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ABSTRACT

A longitudinal approach is demonstrated that allows assessment of: the means by which a student's level and rate of learning in a given subject area compare with the level and rate of learning of other students, and each student's relative strengths and weaknesses across subject areas. The approach involves an individual growth model to estimate performance on each measure. Then, information from multiple measures is combined in profiles to investigate multivariate aspects of status and growth, including intra-individual strengths and weaknesses. A simple straight-line growth model is used. During 1985, the approach was applied to a group of students who attended all four years of high school in the San Francisco Unified School District during the 1980-81/1983-84 period. Data consist of scores on the Comprehensive Test of Basic Skills, Form S, Level 4, administered in the fall of 1980, 1981, 1982, and 1983. Individual and group analyses were conducted. Results indicate that it is possible to provide individualized interpretations of multivariate longitudinal growth in achievement for groups of students. Six tables and five figures are included. (TJH)

LONGITUDINAL PROFILES OF ACHIEVEMENT

by

Gary L. Williamson

Paper presented at the annual meeting of the American Educational Research Association, April, 1988. New Orleans, Louisiana.

Longitudinal Profiles of Achievement

Consider the following motivating situation. Each of a group of students is tested with a standardized achievement test in a given year of the school career. The test battery consists of a number of subtests which may be grouped into substantive areas (e.g., reading, language, mathematics). Tests are administered repeatedly over the course of a student's school career, resulting in a complex student record consisting of multiple measures each taken on several occasions. The data are three-dimensional (persons, measures and occasions), and can be viewed as longitudinal profiles of achievement for a collection of individuals. (See Figure 1.)

If the same attributes are measured at each time and comparable scales are used, it is possible to assemble, for any individual, a record of all test scores over the numerous testings and to attempt a systematic interpretation of individual performance across an array of attributes. Two basic questions may be asked.

(1) How does a student's level and rate of learning (in a given subject area) compare with the level and rate of learning of other students?

(2) What are each student's relative strengths and weaknesses (across subject areas)?

Answering (1) presumes the ability to describe level and change in achievement over time for a single individual; question (2) entails a comparison of an individual's performance in different subject areas at a given point in time. In this paper, an approach is demonstrated that makes it possible to address both questions.

Methodological Background

Davis (1959) and Sakoda, Cohen and Beall (1954) laid the statistical groundwork for identifying ipsative (intraindividual) strengths and weaknesses in a profile of commensurable scores. However, Kaufman (1979) has probably provided the best summary and overview of the issues involved in ipsative test interpretation (which includes the problem of identifying intraindividual strengths and weaknesses). With Reynolds (Kaufman and Reynolds, 1983), he considered both normative and ipsative methods of evaluating test performance. Their work is limited to the investigation of profiles of performance at a given point in time.

The issue of individual strengths and weaknesses takes on additional meaning when academic growth is considered. In order to be able to identify strengths and weaknesses in growth, it is first necessary to describe individual growth and to create individual profiles of growth.

Recent developments in the measurement of change provide the most reasonable framework for describing growth. Rogosa, Brandt and Zimowski (1982) emphasized the modeling of individual change and asserted that individual time paths are the "proper focus for the analysis of change." (p. 744) They stated a statistical model for the individual growth curve and examined various assumptions in the measurement of change literature. In their "Mottos for the Measurement of Individual Change", they summarized key points derived from a conceptual and mathematical framework for the measurement of individual change.

In more recent work, Willett (1985) and Rogosa and Willett (1985) discussed correlates of change via models for systematic individual differences in growth. Their approach incorporates a model for growth and a model for individual differences in growth.

Rogosa, Willett and Williamson (1986) used the above techniques to analyze achievement scores on the Comprehensive Tests of Basic Skills (CTBS) for a cohort of approximately two hundred high school students. They also illustrated how some common indices from the profile similarity literature can be used to help examine growth on multiple measures. Finally, they introduced a prototypical student achievement report that highlights academic growth on multiple measures.

The recent literature on the measurement of change focuses attention on models for individual growth on a single measure. Questions about level and rate of learning on a single measure are phrased in terms of quantities derived from a specified model. This approach, extended to the multivariate case, is a useful one for providing answers to the two substantive questions posed at the beginning of this paper.

In this paper individual growth on multiple measures is summarized by a *profile of progress*: the vector of estimated growth rates obtained by fitting a straight-line growth model to individual longitudinal data sequences on each academic measure. Using this framework, univariate and multivariate descriptions of achievement and growth are provided, and intraindividual strengths and weaknesses are computed for profiles of achievement (i.e., profiles consisting of scores on each measure) and profiles of progress. Data from 200 high school students over four years are used to illustrate the techniques.

Procedure

Considerable technical work underlies the procedure used here. However, a complete description is beyond the scope of this paper. A brief summary of some of the significant steps follows. The approach is in two phases. First, an individual growth model is used to estimate each individual's performance on each measure, in turn. Then information from multiple measures is combined in profiles to investigate multivariate aspects of status and growth, including intraindividual strengths and weaknesses.

A simple straight-line growth model is used in this investigation. The model for an observed achievement score is.

$$X_{pm}(t_i) = \xi_{pm}(t^*) + \theta_{pm}(t_i - t^*) + \epsilon_{pmi}, \qquad (1)$$

where $X_{pm}(t_i)$ is the observed score for person p on measure m at time t_i (the time on the i^{th} occasion of measurement). True scores are indicated by ξ, and t^* is an arbitrary origin for the time scale. θ_{pm} is the true rate of change for person p on measure m. Errors of measurement are denoted by ϵ. Ordinary least squares is used to fit the model to each individual's longitudinal data on each measure.

The adequacy of the straight-line model can be examined by computing the squared multiple correlation (R^2) for each individual regression. Low R^2 indicates the possibility of poor model fit. In some cases, a straight line may still be a reasonable summary of performance, and there may be no cause for concern (as, for example, is the case when R^2 is low because the slope of the regression is zero; scores are not changing over time) However, individual scores can be examined to determine if this is so

Another indication of poor model fit can be found by examining the sums of squared residuals (SSRES) for each individual. High values of SSRES support the contention that the observed scores of an individual are not well modeled by a straight line. By listing the identification numbers and scores of persons with the highest values of SSRES, it is possible to detect cases where the linear model is not adequate.

Given that the model is appropriate for the majority of individuals, fitting straight-line growth curves to each measure results in estimates of rates of growth that can be summarized separately for each measure. One can quickly identify those persons who grow fastest and those who grow slowest on each measure. Constructing empirical distributions, it is possible to determine an individual's relative standing in a group of individuals. By combining information from different measures into individual profiles, consistency of individual performance across measures is apparent

Any potentially interesting aspect of the data can be examined. In particular, any multivariate index of growth or status (for example, the average of the components of a profile, also called the profile *elevation*) can be computed, displayed and ranked to provide insights into growth or status on multiple measures. This provides one way of answering the question "How does a student's level and rate of learning in a given area compare with the level and rate of learning of other students?"

In addition to interpretations of performance for individual students, estimates of various properties of the collection of growth curves are available based on theory presented by Blomqvist (1977) and Willett (1985) Maximum likelihood estimates are possible for several relevant parametric properties of the collection of growth curves. Those reported here include: t^o, the time at which true score variance is at a minimum; $VAR_p(\theta_{pm})$, the variance of the true rates of change; the variance of $\xi_{pm}(t^c)$ (the true score of person p on measure m at time, t^o); and, $\rho(\hat{\theta})$, the reliability of the estimated rates of change

Finally, intraindividual strengths and weaknesses can be computed. Traditionally, intraindividual strengths and weaknesses (at a given time) are identified by significant deviations from the elevation of the profile of achievement. That is, observed score profiles are used, the deviation of each component from the profile average is calculated, a critical ratio is computed and compared with a normal probability table, and those components that deviate significantly from the average are labeled a strength or a weakness depending on the sign of the deviation.

The identification of strengths and weaknesses is improved by using profiles of fitted scores derived from the straight-line growth models rather than using the original profiles of observed scores

(This is because the fits from the model yield more reliable estimates of the true score profiles.) If growth is of particular interest, then using the profile of progress and the profile of achievement at one or more times has intuitive appeal. An example is considered next.

Example

Source and Nature of the Data

In this section analyses are presented for a group of students who attended high school in the San Francisco Unified School District during school years 1980-81 through 1983-84. The data were obtained in 1985 while working on the Measurement of Student Progress component of the Teaching and Testing Group, one of several major research efforts sponsored by the Study of Stanford and the Schools. (The cooperation of Dr. Don Barfield of the San Francisco Unified School District is acknowledged.)

The data consist of scores on the Comprehensive Tests of Basic Skills (CTBS), Form S, Level 4 for the cohort of students who matriculated in one high school in the fall of 1980. Test administrations were conducted each fall of the years 1980, 1981, 1982 and 1983. Only students who attended the high school all four years are included in the data. The same form and level of the test was administered each year.

The CTBS is an achievement test battery published by CTB/McGraw-Hill. It consists of several subtests, grouped into three broad categories: Reading (including Vocabulary and Comprehension), Language (including Mechanics, Expression and Spelling), and Mathematics (including Computation, Concepts and Applications) A variety of scores are provided for each subtest, for each "Total" test (i.e., each of the three broad subject areas) and for the "Total Battery". The score types include raw scores, grade equivalents, national percentiles and scale scores.

Reading Total scale scores (RTSS), Language Total scale scores (LTSS) and Mathematics Total scale scores (MTSS) are chosen for the analyses. Scale scores reportedly have characteristics that make them well suited for comparisons across forms and levels of tests. For Level 4 of the CTBS, Form S, scale scores are suitable for comparisons across subject areas as well (CTB/McGraw-Hill, 1974a, 1974b). The total test scores are chosen largely for convenience. They provide a profile of student performance across three broad areas of achievement.

With real data, a number of problems arise. First, incomplete data are common. Complete profile information was unavailable for approximately 15% of the individuals. Though there are numerous imputation procedures that could be used to supply missing data, to simplify matters some analyses (e.g., the identification of strengths and weaknesses) used only cases with complete information.

Outliers are a problem for many statistical procedures. It is well-known that outliers can have serious consequences for linear regression. It is thus wise to carefully scrutinize the fitted regressions. For these data, few points are available and it is possible to quickly scan the observed scores across time to get a feel for the presence of outliers With a larger number of points in time, this may be harder to do. If a more sophisticated analysis is desired, standard techiniques for investigating outliers and their influence are available

Another problem is the presence of ceiling and floor effects in the measurement scales. It seems likely, judging from some of the results, that such effects were operating in these data. Thus some of the results derived from the straight-line growth model may be artifacts of the measurement scale. Such instances are noted in the examples.

Results of Fitting Growth Curves

The means and standard deviations of the scale scores are shown in Table 1. There is an increase in mean scale score each year from ninth grade to twelfth grade on each of the three tests. The mean achievement level is nearly equal for reading and language; it is somewhat higher on mathematics. There seems to be a slight tendency for the scale score variance to increase from ninth to twelfth grade, suggesting that t° may be less than or equal to nine. The number of students is different for each test since only students with complete longitudinal information were included in the averages.

Straight-line growth curves were fit to each student's longitudinal data on each measure using ordinary least squares. Table 2 shows summaries of R^2 and SSRES. The traditional five-number summaries have been modified to include the fifth and ninety-fifth centiles. The median value of R^2 is approximately .83 on all three measures. Nearly a fourth of the students in the cohort have R^2 values above 93 The lower quartile is around 6, so most of the students in the cohort have reasonably good fits as indicated by R^2.

The summaries of SSRES generally confirm the fact that straight lines are appropriate summaries of the longitudinal information in the data. However, the dramatic jump in SSRES between the ninety-fifth centile and the maximum value of SSRES indicates that there are some individuals for whom the straight-line model is very inappropriate. Consequently, students with the highest values of SSRES were eliminated from subsequent analyses (e.g., estimation) where serious lack of fit would jeopardize the results. (Approximately three percent of the total number of students were deleted. Three percent was chosen arbitrarily, based on previous experience For these data, deleting the highest three percent of SSRES's seemed to produce satisfactory estimation while retaining as many of the students as possible.)

Table 3 summarizes the estimated rates of change in the cohort for each measure. The median rate of growth ranges from 32 to 35 scale score points per unit change in time. Except for the upper extremes, the distributions of growth rates are roughly comparable on the three measures. However, there is a marked difference at the ninety-fifth centile between mathematics on the one hand, and reading and language on the other. The fastest rate of growth in mathematics is 86.6, while for language the fastest is 116.8 and for reading, it is 127 (They are not achieved by the same individuals, however.)

Table 4 shows maximum likelihood estimates of certain properties of the collections of growth curves. Note that the estimated values of t° (RTSS 8.975; LTSS 9.272, MTSS. 7.933) are approximately less than or equal to nine, as suggested by the standard deviations in Table 8. The estimated reliabilities of $\hat{\theta}$ range from .541 (RTSS) to .636 (LTSS)

Analyses of Individual Students

Fiv. individuals are selected for examination in this section. A brief summary is given on each individual and an interpretation of performance is offered.

In the following discussion, reading, language and mathematics will be regarded as measures 1, 2 and 3, respectively. The occasions of measurement correspond to the autumn of each high school grade. So t_1=9, t_2=10, t_3=11 and t_4=12 When a vector or matrix is written, the dimensions are ordered according to the indices *m* and *i*. Thus the scores will always appear ordered as Reading Total, Language Total and Mathematics Total, and the times are ordered 9, 10, 11 and 12.

Student #08114754. This student shows an interesting pattern of performance, illustrated well by the fitted growth curves in Figure 2. Her achievement level and rate of growth are very similar for RTSS and LTSS. She starts with moderate to low scores and grows rapidly. Her achievement level is much higher in MTSS, but her rate of growth is less than half as fast as on the other two tests. The profile of progress, $\hat{\theta}_{p\cdot}$=(80.5 82.3 34.8)′. The first two components place this student in the fourth quartile of the empirical distributions of reading and language growth rates The last component lies in the third quartile among mathematics growth rates. The average level of achievement is $\xi_{p}.(10.5)=(645.3\ \ 646.2\ \ 690.5)'$ All of these scores are in the third quartile of their respective empirical distributions

Turning now to the question of relative strengths and weaknesses, critical ratios were calculated for each component of the profiles of fitted scores and a family-wise error rate of .05 was ensured for each profile of achievement by the use of the Bonferroni inequality. The pattern of strengths and weaknesses is represented below by a 3 x 4 matrix of letters. "S" stands for "Strength"; "W" stands for "Weakness"; and, "0" stands for "Neither Strength nor Weakness". The first row represents Measure 1 over the four occasions and so forth

Her pattern of strengths and weaknesses in achievement is

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ S & S & 0 & 0 \end{bmatrix}.$$

A weakness was identified for MTSS in the profile of progress. Thus this student starts out with relative strengths in MTSS achievement relative to her achievement on RTSS and LTSS. However her rate of growth on MTSS is significantly slower, so by the last two occasions her achievement level is comparable on all three tests.

Student #08137927. This student provides a more extreme exampl of the same basic pattern of performance observed with Student #08114754. That is, his achievement level and rate of growth is comparable on RTSS and LTSS (i.e., moderate level, fast growth), his performance on MTSS consists of very high achievement and a low growth rate. In fact, his *raw* scores (96, 98, 97, 98 at grades 9, 10, 11, and 12, respectively) show that this student is topping out on the Mathematics Total test. (The maximum raw score is 98.) As might be expected, the R^2 for MTSS is relatively low at .44 because the fitted growth curve is nearly flat. There is no growth on mathematics because of the nature of the test. Growth on the other two tests is respectable. This interpretation is illustrated by the fitted growth curves in Figure 3. The interpretation is also supported by the profile of progress and the average profile of achievement $\hat{\theta}_{p\cdot}=(71.4\ \ 53.7\ \ 8.9)'$; $\hat{\xi}_{p\cdot}(10.5)=(550.5\ \ 542.7\ \ 842.2)'$.

The pattern of intraindividual strentghs and weaknesses in achievement level is

$$\begin{bmatrix} W & W & W & O \\ W & W & W & W \\ S & S & S & S \end{bmatrix}.$$

Note that there is so much discrepancy between the achievement level in mathematics and the achievement level in the other two areas, that nearly all of the RTSS and LTSS results are labeled as weaknesses All of the mathematics scores represent strengths in achievement level. The rate of growth in MTSS is identified as a weakness.

However, this is clearly an artifact of the ceiling effect of the mathematics test

Student #08129311. This student has a substantially different pattern of performance from those seen thus far The observed scale scores are

$$X_{p\cdot\cdot} = \begin{bmatrix} 474 & 383 & 474 & 383 \\ 265 & 346 & 386 & 405 \\ 421 & 526 & 536 & 593 \end{bmatrix}$$

and the profile of progress is $\hat{\theta}_{p\cdot}=(-18.2 \;\; 46.0 \;\; 52.6)'$. The average achievement level is in the first quartile on all three tests. Relative to other individuals, the rates of growth are in the first, third and fourth quartiles respectively. Figure 4 shows the fitted growth curves. The R^2 for the fit to RTSS is .20, because the scale scores alternate about a nearly flat line.

The growth rate on RTSS appears negative, and is identified as a weakness In the figure, the fitted line for RTSS actually crosses both the other growth curves and reflects the fact that the RTSS is lowest at t=12, even though it was highest at t=9. Growth on LTSS occurs at nearly the same rate as growth on MTSS. However, language achievement is at a considerably lower level. The pattern of strengths and weaknesses in achievement is

$$\begin{bmatrix} S & O & O & W \\ W & W & W & W \\ O & S & S & S \end{bmatrix}.$$

The pattern accurately reflects the fluctuation in the scores and shows that this student's performance is generally somewhat weak except on MTSS This pattern of performance suggests a person who is weak in verbal and language skills, but is able to make good progress in mathematics With appropriate background information about this student, a hypothesis might be generated to account for the performance. For example, this individual might be a non-English speaking student who has trouble with communication skills, but who has no trouble with the computational skills represented in the mathematics test.

Student #71074927 This student exemplifies the case where consistent strengths and weaknesses are maintained over time, i.e., the growth curves are roughly parallel and separated by considerable differences in level. Figure 5 shows the plotted straight-line time paths. The average level of achievement is moderate to high, relative to other students. $\hat{\xi}_{p\cdot}(10.5)=(614.0\ \ 659.5\ \ 769.3)'$ and the scores are in the third, third and fourth quartiles, respectively. The growth rates are relatively low compared to other students except on MTSS. $\hat{\theta}_{p\cdot}=(15.0\ \ 20.0\ \ 57.5)'$. The first two rates are in the first quartile, the last is in the fourth quartile.

Intraindividual strengths and weaknesses in achievement are

$$\begin{bmatrix} 0 & W & W & W \\ 0 & 0 & 0 & 0 \\ 0 & S & S & S \end{bmatrix}.$$

A strength is identified in the profile of progress for MTSS. Unlike most of the other cases that have been presented, in this case, the student's intraindividual strengths and weaknesses do not change very much over time.

Group Analyses

Summaries can be computed for the total group of students. Generally, means or medians of individual quantities are informative. For example the average rates of change are 38.9, 36.0 and 33.8 on RTSS, LTSS, and MTSS, respectively. The elevation of the group mean profile of progress is thus 36.2. The median values of R^2 are .828, .835, .835 for RTSS, LTSS, and MTSS, respectively, indicating that the straight-line model is reasonably good for most students. Subgroup statistics could be included for any subgroups of interest where relevant grouping information has been collected.

Even more useful are distributional summaries of interesting quantities. For example, a five number summary of the elevations of the profiles of progress in the group can be computed. The lower extreme, fourth, median, upper fourth and upper extreme are -14.6, 24.4, 35 0, 46 1, and 104.0, respectively. The median of the elevations (35.0) is relatively close to the elevation of the group profile of rates (36.2), which is to be expected because of the normality of the estimated rates of change

The observed and disattenuated correlations among the estimated rates of change are shown in Table 5. The observed correlations are computed using the full data (i.e., with only pairwise deletion of cases for which one or both rates are missing). The disattenuated correlations use the reliabilities from the "trimmed" analysis (i.e., cases with high SSRES were deleted before the reliability estimates were calculated). Still, one of the disattenuated correlations exceeds one

The relationships between the rates are as follows. There is a very high positive correlation between Reading Total and Language Total. There is also a high correlation between Language Total and Mathematics Total. There is a moderate correlation between Reading Total and Mathematics Total.

Finally, it is interesting to summarize the individual strengths and weaknesses in the group. Table 6 shows the number of strengths and weaknesses in achievement that were identified on each occasion for each measure. The number of strengths and weakesses in rate of change is also shown. The are [illegible] students for whom strengths and weaknesses are calculated. This represents the number of students in the group who have complete achievement profiles on all four occasions.

Few strengths and weaknesses in rate of change are identified. However, quite a few strengths and weaknesses in achievement level are identified. Note that most of the strengths occur in Mathematics Total scores. There are very few weaknesses in mathematics Conversely, most of the weaknesses occur in Language Total scores. There are very few strengths in language. Such a summary seems very informative about group performance. It shows that about half of the students in the school have relative strengths and weaknesses in achievement at any given time. This means that their performance across measures is disparate. They are not showing comparable levels of achievement in all three subjects If one of the goals of the educational program is to assure each student attains comparable achievement in all curricular areas then these results suggest that the goal was perhaps not met with this cohort.

Summary and Conclusions

These examples show that it is possible to provide individualized interpretations of multivariate longitudinal growth in achievement for groups of students. The particularly significant aspect of these results is that ipsative interpretations can be provided just as easily as the more usual normative interpretations.

Any investigation of individual performance on multiple measures should include analyses of strengths and weaknesses, such as the ones discussed here. Patterns of strengths and weaknesses over time, supplemented with the profile of progress and plots of the fitted growth curves, open a rich new avenue for understanding student achievement and growth in achievement. This type of information has never before been used in reporting student achievement test results. It is possible only when test results are managed appropriately and consolidated in a way that takes advantage of the multivariate, longitudinal nature of the data.

The implementation and reporting of academic growth on multiple measures is a new and evolving endeavor. While the reporting of academic achievement *level* on multiple measures is commonplace, the additional data management necessary for exploring *growth* is rarely if ever done in practice. There are several reasons: the paucity of theoretical models; the lack of technical skills and knowledge; lack of available computer facilities; and, perhaps, insufficient numbers of committed administrators. The situation is beginning to improve with recent work in the measurement of change and advances in technology. When the current work was begun, the analyses were only feasible on large mainframe computers. However, recently, with the advent of new computer software and hardware, the techniques employed in this paper are possible with a microcomputer.

Table 1

Means and Standard Deviations of CTBS Scale Scores

	Score					
	RTSS		LTSS		MTSS	
Grade	Mean (N=228)	SD	Mean (N=198)	SD	Mean (N=210)	SD
9	539.1	91.8	533.5	96.5	597.0	78.6
10	583.2	93.1	573.4	93.5	635.7	84.7
11	616.0	101.7	606.9	99.3	670.9	96.9
12	658.0	111.8	642.5	112.1	697.7	100.4

Table 2

Summaries[a] of R^2 and SSRES for High School Data

R^2

RTSS (#228)

M	828	
F	622	928
5%	.064	.992
1	000	998

LTSS (#198)

M	.835	
F	.572	930
5%	.155	.991
1	.007	.999

MTSS (#210)

M	.835	
F	.609	.936
5%	.053	.988
1	.000	999

SSRES

RTSS (#228)

M	1291	
F	539	3607
5%	63	15,737
1	18	31,262

LTSS (#198)

M	1106	
F	344	2799
5%	59	8395
1	6	37,122

MTSS (#210)

M	1106	
F	382	2452
5%	47	8549
1	6	19,414

[a]M=median; F=fourths; 5%=5th and 95th quantiles; 1=extremes

Table 3

Summary[a] of $\grave{\theta}$ for High School Data

$\hat{\theta}$

RTSS

#228		
M	35.4	
F	21.3	51.6
5%	3.6	85.8
1	-27.0	127.0

LTSS

#198		
M	32.1	
F	20.3	47.5
5%	6.6	82.4
1	-14.2	116.8

MTSS

#210		
M	33.9	
F	18.6	48.6
5%	0.8	69.6
1	-24.6	86.6

[a]M=median, F=fourths; 5%=5th and 95th quantiles, 1=extremes

Table 4

Maximum Likelihood Estimates of Properties of the Collections of Growth Curves. Reading Total, Language Total and Mathematics Total

Parameter	Estimate[a]		
	RTSS	LTSS	MTSS
t°	8.975	9.272	7.933
$VAR_p(\theta_{pm})$	306.919	325.636	246.835
$VAR_p(\zeta_{pm}(t^{\circ}))$	7345.319	8061.288	5328.842
$\rho(\hat{\theta})$	.541	.636	589
$SEM(\hat{\theta})$	16.123	13.646	13 133

[a].03 of cases deleted due to high SSRES

Table 5

Observed and Disattenuated Correlations Among Estimated Rates

High School Data

Measure	RTSS	LTSS	MTSS
		Observed Correlations	
RTSS	1	.598	.391
		(n=197)	(n=195)
LTSS		1	.498
			(n=194)
MTSS			1
		Disattenuated Correlations[a]	
RTSS	1	1.000[b]	.693
LTSS		1	.814
MTSS			1

[a] Reliabilities for disattenuated correlations based on n=193

[b] Algorithm produced impossible value, 1.019.

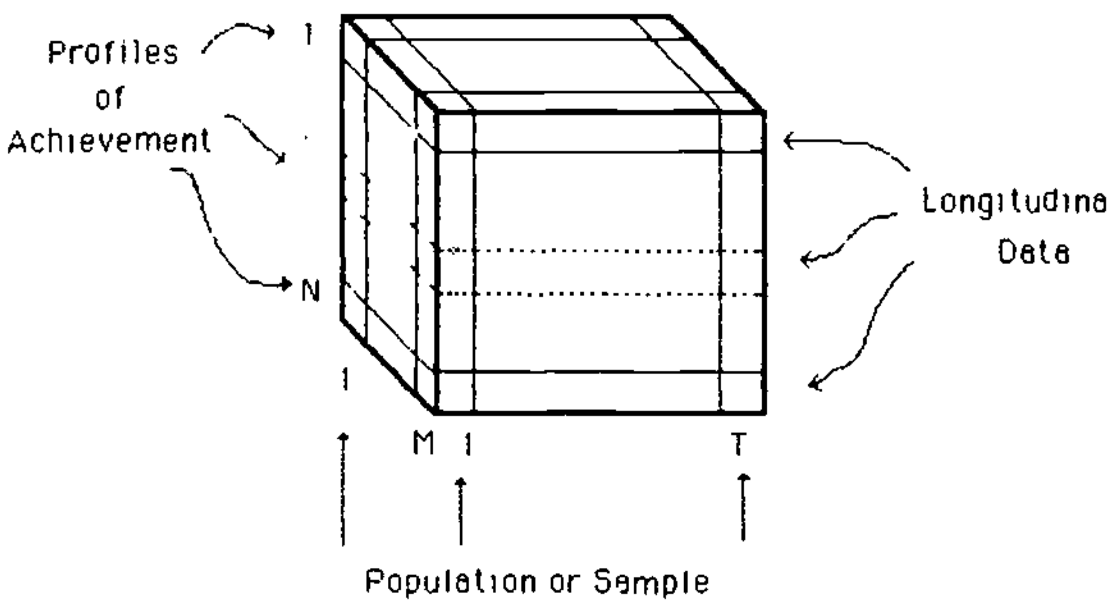
Table 6

Number of Strengths and Weaknesses in Achievement Level and Rate of Change

	Achievement Level in Grade				
Measure	8	9	10	11	12
			Strengths		
RTSS	9	7	16	18	10
LTSS	0	3	8	8	1
MTSS	6	80	104	101	72
			Weaknesses		
RTSS	2	38	66	55	29
LTSS	5	51	81	81	47
MTSS	10	8	14	12	11

Figure 1

Slices of the Data Cube That Correspond to Profiles of Achievement



N=number of individuals
M=number of measures
T=number of occasions

Figure 2

Fitted Growth Curves for Individual 08114754

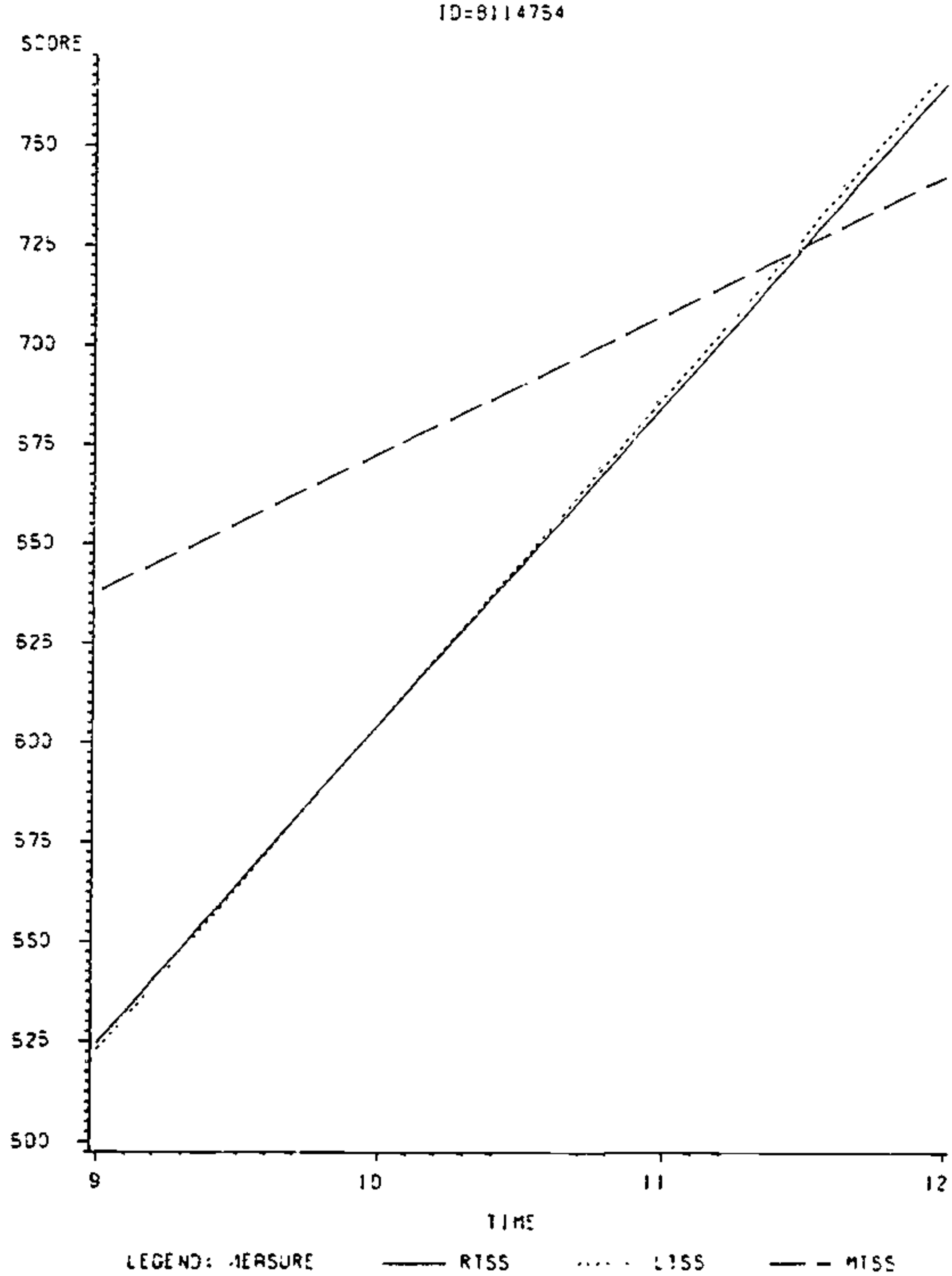


Figure 3

Fitted Growth Curves for Individual 08137927

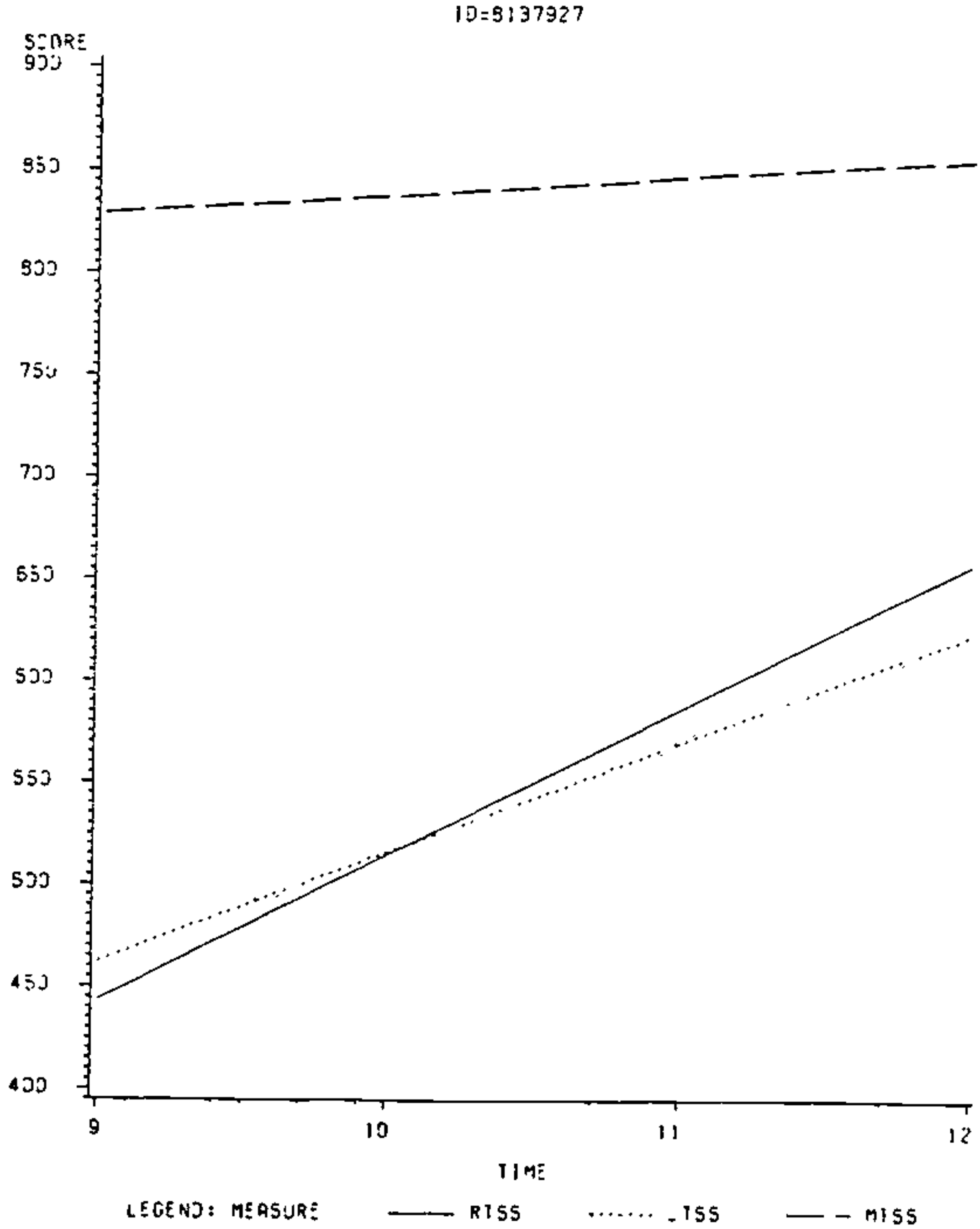


Figure 4

Fitted Growth Curves for Individual 08129311

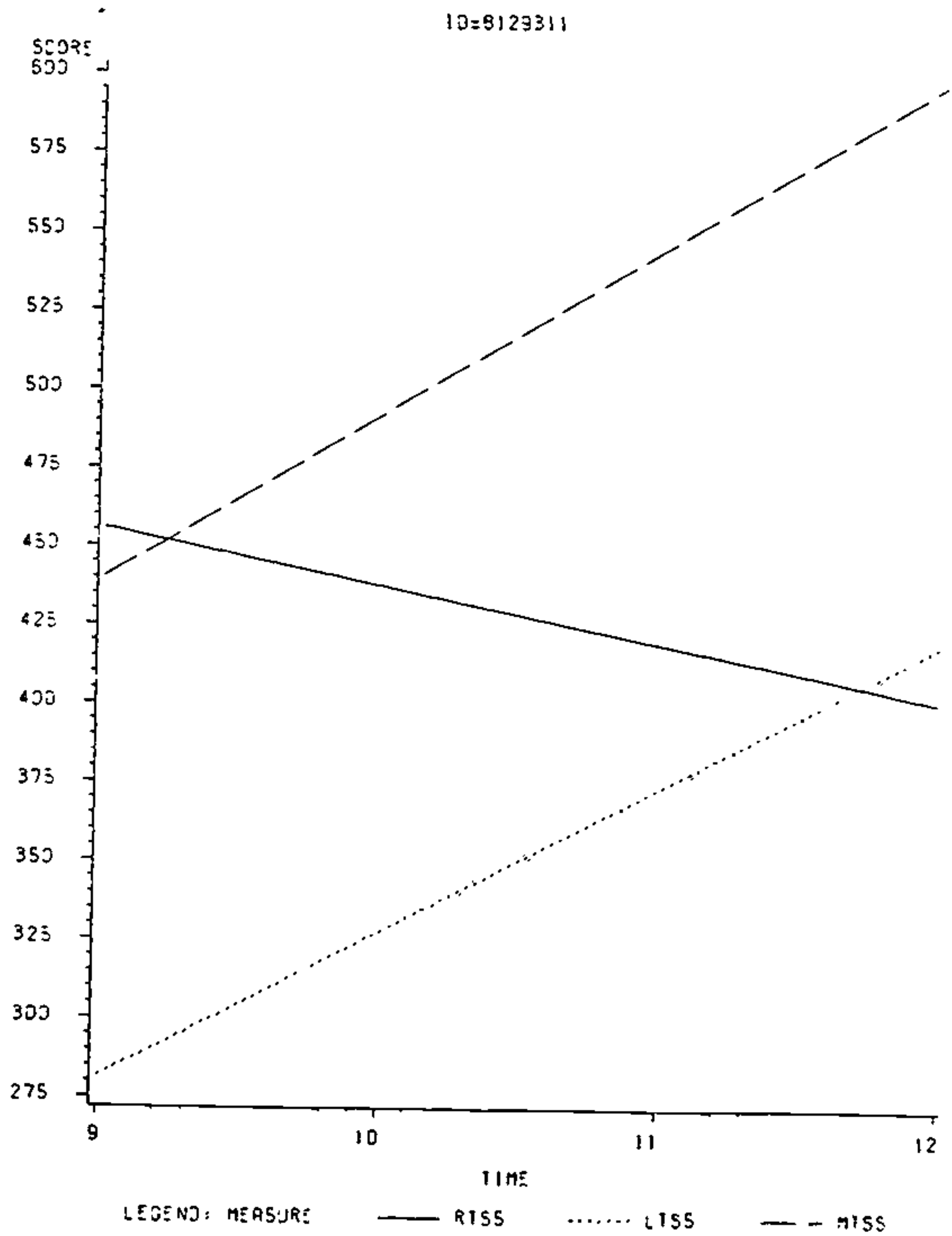
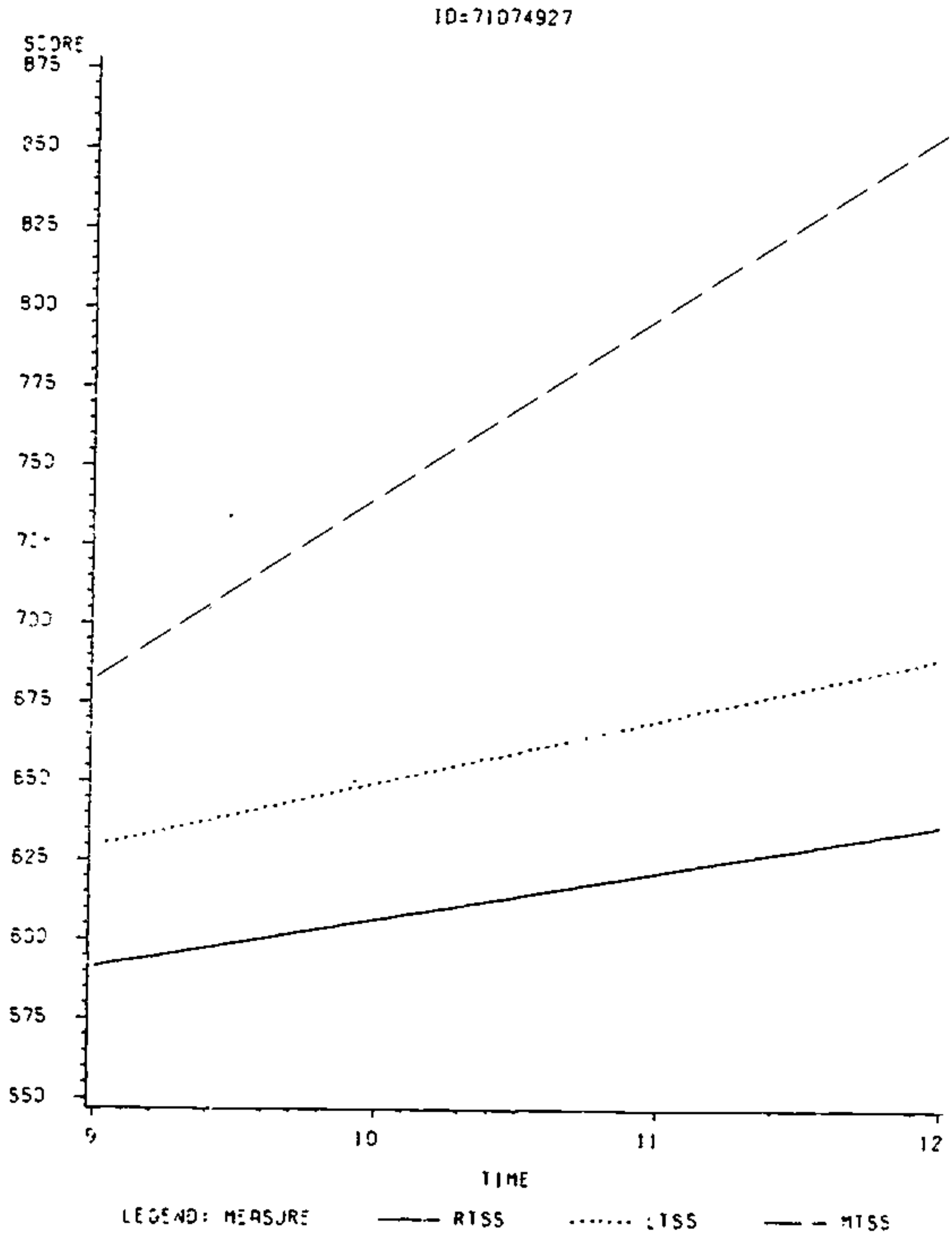


Figure 5

Fitted Growth Curves for Individual 71074927



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