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ABSTRACT

This issue contains abstracts and critical comments for 10 papers. The reports are concerned with: (1) children's inferencing behavior; (2) instruction related to problem-solving and basic skills for seventh grade students; (3) remediation of children's subtraction errors; (4) investigation of young children's academic arithmetic contexts; (5) spatial training for calculus students; (6) factors and features affecting the use of mathematical strategies; (7) a linguistic approach to learning numbers; (8) prediction of achievement and attitudes in mathematics and reading; (9) ordering decimals and fractions; and (10) achievement, understanding, and transfer in a learning hierarchy. Research references from "Current Index to Journals in Education" (CIJE) and "Resources in Education" (RIE) for January through March, 1987 are also listed. (RH)

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Behr, Merlyn J. and Khoury, Helen Adi. CHILDREN'S INFERENCING BEHAVIOR. Journal for Research in Mathematics Education 17: 369-381; November 1986.

Abstract and comments prepared for I.M.E. by LARS C. JANSSON, University of Manitoba.

1. Purpose

This study investigated third- and fifth-grade children's inferencing behavior--the process of generating, testing, and generalizing hypotheses to discover a general rule and a relationship between given sets of data. Inferencing behavior also includes demonstrating that a rule was discovered.

2. Rationale

Inferencing behavior has been investigated by means of the concept-attainment paradigm in which the student identifies a positive or negative instance of a given rule, and by means of the information processing paradigm which has focused on the ability to generate a rule description from a set of positive instances and to use this rule to extrapolate to find additional instances.

The present study explored the steps in this sequence: (a) the ability to discover a mathematical rule, (b) hypothesis-testing performance, and (c) generalization ability. Hypothesis-testing performance was defined in terms of children's willingness to change their hypothesized rule in the light of contrary evidence or to maintain it in the presence of confirming evidence.

Specifically the study set out to answer four questions:

1. Do children discover correct mathematical rules?
2. What is children's hypothesis-testing ability?

3. What is children's hypothesis-generalization ability?
4. What types of correct hypothesized rules do children generate?

3. Research Design and Procedures

Twenty-five children in each of grades 3 and 5 were randomly assigned in equal numbers to five experimental groups. Each group was given five tasks: one for each of five relation rules ($y = 2x$, $y = x + 1$, $y = 3x - 2$, $y = x^2$, and $y < 2x$) in one of five presentation modes. A Latin square design was employed. Presentation modes were: a) graphic/graphic, b) symbolic/symbolic, c) pictorial/symbolic, d) symbolic/graphic, and e) pictorial/pictorial, where the first element is the mode of presentation for the abscissa, the second is the presentation mode for the ordinate.

The tasks were randomly sequenced for each child and presented in a one-to-one interview that was audiotaped. For each function rule, two positive instances were presented and the abscissa of a third instance was given. The children were asked to write or draw the ordinate for the third instance. Feedback was provided, and the correct ordinate was given in case of an error. A fourth ordered pair of the function was given, plus the abscissa of a fifth instance. The procedure was repeated for a sixth and seventh instance and then the children were asked to describe verbally the function rule and explain how they obtained it. A scoring system was developed for each of the three phases: Rule-Discovery, Hypothesis-Testing, and Hypothesis-Generalization.

Results were analyzed using a Grade X Mode X Function ANOVA on each set of scores. One relation ($y < 2x$) was omitted from the analysis because it "differed from the other four in not having one or two abscissas for a given ordinate. It thereby presented the children with a task that was very different from the others."

All subjects were also administered Shipley's Series Completion Test as a measure of sequential reasoning and Raven's Progressive Matrices Test as a measure of intellectual ability.

4. Findings

The results and findings are organized by the research questions:

1. Grade and function showed significant main effects for the rule-discovery scores, and a significant interaction effect was found for Function X Mode.

2. Similar ANOVA findings occurred for the hypothesis testing scores. Scores for the three linear functions were significantly correlated, but none of these were correlated significantly with the function $y = x^2$.

3. Similar ANOVA results were found for the hypothesis-generalization scores.

4. Not only were the children able to discover the mathematical rules, they were also able to describe them. (A short list of these descriptions is provided).

Students in both grades successfully generated hypotheses, although the fifth graders were better at generating correct rules. Furthermore, no evidence was found the children were unwilling to relinquish their hypotheses in the presence of contradictory evidence. They tended to view their rules as their "own" and would willingly replace them with another of their own.

The hypothesis testing score correlated significantly and negatively with all other measures, including Shipley's and Raven's tests. All other intercorrelations were positive and significant.

Abstractor's Comments

1. The research appears to have been carefully carried out with clear task definitions and a well-defined interview protocol.

2. The questions asked in this investigation are clearly relevant ones, as is the sequence in which they are dealt with. However, the report does not say very much about the performance of individual students through the sequence. Only the intercorrelation matrix of scores gives any hint of relationships among the various scores (and this only for the two grades combined).

3. The interactions between mode and function rule were evident throughout. I am not entirely surprised by this, and it confirms earlier findings by the researchers. They tentatively suggest some function characteristics which may relate to performance in particular modes. It is obvious that more research would be needed to further develop these relationships. Unless such relationships were simple, I am unclear as to their value to the teacher, although perhaps the information would be useful to curriculum writers.

4. The authors suggest that elementary teachers introduce the graphic mode of presentation as early as they do the symbolic mode. While this may be a good suggestion for many reasons, it does not follow strongly from the research (it appears to be based on the results for one of the function rules).

5. A Latin square design was reported to be used, although the design really may be 5 x 4 since one of the original function rules was dropped from the analysis. It was not

clear to this reader how the ANOVAs were carried out. If the original square was used and the one set of data not reported, then degrees of freedom for function should be 4 (not 3 as reported). If the design was truly not square, what was done?

6. The sequence of presentation to students was random (5 instances), perhaps appropriate in this context, although the numbers of all elements are small. However, order of presentation could be an important variable, particular for function rule, and maybe even for presentation mode. This should be investigated further.

7. The descriptions of the rules given by the students was one of the most interesting aspects of the study to this reviewer. The authors do state that in some situations students focused on the relation of the sequence of y values rather than on the relationship between ordinate and abscissa. It would be useful to see more of this kind of description and more commentary on it. This shows how the children are thinking.

Burns, Robert B. and Lash, Andrea A. A COMPARISON OF ACTIVITY STRUCTURES DURING BASIC SKILLS AND PROBLEM-SOLVING INSTRUCTION IN SEVENTH-GRADE MATHEMATICS. American Educational Research Journal 23: 393-414; Fall 1986.

Abstract and comments prepared for I.M.E. by DALE BURNETT, University of Lethbridge, Lethbridge, Alberta.

1. Purpose

The abstract states, "The primary purpose of this study was to describe problem-solving instruction and examine whether it was different from basic skills instruction."

2. Rationale

Educational research may be approached from many viewpoints. For example, a study may focus on human factors such as the learner, the teacher, the administrator, or the community, either singly or in combination. Alternatively, a study may be oriented around more abstract considerations such as the curriculum. Subjects may be viewed in isolation or in groups, and the setting may be some variation of a laboratory setting or it may include more naturalistic classroom situations. Issues such as generalizability and external validity are often a major consideration in determining which research procedures will be followed, particularly when the researcher has an education interest.

The authors of this article thus point out, "A surprising amount of the research on problem solving has been conducted outside of or tangential to the field of research on teaching." It is noted that much of the problem-solving research to date has focused on the student, and "has ignored the teacher and the difficulties and concerns teachers face in implementing problem-solving instruction." This study attempts to redress this imbalance.

The decision to examine problem solving as experienced by the teacher is further supported by the contention that teachers are continually being asked to incorporate more problem solving in their activities. In addition to psychometric concerns such as validity, timeliness and relevance are also invoked in determining the parameters of the study to be conducted.

3. Research Design and Procedures

The basic sample consists of 9 grade seven mathematics teachers (and their students).

The research design contained three components: Phase I, where the teachers were observed for 5 consecutive days while they provided their "basic skills curriculum"; an intervention consisting of a 3-hour workshop for the teachers on problem solving where they were "introduced to four problem-solving skills," followed by an elapsed time of 1 month which could be used to plan a unit on problem solving where they would teach those four problem-solving skills to their students; and Phase II, where the same teachers were observed for 6 consectuve days while giving their "problem-solving instruction."

The actual data used in the analysis came from four sources: (1) narrative records of each lesson: these were obtained by combining field notes taken during the lesson with audiotapes of the lesson. "This procedure resulted in a very detailed description of verbal and behavioral action that occurred during the lesson and the time that it occurred," (2) two observer instruments, completed following each lesson, (3) one teacher-rating instrument, answered by each teacher at the end of each lesson, and (4) student work that was completed during the class.

The two observer instruments were based on the concept of lesson segment, analysis that typically lasts minutes rather than seconds

A total of seven segment purposes were identified: development, directions, practice, review, enrichment, testing, and other. It was not clear from reading the article where these categories came from (i.e., from related literature or empirically, by looking at the data), although it appeared that two unpublished reports by the authors would clarify this.

4. Findings

The main summarization for all of this data collection is in a 7x9x2 table containing the minutes and percentages of time spent on each of the seven types of segment purposes for each of the nine teachers in both instructional settings ("Regular" and "Problem-solving"). The three major purposes (development, practice and review) were then highlighted with a graphic representation of the same data using histograms. The resulting graphs clearly show noticable differences among the teachers for each of the two instructional processes as well as some differences within each teacher across the two lesson formats. The next task was to tease out of these visual differences some form of statistical test to verify that the apparent differences are statistically significant.

The authors take the data for the three major purposes, compute a difference score (time spent on purpose during problem solving minus time spent on purpose during regular instruction) for each purpose and then proceed to compute a t-statistic for the differences between the two lesson types. The difference between types of problem solving is significant at the .05 level for development time but not for practice or review.

Another comprehensive 2x9x8 table shows the final tallies for each teacher in each setting by four variables (number of lessons, number of segments, number of assignments, and number of problems (further classified into computation, skill, routine, non-routine, and other)). Once again, noticable differences occur both among teachers and between type of instruction, but there is no indication of any statistical analysis.

Finally, mean ratings by the observer using the Lesson Rating Instrument were summarized in a table for each of the nine items of the instrument. The mean values for each item for the two processes were compared using a paired t-test. Three of the nine items were significant at the .05 level (if you accept a p-level of .053 as being less than .05). Only one item out of five from the teacher rating scale was significant at the .05 level (if you accept a p-level of .058 as being greater than .05).

5. Interpretations

The main conclusion reached by the authors is that "The teachers in this study organized instruction during the problem-solving unit in much the same way as they did their regular curriculum." Thus, "the teachers in this study typically reviewed the previous day's assignment, showed students how to do problems on today's assignment, and then gave them the opportunity to practice more problems just like the ones they were shown how to do."

The authors then proceed to ask "why the teachers approached the problem-solving unit in the way that they did." The authors appear to recognize that they have no actual data on this question, and after noting two articles by Olson on the teachers' perspective, they state "this study argues for more research on the opportunities and constraints that shape teacher behavior in classrooms."

Abstractor's Comments

Was the study worth doing?

Yes. The two main features of this study, problem solving and classroom instruction, are both timely topics in educational research.

The question of worth comes from a saying that "a job not worth doing is not worth doing well." Accepting the value-laden nature of the question, I believe that research into the ways that teachers approach problem-solving lessons is appropriate at this time.

Was the study well carried out?

Yes and no. We now have the converse of the earlier saying: "a job worth doing is worth doing well." The use of lesson segments and of four different sources of data are excellent ways to quadrangulate onto the issues of interest. But the published report leaves out much of the detail of both the nature of the problem solving activities as well as what actually transpired in the classroom. Perhaps this was necessary for publication but it raises questions about what we gain and what we lose with our current editorial policies. Perhaps another report that provides more of the detail that was apparently collected should also be available to interested readers. Reflecting on what I have just written, I notice that we would then have: the detailed report, the abbreviated report (published), and the review of the abbreviated report. In a twist of normal priorities, educational research seems to value the Reader's Digest version over the original novel.

Do the conclusions follow from the results?

Yes. I think. One of the problems I had was trying (unsuccessfully, I admit) to get a sense of whether the particular perspective taken (that of lesson segments and nine particular purpose categories) forced the data to be viewed in a particular manner that prevented one from noting other meaningful differences. The authors also appear to recognize the potential problem: "...we have presented data only on the organization of instruction for basic and problem-solving skills. We have not presented data on the actual teaching techniques that were used during instructional segments.

Since teaching techniques are closely tied to the actual content of instruction..., it is possible that there were differences that we did not detect."

Suppose that one were to take the seven purp· · categories of development, directions, practice, review, enrichment, testing, and other and use these categories to examine a variety of classroom situations. Under what conditions would one expect to find significant differences among the percentages of time spent on each? In other words, are the procedures adopted sufficiently sensitive to notice differences that actually exist? I am not sure.

Rather than imagine possible next steps as a metaphor of linear progress, I prefer to think in terms of iterations and of how the same study could be redone with various modifications.

(1) Are there related questions that are worth asking?

One suggestion would be to collect data on "why the teachers do what they do." Another would be to obtain more information on the students' perceptions.

(2) Are there variations in the methodology that could be considered?

The study collapsed all data into total frequencies over the observation period (5 or 6 classes). There was no attempt to provide any form of temporal analysis. One should ask oneself how close such a set of frequencies comes to capturing the important features of the event under scrutiny. The study had a number of excellent features of a time-series design. Yet the data analysis ignored this feature.

The falling back to a few independent t-tests failed to do justice to the data. More sophisticated approaches such as ANOVA or regression analysis should also be tried.

The article failed to provide any real descriptive information of what occurred. Problem solving admits to many interpretations. Without more detail it is difficult to appreciate what is actually being tested.

I believe that many educational research questions that are worth asking are indeed difficult to answer. Nonetheless, I believe that we should not shy away from them, but that we should attempt to make progress on them, both procedurely as well as substantively. I applaud the authors for making a beginning in this area and hope that others will follow their lead.

Cebulski, Larry A. and Bucher, Bradley. IDENTIFICATION AND REMEDIATION OF CHILDREN'S SUBTRACTION ERRORS: A COMPARISON OF PRACTICAL APPROACHES. Journal of School Psychology 24: 163-180; Summer 1986.

Abstract and comments prepared for I.M.E. by A. EDWARD UPRICHARD, University of South Florida.

1. Purposes

Three studies were conducted to determine the errors committed by children on subtraction problems requiring borrowing and to examine the effectiveness of four remedial techniques that varied in their sophistication and ease of presentation. Study I was designed to determine which type of subtraction error occurs most frequently on problems that require borrowing. The purpose of Study II was to examine the efficacy of instructions to borrow and the promise of reward as remedial approaches. In Study III, the effectiveness of two remedial techniques (Component Skills Training versus Criterion Training) were examined.

2. Research Design, Procedures, and Findings

STUDY I

The sample for Study I was comprised of 56 third-grade children (28 male and 28 female) selected from two elementary schools in lower middle-class areas. All children were expected to have learned subtraction facts to 18 and had been given experience with subtraction problems involving borrowing. Each child was tested individually, one item at a time, using tw forms (A and B) of a 20-item subtraction test. Forms A and B contained the same 20 subtraction problems arranged in a different random order. For initial testing, half the children were randomly assigned Form A and the other half, Form B. Following a three- to four-week interval, all children were retested with the alternate form. All substraction problems used consisted of three-digit minuends and three-digit subtrahends and required borrowing from both the tens and hundreds columns.

To categorize errors, verbal reports were compared with the child's actual written performance. The error categories used were: (1) counting error, (2) number fact error, (3) inversion error, (4) borrowing error, and (5) other. The dependent measure for each type of error was the number of errors divided by the number of opportunities for the error to occur. Inter-observer agreement for each type of error was established to be .938 or greater.

Of the 56 children tested, 11 failed to solve a single subtraction correctly; 27 children made more than one error, but solved at least one problem correctly. The data for children who solved no problems correctly (Group 1) and data for those who solved at least one problem correctly (Group 2) were analyzed separately. For Group 1, inversion errors occurred in over 90% of all opportunities and the proportion of inversion errors was significantly greater than that of all other error types combined ($p < .001$). For Group 2, more non-conversion errors were committed than inversion errors ($p < .001$). Counting was the most frequent source of error for Group 2 children. While there was no significant difference between the proportion of counting and number fact errors, there was a significant difference between the proportion of counting errors and each of the other type of errors.

Subjects were retested after a period of three to four weeks in order to study changes in subtraction performance. The probability that a child initially categorized as Group 1 would remain so after a one-month interval was .91. The probability that a Group 2 child would remain so classified after one month was .59.

STUDY II

Study II was designed to examine the efficacy of instructions to borrow and the promise of reward as remedial approaches for Group 1 (no problems correct) and Group 2 type children. The error categories used in Study I were modified so that direct observation could be eliminated. The sample consisted of 80 (30 males and 50 females) third-grade children who were expected by teachers to have learned their subtraction facts to 18 as well as how to solve the type of subtraction problems used in this study.

Study II used a pretest-posttest design (3 x 2 factorial). The subtraction problems used were of two types. The first type required borrowing from the tens column and the second type required borrowing from the hundreds column (five of each type per test). On the basis of the pretest, children were classified as either Group 1 (n = 31*) or Group 2 (n = 49) and, within groups, randomly assigned to either a motivation, instructions, or control condition. Children were tested on both the pretest (Form A) and posttest (Form B) in groups of four to six. The experimenter presented the pretest, along with instructions to solve problems. Following the pretest, children were given the alternate test form and the instructions for the selected experimental conditions. The error categories used in Study II were the same as those used in Study I, with the exception that number fact and counting errors were combined to form a new category called computational errors.

As in Study I, Group 1 children committed a greater proportion of inversion errors relative to total errors than Group 2 ($p < .003$), and Group 2 children made a greater proportion of computational errors relative to total errors than Group 1 ($p < .003$). The proportion of borrowing errors relative to total errors was not significant for either of the groups. There was no treatment effect relative to the number of problems solved correctly for either group when compared to scores of controls.

An ANCOVA followed by Bonferroni t comparisons indicated that Group 1 children assigned to the instructions condition committed

*The text of the article reports that 32 children were assigned to Group 1 (p. 169). The 32 is a misprint.

fewer inversion errors than Group 1 children assigned to the control condition ($p < .05$) and to the motivation condition ($p < .05$) Inversion errors committed by Group 2 children were not affected by experimental conditions. A one-way ANCOVA followed by Bonferroni t comparisons indicated that Group 1 children assigned to the instructions condition made more borrowing errors than those subjects assigned to the control conditions ($p < .005$). No effects of either experimental condition relative to controls were obtained for computation or other errors.

STUDY III

The purpose of Study III was to compare the effectiveness of two remedial techniques: Component Skills Training (CST) versus Criterion Training (CRT). CST used a step-by-step presentation of borrowing procedures, and the CRT presented children with 100 (10 groups of 10) subtraction problems, most requiring borrowing, followed by performance feedback in the form of correctly worked solutions. Both treatments were administered using a booklet format over a three-day period. The sample comprised those children who solved less than 80% of the problems correctly on the pretest. Of the 64 children selected, 32 were assigned to a satisfactory group (60-80% of the problems correct on pretest) and 32 to an unsatisfactory group (less than 60% of the problems correct on the pretest). Within each group, children were assigned to one of three conditions: CST, CRT, and control. A pretest and two posttests, each containing 24 different problems, were administered to all children. Children received Posttest 1 two to three days following the remedial phase of the study and Posttest 2, two weeks later. The error categories used in Study III were the same as those used in Study II. Since the remedial programs were designed to improve borrowing procedures, only those problems requiring borrowing were used in the data analysis.

For the unsatisfactory group at Posttest 1, those children assigned to CST and CRT conditions solved more borrowing problems correctly than did controls ($p = .02$ and $p = .049$, respectively).

There were no significant differences between treatment conditions and controls at Posttest 2. Also, no significant effects of treatment condition were found at Posttest 1 or Posttest 2 for children assigned to the satisfactory group. Neither CST nor CRT resulted in significant reductions of any type of error relative to controls for either the satisfactory or unsatisfactory group.

Since some children did not benefit from training, the investigators explored children's performances throughout training. The last ten problems administered to children in the CRT program were used as an index of successful program completion. A median split based on the number of these problems solved correctly was used to identify tutorial-high and tutorial-low children. The performance of tutorial-high and tutorial-low children within the satisfactory and unsatisfactory groups was examined using nine blocks of ten problems each. For tutorial-low children in both the CRT satisfactory and unsatisfactory groups, there was a significant decrease ($p = .028$ and $p = .007$, respectively) in the number of problems solved correctly from Block 1 to Block 9.

3. Interpretations

This research represents a systematic attempt to identify and remediate subtraction difficulties with techniques that require progressively greater amounts of time and preparation. The finding that simple remedial techniques such as promise of reward or promise to borrow fail to assist performance highlights the enduring and robust nature of subtraction difficulties. That feedback appeared to be as effective as a more carefully programmed instructional package was an unexpected but welcome finding from a cost-effectiveness viewpoint. (Cebulski and Bucher, 1986, pp. 177-178)

Abstractor's Comments

I would rate the three studies reported in this article as good from a technical aspect. The investigators have made a good effort to use traditional textbook designs and procedures, along with appropriate statistical analysis. However, my rating of these studies in terms of their substance and contribution to the field of mathematics education would be less than good. Following are some general comments that relate to each of the three studies.

Study I

The need for conducting Study I is questionable given the research literature on subtraction of whole numbers and the stated interests of the investigators in seeking remedial treatments to correct borrowing errors. Study I results provide no new findings on the types of subtraction errors made by young children. Further, it would appear that the remedial treatments designed for use in Studies II and III do not reflect directly the results from Study I at all. They are grounded in theories related to learning and motivation.

My main concern with Study I is with the criterion used by the investigators to partition the sample of 56 children into Groups 1 and 2 for purposes of data analysis. Children assigned to Group 1 did not solve any problems correctly on the pretest; those assigned to Group 2 solved at least one problem correctly. The delayed test results for Group 2 children suggest that some other criterion be used to partition the sample; the probability that a Group 2 child would be classified the same after one month was .59. If the investigators had provided a distribution of pretest scores (number of correct responses), it might be easier to understand the rationale for using the criterion they did. I would hypothesize that the more erratic error pattern and the inconsistent performance of Group 2 children were more a function of grouping than low motivation, as suggested by the investigators.

Study II

The design used in Study II was a 3 (experimental condition--Motivation, Instructions, Control) x 2 (Groups 1 and 2) factorial. The same criterion used to assign children to groups in Study I was, again, used in Study II. It is my belief that this grouping procedure is inappropriate and more than likely confounded Study II results.

The fact that there were no significant results between or among experimental conditions relative to the number of subtraction problems solved correctly is not surprising given the simplistic nature of the motivation and instructions conditions. Also, the statistical analysis of inversion and borrowing errors yielded expected results for Group 1 children. Obviously, if you tell a group of slower children (instructions condition) that they must borrow in order to solve problems correctly, they will attempt to borrow on practically every problem. If they make more borrowing errors, the chances are excellent that they will make fewer inversion errors. The analysis of errors in Study II is somewhat interesting but not nearly as important as the analysis of the dependent measure (number of correct responses). It would have been useful if the investigators reported the number of children assigned to each experimental condition within each of Groups 1 and 2.

Study II results would seem to contradict the investigator's notion (from Study I) that the more erratic error pattern and the inconsistent performance of Group 2 children in solving subtraction problems is a function of low motivation.

Study III

Of the three studies reported, Study III was the most interesting. Although the remediation phase of this study was probably too short (two to three days) to yield significant results, it appears that the investigators gave careful thought to designing the Component Skills Training and the Criterion Training treatments. Also, the procedures used to identify unsatisfactory group children and satisfactory group children were much more realistic than the procedures used in Study I and Study II.

The investigators claim that: "While program modifications are desirable to increase the number of children who benefit from training, Component Skills Training and Criterion Training appear to be two effective and efficient remedial strategies for teaching borrowing skills" (p. 178). The weight of the evidence (the statistical analysis of Posttest 1 and Postt st 2 data and the descriptive data in Table 3--mean number of borrowing problems solved correctly) do not strongly support this notion.

The investigators' analysis of children's performances throughout the CRT training condition was interesting and useful. However, it was never clear to me how the children's performance throughout training was analyzed within the CST program. It was my understanding from the CST treatment description given by the investigators that the children assigned to this condition would not have solved 90 subtraction problems over the three-day treatment period.

Cobb, Paul. AN INVESTIGATION OF YOUNG CHILDREN'S ACADEMIC ARITHMETIC CONTEXTS. Educational Studies in Mathematics 18: 109-124; May 1987.

Abstract and comments prepared for I.M.E. by JAMES SHERRILL, University of British Columbia.

1. Purpose

The study was designed to a) investigate whether and to what extent the subjects operated in different contexts in their approach to the equals sign and b) if such contexts existed, to investigate the nature of the contexts and their possible experiential origins.

2. Rationale

Several studies indicate that primary school children interpret the equals sign in addition and subtraction sentences in terms of actions to be performed, i.e., "to do something", as opposed to an expression fo relational equivalence between numbers, i.e., "is the same as". Based on subsequent research and suggested models the hypothesis is put forward that one must look beyond cognitive limitations to explain the persistence of enactive interpretations of the equals sign to the constraints of the students' academic mathematics contexts.

3. Research Design and Procedures

There were three distinct phases to the study: interview (part 1) to investigate students' self-generated methods; interview (part 2) to investigate students' behavior in academic arithmetic; and 3) teacher interviews to investigate what happened in the students' arithmetic classes.

Clinical interviews were conducted with 34 children drawn from five classrooms in May of their first-grade year. All interviews, which lasted approximately 30 minutes, were video-taped for later analysis.

1. Self-generated Methods: Counting, subtraction, and thinking strategy tasks were administered in the first part of the interview. Each counting task involved using visible and screened collections of felt squares to present either an addition or a missing addend problem. The children were also asked to solve subtraction sentences in which the subtrahend was relatively large when compared with the minuend. The thinking strategy task involved presenting a sequence of addition sentences in which the second addend was increased by one or two. The child was asked to explain all solutions that appeared to involve the use of a prior result.

2. Worksheet Tasks: Two worksheet tasks were presented at the end of the interviews. In the first, the equals sign task, the child was shown a worksheet containing typical and atypical (reverse: 10 = 6 + 4, 6 = 10 - 4, and symmetric: 6 + 4 = 4 + 6, 10 - 4 = 10 - 4, 9 - 4 = 4 - 9) symbolic forms. The child was asked whether the sentence was correct and, if necessary, to fix it. The second task, the addend-increasing task, required the child to solve one of the following two sequences of sentences depending on their knowledge of the basic additions facts:

LOW FORM	HIGH FORM
4 + 0 = ____	14 + 0 = ____
4 + 1 = ____	14 + 1 = ____
4 + 2 = ____	14 + 2 = ____
4 + 3 = ____	14 + 3 = ____
4 + 4 = ____	14 + 4 = ____
4 + 5 = ____	14 + 5 = ____
4 + 7 = ____	14 + 7 = ____
4 + 8 = ____	14 + 8 = ____
4 + 6 = ____	14 + 6 = ____

The relationships between successive sentences were varied to distinguish between thinking strategy and number word solutions.

3. Teacher Interviews: Based on the results of the student interviews and worksheet responses, unstructured individual interviews were conducted with each of the five teachers from whose classrooms the children were drawn. The interviews focused on the arithmetical topics that the teachers had covered, with attention given to the extent to which the teachers relied on a textbook, the manner in which they organized their mathemaitcs instruction, the degree to which they attempted to teach relationships between number facts, their views concerning the use of fingers and other concrete objects to find sums and differences, and their awareness of the self-generated methods that their students used.

4. <u>Findings</u>

(A) Self-generated Methods: All 34 children were able to solve both addition and missing addend counting tasks. On the thinking strategy task, 24 children used the addend-increasing strategy spontaneously and a further four did so after they were prompted to use a prior result.

(B) Worksheet Tasks: The children's performance on the addend-increasing worksheet contrasted sharply with that on the thinking strategy task. Nineteen children solved all nine sentences independently of each other, eight used a thinking strategy, and seven produced number word solutions. Only eight of the 24 children who had spontaneously used a strategy when they engaged in a dialogue with the interviewer reflected to the degree necessary to use the most elementary type of thinking strategy in the worksheet setting.

The children's performance on the equals sign task was compatible with the findings of previous investigations into children's interpretations of this sign. Nine of the children were inferred to have meaningfully reversed the form. For half the sample, however, the task seemed to be to produce a string that looked right in the sense that it comprised three numerals punctuated first by an operation sign and then by the equals sign.

Of the 21 children presented with the low form of the equals sign task, 19 repeatedly produced the most primitive possible solution methods - direct modeling using fingers. Of these children 12 had used counting-on and counting backwards strategies without being prompted in the first part of the interview. All 13 children presented with the high form of the task checked or generated sentences that were beyond the range of finger patterns and, as a consequence, used more mature counting methods.

The children's tendency to use primitive methods whenever possible was accompanied by the unreflective attitude that was also evidenced by their performance on the addend-increasing worksheet task. Twenty-six of the 34 children checked or generated and solved the same sentence by counting at least three times.

(C) Teacher Interviews: The interviews indicated that there was considerable uniformity in arithmetic instruction across classrooms. All five teachers said that their implemented mathematics curricula were largely determined by a textbook; all classes had covered the same relevant content; all teachers reported that they had repeatedly stressed the inverse relationship between addition and subtraction and the commutative property of addition; all classes spent the bulk of their arithmetic time completing worksheets or textbook exercises individually. None of the teachers taught any numerical relationships except sentences grouped by fact families; none of the teachers were aware of the children's self-generated methods (other than using fingers); none of the teachers differentiated among different methods which involved using fingers. Four of the five teachers felt that using fingers to find sums and differences was undesirable.

5. Interpretations

(A) Students: The contrast between the solution methods used in the first and second parts of the interview, the children's generally unreflective attitude in worksheet settings, and the occurrence of syntactic responses all suggest that the two parts of the interview were different contexts for most of the children. Their arithmetical activity in the two situations seemed to be guided by differing anticipations and expectations.

(B) Teachers: The five teachers' implemented curricula seemed to be characterized more by imposition than by negotiation. There was no indication that they viewed teaching as a process in which one attempts to develop shared meanings with students.

(C) Both Parts of the Study: The children's activity in the first part of the interview, like that in the worksheet settings, was constrained by the contexts they established. The interviewer intervened to help the children to establish the goal of construction of numerical relations rather than to find ways to produce behaviors they inferred he expected.

The children's generally unreflective attitude is reasonable when one considers they are behaving based on the assumption that academic arithmetic involves completing a sequence of isolated, unrelated tasks; reflecting on prior or potential problem-solving activity is not relevant to generating correct answers.

Symbolic forms can only make sense to these students if they can be interpreted as directions to act. The "do something" interpretations of equals signs is related to the ways children make sense of typical interactions in classrooms characterized by imposition. "Do something" meanings appear to have social as well as cognitive origins.

Abstractor's Comments

The abnormal length of this abstract is directly due to the type of article involved. The article describes the study in great detail.

The reader must have much of that detail to see where the author is coming from and going. The author has published several other studies (referenced in the present article) on the same topic and I found it extremely helpful to read those in trying to put the current article in perspective.

As with any article there is a limited amount of space to present the details. It is likely the case that the limit on space explains why some discussion is missing. The article lists in great detail the performance of the students. The article does not give any discussion as to how two truly different contexts can be created in 30 minutes.

Why weren't the students selected from classes which represented different contexts? Can grade 1 students who have, rightly or wrongly, been studying the "do something" interpretation of the equals sign change their ways in half an hour? If this study is an existence proof (i.e., students learning in one specified context produce certain errors), then the goal was accomplished.

Is this a study to lend support to the statement that teachers do make a difference? The kernel of the study appears to be that findings concerning older students should be interpreted in the light of how they were taught. Students learning in one context make different errors from students learning in a different context.

My main concern is the strength of conviction with which the conclusions are made based on the study described. The study does show that students learning in classrooms characterized by imposition produce certain types of errors when dealing with the equals sign. The study does not show, as is implied in the conclusions, that teaching grade 1 students in classes characterized by negotiation will produce students who will avoid the same or a different, but well-defined, set of errors.

While the results sound very "common sensical", this study should be of interest to those studying the meanings children give the equals sign.

Ferrini-Mundy, Joan. SPATIAL TRAINING FOR CALCULUS STUDENTS: SEX DIFFERENCES IN ACHIEVEMENT AND IN VISUALIZATION ABILITY. Journal for Research in Mathematics Education 18: 126-140; January 1987.

Abstract and comments prepared for I.M.E. by TOMMY DREYFUS, Center for Technological Education, Holon, Israel.

1. Purpose

Mathematics achievement in general and sex differences in mathematics achievement in particular are known to be related to spatial abilities. It was the purpose of the research reviewed here to investigate more specific aspects, namely to relate to achievement in college calculus rather than mathematics generally, and to investigate the effects of a training program with and without physical models designed to improve spatial skills.

The specific research questions of the study were:

1. Are there sex differences in spatial visualization ability, in calculus achievement, or in the use of visualization for the solution of a particular type of calculus problem?
2. Does a spatial visualization training program affect the answers to question 1?
3. Is the answer to question 2 different for men and for women?

2. Rationale

Spatial abilities have been extensively researched, and although there is no agreement among specialists on whether spatial abilities can be categorized into subclasses, there is no doubt that a strong connection exists between spatial ability and mathematics achievement. Three aspects of this relationship form the background against which this research should be seen: mathematical topics, sex differences, and training on spatial skills. The connection between spatial skills and mathematics achievement is a topical one. On the one hand, spatial skills, operations, and transformations are actually used in learning and doing mathematics. The connection is a statistical one on the other hand: positive, moderately significant correlations have been found between spatial ability and the acquisition of high-level mathematical concepts. The available results focus on global relationships. Little is known about the influence of spatial abilities on particular mathematical disciplines and even less is known about the cognitive connections between spatial ability and mathematics achievement. With respect to sex differences in mathematics achievement it is questionable whether they can be attributed to sex differences in spatial ability or not, because of the dependence of the corresponding results on age and other factors. It has been shown that spatial skills can be trained. Hereby, age might be a factor, as might the mode of training--in particular, whether the training includes the manipulation of three-dimensional physical models or not.

3. Research Design and Procedures

The study involved 334 college students in seven sections of a standard calculus course. The course includes as a standard feature remediation modules in algebra and trigonometry, to which students were assigned according to need. Within this framework, two sets of spatial training modules were added, one with and one without manipulative aids. Otherwise the two modules were identical; they emphasized visual estimation, discrimination among images, mental manipulation of images, and reflection, among others. The students were assigned randomly to training with models, training without models, and control groups. Seven measures were taken on the students: one measure from each of three pretests on precalculus background, calculus background, and spatial visualization background; a measure from a posttest on spatial visualization which was identical to the pretest; and three measures from four calculus tests that were given in the course of the semester: the overall course grade, the grade on Unit 3 which was of special interest because of the possible spatial nature of some of the test questions, and a measure VSRSC which was a combination of the tendency of students to sketch the solid of revolution whose volume they had to compute in the Unit 3 test and of their success in identifying and sketching solids of revolution from the plane figure to be rotated.

The results were analyzed by a multivariate analysis of covariance with treatment and sex as the independent variables, the three pretests as covariates, and three of the four remaining measures as dependent variables. The fourth, VSRSC, had to be eliminated from the model because it destroyed the parallelity of the regression surfaces. A separate analysis of variance was performed for VSRSC; in this analysis the two types of training were identified but no reason for this is given.

4. Findings

Few significant differences were found. The multivariate analysis of covariance yielded a significant effect of sex which resulted from women performing better than men on the Unit 3 test.

One out of several (many?) univariate follow-up tests that were performed seems to show that for men the training with manipulatives was more helpful, whereas for women the training without manipulatives was more helpful. This is an interaction effect which appears in the overall course grade if two of the three covariates are used. It disappears when all three covariates are used. It does not appear in any of the other dependent variables. Moreover, the corresponding multivariate interaction effect is not significant.

The analysis of variance for VSRSC yielded a significant main effect of training as well as a significant training-by-sex interaction. It shows that trained students scored higher on VSRSC than untrained ones and that women responded better to the training than men.

5. Interpretation

The significant sex difference on the Unit 3 test is ascribed by the author to the high mathematics ability of women who elect college calculus. For these women training also contributed to their ability and readiness to use visualization in solving the solid-of-revolution problem on the Unit 3 test. According to the author, although the training program was not successful in improving students' spatial visualization scores, "the feasibility of fostering improvement in college students' spatial scores through other means is clearly indicated." To achieve this aim, there is a "need to understand more fully how components of the calculus semester experience might be contributing to spatial test score improvement."

Abstractor's Comments

This research is a contribution to the literature on the interrelationship between spatial ability and mathematics achievement, with particular attention paid to sex differences. The main value of the study lies in its attempt to focus on a particular topic in mathematics and to investigate the effects of specific training on achievement in this particular topic. The influence of spatial ability on mathematics learning, and the use of spatial techniques in mathematics teaching, learning and problem solving, merits researchers' attention. This is particularly true for studies which deal with the finer aspects of this relationship, analyzing the specific mechanisms that are responsible for the influence of spatial abilities on mathematical thinking.

The research has been carefully planned and executed. The statistical results have been interpreted with care and presented clearly. The figures, as often, give the reader quicker access to the main features of the results than the tables. The two figures in the paper were, however, interchanged by mistake. This caused me a lot of trouble understanding what is going on, and is likely to cause the casual reader even more trouble. Also, there is an unfortunate misprint at an essential entry in Table 5.

The statistical results did not show many interesting effects. The sex difference in the multivariate analysis appears to be due to the Unit 3 results only; it is not present in the overall course grade (of which Unit 3 is a part) nor in VSRSC, nor is it related to the training. In fact, the control group performed consistently, although not significantly, better on Unit 3 than the trained groups. This sex difference is therefore inconsequential. This leaves us with the result that training helped for VSRSC, and that it helped women more than men. This result has to be interpreted with care for several reasons; first of all, it concerns the variable that had to be eliminated from the multivariate analysis because of non-parallelity of regression surfaces; the nature of such non-parallelity is difficult to describe, and the author has chosen not to describe it. Without such a description, however, the above significance result cannot be taken at face value. More importantly, the VSRSC score is based on two components: the tendency to sketch in one problem on the Unit 3 test, and the ability to identify and sketch solids of revolution. The ability part of VSRSC was directly taught in the training modules; therefore the tendency part, although based on a single problem, is the more interesting one. Unfortunately, the paper gives no indication which of the two parts of VSRSC was responsible for the significance of the result.

With respect to VSRSC, several questions should be addressed:

- Are solid-of-revolution problems a good choice in the sense that sketching the solid helps solving the problem? Personally, I sketch only the two-dimensional region that is being rotated.
- What happens in other calculus problems? The Unit 3 test results did not show the same effects as the VSRSC results; does this imply that the results are specific for solids of revolution?
- What was the solid-of-revolution problem that happened to be on the Unit 3 test? If there were different problems on different versions of the test, did all of them produce equivalent measures of sketching?

- Is there any evidence that the students who did sketch the solid in any detail actually related to their sketch when setting up the integral for computing the volume? Did the students who sketched do any better in setting up the integral than those who did not? (This reviewer has evidence from his own students that for other three-dimensional volume calculations this is not the case.)

In summary, the research questions asked in this work are important questions; the methodology used was carefully executed but not ideally suited for answering the questions. This reviewer agrees with the author that there is a need for understanding more fully the connection between spatial abilities and calculus problem solving. In this endeavor, cognitive results might be more revealing than statistical ones.

Galbraith, P. L. THE USE OF MATHEMATICAL STRATEGIES: FACTORS AND FEATURES AFFECTING PERFORMANCE. Educational Studies in Mathematics 17: 413-441; November 1986.

Abstract and comments prepared for I.M.E. by CHARLES E. LAMB, The University of Texas at Austin.

1. Purpose

The purpose of this piece of work was to investigate the use of process by pupils in various contexts. The quality of pupil response was considered in terms of several factors. Among them were mathematical attributes, number of years in mathematics study, sex, and cognitive levels.

2. Rationale

A review of the literature had revealed several studies of student performance in the process area. In all studies, emphasis had been placed on the qualitative nature of student responses. Most studies had given results which indicated a high degreee of variability. Some had made attempts to relate response made to mathematical and/or developmental factors. The current study was designed to explore this variability in terms of process attributes, levels of response and in the development and use of strategies.

3. Research Design and Procedures

The study was conducted in two Australian state high schools of a nonselective type. Of the 334 pupils in the study 157 were male and 177 female. The pupils were enrolled in years 8, 9, and 10 for which the respective numbers were 121, 102, and 111. Since the testing took place during the end-of-year examination period, the pupils could be regarded as having completed one, two, and three years of secondary school according to year level. The subjects formed a representative though not random sample. They were chosen by the respective subject masters to proportionally represent the range of mathematics classes at each level.

Tw tests were given on the same morning. The first of these was the Operations test (ACER) which provided a measure of cognitive operational level for each student. Scores on this test were in the range 0-80. The criterion test contained 12 pilot-tested questions which required the pupils to respond with reasons to particular pieces of mathematics presented to them. The time allowed for completion of this test was 1 1/4 hours. The test was scored in two ways:

(a) A numerical score was assigned to each response and an aggregate score computed for each student.

(b) A categorical score was assigned on the basis of whether contextually appropriate strategies had appeared in the response. For this dimension the scoring was on the basis of yes/no/not applicable. The numerical score provided data for statistical analysis while the categorical score contributed to the identification of process attributes.

Table 1 contains a list of the strategies that comprised the process component of the study.

The strategies were determined within a definitional framework chosen for the study. The fundamental criterion used was universality; i.e., choice was based upon the premise that modes of mathematical reasoning are not context specific and that strategies have embodiments across content and levels. Each process attribute in Table 1 satisfies this condition. As an example Attribute 2 appears at kindergarten level in the sorting of blocks, at primary level in the distinction between different kinds of numbers, at secondary level in the classification of geometrical properties, and at advanced and research level in the distinguished properties of particular sets and in the identification and treatment of singular cases. This

TABLE 1

Characteristics of process test items

The items embodied process attributes that included the ability to

1. Demonstrate system and variety in the checking of finite cases.
2. Identify classes which require separate subsequent examination.
3. Interpret mathematical statements literally.
4. Explain contradictions.
5. Appeal to a general principle to justify a conclusion.
6. Correctly use transitive inference.
7. Carry out the essentials of an implicarion argument.
8. Distinguish between equivalence and implication.
9. Return an open finding in the absence of sufficient data to close a mathematical argument.
10. Generalize from a set of special cases that embodies a logical pattern.
11. Select and use appropriate data in context.
12. Recognize and expose a circular argument.
13. Analyse an explanation into components to be tested.
14. View proof as a logically coherent sequence of steps.
15. Examine the domain of validity of a generalization.
16. Recognize the superiority of a logical principle over empirical evidence.
17. Maintain consistency; e.g., recognize the inconsistency of contradictory conclusions.
18. Assess the relevance of data.
19. Distinguish between inclusion and equality.
20. Separate data from conclusion in an inferential statement.

distinguishes the purpose of the present study from those that focus upon the performance of an algorithm (skill based) or upon the development of a concept such as ratio (concept based). The items have been designed using thoroughly familiar mathematical content. This is to prevent concern about content from interfering with the purpose of the items; i.e., to focus upon the use of strategies. (pp. 422-423)

4. Findings

Results indicated a strong correlation (Pearson 0.65) between the operations test score and the process test score. Because sex, prior mathematical experience, and cognitive level were of particular interest in this study, analysis of covariance was run. The operations test score was used as a covariate and the independent variables were sex and year level. Main effects were found to be significant ($p < .001$) for grade level, sex, and operational level. No significant interactions were found. Performance improved from year to year and females outperformed males.

Having found no significant interaction effect, a multiple classification analysis was performed. A major finding of this analysis dealt with the sex variable. The sex factor in performance was consistent at each level. This result is in contrast to many previous studies which had shown boys to perform better than girls.

Process data was placed into categories. The data supported the results of the prior quantitative analysis. That is, the use of strategies increased with cognitive level as measured by the operations test scores.

The SOLO (Structure of Observed Learning Outcomes) taxonomy was employed in order to provide a framework for the classifications of response quality. There was a regularity in the appearance of particular answer forms. The answer clusters tend to show varied levels of thinking.

5. Interpretations

(a) The proper use of a strategy in one case does not insure its use in another situation.

(b) Students must appreciate meaning in order to effectively and consistently use strategies.

(c) Operational level exerts a dominant influence on process facility.

(d) Most students did not appear to possess generalized thinking and abstraction processes.

(e) Girls outperformed boys consistently. This is probably related to the fact that girls outperform boys in language arts related activities (higher-level mathematics has more verbal requirements).

(f) The SOLO taxonomy provided a method for classification for looking at the quality of responses.

(g) Implications for instruction includes:

    i) Recognize the concrete/formal nature of mathematics in the use of processes;

    ii) Include opportunities for class discussion (teacher-student and student-students); and

    iii) Apply the SOLO taxonomy to generate examples for (ii).

Abstractor's Comments

1) The study was very well thought out. The report was clear, detailed, and well written.

2) The study's report provides a great deal of background literature on the variable of interest.

3) The quality of student response when using strategies is an extremely important topic for mathematics education.

4) The author's detailed description of items, etc. provides researchers and teachers alike with much information for educational use.

5) The implications for instruction are well-founded and need to be given serious thought for implementation.

6) Of the several articles I have abstracted for I.M.E., I believe this is the overall finest.

Nesher, Pearla and Katriel, Tamar. LEARNING NUMBERS: A LINGUISTIC PERSPECTIVE. Journal for Research in Mathematics Education 17: 100-111; March 1986.

Abstract and comments prepared for I.M.E. by THOMAS C. GIBNEY, University of Toledo.

Through a contrast between the natural language system and the formal language of mathematics, the authors investigate the basic question of how primary students read a word problem (in an Israeli textbook) in which the numeral denotes a number functioning as a predicate. Many interesting questions for further research were mentioned in the article, with some hints of possible results. It is difficult to generalize the results from the study reported in the article when the subjects were selected only on the basis of their willingness to participate rather than in a controlled design.

Students appear to learn words at a rapid rate. They may add as many as 3,000 words annually to their reading vocabularies between third and twelfth grades (Nagy and Herman, 1984). Only a small proportion of this growth, perhaps 200-300 words per year, could be attributed to vocabulary instruction (Durkin, 1979; Jenkins and Dixon, 1983). Therefore, students must learn most new words incidentally from context while reading and by listening.

Results from the study reported in this article indicated an overlap of the linguistic and mathematical systems from students in grades one through three as they read problems in Hebrew. The distinction between textbook exercises or problems written with words or numerals also is noteworthy. When numerals are the symbols we write to convey numbers, they are usually called a phrase or an expression (May, 1971). Concern with arithmetic vocabulary was also expressed by Wilmon (1971). This survey revealed that students are introduced to approximately 500 new technical words and phrases by the time they reach fourth grade. These findings support the contention by Nesher and Katriel that the linguistic as well as the mathematical systems must be taught. Students in the primary grades must develop problem-solving strategies specifically designed to bridge the gap between the two systems.

The relationship between mathematical development and language development, and particularly the transition from the conception of number as a predicate in natural languages to its conception as an object in the mathematics language, needs careful examination through controlled research projects. In addition controlled experiments are needed to substantiate the difference found in this study between the errors related to the use of masculine and feminine nouns. Also, additional documentation is needed to substantiate the authors' contention that in the initial stages of instruction, numbers in primary grade textbooks should be presented in their predicative form, written as words, rather than as numerals.

This article should be of interest to mathematics teacher educators who are interested in research related to the linguistic perspective of mathematics and to mathematics textbook publishing editors and authors who strive to publish a mathematics textbook in the primary grades that can be read and understood by primary students.

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Abstract and comments prepared for I.M.E. by JANE O. SWAFFORD, Illinois State University.

1. Purpose

The purpose of this longitudinal study was to investigate the prediction of (a) fifth- and tenth-grade students' academic achievement from cognitive tasks given prior to kindergarten and (b) tenth-grade students' academic attitudes from measures of achievement taken in grades 2, 3, and 5 and ratings of general cognitive abilities made by the students' mothers and teachers during those same years.

2. Rationale

This study is a continuation of an earlier study that focused on the prediction of performance in the first three grades. The present study relates these prediction tasks to achievement in the fifth and tenth grades. Although there is a large literature on screening tasks used in the prediction of early achievement, little is known about the usefulness of early assessment tasks over long periods of time. The study secondly investigates the relationship of teachers' and parents' ratings and students' earlier achievement to attitudes in high school. It is suspected that students' attitudes depend both on past performance and on the perceptions of significant others.

3. Research Design and Procedures

Initially 255 children were administered a battery of 25 cognitive tasks before entering kindergarten. By grades 5 and 10, 153 and 105 students, respectively, remained in the sample and were tested again. At the end of kindergarten and grades 2, 3, and 5, teachers' ratings of students on a variety of characteristics that appear to be important for successful performance in school were obtained. Also during grades 2, 3, and 5, mothers were asked to rate their children on a subset of the scales used by the teachers. Each year students were also asked to rate themselves on five characteristics.

A variety of achievement measures were taken each year. These consisted of the Wide Range Achievement Test (grades 1, 2, 3, 5, and 10), the Gray Reading Comprehension Test (grades 2 and 5), the Wechsler Intelligence Scale for Children (grades 2 and 5), the Stanford Reading Comprehension Test (grade 3), the Gates-MacGintie Reading Test (grade 10), and a series of standard textbook arithmetic and algebra word problems (grade 10). At grades 2, 3, and 5 scores were combined into a single index of mathematical achievement and an index of reading achievement.

In grade 10, students were administered four attitude scales for each of mathematics and reading. These covered self-concept of ability, expectancy for success, value of success, and perception of task difficulty.

Correlational and stepwise regression analyses were performed.

4. Findings

From the previous study, four of the 25 prekindergarten tasks had been identified as the most optimal predictors. For mathematics, the four tasks were: verbal recall, paired associates, perceptual learning, and coding. There was a decline after grade 2 in the correlations, however; the tasks (especially paired associates) consistently related to later mathematics achievement in grades 5 and 10. For reading, the four tasks (naming letters, paired associates, reversals, and category naming) maintained a high relation to later achievement scores.

On the attitude scales, males generally had a more favorable attitude toward mathematics and females a more favorable attitude toward reading. Hence, results on predictors were reported by sex. Males' attitudes about mathematics were related only to their prior performance at grades 3 and 5. Mothers' and teachers' ratings rarely were related to boys' later attitudes in either subject area. For females a much more complex picture emerged. Their actual achievement in earlier grades was positively related to attitudes, but mothers' rating of their daughters' cognitive abilities in grade 5 was negatively related to attitudes toward mathematics.

5. Interpretations

A small set of cognitive tasks administered before children entered kindergarten maintained a remarkably high relation of high school scores in mathematics and reading. Long-term prediction, however, was less effective for mathematics scores than for reading. It may be that the content of mathematics and the cognitive processes required differ more for mathematics in successive years than do those in reading.

Predictions of tenth-grade attitudes were not as effective for males as for females, especially for reading. The finding suggests that mothers' perceptions have an important influence on the development of females' achievement-related attitudes positively for reading and negatively for mathematics. Other studies have documented the influence of parents on students' achievement and attitudes. The present study extends this literature by contrasting the influence of parents and teachers and by contrasting the academic subjects of mathematics and reading for boys and girls. Failure to consider the domain specificity of academic activities may mask important features of the socialization process by which students develop attitudes related to achievement.

Abstractor's Comments

This study is technically very sound. No statistical tool was left unturned. In all, nearly 400 correlation coefficients are presented. The tables are impressive. The significance, however, escapes me.

It is neither surprising, interesting, nor useful to find out that tenth-grade achievement is related to four prekindergarten tasks. It is curious but of little use to find that for females, mothers' earlier ratings of their daughters' academic characteristics was later related (negatively) to mathematics attitudes.

As an educator, I am interested in those factors that the schools can influence. The others have no significance for me.

Vance, James H. ORDERING DECIMALS AND FRACTIONS: A DIAGNOSTIC STUDY. Focus on Learning Problems in Mathematics 8: 51-59; Spring 1986.

Abstract and comments prepared for I.M.E. by AARON D. BUCHANAN, Southwest Regional Laboratory, Los Alamitos, California.

1. Purpose

This study sets out to improve what is generally known about abilities of students to order and compare fractions and decimals. In particular, the investigator examines performances of sixth- and seventh-grade students on two different kinds of ordering tasks. The first is fairly straightforward: arranging three decimals or fractions in their proper order. The second task is a little more indirect. Students are given two decimals or fractions and asked to generate a third that comes between them. In addition, the study sought to identify mental strategies or heuristics, some of them appropriate, others faulty, that students employ to help them deal with fractions and decimals.

2. Rationale

Students first learn about rational numbers in the middle grades. In fact, mastery of basic processes involving the use of fractions and decimals in everyday situations constitutes a great deal of the entire mathematics curriculum for grades 4, 5, and 6, and in some cases grades 7 and 8. In spite of all that time and attention, student achievement in the area of rational numbers is generally spotty and remains that way well into senior high school.

How much students really learn about fractions and decimals is a serious issue. They seem to know a little about the notation systems for fractions and decimals, but the tasks that students generally know how to perform are few and have very little depth. What is lacking is an understanding of concepts--even the most rudimentary concepts that lie at the heart of what rational numbers are and, more important, how they are different from whole numbers.

The conceptual base that students generally have to work with is too weak to support many of the kinds of things we would like them to be able to do. Processes such as estimation that require much intuition are out of the range. Students can't make estimates involving decimals and fractions because, for starters, they seem to have very little intuition about number sizes. Far worse than whole numbers, the symbols for fractions and decimals are just that--symbols. The relative sizes of the numbers that the symbols represent are a mystery for most students in the middle grades, and a continuing source of frustration for their teachers.

3. Research Design and Procedures

The main part of the study consisted of a two-part test on ordering decimals and common fractions which was administered to 129 students in grades 6 and 7. All students were from the same school. Part 1 consisted of 10 items that required students to identify the least and the greatest of three numbers. Part 2 consisted of 10 items where students were given two numbers and asked to write down a third number that came between them. The first five items in each part dealt with decimals; the next five dealt with fractions. All numbers in both parts of the test ranged from 0 to 1.

As a followup, the same test was administered individually to 12 students at grades 6 and 7 from another school. Each student was taken through the test items one at a time and asked to think out loud as they attempted to work out an answer.

Students in the main study and the followup all came from mid-socioeconomic neighborhoods. Both the study and the followup were conducted during the last month of the school year.

4. Findings

Major comparisons in test results were made between: grade 6 (N = 62) and grade 7 (N = 69); Parts 1 and 2 of the test; items involving decimals and those involving common fractions; and boys and girls.

Grade 7 students scored significantly higher than sixth graders on the test overall. They also scored higher on each of the four subtests (Part 1: decimals or fractions; Part 2: decimals or fractions), but some differences between grades 6 and 7 were not significant. Differences were greatest on Part 1, where the average score for grade 7 was 64% versus an average score of 54% for grade 6. Part 2 was harder and the difference in performance between grades was cut in half.

Average scores for combined grade levels were about 60% on Part 1 and 50% on Part 2. Seventh graders tended to bring up the average on Part 1. There were more of them (69 versus 62), and they scored relatively high on Part 1 compared to sixth grade students and compared to their own performance on Part 2. There was less difference between parts of the test for sixth graders than for seventh graders. In fact, seventh grade students scored about the same on Part 2 as sixth graders did on Part 1.

Decimals were considerably harder than common fractions for grade 6 students on Part 1 of the test. Everywhere else in the study, overall performances on decimals and fractions differed by only one or two percentage points. On the average, seventh graders answered about two-thirds of the items on Part 1 correctly, regardless of whether the items involved fractions or decimals, and both sixth and seventh graders answered about half of the items correctly on Part 2.

Boys scored higher than girls on both parts of the test. Differences between sexes were greatest (58% versus 51%) among results for sixth grade students on Part 1 and seventh graders (61% for boys versus 49% for girls) in Part 2. Part 2 was fairly difficult for all sixth-grade students, boys and girls alike. In the same vein, Part 1 was relatively easy for all seventh graders.

Error analysis among individual items revealed several strategies that were leading students to the wrong answer. In Part 1, for example, students seemed to think that a longer string of digits in a decimal represented a greater number. In fact, about half of the sixth graders and one-third of the seventh-grade students consistently used the number of digits as a cue for size. Many students also thought that larger denominators in items involving fractions represented numbers of greater magnitude. In Part 2, students at both grades 6 and 7 scored fairly well as long as the two numbers they were given seemed to leave room for them to write a third one. All students had trouble when the two numbers they were given seemed to be next to one another (for example 3/5 and 4/5), or they asked to deal with fractions or decimals that were very close to 0 or 1 (for example 8/9 or 1/7). Many students tried to halve the difference between pairs of numbers in Parts 1 and 2, but didn't know how to express the result. As a consequence, they often gave numbers such as

$$0.1/2 \text{ or } \frac{3\ 1/2}{7}$$

which were not accepted by the investigator as adequate responses.

Results from followup testing, where 12 students were interviewed as they worked through each item in the test, generally confirmed the analysis of errors committed by students during the main part of the study.

Abstractor's Comments

Results of this study confirm that students who are near the end of the pre-algebra curriculum are still pretty shaky in their abilities to deal with fractions and decimals. In fact, their facility with the most basic concepts needed to compare fractions and decimals is rigid and unreliable. Students do fairly well in ordering decimals as long as they can use their "rule" based on number of digits that many of them acquired when they were learning to order whole numbers. (Longer digit strings mean greater numbers.) Students do better with fractions, but there are still discomfiting gaps in what they know. At both grade levels almost 70% of the students somehow misled themselves into believing that 1/2 is greater than 4/7.

In general, the investigator should get points for carefully designing a test that, in a few items, teases out a lot of basic information about faulty knowledge about fractions and decimals. Items were chosen carefully so that students could think through their comparisons of numbers rather than grind out equivalent fractions or decimals that require lots of recall but not much ingenuity or cleverness. I should think that a test no more complicated than this one is all that most classroom teachers need as a diagnostic tool to show them what building blocks in the way of existing student proficiencies they have to work with. In this study, results from this test sketch out prospects that are pretty sobering, but a longer test wouldn't make things look any better and it couldn't make them look much worse. Students do need a lot of new experiences in reconstructing what they know, and teachers must give them a lot of coaching and support. It's a serious challenge and a lot of work, but that's what schools are for.

One thing this study tells us is that students have acquired quite a lot of inflexible concepts and rules for manipulating whole numbers that have to be manually overridden before they can make much headway with fractions and decimals. I purposely use a human-versus-machine metaphor because it seems to me to describe what students are doing. Too often they learn to deal with fractions and decimals by first learning, somewhat mechanistically, that most of what they know about whole numbers can't be trusted. This isn't the kind of integrated, holistic experience we planned for, but it seems to represent a lot of the kind piecemeal result that we're getting.

I was pleasantly surprised at the number of students in Part 2 who had a basic understanding that fractions or decimals are dense. Any two of them are never so close that there isn't another number between them. Moreover, students intuitively sought to split the distance between the two numbers, but didn't know how to express the results. The kind of task represented in items on Part 2 might not be a bad place for instruction to begin.

Finally, I was interested in results on individual test items that discriminated fairly well between sixth and seventh graders. The bits of content that discriminate between grade levels are interesting indicators of schooling effects. Ceilings and plateaus are everywhere in student performance. So content that can discriminate between adjacent grade levels by as much as 10 percentage points (without resorting to trickery that calls on intelligence or on information acquired in an enriched environment outside the school) is pretty interesting stuff. In Part 1 of this test, the content that discriminated best between grades 6 and 7 was decimals. That's reasonable. Most students don't get much of an introduction to decimals until well into grade 6, so they're not likely to have had much of the varied practice that's needed to strengthen true generalizations and reconstruct false ones. The items in Part 2 represented content that was unfamiliar to both sixth and seventh grade students. As a result, nobody did very well. Items that did discriminate between grades 6 and 7 mostly involved the comparisons of decimals and fractions with 1 or 0. Was the difficulty primarily in working in close to 0 or 1, in thinking of 0 or 1 as fractions or decimals, or in dealing with two different representations of numbers at the same time? I don't know, but it would be interesting to find out.

Winkles, Jim. ACHIEVEMENT, UNDERSTANDING, AND TRANSFER IN A LEARNING HIERARCHY. American Education Research Journal 23: 275-288; Summer 1986.

Abstract and comments prepared for I.M.E. by CLYDE WILES, Indiana University Northwest.

1. Purpose

This report of two experimental studies investigates the effects of the independent variable Instructional Strategy, with levels AU and A, upon the dependent variables, Vertical Transfer and Lateral Transfer. All of this was done within the context of a validated learning hierarchy.

2. Rationale

Assuming the validity of learning hierarchies (see White and Gagne, 1974), these studies attempt to see how validated learning hierarchies may be used to promote the retention and transfer of intellectual skill learning. Intellectual skills, after White (1974), are associated with the descriptors: knowing how, algorithmic knowledge, procedural knowledge, and syntax as used by others. The earlier work of Mayer and Greeno (1972), involving emphases upon "internal connections" between objects used in skills and "external conections" between skills, formed the basis of the instructional treatments. Instruction that simply practices subordinate skills of a learning hierarchy with no reference to any connections is called A (for Achievement only), while instruction that emphasizes these connections is called AU (for Achievement and Understanding).

The purpose of this research then is to "advance the earlier work (of Mayer and Greeno (1972) as it relates to intellectual skills by using a learning hierarchy to guide the instruction and to define the transfer tasks. ... The research reported (in the present study) concentrates on lateral and vertical problem-solving transfer within a validated learning hierarchy." Problem solving means that students are asked to do transfer tasks (laterally or vertically related) without specific instructions for the tasks assessed.

3. Research Design and Procedures

Two experiments are reported. They both involved convenience samples. The first involved 90 grade 8 students from a metropolitan high school in Australia, who were taught in four classes. The second involved 87 grade 9 students from the same or similar school situation. The grade 9 students were also taught in four classes, but treatments were changed somewhat so as to provide two sub-experiments. The experiments involved heterogeneous groups that were formed using some kind of matching scheme based upon a 30-item mathematics test. Care was taken to remove students who had formally studied trigonometry. All classes, each involving three weeks of "ordinary, 40-minute mathematics classes," were taught by the experimenter following either instructional pattern A or AU described above.

Both experiments involved a hierarchy of 13 skills related to the solution of right triangles using sine or tangent ratios only. The instructional treatments involved reviewing/teaching the two prerequisite skills (dealing with the sum of the acute angles of a right triangle) and extensive teaching of some subset of the six middle level skills.

Experiment 1

All six of the middle-level skills of the hierarchy were taught to all groups. The tasks presented all of the problems in a standard pictorial form. This provided the opportunity to use rearrangements of the elements to provide "rotated items" to form a test of transfer. Following instruction a 21-item test was given. It contained 4 items to test the two prerequisite skills and validate the hierarchy, 6 items to test the six skills taught, 6 items to test the "rotated tasks", and 5 items to test the five vertically related tasks. The 6 rotated items were determined to be measures of lateral transfer, and the 5 items were used to test for vertical transfer. Eight and one-half weeks following the posttesting, a six-item retention test of the six tasks that were taught was given to all of the students.

Experiment 2

Only five of the middle level tasks were taught to the students in Experiment 2. The remaining task, and one task that was vertically related to it, were taken as measures of "more distant lateral transfer." One branch of the hierarchy involved three middle-level skills relating to the sine function, and another branch involved three such skills relating to the tangent function. Therefore it was decided to omit one of the six tasks in two different ways. This was seen as being symmetrical within the hierarchy, and the two "sub-experiments" that resulted were taken as equivalent. The 10-item posttest for Experiment 2 then involved 5 items for an achievement test, 2 items for lateral transfer, and 3 items for vertical transfer. The original five vertically related tasks were reduced to three by including one of them as a lateral transfer task and by omitting one of them (given the legs, find the hypotenuse) that depended upon a skill dropped from the instruction of one sub-experiment.

The instruction for Experiment 2 differed from that of Experiment 1 in that it was arranged to disguise the fact that the five skills taught did not in themselves provide all the information needed to solve the general right triangle. The last two days of instruction also included instruction relating to the vertical transfer tasks for the AU instructional groups. This included teacher presentations of solutions to these non-taught tasks.

4. Findings

Experiment 1

Using the pretest as a covariate, the adjusted means for all of the posttest and retention scores for the students in the AU groups were greater than those for the A groups. Using the individual as the unit of analysis, the differences for the lateral transfer tasks and the vertical transfer tasks were found to be significant. The differences on the vertical transfer tasks were entirely due to differences among the scores of nine students in the upper half of ability. These nine students were successful with at least one of the vertical transfer tasks. Eight of these were in the AU group and one was in the A group.

The retention means for the six tasks taught for eight and one-half weeks fell from almost 4 of a possible 6 to little more than 1.

Experiment 2

The combined differences for the two subexperiments revealed no significant mean differences for the five achievement items. Although mean differences for vertical transfer again favored the AU subjects, they were not significant. The lateral transfer means were found to be significant.

5. Interpretations

"If straightforward performance of explicitly taught skills is the main concern, then these experiments suggest that the relational style of instruction may have no advantage over the instrumental style, either for immediate post-instructional performance or for retention. If lateral transfer is an important aim, then Experiment 1 suggests that the relational style of instruction has advantages for near transfer, and Experiment 2 suggests that the advantages persist for more distant transfer."

"If problem-solving vertical transfer is a key objective, then these experiments suggest that the relational style of instruction offers more potential than the instrumental style. However, that potential will be realized more often with students who have higher-than-average mathematical ability."

The author further notes that "When the likely vertical transfer tasks can be identified at the time of instruction ... a small amount of instruction on the possibility of combining two subordinate skills may greatly improve vertical transfer..." Furthermore, though the strictures of the experimental design did not allow it, it was felt that a great deal more could have been realized in the AU approach with just a few more days of instruction focused upon the possibilities of lateral and vertical transfer. The piece concludes with: "The ... significant results obtained on transfer tests, with relatively small differences between treatments, suggests that teachers committed to teaching for relational understanding over extended periods of time are entitled to hope for superior results."

Abstractor's Comments

There are difficulties with these studies.

1. The content was restricted in a way that I find worrisome. It supposes that students know nothing at the application level of the theorem of Pythagoras. The two highest level tasks involved finding the third side of a right triangle when the other two are given. In one case the hypoteneuse is missing, and in the other a leg is missing. Since this is routinely presented at much lower grade levels of my experience (even though few seem to master its rudiments), I find it surprising that apparently none of the students at either grade level used it to solve the problems.

2. The hierarchy is presented as validated, and as such is used to define lateral and vertical transfer. However, in

Experiment 1 tasks given as vertically related are in Experiment 2 taken as laterally related. And these newly identified, laterally related tasks are displayed with a prerequisite task that was not taught. This makes the practical distinction between lateral and vertical in this experiment largely empty. Furthermore, the drastic drop-off in achievement levels for all of the grade 8 students leads me to doubt the mastery of any of the learning. And, since the drop-offs were so sharp for the grade 8 students, why was no retention measure taken for Experiment 2? And, if the material that was "intensely taught" for three weeks was not retained, what practical meaning could there be for transfer to other tasks?

3. The test items are valid as the tasks are practically defined in terms of them. However, there was no mention of reliability. Mastery testing involving three to four items can produce remarkable reliability coefficients. However, with a single measure of a given skill I worry about this. While the test for the six achievement items might be expected to have high reliability, a transfer test of two items seems quite likely to have a low reliability. Furthermore, it was on the two- and three-item transfer tests that differences were found. I would suggest that an achievement test that covers a much broader range of tasks of, say, 20 items would provide a more reliable, and so a more valid, measure of the relative effectiveness of the two instructional approaches. Since the tasks could admittedly be approached in a number of ways (e.g., use the Theorem of Pythagoras), and since the horizontal and vertical relationships were sufficiently unclear as to allow for the reclassification of tasks at will, I would suggest that a more broadly defined achievement test might be the only truly valid way to approach the question of who learned what.

4. The internal validity of the treatments is also at issue. The treatments were not independent as required by the analysis. The differences in treatment are not simply ones of instructional approach. There are very real content differences. The AU groups in Experiment 2 were exposed to more information. They not only saw all of the material that the A groups encountered, but they were also provided with the information that these tasks could be related in important ways, and they were specifically shown examples of the "transfer tasks". It was reported to be "unacceptable" to keep the students in the A groups of Experiment 2 at "routine practice" any longer. This suggests to me that instructional time might well have been wasted for these students. And finally, the instructor is the experimenter with very definite expectations about what is supposed to happen. If a person was committed to a disciplined, rote instructional plan this would not be tolerated. Such a person would certainly not find this study persuasive. In sum, the problem area might be generally identified as an investigation of meaningful instruction organized about a learning hierarchy. While the meaningful-rote controversy has, I think, been settled in theory, this approach provides a way of thinking about how meaning can be provided in practice. The practitioner who is looking for help in defining and delivering relational, meaningful instruction will find some help and encouragement here. To the extent that a rote versus relational controversy is in view, this study stands as an action piece of research that is not very persuasive. I'm convinced. But then I was to start with.

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