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ABSTRACT

Comparisons of expert and novice problem solving in physics have helped characterize some of the key features of expert behavior. There is considerable debate, however, as to whether these characteristics are specific to the field of expertise (physics) or exportable to other fields. While the question seems difficult to answer in general, at least three skills frequently observed in physics "experts" seem to have broad application to a variety of fields. However, the transfer to other fields may require a more deliberate and conscious awareness of the use of these skills than normally accompanies the acquisition of expertise in physics. This paper defines "novice" and expert and describes the strategies employed by each in a problem solving situation. The performance of experts is described in detail including the choice of appropriate principles, the use of chains of analogies and the ability to transfer previous knowledge to areas of little experience. (CW)

TOWARD A UNIFIED THEORY OF PROBLEM SOLVING: A VIEW FROM PHYSICS

By

Klaus Schultz and Jack Lochhead

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Comparisons of expert and novice problem solving in physics have helped characterize some of the key features of expert behavior. There is considerable debate, however, as to whether these characteristics are specific to the field of expertise (physics) or exportable to other fields. While the question seems difficult to answer in general, at least three skills frequently observed in physics "experts" seem to have broad application to a variety of fields. However, the transfer to other fields may require a more deliberate and conscious awareness of the use of these skills than normally accompanies the acquisition of expertise in physics.

It behooves us first to supply brief descriptions of "novices" and "experts" for the purposes of this paper. Novices are not blank slates; we should assume that they ha-- at least been exposed to (if we may use a term from the medical field) the laws, concepts, etc. relevant to a given problem space. Experts are knowledgeable in the general domain in question, but not so familiar with specific problems in the domain that they can solve them automatically, from memory or by application of a memorized algorithm.

The three competencies that are candidates for generalized expert problem-solving skills are:

1. The ability to organize quantitative calculations through an understanding of qualitative relations.
2. The ability to organize one's knowledge according to principles selected to fit the current problem's anticipated solution.
3. The ability to evaluate the probable validity of a physical (or other) model through an analogy or chain of analogies.

Each of these will be discussed in turn, with an illustration. None of them are new. The second item is closely related to the notion of "chunking" proposed some years ago. The first was neatly summarized by Champagne, Gunstone, and Klopfer (1983), in a study of expert and novice behaviors, with the diagram of Figure 1.

From Qualitative to Quantitative

How might this expert/novice difference show up in a problem-solving situation? Imagine a problem like this: one is about to cut down a 100-foot tall telephone pole, and wants to know how fast it would be moving when it hits the ground. (You might well ask why there is any interest in this problem. Suppose you want to film a commercial for a certain kind of pickup truck, to show how tough it is. Before you drop the telephone pole on top of it, you want to estimate the damage, to see whether the pickup will survive well enough to warrant filming the commercial, or whether you need to fake the sequence.) A typical novice might search in his or her memory for potentially relevant equations, especially those related to the rotational motion of rigid bodies, and focus on those in which there is a good match between the symbols in the equations and the givens and unknowns of the problem statement. In this example, what might first come to mind is the rotational analog of F = ma, i.e. the equation linking torque and angular acceleration (see Fig. 2). This equation looks simple, but the torque varies with the angular position as the pole falls to the ground. This leads to a difficult second-order differential equation. Our typical novice might well give up at the mere sight of this equation, or perhaps give the old college try at solving it before giving up. A "better" novice, knowing that introductory physics problems never call for such hard mathematics, might realize that there must be a more straightforward approach, and eventually arrive at one of the equations expressing the principle of conservation of energy.

An expert would likely start with a qualitative approach, recognizing that the potential energy convertible to kinetic energy is the same as if the entire mass of the pole were concentrated at its center, 50 feet above ground. A first approximation would then lead to the equation $mgh = 1/2\ mv^2$ (with $h = L/2$, where L is the length of the pole), which can be easily solved for v, the velocity of the center of mass, to give the expression $\sqrt{gL}$. (Other parts of the pole would be moving at speeds greater or smaller than this, depending on their location relative to the pivot point at the bottom of the pole.)

However, the kinetic energy of the telephone pole consists of both translational energy (the motion of the center of mass) and rotational energy (around the center of mass). The exact calculations, taking into account the moment of inertia, involve the very same equations dragged up from memory by the novice. Even before writing down the relevant equations, the expert would likely make a mental note that the falling velocity will be less than $\sqrt{gL}$ because the available potential energy must be shared between translational and rotational kinetic energies.

So far the expert has been using general problem-solving skills: start with a simpler situation to get an approximate solution, then consider how the full solution would differ, without resorting to numbers or even equations. A physics expert might also make use of domain-specific experience, and recall that in such situations the rotational energy is usually of the same order of magnitude as the translational. The exact value of v is then expected to differ from $\sqrt{gL}$ by no more than about a factor of 2 (in fact, no more than about a factor $\sqrt{2}$, because of the quadratic relation between energy and speed; this latter point would stem from general mathematical sophistication rather than domain-specific knowledge or skills).

Note, in Fig. 2, that while an expert in biology might not remember the expression for the moment of inertia for a uniform rod, he or she can make a good start by applying general principles such as conservation of energy, which can give at least an "order of magnitude" answer. Also, an expert would almost surely make use of a hand-drawn diagram (not necessarily carefully drawn to scale) to help identify the relevant principles before writing down any equations.

Choice of Appropriate Principles

For experts to perform in this manner, they need what Champagne et al call a "comprehensive and integrated motion-of-object schema." One characteristic of such a schema is that it has a hierarchical structure. Mestre et al (1988) illustrate how such a structure can be used to teach students to categorize physics problems for puposes of identifying effective solution paths. Their Hierarchical Analyzer is designed to lead students through a sequence of questions (see Fig. 3 a & b for two examples) that help them focus on the relevant physical principles. Experts probably go through a process like this, even if unconsciously. By contrast, left to their own devices, novice students tend to use a nonhierarchical "laundry list" (Fig. 3 c & d) of physics terms, problem types, and variable names, and match these to superficial aspects of the problem statement.

Inherent in the hierarchical structure are a set of physics concepts (e.g. energy, momentum, force) which serve to organize physical situations into categories with well-defined solution strategies. Each of the physical concepts (e.g. energy) was constructed, and its definition refined through a centuries-long process, to make description and solution of problems as simple as possible. The resulting system of concepts is redundant - in the good sense of the term (like the redundancy in language or in mechanisms of depth perception). In the example of the falling telephone pole, it is possible to apply either energy or force and torque considerations; but one view leads to a much simpler solution path. "Expertise" is intimately connected with the ability to cast the problem in terms of those concepts that make the problem relatively simple to solve.

Chains of analogies

Among experts, the process of selecting a convenient representation (choosing the most advantageous conceptual perspective) may proceed via analogies to previously-solved problems or to situations where the expert feels confident. Novices do not generally make use of such a strategy. For example, most novices are stumped by the question of how much force a table applies to an object resting on top of it; usually they don't believe that there is any such force. Viewed from an energy perspective, one is apt not to see a need for such a force, since there are no perceptible changes in energy. The need for an upward force exerted by the table can be generated by recourse to an atomic model of solid matter, with spring-like forces between neighboring atoms - in effect viewing the table surface as a compressible spring. For most novices, this analogy requires too great conceptual leap; they may accept it from the voice of authority, and may even parrot it back, but it does not make much sense to them. Clement (in press) has shown how experts spontaneously use chains of analogies to expand their understanding of novel situations. Figure 4 contains segments of one expert's attempt to understand the force exerted by a stretched helical spring through constructing analogies to square and hexagonal springs. It appears that this strategy can be used to help novices gain a deeper understanding of technical concepts in physics. Clement (1987), Brown (1987), and Schultz et al (1987) have demonstrated in one-on-one tutoring mode and in classroom situations that a chain of analogies (Fig. 5, the middle row of examples) can be used to help students understand the nature and origin of forces exerted by "rigid," stationary bodies such as tables.

In addition to the above three aspects of expertise which are prominent in the literature of problem solving in physics, there is a fourth, probably more critical, component. None of the three techniques outlined above are guaranteed to yield a correct solution to a given problem. Rather they are heuristics, suggesting promising paths while minimizing the chance of a gross error. While using them, the expert continuously monitors and questions his or her reasoning. The key to expert scientific behavior is doubt, resulting in a constant searching for other perspectives that may support or disconfirm previous ones.

Transfer

Expert knowledge appears to consist of a complex network of multiply-interconnected concepts, organized so that there may be many potential (and several actual) paths to correct conclusions. Expertise is the ability to reach conclusions with confidence, but without disregarding the possibility of error. Experts are often reluctant to apply their skills to areas in which they have little experience. Experience is the yardstick by which experts try to determine the degree of confidence they have in their conclusions.

Transfer, as it is traditionally described, requires a willingness to operate in areas where one has little experience. The wise expert recognizes that this imposes a major handicap. But there is a big difference between having the ability to operate as an expert on the one hand, and possessing a detailed self-knowledge of the strategies used on the other. In the latter case, it is possible to deliberately apply the strategies to novel situations, keeping in mind that without extensive experience one's judgement will be hampered. Creative leaps, however, do not require this kind of judgement; in fact, there are examples in the history of science where great creative advances were made precisely because the scientist did not "know too much" in the domain.

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Champagne, Gunstone, & Klopfer (1983) summarize a considerable array of expert/novice studies in the field of Mechanics with the following diagram

EXPERT

Problem Statement ——— Qualitative Analysis ——→ Equation

NOVICE

Problem Statement ————————→ Equation

From Champagne et al (1983)

Figure 1

FALLING TELEPHONE POLE PROBLEM

Novice's procedure:
Select equation of motion: $\tau = I\alpha$

(τ = torque; α = angular acceleration; I = moment of inertia)

$$\tau = I\, d^2\theta/dt^2$$

But τ varies with θ. So above equation is difficult to solve.

Expert's procedure:

For a pole of length L.

Conservation of energy:

initial potential energy = final kinetic energy

$$mgh = (1/2)mgL \sim (1/2)mv^2 \quad \text{(rough approx.)}$$

More precisely:
Kinetic energy = translational energy of center of mass +
rotational energy about center of mass

$$= (1/2)mv^2 + (1/2)I\omega^2$$

(v = velocity of center of mass; ω = angular velocity of pole = $v/(1/2)L$;

I = moment of inertia about the center of mass.

$$(1/2)mgL = (1/2)mv^2 + (1/2)I \cdot 4v^2/L^2$$

Look up or calculate: $I = mL^2/12$

A bit of algebra:

$$mgI = (4/3)mv^2$$

$$v = \sqrt{3gL/4}$$

Fig. 2

FIGURE 1. Hierarchical Analyzer Menus & Choices for Problem 1

1 Which principle applies to this part of the problem solution?

1. Newton's Second Law or Kinematics
2. Angular Momentum
3. Linear Momentum
4. Work and Energy

Please enter your selection: [4]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

2 Describe the system in terms of its mechanical energy

1. Conservative system (conservation of energy)
2. Non-Conservative system (work-energy exchange)

Please enter your selection: [1]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

3 Describe the changes in mechanical energy. Consider only the energy of one body at some initial and final state

1. Change in kinetic energy
2. Change in potential energy
3. Change in potential and kinetic energies

Please enter your selection: [3]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

4 Describe the changes in kinetic energy

1. Change in translational kinetic energy
2. Change in rotational kinetic energy
3. Change in translational and rotational kinetic energies

Please enter your selection: [1]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

5 Describe the boundary conditions

1. No initial translational kinetic energy
2. No final translational kinetic energy
3. Initial and final translational kinetic energies

Please enter your selection: [1]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

6 Describe the changes in potential energy

1. Changes in gravitational potential energy
2. Changes in spring potential energy
3. Changes in gravitational and spring potential energies

Please enter your selection: [1]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

7 Describe the boundary conditions

1. No initial gravitational potential energy
2. No final gravitational energy
3. Initial and final gravitational energy

Please enter your selection: [2]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

8 Is there another body in the system which has not been examined?

1. Yes
2. No

Please enter your selection: [2]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

9 The Energy Principle states that the work done on the system by all non-conservative forces is equal to the change in the mechanical energy of the system:

$$W_{nc} = E_f - E_i$$

According to <u>your</u> selections,

$W_{nc} = 0$ (Conservative system: mechanical energy conserved)

$E_f = (\frac{1}{2}Mv^2)_{1f}$

$E_i = (Mgy)_{1i}$

Please press any key to continue

10 *** Work and Energy ***

1. Problem solved
2. Return to Main Menu to continue solution
3. Review previous solution screens

Please enter your selection:

FIGURE 3a. From Mestre et al (1988)

FIGURE 2. Hierarchical Analyzer Menus & Choices
For Second Part of Problem #2

1

Which principle applies to this part of the problem solution?

1. Newton's Second Law or Kinematics
2. Angular Momentum
3. Linear Momentum
4. Work and Energy

Please enter your selection: [3]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

2

Describe the system in terms of its linear momentum

1. Momentum conserved (external forces do no work)
2. Momentum not conserved (external force does work)

Please enter your selection: [1]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

3

Describe the system at some initial state

1. One particle
2. Two particles
3. More than two particles

Please enter your selection: [2]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

4

Describe all motion within the system at some initial state

1. No motion
2. One particle in motion
3. Two particles in motion

Please enter your selection: [2]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

5

Describe the system at some final state

1. One particle
2. Two particles
3. More than two particles

Please enter your selection: [1]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

6

Describe all motion within the system at some final state

1. No motion
2. One particle in motion

Please enter your selection [2]

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

7

The Impulse-momentum theorem states that the impulse delivered to a system is equal to the change in momentum of the system:

$$\int F_{ext}dt = P_f - P_i$$

According to your selections,

$\int F_{ext}dt = 0$ (conservation of momentum)

$P_i = M_1 V_{1i}$

$P_f = (M_1 + M_2)V_f$

Please press any key to continue

8

*** Final Menu ***

1. Problem solved
2. Return to Main Menu to continue solution
3. Review previous solution screens

Please enter your selection

FIGURE 3b.

Terms Used for Sorting Equations in the Equation Sorting Tool

---

Physics Terms

acceleration
angular acceleration
angular displacement
angular momentum
angular velocity
center of mass
centripetal acceleration
centripetal force
circular motion
conservation of angular momentum
conservation of energy
conservation of momentum
conservative forces
conservative systems
equilibrium of rigid bodies
frictional force
gravitational force
impulse
impulse & change in momentum
kinematics
kinetic energy
mechanical energy
moment of inertia
moment of inertia (disk)
moment of inertia (hoop)
moment of inertia (rod)
moment of inertia (sphere)
momentum
Newton's Second Law (definition)
Newton's Second Law (dynamics)
nonconservative forces
nonconservative systems
parallel-axis theorem
potential energy
power
rotational dynamics
rotational kinematics
speed
spring force
statics
torque
uniform circular motion
velocity
weight
work
work-energy theorem

Problem Types

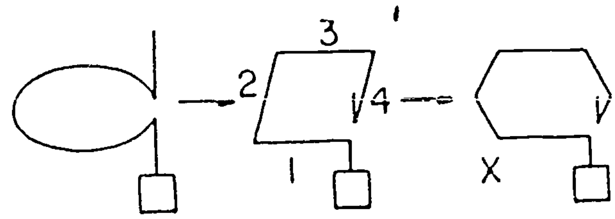
angular motion
Atwood's machine
ballistic pendulum
blocks and planes
circular motion
collisions (elastic)
collisions (completely inelastic)
conveyor belts
energy
equilibrium of rigid bodies
freely falling bodies
friction
frictional forces
gravity
hanging bodies
impulse
inclined planes
kinetic energy
linear motion
motion in a plane
motion in one dimension
potential energy
projectile motion
pulleys
rockets
rolling bodies on planes
rolling without slipping
rotating bodies
springs
statics
strings and ropes
trajectories
uniform circular motion
variable mass
vertical motion
work
work done by friction

FIGURE 3c.

Variable Names

acceleration
angle
angular acceleration
angular displacement
angular momentum
angular velocity
arc length
center of mass coordinates
centripetal acceleration
coefficient of kinetic friction
coefficient of static friction
displacement, distance, position
distance, displacement, position
force
friction
gravitational acceleration
height
impulse
kinetic energy
length
mass
mechanical energy
moment of inertia
momentum
normal force
potential energy
position, displacement, distance
power
radial acceleration
radius
speed, velocity
spring constant
tension
time
torque
velocity, speed
weight
work

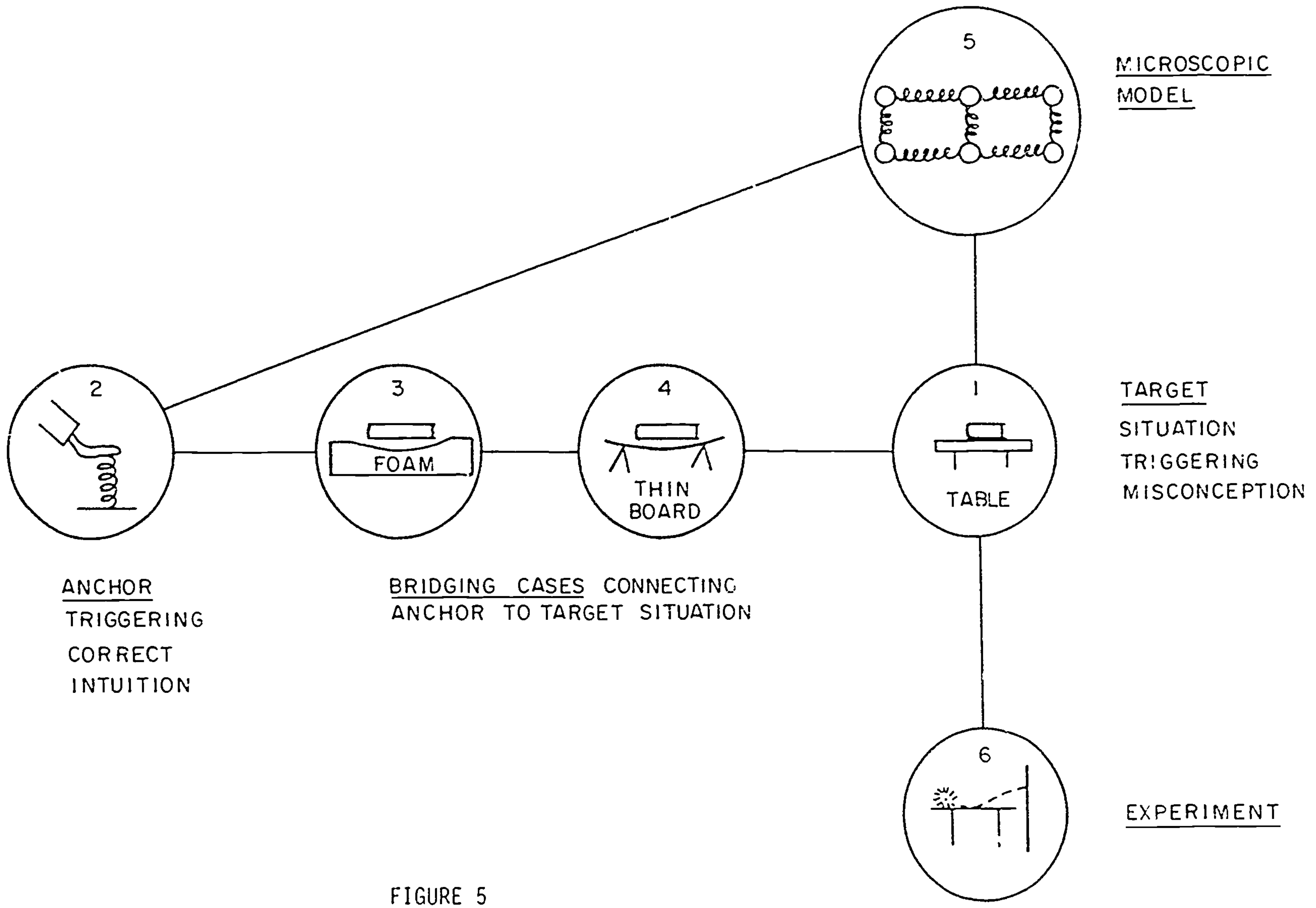
FIGURE 3d.



S: Darn it, darn it, darn it...What could the circularity [in contrast to the straight rod] do? Why should it matter? How would it change the way the force is transmitted from increment to increment of the spring? Aha. Now let me think about - aha. Now this is interesting. I imagined - I recalled my idea of the square spring and the square is sort of like a circle and I wonder...what if I start with a rod and bend it once (places hands at each end of rod in drawing and motions as if trying to bend a rod) and then I bend it again. What if I produce a series of successive approximations to the circle by producing a series of polygons?...Clearly there can't be a hell of a lot of difference between the circle and say, a hexagon... (Draws hexagonal coil in Figure 4a) Now that's interesting. Just looking at this [hexagon] it occurs to me that when force is applied here, you not only get a bend on this segment, but because there's a pivot here (points to X in Figure 4a), you get a torsion effect... Aha! Maybe the behavior of the spring has something to do with twist forces (moves hands as if twisting an object) as well as bend forces (moves hands as if bending an object). That's a real interesting idea...That might be the key difference between this [bending rod] which involves no torsion forces, and this [hexagon]. Let me

FIGURE 4

FROM CLEMENT (IN PRESS)



5
MICROSCOPIC
MODEL
2
3
FOAM
4
THIN
BOARD
1
TABLE
TARGET
SITUATION
TRIGGERING
MISCONCEPTION
ANCHOR
TRIGGERING
CORRECT
INTUITION
BRIDGING CASES CONNECTING
ANCHOR TO TARGET SITUATION
6
EXPERIMENT

FIGURE 5