DOCUMENT RESUME

ED 299 158 SE 049 694

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TITLE Lesson Complexity, Student Performance, and Student Perception in Mathematics.
PUB DATE 88
NOTE 18p.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Algebra; *Difficulty Level; Educational Research; Geometry; High Schools; Mathematical Applications; *Mathematics Achievement; Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; *Secondary School Mathematics; Student Attitudes; *Student Reaction

ABSTRACT

First-year algebra high school students of various ability levels were presented lessons concerning the application of geometry theorems. The lessons varied in terms of the complexity of the examples that were shown. After the lessons, students completed a questionnaire concerning their perceptions of the lessons, and then they were tested over the material covered in the lessons. With test scores as the dependent variable, the main effect due to lesson complexity was not significant. With student perception as the dependent variable, significant main effects due to lesson complexity were identified. Significant interactions between lesson complexity and student ability level also were identified. These results are discussed in terms of teaching secondary school mathematics. (Author)

Lesson Complexity, Student Performance, and Student Perception in Mathematics

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Running Head: Lesson Complexity

Abstract

First-year algebra high school students of various ability levels were presented lessons concerning the application of geometry theorems. The lessons varied in terms of the complexity of the examples that were shown. After the lessons, students completed a questionnaire concerning their perceptions of the lessons, and then they were tested over the material covered in the lessons. With test scores as the dependent variable, the main effect due to lesson complexity was not significant. With student perception as the dependent variable, significant main effects due to lesson complexity were identified. Significant interactions between lesson complexity and student ability level also were identified. These results are discussed in terms of teaching secondary school mathematics.

Lesson Complexity, Student Performance,
and Student Perception in Mathematics

Typical high school mathematics textbooks contain a variety of experiences at various levels of difficulty or complexity. Should the teacher present the challenging exercises only to the better-than-average students? Should below average students only deal with the simple exercises? What level of presentation should the teacher select for classes of students with a wide variety of ability levels? This study focuses on the complexity of lesson presentation, the ability levels of students, and their combined effect on student achievement and student perception in mathematics.

Mayer (1982) lamented our economic health and strength as a nation because assessments of student achievement in mathematics have been so discouraging. Literally hundreds of studies have been conducted in an attempt to determine ways to improve mathematics achievement scores. Lester and Garofalo (1982), Lindquist (1980), Lochhead (1981), Shumway (1980), Silver (1985), and Silver and Thompson (1984) have presented a variety of perspectives on the many directions such research has taken. One particular area of study has been on the complexity level of mathematics problems and its relation to student learning. For example, Linville (1970) reported that students solve more problems when the syntax of the problem statements is not complex. Other researchers (e.g., Jerman, 1973; Lester, 1980; Silver, & Thompson, 1984; Zweng, Turner, & Geraghty, 1979) have found that mathematics problems are more complex and more difficult to solve when they require several steps to obtain a solution, when subgoals must be reached before a solution can be obtained, and when the problems contain numbers that are of high computational complexity. Problems are easier to solve when they are straightforward, when they require only a few steps to reach a solution, and when they contain numbers that are easy to work with. In the present study, lesson complexity is studied in terms of these criteria.

Method

Subjects

The subjects were 250 students enrolled in 10 first-year algebra classes in three Georgia high schools. A total of 128 of the students were females and 76% of the students were of Caucasian ancestry. The students represented a wide range of mathematics ability levels.

To determine the ability levels of the students, a test made up of 20 mathematics problems from previous SAT tests was administered. The tests focused on concepts normally covered in high school algebra and geometry. The students were given one 50 minute period to complete the test. The mean of the test was 10.13 and the standard deviation was 2.45. The split-half reliability of the test was .82. Of the 250 students, 66 made scores that were higher than three-fourths of a standard deviation above the mean. These students were referred to as the "above average" group and their raw scores on the test were 12 or higher. A total of 106 students had scores ranging from three-fourths of a standard deviation below the mean to three-fourths of a standard deviation above the mean. Their raw scores ranged from 9 to 11 inclusive. These students were referred to as the "average" group. Finally, 78 students scored lower than three-fourths of a standard deviation below the mean. Their raw scores were 8 or below. These students comprised the group referred to as "below average."

Procedure

The 66 students in the above average group were randomly assigned to one of two lesson complexity conditions (high complexity, low complexity), thus dividing this group into two subgroups of 33 students. The 106 students in the average group also were randomly assigned to the two complexity conditions, forming two subgroups of 53 students. Similarly, the below average students were randomly assigned to the two complexity conditions, creating subgroups of 39 students.

Five days after the 20-item test was administered, students were placed in their assigned groups and were presented a geometry lesson. On the day of the lesson, 12 of the 250 students were absent. Two of the absentees were in the above average group, four were in the average group, and six were in the below average group. Thus, a total of 238 students were presented the geometry lesson.

The lesson content was chosen for two reasons. First, the content is covered in the geometry courses that are offered in the high schools that participated in this study. The high schools used the same geometry textbook (Jurgensen, Brown, & King, 1983). It is very unlikely that the first-year algebra students comprising the study sample had prior exposure to the content because students are required to complete the first-year algebra course before they enroll in geometry. The second reason the content was selected is that, even though it focuses on geometry, standard procedures learned in first-year algebra can be used to apply the content in solving problems.

The lesson was about chords, tangents, and secants of circles. The lesson was begun by defining line sements, chords, tangents, secants, and external segments of secants. Then three theorems were stated and three examples or applications for each theorem were presented. The first theorem says that "when two chords intersect inside a circle, the product of the segments of one chord equals the product of the segments of the other chord." The second theorem states that "when two secants are drawn to a circle from an outside point, the product of one secant and its external segment equals the product of the other secant and its external segment." The third theorem says that "when a tangent and a secant are drawn to a circle from an outside point, the square of the tangent is equal to the product of the secant and its external segment." The examples provided for each theorem involved solving algebraically for the lengths of certain segments when sufficient information was given. For instance, for the third theorem, a typical example would involve a tangent and a secant drawn to a circle from a common point outside the circle. If the lengths of the secant and its external segment are given, then it is possible to solve for the length of the tangent. After three examples for each theorem were shown, the lessons were concluded with a summary of the three theorems and a reminder about the types of problems that can be solved by using the theorems.

The lessons were presented in exactly the same way, except that half of the groups were presented a lesson containing examples of "low complexity", and the other half of the groups were presented a lesson containing examples of "high complexity." As discussed previously, low complexity examples are straightforward, they require only a few steps, and they have numbers that can be manipulated easily. High complexity examples contain numbers that are more difficult to work with, they involve many steps, and attainment of subgoals is a requisite of reaching solutions.

The following excerpt is from the low complexity lesson. The excerpt involves two of the examples related to the second theorem, which concerns two secants drawn to a circle from a common external point.

"Let's consider another application of this generalization. Look at the circle in figure 5 on your handout. Line segment AC is a secant of the circle and AB is the external segment of secant AC. Line segment AE is a secant of the circle and AD is the external segment of secant AE. If AC is 20 cm., and AB is 3 cm., and AE is 15 cm., then we can find out how long AD is. We have AC times AB equals AE times AD. Therefore, 20 cm. times 3 cm. equals 15 cm. times AD. So 60 square cm. equals 15 cm. times AD, and AD equals 4 cm.

Let's do one more example concerning the theorem involving two secants. Look at figure 6 on your handout. If secant MD is 16 feet, and secant CD is 13 feet, and external segment MA is 4 feet, what is the length of secant MB? We know that MD times MC equals MB times MA. We also know that MD is 16 feet and we can find MC, because MD equals MC plus CD. So 16 feet equals MC plus 13 feet, and therefore MC equals 3 feet. By our theorem, we have 16 times 3 equals MB times 4, so MB equals 12 feet."

The following excerpt is from the corresponding portion of the high complexity lesson.

"Let's consider another application of this generalization. Look at the circle in figure 5 on your handout. Line segment AC is a secant of the circle and AB is the external segment of the secant AC. Line segment AE is a secant of the circle and AD is the external segment of secant AE. If AC equals 6 yards and 2 feet, and AB equals 1 yard, and AD is one fourth as long as AC, we can find out how long AE is. First we need to convert our lengths to the same unit. Let's convert all lengths to feet. We have AC equals 6 yards and 2 feet, which is 20 feet. We have AB equals 1 yard, which is 3 feet. Also, AD is one fourth of AC, so AD is 5 feet. Therefore, AC times AB equals AE times AD, so 20 feet times 3 feet equals AE times 5 feet. We get 60 square feet equals AE times 5 feet, so AE is 12 feet.

Let's do one more example concerning the theorem involving two secants. Look at figure 6 on your handout. It is 160 miles from M to D, 130 miles from C to D, and 40 miles from M to A. How many gallons of gasoline would it require to drive from M to B in a car that averages 20 miles per gallon of gas? We know that MD times MC equals MB times MA. We know that MD is 160 miles and we can find MC, because MD equals MC plus CD. So 160 miles equals MC plus 130 miles, and therefore MC equals 30 miles. By our theorem, we have 160 times 30 equals MB times 40, so MB equals 120 miles. Since the car gets 20 miles to the gallon, it would take 6 gallons to drive from M to B."

To ensure control of the lesson complexity level, the lessons were audiotaped. Overhead projections of figures and computations accompanied the audiotaped presentation and students were given handouts that contained diagrams related to the examples. The handout contained the same diagrams and computations as the overhead projections contained. This gave the students ready access to a means of taking notes and performing calculations as the lesson progressed. The classroom teacher monitored the students as the lesson progressed and ensured that the audiotape, the overhead transparencies, and the handout diagrams were synchronized. The teacher was not allowed to stop the audiotape to answer questions during the presentation. Such a technique was necessary in order to assure that extraneous variables were not introduced during the presentation. The recorded lessons were constructed to represent natural classroom instruction and it is reasonable to assume that the results of this study can be generalized to secondary school mathematics classrooms. Factors such as rate of speech, tone of voice, and variance of voice pitch were virtually the same for the low complexity lesson and the high complexity lesson. The low complexity lesson lasted 17 minutes and the high complexity lesson was 19 minutes in length.

Immediately after each lesson was completed the students were administered a six-item lesson evaluation (see Table 1). A Likert-type scale, ranging from 1 to 5 for each item, was used. Students were given 5 minutes to complete this form.

Insert Table 1 about here.

After students completed the lesson evaluations, a 12-item test was administered to test for comprehension of the lesson. Students were not allowed to use notes handouts or personal notes during the test. The split-half reliability of the test was .79. Six of the test problems were of "low complexity" and six were of "high complexity", where the complexity level was a previously defined. Students were given 35 minutes to complete the test.

Results

A 2(Lesson: high complexity vs. low complexity) X 3(Student ability: above average vs. average vs. below average) analysis of variance was performed on the student achievement scores as well as on each of the six lesson evaluation scores. The results are shown in Table 2. With achievement as the dependent variable, the main effect due to student ability level was significant, $\underline{F}(2, 232) = 18.50$, $\underline{p} < .001$. Scheffe' specific comparison tests showed that the above average group scored significantly higher (beyond the .05 level) than did the average group and the below average group. Although the mean of the average group was higher than the mean of the below average group, the difference between the two means was not statistically significant. Neither the main effect due to lesson complexity nor the interaction between lesson complexity and student ability was significant.

For student perception Item a (easy to understand), the main effect due to lesson complexity was significant, $\underline{F}(1, 232) = 12.55$, $\underline{p} < .001$, with the low complexity lesson receiving significantly higher ratings than the high complexity lesson. The main effect due to student ability level was significant, $\underline{F}(2, 232) = 4.66$, $\underline{p} = .010$. Scheffe' tests indicated that the above average students gave higher ratings then did the below average students. Finally, the interaction between lesson complexity and ability level was significant, $\underline{F}(2, 232) = 4.30$, $\underline{p} = .014$. Specific comparisons showed that the below average students who received the lesson of high complexity rated the lesson significantly lower than did the below average students who were presented the low complexity lesson and the above average students who received the low complexity presentation.

For Item b (organized) and Item c (prepared), there were no significant main effects or interactions.

Using Item d (confident) as the dependent variable resulted in a significant difference in favor of the low complexity lesson over the high complexity lesson, $\underline{F}(1, 232) = 16.57$, $\underline{p} < .001$. Similarly, for Item e (enjoyed), the main effect due to lesson complexity was significant in favor of the low complexity lesson over the high complexity lesson, $\underline{F}(1, 232) = 7.21$, $\underline{p} = .007$.

For Item f (pace), the main effect due to lesson complexity was significant, $\underline{F}(1, 232) = 19.39$, $\underline{p} < .001$, in favor of the low complexity lesson over the high complexity lesson. The interaction between lesson complexity and student ability also was significant, $\underline{F}(2, 232) = 6.48$, $\underline{p} = .002$. The below average students who were given the high complexity lesson rated the lesson significantly lower than did the below average students who received the low complexity lesson and the above average students who were presented the low complexity lesson.

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Insert Table 2 about here.

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An intercorrelation matrix for the 20-item pretest and the seven dependent variables is presented in Table 3. The pretest was correlated significantly with the 12-item posttest, Item a (easy), and Item d (confident). The posttest was correlated significantly with all six lesson evaluation items. The six evaluation items were highly correlated with one another, with correlations ranging from .405 to .718.

Discussion

It is necessary to bear three cautions in mind when interpreting these results. First, the reaction questionnaire and the posttest were administered immediately after presentation of the lessons and no time for reflection or study was available to students. Second, students were not allowed to make comments or ask questions during the lessons. This was because the lessons were taped and students were given handouts and were shown overhead transparencies as the lessons progressed. Although every effort was made to present the lessons in as natural a way as possible, no teacher-student interaction took place as in a typical classroom. The third caution concerns the selection process that was used to obtain the sample. Students participated in the study by virtue of their teachers' willingness to volunteer their time from the regularly scheduled class meetings. Therefore the subjects for this study were not randomly selected from a population. However, the students were randomly assigned to treatment groups once they were identified as participants.

With these cautions in mind, the following conclusions are made. First, although achievement scores were slightly higher under the low lesson complexity condition than under the high complexity condition, the difference was not statistically significant. Therefore, no claims can be made concerning the complexity level teachers should use. However, it should be noted that students rated the low complexity presentation significantly higher than the high complexity lesson in terms of ease of understanding, development of confidence, enjoyment of the lesson, and proper pacing of the lesson. Furthermore, students of above average ability rated the lessons significantly higher than students of below average ability in terms of ease of understanding. The lowest ratings in terms of ease of understanding, and pacing of the lesson were given by below average students in the high lesson complexity condition.

Common sense dictates that teachers should not avoid relevant content that is of a high complexity level. But such content should be presented in as clear and concise a way as possible with extra time allowed for student questions and practice.

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Table 1

Lesson Evaluation

Item	Score 5	4	3	2	1	Item
a. Easy to understand						a. Hard to understand.
b. Well organized.						b. Poorly organized.
c. Well prepared.						c. Poorly prepared.
d. I am confident of the material.						d. I am not confident of the material.
e. I enjoyed the lesson.						e. I did not enjoy the lesson.
f. The teacher presented the lesson at a good pace.						f. The teacher presented the lesson too fast.

Table 2

Results of Analysis

Lesson Complexity		High	High	High	Low	Low	Low
Student Ability		High	Average	Low	High	Average	Low
n		32	50	34	32	52	38
Achievement	M	6.09	5.26	4.12	7.00	5.00	4.89
	SD	2.44	2.16	1.89	2.09	1.97	2.20
Item a (easy)	M	3.44	3.16	2.44	3.69	3.35	3.58
	SD	1.13	1.18	1.02	1.06	1.27	0.92
Item b (organized)	M	4.03	3.96	4.06	4.09	3.98	4.37
	SD	0.90	0.86	1.00	0.82	1.08	0.97
Item c (prepared)	M	4.19	4.04	3.94	4.22	4.06	4.32
	SD	0.86	0.95	0.98	0.87	1.11	1.02
Item d (confident)	M	2.75	2.88	2.29	3.53	3.04	3.24
	SD	1.30	1.15	1.09	1.24	1.19	1.02
Item e (enjoyed)	M	2.56	2.72	2.59	2.94	3.06	3.16
	SD	1.24	1.07	1.31	1.19	1.29	1.13
Item f (pace)	M	3.06	3.42	2.56	3.81	3.42	4.08
	SD	1.37	1.39	1.37	1.06	1.35	1.17

Table 3

Intercorrelation Matrix

Variable	Ach	a	b	c	d	e	f
Pretest	.426	.211	-.002	.060	.140	-.014	.042
Achievement		.357	.197	.229	.371	.145	.215
em a			.499	.487	.718	.539	.606
Item b				.747	.447	.471	.405
Item c					.454	.457	.425
Item d						.518	.545
Item e							.428

Note. Correlation significant at .05 level for r $\geq$ .130. Significance at .01 level for r $\geq$ .141.