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AUTHOR Dossey, John A.; And Others  
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ABSTRACT

This executive summary presents key findings from the 1986 National Assessment of Educational Progress (NAEP) in mathematics. It is designed to alert leaders in classrooms, families, and councils of government to the state of mathematics education in the United States. "Why Mathematics Counts" is summarized in the first section. Then, highlights from the assessment are given: the trend in mathematics performance is encouraging, particularly for students at ages 9 and 17 and for Black and Hispanic students. However, the gains have been confined primarily to lower-order skills. Other findings concerning achievement, instructional patterns, technology, course-taking, and attitudes are succinctly presented. Next, the assessment procedures are summarized, followed by some reflections on the findings. Trends in mathematics proficiency is the concern of the next section, with a graph highlighting overall trends and a chart showing the percentage of students in each age group (9, 13, and 17) in the last three assessments (1978, 1982, and 1986). Implications for instruction are considered in terms of students' perception of mathematics, patterns of classroom instruction, and the place of mathematics in the curriculum. Finally, a summary stresses the need to teach not only skills, but also higher-order thinking strategies. (MNS)

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# Mathematics:

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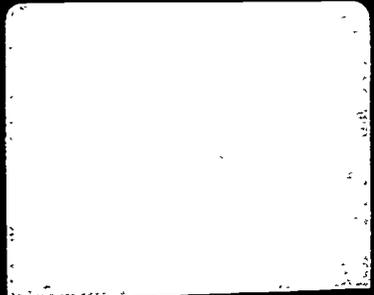
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THE NATION'S  
REPORT  
CARD



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*Mathematics Report Card*, drawing from four national surveys over 13 years, is the most thorough National Assessment profile of young Americans' mathematical skills and knowledge. This executive summary of the key findings is designed to alert leaders in classrooms, families, and councils of government to the state of mathematics education in the United States.

Time flies. Every year, nearly 1.5 million American 17-year-olds near the end of high school without much-needed mathematical reasoning skills. Fully a third of our 13-year-olds haven't mastered skills universally taught in elementary school. Few youngsters can put mathematics to work effectively in solving everyday problems. Such practical activity is absent from most classrooms. As a society on the threshold of the 21st century, are we measuring up?

In a word, no. But America's vibrant education system has risen to challenges before, providing basic education to more of its population than any country in the world. Now, at this critical point in time, I urge you to consider NAEP's mathematics findings in light of the human potential we stand to lose or use as a country.

Gregory Anrig  
President  
Educational Testing Service



# Mathematics:

## Are We Measuring Up?

### THE MATHEMATICS REPORT CARD EXECUTIVE SUMMARY

## Why Mathematics Counts

The skills and expertise of a country's workforce are the foundation of its economic success. Lately, in our country, this foundation appears too fragile to withstand the challenges of the 21st century.

- The most recent international mathematics study reported that average Japanese students exhibited higher levels of achievement than the top 5 percent of American students enrolled in college preparatory mathematics courses.<sup>1</sup> As a case in point, a Japanese semiconductor company recently opened a plant in the southeastern United States had to use college students at the graduate level to perform statistical quality control functions; the same jobs were performed by high-school graduates in Japan.<sup>2</sup>
- One out of three major corporations already provides new workers with basic reading, writing, and arithmetic courses. If current demographic and economic trends continue, American businesses will hire a million new people a year who can't read, write, or count. Teaching them how, and absorbing the lost productivity while they are learning, will cost industry \$25 billion a year for as long as it takes — and nobody seems to know how long that will be.<sup>3</sup>
- American colleges have reported a 10-to-30 percent rise in demand over the past several years for remedial coursework in mathematics for incoming freshmen. As diagnosed in one

<sup>1</sup>Curtis McKnight, et. al., *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective*. International Association for the Evaluation of Education Achievement, Stipes Publishing Company, Champaign, IL, 1987.

<sup>2</sup>George Gilder, "Chip Sense and Nonsense," *Wall Street Journal*, April 2, 1987.

<sup>3</sup>David Kearns, Chairman and Chief Executive Officer, Xerox Corporation, *Learning to Be Literate in America*, Foreword, 1987.

study, these young people are not defined as at-risk, yet they are not workforce-ready. For the at-risk populations, the mismatch between workplace needs and workforce skills is even greater.<sup>4</sup>

- Looking toward the year 2000, the fastest-growing occupations require employees to have much higher math, language, and reasoning capabilities than do current occupations.<sup>5</sup>

Too many students leave high school without the mathematical understanding that will allow them to participate fully as workers and citizens in contemporary society. As these young people enter universities and businesses, American college faculty and employers must anticipate additional burdens. As long as the supply of adequately prepared precollegiate students remains substandard, it will be difficult for these institutions to assume the dual responsibility of remedial and specialized training; and without highly trained personnel, the United States risks forfeiting its competitive edge in world and domestic markets.

Even for those working in less scientifically specialized areas, technological innovations require the ability to learn and adapt to new conditions. Studies of technological change have reached differing conclusions as to the nature and extent of their impact on job skill requirements, but it is certain that the current generation of students will need to work with increasingly large and complex bodies of information in performing even basic tasks. From the basic computational skills required to organize and track large-scale shipments of merchandise to the higher-level expertise necessary to make technological discoveries, it is clear that mathematical abilities will be critical to our nation's continued economic success.

## Highlights from NAEP's Mathematics Assessments

NAEP's 1986 mathematics assessment provides a timely account of student achievement in this vital subject, and the results highlight the need for even greater commitment to school mathematics programs. Trends across four assessments since 1973 offer a comprehensive view of achievement patterns for students at ages 9, 13, and 17.

- Recent national trends in mathematics performance are somewhat encouraging, particularly for students at ages 9 and 17. Subpopulations of students who performed comparatively

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<sup>4</sup> *The Fourth R: Workforce Readiness*. National Alliance of Business, November 1987, p. 5.

<sup>5</sup> *Workforce 2000: Work and Workers for the 21st Century*. Hudson Institute, Indianapolis, IN, June 1987.

poorly in past assessments have shown significant improvement in average proficiency since 1978: at all three ages, Black and Hispanic students made appreciable gains, as did students living in the Southeast.

- While average performance has improved since 1978, the gains have been confined primarily to lower-order skills. The highest level of performance attained by any substantial proportion of students in 1986 reflects only moderately complex skills and understandings. Most students, even at age 17, do not possess the breadth and depth of mathematics proficiency needed for advanced study in secondary school mathematics.

While we may be recovering from the doldrums of poor performance that characterized the 1970s, it is crucial that we do even better to reach expected or hoped-for levels of achievement. Improvements are needed, not only in average proficiency, but also in the number of students who reach the upper levels of performance.

## Other Findings

- Discrepancies between the level of mathematics commonly taught in elementary, middle, and high schools and what students know and can do in the subject appear to increase over the school years, especially for Black and Hispanic students. Only about half of all 17-year-olds in the 1986 assessment reached a level of proficiency associated with material taught in junior high school mathematics.
- Mathematics instruction in 1986, as in previous years, continues to be dominated by teacher explanations, chalkboard presentations, and reliance on textbooks and workbooks. More innovative forms of instruction — such as those involving small group activities, laboratory work, and special projects — remain disappointingly rare.
- Students reported more homework and testing in mathematics in 1986 than in previous assessments, perhaps indicating a growth in academic expectations in schools.
- Students appear to gain basic mathematical knowledge and skills in numbers and operations between grades 3 and 7, while higher-level applications in numbers and operations develop steadily across the three grade levels. Females

outperformed males in the area of basic knowledge and skills, while males had the advantage in higher-level applications.

- Although the role of technology in the mathematics classroom appears to be changing, the benefits of using computers and calculators seem to be available primarily to small proportions of students who are in the upper range of ability or in the upper grades.
- Although more high school students in 1986 than in previous years reported taking higher-level mathematics courses, including Algebra II, Geometry, and Calculus, the overall percentage of students taking these advanced courses remains disappointingly low.
- High school students whose parents encourage mathematics course-taking and have higher levels of education tend to exhibit higher mathematics proficiency than those who lack this home support.
- Students who enjoy mathematics and perceive its relevance to everyday life tend to have higher proficiency scores than students with more negative perspectives. At the same time, students' enjoyment of and confidence in mathematics appear to wane as they progress through their schooling. Most perceive that the subject is composed mainly of rule memorization and expect to have little use for mathematical skills in their future work lives.

## Summary of Assessment Procedures

*The Mathematics Report Card* chronicles trends in proficiency across four mathematics assessments conducted in 1972-73, 1977-78, 1981-82, and 1985-86. (For convenience, each assessment will be referred to by the last half of the school year in which it occurred — 1973, 1978, 1982, and 1986.) Each of the four mathematics assessments involved nationally representative samples of 9-, 13-, and 17-year-olds, and together the assessments generated data from a total of 150,000 students for the examination of trends. In the 1986 assessment, NAEP sampled students by grade as well as age, making available a second data set based on 34,000 additional students in grades 3, 7, and 11.

The mathematics assessments included both open-ended and multiple-choice questions covering a wide range of content and process areas. Student background information gathered during each administration permits consideration of trends in relation to school, home, and attitudinal factors.

The data were analyzed using Item Response Theory (IRT) scaling technology and are summarized on a common scale (0 to 500) to facilitate direct comparisons across assessment years for age groups and subpopulations. To provide a basis for interpreting the results, the report describes what students attaining different proficiency levels on the scale are able to do. Based on the assessment results, five levels of proficiency have been defined:

**Level 150** — Simple Arithmetic Facts

**Level 200** — Beginning Skills and Understanding

**Level 250** — Basic Operations and Beginning Problem Solving

**Level 300** — Moderately Complex Procedures and Reasoning

**Level 350** — Multi-step Problem Solving and Algebra

NAEP's mathematics scale was computed as a weighted composite of proficiency on five content area subscales — knowledge and skills, higher-level applications, measurement, geometry, and algebra. Thus, for the most recent assessment, results are also available indicating students' relative strengths and weaknesses across these content areas.

## Reflections

The assessment findings show both encouraging and discouraging trends for mathematics education in the United States. It is encouraging to see improvements in performance occurring across such a wide segment of the student population, especially among Black and Hispanic students and those in the Southeast. However, this good news must be tempered by continuing concern over the generally low levels of performance exhibited by most high school students and by the fact that the majority of improvement shown resulted from increased performance in low-level skills.

Evidence concerning the nature of mathematics education suggests that the curriculum continues to be dominated by paper-and-pencil drills on basic computation. Little evidence appears of any widespread use of calculators, computers, or mathematics projects. This picture reflects classrooms more concerned with students' rote use of procedures than with their understanding of concepts and development of higher-order thinking skills. The continuance of such a pattern offers little hope that the mathematics education of our children will achieve the goals being set by the recent educational excellence movement.

Findings from the 1986 assessment, however, indicate that recent reforms directed toward increasing requirements in high school mathematics education, and schooling in general, may be beginning to have some

effect in raising the overall performance of our students.

Achieving a higher-quality mathematics curriculum across schools in the United States will require new materials, effective instructional methods, and improved means of evaluating student performance. There are many well-qualified and dedicated teachers in our classrooms capable of promoting improved ways of learning.<sup>6</sup> In order to do so, our teachers will need the support of administrators, parents, and the public at large. No longer can society afford to view mathematics as a subject for a chosen few or as a domain solely comprised of arithmetic skills. Students must come to see it as a way of thinking, communicating, and resolving problems. Until American schools move toward these more ambitious goals in mathematics instruction, there is little hope that current levels of achievement will show any appreciable gain.



## Trends in Mathematics Proficiency

### Overall Trends in National Performance

Figure 1 provides an overall index of trends in national mathematics performance for 9-, 13-, and 17-year-olds. Based on assessments conducted in 1973, 1978, 1982, and 1986, the data provide an opportunity to examine both relatively long-term trends and recent changes. (The dotted line from 1973 to 1978 reflects a rough estimate of extrapolated results based on previously reported NAEP data.<sup>7</sup>)

**Nine-year-olds** — As a result of recent improvements, 9-year-olds showed significant gains in mathematics proficiency during the eight-year period from 1978 to 1986. In the 13-year span covered by NAEP's four mathematics assessments, their performance was quite stable across the 1970s, but improved between 1982 and 1986.

**Thirteen-year-olds** — Thirteen-year-olds also showed significant improvement during the eight-year period from 1978 to 1986, but the pattern differed from that shown by 9-year-olds. After a slight decline between 1973 and 1978, student performance improved between 1978 and 1982, and then leveled off in 1986.

<sup>6</sup>And Gladly Teach: A Ford Foundation Report on the Urban Mathematics Collaborative. New York: The Ford Foundation, 1987.

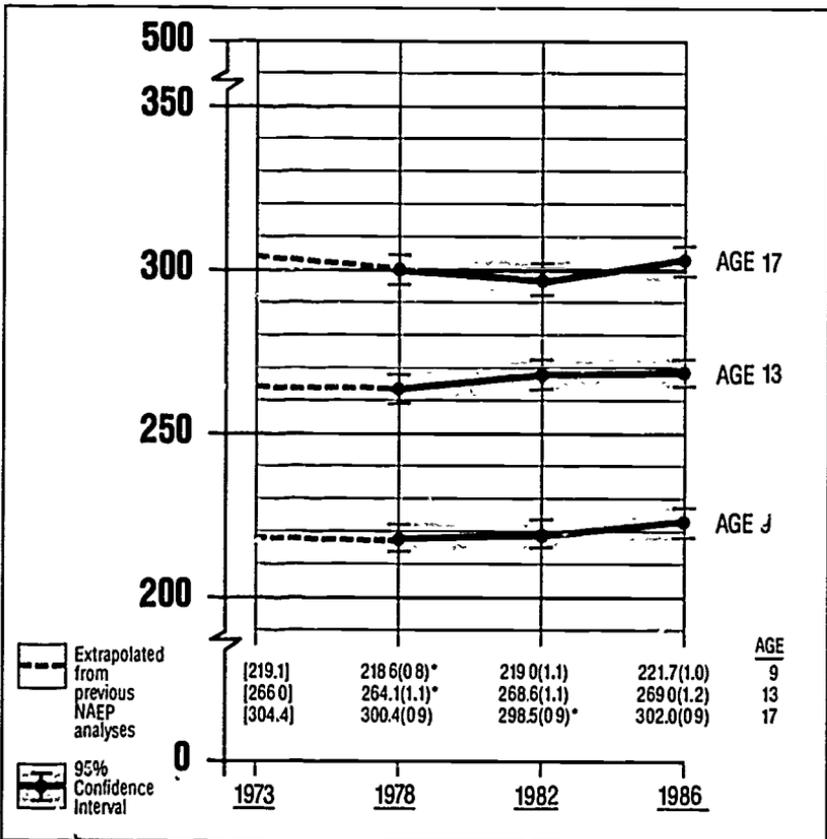
<sup>7</sup>The Third National Mathematics Assessment: Results, Trends and Issues. National Assessment of Educational Progress, Education Commission of the States, 1983.

**Seventeen-year-olds** — The mathematics performance of 17-year-olds declined from 1973 to 1978, and the negative trend continued, although abated, into the early 1980s. However, the most recent assessment showed a significant upturn between 1982 and 1986.

Although not particularly dramatic, national trends indicate recent improvements at all three age groups assessed. For the two younger age groups, the significant increases between 1978 and 1986 indicate that performance is gradually improving, although somewhat unevenly. The signs of recovery at age 17 appear to correspond with the findings of other large-scale studies. However, the question remains as to whether the recent upturn in performance represents the beginning of a positive trend back to and even beyond previous achievement levels or simply an abatement of earlier declines.

National Trends in Average Mathematics Proficiency for 9-, 13-, and 17-year-olds: 1973-1986

FIGURE 1



\* Statistically significant difference from 1986 at the .05 level.  
 \*\* Unrounded standard errors are presented in parentheses.



## Overall Trends for Subgroups

In several instances, trends for particular demographic subpopulations of students vary from the national trends. Generally, these variations have the effect of narrowing differences in performance between traditionally advantaged and at-risk groups. For example, while White students' performance remained relatively static between 1978 and 1986, Black students and to some extent Hispanic students have reduced the performance gaps relative to their White peers, although the differences still remain substantial. Similarly, the Southeastern region showed comparatively larger and more consistent gains than the other three regions of the country. Students in that region showed significant improvement between 1978 and 1986 at all three age levels.



## Levels of Mathematics Proficiency

While the trends in average mathematics achievement are generally encouraging, especially for subpopulations of students whose performance was relatively low in previous assessments, they do not provide information about what students know and can do in the subject. To describe the nature of mathematics performance, NAEP used the assessment results to define five levels of mathematics proficiency and provide trends in the percentages of students attaining each level. The five levels are described in Figure 2.

In analyzing the NAEP results, three factors appeared to characterize student performance: the kind of mathematical operations students were asked to perform, the type of numbers or number system involved, and the problem situation. As the operations grew more involved and moved out of the realm of whole numbers, performance levels decreased. Similarly, students had more difficulty with questions requiring the application of concepts and with those requiring problem-solving strategies, particularly in less frequently encountered situations.

**Level 150 — Simple Arithmetic Facts**

Learners at this level know some basic addition and subtraction facts, and most can add two-digit numbers without regrouping. They recognize simple situations in which addition and subtraction apply. They also are developing rudimentary classification skills.

**Level 200 — Beginning Skills and Understanding**

Learners at this level have considerable understanding of two-digit numbers. They can add two-digit numbers, but are still developing an ability to regroup in subtraction. They know some basic multiplication and division facts, recognize relations among coins, can read information from charts and graphs, and use simple measurement instruments. They are developing some reasoning skills.

**Level 250 — Basic Operations and Beginning Problem Solving**

Learners at this level have an initial understanding of the four basic operations. They are able to apply whole number addition and subtraction skills to one-step word problems and money situations. In multiplication, they can find the product of a two-digit and a one-digit number. They can also compare information from graphs and charts, and are developing an ability to analyze simple logical relations.

**Level 300 — Moderately Complex Procedures and Reasoning**

Learners at this level are developing an understanding of number systems. They can compute with decimals, simple fractions, and commonly-encountered percents. They can identify geometric figures, measure lengths and angles, and calculate areas of rectangles. These students are also able to interpret simple inequalities, evaluate formulas, and solve simple linear equations. They can find averages, make decisions on information drawn from graphs, and use logical reasoning to solve problems. They are developing the skills to operate with signed numbers, exponents, and square roots.

**Level 350 — Multi-step Problem Solving and Algebra**

Learners at this level can apply a range of reasoning skills to solve multi-step problems. They can solve routine problems involving fractions and percents, recognize properties of basic geometric figures, and work with exponents and square roots. They can solve a variety of two-step problems using variables, identify equivalent algebraic expressions, and solve linear equations and inequalities. They are developing an understanding of functions and coordinate systems.

Table 1 shows the percentage of students in each age group who attained each level of proficiency in the 1978, 1982, and 1986 assessments. (The highest mathematics levels attained across the three assessments by most 9-, 13-, and 17-year-olds are highlighted, as are the 1986 percentages of 17-year-olds achieving the two highest proficiency levels.)

Trends in Percentages of 9-, 13-, and 17-Year-Old Students at or Above the Five Proficiency Levels

TABLE 1

Proficiency Levels	Age	Assessment Year		
		1978	1982	1986
Level 150 Simple Arithmetic Facts	9	<b>96.5</b> (0.2)	<b>97.2</b> (0.3)	<b>97.8</b> (0.2)
	13	99.8 (0.0)	99.9 (0.0)	100.0 (0.0)
	17	100.0 (0.0)	100.0 (0.0)	100.0 (0.0)
Level 200 Beginning Skills and Understanding	9	70.3 (0.9)*	71.5 (1.1)	73.9 (1.1)
	13	<b>94.5</b> (0.4)*	<b>97.6</b> (0.4)	<b>98.5</b> (0.2)
	17	99.8 (0.0)	99.9 (0.1)	99.9 (0.1)
Level 250 Basic Operations and Beginning Problem Solving	9	19.4 (0.6)	18.7 (0.8)	20.8 (0.9)
	13	64.9 (1.2)*	71.6 (1.2)	73.1 (1.5)
	17	<b>92.1</b> (0.5)*	<b>92.9</b> (0.5)*	<b>96.0</b> (0.4)
Level 300 Moderately Complex Procedures and Reasoning	9	0.8 (0.1)	0.6 (0.1)	0.6 (0.2)
	13	17.9 (0.7)	17.8 (0.9)	15.9 (1.0)
	17	51.4 (1.1)	48.3 (1.2)	<b>51.1</b> (1.2)
Level 350 Multi-step Problem Solving and Algebra	9	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
	13	0.9 (0.2)	0.5 (0.1)	0.4 (0.1)
	17	7.4 (0.4)	5.4 (0.4)	<b>6.4</b> (0.4)

\* Statistically significant difference from 1986 at the .05 level. (No significance test is reported when the proportion of students is either >95.0 or <5.0.) Jackknifed standard errors are presented in parentheses.

**Level 150 — Simple Arithmetic Facts.** Across all three ages and across subgroups, students know elementary addition and subtraction; however, their ability to apply these simple arithmetic procedures is likely to be quite constrained. Virtually all students performed at or above Level 150 in the most recent as well as in previous mathematics assessments.

**Level 200 — Beginning Skills and Understanding.** Students performing at Level 200 demonstrate increasing number sense for two-digit whole numbers, although their use of basic mathematical skills is still imperfect and relatively inflexible. Learners at this level would have difficulty with operations that require more than simple numerical reasoning. In 1986, almost all of the 13- and 17-year-olds as well as 74 percent of the 9-year-olds performed at or above Level 200. These levels of success tended to hold true across most subpopulations at the two older age levels, and across gender and region at age 9. However, a smaller

percentage of Black and Hispanic 9-year-olds attained this level than White 9-year-olds.

### **Level 250 — Basic Operations and Beginning Problem Solving.**

Students reaching Level 250 have a surface understanding of the four basic operations, and are beginning to acquire more developed reasoning skills. At this level, there were substantial differences in performance across the three age groups and across various subpopulations. Less than one-quarter of the 9-year-olds reached this level (or higher) in any of the three most recent assessments and the percentages were substantially lower for some subpopulations, particularly Black and Hispanic students.

Although significantly more 13-year-olds performed at or above Level 250 in 1986 (73 percent) than in 1978 (65 percent), most of the gain occurred between 1978 and 1982. It is alarming that one-third of these students, primarily in the seventh and eighth grades, did not attain this level of proficiency, because basic whole-number computational skills are universally taught in the elementary grades.

The percentage of 17-year-olds performing at or above Level 250 also increased (from 92 percent in 1978 to 96 percent in 1986). Disappointingly, fewer students from historically at-risk populations demonstrated skills with basic operations and beginning problem solving. For example, roughly 35 percent of the Black and 90 percent of the Hispanic 17-year-old students performed at or above this level compared to nearly 100 percent of the White 17-year-olds attending school.

### **Level 300 — Moderately Complex Procedures and Reasoning.**

Students performing at this level demonstrate more sophisticated numerical reasoning and are beginning to draw from a wider range of skill areas, including algebra and geometry.

In 1986, only about 1 percent of the 9-year-olds, 16 percent of the 13-year-olds, and 51 percent of the 17-year-olds were able to perform at or above this level. Further, at age 13, this reflects a decrease from 1978. Although the knowledge and problem-solving skills required to complete items at Level 300 are considered too advanced for 9-year-olds, it is troubling that more 13- and 17-year-olds have not attained this level of performance. While Black and Hispanic 17-year-old students did not show signs of declines at this level of performance, across time their performance levels remained particularly low.

**Level 350 — Multi-step Problem Solving and Algebra.** Performance at this level is characterized by the capacity to apply mathematical operations in a variety of problem settings. As might be expected, virtually no 9- or 13-year-olds attained this level of performance. However, in 1986 only a small proportion (6 percent) of the 17-year-olds reached this level.

The proficiency results are distressing from two perspectives. First, too few students in each age group attain the level associated with material covered in elementary, junior high, and high school respectively. Second, the discrepancy between students' expected and actual mathematics performance increases as they progress through school.



## Mathematics Content Areas

NAEP analyzed mathematics proficiency across grades 3, 7, and 11 in three broad content areas — knowledge and skills in numbers and operations, higher-level applications in numbers and operations, and measurement. Not surprisingly, the largest relative increase in performance occurred on the knowledge and skills subscale from grade 3 to grade 7. Although this rapid growth in computational skills could be expected to pave the way for students to pursue more complex mathematics content (e.g., geometry and algebra) in the higher grades, the descriptions of what older students know and can do in mathematics belie this theory.

In addition to the three content areas listed above, NAEP analyzed geometry proficiency at grades 7 and 11, and algebra proficiency at grade 11. The results for demographic subpopulations defined by race/ethnicity, region, and gender tend to follow national patterns across the five content areas. However, some interesting variations by gender did occur in the results. Reinforcing existing research that posits gender differences in spatial abilities, males showed higher achievement than females in geometry and measurement. Within the area of numbers and operations, females showed superior performance compared to males in knowledge and skills, but weaker performance in higher-level applications.



# Implications for Instruction

## Students' Perceptions of Mathematics

The National Council of Teachers of Mathematics (NCTM) has stated that successful learning in mathematics involves not only the acquisition of essential skills and concepts, but also the development of positive views of mathematics as a discipline.<sup>8</sup>

At grades 3, 7, and 11, students who hold affirmative beliefs, values, and perceptions of mathematics tend to have higher proficiency scores. Although the study design did not permit NAEP to answer the "chicken and egg" question of which comes first — students' positive views, or their mathematical abilities — others have theorized that early attitudes toward mathematics set the stage for later interest and performance.

Although both 13- and 17-year-olds approach mathematics with greater confidence than affinity, confidence and enjoyment decline with years in school. This pattern was consistent across the assessments in 1978, 1982, and 1986.

More than 80 percent of the students in grades 7 and 11 perceive mathematics as a rule-governed subject, and nearly half believe that it involves mostly memorizing. Students in the lower quartile of proficiency appear more likely to hold these views, perhaps because they have had fewer opportunities to explore the creative aspects of mathematics.

Approximately three-quarters of the students in grades 7 and 11 believe that mathematics has practical value in their lives, that its skills are useful in solving everyday problems, and that the discipline helps an individual to think logically. However, less than half of the students envision themselves in a future job that will require mathematical knowledge.

## Patterns of Classroom Instruction

According to student reports, mathematics instruction consists primarily of teacher explanations, reliance on textbooks and chalkboard demonstrations, regular homework assignments, and routine testing. This pattern remained consistent from earlier assessments, with the exception of significant increases in homework assignments and testing in 1986. Given current concerns over the quality of American education in general, and mathematics education in particular, this recent shift may be a response to demands for increased academic rigor in the schools.

<sup>8</sup> *Curriculum and Evaluation Standards for School Mathematics*. National Council of Teachers of Mathematics, Inc., Working Draft, October 1987.

Although the reemphasis on skill development and testing is perhaps warranted, the absence of innovative instructional approaches is cause for concern. A majority of students indicated that they rarely engage in projects, reports, or laboratory activities; thus, they enjoy few opportunities to apply their mathematical skills in real world or experimental situations.

Another cause for concern lies in the unavailability and infrequent use of calculators in the mathematics classroom. To expand the range of tools with which they can approach mathematical problems, students should be given the opportunity to investigate numerical patterns with calculators and use calculators in applied problem solving.

Although calculators have not played a major role in the mathematics classroom, computer usage has risen considerably since previous assessments, particularly for students in the upper grade and the upper range of proficiency. NAEP's studies of the relationship between computer use and level of mathematics proficiency thus far have been inconclusive; however, a strong relationship has been found between positive attitudes toward computer technology and higher levels of mathematics proficiency.

Although students report greater participation in computer programming courses in 1986 than in previous years, this primarily represents students who have taken higher-level mathematics courses. It appears that computers have yet to be well-integrated into the lower levels of instruction.

## **The Place of Mathematics in the Curriculum**

The development of an organized body of mathematical knowledge depends on students' acquisition of prerequisite skills and concepts needed to perform higher-level operations. However, overemphasis on basic skills may leave students unprepared for more advanced studies in mathematics, which require more inventive strategies and skills. Findings from the 1986 assessment reinforce those of previous years: While most children develop an adequate understanding of basic mathematical skills and concepts, too few move into the higher levels of proficiency. Despite our best efforts to revitalize the high school curriculum, many secondary school students avoid upper-level mathematics courses.

There are some signs of encouragement from the recent assessment. Slightly more high school students than in previous years reported taking courses in Algebra II and Pre-calculus or Calculus, perhaps a response to more stringent college entrance and graduation requirements recently implemented in a number of states. Given the positive relationship between advanced course taking and mathematics proficiency, this trend

corresponds to the recent upturn in levels of mathematics performance among 17-year-olds.

As might be expected, students enrolled in academic or college preparatory high school programs were more likely to have taken advanced mathematics courses than students in either general or vocational/technical school programs. Their levels of proficiency varied accordingly, with students in academic or college preparatory programs exhibiting the highest proficiency levels.



## Summary

The encouraging outlook suggested by the trends in mathematics performance must be considered in light of the dismal levels of proficiency shown by many students. Only about three-fourths of the 9-year-olds (third and fourth graders) showed beginning skills and understanding, only about two-thirds of the 13-year-olds (seventh and eighth graders) demonstrated skills universally taught in elementary school, and only about half the 17-year-olds performed at a level suggesting any sophisticated understanding of mathematics. The further students progress through school, the fewer attain what might be expected in terms of the curriculum presumed to be in place at their grade levels. By the high school years, this translates into nearly 1.5 million 17-year-old students each year who are leaving their secondary school experience unlikely to be able to reason mathematically. Moreover, while some significant improvements occurred at the lower to mid-ranges of the scale, there were few signs of progress at the higher end of the scale. This level of mathematical resource is not likely to be adequate for a nation that wants to continue to reap the benefits of modern technology and to compete in the future global economy.

Improving mathematics performance will require educators' best efforts to upgrade the curriculum, modify classroom instruction, and use new teaching materials, including technological resources. The existing curriculum may place far too much emphasis on learning basic computational skills, and teachers may rely too much on explanations and textbooks. The rapid pace of technological progress necessitates a revised set of priorities for mathematics instruction. To improve their understanding of mathematics and their ability to solve mathematical problems, students need the benefit of instruction that emphasizes practical experience in solving problems and the opportunity to use calculators and computers.

The impact of increased course-taking requirements implemented by state legislatures across the country appears to be reflected in the assessment results, yet far too many high school students still elect to avoid advanced mathematics classes. In order to experience the richness and power of the discipline, students from the earliest grades need to see the place of mathematics in solving everyday problems. They will then possess the abilities and confidence they need to explore mathematics as they progress through school, and take full advantage of advanced coursework.

It is the responsibility of educators to see that students acquire the mathematical tools and understanding necessary to negotiate the challenges of work and daily life. Not only must students achieve mastery of essential computational skills, but they must also acquire the higher-order thinking strategies needed to match these skills to the demands of the 21st century.

This Executive Summary presents in capsule form the findings from *The Mathematics Report Card*, which can be ordered for \$14.00, including shipping and handling, from the National Assessment of Educational Progress at Educational Testing Service, Rosedale Road, Princeton, New Jersey 08541-0001.

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