Discussion** Critical thinking leads to the answer. The first two clues limit the number to either 5 or 6. The third clue eliminates the 5.

**PROBLEM 58** Jeff's plant is shorter than Nancy's. Danny's plant is taller than Nancy's. Jeff's plant is taller than Brad's. Whose plant is the tallest? Whose is the shortest?

**Discussion** Vertical line segments representing each person's plant will enable the students to discover that Danny's plant is the tallest and Brad's plant is the shortest.




**PROBLEM 59** You get two tosses with a beanbag at the target shown in Figure B–8. How many different scores can you get? What are they?

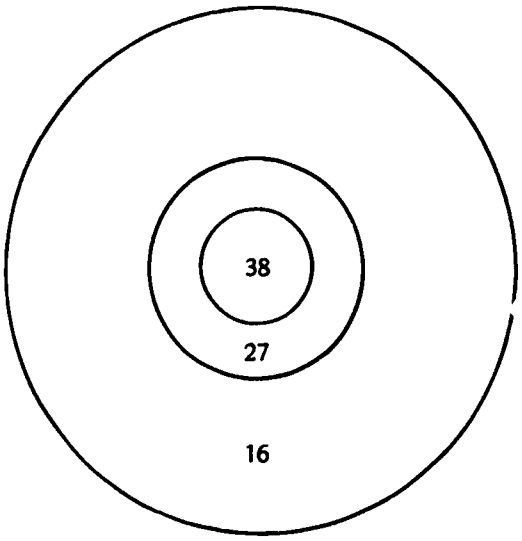



Figure B–8

**Discussion** Make an exhaustive list of all possible scores. Organize your list to be certain that you have all possible scores.

| | | |
|---|---|---|
| 16 + 16 = 32 | 27 + 27 = 54 | 38 + 38 = 76 |
| 16 + 27 = 43 | 27 + 38 = 65 | |
| 16 + 38 = 54 | | |

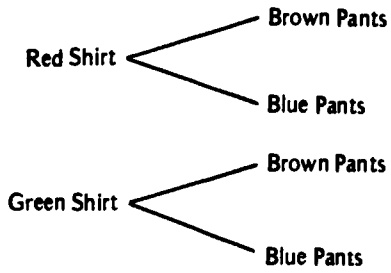
Notice that there are two different ways to score 54. We must only count one of these. Thus, there are 5 different scores possible: 76, 65, 54, 43, 32.

**PROBLEM 60** There are 3 houses on Clara Street. Sue, David, and Barbara live in the 3 houses. Sue does not live next to David. David lives on a corner. Who lives in the middle house?

**Discussion** Have the children use points to represent the houses on a line segment, which represents the street. Since David's house is on a corner, put the point that represents David's house at one end of the line segment. Barbara's house will be in the middle.

**PROBLEM 61** Billy got out of bed early this morning. He put on a shirt and pants. Billy has a red shirt and a green shirt. He has brown pants and blue pants. He can get dressed in 4 different ways. One way is red shirt with brown pants. Find the other ways.

**Discussion** This is a convenient way to introduce "tree diagrams" to the children:




These show the four different ways that Billy can get dressed.

**PROBLEM 62** Find all of the two-digit numbers for which the sum of the two digits is 10.

**Discussion** Make a list and notice the pattern. The list *is* the answer.

19, 28, 37, 46, 55, 64, 73, 82, 91

**PROBLEM 63** Select three of these numbers whose sum is 17:

3, 4, 5, 6, 7, 8, 9

**Discussion** This is another example of the use of an organized list. There are only four possibilities that satisfy the conditions of the problem. These are (3,5,9)–(3,6,8)–(4,5,8)–(4,6,7).

**PROBLEM 64** At which step do you go over 100?

| *Step 1* | *Step 2* | *Step 3* | *Step 4* |
|---|---|---|---|
| 1 | 2 | 4 | 8 |
| + 1 | + 2 | + 4 | + 8 |

**Discussion** Use a calculator. Students should continue the pattern until they reach the step where the answer becomes a 3-digit number. Step 5 is 16 + 16 = 32; Step 6 is 32 + 32 = 64; Step 7 is 64 + 64 = 128, which is over 100.

**PROBLEM 65** The faces of the cube in Figure B–9 are numbered consecutively. What is the sum of the numbers not shown in the figure?

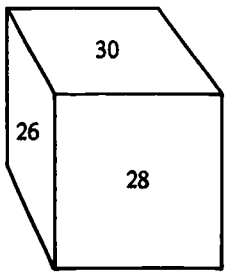



Figure B–9

**Discussion** First of all, be certain that the students realize that there are 6 faces on a cube. There are two different answers to this problem depending on whether the cube is numbered from 26 to 31, or from 25 to 30. (There is nothing in the problem to indicate which is the case.) One way to attack the problem is to determine the numbers not shown on the faces and add:

| *Case I* | *Case II* |
|---|---|
| 26 | 25 |
| 27 | 26 |
| 28 | 27 |
| 29 | 28 |
| 30 | 29 |
| 31 | 30 |
| Sum of missing faces = 87 | Sum of missing faces = 81 |

Another interesting method would be to add the numbers

shown. Subtract this from either, the sum of the numbers from 25 to 30, or the sum of the numbers from 26 to 31. A calculator would be of great help here.

**PROBLEM 66** How many different ways can you make change for a 50¢ piece without using any pennies?

**Discussion** In order to organize the work, prepare a table:

| *Nickels (5¢)* | *Dimes (10¢)* | *Quarters (25¢)* | |
|---|---|---|---|
| 10 | 0 | 0 | |
| 9 | – | – | (cannot be done) |
| 8 | 1 | 0 | |
| 7 | – | – | (cannot be done) |
| 6 | 2 | 0 | |
| 5 | 0 | 1 | |
| 4 | 3 | 0 | |
| 3 | 1 | 1 | |
| 2 | 4 | 0 | |
| 1 | 2 | 1 | |
| 0 | 5 | 0 | |
| 0 | 0 | 2 | |

There are a total of 10 ways to satisfy the given conditions.

**PROBLEM 67** Complete the pattern:

3 → 3
26 → 8
37 → 10
18 → 9
62 → 8
49 →
31 →

**Discussion** This pattern may not be easy to see. However, the output is the sum of the digits of the input number. Thus, the missing outputs are 13 (4 + 9) and 4 (3 + 1). It is interesting for the students to notice that both 62 and 26 go to an output of 8.

**PROBLEM 68** Fill in the squares with the numbers 2, 3, or 4 so that the

numbers in each row across, down, and diagonally must add up to 9.

| | | |
|---|---|---|
| 3 | 4 | |
| | | |
| | | 3 |

**Discussion** This is an exercise working with number facts. Horizontally we have 3 + 4 = 7; thus, we need a 2 for a sum of 9. Now, we add vertically in the right-hand column, 2 + 3 = 5; thus, we need a 4 in the missing box. Continue in a similar manner.

**PROBLEM 69** A football team won 3 more games than it lost. The team played a total o: 11 games. How many games did they lose?

**Discussion** Some students may solve this problem using the guess and test strategy. Others may subtract the 3 games that represent the won–lost difference. Now, with the number of wins and losses being equal, we merely divide by 2 to get 4 losses. To test this result, we subtract the losses (4) from the total (11), and see if the result (7 wins) is indeed 3 more than the number of losses.

**PROBLEM 70** Gail's pencil is 7 inches long. Stan's pencil is 3 inches shorter than Gail's. Mickey's pencil is 4 inches longer than Stan's. Whose pencil is the longest?

**Discussion** This problem can be done by laying the various lengths out on a number line. However, if we follow each statement in turn, we find that:

Gail's pencil is 7 inches long;
Stan's pencil is 4 inches long;
Mickey's pencil is 8 inches long.

Thus, Mickey's pencil is the longest.

**PROBLEM 71** A special plant doubles its height each day. On Monday, it was 2 inches tall. On Tuesday, it reached 4 inches tall. How tall will the plant be on Friday?

**Discussion** Making a table will reveal a pattern of the powers of 2:

| | |
|---|---|
| Monday | 4 inches |
| Tuesday | 4 inches |
| Wednesday | 8 inches |
| Thursday | 16 inches |
| Friday | 32 inches |

The plant will be 32 inches tall on Friday.

**PROBLEM 72** How long is a row of 24 pennies placed end to end so that they touch?

**Discussion** Take 24 pennies. Place them end to end as shown in Figure B–10. Measure the length of the line with a ruler. Some students might measure one penny and multiply by 24. This could reveal a slight error, or difference in the answer due to an error of precision in measuring the penny. The line should be 18 inches long.

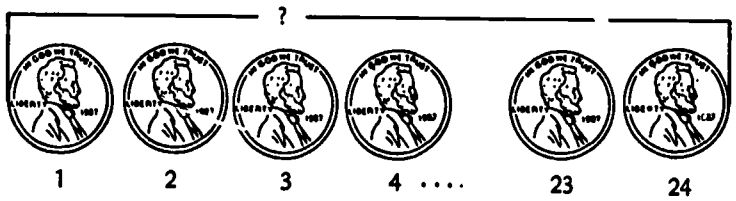



Figure B–10

**PROBLEM 73** Circle two numbers whose quotient is 8.

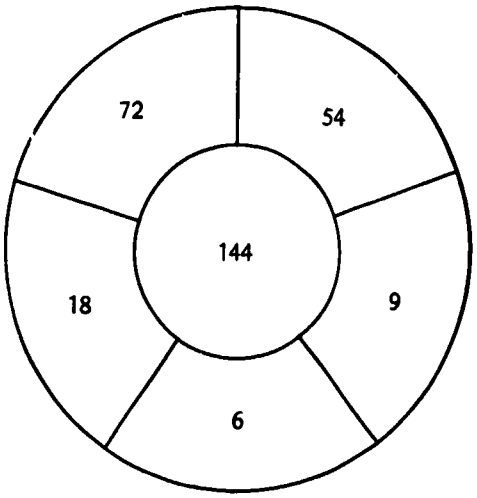



Figure B–11

**Discussion** Guess and test. The problem has two answers.

**PROBLEM 74** During the softball season, Steve and Amy had a total of 80 hits. Steve had 10 more than Amy. How many hits did each have?

**Discussion** The method of solution is similar to that used in Problem 69. In both cases, students should use the guess and test procedure. Steve had 45 hits; Amy had 35 hits.

**PROBLEM 75** Add 5 to the mystery number. Then subtract 7. The result is 10. What's the mystery number?

**Discussion** Work backward. This means using inverse operations in the reverse order. Thus, we begin with the final situation (10), and we *add* 7 (17). Now subtract 5. The result is 12, which is the mystery number. Students may also decide to guess and test their guesses until they reach the correct number, 12.

**PROBLEM 76** If you and 3 friends share this money equally, how much will each of you get?

Figure B–12

**Discussion** Some students will add up all of the money ($1.60) and then divide by 4 to get the answer, 40¢. Others may recognize that there are 4 of each coin. Thus, each person gets 1 quarter, 1 dime, and 1 nickel, or 40¢.

**PROBLEM 77** Tim lives 8 blocks from school. How many blocks does he

walk if he goes to school, goes home for lunch, and then goes right home after school?

**Discussion** In order to solve the problem, the reader must infer from the last sentence that four trips were made between Tim's home and school (Tim must have gone *back* to school after lunch).

**PROBLEM 78** There are 5 students in Mrs. Martin's class who wish to ride on a "bicycle built for two." How many rides must they take so that each person rides with each other person just one time?

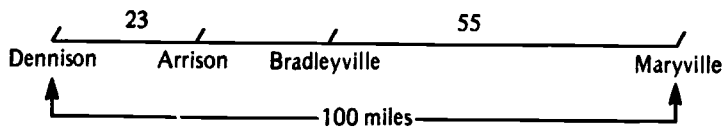
**Discussion** Make an organized list of all possible pairs of students:

| | | | |
|---|---|---|---|
| A–B | B–C | C–D | D–E |
| A–C | B–D | C–E | |
| A–D | B–E | | |
| A–E | | | |

There will be 10 rides needed. Notice that this list exhausts all the possible ways in which the rides can be taken. The list does not include B–A, since A–B and B–A are the same two students. This is typical of the combinatorial problems that students encounter more formally later in their mathematics program.

**PROBLEM 79** Arrison, Bradleyville, and Cork are 3 towns on the road between Maryville and Denniston. The road from Maryville to Denniston is a straight, 100–mile road. From Arrison to Denniston is 23 miles. From Maryville to Bradleyville is 55 miles. From Maryville to Cork is 30 miles. How far is Arrison from Bradleyville?

**Discussion** Make a drawing from the given information. (The information about Cork is not needed.)




The distance from Arrison to Bradleyville is 22 miles.

**PROBLEM 80** Janice tossed three darts at the dart board shown in Figure B–13. Make a list of all the ways Janice could score 40 points.

**Discussion** Listing is an important problem-solving skill. This problem forces children to make an exhaustive, organized list. It

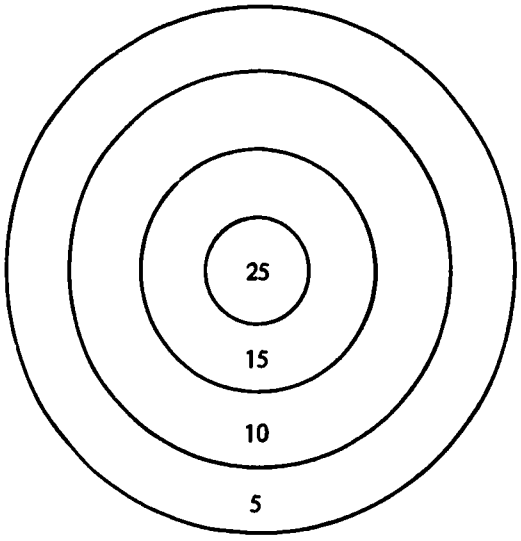



Figure B–13

also provides practice in arithmetic computation. The possible answers are:

15 + 15 + 10
25 + 10 + 5
25 + 15 + miss

**PROBLEM 81** Marbles cost 2 for 25¢. Luis had one dollar. He bought 6 marbles. How much money does Luis have left?

**Discussion** This is an example of a multi-stage problem. Since the marbles cost 2 for 25¢, Luis spent 75¢ for the 6 marbles. Thus, he had 25¢ left after his purchase.

**PROBLEM 82** How would you make 7 quarts?

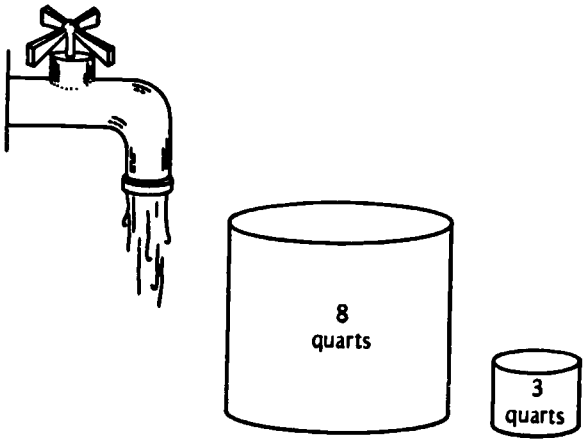



Figure B–14

**Discussion** This transfer problem requires a little thought and simulation. First fill the 3-quart container 3 times, pouring each into the 8-quart container. This will leave 1 quart in the 3-quart container. Now empty the 8-quart container and pour the 1 quart from the 3-quart container into the 8-quart container. Now fill the 3-quart container twice and empty it into the 8-quart container. This will produce the required 7 quarts (1 + 3 + 3).

**PROBLEM 83** Rex tossed five number cubes. All the cubes have three 4s and three 5s on them.

(a) What is the smallest sum that Rex could obtain by adding the faces that are "up"?
(b) What is the largest sum that Rex could obtain by adding the faces that are "up"?
(c) Rex added up his score and got a 22. How many 4s and how many 5s were there?

**Discussion** Give the students five cubes numbered as the problem stated. Have them arrange the cubes to find the smallest sum, the largest sum, and a sum of 22. The smallest sum is 20 (5 × 4). The largest sum is 25 (5 × 5). To obtain a sum of 22, we would need three 4s and two 5s. Some children may not need the physical aid of the actual cubes—they can mentally solve the problem.

**PROBLEM 84** It is possible to make each of the amounts of money listed with exactly six coins. Record your answers on the given table.

| Amount | 1¢ | 5¢ | 10¢ | 25¢ | 50¢ |
|---|---|---|---|---|---|
| .42 | | | | | |
| .85 | | | | | |
| $ 1.26 | | | | | |
| $ 1.70 | | | | | |

**Discussion** This is a problem that can be solved by guess and test. There may be multiple answers for each amount. One possible set is shown:

| Amount | 1¢ | 5¢ | 10¢ | 25¢ | 50¢ |
|---|---|---|---|---|---|
| .42 | 2 | 0 | 4 | 0 | 0 |
| .85 | 0 | 1 | 3 | 2 | 0 |
| $ 1.26 | 1 | 0 | 0 | 5 | 0 |
| $ 1.70 | 0 | 2 | 1 | 0 | 3 |

**PROBLEM 85** Lonny has 2 bats and 1 ball that cost him $11. Andy has 1 bat and 2 balls that cost him $7. How much should 1 bat and 1 ball cost?

**Discussion** Guess and test. The cost of 1 bat is $5 and the cost of 1 ball is $1. Recording the guesses in a table helps the students to refine succeeding guesses.

**PROBLEM 86** Nina asked her dad how old he was. He told her, "If I add 10 to my age and double the result, I will get 84." How old is Nina's dad?

**Discussion** Work backward and reverse the operations. Thus, we begin with 84, and *divide* by 2 to get 42. Now *subtract* 10 and get Nina's dad's age as 32.

**PROBLEM 87** The Whip ride at the amusement park takes a new group of 15 people every 10 minutes. There are 70 people who want to ride. It is now 2:00 P.M. At what time will the 70th person complete the ride?

**Discussion** Simulate the action with a clock or with a table.

2:00 – 2:10 riders 1 – 15
2:10 – 2:20 riders 16 – 30
2:20 – 2:30 riders 31 – 45
2:30 – 2:40 riders 46 – 60
2:40 – 2:50 riders 61 – 70 (75)

The 70th person will complete the ride at 2:50 P.M.

**PROBLEM 88** Ira wants to mail 2 letters and a postcard. One letter needs 39¢ worth of stamps while the other needs only 22¢. The postcard needs 14¢. He has the stamps shown in Figure B–15. Show how he should put the stamps on the letters and the postcard so that they can be mailed.

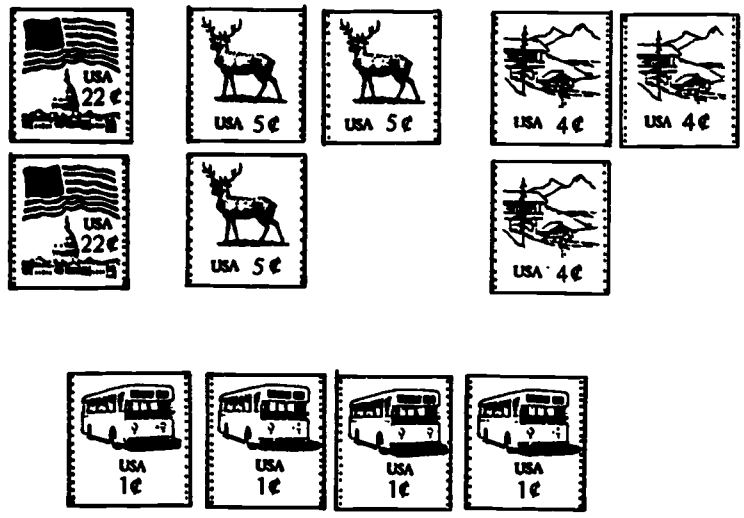



Figure B–15

**Discussion** Cut out the "stamps" (experiment) or simulate with a paper and pencil. Either way, the strategy used will be guess and test.

22¢ = 1 @ 22¢
39¢ = 1 @ 22¢ + 3 @ 5¢ + 2 @ 1¢
14¢ = 3 @ 4¢ + 2 @ 1¢

**PROBLEM 89** A triangular shape is made by placing a row of blocks on a table and then a row containing one less block on top of that row. Continue this procedure until 1 block is on the very top. If a total of 15 blocks are used, how many rows are in the triangular shape?

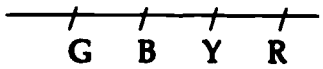
**Discussion** Work backward. Start with the top row of 1 block. If there were only 1 row, there would be only 1 block. Make a table.

| *Number of Rows* | *Total number of Blocks* |
|---|---|
| 1 | 1 = 1 |
| 2 | 1 + 2 = 3 |
| 3 | 1 + 2 + 3 = 6 |
| 4 | 1 + 2 + 3 + 4 = 10 |
| 5 | 1 + 2 + 3 + 4 + 5 = 15 |

There are 5 rows in the shape. Notice that this problem can be extended very nicely. Suppose there were 21 blocks, 55 blocks, etc.

**PROBLEM 90** There are four boats on the river. The yellow boat is in front of the red boat. The blue boat is behind the green boat. The yellow boat is behind the blue boat. In what order are the boats?

**Discussion** Draw a line. Place the boats on the line according to the clues, one at a time.




**PROBLEM 91** Dan has a bad cold and has to take 1 teaspoon of cough syrup every 2 ½ hours. He took his first dose at 9:00 A.M. He is supposed to take 6 doses before he goes to bed at 8:00 P.M. Can he do it?

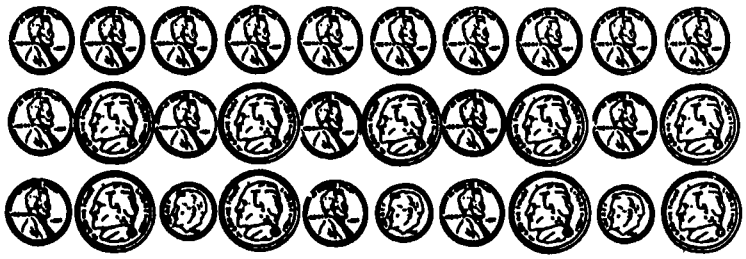
**Discussion** Make a table showing the time at which Dan takes each dose. The table reveals the answer.

| *Time* | *Dose Number* |
|---|---|
| 9:00 | 1 |
| 11:30 | 2 |
| 2:00 | 3 |
| 4:30 | 4 |
| 7:00 | 5 |
| 9:30 | 6 |

He cannot take the 6th dose in time before going to bed.

**PROBLEM 92** Put 10 pennies in a row on your table. Now replace every other coin with a nickel. Next replace every third coin with a dime. What is the value of the 10 coins now on the table?

**Discussion** Act it out with coins or simulate the action with materials or pencil and paper:



53¢

Figure B–16

**PROBLEM 93** Peter, Stuart, and Oliver are tossing a football. Peter tosses the ball 3 feet further than Stuart. Oliver tosses the ball 2 feet less than Peter. Who tossed the football the shortest distance?

**Discussion** Use a number line and simulate the tosses by segments. Begin by placing Stuart anywhere on the line. The line shows that Oliver threw the football the shortest distance.

**PROBLEM 94** Take two consecutive numbers. Multiply each number by itself. Add the products. Do it several times with different numbers. What can you tell about the results.?

**Discussion** When discussing two consecutive numbers, one will always be even and one will always be odd. The product of even numbers is always even, while the product of odd numbers is always odd. Thus, the sum of the resulting even number and odd numbers will always be an odd number.

**PROBLEM 95** Amy and Patti have a piece of rope that is 24 feet long. They want to cut it in order to make two jump ropes. Amy's rope is 6 feet longer than Patti's. How long is each rope?

**Discussion** Guess and test. List all number pairs whose sum is 24, until you find the pair whose difference is 6. The answers are 9 feet and 15 feet.

**PROBLEM 96** Sam, Kim, and Helen played a number guessing game. Sam wrote three numbers on a piece of paper and gave Kim and Helen the following three clues:

(a) The sum of the numbers is 17.
(b) All the numbers are different.
(c) Each number is less than 8.

Which three numbers did Sam write down?

**Discussion** Guess and test. Make a list. Begin with the largest possible number, 7. The only set of three numbers that satisfies all three clues is 7 + 6 + 4.

**PROBLEM 97** Ann, Beth, Carol, and David are throwing a ball. Each person throws the ball to the other three children. How many times is the ball thrown?

**Discussion** Make a drawing of the four people standing as vertices of a rectangle. The sides and diagonals of the rectangle represent the paths of the ball. Since each throws the ball to

each other, each line must be counted twice. There will be 12 tosses of the ball.

**PROBLEM 98** Karen has three different teachers for science, mathematics, and music. Mrs. Alexander enjoys her work as a music teacher. Mr. Brown used to teach science, but doesn't any more. Mrs. Carlton was absent last Tuesday. Who teaches each subject?

**Discussion** Use logic. Clue #1 establishes Mrs. Alexander as the music teacher. Clue #2 tells us that Mr. Brown is not the science teacher, and therefore must be the mathematics teacher. Notice that Clue #3 is not needed.

**PROBLEM 99** The six students in Mr. Charnes' biology class were arranged numerically around a hexagonal table. What number student was opposite number 4?

**Discussion** Draw a diagram showing the six students around the hexagonal table. Number 1 is opposite number 4.

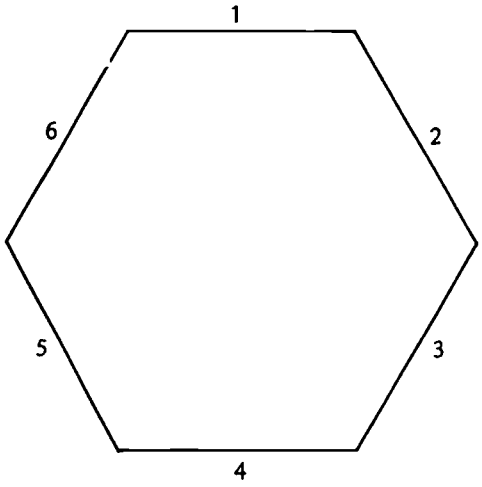



Figure B–17

**PROBLEM 100** The club members are saving to buy records. The records cost $5 each. The club treasurer puts money into an envelope until the envelope has $5 in it. Then she starts another envelope. The members of the club have saved $23 so far. How many envelopes do they have?

**Discussion** This problem can be acted out. However, it can be done by division with an understanding of the meaning of the remainder, a concept that is important in division. They have 5 envelopes.

**PROBLEM 101** How many squares of all sizes are on the checkerboard shown in Figure B–18?

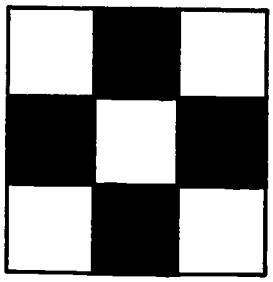


Figure B–18

**Discussion** Reduce the complexity of the problem. Consider a 1 × 1 square checkerboard, then a 2 × 2 square checkerboard.

| | *1 × 1* | *2 × 2* | *3 × 3* | *Total* |
|---|---|---|---|---|
| 1 × 1 checkerboard | 1 | — | — | 1 |
| 2 × 2 checkerboard | 4 | 1 | | 5 |
| 3 × 3 checkerboard | 9 | 4 | 1 | 14 |

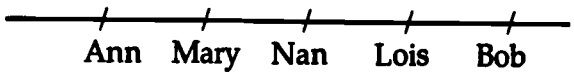
There are 14 squares of all sizes on the checkerboard.

**PROBLEM 102** At the record store, Carol bought the same number of tapes as records. She bought the same number of Western records as all the other records she bought. How many records and how many tapes did she buy if she bought 5 Western records?

**Discussion** Work backward. Carol bought 5 Western records; thus she bought 5 other records as well, or 10 records altogether. Since she also bought the same number of tapes as records, her total was 20 records and tapes.

**PROBLEM 103** Waiting in line to buy movie tickets, Lois was behind Nan. Mary was in front of Nan and behind Ann. Lois was between Nan and Bob. Who is in the middle of the line?

**Discussion** Draw a diagram consisting of a "number line":



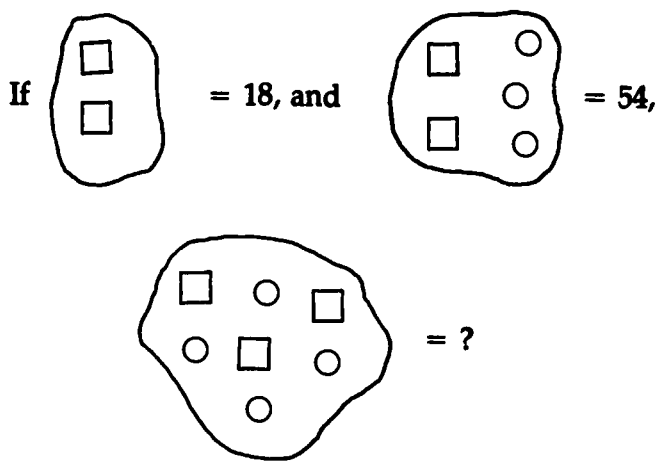
Use each clue to place the people in line. The drawing shows that Nan is the middle person in line.

**PROBLEM 104** Bill needs 39¢ worth of stamps to mail a package. He has only 5¢, 6¢, 7¢, and 8¢ stamps. He wants to use only two different kinds of stamps on each package. He could use 4 stamps at 8¢ each and 1 stamp at 7¢ to make up the 39¢ on one package. Find other ways he might mail the packages.

**Discussion** Guess and test. Make a table to keep track of your guesses. There are five ways:

| | |
|---|---|
| 5 @ 5¢ + 2 @ 7¢ | 3 @ 5¢ + 3 @ 8 ¢ |
| 3 @ 5¢ + 4 @ 6¢ | 3 @ 6¢ + 3 @ 7¢ |
| | 1 @ 7¢ + 4 @ 8¢ |

**PROBLEM 105**




**Discussion** If 2 squares = 18, then each square = 9. Thus, 3 circles + 18 will equal 54, and each circle = 12. Then 3 squares plus 4 circles equals 3(9) + 4(12) = 27 + 48 = 75.

**PROBLEM 106** In Panacola's Restaurant, a circular table seats 4 people. A rectangular table seats 6 people. There are 18 people waiting to be seated. How can it be done?

**Discussion** Make a list of the multiples of 4 and a list of the multiples of 6. See what numbers occur in both lists that give a sum of 18.

| *Multiples* of 4 | *Multiples* of 6 |
|---|---|
| 4 | 6 |
| 8 | 12 |
| 12 | 18 |
| 16 | |

Thus, we can seat them in two different ways: 3 circular tables and one rectangular table, or 3 rectangular tables.

**PROBLEM 107** From Figure B–19, select 2 strips of paper whose lengths have a sum of 15 centimeters and a difference of 3 centimeters.

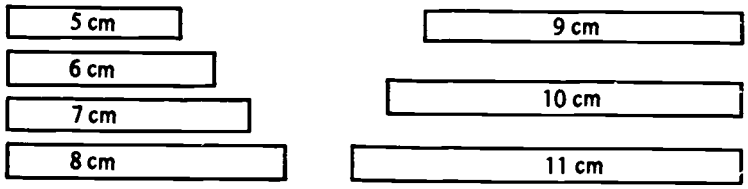



Figure B–19

**Discussion** Students may begin by examining pairs of the paper strips, the sum of whose lengths is 15 centimeters (10, 5; 9, 6; 8, 7), and then checking for a difference of 3 centimeters. Others may decide to list all pairs of numbers together with their sums and differences.

**PROBLEM 108** What's my number?

(a) I am a two-digit number.
(b) I am a multiple of 6.
(c) The sum of my digits is 9.
(d) My tens' digit is one-half of my units' digit.

**Discussion** Make a progressive list of all numbers that satisfy the first two clues and check each number against the remaining clues.

12 (does not satisfy clue (c)
18 (does not satisfy clue (d)
24 (does not satisfy clue (c)
. . .
Only 36 satisfies all four clues.

**PROBLEM 109** A spider wishes to crawl from point H to point B (see Figure B–19). How many different "trips" can he crawl, if each trip is exactly three edges long?

**Discussion** Simulate the trips with pencil and paper, and make a record of the paths covered:

| | | |
|---|---|---|
| H–E–F–B | H–G–C–B | H–D–A–B |
| H–E–A–B | H–G–F–B | H–D–C–B |

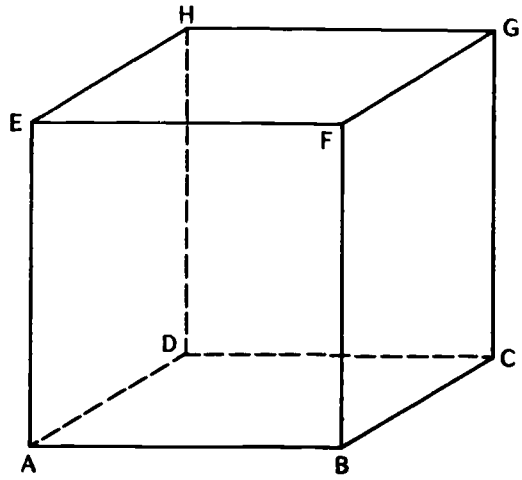



Figure B–20

**PROBLEM 110** A bus with 53 people on it makes two stops. At the first stop, 17 people get off and 19 people get on. At the second stop, 28 get off and 23 get on. How many people are now on the bus?

**Discussion** This problem requires careful reading and careful record keeping. Make a table.

| *Stop #* | *People Off* | *People On* | *On Board* |
|---|---|---|---|
| — | — | — | 53 |
| 1 | 53 − 17 = 36 | 36 + 19 = | 55 |
| 2 | 55 − 28 = 27 | 27 + 23 = | 50 |

There are now 50 people on board.

**PROBLEM 111** Jesse bowled 139, 196, and 154 in his first three games in the Bowling League. He got a 159 in his fourth game. Was this above or below his average for the first three games?

**Discussion** Find the average for the first 3 games and compare this to 159.

$$139 + 196 + 154 = 489$$
$$489 \div 3 = 163$$

The fourth game was 4 pins below his average for the first three games.

**PROBLEM 112**

$1 + 2 + 3 = 6$

$2 + 3 + 4 = 9$

$3 + 4 + 5 = 12$

$4 + 5 + 6 = 15$

Find the 3 consecutive numbers that add up to 24

**Discussion** Each set of 3 numbers begins with the next counting number. At the same time, each sum increases by 3. Thus, we will use $7 + 8 + 9 = 24$ to satisfy the problem. Some students may continue the entire pattern series until they reach the answer:

$$\begin{aligned} 1 + 2 + 3 &= 6 \\ 2 + 3 + 4 &= 9 \\ 3 + 4 + 5 &= 12 \\ 4 + 5 + 6 &= 15 \\ 5 + 6 + 7 &= 18 \\ 6 + 7 + 8 &= 21 \\ 7 + 8 + 9 &= 24 \end{aligned}$$

**PROBLEM 113** Alim, Brenda, and Carol are all selling fruit at the school carnival. They sold oranges, apples, and pears.

(a) Alim and the orange seller are sisters.
(b) The apple seller is older than Brenda.
(c) Carol sold the pears.

Who sold which kind of fruit?

**Discussion** Clue (c) tells us that Carol sells the pears. Since Alim cannot be the orange seller (clue (a)), she sells the apples. Thus, Brenda sells the oranges.

**PROBLEM 114** Ursula is in training. She did 5 sit-ups the first day. She did 6 sit-ups the second day, 7 the third day, and so on. How many sit-ups did she do on the 14th day?

**Discussion** Write out 14 counting numbers beginning with 5.

**PROBLEM 115** A fancy bottle of perfume costs $25. The bottle can be purchased by collectors without the perfume. When purchased this way, the bottle alone costs $15 less than the perfume. How much does the bottle cost alone?

**Discussion** Guess and test provides an alternative to an algebraic solution. Since the total for the bottle and perfume is $25, one could guess $1 for the bottle which leaves $24 for the perfume. Listing is an important skill.

| *Bottle* | *Perfume* | *Total* |
|---|---|---|
| 1 | 24 | 25 |
| 2 | 23 | 25 |
| 3 | 22 | 25 |
| • | • | • |
| • | • | • |
| • | • | • |
| 5 | 20 | 25 |

**PROBLEM 116** How many 2s must you multiply together to reach a 3-digit number?

**Discussion** Use a calculator. Continue multiplying by 2 until you go from 64 to 128. There will be seven 2s.

**PROBLEM 117** I have two children. The product of their ages is 24. The sum of their ages is 11. Find the ages of my children.

**Discussion** Make a list of all pairs of numbers whose product is 24.

| | |
|---|---|
| 24 | 1 |
| 12 | 2 |
| 8 | 3 |
| 6 | 4 |

Now find which of these pairs has a sum of 11. The children's ages are 8 and 3.

**PROBLEM 118** Jim is in line at the bridge waiting to pay his toll. He counts four cars in front of him and six cars behind him. How many cars are there in line at the bridge?

**Discussion** Act it out or draw a diagram using Xs to represent the cars in line. Don't forget to count Jim's car, too.

**PROBLEM 119** One paper clip is 3 centimeters long and weighs 1 gram. Joan made a chain of these paper clips that was 300 centimeters long. How many grams does the chain weigh?

**Discussion** Find the number of paper clips by dividing 300 centimeters by 3 centimeters. There are 100 clips. To find their total weight, multiply the number of clips by the weight of each clip.

**PROBLEM 120** You are waiting for the elevator to take you to the observation tower on the 70th floor of the Hancock building. There are 45 people in line ahead of you. If each elevator can carry 10 people, on which trip will you be?

**Discussion** The fact that the observation tower is on the 70th floor is excess information. If 10 people go on each trip, the first

four trips of the elevator will take 40 people. You will be on the 5th trip. Some children may begin with 46 and repeatedly subtract 10.

**PROBLEM 121** You had 7 dimes and 7 pennies. You bought a comic book for 49¢. You give the clerk 5 coins and she gives you one coin back. What coins do you now have?

**Discussion** Some children will need the actual coins to solve this problem. Others may simulate the situation with a paper and pencil. There is only one way to pay 49¢ and receive one coin in change, and that is with 5 dimes and 1¢ in change. Thus you now have 2 dimes and 8 pennies.

**PROBLEM 122** Gail bought 5 pencils that cost 12¢ each and 3 erasers that cost 8¢ each. She gave the clerk a $1 bill. How much change did she get?

**Discussion** This is another example of a multi-stage problem. The problem should be carefully worked in stages.

Stage 1: 5 pencils = 5 × 12¢ = 60¢
Stage 2: 3 erasers = 3 × 8¢ = 24¢
Stage 3: Amount spent = 60¢ + 24¢ = 84¢
Stage 4: $1.00 − .84 = 16¢
She received 16¢ in change.

**PROBLEM 123** Nicole has a package of 48 silver stars. She wants to arrange them in rows, so that each row has the same number of stars. How can she arrange them so that the number of stars in each row is an odd number?

**Discussion** Have the children make a list of all the number pairs whose product is 48:

| *Number of Rows* | × | *Number of Stars* |
|---|---|---|
| 48 | × | 1 |
| 24 | × | 2 |
| 16 | × | 3 |
| 12 | × | 4 |
| 8 | × | 6 |
| 6 | × | 8 |
| 4 | × | 12 |
| 3 | × | 16 |
| 2 | × | 24 |
| 1 | × | 48 |

The list shows two series in the "stars" column that are

odd numbers. That is, 48 rows of 1 star, or 16 rows of 3 stars.

**PROBLEM 124** Luisa was playing darts. She threw 3 darts and all 3 hit the target shown in Figure B–21. Which of the following could be her score?

4, 17, 56, 28, 29, 31

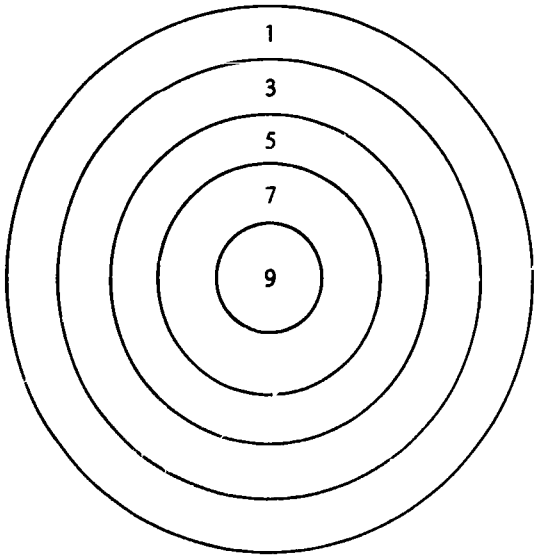



Figure B–21

**Discussion** Since all 3 darts hit the target, Luisa's highest score could only be 3 × 9 or 27; her lowest possible score could be 3 × 1 or 3. Furthermore, since there are only odd numbers on the target, the 3 hits must have an odd sum. Thus, only 17 is a possible score for Luisa. She could have scored this in several ways:

$$9 + 5 + 3 = 17$$
$$7 + 5 + 5 = 17$$

**PROBLEM 125** Here is a menu for lunch in school:

Hamburger ..... 38¢
French fries ..... 15¢
Malted milk ..... 35¢
Milk ..... 25¢
James spent 78¢. What did he buy?

**Discussion** By guess and test, we arrive at 1 hamburger, 1 french fries, and 1 milk. (Ask the students if other answers are possible.)

**PROBLEM 126** The town of Graphville has intersections formed by 27 avenues that run north–south and 31 streets that run east–west. If we plan one traffic light at each intersection, how many traffic lights do we need?

**Discussion** The most direct way of dealing with this problem would be to actually draw the 31 by 27 line grid and count the intersections. However, the complexity of the numbers can be reduced to a $2 \times 2$ grid, then a $2 \times 3$ grid, then a $2 \times 4$ grid, a $3 \times 3$ grid, etc., until we see that the product of the two numbers is the number of intersections.

**PROBLEM 127** Last Saturday, George and his friend Mike went to a big-league baseball game. After the game, they went to the locker room to collect autographs of their favorite players. Together they collected 18 autographs, but Mike collected 4 more than George. How many did George collect?

**Discussion** Although this problem in an algebra class would provide a classic example of the simultaneous solution of two linear equations, it also provides an excellent opportunity for younger students to practice guess and test in conjunction with organized listing. A series of carefully chosen recorded guesses adding to 18 leads to the numbers 11 and 7.

| *George* | *Mike* | *Total* |
|---|---|---|
| 0 | 18 | 18 |
| 1 | 17 | 18 |
| 2 | 16 | 18 |
| • | • | • |
| • | • | • |

**PROBLEM 128** I have five coins: quarters, nickels, and dimes. The total value of the coins is 50¢. How many of each coin do I have?

**Discussion** Make an exhaustive list:

| *Quarters* | *Dimes* | *Nickels* | *Total* | *Number of Coins* |
|---|---|---|---|---|
| 2 | 0 | 0 | 50¢ | 2 |
| 1 | 2 | 1 | 50¢ | 4 |
| 1 | 1 | 3 | 50¢ | 5 |
| 0 | 5 | 0 | 50¢ | 5 |

Since all the coins were to be represented, only the third row is a correct answer.

**PROBLEM 129** Here are the designs drawn on the six faces of a cube:

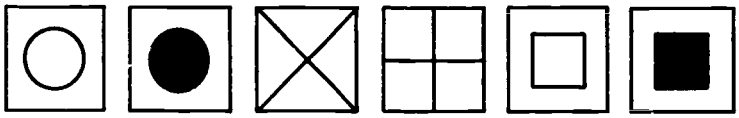


Figure B–22

Here are three views of the same cube. Which designs are on opposite faces of the cube?

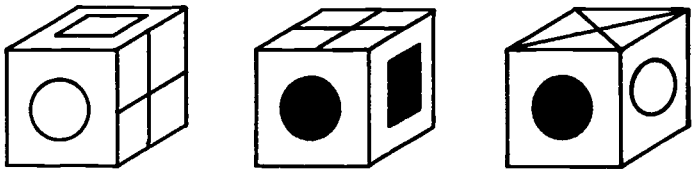


Figure B–23

**Discussion** From the second and third views of the cube, we can see that neither the "+" sign, the solid square, the "X" sign, nor the open circle can be *opposite* the solid circle, since they are all shown to be *adjacent* to the solid circle. Therefore, only the open square can be opposite the solid circle. In a similar manner, it can be shown that the open circle is opposite the solid square and the "+" sign is opposite the "X" sign.

**PROBLEM 130** There were 8 girls and 16 boys at a meeting of the June Fair Planning Committee of the third grade. Every few minutes, one boy and one girl leave the meeting to go back to class. How many of these boy and girl "pairs" must leave the meeting so that there will be exactly five times as many boys as girls left at the meeting?

**Discussion** Make a list:

| *Boys* | *Girls* |
|---|---|
| 16 | 8 |
| 15 | 7 |
| 14 | 6 |
| 13 | 5 |
| 12 | 4 |
| 11 | 3 |
| 10 | 2 |

5 times as many; six pairs must leave.

**PROBLEM 131** Put a single digit into each box and make the problem correct.

$$\begin{array}{r} \square\,\square\,\square \\ \times\ \square \\ \hline 1\,0\,9\,0 \end{array}$$

**Discussion** There are two possible answers:

$$\begin{array}{r} 545 \\ \times 2 \\ \hline 1090 \end{array} \qquad \begin{array}{r} 218 \\ \times 5 \\ \hline 1090 \end{array}$$

**PROBLEM 132** Stanley makes extra money by buying and selling comic books. He buys them for 7¢ each and sells them for 10¢ each. Stanley needs 54¢ to buy some batteries for his calculator. How many comic books must Stanley buy and sell to earn the 54¢?

**Discussion** Some students will realize that Stanley earns 3¢ profit on each comic book. Thus, they can divide 54¢ by 3¢ to find the number of comic books he must sell (18). Other students will want to make a table:

| Number of comics | 1 | 2 | 3 | 4 | 5 | 6 | 7 . |
|---|---|---|---|---|---|---|---|
| Profit | 3¢ | 6¢ | 9¢ | 12¢ | 15¢ | 18¢ | 21¢ . |

**PROBLEM 133** How many paths are there from Start to Finish?

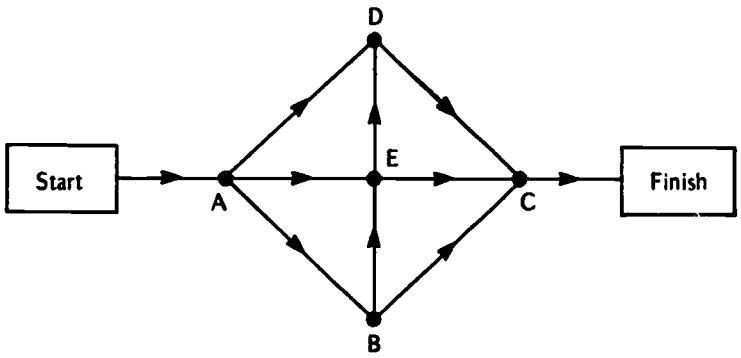



Figure B–24

**Discussion** Make an exhaustive list of all the possibilities. Organize the list beginning with A–B.

S–A–B–C–F S–A–D–C–F S–A–E–C–F
S–A–B–E–C–F S–A–E–D–C–F
S–A–B–E–D–C–F

There are 6 different paths from S (Start) to F (Finish).

**PROBLEM 134** In January, our team won 2 games and lost the same number. In February, the team lost 3 more games than it did in January, but won the same number it lost. In March, it won the same number of games as it did in February but lost 2 fewer games than it did in February. What was its record at the end of March?

**Discussion** Organize the data. Put the information into a table as you read it.

| | *January* | *February* | *March* | *Total* |
|---|---|---|---|---|
| Win | 2 | 5 | 5 | 12 |
| Lose | 2 | 5 | 3 | 10 |

The team's record was 12 wins and 10 losses at the end of March.

**PROBLEM 135** What is the smallest number of pennies that can be arranged into 6 equal piles and also into 8 equal piles?

**Discussion** Some students will multiply 6 × 8 and give 48 as their answer. This would be a correct solution if it were not for the requirement that the answer be the *smallest* number. Thus, 24 is the correct answer. Notice that this problem can be done using chips, marbles, or bottle caps for the students who wish to work with physical objects.

**PROBLEM 136** Lucy has a dog, a parrot, a goldfish, and a Siamese cat. Their names are Lou, Dotty, Rover, and Sam. The parrot talks to Rover and Dotty. Sam cannot walk nor fly. Rover runs away from the dog. What is the name of each of Lucy's pets?

**Discussion** Prepare a logic matrix as shown. As each clue is given, record the information in the matrix. The first clue, "the parrot talks to Rover and Dotty," tells us that the parrot cannot be Rover nor Dotty. Place an X in the appropriate

boxes in the matrix. The second clue, "Sam cannot walk or fly" establishes Sam as the goldfish. Put a checkmark in the appropriate box, and Xs in all the remaining boxes in the Sam–goldfish row and column. Continuing this process establishes that the parrot is Lou, the dog is Dotty, the goldfish is Sam, and the cat is Rover.

| | Dog | Parrot | Goldfish | Cat |
|---|---|---|---|---|
| Lou | | | x | |
| Dotty | | x | x | |
| Rover | | x | x | |
| Sam | x | x | ✓ | x |

**PROBLEM 137** A farmer has 15 animals, some pigs and some chickens. Together they have a total of 40 legs. How many pigs and how many chickens does the farmer have?

**Discussion** We have the restriction that pigs have 4 legs and chickens each have 2 legs. Guess and test. Prepare a table to record our guesses and to refine each guess as we proceed.

| *Pigs* | *Legs* | *Chickens* | *Legs* | *Total Number of Legs* |
|---|---|---|---|---|
| 1 | 4 | 14 | 28 | 32 |
| 2 | 8 | 13 | 26 | 34 |
| 3 | 12 | 12 | 24 | 36 |
| 4 | 16 | 11 | 22 | 38 |
| 5 | 20 | 10 | 20 | 40 |

The farmer had 5 pigs and 10 chickens.

**PROBLEM 138** How many lengths of rope each 3 feet long can be cut from a roll of rope that contains 50 feet of rope?

**Discussion** If we divide 50 by 3, we obtain 16 with a remainder of 2. Thus the correct answer is 16. Disregard the remainder, since we need each length to be exactly 3 feet long.

**PROBLEM 139** A taxi charges 90¢ for the first one-quarter mile, and 25¢ for each additional quarter mile. How much did Leon pay for a ride of 1 mile?

**Discussion** This is a multi-stage problem. The first quarter mile costs 90¢ (Stage 1). This leaves 3 quarters (Stage 2). These 3

quarters cost 25¢ each, or 75¢ (Stage 3). The total cost is 90¢ + 75¢ or $1.65 (Stage 4).

**PROBLEM 140** A taxi charges 90¢ for the first one-quarter mile and 25¢ for each additional quarter mile. Sandy paid $2.90 for her ride. How far did she travel?

**Discussion** Sandy spent 90¢ of the $2.90 on the first quarter-mile. The remaining $2 when divided by 25¢ for each quarter mile yields 8 additional quarter miles. Thus, she paid for a ride of 9 quarter miles or 2 ¼ miles. Note: since the meter "clicks" at the beginning of the quarter-mile segment, the actual ride must have been somewhere between 2 miles and 2 ¼ miles. (This problem is typical of a class of problems whose graph yields a step-function. Other problems in this class include postage rates, sales tax, etc.)

**PROBLEM 141** Given the sequence of numbers,

2, 3, 5, 8,

explain why the next number might be 12, or 13, or 2, or 5.

**Discussion** There are a variety of ways in which the four given terms might have been arrived at. For example, if we regard these as members of a Fibonacci sequence, each term was arrived at by adding the preceding two terms. Thus, 2 + 3 = 5, 3 + 5 = 8, 5 + 8 = 13, and so on. On the other hand, we might view the sequence as having been generated by adding increasing differences. Thus, 2 + 1 = 3, 3 + 2 = 5, 5 + 3 = 8, 8 + 4 = 12, and so on. Then, too, the sequence might be viewed as a cyclical sequence in which the four terms repeat. Thus, the next term would again be a 2. Finally, the series might be a 7-term series that is symmetric about the middle term, 8. Thus, the next term would be 5.

**PROBLEM 142** How many breaths do you take in one 24-hour day?

**Discussion** Have the students first determine how many breaths they take in one minute. Then use a calculator. Multiply by 60 (to find the number of breaths in one hour) and then by 24 (for one day). For example, if a student takes 20 breaths in one minute, he or she would take 20 × 60 × 24 or 28,800 breaths in one day. Students may be amazed at the size of the final answer.

**PROBLEM 143** A city block is about 270 feet long. If cars are parked bumper-to-bumper, and a small car is 15 feet, while a large car is 18 feet,

(a) What is the smallest number of cars that can be parked on one block?
(b) What is the largest number of cars that can be parked on one block?
(c) If we park an equal number of large and small cars in one block, how many would fit?

**Discussion**

(a) The smallest number of cars occurs when all the cars are large cars: 270 ÷ 18 = 15. The smallest number is 15 cars.
(b) The largest number of cars occurs when all the cars are small cars: 270 ÷ 15 = 18. The largest number is 18 cars.
(c) Since there are an equal number of each size, a pair of cars will total 33 feet: 270 ÷ 33 = 8.1. There will be 8 cars of each size.

Since 270 is the product of 18 and 15, parts (a) and (b) can be done mentally.

**PROBLEM 144** What is the greatest number of coins you can use to make 35¢? What is the smallest number of coins you can use? In how many different ways can you make 35¢?

**Discussion** The greatest number of coins is obviously 35 pennies. The smallest number of coins is 2 (1 dime and 1 quarter). To find the number of *different* ways change can be made, we can make a table.

| *Pennies* | *Nickels* | *Dimes* | *Quarters* |
|---|---|---|---|
| 35 | — | — | — |
| 30 | 1 | — | — |
| 25 | — | 1 | — |
| 25 | 2 | — | — |
| • | • | • | • |
| • | • | • | • |
| • | • | • | • |
| — | — | 1 | 1 |

**PROBLEM 145** Mrs. Lewis bought 6 cards. Mr. Lewis bought 6 cards that same day. How much would they have saved if they had bought 12 cards and shared them equally?

| Number of Cards | 1–3 | 4–6 | 7–9 | 10–12 | 13 or more |
|---|---|---|---|---|---|
| Cost for Each Card | $1.00 | 90¢ | 85¢ | 80¢ | 75¢ |

**Discussion** The necessary information for obtaining the answer is in the table. First find the cost for each person to buy 6 cards:

$$6 \times 90¢ = \$5.40$$

They spent a total of \$10.80 for their 12 cards. Now find the cost for 12 cards as a single purchase:

$$12 \times 80¢ = \$9.60$$

They would have saved \$10.80 − \$9.60 or \$1.20.

**PROBLEM 146** Melinda bought some peanuts for 35¢ and an apple for 20¢. She paid for her purchases with 3 coins of the same amount. How much change did she receive?

**Discussion** When we add 35¢ + 20¢ we get 55¢. Melinda could not have paid her bill with 3 dimes (30¢) or 3 nickels (15¢). She would not have had to pay with 3 half-dollars, since 2 would have been enough. Thus she must have paid with 3 quarters, or 75¢. Her change was 75¢ − 55¢ = 20¢.

**PROBLEM 147** What was the final score of the Tigers – Sharks baseball game?

(a) If their scores are added, the sum is 8.
(b) If their scores are multiplied, the product is 15.
(c) The Sharks won the game.

**Discussion** Make a list of all the number pairs whose sum is 8.

8–0
7–1
6–2
5–3
4–4

Now find the pair of numbers on the list whose product is 15. Since the Sharks won the game, the final score must have been Sharks 5, Tigers 3.

**PROBLEM 148**

*Table of Moon Facts*

The moon is smaller than the earth.
People weigh 6 times as much on earth as on the moon.
The moon goes around the earth once in 28 days.
The moon is about 240,000 miles from the earth.

(a) Peter figures that he would weigh 14 pounds on the moon. What does Peter weigh on Earth?
(b) Peter's mother weighs 120 pounds on Earth. How much would she weigh on the moon?
(c) About how long does it take the moon to go around the earth four times?

**Discussion** This three-part problem involves obtaining facts from a table.

(a) If Peter weighs 14 pounds on the moon, he must weigh 84 pounds on Earth.
(b) If Peter's mother weighs 120 pounds on Earth, she would weigh ⅙ as much or 20 pounds on the moon.
(c) If the moon goes around the earth once in 28 days, it would take approximately 112 days to go around the earth four times.

**PROBLEM 149** How far are you from Tamar when you are on the road and midway between Tamar and Cass?

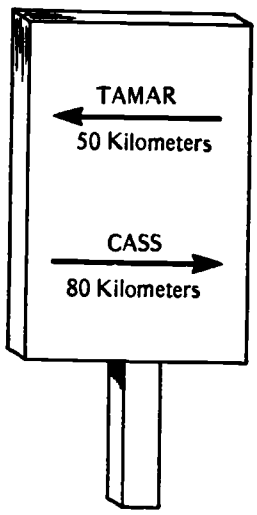



Figure B–25

**Discussion** Examine the question carefully. It asks how far are you when you are halfway. Thus the starting point is irrelevant. Since the total distance is 130 kilometers, the midpoint is 65 km.

**PROBLEM 150** Sandra owes Charlene $1.35. Sandra and Charlene agree to split equally the cost of a $2.00 comic book. Sandra pays the $2.00 for the book. How much does Sandra now owe Charlene?

**Discussion** Act it out or think it through. Sandra paid $2 for the comic book. Thus Charlene's share was $1.00, which she owes Sandra. Since Sandra owed Charlene $1.35, she now owes her only 35¢.

**PROBLEM 151** Mitch and his sister Pauline went to visit a friend who lives

12 blocks away. They walked 6 blocks when they realized that they had dropped a book. They walked back and found the book. Then they walked the 8 blocks to their friend's house. How far from their home did they drop the book?

**Discussion** Some children will draw a diagram to illustrate the problem. However, what is really needed is to subtract the 8 blocks they finally walked after finding the book from the 12-block trip. They dropped the book 4 blocks from their home.

**PROBLEM 152** "I want you to go shopping for me," said Jimmy's mother. "First go 5 blocks west to the grocery store. Then go 3 blocks east to the fruit store. Then go 5 blocks east to the candy store." Which store is closest to Jimmy's house?

**Discussion** Simulate the action with a series of drawings of a number line.

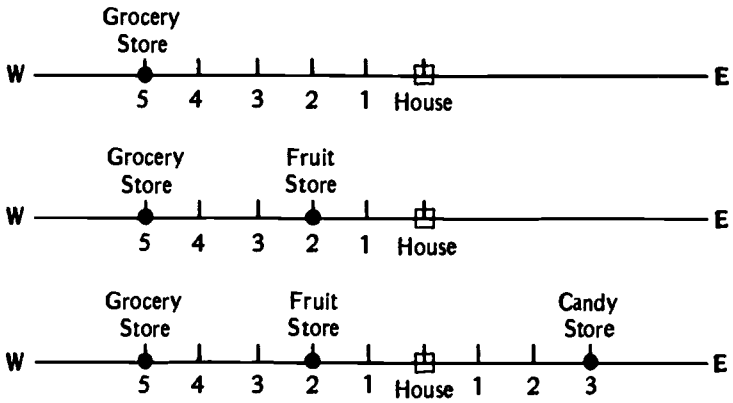
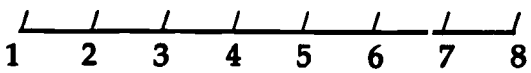



Figure B-26

The drawing shows that the fruit store is only 2 blocks from Jimmy's house, due west.

**PROBLEM 153** A circus tent has 8 poles from one end to the other, in a straight line. The poles are 20 meters apart. How long is the tent? What if there were 11 poles?

**Discussion** A drawing reveals that there are 7 "spaces" between 8 poles.




Thus, there are 7 × 20 or 140 meters as the length of the

tent. If there had been 11 poles, we would have had 10 spaces, or 200 meters from end to end.

**PROBLEM 154** July has 5 Tuesdays. Three of them fall on even-numbered dates. What is the date of the third Tuesday in July?

**Discussion** July has 31 days. In order to have 5 Tuesdays, they would fall on the following dates:

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 8 | 9 | 10 |
| 15 | 16 | 17 |
| 22 | 23 | 24 |
| 29 | 30 | 31 |

Since 3 of the dates must fall on even-numbered dates, the Tuesdays would fall on the 2nd, 9th, 16th, 23rd and 30th. The third Tuesday would be July 16.

**PROBLEM 155** Mary bought a candy bar for 29¢. She gave the clerk a $1 bill and received 5 coins in change. What 5 coins did she receive?

**Discussion** Since she gave the clerk 29¢, she received 71¢ in change. Make a list showing the possibilities. (She obviously receives 1¢, leaving 4 coins to make 70¢.)

| 50¢ | 25¢ | 10¢ | 5¢ | 1¢ |
|---|---|---|---|---|
| 1 | | 1 | 2 | 1 |
| | 2 | 2 | | 1 |

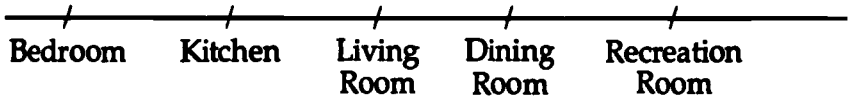
She received either 2 quarters, 2 dimes, and 1 penny, or 1 half-dollar, 1 dime, 2 nickels, and 1 penny.

**PROBLEM 156** A rabbit ate 32 carrots in 4 days. If he ate 2 more carrots each day than he did the day before, how many carrots did he eat each day?

**Discussion** Guess and test. Some children may require hands-on material. Give them 32 tokens, chips, or other materials. Have them separate the chips into 4 piles each of which contains 2 more than the preceding one. The answer is 5 the first day, 7 the second day, 9 the third day, and 11 carrots the fourth day.

**PROBLEM 157** Nan has a 5-room apartment. The bedroom is next to the kitchen. The living room is between the kitchen and the dining room. The recreation room is farthest from the bedroom. Which room is in the middle?

**Discussion** Make a drawing to show the given information.




The living room is in the middle.

**PROBLEM 158** What is the sum of the numbers in this table?

| | | | | |
|---|---|---|---|---|
| 3/4 | 3/8 | 3/7 | 3/5 | 3/13 |
| 2/5 | 4/7 | 10/13 | 5/8 | 1/4 |

**Discussion** A careful examination of the table reveals that there are five pairs of fractions, each of which has a sum of 1. Thus, the sum of the ten numbers is 5.

**PROBLEM 159**

3 yuchs = 2 ughs
4 ughs = 6 wims
2 yuchs = ? wims

**Discussion**

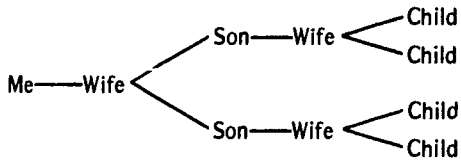
3 yuchs = 2 ughs
6 yuchs = 4 ughs = 6 wims

If 6 yuchs = 6 wims, then 2 yuchs will equal 2 wims.

**PROBLEM 160** I am taking these people to dinner:

(a) me
(b) my wife
(c) my 2 sons and their wives
(d) each son's 2 children
How many reservations should I make?

**Discussion** Here is an opportunity to make a tree drawing:




4 Children
2 Wives
2 Sons
1 Wife
+ 1 Me
10

I must make a reservation for 10 people.

**PROBLEM 161** If a pound of plums contains 4 to 6 plums, what is the least possible weight in pounds of 3 dozen plums?

**Discussion** Since a pound will contain 4, 5, or 6 plums, three dozen plums (36 plums) will weigh 9 pounds, 7 1/5 pounds, or 6 pounds. The least possible weight is 6 pounds.

**PROBLEM 162** How would you make 5 liters?

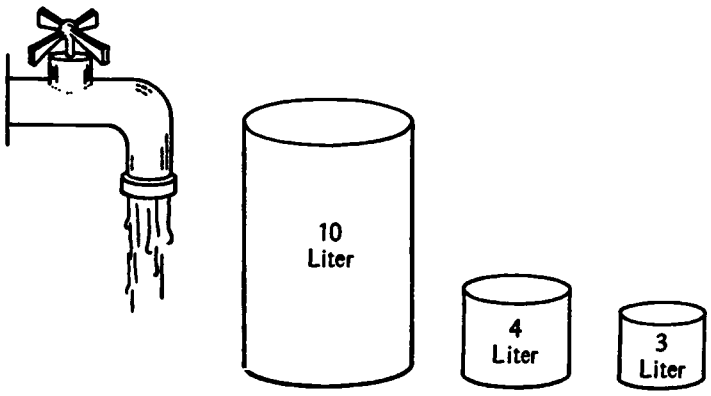



Figure B–27

**Discussion** There are several ways to do this.

(a) Fill the 10-liter pail and pour it into the 4-liter pail. This leaves 6 liters. Empty the 4-liter pail, and fill it again from the 6 liters that remain in the 10-liter pail. There are now 2 liters in the 10-liter pail. Now fill the 3-liter pail and pour it into the 10-liter pail along with the 2 liters already there.
(b) Fill the 4-liter pail and pour it into the 3-liter pail. This will leave 1 liter in the 4-liter pail, which should be poured into the 10-liter pail. Now refill the 4-liter pail and add it to the 1 liter already in the 10-liter pail.
(Ask your students to find additional ways.)

**PROBLEM 163** My license tag is a 3-digit number. The product of the digits is 216; their sum is 19; and the numbers appear in ascending order. Find my license plate number.

**Discussion** Make a list of all the number triples whose product is 216 and which are single digits. There are only three such triples:
3, 8, 9—the sum of these is 20
4, 6, 9—the sum of these is 19
6, 6, 6—the sum of these is 18

Only 4, 6, and 9 satisfy the given conditions. The license plate number is 469. Note that this problem also provides a considerable amount of drill and practice in factors, multiplication, and division.

**PROBLEM 164** The cost of a concert ticket and a football ticket is $14. The cost of a movie ticket and a football ticket is $11. The cost of a concert ticket and a movie ticket is $7. Find the cost of each ticket.

**Discussion** A concert ticket and a football ticket cost $14. A movie ticket and a football ticket cost $11. Thus, the concert ticket is $3 more than the movie ticket. Since the concert ticket and the movie ticket cost $7, we need two numbers whose sum is 7 and whose difference is 3. Guess and test. The concert ticket costs $5; the football ticket costs $9; the movie ticket costs $2.

**PROBLEM 165** Norene set her wristwatch when she left for school at exactly 7:30 A.M. on Monday. At 1:30 P.M. on Monday, she noticed that her watch had lost 4 minutes. At this same rate, how many minutes will the watch lose by the time Norene resets it when she leaves for school at 7:30 A.M. on Tuesday?

**Discussion** Although this problem can be solved by many students by counting, since clock arithmetic is in base 12, others may need a picture of a clock or a model of a clock with moveable hands to illustrate the situation. From the drawing, students should see that the elapsed time between 7:30 A.M. and 1:30 P.M. is 6 hours. Since there are 24 hours until 7:30 A.M. on Tuesday, we need a number 4 times the 6 hours. Thus 4 times the 4 minutes will make her watch lose 16 minutes.

**PROBLEM 166** Laura jogs 7 blocks the first day of her training program. She increases her distance by 2 blocks each day. On the last day, she jogs 25 blocks. How many days was she in training?

**Discussion** Make a list.

| *Day* | *Number of Blocks* |
|---|---|
| 1 | 7 |
| 2 | 9 |
| 3 | 11 |
| 4 | 13 |
| • | • |
| • | • |
| • | • |

**PROBLEM 167** How much will it cost to cut a log into 8 equal pieces if cutting it into 4 equal pieces cost 60¢? There is no stacking of the pieces.

**Discussion** Make a drawing of the log. It is easy to see that cutting the log into four pieces requires only three cuts. Thus, each cut costs 20¢. To cut the log into eight equal pieces, we need only seven cuts at 20¢ each, or $1.40.

**PROBLEM 168** The listed price for *Sports Magazine* is $1.25 a copy. You pay $16.56 for a 24-issue subscription. How much do you save by buying the subscription?

**Discussion** This is an example of a two-stage problem. Students first find the total cost of 24 copies at the per issue rate. Then they subtract the subscription price from this total.

**PROBLEM 169** Pat and Mike are having a contest. They will shovel snow to clear a 21-foot path. Pat shovels 3 feet with each push of the shovel. Mike shovels 1 foot on the first push, 2 feet on the second push, 3 feet on the third push, and so on. He will shovel 1 foot more on each push than on the push before. Who wins the contest?

**Discussion** Make a table to simulate the action.

| *Pat* | | | *Mike* | |
|---|---|---|---|---|
| *Total Distance* | *Distance* | *Push #* | *Distance* | *Total Distance* |
| 3 | 3 | 1 | 1 | 1 |
| 6 | 3 | 2 | 2 | 3 |
| 9 | 3 | 3 | 3 | 6 |
| 12 | 3 | 4 | 4 | 10 |
| 15 | 3 | 5 | 5 | 15 |
| 18 | 3 | 6 | 6 | 21 |

Mike wins the contest, since he shoveled the 21 feet on the 6th push, while Pat only shoveled 18 feet.

**PROBLEM 170** Four people enter a clubroom. Each person shakes hands with each of the other people. How many handshakes are there?

**Discussion** You can act out this problem. Select 4 students and have them each shake hands while the class keeps count. Or, make an exhaustive list.

| *A shakes* | *B shakes* | *C shakes* | *D shakes* |
|---|---|---|---|
| A–B | ~~B–A~~ | ~~C–A~~ | ~~D–A~~ |
| A–C | B–C | ~~C–B~~ | ~~D–B~~ |
| A–D | B–D | C–D | ~~D–C~~ |

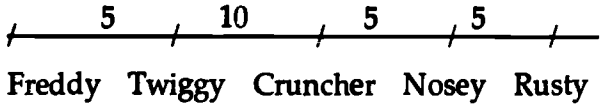
Notice that several of the handshakes are the same. That is, if A shakes hands with B, that is the same as B shakes hands with A. Thus, repeats are crossed out in the list. There will be 3 + 2 + 1 = 6 handshakes.

**PROBLEM 171** The 3-digit number 53A is exactly divisible by 6. Find the value of A.

**Discussion** To be exactly divisible by 6, a number must be divisible by 2 (an even number) and divisible by 3. If we try A = 0, 2, 4, 6, or 8, we see that A must be 4. Thus, 534 ÷ 6 = 89. A more direct procedure would be to use the division algorithm, replacing the third digit with 0. Then 530 ÷ 6 = 88 with a remainder of 2. But the remainder must be a 6 or a 0 to be divisible by 6. Thus, we need A = 0 + 4 or 4. The number is again 534.

**PROBLEM 172** Five bookworms have eaten into the big dictionary on the teacher's desk. Twiggy is 20 mm ahead of Rusty. Cruncher is 10 mm behind Twiggy. Rusty is 5 mm behind Nosey. Freddy is 15 mm ahead of Cruncher. Nosey is 20 mm behind Freddy. List the five bookworms in order.

**Discussion** Draw a number line and use the clues place the bookworms on the line.




Notice that the final clue, Nosey is 20 mm behind Freddy, is not needed to solve the problem.

**PROBLEM 173** I have an apple, an orange, and a peach. I weighed them two at a time. The apple and the orange weigh 14 ounces; the apple and the peach weigh 18 ounces; the orange and the peach weigh 20 ounces. How much does the apple weigh?

**Discussion** The apple and the orange weigh 14 ounces. The apple and the peach weigh 18 ounces. This tells us that the peach weighs 4 ounces more than the orange. However, the orange and the peach together weigh 20 ounces. Thus, we are looking for two numbers whose sum is 20 and whose difference is 4, namely 8 and 12. The weights are: orange = 8 ounces; peach = 12 ounces; apple = 6 ounces.

**PROBLEM 174** Sol gave away half of his marbles, dividing them equally among Mary, Doug, and Linda. Linda took her share of

the marbles and shared them equally among herself and 4 friends. Each friend got 4 marbles. How many marbles did Sol start with?

**Discussion** Work backward. Linda shared her marbles with 4 other people. Each of them received 4 marbles, so Linda must have had 20 marbles. Since Sol gave marbles equally to 3 people (one of whom was Linda), he must have given away 60 marbles. Thus he started with 120 marbles

**PROBLEM 175** In a recent sale at the local stationery store, the following sign appeared:

| | |
|---|---|
| ERASERS | 5¢ |
| PENCILS | 7¢ |
| LIMIT: 3 OF EACH TO A CUSTOMER | |

If you had 20¢ to spend, what different combinations of pencils and erasers could you buy?

**Discussion** Make a list. The following combinations are possible:

| *Erasers* | *Pencils* |
|---|---|
| 0 | 1 |
| 0 | 2 |
| 1 | 0 |
| 1 | 1 |
| 1 | 2 |
| 2 | 0 |
| 2 | 1 |
| 3 | 0 |
| 4 | 0 |

**PROBLEM 176** You bought 10 comic books at 5 for $9. You then sold them all at $2 each. How much profit did you make?

**Discussion** Do this problem by the divide and conquer strategy. Work each part separately. If you bought 10 comic books at 5 for $9, then you bought 2 × 5 for 2 × $9 or $18. If you sold all 10 comic books for $2 each, you received $20. Your profit was $20 − $18 or $2.

**PROBLEM 177** Jeremy worked a math problem and got 16 as his answer. However, in the last step, he multiplied by 2 instead of dividing by 2. What should have been the correct answer?

**Discussion** Work backward! If Jeremy multiplied by 2 as the last step, he must have had 8, just prior to that. If he correctly divided by 2, he would have had 4, the correct answer.

**PROBLEM 178** With 3 minutes left to play in the game between the Cougars and the Hawks, the Cougars were ahead by 10 points. In those last 3 minutes, the Cougars scored 6 points per minute, while the Hawks scored 99 points per minute. Who won the game, and what was the final score?

**Discussion** Examine the scoring during those last 3 minutes. The Cougars scored $3 \times 6$ or 18 points. The Hawks scored $3 \times 9$ or 27 points. The Hawks scored 9 points more than the Cougars. But they had trailed by 10 points. Thus, the Cougars won by 1 point. An alternate method might be to pick any scores with the Cougars ahead by 10, say 48 to 38. In 3 minutes, the Hawks score $3 \times 9$ or 27 points. Their score will be $38 + 27 = 65$. But, the Cougars score $3 \times 6$ or 18 points. Their final score is $48 + 18$ or 66 points. So, the Cougars won by 1 point, 66 to 65.

## SECTION C

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## SECTION D

# Masters for Selected Problems

This section contains the problems from Section C in reproducible form. They may be used in a variety of ways in your classroom. Among them, we suggest the following:

(1) *Problem Sheets for the Classroom*
Duplicate the individual pages as you wish to use them. Distribute the sheets in class as they are needed.

(2) *Student Problem Decks*
Duplicate the sheets and distribute them to the children. Have them attach each to a 5″ × 8″ card. This will provide each student with his or her own deck of problems.

(3) *Teacher Resource Deck*
Cut out the individual problems as shown and paste each on a 5″ × 8″ card. The discussions and/or solutions can be obtained from Section C, and placed on the back of the card. Next, laminate the individual cards and place them in a box to be used as you need them.

What's next?

(a) 1, 2, 3, 4, _____.

(b) 2, 4, 6, 8, _____.

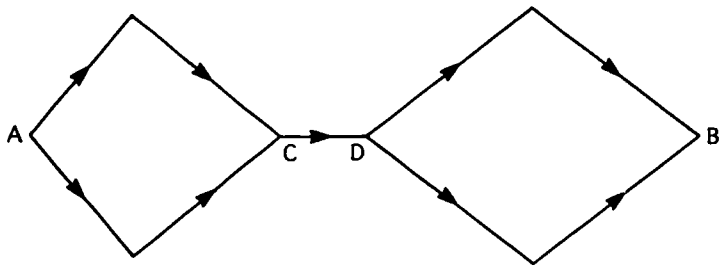
(c) △, □, ○, △, □, ○, △, _____.

John is taller than Alex. Lucy is shorter than Alex. Arrange the three children in order of size with the shortest child first.

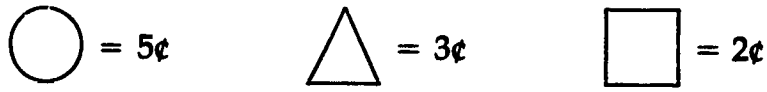
In our classroom, I have Spelling before Art. I have Math right after Art. Which class comes first?

Bianca jumped from the 4-foot line and landed on the 9-foot line. Joanne jumped from the 2-foot line and landed on the 6-foot line. Who had the longer jump?

How many ways can you get from A to B in the figure shown below?



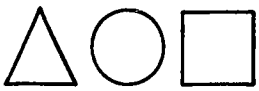

David woke up at 7:00 A.M.
Barbara woke up one hour after David.
Suzie woke up two hours before Barbara.
At what time did Suzie wake up?



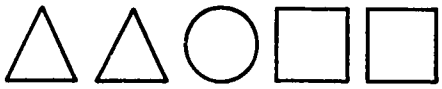

Find the value of:

(a)

(b)

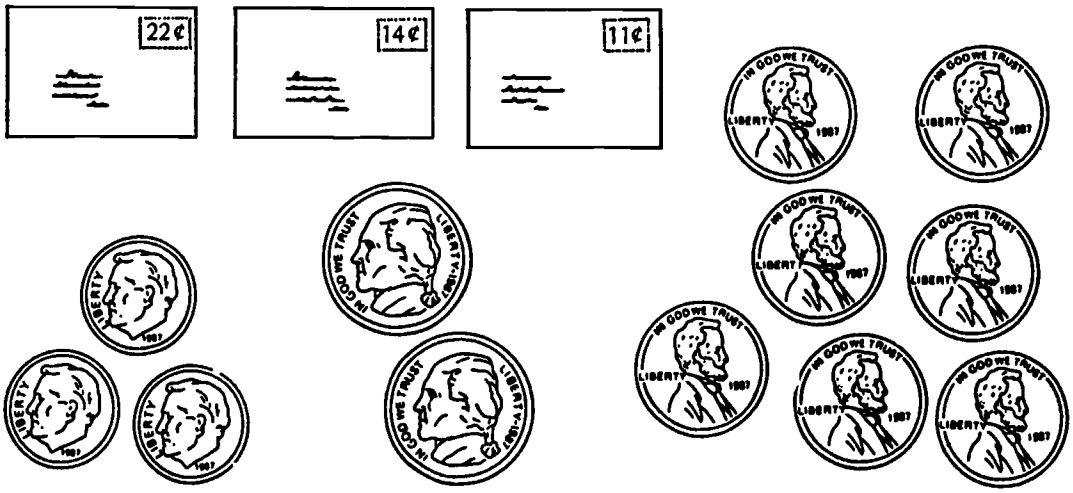


(c)



(d)

You want to buy each of the three stamps in order to mail the letters shown. You have the coins that are shown. Which coins would you need to buy each stamp? You will use all of the coins.



How many days are there from May 5 through May 20?

Move only one block to another stack and make the sum of the numbers in each stack be 12.

| | | |
|---|---|---|
| 5 | 6 | 3 |
| 3 | 2 | 4 |
| 2 | 6 | 5 |

Kay's pencil is 7 inches long.
Ray's pencil is 2 inches shorter than Kay's.
May's pencil is 3 inches longer than Ray's.
Whose pencil is the longest?

July 4th is a Tuesday. Your birthday is on July 23rd. On what day of the week is your birthday?

I put my 10 checkers into two stacks. One stack has 4 more checkers than the other has. How many checkers are in each stack?

Last week the Giants played the Dodgers. There were a total of 7 runs scored in the game. What could have been the final score?

I have nine bills in my wallet. Five of them are $1 bills, and the rest of them are $5 bills. How much money do I have in my wallet?

In a line, there is a rabbit in front of two rabbits. There is a rabbit behind two rabbits. There is a rabbit between two rabbits. What is the smallest number of rabbits in the line?

Here is a table showing the runs scored by two teams in three baseball games played against each other. If this scoring pattern continues, what will be the score of the 5th game that they play?

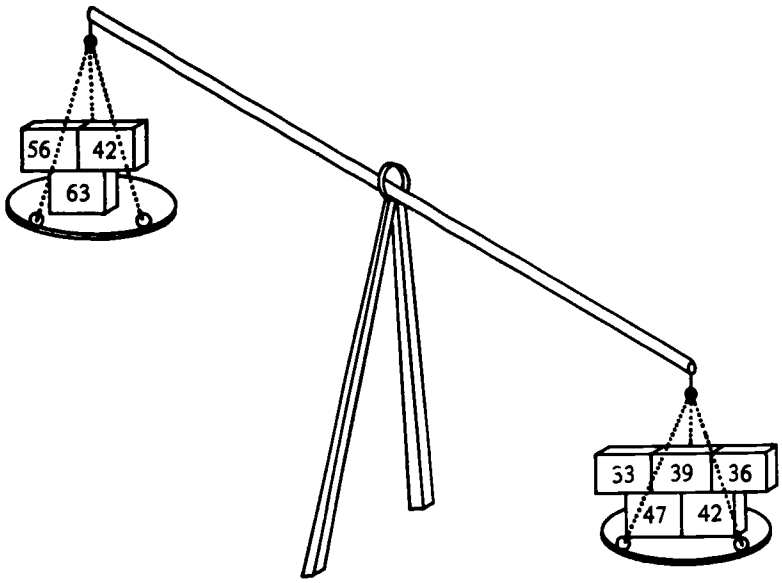
| Game | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Robins | 2 | 4 | 6 | | |
| Crows | 5 | 6 | 7 | | |

Jeanne has $8. Grace has $6. Trish has $7. Ann has $4. Two of the girls put their money together and had a total of $12. Who were the two girls?

How far is it from Corcoran to Millville?


Ricardo's guppies had baby fish. He gave 6 of them to Marlene. He gave 5 of them to Sonja. If there were 18 baby fish to start, how many does Ricardo keep?

Which box would you take off the balance scale to make it balance?




Peter, Paul, and Mary have 5 cookies. How many ways can they divide the cookies if each person must get at least one cookie?

The houses on Whitehall Street all have odd numbers. The first house is number 3, the second house is number 5, the third house is number 7, and so on. What is the number of the 10th house?

Find the number to fill the spots:

(a)
$$\begin{array}{r} 203 \\ +\,47\bigcirc \\ \hline 675 \end{array}$$

(b)
$$\begin{array}{r} 368 \\ -\,\bigcirc 4\bigcirc \\ \hline 226 \end{array}$$

(c)
$$\begin{array}{r} 25 \\ 2\bigcirc \\ +\,\bigcirc 7 \\ \hline 75 \end{array}$$

The number of my classroom is odd, and is between 20 and 30. It does not end in either a 7 or a 9. It is more than 23. What is my room number?

Every bike slot in the bicycle rack was filled. Donna's bike is in the middle. There are 6 bikes to the right of Donna's bike. How many bicycles are in the rack?

Irv has 6 baseball cards. Bob has 4 baseball cards. Steve has 3 baseball cards. Sandra has 7 baseball cards. And Marcella has 9 baseball cards. Three of them put their cards together and had a total of 18 cards. Who put their cards together?

Which of the numbers 4, 7, or 9 is the mystery number?

(a) It is more than 3.
(b) It is less than 8.
(c) It is more than 5.

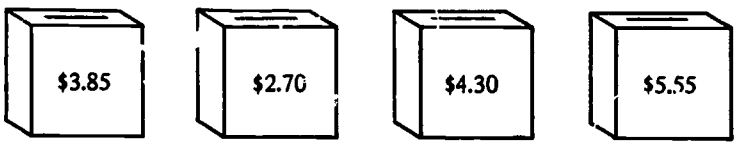
Mitch bought three different toys for his children. The gifts cost him $12.00. What did he buy?

| | |
|---|---|
| Football | $6.00 |
| Soccer ball | $4.00 |
| Book | $5.00 |
| Puppet | $2.00 |

Arthur is making lunch. He makes sandwiches with white bread or rye bread. He uses either cheese, jelly, or lunch meat. How many different sandwiches can he make?

The Little League scores are on two facing pages of the local newspaper. The sum of the page numbers is 13. What are the page numbers?

Which two banks have a total of $8.25?



Janet bought her goldfish on Thursday, July 10th. On what day of the week was the first day of the month?

Which of the following sums of money could you pay with exactly three coins? Tell how you would do it.

7¢ 16¢ 22¢ 56¢

Mrs. Chen has lost the middle digit from her house number:


She knows that it is greater than the last number, and smaller than the first number. It is an even number. What is the missing number?

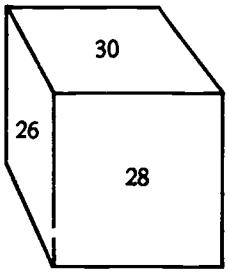
Jeff's plant is shorter than Nancy's. Danny's plant is taller than Nancy's. Jeff's plant is taller than Brad's. Whose plant is the tallest? Whose is the shortest?

Find all of the two-digit numbers for which the sum of the two digits is 10.

At which step do you go over 100?

| *Step 1* | *Step 2* | *Step 3* | *Step 4* | . . . |
|---|---|---|---|---|
| 1 | 2 | 4 | 8 | |
| + 1 | + 2 | + 4 | + 8 | |

The faces of the cube are numbered consecutively. What is the sum of the numbers not shown in the figure?




How many different ways can you make change for a 50¢ piece without using pennies?

Complete the pattern:

3 → 3
26 → 8
37 → 10
18 → 9
62 → 8
49 → ?
31 → ?

Fill in the squares with the numbers 2, 3, or 4 so that the numbers in each row across, down, and diagonally must add up to 9.

| | | |
|---|---|---|
| 3 | 4 | |
| | | |
| | | 3 |

How long is a row of 24 pennies placed end to end so that they touch?

During the softball season, Steve and Amy hit a total of 80 hits. Steve had 10 more than Amy. How many hits did each have?

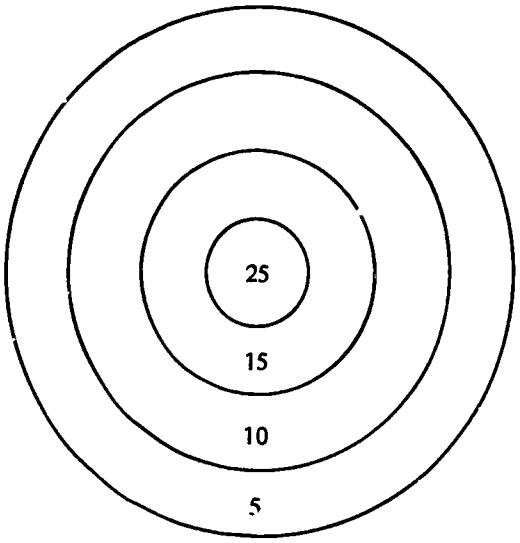
If you and 3 friends share this money equally, how much will you get?

Tim lives 8 blocks from school. How many blocks does he walk if he goes to school, goes home for lunch, and then goes right home after school?

There are 5 students in Mrs. Martin's class who wish to ride on a "bicycle built for two." How many rides must they take so that each person rides with each other person just one time?

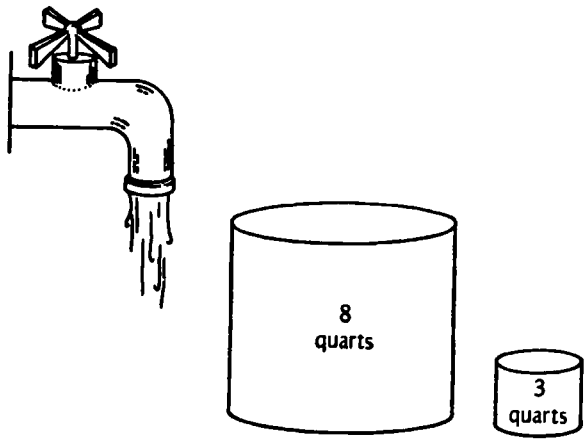
Arrison, Bradleyville, and Cork are 3 towns on the road between Maryville and Denniston. The road from Maryville to Denniston is a straight, 100-mile road. From Arrison to Denniston is 23 miles. From Maryville to Bradleyville is 55 miles. From Maryville to Cork is 30 miles. How far is Arrison from Bradleyville?

Jan tossed three darts at the dart board shown below. Make a list of all the ways Jan could score 40 points.




Marbles cost 2 for 25¢. Luis had one dollar. He bought 6 marbles. How much money does Luis have left?

How would you make 7 quarts?




Rex tossed five number cubes. All the cubes have three 4s and three 5s on them.

(a) What is the smallest sum that Rex could obtain by adding the faces that are "up"?
(b) What is the largest sum that Rex could obtain by adding the faces that are "up"?
(c) Rex added up his score and got a 22. How many 4s and how many 5s were there?

It is possible to make each of the amounts of money listed with exactly six coins. Copy the table and record your answers.

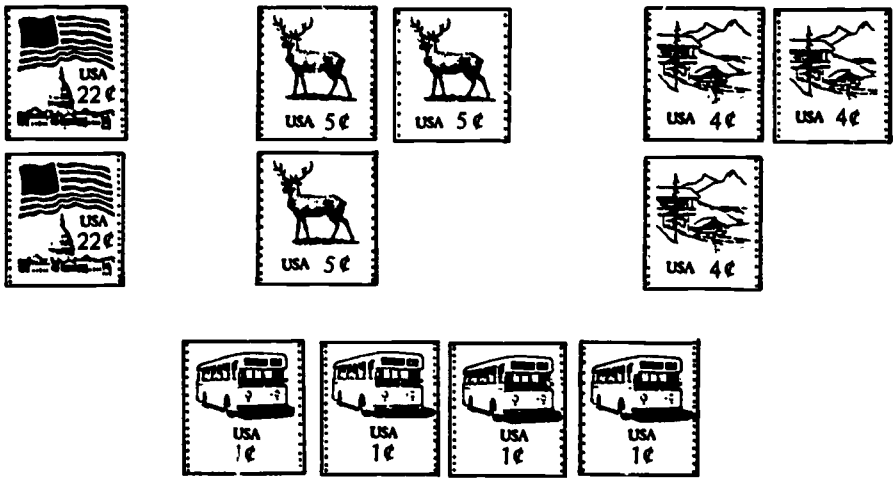
| Amount | 1¢ | 5¢ | 10¢ | 25¢ | 50¢ |
|---|---|---|---|---|---|
| .42 | | | | | |
| .85 | | | | | |
| $ 1.26 | | | | | |
| $ 1.70 | | | | | |

Lonny has 2 bats and 1 ball that cost him $11. Andy has 1 bat and 2 balls that cost him $7. How much should 1 bat and 1 ball cost?

Nina asked her dad how old he was. He told her, "If I add 10 to my age and double the result, I will get 84." How old is Nina's dad?

The Whip ride at the amusement park takes a new group of 15 people every 10 minutes. There are 70 people who want to ride. It is now 2:00 P.M. At what time will the 70th person complete the ride?

Ira wants to mail 2 letters and a postcard. One letter needs 39¢ worth of stamps, while the other needs only 22¢. The postcard needs 14¢. He has the stamps shown below. Show how he should put the stamps on the letters and the postcard so that they can be mailed.




A triangular shape is made by placing a row of blocks on a table and then a row containing one less block on top of that row. Continue this procedure until 1 block is on the very top. If a total of 15 blocks are used, how many rows are in the triangular shape?

There are four boats on the river. The yellow boat is in front of the red boat. The blue boat is behind the green boat. The yellow boat is behind the blue boat. In what order are the boats?

Dan has a bad cold and has to take 1 teaspoon of cough syrup every 2½ hours. He took his first dose at 9:00 A.M. He is supposed to take 6 doses before he goes to bed at 8:00 P.M. Can he do it?

Put 10 pennies in a row on your table. Now replace every other coin with a nickel. Next replace every third coin with a dime. What is the value of the 10 coins now on the table?

Peter, Stuart, and Oliver are tossing a football. Peter tosses the ball 3 feet further than Stuart. Oliver tosses the ball 2 feet less than Peter. Who tossed the football the shortest distance?

Take two consecutive numbers. Multiply each number by itself. Add the products. Do it several times with different numbers. What can you tell about the results?

Amy and Patti have a piece of rope that is 24 feet long. They want to cut it in order to make two jump ropes. Amy's rope is 6 feet longer than Patti's. How long is each rope?

Sam, Kim, and Helen played a number guessing game. Sam wrote three numbers on a piece of paper and gave Kim and Helen the following three clues:

(a) The sum of the numbers is 17.
(b) All the numbers are different.
(c) Each number is less than 8.

Which three numbers did Sam write down?

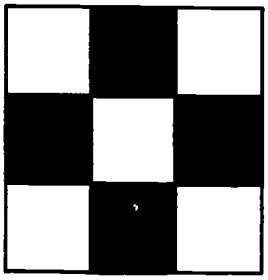
Ann, Beth, Carol, and David are throwing a ball. Each person throws the ball to the other three children. How many times is the ball thrown?

Karen has three different teachers for science, mathematics, and music. Mrs. Alexander enjoys her work as a music teacher. Mr. Brown used to teach science, but he doesn't any more. Mrs. Carlton was absent last Tuesday. Who teaches each subject?

The six students in Mr. Charnes' biology class were arranged numerically around a hexagonal table. What number student was opposite number 4?

The club members are saving to buy records. The records cost $5 each. The club treasurer puts money into an envelope until the envelope has $5 in it. Then she starts another envelope. The members of the club have saved $23 so far. How many envelopes do they have?

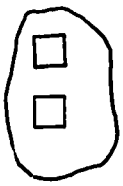
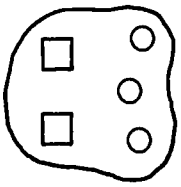
How many squares of all sizes are on the checkerboard shown below?

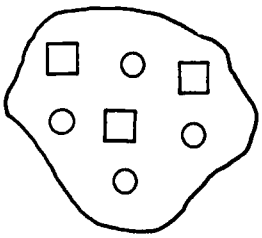


At the record store, Carol bought the same number of tapes as records. She bought the same number of Western records as all the other records she bought. How many records and how many tapes did she buy if she bought 5 Western records?

Waiting in line to buy movie tickets, Lois was behind Nan. Mary was in front of Nan and behind Ann. Lois was in between Nan and Brad. Who is in the middle of the line?

Bill needs 39¢ worth of stamps to mail a package. He has only 5¢, 6¢, 7¢, and 8¢ stamps. He wants to use only two different kinds of stamps on each package. He could use 4 stamps at 8¢ each and 1 stamp at 7¢ to make up the 39¢ on one package. Find other ways he might mail the packages.

If  = 18, and  = 54,

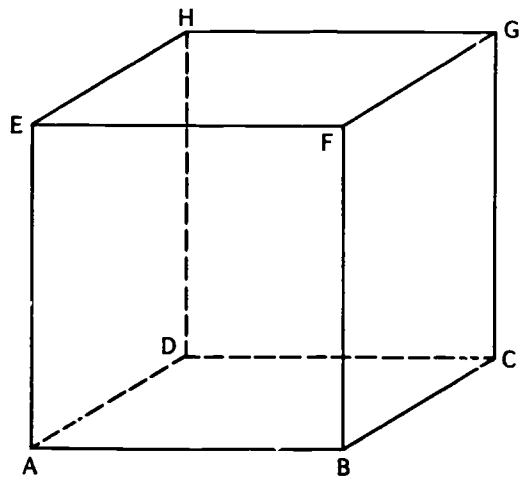
 = ?

What's my number?

(a) I am a two-digit number.
(b) I am a multiple of 6.
(c) The sum of my digits is 9.
(d) My tens' digit is one-half of my units' digit.

In Panacola's Restaurant, a circular table seats 4 people. A rectangular table seats 6 people. There are 18 people waiting to be seated. How can it be done?

A spider wishes to crawl from point H to point B. How many different "trips" can he crawl, if each trip is exactly three edges long?




A bus with 53 people on it makes two stops. At the first stop, 17 people get off and 19 people get on. At the second stop, 28 get off and 23 get on. How many people are now on the bus?

Jesse bowled 139, 196, and 154 in his first three games in the Bowling League. He got a 159 in his fourth game. Was this above or below his average for the first three games?

$$1 + 2 + 3 = 6$$
$$2 + 3 + 4 = 9$$
$$3 + 4 + 5 = 12$$
$$4 + 5 + 6 = 15$$

Find the three consecutive numbers that add up to 24.

Alim, Brenda, and Carol are all selling fruit at the school carnival. They sold oranges, apples, and pears.

(a) Alim and the orange seller are sisters.
(b) The apple seller is older than Brenda.
(c) Carol sold the pears.

Who sold which kind of fruit?

Ursula is in training. She did 5 sit-ups the first day. She did 6 sit-ups the second day, 7 the third day, and so on. How many sit-ups did she do on the 14th day?

A fancy bottle of perfume costs $25. The bottle can be purchased by collectors without the perfume. When purchased this way, the bottle alone costs $15 less than the perfume. How much does the bottle cost alone?

How many 2s must you multiply together to reach a 3-digit number?

I have two children. The product of their ages is 24. The sum of their ages is 11. Find the ages of my children.

Jim is in line at the bridge waiting to pay his toll. He counts four cars in front of him and six cars behind him. How many cars are there in line at the bridge?

One paper clip is 3 centimeters long and weighs 1 gram. Joan made a chain of these paper clips that was 300 centimeters long. How many grams does the chain weigh?

You are waiting for the elevator to take you to the observation tower on the 70th floor of the Hancock building. There are 45 people in line ahead of you. If each elevator can carry 10 people, on which trip will you be?

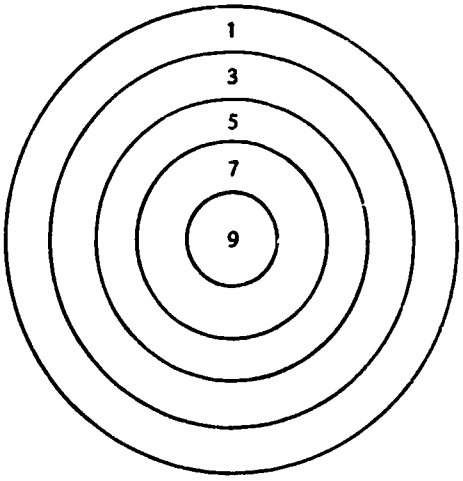
You had 7 dimes and 7 pennies. You bought a comic book for 49¢. You give the clerk 5 coins and she gives you one coin back. What coins do you now have?

Gail bought 5 pencils that cost 12¢ each and 3 erasers that cost 8¢ each. She gave the clerk a $1 bill. How much change did she get?

Nicole has a package of 48 silver stars. She wants to arrange them in rows, so that each row has the same number of stars. How can she arrange them so that the number of stars in each row is an odd number?

Luisa was playing darts. She threw 3 darts and all 3 hit the target. Which of the following could be her score?

4, 17, 56, 28, 29, 31



The town of Graphville has intersections formed by 27 avenues that run north–south and 31 streets that run east–west. If we plan one traffic light at each intersection, how many traffic lights do we need?

Here is a menu for lunch at school:

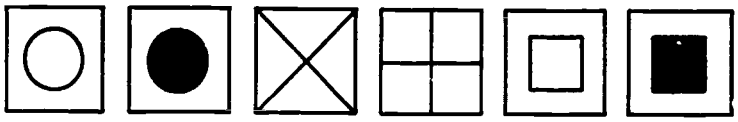
| | |
|---|---|
| Hamburger | 38¢ |
| French fries | 15¢ |
| Malted milk | 35¢ |
| Milk | 25¢ |

James spent 78¢. What did he buy?

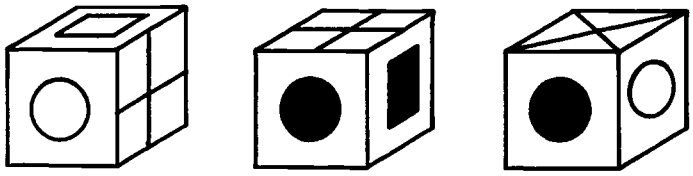
Last Saturday, George and his friend, Mike, went to a big-league baseball game. After the game, they went to the locker room to collect autographs of their favorite players Together they collected 18 autographs, but Mike collected 4 more than George. How many did George collect?

I have five coins: quarters, nickels, and dimes. The total value of the coins is 50¢. How many of each coin do I have?

Here are the designs drawn on the six faces of a cube:



Here are three views of the same cube. Which designs are on opposite faces of the cube?



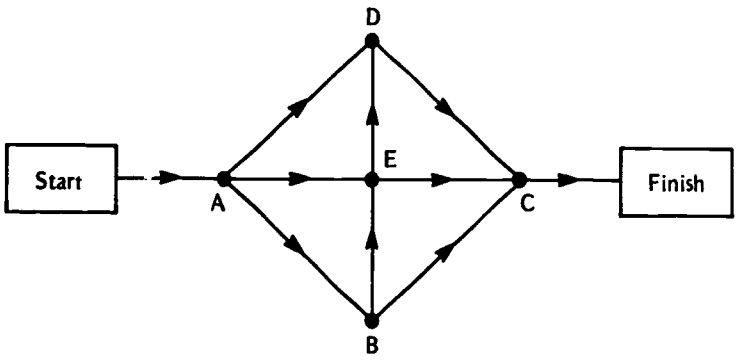
There were 8 girls and 16 boys at a meeting of the June Fair Planning Committee of the third grade. Every few minutes, one boy and one girl leave the meeting to go back to class. How many of these boy and girl "pairs" must leave the meeting so that there will be exactly five times as many boys as girls left at the meeting?

Put a single digit into each box and make the problem correct:

$$\begin{array}{r} \square\,\square\,\square \\ \times\ \square \\ \hline 1\,0\,9\,0 \end{array}$$

Stanley makes extra money by buying and selling comic books. He buys them for 7¢ each and sells them for 10¢ each. Stanley needs 54¢ to buy some batteries for his calculator. How many comic books must Stanley buy and sell to earn the 54¢?

How many paths are there from Start to Finish?




In January, our team won 2 games and lost the same number. In February, the team lost 3 more games than it did in January, but won the same number it lost. In March, it won the same number of games as it did in February but lost 2 fewer games than it did in February. What was its record at the end of March?

What is the smallest number of pennies that can be arranged into 6 equal piles and also into 8 equal piles?

Lucy has a dog, a parrot, a goldfish, and a Siamese cat. Their names are Lou, Dotty, Roger, and Sam. The parrot talks to Rover and Dotty. Sam cannot walk nor fly. Rover runs away from the dog. What is the name of each of Lucy's pets?

A farmer has 15 animals, some pigs and some chickens. Together, they have a total of 40 legs. How many pigs and how many chickens does the farmer have?

How many lengths of rope, each 3 feet long, can be cut from a roll of rope that contains 50 feet of rope?

A taxi charges 90¢ for the first one-quarter mile, and 25¢ for each additional quarter mile. How much did Leon pay for a ride of 1 mile?

A taxi charges 90¢ for the first one-quarter mile and 25¢ for each additional quarter mile. Sandy paid $2.90 for her ride. How far did she travel?

Given the sequence of numbers,

2, 3, 5, 8, . . .

Explain why the next number might be 12, or 13, or 2, or 5.

How many breaths do you take in one 24-hour day?

A city block is about 270 feet long. If cars are parked bumper-to-bumper, and a small car is 15 feet, while a large car is 18 feet,

(a) What is the smallest number of cars that can be parked on one block?
(b) What is the largest number of cars that can be parked on one block?
(c) If we park an equal number of large and small cars in one block, how many would fit?

What is the greatest number of coins you can use to make 35¢? What is the smallest number of coins you can use? In how many different ways can you make 35¢?

Melinda bought some peanuts for 35¢ and an apple for 20¢. She paid for her purchase with 3 coins of the same amount. How much change did she receive?

Mrs. Lewis bought 6 cards. Mr. Lewis bought 6 cards that same day. How much would they have saved if they had bought 12 cards and shared them equally?

| Number of Cards | 1–3 | 4–6 | 7–9 | 10–12 | 13 or more |
|---|---|---|---|---|---|
| Cost for Each Card | $1.00 | 90¢ | 85¢ | 80¢ | 75¢ |

*Table of Moon Facts*

The moon is smaller than the earth.
People weigh 6 times as much on Earth as on the moon.
The moon goes around the earth once in 28 days.
The moon is about 240,000 miles from the earth.

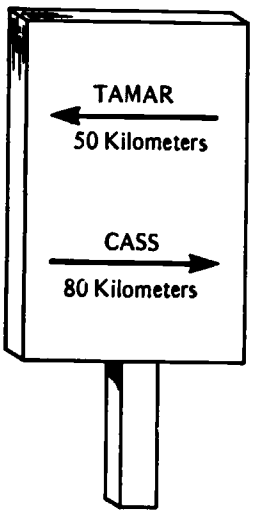
(a) Peter figures that he would weigh 14 pounds on the moon. What does Peter weigh on Earth?
(b) Peter's mother weighs 120 pounds on Earth. How much would she weigh on the moon?
(c) About how long does it take the moon to go around the earth four times?

What was the final score of the Tigers–Sharks baseball game?

(a) If their scores are added, the sum is 8.
(b) If their scores are multiplied, the product is 15.
(c) The Sharks won the game.

Sandra owes Charlene $1.35. Sandra and Charlene agree to split equally the cost of a $2.00 comic book. Sandra pays the $2.00 for the book. How much does Sandra now owe Charlene?

How far are you from Tamar when you are on the road and midway between Tamar and Cass?



"I want you to go shopping for me," said Jimmy's mother. "First go 5 blocks west to the grocery store. Then go 3 blocks east to the fruit store. Then go 5 blocks east to the candy store." Which store is closest to Jimmy's house?

Mitch and his sister Pauline went to visit a friend who lives 12 blocks away. They walked 6 blocks when they realized that they had dropped a book. They walked back and found the book. Then they walked the 8 blocks to their friend's house. How far from their home did they drop the book?

July has 5 Tuesdays. Three of them fall on even-numbered dates. What is the date of the third Tuesday in July?

A circus tent has 8 poles from one end to the other in a straight line. The poles are 20 meters apart. How long is the tent? What if there were 11 poles?

A rabbit ate 32 carrots in 4 days. If he ate 2 more carrots each day than he did the day before, how many carrots did he eat each day?

Mary bought a candy bar for 29¢. She gave the clerk a $1 bill and received 5 coins in change. What 5 coins did she receive?

What is the sum of the numbers in this table?

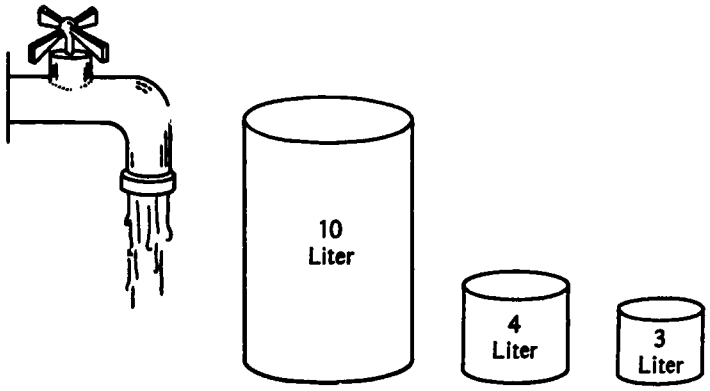
| | | | | |
|---|---|---|---|---|
| 3/4 | 3/8 | 3/7 | 3/5 | 3/13 |
| 2/5 | 4/7 | 10/13 | 5/8 | 1/4 |

Nan has a 5-room apartment. The bedroom is next to the kitchen. The living room is between the kitchen and the dining room. The recreation room is farthest from the bedroom. Which room is in the middle?

If a pound of plums contains 4 to 6 plums, what is the least possible weight in pounds of 3 dozen plums?

3 yuchs = 2 ughs
4 ughs = 6 wims
2 yuchs = ? wims?

How would you make 5 liters?




I am taking these people to dinner:

(a) me
(b) my wife
(c) my 2 sons and their wives
(d) each son's 2 children.

How many reservations should I make?

The cost of a concert ticket and a football ticket is $14. The cost of a movie ticket and a football ticket is $11. The cost of a concert ticket and a movie ticket is $7. Find the cost of each ticket.

My license tag is a 3-digit number. The product of the digits is 216; their sum is 19; and the numbers appear in ascending order. Find my license plate number.

Laura jogs 7 blocks the first day of her training program. She increases her distance by 2 blocks each day. On the last day, she jogs 25 blocks. How many days was Laura in training?

Norene set her wristwatch when she left for school at exactly 7:30 A.M. on Monday. At 1:30 P.M. on Monday, she noticed that her watch had lost 4 minutes. At this same rate, how many minutes will the watch lose by the time Norene resets it when she leaves for school at 7:30 A.M. on Tuesday?

The 3-digit number 53A is exactly divisible by 6. Find the value of A.

The listed price for *Sports Magazine* is $1.25 a copy. You pay $16.56 for a 24-issue subscription. How much do you save by buying the subscription?

Pat and Mike are having a contest. They will shovel snow to clear a 21-foot path. Pat shovels 3 feet with each push of the shovel. Mike shovels 1 foot on the first push, 2 feet on the second push, 3 feet on the third push, and so on. He will shovel 1 foot more on each push than on the push before. Who wins the contest?

How much will it cost to cut a log into 8 equal pieces if cutting it into 4 equal pieces costs 60¢? There is no stacking of the pieces.

Five bookworms have eaten into the big dictionary on the teacher's desk. Twiggy is 20 mm ahead of Rusty. Cruncher is 10 mm behind Twiggy. Rusty is 5 mm behind Nosey. Freddy is 15 mm ahead of Cruncher. Nosey is 20 mm behind Freddy. List the five bookworms in order.

Four people enter a clubroom. Each person shakes hands with each of the other people. How many handshakes are there?

Sol gave away half of his marbles, dividing them equally among Mary, Doug, and Linda. Linda took her share of the marbles and shared them equally among herself and 4 friends. Each friend got 4 marbles. How many marbles did Sol start with?

I have an apple, an orange, and a peach. I weighed them two at a time. The apple and the orange weigh 14 ounces; the apple and the peach weigh 18 ounces; the orange and the peach weigh 20 ounces. How much does the apple weigh?

You bought 10 comic books at 5 for $9. You then sold them all at $2 each. How much profit did you make?

In a recent sale at the local stationery store, the following sign appeared:

| | |
|---|---|
| ERASERS | 5¢ |
| PENCILS | 7¢ |

LIMIT: 3 OF EACH TO A CUSTOMER

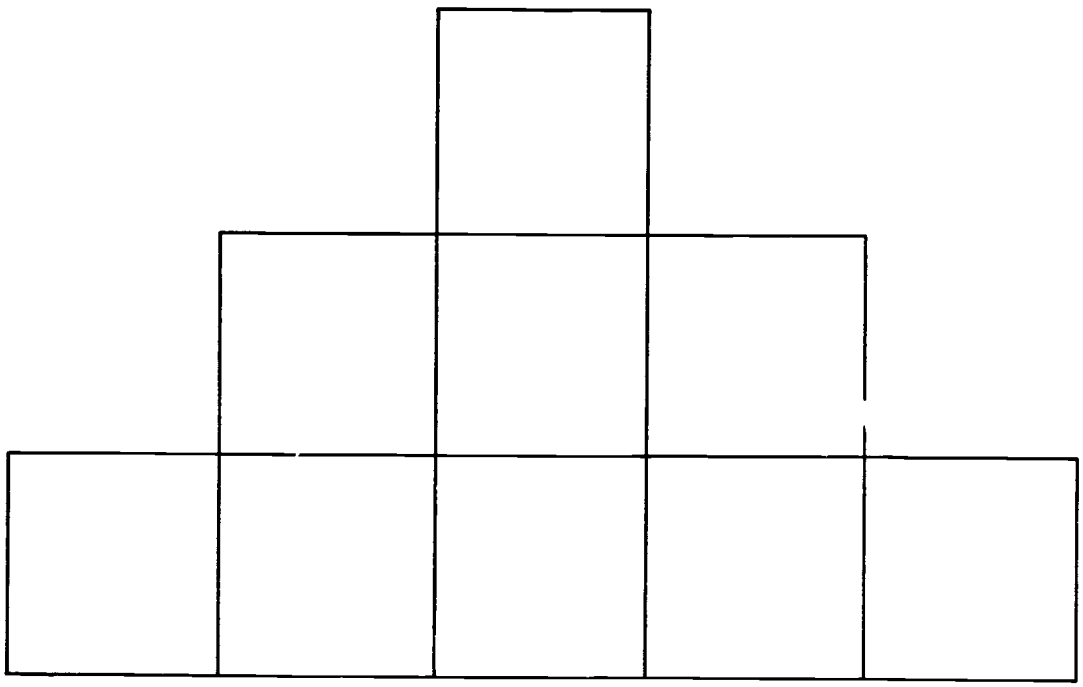
If you had 20¢ to spend, what different combinations of pencils and erasers could you buy?

With 3 minutes left to play in the game between the Cougars and the Hawks, the Cougars were ahead by 10 points. In those last 3 minutes, the Cougars scored 6 points per minute, while the Hawks scored 9 points per minute. Who won the game, and what was the final score?

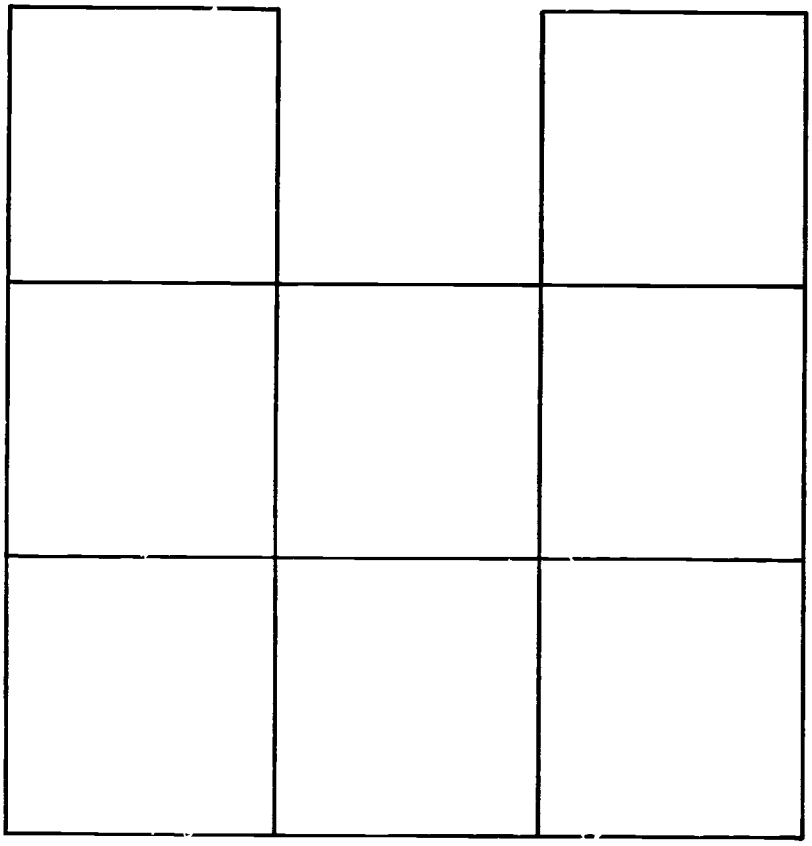
Jeremy worked a math problem and got 16 as his answer. However, in the last step, he multiplied by 2 instead of dividing by 2. What should have been the correct answer?

## SECTION E

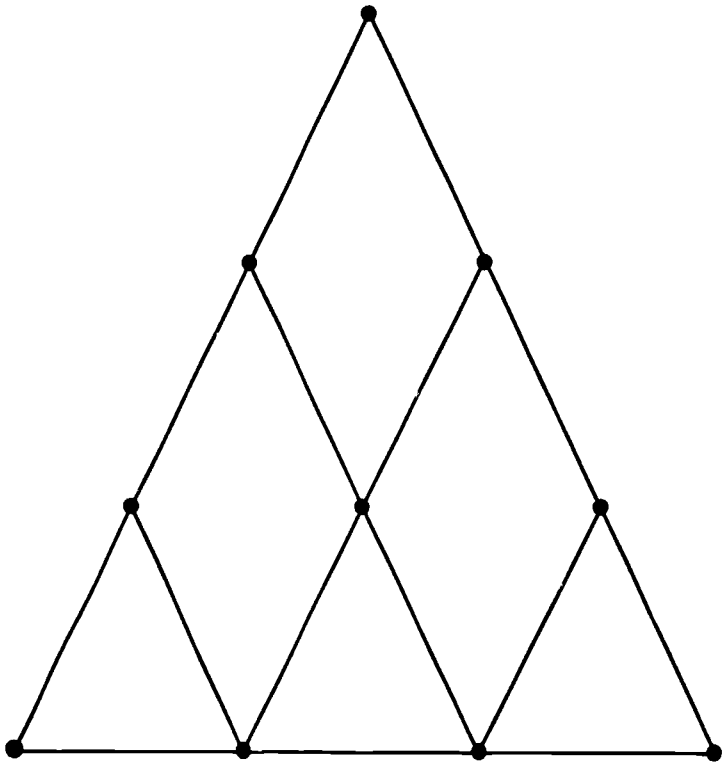
# Masters for Strategy Game Boards



Mountain Tic-Tac-Toe



Valley Tic-Tac-Toe



Dots-in-a-Row Tic-Tac-Toe

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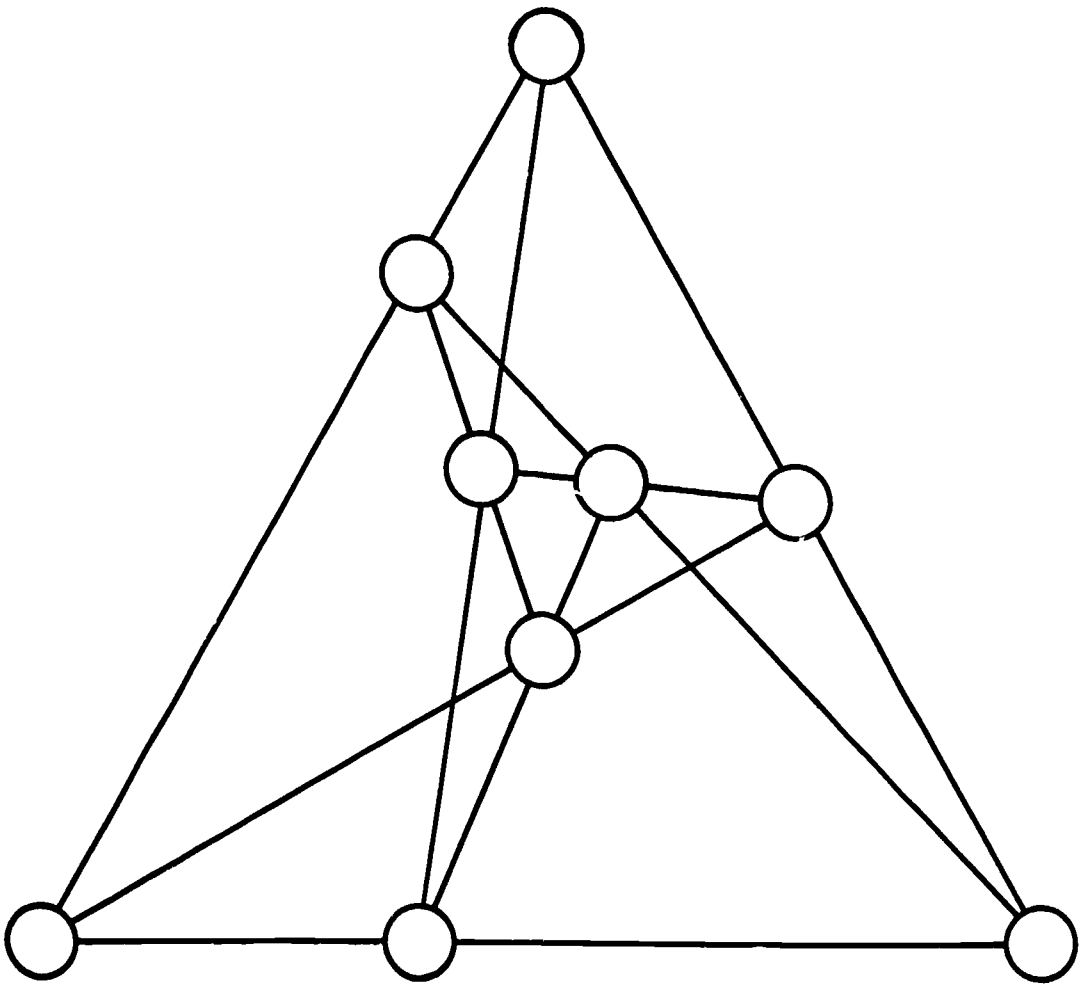
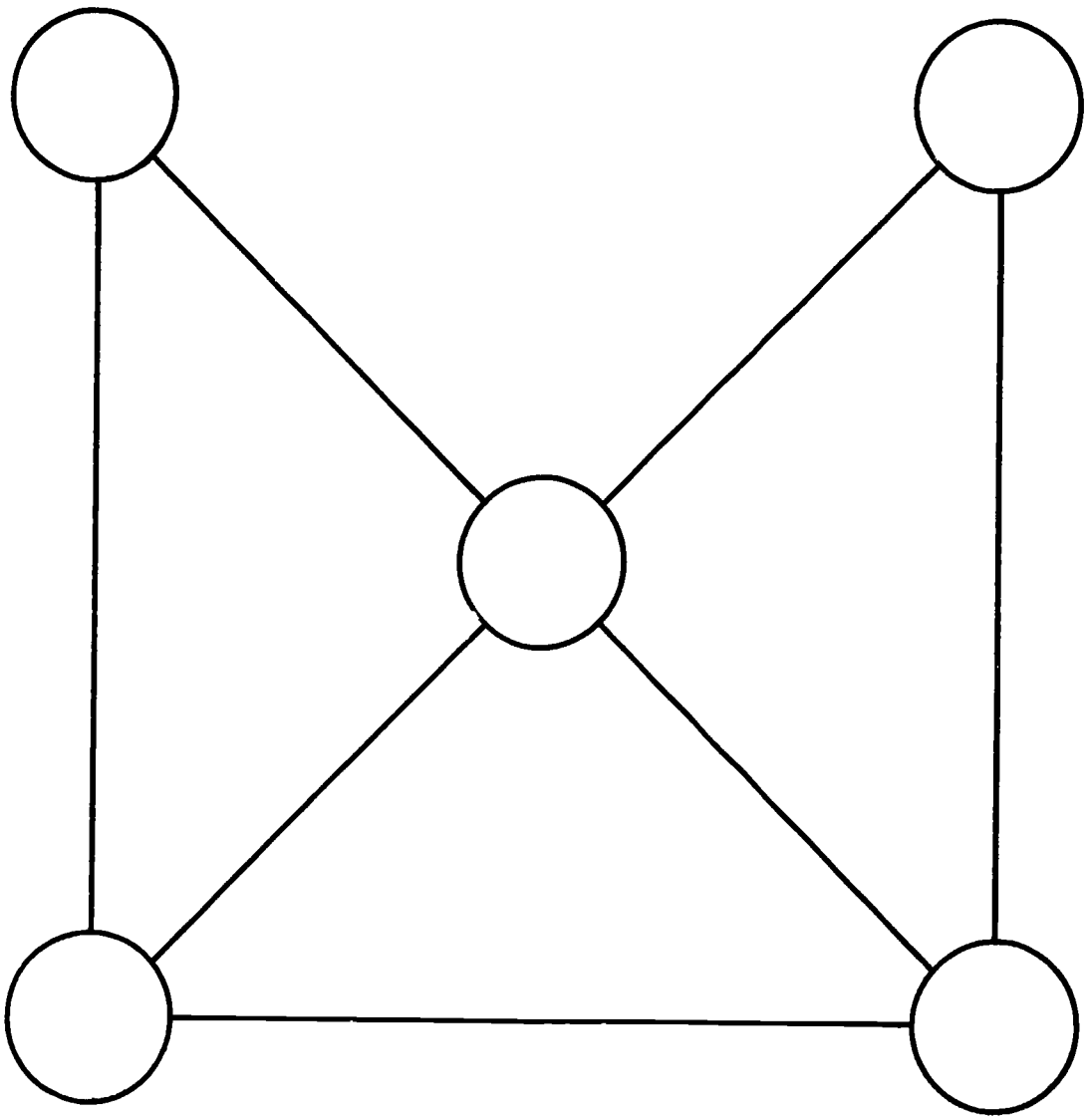
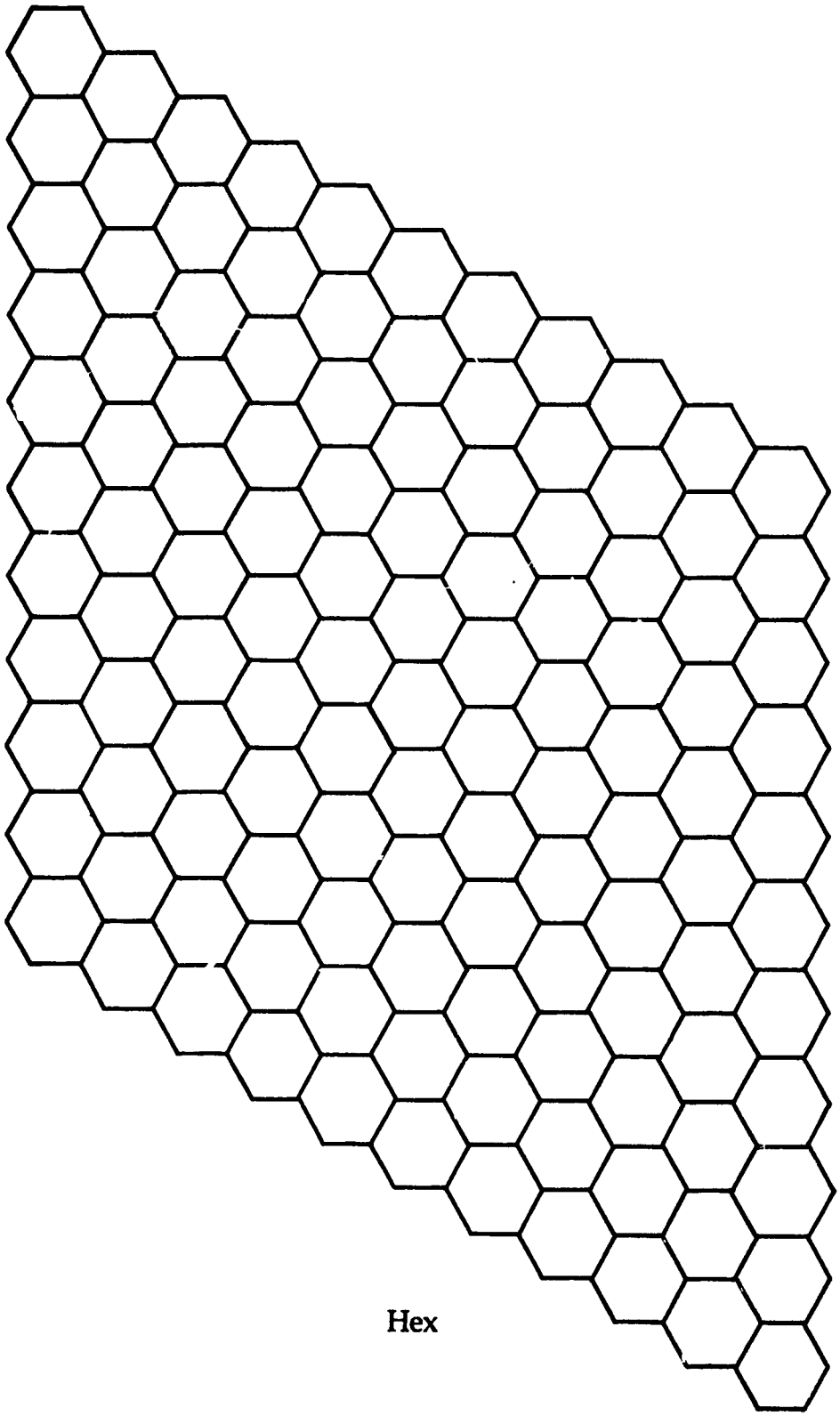
Tac-Tic-Toe



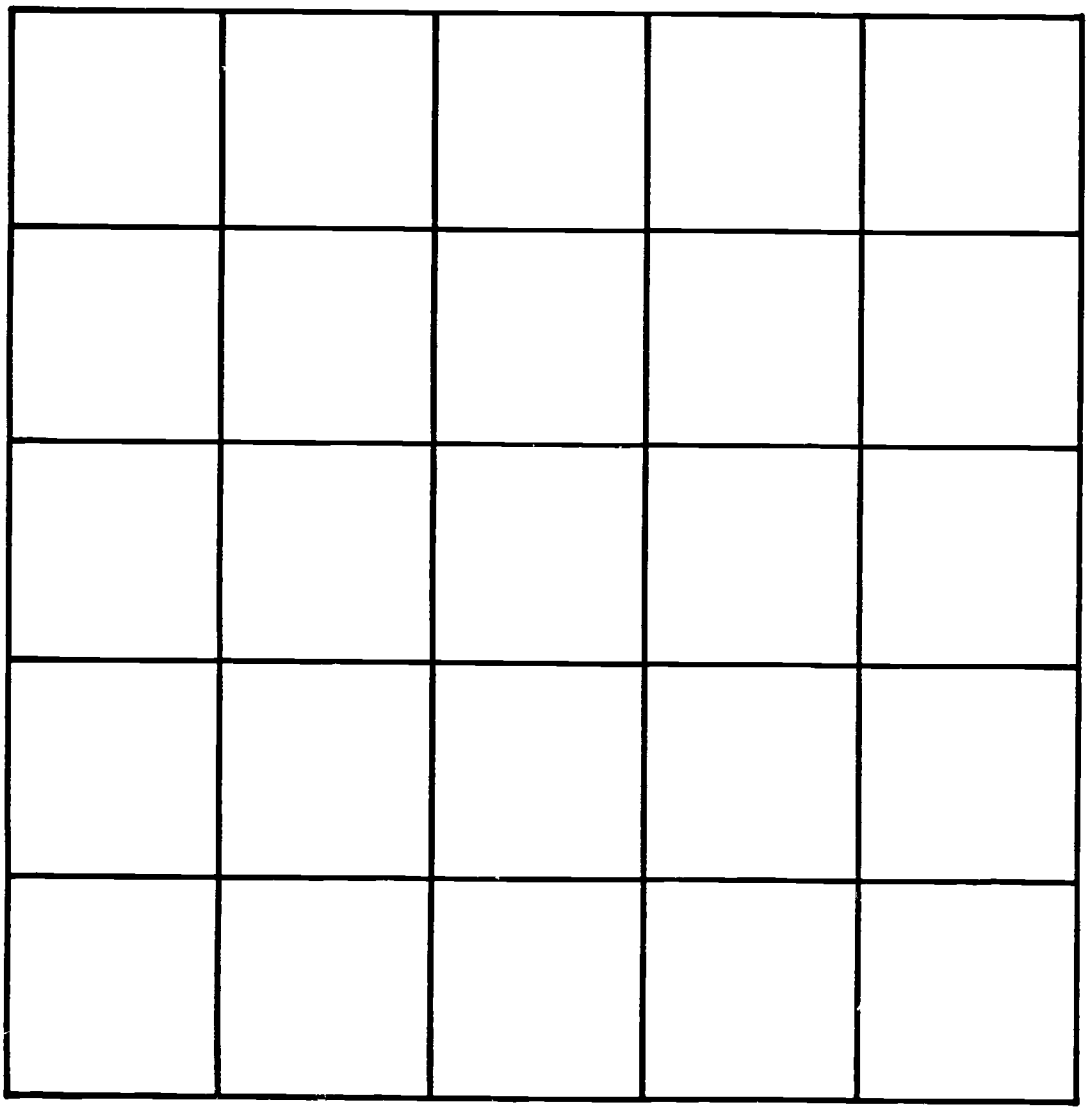
Triangular Tic-Tac-Toe



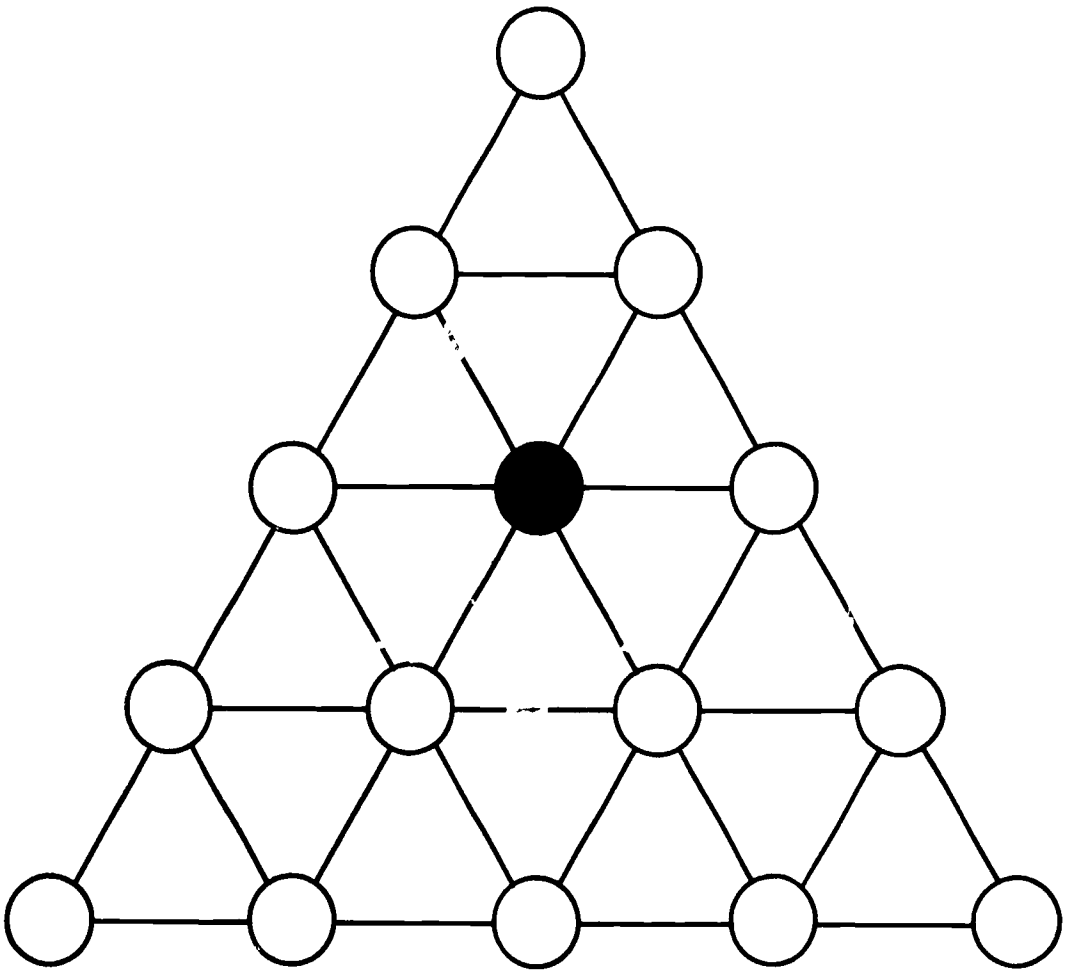
Blockade



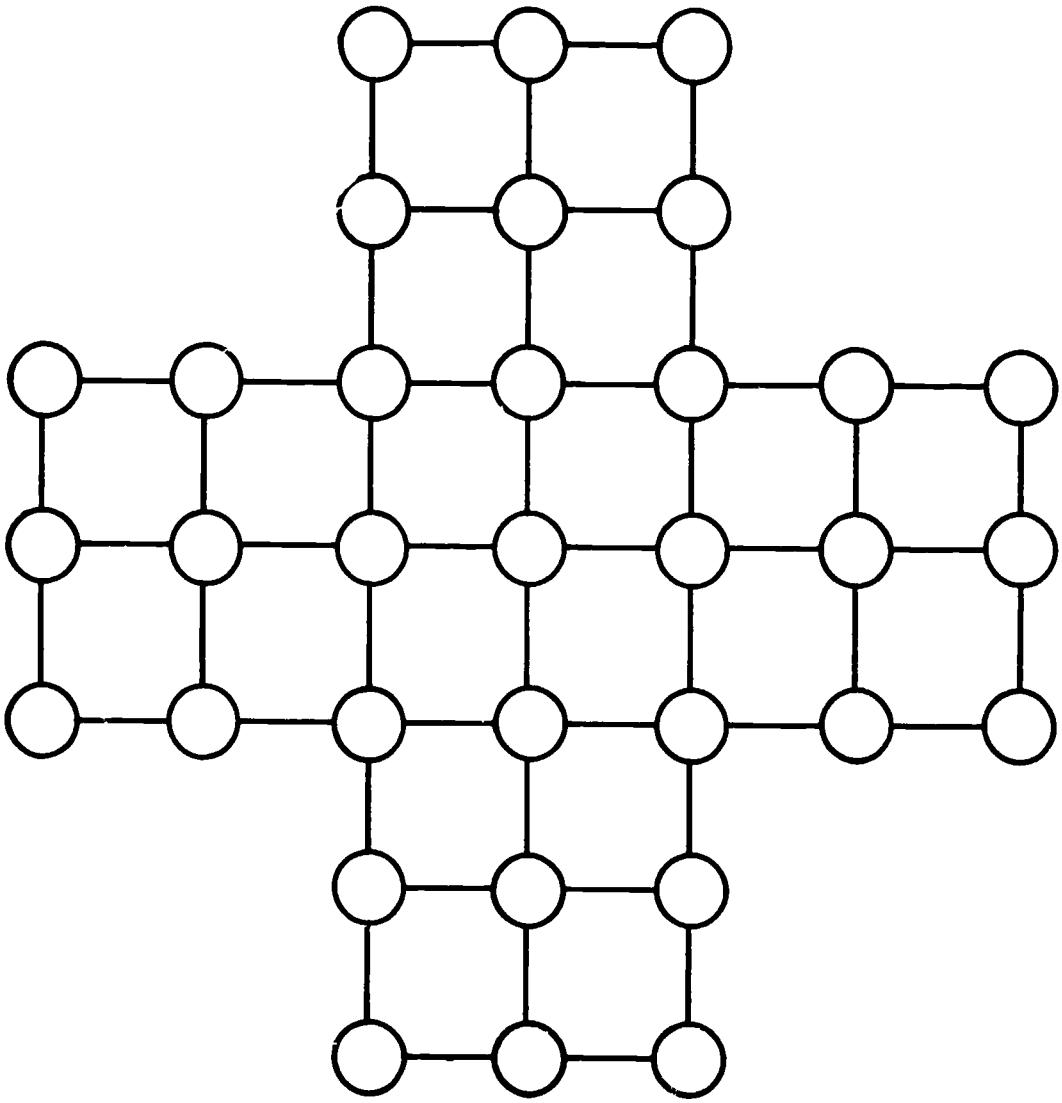
Hex



Tromino Saturation



Solitaire



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