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ABSTRACT

This is the final report of a year-long research project on the implementation of a guided inquiry approach using the Geometric Supposer, a microcomputer software series, to teach high school geometry in three Boston area suburbs during the 1985-1986 school year. The project assessed student learning and examined the many and varied issues involved in the implementation of such an approach using a variety of data sources. The report is divided into four sections: (1) the first section describes the overall objectives of the study, the pilot research on which this study is based, and the specific research questions related to student learning and implementation; (2) the second section describes the general research strategy, the intervention including an overview of our guided inquiry approach, a brief look at the three sites and classes, training and support at the sites, data sources and data analysis; (3) the third section is organized by classroom perspective, teacher perspective, and student perspective in order to provide a coherent portrait of the experience and insights into the perspectives of the key participants, closing with a report on the pretests and posttests of mathematics learning; and (4) the last section integrates the data in terms of student learning and implementation issues. This section also presents recommendations for altering the intervention and for future research. (TW)

TR88-6

GUIDED INQUIRY AND TECHNOLOGY.
A YEAR LONG STUDY OF CHILDREN AND TEACHERS USING THE GEOMETRIC SUPPOSER

Technical Report

January 1987

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GUIDED INQUIRY AND TECHNOLOGY: A YEAR LONG STUDY OF CHILDREN AND TEACHERS USING THE GEOMETRIC SUPPOSER

ETC Final Report

Technical Report
January 1987

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Preface

This is the final report of a year-long research project on the implementation of a guided inquiry approach using the GEOMETRIC SUPPOSER, a microcomputer software series, to teach high school geometry in three Boston area suburbs during the 1985 - 1986 school year. The project assessed student learning and examined the many and varied issues involved in the implementation of such an approach using a variety of data sources.

Below is a description of the contents of each section of the report.

I. GOALS describes the overall objectives of the study, the pilot research on which this study is based, and the specific research questions related to student learning and implementation.

II. METHODS describes the general research strategy, the intervention including an overview of our guided inquiry approach, a brief look at the three sites and classes, training and support at the sites, data sources and data analysis.

III. RESULTS are organized by classroom perspective, teacher perspective, and student perspective in order to provide a coherent portrait of the experience and insights into the perspectives of the key participants. The section closes with a report on the pretests and posttests of mathematics learning.

The classroom perspective first describes the make-up of the student population and the background of the teacher in each experimental class and then looks at the evolution of key instructional elements in each class during the school year.

The teacher perspective includes the writings of the three teachers at mid-year and year-end and a report on teacher interviews.

The student perspective reports on year-end interviews with a sample of students.

The tests of mathematics learning include a test designed to assess student ability to generalize from data or from a description of a geometric situation and a test designed to assess student ability to devise proofs.

IV. CONCLUSIONS integrates the various data in terms of the two major foci of the study: student learning and implementation issues. This section also presents recommendations for altering the intervention and for future research.

I. GOALS

The use of the GEOMETRIC SUPPOSER[1] in the classroom provides an occasion to investigate whether the learning and use of inductive and inquiry skills can deepen student understanding of relationships, evidence, and argument in geometry; whether technology can be used to improve the effectiveness of geometry learning and teaching; and to examine how the use of technology in a guided inquiry approach affects the relationships among students, teachers, and the curriculum.

In 1984-5, M. Yerushalmy conducted a pilot research study (Yerushalmy, 1986) on inductive reasoning in geometry and the use of the GEOMETRIC SUPPOSER in Weston (MA) High School. The research involved two geometry classes (50 students), rated as average by the school, and taught by the same teacher. The SUPPOSER was the key tool in the instructional process. No text was used. Data were collected from class observations, homework of fifteen students, test scores during the year, PSAT scores, and interviews with students about their use of inductive thinking in the class.

Results indicated that the SUPPOSER facilitated the use of an approach which emphasized investigation and inductive thinking. Students were able to "re-invent" definitions and theorems that exist in the traditional curriculum and were capable of exploring new, interesting, and complex geometric ideas. Students demonstrated an ability to formalize their intuition into concepts that are integral to the Euclidean system.

Also during the 1984-1985 school year, a number of teachers from other Boston area school districts taught with the SUPPOSER and participated in a users group. Anecdotal data reported by members of this group were consistent with and reinforced the findings of the Weston pilot study.

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[1] The GEOMETRIC SUPPOSER is a series of tool software programs which enables users to carry out with ease constructions that are possible using straightedge and compass. Users can construct geometric shapes (e.g., triangles, quadrilaterals, and circles), draw a range of constructions (e.g., segments, medians, altitudes, angle bisectors, etc.) on those shapes, and make measurements on shapes and constructions. In addition, users can repeat constructions carried out on one shape on another shape of their choosing. The GEOMETRIC SUPPOSER is published by Sunburst Communications, Inc. .

Based on the results of the pilot research study and the experience of the users group, the research team decided to look in more detail at two transitions that students must make as they learn geometry in a guided inquiry context using the SUPPOSER: moving from the specific to the general and moving from conjecture to proof. In addition, an exploratory study was initiated to examine the roles of teachers and students in this approach and the relationships among student, curriculum, teacher and technology in the implementation of guided inquiry in geometry instruction. This research was carried out in three classrooms in three school districts. The research team introduced the SUPPOSER to the teachers and provided them with materials and support throughout the school year (see II.B. Intervention for details).

More specifically, the research examined the following questions:

<u>Implementation</u>

The SUPPOSER as a software program

How do students perceive the SUPPOSER? Do they find it difficult to use the SUPPOSER?

How do teachers perceive and use the SUPPOSER? When do they use it? When do they put it aside?

The guided inquiry approach

What effect does a guided inquiry approach have on the teaching of geometry -- the content, the order in which it is taught, the style in which non-inductive material is taught?

What effect does the guided inquiry approach have on the teacher-student relationship?

Do students' expectations of teacher roles and teachers' expectations of student roles change?

Is there a shift in control over the content and process of learning?

Do roles and relationships among students change?

What difficulties do teachers experience?

What difficulties do students experience?

Student Learning

The transition from the specific to the general

Are students able to generalize from data collected about specific figures to conjectures about classes of figures?

Over the course of the year, what is the evolution of students' ability to generalize?

What difficulties do students encounter in moving from the specific to the general?

At the end of the year, to what extent are students able to develop generalizations when presented with geometric situations and data?

The transition from generalization to proof

At what point do students seek to formalize their conjectures?

Over the course of the year, what is the evolution of students' ability to formalize their knowledge?

What kinds of evidence or process do they consider sufficient to establish "proof?"

At the end of the year, to what extent are students able to produce informal and formal proofs in response to true geometric statements?

The research team began the school year with the software series, some problem sets for teachers and students, and some ideas about guided inquiry and technology. Over the course of the year, working with teachers and students we learned what it takes to implement a guided inquiry approach using tool software in geometry instruction. We are grateful to the schools, the teachers, and the students for their readiness to try something new, for their patience as we learned together, and for their valuable contributions. Our insights and understanding are the product of their experiences, hard work, and occasional frustrations.

II. METHODS

A. Research Approach

To address the ambitious set of questions outlined in the Goals section above, the research strategy was to collect data from a variety of sources that would enable the project to assess student learning and to begin to understand the many and varied issues involved in the implementation of a guided inquiry approach using the SUPPOSER. Critical to the study were data on:

- student skills at the beginning and the end of the school year,
- the evolution of student skills and understanding over the course of the year,
- the evolution of teacher performance and understanding over the course of the year,
- student and teacher perceptions.

The intent was to provide a description of the year's experience built from several kinds of data and perspectives, providing a stronger basis for interpretation of both learning and implementation issues.

Below we list data sources by area of research and then in section II.C we describe the sources individually.

- Implementation: Data on implementation of the guided inquiry approach were collected from classroom observations, teacher meetings, teacher interviews and written reflections, and year-end student interviews.
- Student Learning: Data on student learning were collected from classroom observations, student work on problems, teacher meetings, teacher interviews and written reflections, year-end student interviews, and conjecture and argument tests.

B. The Intervention

1. PEDAGOGICAL GOALS AND ASSUMPTIONS

The rationale for teaching geometry as part of the standard high school curriculum is twofold: 1) to teach students about the measurement, properties, and relationships of points, lines, angles, surfaces, and solids; and 2) to teach students deductive reasoning by exposing them to classical Euclidean geometry, the archetype deductive system. Most geometry courses come up short on both counts.

The centerpiece of most geometry instruction is neither the "stuff" of geometry nor deductive thinking, but the two column geometric proof -- which in many respects seems to be beyond the grasp of many students. Students cope by memorizing theorems and proofs and come away from these experiences with no understanding and appreciation of either geometry or deductive reasoning and proof.

We argue that geometry instruction would be more effective if, rather than teaching definitions and theorems as givens and concentrating on proof, it were to give students an opportunity to experiment with geometric shapes and elements, to move from the particular to the general, and to make conjectures before grappling with proofs. This approach to geometry is absent from the "formal" secondary geometry curriculum. It is more common and more accepted in the "informal" geometry curriculum taught at lower grade levels -- in part because it is viewed as intuitive and lacking rigor. This division of reasoning skills and the implied hierarchy does not further the causes of geometry learning and teaching or students' development of reasoning skills.

We want to test the notion that by asking and enabling students to "explore" geometry, they will become more engaged in the subject matter, will approach the task of devising proofs with greater motivation and understanding, will become more skilled at inductive and deductive reasoning, and will learn more geometry. With the infusion of inquiry skills and a tool such as the GEOMETRIC SUPPOSER into geometry learning and teaching such an approach is feasible.

The GEOMETRIC SUPPOSER (Schwartz and Yerushalmy, 1984) is a series of microcomputer programs, each of which deals with a family of geometric shapes (Triangles, Quadrilaterals, Circles). The SUPPOSER allows users to make on the computer screen any construction that can be carried out with a
tedge and compass on a random shape or a shape of
n making. The program also includes supporting
es with which users can make measurements on any
in the construction, can make computations with those

measurements, can rescale, and can return to previous shapes with the same or different constructions. Perhaps most importantly, the SUPPOSER allows users to repeat constructions as procedures on other shapes. With these features, the SUPPOSER is a tool which enables students to explore the properties of shapes and geometric elements and to investigate whether the properties and consequences of a given construction on a given shape are dependent on some particular property of that shape, or if the result can be generalized.

In our instructional approach which we call "guided inquiry," the content of the curriculum is the same as a standard geometry course with only minor variations in sequencing. The goals are different. Rather than focussing only on deductive reasoning and proof, the guided inquiry approach calls for students to integrate inductive reasoning with deductive reasoning and empirical work with conceptual work in solving problems and devising proofs.

The pedagogy of guided inquiry can be differentiated from both the traditional lecture format and from discovery learning. While new material is often introduced by teacher lecture as in traditional pedagogy, guided inquiry emphasizes laboratory work and class discussion in which students take a more active and responsible role in the learning process. During a lab period, students work, usually in pairs, on a given task or problem. Discussion periods focus on the sharing of student data, conjectures, and supporting arguments generated from the lab work. In contrast to a discovery approach, guided inquiry does not call for students to discover every theorem in the year's curriculum on their own. In some cases, theorems are presented by the teacher and students investigate problems related to these theorems in the lab. In other cases, theorems are in fact brought to light by students in the course of manipulating data and making conjectures. Student inquiry is encouraged and aided by teachers modeling inquiry skills and supporting and guiding investigation in the lab.

2. SETTING AND SAMPLE

Experimental Classes

The sample for the study was three high school geometry classes in three different Boston area suburbs. The number of students was 44 (22 males, 22 females; 18 freshmen, 18 sophomores, 2 juniors, 1 senior). One class was rated by the school as an honors class; two classes were rated by their schools as low level. Two of the teachers had more than twelve years teaching experience; one had two years experience.

For more detailed descriptions of the three sites, see section III.A.

Comparison Classes

At each site there was a comparison class. In contrast to the experimental classes, the comparison classes were taught primarily from the text with an approach that focused on deductive reasoning and the two column proof. Comparison classes were selected by school administrators to match the SUPPOSER-using classes in ability and academic level. Since the expectation was that the instructional style of SUPPOSER-using teachers would change over the course of the year, comparison classes were taught by other teachers.

3. ENTREE AND SUPPORT

We began discussions with school systems about participating in the research in the spring of 1985, but it was not until the middle of September that the last of the site arrangements was completed. Identifying interested teachers who had the support of mathematics department chairpersons and principals, and who worked in schools which could provide students with access to computers (average two students per computer) as often as two or three periods per week was a challenge. We did not provide hardware to schools for instructional use.

Given the demands and extent of this study (entire school year, as many as two class periods per week), teachers and administrators expressed concerns about what students would learn and how much of the curriculum would be covered. In dealing with each school system, we assured the mathematics department that participating students would cover the same material as students in non-SUPPOSER geometry courses. From the outset, we took the position that we had a responsibility to the schools, the teachers, and the students and their parents to make this experience as productive as possible for each student and teacher.

Mid-September was a late start for a yearlong teaching experiment. We put aside our original plans for a week of training in August. We prepared a topic outline for the year's curriculum (see Appendix A) paralleling the traditional curriculum in content (with only one minor variation in sequence) and we prepared problem sets geared to the major topics in the curriculum (see Appendix A and Yerushalmy and Houde (1987) for examples of problems and projects). Each teacher received a computer for home use and the SUPPOSER programs and was asked to explore the software and problems on their own.

The teachers met with the researchers three times in the evening (2 - 2 1/2 hours) during the first month. We then assumed a regular meeting schedule, one evening approximately every three weeks. The agenda for these meetings included: teacher reports on the material covered since the last meeting, where and how they and their students experienced success or failure, sharing of suggestions and ideas, in-depth examination of topics that were of common concern, and plans and materials for future weeks.

In addition to these meetings, the most regular form of advice and technical assistance came from members of the team who functioned as classroom observers. In addition to collecting data, the observer discussed the class or lab session with the teacher and offered suggestions for dealing with specific problems or geometric concepts, for using the software more effectively, and for assisting students with their work.

At the outset, we envisioned two intensive periods of research and SUPPOSER use, each six to eight weeks in duration. The plans called for the first period in the fall to focus on how students collect data and make the transition to conjecture. In the middle of the academic year, there would be a less intensive period during which use of the SUPPOSER would be at the teachers' discretion. During the spring, the focus would turn the transition from conjecture to proof.

C. Data Sources

In this section, we describe each source of data.

1. Classroom Observations: Each class was observed approximately once every three weeks from October through mid-June. To ensure consistency of observation and reporting, observers initially visited classrooms together and in discussion developed an approach to observation. To check perceptions, each wrote up his/her observations of the same class, and then discussed the individual reports. The observers wrote reports on the class following each visit. These reports were reviewed regularly by the team and in response, were clarified or elaborated. The two primary observers were the project leader and researcher.

2. Student SUPPOSER Work: All written student work on SUPPOSER problems was collected. Each student maintained a three ring binder of his/her work and these materials

were collected and reviewed by teachers and then turned over to other members of the research group.

3. Teacher Meetings: Notes were taken at each group meeting and were shared with participants for their review.

4. Teacher Interviews (see Appendix B for questions): Teachers were interviewed in January for their perceptions of the experience to date. Questions asked them to describe their experiences, their expectations, the reactions of students, and their plans for the remainder of the year.

5. Teacher Reflections: In January and in June, teachers were asked to reflect on their experiences in writing with little or no direction. The only suggestion was that they write about issues that might be of interest to other teachers considering this approach.

6. Student Interviews (see Appendix C for questions): Eleven students from the three classes were interviewed in June. Questions were asked about using the SUPPOSER, the experience of learning and being taught with the SUPPOSER, what they would do differently if they were teaching with this approach, and whether this approach had any impact on their thinking or work outside geometry.

7. Generalization/Conjecture Tests (see Appendix D): A pretest and a posttest were designed to assess students' ability to make conjectures or general statements (see Yerushalmy 1985 for complete details). The tests present students with problems that are composed of a statement and diagram(s) that illustrate the statement. Problems on the tests are posed as data formulations and abstract formulations. In data formulations, the statement contains data and is designed to provide insight into students' ability to generalize from data. In the abstract formulations, the statement contains a generalization and is designed to provide insights into students' ability to derive "higher level generalizations" from the statements given in the problem. The tests ask students to "write significant connected statements" in response to the problem. While it is important to note what students consider to be "significant" and "connected" statements, the variable of greatest interest is the level of generality of the conjectures that students generate. These tests were administered to experimental and comparison classes.

8. Argument/Proof Test (see Appendix I): Designed to assess students' ability to produce proofs for true

statements (see Yerushalmy 1985 for details), this test presents three problems with two true statements accompanied by diagrams. It asks students to provide arguments or support for one of the two statements. Students are free to choose whichever statement they find more convincing. One problem contained statements which were familiar to all students and had been studied in class. Statements in the second were generalizations of material studied by the students. The third problem introduced unfamiliar material. For each problem, the focus was on the type of proof provided by the students. Are there proofs? Are they well done? Are they formal or informal in style? This test was administered to the experimental classes and to two of the comparison classes at the end of the school year.

D. Data Analysis

Since the results of the study are reported in terms of the different perspectives on the experience (the classroom, the teacher, the student, and the pretests and posttests), this section describes the procedures for data analysis in terms of the perspectives.

1. <u>The Classroom Experience</u>: This section is based on observers' classroom observations, analysis of student written work, teacher notes, and minutes of teacher meetings. The content of the observations was analyzed and themes were identified for each classroom. The other sources were analyzed for corroborating, contradictory, or illuminating data. As the themes were elaborated for each class, class approaches and style were characterized and contrasted with each other. The main theme in the three classroom descriptions is the difficulties students experienced making conjectures and the response of the teachers to these difficulties. Other themes include: skill in using the SUPPOSER, evolution of the lab and classroom sessions, working with visual data, attention to proof and the year-end projects done by students.

2. <u>Teacher Perspective</u>: Teacher meeting notes were summarized by issue (reports on classroom experience, introduction of problems, curriculum and teaching) and by date. These summaries were then analyzed to identify significant teacher concerns. Teachers' writing are presented as written with only minor typographical corrections. Midyear teacher interviews were analyzed by question.

3. <u>Student Perspective</u>: Year-end interviews with students were analyzed by question and across classes.

4. <u>Tests</u>: Results compare the performance of experimental and comparison class students. Chi-square analyses were performed to check for significant differences between these groups. The differences that were found were then interpreted in light of all of the other types of data reported earlier.

III. RESULTS

The results of the study reported here are organized in terms of the classroom perspective, the teacher perspective, the student perspective, and the conjecture and argument tests. This framework and order is designed to provide a coherent portrait of the experience of each of the classes, and at the same time, to offer a context in which to consider the perspectives of the key participants, teachers and students, and the performance of experimental and comparison classes on the tests. In the conclusion, these results will be integrated in terms of the research areas (implementation and student learning) and the research questions.*

A. The Classroom Experience

This section describes each of three classes and their experiences with the SUPPOSER over the course of the year. The descriptions are drawn from classroom observations, student work, and teachers' meetings and notes. Within the description of each class, the focus is on the evolution of key instructional elements, teacher skills, and student skills over the course of the year. These key elements include: conjecture-making, SUPPOSER use, the lab experience, classroom discussion, working with visual data, proof, and year-end projects. In addition, these descriptions are designed to provide a setting for other results. Each description concludes with a brief characterization of the year's experience in the class.

1. COUNTRYTOWN

Background

The class was made up of eighteen students: twelve sophomores, five juniors, and one senior; eight males and ten

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* The results of two other facets of this study, student ability to manipulate visual images and an analysis of factors involved in the structuring and posing of problems, will be alluded to in the conclusion and summary of this report and will be reported in detail in two papers which are now in preparation.

females. Two of the students were taking geometry for the second time. This class was considered the lowest level geometry class in the school.

This class only met four times a week, although each period was fifty two minutes long.

The computer lab was two doors away from the classroom. Computers were on desks in a horseshoe arrangement in the center of the room with desks around the perimeter. Students were able to use the lab at any time by getting the key from a teacher.

The teacher had two years of experience teaching mathematics at the high school level and was a product of a masters program in mathematics, science, technology and education which combined teaching in the public schools with experience in industry.

Unlike the other teachers, the Countrytown teacher did not hand out a textbook. He preferred to photocopy packets of homework problems from a number of texts for a period of a week or two.

On the midyear exam, the class scored better than some of the other geometry classes taught by the same teacher. At the end of the year, they did just as well as these other classes.

Use of the SUPPOSER

At the beginning of the year, the teacher in Countrytown reported being concerned that his students would not take the SUPPOSER and the lab periods seriously; the students in Countrytown thought that finding patterns of numbers in their data was the goal of the lab. Both of these initial perceptions changed dramatically over the course of the year.

In October and November, students concentrated only on measurements and numerical data and were not concerned with the geometric relations that these numbers represented. They were not selective about what they measured. Often they worked on a trial and error basis. They did not work systematically with variables and keep track of the data. For example, one problem asked students to draw the median from each vertex in each kind of triangle and to look at the triangles created by the construction. Students had difficulty sorting out which properties were related to the vertex from which the median was drawn and which related to the triangle type. In the end, the teacher wrote out the problem with a page for each triangle, thus isolating the variables and providing a structure for organizing the data and its collection.

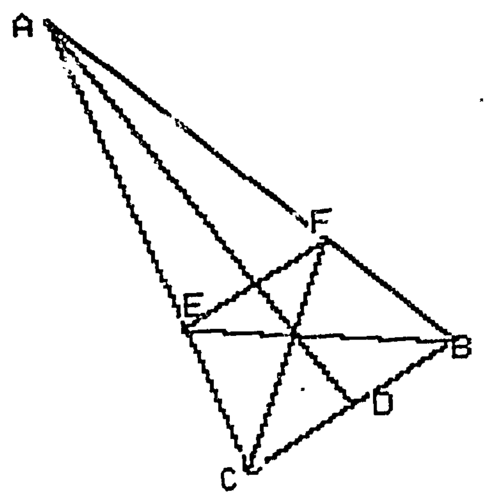
Students also seemed to be held captive by the particular diagram presented in a problem. In the lab, we observed that students' first step in solving a problem was to search for a triangle on the computer screen that matched the triangle on the worksheet in shape and orientation. If an exact match could not be found, students might rotate the paper until the orientations matched.

Class discussions got off the ground slowly. Initially, the discussions centered on the data and its relationship to conjectures. To make the discussions run more smoothly, the teacher prepared summary sheets of student data and conjectures.

In November, the focus moved gradually to generating convincing arguments for conjectures. The students in this class had weak algebra backgrounds. Early on when the teacher based many of the arguments and proofs on algebraic manipulations, the students had difficulty following the algebra and therefore the arguments. A lack of mastery and skill from previous courses confounded their first experience with informal arguments.

By midyear, we witnessed considerable growth and development in the lab and in class discussions. The differences were striking. Students were no longer bound by diagrams; they were now able to visualize and manipulate relationships in their heads. In discussions, they were able to look at a diagram and imagin transformations without requiring actual drawings.

For example, the class was working with a drawing which included two parallel lines, BC and EF.



The students were not told that the lines were parallel, they had not yet conjectured that they were parallel, and with the knowledge they had at that point in the year, they could not prove that these lines were parallel.

It was however a legitimate and verifiable conjecture. AD was an altitude and thus perpendicular to BC. One student argued that AD was also perpendicular to EF, implying that EF and BC were parallel, but not mentioning that fact. Another student argued that "it doesn't work if EF is tilted". In other words, it doesn't work if EF is not parallel to BC. This assertion was not based on the way the diagram looked. The student was manipulating the diagram and considering an alternative in her "mind's eye."

In November, two column proofs became a part of the class discussions and the teacher asked students for their assistance in writing out their informal arguments in this form. Students began to call for proofs when a proof was in order. They knew the difference between verifying a conjecture and a proof, although they would sometimes use the word "proof" for both types of arguments.

Starting in November, the teacher made regular use of visual aids and concrete models in class. Transparencies and an overhead projector were a fixture in discussions. He brought some cardboard triangles, a weight, and some string for the discussion of altitudes, coffee stirrers for diagonals in quadrilaterals, and "Miras" to clarify the idea of reflection. At times, even compass and straightedge made an appearance and students worked on constructions at their desks as part of the class discussions.

By December, students had yet to use the REPEAT key on the SUPPOSER as a strategy for thinking about a problem (seeing how a given construction behaves on other triangles) before making measurements. However, when the teacher suggested to some students that this might be a profitable strategy, many students picked it up quickly. They became sensitive to and interested in visual clues that appeared as a construction was repeated on different types of triangles.

In the lab, the style of work changed. Students were beginning to approach problems systematically. For example, they were able to articulate when it was a good strategy to begin their work in an equilateral triangle, and not in an acute triangle. The teacher had always been meticulous in making his drawings in class; now many students practiced the same care in their work. They began to use compass and straightedge to help record their data from the SUPPOSER screen and to produce work that was neat and easy to read.

The teacher reported that students considered the lab and class discussions about their conjectures as both hard work and fun. He felt that when he did algebraic textbook problems in class students showed less interest. They did not find these problems as difficult or as exciting, which may have been in part the byproduct of their dislike of algebra.

At the end of December, expectations changed. Lab problems now demanded proof before class discussion, not just a convincing argument. Finding conjectures was no longer considered difficult nor was it sufficient. Proof was important. Data receded into the background.

Throughout the year, the teacher made extensive comments on student papers. Now he wrote, "You must have all three: data, conjectures, and proof."

In March, expectations changed again. The teacher gave out an assignment that he called a "project." The content of the project was reflections. With these larger problems, students had to work in the lab on their own for a longer period of time and to write up their work neatly and hand it in before the class discussion Instructions were explicit: data was not important; conjectures and proofs, where possible were the focus.

While the students' lab work focussed on the projects for three or four days running, they still had homework. Their homework did not directly relate to the work in the lab. It consisted of sheets of problems photocopied from a textbook on the topic of special right triangles.

The students took the first project very seriously and produced good work. There were many conjectures, but few students came up with proofs. However, this may have been due to the problem and its presentation. The problem emphasized making constructions and did not present a clear definition of a reflection and so, students lacked the necessary tools to make formal proofs.

In general, March and April lab periods were extremely productive. Students made predictions about data and outcomes before making measurements. They worked hard outside of class, coming into discussions ready to prove simple conjectures. Many students, even the quiet ones, participated in proving these conjectures. On occasion, students came up with different proofs. When a problem and a proof were more complex, the teacher stepped through the proof with the help of his students.

In May, the teacher gave students a second project. He encouraged students to collect only a minimum amount of data and to report those data that were essential. He asked for a list of conjectures from each student after the lab work. A week after collecting the conjectures, students turned in proofs. Students worked diligently and produced in-depth reports. Every student produced at least three proofs. Some of the proofs were complicated, but the majority of the proofs were simple and small in scale. Students still had difficulty getting an involved formal proof off the ground and giving formal written form to ideas that were voiced easily and informally in class discussion.

Conclusion

On the whole, students in this class were not able to prove general results, or to generalize their conjectures from triangles to quadrilaterals. Yet at year's end, their inquiry was systematic and thorough, they were able to work on large scale problems, to generate conjectures, and to prove some of their conjectures.

2. RIVERTOWN

Background

The class was composed of eighteen students: eleven males and seven females, all freshmen. The school considered this class an honors class. The teacher did not consider them to be "honors" caliber.

The class met during the last period of the day. The computer lab was two floors below the regular classroom. The computers were arranged along the walls and there were no desks or other work surfaces in the room. A teacher had to open the computer lab and be present for students to use the lab. The teacher had twelve years experience, teaching all levels and subjects in high school mathematics, and held an undergraduate degree in mathematics and an M.A.T.

Use of the SUPPOSER

In the beginning of the year, the experience of students in Rivertown paralleled that of students in Countrytown. They, too, focused on numerical data, giving little thought to the context or the diagram, were not selective about what they measured, and worked on a trial and error basis. However, they had a more difficult time producing conjectures than the students in Countrytown. The teacher was convinced that this difficulty stemmed from an inability to see patterns in the numerical data. So she carefully structured their inquiry, rewriting problems, providing explicit directions and charts for recording data. She hoped that this structured approach would make patterns in the data apparent and lead students to conjectures.

During the early part of the year, this produced more complete and better organized numerical data, but no improvement in conjecturing. Students continued to focus on the numbers and not on the geometric relationships.

Class discussions also started off slowly. The problem seemed to be in part how to blend student data into the discussion. (Unless the characteristics of a triangle are specified (SSS, SAS, ASA), the SUPPOSER will draw a random acute, obtuse, right, isosceles, or equilateral triangle after the student identifies the type of triangle to be drawn. So each student or pair of students, though working on the same problem, is working on a unique triangle and brings a unique set of data from the lab to the class discussion.) Asking students as a class to reason with their data turned out to be awkward and difficult. The teacher took alternative paths -- discussion without reference to student data or a dialogue with a single student.

As a result, the first discussion of the year looked a lot like a traditional geometry class. The teacher introduced formal proofs for the students' conjectures. Later in the period, she took those proven conjectures and she posed numerical problems based on the relationships established in the proofs. These were similar to those found in a textbook. An example of a typical textbook problem is:

> If $\angle$ACB is 51 degrees and the opposite exterior angle is 112 degrees, what is the size of $\angle$ABC?

Recognizing that such a strategy inhibited student discussion and conjecture making, she cut back on the amount of formal proof in the following sessions. The next discussion centered on medians in right triangles. Students participated enthusiastically. They arrived at conjectures that were true for the right angle vertex and not true for the other vertices. They also developed separate conjectures for an isosceles right triangle.

As the year rolled on, the links between numerical data and the geometric construction, or the visual data, were never forged. Conjectures, for the most part, continued to focus on numerical relationships. Students did not seem to make the connection between formal proof and the conjectures developed in the lab. They did not see the potential role and value of deductive/textbook knowledge in the lab. In the lab, students were convinced that if the numerical data supported the conjecture, that proved the conjecture. In class, proof meant a two column proof. One day a quiz was given in the lab and it asked for proofs. At first the students were stymied. When the teacher explained that they were being asked to do proofs "like in class," they immediately produced two column proofs. The differences and relationships between class and lab, and among data, conjecture, and proof were never clear.

In discussions, students did not seem to benefit from the work or insights of the rest of the class. The discussions that followed work in the lab were not occasions for community inquiry.

How to share student data and how to verify conjectures based on data continued to be an issue. During one discussion on ratios in similar triangles, the teacher had students regroup to sit next to their lab partners to make it easier to link conjectures to specific data. They were then asked to manipulate their data and verify the conjectures that had been discussed. Some students had difficulty manipulating the ratios; some were able to verify the conjectures. With the students working in twos, it was difficult for the teacher to help the class as a whole and there was not enough time for her to work with each pair.

In the lab, students had difficulty understanding instructions and getting down to work. The teacher spent a lot of time going over instructions, but it seemed to have little effect. During the lab, many students had their hands up, constantly calling for the teacher's attention. This detracted from the amount of work students accomplished in the lab and was hard on the teacher.

As in Countrytown, students did not discover the use of the REPEAT key. When its use was demonstrated, they too incorporated it into their lab work.

At this point, students did what they were asked to do, but no more. When asked to put numbers in a table, they complied. On the whole, they limited their inquiry to the numerical information requested in the problem. On one problem, after reading the directions, students had no idea of what to do, so they turned to the back of the packet of papers, found a chart, and measured the elements listed on that chart. They did not go on to make conjectures or proofs.

During the months of March and April, when teachers were given more discretion about how and when to use the SUPPOSER, this class used the SUPPOSER infrequently. They used the SUPPOSER to investigate similar triangles, but did very little work with the SUPPOSER on quadrilaterals or circles.

At the end of the year, the teacher gave the students a larger scale problem, "a project," to work on. As preparation for this problem, the teacher wrote explicit instructions that detailed a method for approaching a larger problem. She asked students to rewrite the problem in their own words and to outline the way in which they planned to solve it before beginning the problem. The teacher from Countrytown borrowed this approach as well. Classes in Rivertown and Countrytown worked on the same project.

Students practiced this structured approach on a smaller problem before tackling the larger problem. Having collected their data during four lab periods, they were given time to make conjectures from the data. A week after handing in their conjectures, they were asked to produce proofs for three conjectures. An "A" student paper from an honors class (not in the study) in Countrytown was shared with the students as a model of excellent work. To further motivate students, the work on this large problem counted as the equivalent of two test scores on students' final grades.

After two weeks without a lab session, students seemed to enjoy returning to the lab. They worked hard on the problem. In the lab this time, there were few questions for the teacher. She was able to circulate around the room at a more leisurely pace. For the first time in the lab, she felt that she had the time and the opportunity to answer student questions and monitor their progress. The students collected large amounts of numerical data and collected it neatly. They used compass, straightedge, and a circle drawing tool to record their diagrams neatly. Many of them worked on the problem after school or during their free periods. They traded telephone numbers and worked at home. There was an element of excitement in the air. One student remarked that the problem was exciting because there was a lot to work on and write about. She explained that she just had to figure out which ideas were important and which were trivial and therefore not good material for conjectures or further work and writing.

With these problems, students exhibited new skills in the lab. Some students paid careful attention to visual data and noticed quadrilaterals in their constructions as well as triangles. One student, using deductive knowledge to check her measurements, noticed that she had mistaken an area measurement for an angle measurement. She presented an argument that showed why that measure could not possibly be the measure of the angle.

Conclusion

In general, students in Rivertown had difficulty dealing with a lack of structure and with independent work. In an effort to provide students with structure and clarity, the teacher focussed their attention on numbers and tables, not on figures and geometry. Perhaps this emphasis on the numerical data prevented them from looking at patterns of visual data, an alternative route to conjecture which was effective in other classes. Students had difficulty making the connection between visual and numerical data and the inductive work with the SUPPOSER was not well integrated with the deductive work in the classroom. However, by the end of the year there was some improvement in these two areas as evidenced by the project work.

3. TECHTOWN

Background

This class was composed of eight students: six sophomores and two juniors, three males and five females. It was considered a geometry class of low ability. The class met in the computer laboratory during the middle of the day. The computers were against the walls and the rest of the room was organized and furnished as a regular classroom with desks in rows. The teacher had over sixteen years experience teaching math at the middle and secondary school levels including five years in England. She held an undergraduate degree in mathematics and an M.A. in mathematics and science education. She was a member of the EDC-sponsored GEOMETRIC SUPPOSER Users' Group during the 1984-1985 school year.

Use of the SUPPOSER

Like the students in Rivertown and Countrytown, the students in Techtown were not able to make conjectures on their own from data at the beginning of the year. They concentrated on numbers and not on geometric relationships. For example, they seemed not to notice changes in visual data. In their diagrams, they often drew what they thought was or should be true, rather than copying accurately what appeared on the screen. When working on the definition of altitudes, one student presented the following drawing as evidence to support her definition. She claimed that her drawing was an accurate copy from the screen of the SUPPOSER.

Her definition of an altitude was "a line in a triangle that makes ninety degrees."

The Techtown teacher reacted differently to such misconceptions, which surfaced in all three classes. In discussions, she took the students' data and helped them make generalizations from their data during class time. She used several techniques to deal with the problem of organizing, sharing, and discussing student data. She had students collect their data on ditto sheets and then ran off copies before the class discussions. At times, she recorded data on an overhead so that everyone could see her work and then demonstrated how she moved from the data to conjectures. During some classes she worked with a computer and a large monitor at the front of the class. One student sat at the keyboard, creating the constructions and making the measurements that arose during the discussion. If she felt that all of the students needed to do more work on their own, she simply stopped, sent them off to work on the computers and reconvened the class when they finished their investigations.

Discussions in Techtown were productive from the outset. This was no doubt in part a function of the teacher's previous work with SUPPOSER and the size of the class. Many students participated. Together, students and teacher made conjectures, discussed criteria for "good" conjectures, worked on convincing arguments, and talked about what "good" and "suspicious" data were. In these discussions, students listened attentively to one another.

Another feature of these discussions was the number of visual arguments that were considered. To show that a student's definition was inadequate, the teacher made a drawing that conformed to the definition, highlighting its inadequacy. In one case, the student quickly saw that the words of the definition did not conform to her intent or to the figures in her head. She was then able to modify her definition appropriately. The teacher was constantly modeling an approach that made apparent the relationship between numerical and visual data. To reinforce this, a test early in the year contained a number of questions in which students were asked to define a term with a diagram rather than a written definition.

In the lab, students were encouraged to collect their data and write them on drawings of the figures that appeared on the screen. Most of the students did not use tables or charts.

As the year progressed, the teacher felt that the students needed less direction in the lab. She used the lab time as an opportunity to work with students individually on their textbook homework or on topics in which they seemed to need additional help. If students on the computers did need assistance, she stopped the one-on-one work and attended to the lab.

Students also collected less data in the lab. They tried to develop conjectures first and used data to check their conjectures. In that sense, they were more systematic. Students were free to develop their own formats for collecting data and although this might have created problems, the recording of data was very good. One student recorded in a mosaic fashion, placing data from each triangle in or around the drawing of the triangle. Each triangle and its data were then enclosed by a circle. Another student recorded data in lists and a third collected very little data once she had a conjecture.

In talking with the teacher about the students' abilities to devise proofs, she indicated that the students could produce proofs if they were led in that direction.

During the months of February, March and April, the class did less individual work in the lab. They investigated as a group under the direction of the teacher. Students participated actively in these discussions. Discussions focussed on examples prepared by the teacher, creating formal proofs for conjectures, and conversations about necessary and sufficient conditions and counter arguments. The topics covered were similarity, quadrilaterals and circles.

Visual data continued to be emphasized. Students were more careful about their drawings. In one discussion, a student was asked to go to the board to draw a parallelogram. She drew a rectangle with the short legs horizontal. Students pointed out that her diagram was a parallelogram, but that it was not a general one and therefore not appropriate for the discussion. In a later discussion, a student was to go to the board to draw a trapezoid from her data. Her drawing looked like a right trapezoid. When asked about her drawing, she responded that her drawing was to illustrate her data and that the angle in this case was 94 degrees. With this emphasis on visual data, it seems that some of the students were aware of the differences between a drawing and a schematic diagram and when each was appropriate.

When students worked on their own in the lab, their conjecture making and data collecting were integrated. They used their ideas to decide what to measure and what not to measure.

In May, the teacher gave students a set of three projects and allowed them to choose among them. One was similar to the first project assigned in Rivertown and Countrytown. The second focussed on points of concurrence and the third asked for methods to create similar triangles inside a larger triangle.

Techtown students were not provided with detailed instructions and were not asked to rephrase the question or to outline their approach to the problem.

The students had many questions about what they were supposed to do. The teacher clarified the project for each student or pair of students. The results were uneven. Some students worked towards generalizations, while others had difficulty getting started. Some were able to formalize their conjectures and others were not. All of the students were asked to provide informal proofs or an outline of a formal proof. Many were able to do so.

Conclusion

In Techtown, the teacher responded to the class' initial difficulties with data and conjecture making with group work and modelling. She exposed students to effective mathematical thinking skills and problem-solving strategies. She focussed on visual skills, logical arguments, and the criteria for doing good inductive work. Students participated actively in these discussions. While they got off to a slow start and continued to experience some trouble in the initial stages of tackling a problem, by the end of the year they were able to generate conjectures and to prove some of those conjectures in the context of a large problem. They had internalized some of the strategies that had been modelled for them.

B. Teacher Perspective

To provide insights into the teacher perspective on the experience, we present in this section results from two research activities: an overview of monthly teacher meetings and efforts to capture teachers' reflections including mid-year and year-end written statements and a report of mid-year interviews.

1. TEACHER MEETINGS

During the school year, the research team met every three weeks or so with the teachers. Below are first an overview of the content and tone of the meetings and then an outline of key issues that surfaced during the sessions.

The agenda for each meeting was established at each session, but in general centered on three items: teacher reports on their experiences since the previous session, discussion of problems and problem sets, and discussion of curriculum and teaching issues.

Reporting on Class Work

Every meeting opened with a round robin reporting by the teachers on their experiences with some additional commentary by members of the team who had visited the classes. Reported were triumphs and failures: when a class soared with a problem, when and how similarity went over like a lead balloon, how the pep rally preempted geometry, and when the kids showed up in the lab when they were supposed to be in the classroom. The three teachers covered the curriculum at about the same rate. People listened carefully and learned from each other. The spirit of the meetings was consistently open, frank, and supportive.

Introducing Problem Sets

The research team prepared problem sets which addressed the major topics in the geometry curriculum and followed the sequence in the most commonly used texts. The expectation was that classes would go through the sets in two or three weeks, but it quickly became apparent that these classes were moving more slowly and that there were many more problems in a set than teachers and students could handle. It was clear that teachers had to pick and choose.

Problem sets were distributed about a month before they were scheduled to be used, thinking that teachers would have plenty of time to work on them. In fact this was unrealistic given the day-to-day demands of teaching and teachers ended up reading the problems just before the meeting. So the research team presented the intent and the potential for each problem and the teachers commented. The problems were discussed again when they were used in class. These discussions were most productive when teachers were covering the same material in their classes (even when they were using different problems). When the teachers were covering different material, conversation was limited.

There was always a computer available to try out the problems when they were distributed, but rarely was it used during the meetings. People preferred to look at the problems and solve them in discussion, on paper, without the computer. The computer was used only to demonstrate one of the SUPPOSER procedures or to introduce another disk, e.g., Circles, in the SUPPOSER series.

At the outset, teachers used the problems as they were written. Late in the fall, we started using the meeting as an occasion to rewrite problems as a group, and then teachers began redrafting problems on their own. Teachers traded problems and formats at the meetings, but chose not to share homework problems and worksheets, or the task of grading.

Discussing Curriculum and Teaching Issues

Using the SUPPOSER posed a variety of challenges for the teachers. They had to teach geometry differently, to use class time differently, to adapt existing materials and to design new materials and strategies for use with the SUPPOSER, and to change their approaches to student evaluation and homework. The experience also gave them new perspectives on geometry and how to teach geometry. Summarized below are five key issues and two observations that were raised in the context of the meetings.

The Structure of Problems: In these meetings, the issue that came up time and again was the structure of problems. One teacher explained that he was not able to define for the students what was required and was not sure when and what to collect from his students (10/9/85). The group struggled with what constituted a good SUPPOSER problem and how to structure SUPPOSER problems. There seemed to be a fine line between providing students with adequate guidance and providing so much direction that the task lost all meaning.* When they rewrote problems, teachers tended to add structure to problems and to be more directive. This was particularly true when they were concerned that the students were not making conjectures.

Teachers reported that construction problems (i.e., problems whose outcome was a construction rather than a conjecture) were very successful. These problems elicited a good deal of quality student work and were popular among the students.

Curriculum Coverage: A second concern was whether there was sufficient time to cover all the content in the geometry syllabus. Teaching with the SUPPOSER in a guided inquiry approach requires more time than a lecture format. Student investigation in the lab and class review of lab work are time-consuming. Also, a more

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* The structure of problems and the construction problem phenomenon is discussed in Yerushalmy and Chazan (1987) and is analyzed in detail in a paper on problem posing that is now in preparation (Yerushalmy, Chazan, and Gordon 1987.)

open-ended approach which invites in-depth study of any topic is likely to take longer. Since class time is limited, all of these demands placed pressure on teachers. One approach to coping with these pressures is to be a good manager of instruction -- having well-defined objectives in posing any problem and an efficient and effective strategy for managing the collection, collation, and discussion of diverse student data and conjectures. Another response is to work on fewer SUPPOSER problems. In the end, all classes covered approximately the same material as their counterparts who did not use the SUPPOSER. Some of the teachers indicated to us and to their students that they rushed through the material at the close of the school year. (1/15/86, 5/28/86)

Similarity: All teachers reported that the concept of similarity was the most difficult in their geometry curriculum. Students did not seem to understand proportions, correspondence or the ratios in right triangles. Teachers and the research team developed a variety of approaches and problems, but this is one concept where the use of the SUPPOSER did not produce greater student understanding.

Integrating Lab and Class Experiences: This issue has several elements. Teachers reported that they, along with their students, sometimes found it difficult to go back and forth between lab and classroom, SUPPOSER and textbook, and inductive and deductive work. One teacher related the story of a student conjecturing (inappropriately) on a cut and dried homework problem. Another teacher tried to do a textbook problem while in the lab and the kids didn't want to do it unless they were in the classroom.

Teachers also found it difficult to communicate to students the relationship between inductive and deductive work, i.e., using deductive knowledge gained in class to guide and to enrich exploration with the SUPPOSER, and bringing conjectures developed in the lab back to the classroom for proof.

This issue has a physical manifestation in the separation of the lab from the classroom in two of the sites. When the computers are not in the class, planning and scheduling class time takes on greater significance and integrating the instructional experiences seems to be a more difficult task.

Homework: Another issue related to the connection between classroom and lab sessions is that of homework. Teachers found that with the additional burdens of lab and discussion periods they were not able to keep on top of student assignments and review of student homework. They were concerned that students interpreted this to mean that homework was not important and therefore fewer students did the homework.

> Discussions of lab work and discussion of homework competed for class time. While teachers felt that lab work did not get adequate attention and discussion, students in some cases felt their homework was being ignored. (1/22/86)

<u>Grading</u>: Grading student work on the SUPPOSER was a complex issue. Setting and communicating appropriate expectations for teachers and students was very tricky. Teachers accustomed to one right answer found grading SUPPOSER papers somewhat arbitrary. The suggestion to give two grades, one for data and conjectures and one for proofs, was helpful, but did not provide sufficient direction. Elaborate schemes would be difficult to explain to students. So when teachers wanted to define a standard for student work, they posted a copy of exemplary student work on a project from an honors class (not in the study).

Teachers were also eager to test their students on the kinds of tasks that they were doing in the lab. This seemed to be related to two concerns: it seemed unfair to evaluate their students on the basis of standard geometry tests alone and it was important to be able to test for SUPPOSER skills. In each of the classes, there was at least one quiz or test that involved use of the computers. In some of the classes, teachers made an effort to create SUPPOSER-like problems for paper and pencil exams.

2. TEACHER REFLECTIONS

To capture teachers' perceptions directly, the team asked the three teachers to write down impressions of their experience with the SUPPOSER and the experiences of their students. This writing took place in January, half way through the school year, and again in June, at the end of school. In addition, one member of the research group interviewed the teachers in January. To provide chronological continuity in this section, the order of material is January teacher writings, January interviews, and June writings. The teachers' written impressions have not been edited.

January Teacher Writings

Countrytown

I found working with the SUPPOSER challenging, stimulating, and sometimes frustrating. I have tried to carry my personal enthusiasm for geometry and the exploratory possibilities available with the SUPPOSER into the class. Overall students' reactions have been mixed. For example, one student enjoys using the SUPPOSER to build convincing arguments for class discussion. Another enjoys the independence of working on her own. Other students have expressed concern about being used as "guinea pigs" in an "experimental" program. In general, student response seems to be dependent on their ability to experience success with the SUPPOSER as a tool of geometry. Those students who feel comfortable with mathematics are more apt to respond positively to the use of the SUPPOSER. I feel some students are confused by the lack of structure evident in a traditional geometry course. The flexibility needed to work with the software sometimes creates confusion and ultimately results in a struggle between teacher and student. I find the most effective and enjoyable classes to be those in which we can "lay back" a bit, discuss lab work, and relate it to outside "book" assignments. However, it is equally important to reinforce structure and bring the class together (e.g., at the conclusion of a unit).

In sum, I am anxious to begin the second half of the school year using my experience with the SUPPOSER to improve classroom activity. I plan to write more lab material that is tailored to the needs of my students and to more effectively coordinate lab, class, and outside assignments.

Rivertown

Working with the SUPPOSER, I've come to the conclusion that students need a considerable amount of direction before attacking the problems. They need time in the classroom acquiring

1) the basic definitions, techniques, and ideas that will be needed in the unit,
2) an outline of the objectives of the unit to be investigated,
3) rather precise directions -- step by step procedure and format to be used in recording the data and making conjectures (almost a model).

Most students have not been able to read the directions, have them explained, and then understand their responsibility. More often than not, each pair needs individual instruction.

Although students finally may come to some conclusions from their data, they seem to lose something in the transition from the material acquired from the SUPPOSER and the application of these concepts. They still need a great deal of practice and additional explanations. I would have thought that the visual experience would have been more impressive and meaningful.

On the other hand, students working with the SUPPOSER seem to be more relaxed and more willing to explore in collecting data. Nevertheless, they are still not willing to conjecture if they don't think that it's correct. In general, students seem to enjoy the lab more than the classroom. Partly, this is due to their freedom and apparent sense of non-accountability.

Techtown

Students using the SUPPOSER program are more vocal when they do not understand a concept or a problem. This may bring the class to a temporary grinding halt, as happened when we were exploring the proportions formed by the altitude from the right angle of a right triangle. Students could not "see" the similarity between the large triangle and the two smaller triangles because some turning and flipping was necessary to bring all three triangles to the same orientation. A day later, after cutting, flipping, and matching, students seemed satisfied that the triangles were indeed "same shape" similar, but still had trouble formulating proportions in the original diagram. Several students were still hazy about the notions of corresponding sides and of ratio, but they were willing to engage in simpler activities using ratio and to struggle until they were satisfied that they could derive the proportions themselves.

I do not think that this process would have occurred in a traditional geometry classroom. In such a classroom, I believe, students act as if the teacher "owns" the theorem or formula. They are able to do a reasonable job in getting the correct answer to textbook problems by using a rote method of copying the teacher's model. However, in average level classes, there is little understanding, and little long-term retention. Topics appear to students to be a collection of isolated and random results conjured up by teacher or text. In SUPPOSER classes, where students have been encouraged to look for patterns and to formulate conjectures, they are more active, even in teacher-directed lessons. The process of coming to some understanding beyond the rote level of "getting the right answer" is time consuming and sometimes uncomfortable, but I believe that it is valuable.

After work with the altitude in a right triangle, students went on to tackle right triangle trigonometry. They used the

SUPPOSER to rescale a right triangle and to check that the ratios remained constant and matched those in the table in the back of their text. One student remained confused and could not choose the appropriate ratio to use in a numerical problem. Other students joined in clarifying and rephrasing an explanation for her. It seemed that there was a consensus that it was important that she understand what she was doing. There were no complaints and no ridicule. I am happy that we have a class in which students feel free to express their doubts and difficulties, and in which other students feel comfortable when they jump in and offer help.

<u>Mid-year Teacher Interviews</u>

In these interviews, the teachers were asked to reflect on their first four months using the SUPPOSER, their experiences, their expectations, the reactions of their students, and their plans for the remainder of the year.

(To assist the reader in linking statements to classes, we will refer to the Countrytown teacher as teacher C, the Rivertown teacher as teacher R, and the Techtown teacher as teacher T)

All three teachers expressed some misgivings about the experience.

The teachers in Rivertown and Countrytown who were using the SUPPOSER for the first time this year emphasized that they felt inadequately prepared. (The fact is that due to logistical and scheduling problems they received little training.)

Two of the teachers, teacher R and the Techtown teacher who was using the software for the second year, expressed disappointment in the performance of their students. They attributed the problem in part to the characteristics of the students. Teacher R thought that the ability and the age of her all freshman class were major factors. This teacher indicated that students are used to doing more concrete work and to learning in a context where there is one right answer. She also said, "These are ninth graders who seem immature and can't sit still." She reported that in conversation with other teachers in her school that the behavior and performance of this particular group of students was not unique to her geometry class. All of this was exacerbated by the fact that the class is scheduled for the last period of the day and that a number of the boys in the class play freshman sports which means early dismissals and frequent absences.

Teacher T also thought that the lack of progress related to the students' ability level. Based on the performance of last year's SUPPOSER class, an average class, she expected that this year's class, even though below average students, would be farther along and that the SUPPOSER would be a motivating factor for these students. She wondered whether these students simply operate on a totally concrete level and that to move from the specific to the general to proof may just be too much for them.

Both teachers thought that these students needed much more structure and direction than they had anticipated.

While concerned about student learning, both witnessed some growth in the students from October to January. Teacher T noted that students were getting better at organizing data and that some sense of generalization was starting to emerge. Even for those students who needed the most structure, there seemed to be some transfer. She also observed that the SUPPOSER provides no opportunity for evasion. In traditional geometry, students can get by with memorizing theorems, axioms, proofs. With the SUPPOSER, a student cannot fake what he or she knows, and it quickly becomes apparent when and where a student is having trouble. In this respect, the SUPPOSER serves as a diagnostic tool.

Teacher R indicated that the students were not making conjectures, were not listening to one another, or were not working as a group. Also, they only seemed to test their ideas on equilateral and isosceles triangles ("where they know they'll find something"), were not precise with their language, and did not express their generalizations in specific terms. She said that the students were collecting reams of data and making very few if any generalizations. All of which made class discussions about conjectures and generalizations difficult, if not impossible. She did acknowledge, however, that earlier in the year, they were not even collecting any data. She said that she found it difficult to know what students "would be going away with" and thought that to find out, she would have to spend time (which she does not have) working with kids on an individual basis.

Teachers R and T thought that they were seeing progress in class, but that the progress did not seem to get reflected on quizzes and exams, even on so-called "SUPPOSER problems." (All teachers reported that their SUPPOSER students performed as well on the mid-year exam as their comparable non-SUPPOSER classes.)

Teacher C described a shaky start in his class, but while he also had a lower than average class, he attributed the students' problems to his own difficulties. He reported that he was intimidated initially by the SUPPOSER. At the beginning of the year, he said that he was unclear about the goals and the objectives and had no sense of what was going on, and that his students picked up on this lack of clarity. Initially students were withdrawn and quiet. They did not react or engage. Now they were starting to fight back, seemed interested in what they were doing and in how they were doing. Some of the students had reached the stage where they were thinking, not just responding mechanically, and knew that they could not just sit back and let things go by.

Both "new" teachers (R and C) indicated that although they thought that they were comfortable with the inductive process and "teaching inductively," they both reported that using the SUPPOSER requires a lot of preparation time and a different kind of preparation. Beyond getting familiar with the software, teacher R said that ideally you should spend one or two hours every night working on problems and getting data organized -- which is not feasible. Furthermore, it is impossible to lay out lessons in advance or to plan too far ahead. ("Even if you are prepared, something may happen in class that you did not expect.") She indicated that she had not put in sufficient time.

Teacher C said that initially he could have spent (and sometimes did spend) 10 hours per week on the SUPPOSER playing with the software, working through the problems, trying to stay ahead of the class. It required much more preparation than his other classes -- too much more. He also stated that learning intuitively was a learning style that was familiar, enjoyable, and productive for him. After investing a significant amount of time getting up to speed, he now found that preparing for the SUPPOSER class was not taking any longer than for his other classes. On the question of planning ahead, he liked the idea that if something comes up with the SUPPOSER, you don't say, "I'm sorry we don't get to that until Chapter 8," you stop and take it on.

Also, he reported that he is more comfortable now saying "I don't know," in response to a student question -- not only in his SUPPOSER class but in his other classes as well. He also feels comfortable about being flexible, responding to issues as they arise in class, and in writing and organizing his own lessons and problems.

Teachers C and R (using the SUPPOSER for the first time) were asked to reflect on what would have constituted appropriate and useful training. Teacher R said taking a course and working over time with a few students so that when you did something, you would get a response and a sense of what the students were thinking. Teacher C indicated that sitting in on another teacher's SUPPOSER class (which he did last year several times) was inadequate. He suggested that once teachers were thoroughly at ease with the software, they should go through the problems themselves and experience directly the kind of thinking involved. Finally, he emphasized the importance of being able to observe and talk with others using the software. He suggested the idea of identifying mentors or working in pairs or teams within a school. Both teachers emphasized the need for an extra prep period for a SUPPOSER class, at least for the first year.

A key issue for all the teachers was what it means for students to take on some responsibility for the control of the learning process, and that seemed to be a key to student response to the experience as well.

Teacher T thought that part of the problem with her class this year was a reflection of the type of learning experiences that this kind of student (lower level) has at the elementary and middle school levels. In the lower grades, it is only the bright kids, having finished their work early, who have an opportunity to experience what it means to learn on your own by "fiddling around." She thought that for the students in this class who never had that opportunity, learning with the SUPPOSER was more difficult. One student said to her, "The other way is so much easier. I know what I am supposed to do." Still, looking in on other math classes of similar level and ability, this teacher reported that her SUPPOSER class seems to be more on task with less daydreaming and fooling around.

Teacher R said that the students liked the SUPPOSER and liked going to the lab and the sense of freedom and control that it gave them. However, the students lacked discipline -- their response to the freedom was to tune out and walk away (literally sometimes).

Teacher C talked about the need to make it clear that "this is serious business." At the beginning of the year, given his own uncertainty, he distanced himself from this class to make it clear that this was not just playing around. In retrospect it was a good idea and he should have been even more formal. He thinks that students must understand that with this shift in control goes a shift in responsibility and that students must understand that this is work on which performance will be evaluated. He thinks that when the kids are of average ability, learning is as much a matter of organization and discipline as aptitude.

What might they do differently for the rest of the year? Teacher T said she might try breaking the class up and working with the kids in small groups on a regular basis. Teacher R thought that she would use the SUPPOSER less regularly, work on fewer problems -- richer problems broken down into smaller steps, and provide the students with much more direction and structure. Teacher C also said that he would most likely use the SUPPOSER on a more selective basis and try devising problems with the students (i.e., when an appropriate question or problem came up in the class to ask if the SUPPOSER might be helpful, how they might use it, and then go to the lab and work on it.)

June Teachers' Writings

Countrytown

In the second half of the year, I was more selective using the SUPPOSER. I tried to encourage more discussion and focused on individual experiences in the lab. For example, students explored relationships between angles and arcs. A conjecture was made and the class tested and verified that it worked in at least four different cases. Its proof was assigned for homework and discussed within the lab the next day.

Although class participation improved, students continued to have difficulty writing convincing arguments. Putting thoughts in written form was one of the most frustrating experiences for students. For example, while working on final projects one student was so excited by his discovery that he proceeded to work through lunch and his next two free periods. He presented his ideas in class. However, when asked to outline and organize them in writing, he drew a blank. Therefore, areas I would focus on in the future would be modeling of written material and verbal development of conjectures through structured class discussions.

At year end, students have a "review" period (2 days) to pull together the topics and material covered in the course. During this period, I observed two major differences between geometry students who used the SUPPOSER and those who had not. First, students using the SUPPOSER `talked' geometry. Their approach was more visual and manipulative. For example, when working with a 30-60-90 right triangle, students found that by reflecting the hypotenuse through the line containing the long leg, an equilateral triangle could be formed and hence the relationships between the lengths of the sides could be easily derived. Flipping, rotating, and zooming were terms used freely. Second, students seemed more comfortable with the mathematics and less reluctant to explore and make mistakes. In a traditional geometry classroom, I feel students rely heavily on the "right" method or formula. Students tend to spend more time memorizing formulas, looking for the "right" answer, and are often afraid to explore, conjecture, and make mistakes.

In conclusion, the SUPPOSER classroom is active with students learning through inquiry, as opposed to mere memorization of facts or formulas. Although the written work was not as developed as I expected, I was pleased with the sincere efforts students put forth. I feel that the class came a long way from day one.

Rivertown

During the second half of the year, we spent quite a bit of time in the classroom and away from the lab. Nevertheless, the class seemed to appreciate and enjoy the occasions we periodically spent investigating concepts with the SUPPOSER. As the year progressed, the class had a better idea of what kinds of things to look for. However, I am not certain that their concept of proof improved very much.

The culmination of the year for many students was the final project. They began these enthusiastically and with the greatest interest and excitement I had observed to date. Guiding and watching their efforts, I realized that they had a good understanding of how to acquire data; many had as good or better ideas of how to analyze the data. Nevertheless, many still had difficulty expressing their findings as conjectures. The proofs of any conjectures they were able to state were even more difficult to complete. Clearly, the suggestion of having students write their conjectures in "if then" form early in the year might help them better achieve the idea of proof later.

My final observations confirm earlier indications that for the majority of the students there was little correlation between the work done in the lab and the material discussed from the text. Average freshmen seem too immature mathematically, too weak in basic skills, and too dependent on the teacher to be able to work independently with consistency and thoroughness. It appeared to me that certain skills were necessary for students to be successful with the SUPPOSER. Characteristics of orderliness, independence, and inquiry would enable them to produce better conjectures. To help them acquire these traits, perhaps a series of exercises could be developed in which students could learn to propose questions and then draw conclusions. Time devoted to this type of exercise early in the year might provide a stronger foundation for the essential traits of orderliness and. independence.

Techtown

During the last four months of the school year, Techtown students did very little independent work using the SUPPOSER. This came about because of some frustration in the lab setting. Students would gather data diligently, but with no sense of direction. They would ask, "Can you give a hint? ... What do you _want_ us to look for?" I would work with individual students and pairs, asking leading questions, and trying to help them generalize from their observations.

In mid-February I decided that the way I was running the class was, to a great extent, a poor use of time. The class consisted of a small group of students, most of whom were uncomfortable going out on a limb. In a sense they were playing a game in which, by stating that the task I had set was too undefined, or too difficult, they were excusing themselves from spending serious at-home time pulling together facts and coming up with meaningful conjectures and convincing arguments.

In our meetings at EDC there was some discussion of ways to get kids looking for the data which was significant, rather than just anything. The most useful process seemed to be something like this:

1. Make a construction and repeat on several triangles/quadrilaterals, looking for visual clues only.

2. Does anything strike you?

   Figures that may be congruent
   Figures that may be similar
   Segments that may be congruent
   Areas that may be the same
   Parallels

3. Write down the things that you are going to look for, remembering that ratio, as well as equality, may be significant, and that sums of products may produce interesting patterns.

4. Gather numerical data. When comparing quantities that appear to be different, find the ratio of larger to smaller, rather than smaller to larger.

5. Make a formal statement of conjectures.

6. Look for a convincing argument.

This is a way of "doing math" that is totally different from that usually experienced by an average student. At our meetings, Dan [observer from research team] observed that students tend to model the habits of their teachers, using a ruler for diagrams if the teacher does so, using the if...then format for writing conjectures if the teacher does so. Perhaps a student reaches high school being grouped "average" in math partly because he/she is slow to pick up clues and use the teacher's way of working as a model. These students may need more explicit teaching and direction as to what we want them to do (even if what we want them to do is to become creative and independent learners!).

The small size of my class (eight students) gave me one advantage -- it was possible to use a whole group format and have everyone participate. From mid-February to June, the class tackled several SUPPOSER problems as a whole group. We usually proceeded as follows:

1. Give students the problem.

2. If the problem is stated in words with no diagram, have pairs of students try to come up with a paper and pencil diagram.

3. Have a student put the drawing on the screen, and repeat on two or three triangles/quadrilaterals if it is obvious.

4. Brainstorm -- what measurements would be useful? Are there any conjectures at this point?

5. Gather numerical data to check out conjectures. Sometimes we did this as a group; sometimes individuals went to the computers with an assigned task.

6. As a group, state some conjectures. Have we covered all the ground?

7. Convincing arguments.

As well as the whole group work, there were a few (no more than 10) individual lab sessions from mid-February to June. The total time given to SUPPOSER work averaged two periods (of five) per week, but was very uneven. We did very little with the SUPPOSER while covering area (I now have a collection of interesting problems for next time), but, as a group, we spent two full weeks looking at some circle problems that I was putting together for the research group. At the end of the year, every student or pair completed an independent research project.

What worked:

This format produced fairly high involvement and participation. Students became bolder in suggesting possible relationships in figures. They became increasingly confident in justifying their conjectures.

What did not work:

When students had individual projects, they showed little improvement in self-direction. Students who, in class discussion, would suggest measurements to compare, who could think about inequalities and ratio, on their own would go back to recording every length and every angle, then call me over, saying, "I don't have anything. Nothing is the same." I also felt some discomfort because, since students were not meeting my expectations, I had, in one sense lowered them.

What I would do differently:

I think that I overemphasized methodical data collection at the beginning of the year. This became a crutch for the students-- anyone can collect a page of data and avoid the necessity to think. While students must learn to organize numerical data, I would put more emphasis on looking for visual patterns. I would show students exemplary assignments and projects starting at week one. Students were amazed when they saw the work of Richard's [Houde, member of team] best student (in May). Even though I believed that I was articulating what I wanted, they did not really understand what they could do.

This report sounds negative, yet I believe that these students did get more out of their Geometry class than they would have done in a traditional class. They complained that the class was hard, that it would be easier if I would just give all the answers and have them take notes. They admitted that they were being challenged (and they didn't like it!). By the end of the year, though, almost everyone had a sense of how to go about getting a proof. The proofs were often flawed, but there was an understanding of the need for proof and that a proof was a logical sequence and not a list of unrelated statements.

C. Students' Perspective

To investigate the perspective of students, two members of the research team interviewed eleven students from the three SUPPOSER classes at the end of the school year. The students were selected by their teachers guided by the suggestion that those selected be students who might have something to say, would speak up, and would take the task seriously. There were four from each of two classes and three from the third, seven boys and four girls. They included some of the "best" students and some of the "worst," some who prospered with the SUPPOSER and others who found the program very difficult. All the interviews were conducted at the schools by two of us working as a team, talking -- with one exception -- with pairs of students.

Questions included: Was it difficult using the program? What was it like to learn with the SUPPOSER? What was easy? What was hard? What was different? What was it like to be taught with the SUPPOSER? What would you do differently if you were teaching the course? Did this way of learning geometry have anv impact on your thinking or work outside of geometry?

1. USING THE PROGRAM: "... THE COMMANDS ARE ALL THERE."

When the intervention began, there was concern that some students would find working with the SUPPOSER or with the computer difficult, but that was not observed. The interviews were consistent with the observations. Every student interviewed shrugged off the idea that the software was problematic. So it follows that making constructions and making measurements also posed no problem:

> "Basically, the directions are right in front of you. You just press certain buttons."

> "I don't think that the computer program was hard. That was easy to understand. I understood how to find the angles. I understood how to go through finding the areas and all that."

> *(S1) "Oh no, very easy."
> (S2) "No, that wasn't a problem."
> (S1) "It's just label, measure. It's no problem."

> "... the commands are all there. You just have to press one letter commands and then enter numbers and angles. ... If you just picked up the disk and said, `What's this?' and loaded it and just looked at it, you could start doing stuff on your own. All the commands are right there."

2. LEARNING WITH THE SUPPOSER

<u>The Joys: "`Wow! This works so good."</u>

When students were exploring and experienced the joys of discovery and confirmation, they were delighted:

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* The notation S1,S2 stands for Student 1 and Student 2 and is used to differentiate the two student voices in consecutive comments from the same interview.

"Sometimes I can really get into it and really try and solve things. It can be interesting."

"At the beginning of the year, my first packet, I really got into it once. I got two A+ ..."

(S1)"Once you know what you're doing then it's followed very easily."
(S2)" You get all excited and say,`I know what I'm doing. This is great!' When you really finally get it, you're like, `Wow! this works so good.'"

"... I can understand this. The information is so easy to get. I know exactly what's going to happen. You can predict what's going to happen. ... Then it does."

For one student, integrating induction into the learning of geometry fit his learning style like a comfortable pair of old shoes:

"I think that just the SUPPOSER was a parallel to the way I do it [learn]. I see a lot of the way I learn in using the SUPPOSER. .. When you have labs, you learn from your observations. This was the same thing. ...I like learning from experience or from seeing what other people have done. That's what it is. You're doing the stuff yourself, so you're rediscovering stuff. It's really your own learning."

<u>The Hardships</u>: "<u>Conjectures are kind of hard because ...</u>"

For almost all the students, conjectures presented real problems. Knowing what to conjecture about, discerning patterns and relationships, and generating conjectures were all hard work. For some students, conjecture-making was the province of "smart people."

(Interviewer) "Do you like it?"
(S1)"Yeah sometimes. Like I say, you have to get conjectures about all this stuff. That's really hard. That's not to do with a computer really. The computer is just supplying the information. That kind of spoils it with the conjectures.
(S2) The rest of it's pretty good."

" I didn't like making conjectures because I don't think I do very well at that."

"Sometimes it's pretty hard ... not hard with the computer, but hard, like, for a conjecture. You know when you don't really see anything there."

"... Conjectures are kind of hard because you just don't see things the other people do; but when you see them, they're obvious."

This student pointed to the frustration of making conjectures and then finding out that you missed something:

" They don't give you enough. It's too general about what you have to figure out. You can just go through it and figure out everything you can. You get surprised, thinking you did a good job and then you come out knowing you were supposed to find this, and this, and this, and this, and you didn't find it. It was just -- it's weird."

2. BEING TAUGHT WITH THE SUPPOSER

Guidance and Direction

In general, students voiced a need for more guidance and direction, particularly when it came to the difficult task of making conjectures.

These students seemed to feel lost and a little overwhelmed by the responsibility of speculating and generalizing:

(S1)"... You have to get a conjecture out of nowhere. I don't know. What do you think?"
(S2)"... You have to start it all on your own. She'll [the teacher] give us a little something. The sheets that we work on will say a little something. We have to come up with everything. That's what we're supposed to do. We're not complaining. It's just a little hard sometimes. Maybe we are complaining."

For these two students, conjectures were difficult because they didn't understand what a conjecture is and they lacked a model for generating conjectures:

(S1)"... He'll give us two triangles and we're supposed to figure out what's the relationship. He'll go `Think of conjectures.'"
(S2)"Conjectures to me, can be anything. ..."
(S1)" .. He lets us do anything we want for conjectures, but it doesn't help me at all in thinking them up."

This lack of a clear model and insufficient direction was echoed by several students:

" I know they wanted you to think and everything, but I felt like I couldn't do it. We have a lot of smart people in our class, not a lot, but a few smart people and they were always getting it. I'd have to sit here for a half an hour trying to figure out what the problem wants me to do, first, before I can go and work on the computer."

"It was harder to think about what the question wanted you to think about, than the book, where they give you examples and you have that."

"Usually when she gives us hints, we know."

This last student and her partner found this approach to learning satisfying, but ran into difficulties when confronted with a problem that was more open-ended and larger in scope.

(S1)"She never told us what to exactly look for, she just explained it better. She didn't leave us hanging."

(S2)" She told us to check this out and check that out and this worked. You felt like you saw what she wanted you to get. Now we're not seeing it because it's weird."

In one class, the teacher responded to students' need for more direction with detailed step-by-step instructions. But as a student in this class indicated, it wasn't the solution for everybody.

"On the problem sheets we have, it will say, `Step 1: Here's your angles of the triangles.' It will say, `Put an equilateral triangle on the screen.' It will say, `Measure angles, sides, lengths. Get all the data.' Right there it gives you too much. They should just give you the problem and let you figure out what to measure and what to..."

Although they found it difficult to articulate, several students seemed to understand that there is a difference between providing a clear definition of the task and guidance in a problem, on the one hand, and making problems simple and spelling out detailed, step-by-step instructions on the other. The student who was most at home with this style of learning said it best:

" [If I were teaching] When I started out, I would discuss it more, I'd show more of how to do things. Then as you got more and more into the thing, I'd ease off and let people figure it out for themselves, instead of saying ... because then you are back to the textbook which gives you the answers and then asks questions when you know the answer already basically. I would do more talking in the beginning and less in the end instead of sort of steady all the way through."

Integrating the Classroom Experience and the Lab Experience

Another problem that some students perceived was a lack of integration of the classroom experience with the experience in the lab. Deductive learning in the classroom provides a critical framework and an essential foundation for the inductive experience in the lab. If teachers are unable to interweave the two types of learning, there is a greater likelihood that students may find making conjectures difficult and the experience frustrating.

For some students, it appeared to be two parallel curricula:

> (S1)"We're doing completely different things in the classroom [from the lab]."
> (S2)"There is [a relationship]. We just don't see it...
> (S1)"Yeah, the computer is completely different. One day, we'll just go to the computer and start a whole new thing. We'll just forget about what we're doing in the classroom. That's confusing ... It really doesn't go with what we're doing in the book at all."

> "It's like we switch back and forth, back and forth, and back and forth. We step forward and backwards ..."

> "I didn't understand what we were doing when we went back to class. I rather work from the book because then, I think I'd get more out of it."

> " I think it was at least twice as hard. You're trying to learn both from the book and down in the computer lab. It puts a lot of pressure on you."

When the relationship was apparent to this student, the experience was positive:

> "If we had gone over it in class, the next day, sometimes we'd get the same type of problem [in the lab] that we had in class. Then it was easy, but when we hadn't done it in class, I didn't know what I was looking for."

In two of the three classes, this situation was confounded by a physical separation of the classroom from the lab.

> " I think it would be a lot easier if we just stayed in the computer lab and worked, just stayed in the computer lab [rather] than going back up stairs [to the classroom] and working upstairs.

> "Don't just, as we're leaving [the classroom], after the

bell rings, 'By the way, we're going downstairs [to the computer lab] tomorrow.' That happens all the time."

One member of the class in which the computers were in the classroom commented:

"She does a chapter in the book and then we'll go to the computer for awhile. They do relate to each other. You get more of an understanding of little basics."

3. THE DIFFERENCES: "WE'RE COMPLETELY DIFFERENT."

We knew that this approach to learning and teaching geometry was radically different from traditional instruction. These interviews helped us to identify and to characterize some of the differences.

All the students acknowledged that this experience was a significant departure from the "normal" experience in their schools. This was captured in a dramatic exchange between two students:

(S1) "I feel so strange. We're the only class."
(S2)"We can't even associate with other kids in the other classes. We're completely different."

This approach with the SUPPOSER called for new and different behavior on several fronts and levels. These are some of the major differences that students cited.

Grading

There was the matter of what constituted successful performance. By changing the nature of the problems, we changed the criteria for success and the basis for grading. Several students commented that they would get a better grade learning with a textbook since there was less room for making mistakes. With a single right answer to a problem comes a certain precision. Working with the SUPPOSER seemed more of a hit or miss affair.

"I do better in the classroom than I do with the computer. I did better gradewise. I mean in tests. On the computer I would be more likely to get a B than an A. In the classroom I'd get A's. On the computer you can mess up here and there. I make little mistakes that I wouldn't make in class."

The Lab Environment

Another student commented on the informal nature of the lab environment in contrast to the classroom:

> " When you're in the lab, you can make jokes and everything while you're working. In the classroom, if the teacher is giving a lecture, you can't do that. You can say something to a friend and it's not like, 'Shh! This is a classroom.' It's more relaxed. I like that."

Depth and Breadth of Content Covered

As indicated in that dramatic exchange above, students felt isolated from their peers. The classes in this study were the only ones at their ability levels in their schools working with the SUPPOSER. This prompted some teachers and some students to constantly compare their experiences and their work with those of the other geometry classes. For some, when it came to coverage of material and depth of understanding, working with the SUPPOSER was a wash or even a plus:

> "We've kept up with the other classes even going to the computer room and spending this time. We're still on the chapters that they are on. We've spent a lot of time on them but we're not behind."

> "I think you learn pretty much the same. Sometimes we find ourselves ahead of other classes. Then we'll go to the book for a while and they might catch up. You know, like a couple of sections ahead. I think we probably learn the same."

> "Other classes don't go into as much detail as we do about medians and so on. ... We really go in-depth about them. Other people, they just know what they are and know what they do. I think we are more familiar with them."

For others, what they perceived as a lack of linear structure in the content, i.e., not following a text, was disconcerting and learning more about fewer topics than the next class was troublesome.

> (S1)"One day, all my friends, in the other classes, they'd be learning it straight right through the book. They started with page one and go right through to the end. The way ours works out, we skip totally around them and miss whole sections."
> (S2)"I'm afraid we might be behind next year ...
> (S1) "Basic geometry is ahead of us right now."
> (S2)"They're ahead of us right now. I mean,like, we might go into more detail and learn more about it, but they'll

know more. They'll have more knowledge about geometry than we will.

Learning, Teaching, and Knowing

But at the deepest level, the integration of inductive and deductive learning seemed to challenge the fundamentals of the educational experience -- what it means "to learn," "to teach," and "to know."

These differences went well beyond whether the class moved straight through the book:

"It's a lot different from the book -- not different from the book, but taught different. It's understood different, also, from the book. You have to do yourself. It's independent."

For some, there was a loss of certaint and comfort.

"In class you know what you're doing. She just taught you it. It's right there in your notes. It's numbers. How can you go wrong? There [in the lab], it's just like, `Oh, no!'"

With a greater emphasis on student inquiry came what seemed to be a one hundred eighty degree shift in pedagogy.

(S1)"In class, it's like, `This is how it is.'"
(S2)"There [in the lab], it's like, `How is it?'"

Learning was no longer a process in which a teacher who knows all, discourses, and students who are passive, take in the knowledge. One student characterized learning as "absorbing." This student sums it up by contrasting "teach" and "memorize" with "learn" and "think:"

" It's different. It's like abstract thinking. It's different than anything else you've ever done. ... Maybe a little harder than I expected. We have to think about everything that you learn, instead of just having a teacher teach you, memorize it, and just do it. You have to think about it yourself."

In this experience, students had to think:

" You got to use your mind a lot. In class, you really don't. She spells it out."

Students had to work and be active participants in the learning process:

> " ... you're not just sitting back listening. You're doing the work."

Students sensed they had a larger degree of independence and responsibility. There also seemed to be a perception that more independence and responsibility for students meant that less teaching and therefore maybe less learning, was taking place:

> "... you have to learn things that you normally learn from the teacher. You have to figure them out yourself."

> " You have to discover things on your own instead of having it taught to you."

> " If I had Ms. [another teacher], I think my grade would have been a lot better. This lady [the other teacher] was really driving it [the content] constantly."

> (S1) "All the other classes have tons of theorems."
> (S2)"... Yeah, they had the theorems. They're just learning them and they have tests on them. We have like -- you go down there and you learn similar triangles. You have to figure it out yourself. Find out if there are two angles equal and all that, that makes them similar. Stuff like that. The teacher would normally tell you that."

The theme of discovery and invention was echoed by several other students.

> " I think you learn a little less [geometry], but I think what you learn you learn better. You sort of make it up."

One student saw the objective as reinventing geometry and the experience of Euclid.

> "We learn about how the guys who thought up geometry, originally, figured it out with theorems, and stuff like that. A lot more thinking and abstract."

For one student, the one most comfortable with the approach, he thought it was refreshing to have a stake, a direct hand, and a role in the learning process and establishing what is known and true:

> "It takes the place of a textbook. We do our homework at night, then, we come in and we work on -- we reprove some of the theories and stuff, and discover, without the book actually saying, `Oh, this is true,' and show you a proof of how to do it -- but basically you just have to believe

it. Here you have actually proved it. The physical way of doing it, sort of gives you more experience -- you learn it better. Also it gives you sort of a mental -- like, `Oh, I did this.´"

For some students, taking responsibility for learning was not the student´s role:

> " ... we´re only students. There´s a lot of stuff we´re not sure of. At test time, if you get it wrong, who´s to blame? [With the SUPPOSER] You taught it to yourself, basically."

For some students, what they did in the lab was not learning and they looked forward to, as Woodrow Wilson characterized the post-war period, "a return to normalcy:"

> "Algebra from this? ... I think it´s just going to be going back to a regular class. Like we did before, just learn. ... Right from the book and have the teacher teach."

4. TRANSFER BEYOND GEOMETRY: "... I ALWAYS MAKE CONJECTURES NOW ABOUT LITTLE THINGS."

Several students thought that their experience with the SUPPOSER had carried over to other classes and/or affected the way they think and solve problems. Those who experienced this transfer referred to organizing data, to being analytical or solving problems, and to making conjectures or generalizations.

> " You become more organized, especially in your mind after doing all this stuff. Way more organized. Things start to come in columns in your head. Things that you want to look for, you just kind of sort them through."

> "If somebody, a teacher or anybody, tells you something, you think maybe it could be this. You have a bunch of ideas. Not just two, but a bunch of them. You´re thinking what could be the reason for it? You have a list of ideas going through your mind. Then, you sit down and play it out or figure it out."

> " You´ll look at all the stuff you have and then you´ll just kind of put it together and see what you can get out of it. I´m not sure we did that so much before."

> "... I always make conjectures now about little things. I don´t know. It´s very hard to explain. I´ll be in another class. You see how things work, so you make a conjecture and you generalize about other things.

Especially in biology because it's life in general. It's so interesting. You can just make conjectures."

5. CONCLUSION

In these interviews, students were interested, articulate, and insightful.

The interviews confirmed observations over the year that the computer and the SUPPOSER did not trouble students in the least. They are much more at home with the technology than adults and can adapt quite easily and quickly to an unfamiliar interface.

In general, the students seemed to understand the power of the guided inquiry approach and enjoyed learning this way when they were successful. For the most part, they liked learning with the computer and seemed to think that with this approach came a deeper understanding of the content. When it worked for them, they sensed the excitement of discovery and learning. Some students indicated that they were using techniques and strategies from this approach in other classes.

Nearly all the students found guided inquiry more difficult than a traditional approach and experienced some frustration. Some of this can be attributed to a less than optimal implementation of the approach. Some of the frustration can be attributed to the fact that two of the classes were considered low ability level and even the honors class, according to their teacher, was misclassified. Some of the negative remarks can also be attributed to giving adolescents, who normally have no voice, an opportunity to speak out. But there is no question that students, whether they liked or disliked the approach, found it to be very different from what they considered standard, normal practice.

Schools like every other institution have norms, explicit and implicit, codified and uncodified. Teachers are the agents for these norms, socializing students into the culture. As part of that socialization process, students learn what it means to be a member of the culture and what it takes to succeed, and they make accommodations. Students tune their behavior and their expectations to the norms.

From the interviews a composite of "normal" practice as perceived by the students can be constructed: a teacher using a textbook (whose contents and sequence are sacred) conveys "true" information to passive students; the information is received by the students in a lecture format; the student applies it to problems which have a single answer and are similar in form to those solved in class; the student memorizes and absorbs the information, and through performance on tests, demonstrates understanding of the information.

In contrast, the guided inquiry approach with the SUPPOSER calls for teachers to play two key roles. First, teachers provide students with the fundamentals (definitions, concepts) and with the skills (inquiry, inductive and deductive reasoning) necessary for exploring geometric relationships. These tools may be drawn from a text, supplementary materials, or homemade curriculum. Second, teachers serve as guides and models for student inquiry and student movement between the specific and the general, the empirical and the theoretical. Students are active and responsible participants in the learning process. Building on the fundamentals, students should be able to discern relationships, to assert hypotheses, to test their hypotheses, and to prove the general validity of their hypotheses. Students should be able to demonstrate their knowledge by applying the knowledge and skills they learn and develop to new problems that deal with unfamiliar situations.

The critical difference between the two approaches is the added dimension that guided inquiry and the SUPPOSER bring to the learning and teaching of geometry. In traditional geometry instruction, students operate on an abstract level only: they are taught axioms and theorems in order to use them to prove other results using deductive reasoning. Using the SUPPOSER brings an empirical dimension to the geometry experience in which students can construct, manipulate, and measure particular geometric objects. In a guided inquiry approach with the SUPPOSER, when students are presented with a task concerning an axiom or theorem, they can address it on an experiential level, exploring the task empirically as they build their understanding. In this approach, student understanding of axioms and theorems is deepened and clarified by moving back and forth between the empirical and the theoretical.

With the added empirical dimension come new roles for students: students now must take an active part in and some responsibility for the generation and validation of knowledge -- a deviation from the norm and a challenge to the culture and the standard relationships among student-teacher-knowledge. These roles and responsibilities traditionally have been the sole preserve of the teacher in pre-college mathematics (and in most other subjects as well).

Based on this sample of students, the "normal" approach is the standard. Students in all three classrooms in all three schools described this as the way geometry, other mathematics courses, and other subjects are taught. Even the student who embraced the approach with enthusiasm and said that guided inquiry matched his own style of hands-on exploratory learning, indicated that such a match had been rare in his school experience. Students who had trouble with guided inquiry seemed to long for classes where teachers were teachers and students were students.

Many of the concerns voiced by students about guided inquiry were also voiced by teachers. One explanation is that teachers discussed their concerns and their misgivings with the students in class. There probably is some truth to this. Another is that teachers and students in the same class and school are members of one culture and subscribe to the same norms concerning instruction and learning. Any deviation from these norms will surely be perceived and experienced by students and teachers alike.

With the guided inquiry approach, students were well aware of the significant differences in how they learned, how they were taught, how much they learned, and how much they were taught.

This experience violated the norms and students found this disconcerting. It is no wonder that the making of conjectures and the guided inquiry approach proved frustrating and difficult at times.

D. Tests of Student Mathematical Skills

1. STUDENT ABILITY TO MAKE GENERALIZATIONS/CONJECTURES

A pretest and posttest were developed to assess students' ability to make generalizations given data or a description of a geometric situation. This section provides a brief review of the design of the tests, the scoring system, and then the results of student performance. This discussion examines the data in terms of differences in performance on the pretests and posttests between the students who used the SUPPOSER to study geometry in a way that integrates induction and deduction (the experimental group) and the non-SUPPOSER students who studied geometry in a traditional manner (the comparison group).

Test Design

The pretest and posttest consist of four problems and three problems respectively, each presenting a statement, a group of mathematical facts, or a mathematical idea from plane geometry, along with appropriate diagram(s). The first two problems on both tests include numerical data. The remaining problems describe an idea abstractly (see Appendix D for copies of the tests).

The instructions on these tests ask students to list any significant statements connected to the problem. The instructions are deliberately vague in order to ascertain what students consider "significant" and "connected." We asked teachers explicitly to refrain from explaining or elaborating on the instructions when they administered the tests.

In framing the instructions, we wished to invite any plausible idea, not only generalizations, and as many statements as possible. There were no constraints such as demanding that the statements be true or be supported by arguments.

Scoring of the Tests

The tests were scored using five central variables defined by Michal Yerushalmy (1986). These variables were created in order to enable researchers to examine students' generalizations and their ability to generalize from a given statement. Of these variables, the first three are rated on a 0-5 scale. The remaining two are scored on a 0 or 1, exist or don't exist, scale.

- -- Level of generalization (LEVEL)
- -- Originality of generalization (ORIGINALITY)
- -- Correctness of generalization (CORRECT)
- -- Changes made to the original statement in order to create the generalization (CHANGES)
- -- Supporting arguments for the generalizations (ARGUMENTS)

The final category is not relevant to the ability to make generalizations. However, in some instances students provide proofs or arguments spontaneously, and so the variable ARGUMENTS provides important confirmatory evidence for the findings of the proof test.

On each problem, students tend to make more than one statement. Each statement is examined and is assigned a LEVEL value from 0 to 5. Examples of the coding of different level generalizations are presented in Appendix E-1. The LEVEL measure is the value assigned to the most general statement produced by the student on a given problem.

ORIGINALITY, called plausibility in Yerushalmy's study, is a measure of the connection of the statement to the problem and to the school's curriculum. Statements connected to the problem, and not covered by the school curriculum receive a high score of 5, while those that are poorly connected to the problem or that are trivial because they were covered at length in class are rated at a low level of ORIGINALITY. Once again, since students offered many statements, ORIGINALITY is the value of the most original statement on a given problem.

CORRECT is a measure of the percentage of student statements that are true for a given problem.

In addition to these three measures, we examined two other aspects of students' statements--the changes made to the original statement in order to create the generalization (CHANGES) and the arguments given to support the statement (ARGUMENTS). In her study, Yerushalmy calls these "type of variable changed" and "reasoning," respectively.

We were interested in three types of changes:

1. object of interest
2. geometric relationships
3. numerical variables

This variable (CHANGES) was created by noting the presence or absence of the three types of changes. Examples of each of the types of changes appear in Appendix E-2.

In a similar manner, we constructed a variable (ARGUMENTS) from four categories of arguments:

1. an informal written explanation
2. reference to a diagram or explanation via a diagram
3. a formula
4. a formal proof

Thus, ARGUMENTS represents the number of categories of arguments given by the student to support the statements. It does not represent an absolute number of proofs given.

Results

As mentioned above, there were two types of problem formulation: data formulations and abstract formulations. Based on prior work by Yerushalmy, we expected to find that problem formulation would affect all of the students' responses. It is helpful to consider the nature of these differences before examining group differences on each of the five variables.

In fact, both experimental and comparison students responded differently to the two types of formulations. Most students seemed to be less inhibited in their generalizations and experience greater freedom of thought when the question was posed abstractly. Their conjectures were more general and they presented more proofs (see Appendix F for a full analysis of this phenomenon).

One possible explanation for this phenomenon is that the two types of questions elicit different responses. With a data problem, first the data must be manipulated and interpreted before further generalizations or a convincing argument can be tackled. On an abstract question, there is no need to manipulate the data and one can move immediately to abstract thought and concentrate on conclusions and generalizations. This may be particularly significant in a testing situation where time is limited. Therefore, in our discussion we will consider each type of problem formulation separately.

Differences Between Experimental and Comparison Groups

In testing for differences between the experimental and comparison groups, a chi-square analysis was performed to compare the performance of experimental and comparison groups on each pretest and posttest question for each of the five major variables (LEVEL, ORIGINALITY, CORRECT, CHANGES and ARGUMENTS). In reporting the results of these analyses, we explain which cells were collapsed and list a chi-square statistic, a probability measure (either chi-square or Fisher's exact test), and a measure of association (either phi or Cramer's V).

Differences Between Groups on the Pretest: In the beginning of the school year, few students in either the experimental or the comparison classes had any background in geometry. There was very little work on the pretest problems and students didn't have much to write. There was a strong, significant difference in favor of the comparison group on ORIGINALITY in problem one. There were no other statistically significant differences between the comparison and experimental groups on the pretest. (See Appendix G-1 for the statistics on the difference in ORIGINALITY on question one.)

Differences Between Groups on the Posttest: After a full year of geometry instruction, the posttest included many topics which were covered in class and familiar to students. The students wrote many more statements in response to the questions on the posttest. Since there was only one significant difference between the groups at the beginning of the year (in favor of the comparison group), our interest is in the posttest following the year-long intervention.

At the end of the year, there were statistically significant differences between the experimental and comparison groups on question one and question three (see Appendix H-1, H-3). The nature of these differences varied by type of question, as one would expect from the previous discussion of the data and abstract formulations. There were no statistically significant differences between the two groups on question two. However, in retrospect, it was a problematic question since the diagrams contained some irrelevant information.

-- Data Formulation (Question #1)

On question one, it is clear from histograms of the frequencies of responses at the different levels of generalization (the values of LEVEL) that there were differences between the groups (see Figure 1). Fifty-nine percent of the comparison group generalizations were rated zero on the LEVEL variable.

A chi-square analysis reveals that, comparing "high" and "low" LEVEL generalizations on question one, the difference between the two groups was moderately strong and statistically significant (chi-square = 4.07, p-value <0.044 , phi = 0.32). Stated another way, the chances of getting a generalization above level zero were 4 times more likely for the experimental group than for the comparison group.

There were no other statistically significant differences between the experimental and comparison classes on this question. On the data question the sole significant difference between the groups was that in the classes that used the SUPPOSER, students developed more general conjectures.

-- Abstract Formulation (Question #3)

On question three, there were many differences between the response patterns of the experimental and comparison groups. As in question one, there was a difference in the LEVEL variable.

Consistent with the earlier discussion, students in the comparison classes produced conjectures of a slightly higher LEVEL on this question than on the first question (level one generalizations on average in contrast to level zero generalizations). However, the conjectures of the SUPPOSER classes were at even higher levels.

As in question one, the difference on the LEVEL variable between the experimental and comparison classes' responses for question three was significant. However, this time the strength of the difference was much higher (chi- square = 12.98, p-value < 0.0003, phi = 0.59). In fact, SUPPOSER students were 31 times more likely than comparison students to produce a generalization above level one.

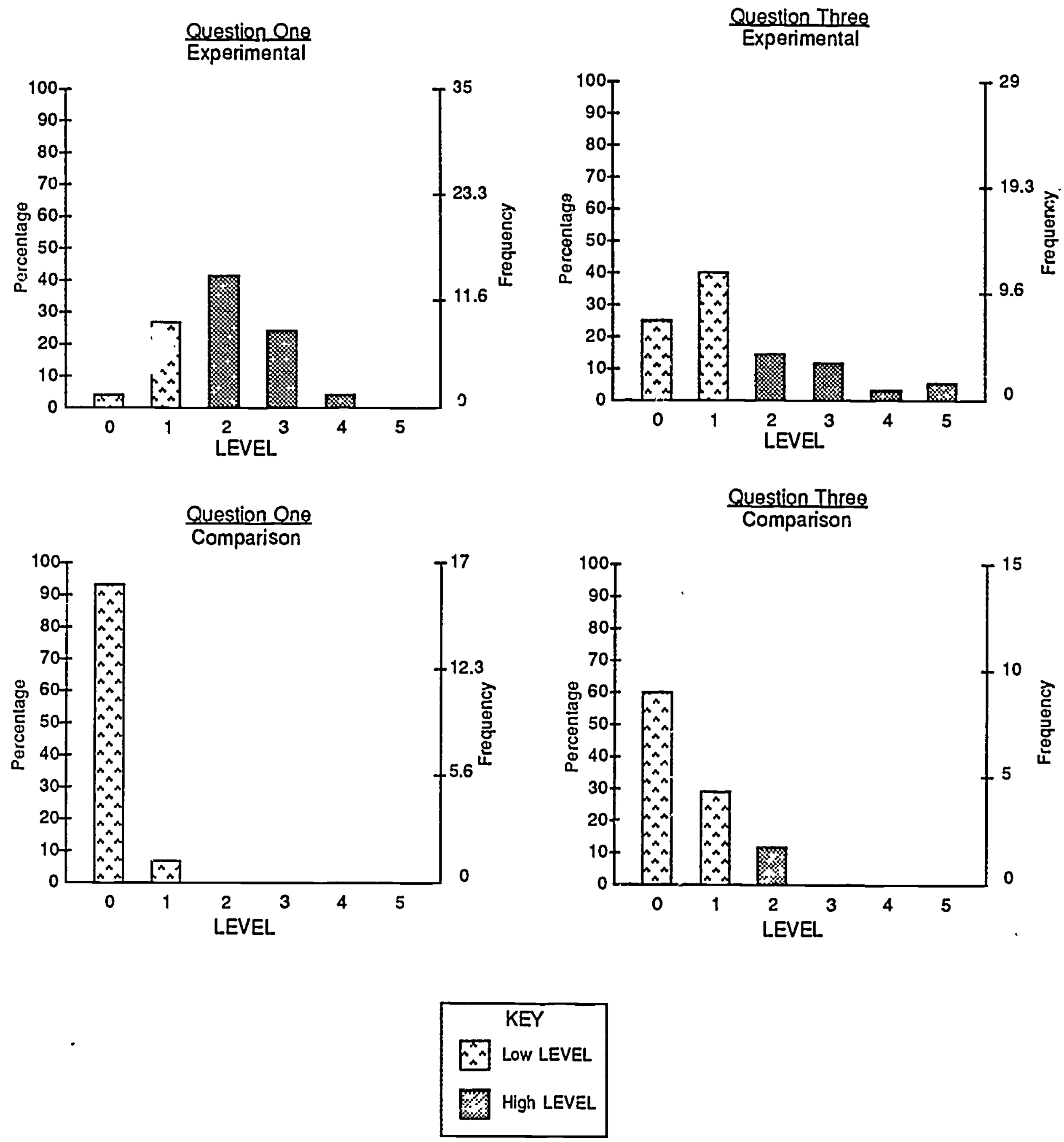


FIGURE 1 - Frequency histograms of generality scores of students' statements on posttest.

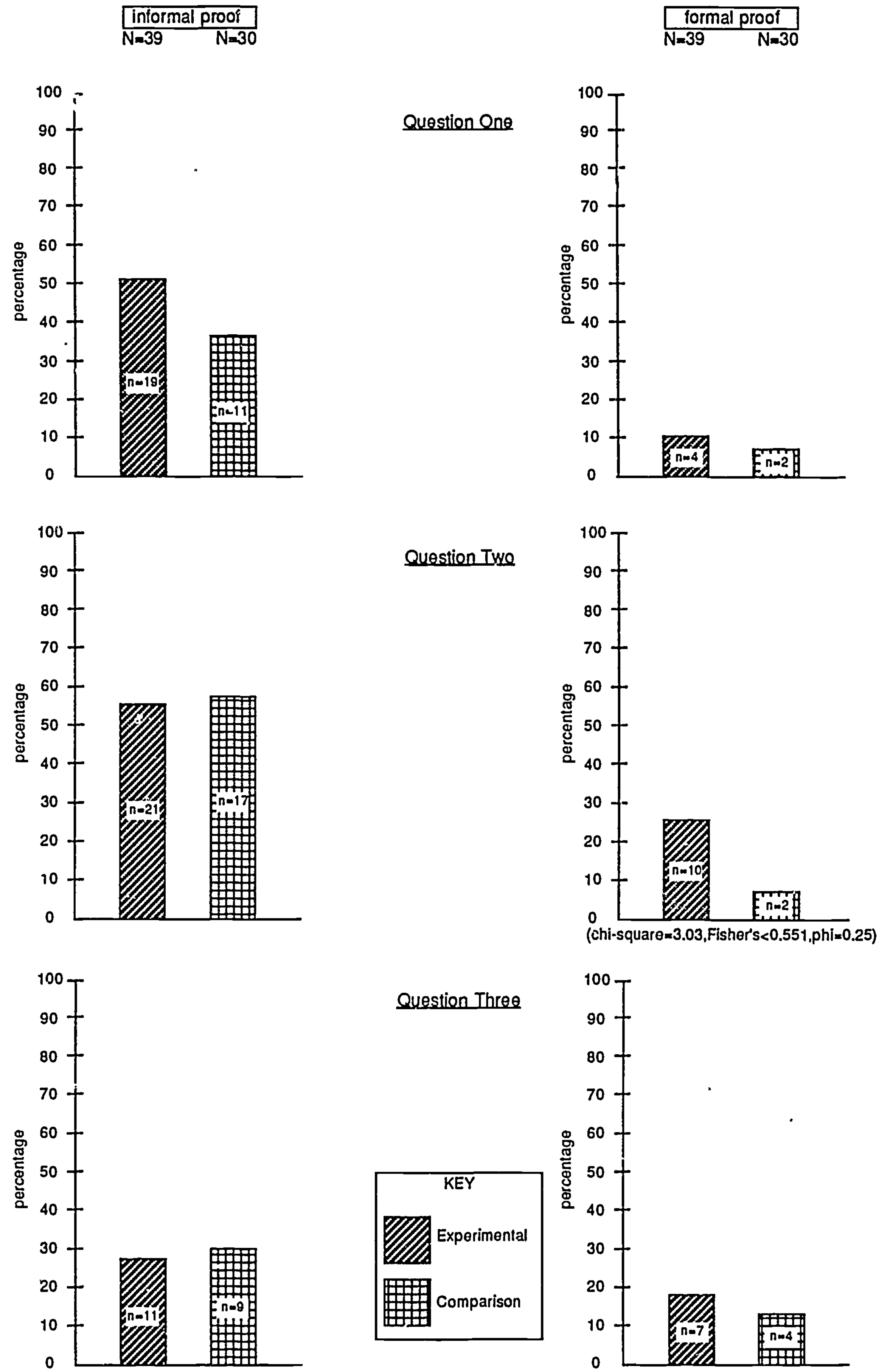


FIGURE 2 - ARGUMENT/PROOF TEST
Frequencies of formal and informal proof in experimental and comparison students' work.

Number of students producing proofs

As these charts reveal, the percentage of students producing informal proofs does not differ significantly from comparison to experimental group on any of the questions. The percentage of students producing formal proofs on questions one and three also does not differ greatly from comparison to experimental groups, though in each case in the experimental group a higher percentage of students produced proofs. However, in problem two, a chi-square analysis reveals that the difference in the frequency of formal proof between the two groups is moderately strong and approaches significance (chi-square = 3.03, p-value < 0.055, phi = 0.25). The chances of a student in one of the experimental classes making a formal proof on problem two were 5 times greater than the chances of a student in one of the comparison classes.

Description of qualitative differences in student work

Looking more closely at the students' papers, two behaviors were exhibited by the experimental group on question two that were not exhibited by the students in the comparison classes. First, many of the experimental students used arguments based on the congruence of quadrilaterals, arguments which are generalizations of congruence arguments for triangles. These arguments were not covered in their course, yet were constructed by the SUPPOSER students in response to the test question.

Second, these students tested the statements in specific cases (i.e., assigning numbers to the lengths and angles) in order to convince themselves that the statement was true before constructing a proof. Students in the comparison group did not exhibit either of these behaviors.

Conclusion

The argument/proof test study reinforces the finding from the generalization/conjecture test study that the experimental group's empirical work with the SUPPOSER during the year did not hinder their ability to make proofs. In fact, the performance of the experimental groups on the argument/proof test indicates that the experience may have fostered an ability to make arguments. These students made as many supporting arguments as the comparison class on two of the questions and more arguments than the comparison class on the third. The students who worked with the SUPPOSER were able to begin with the specific and work their way to a general proof; they were not trapped by the specific.

3. STUDENT SCORES ON MIDYEAR AND FINAL EXAMINATIONS

Neither Rivertown nor Countrytown had departmental midyear or final examinations. Both teachers in these sites however reported that their students scored comparably to other classes on midyears and finals, though these tests were not standard across the departments.

Techtown had departmental midyear and final examinations. The Techtown teacher convinced her supervisor to exempt SUPPOSER students from the departmental midyear. The class did take the departmental final examination. These tests were the same across all "level three" (lowest ability level) geometry classes with one exception -- students in the SUPPOSER class were given one additional test item--a SUPPOSER - like pencil and paper question.

Here are the results of SUPPOSER student performance compared with non-SUPPOSER students:

Table 1

Techtown Final Examination Score Frequencies

SUPPOSER students		non-SUPPOSER students		Fisher's *p*	phi
scores above 60%	scores below 60%	scores above 60%	scores below 60%		
7	1	36	44	<.03	.24
scores above 80%	scores below 80%	scores above 80%	scores below 80%		
4	4	3	7	<.001	.49

These results are particularly striking because at the beginning of the year the SUPPOSER class was considered the weakest of the five classes at this ability level. In addition, they are striking because the SUPPOSER students performed poorly on the "SUPPOSER" question which dampened their scores on the test as a whole.

Several cautions about drawing conclusions from this data. The classes being compared were taught by different teachers and were scheduled at different times of the day. Furthermore, there were only eight students in the SUPPOSER class, while there were thirteen, twenty, twenty-five, and twenty-five students in the other classes.

IV. CONCLUSIONS

From the results of the study, we draw conclusions about student learning and about implementation of a guided inquiry approach. (In the conclusions, we reverse the order in which we deal with the research areas: from implementation and student learning to student learning and implementation.) We then go on to make recommendations for changing the approach and to identify topics and concerns for future research.

As a reference for the reader, we begin each section by repeating the research questions related to student learning and implementation which were outlined in Section I.

A. Student Learning

Conclusions about student learning include mathematical content and skills and insights and skills related to the learning process.

Student Learning - (Research questions)

The transition from the specific to the general

Are students able to generalize from data collected about specific figures to conjectures about classes of figures?

Over the course of the year, what is the evolution of students' ability to generalize?

What difficulties do students encounter in moving from the specific to the general?

At the end of the year, to what extent are students able to develop generalizations when presented with geometric situations and data?

The transition from generalization to proof

At what point do students seek to formalize their conjectures?

Over the course of the year, what is the evolution of students' ability to formalize their knowledge?

What kinds of evidence or process do they consider sufficient to establish "proof?"

At the end of the year, to what extent are students able to produce informal and formal proofs in response to true geometric statements?

1. MATHEMATICAL LEARNING

Although the SUPPOSER-based intervention emphasized skills and activities that are not a part of the traditional approach to geometry, we made a commitment to the schools to attend to the content of standard geometry courses and to integrate this intervention into the standard school setting and framework for geometry instruction.

Coverage of the standard curriculum was a concern throughout the year. Teachers commented on the demands that this approach places on class time and on the pressure that they felt to cover the standard content. By the end of the year, all three classes had covered the bulk of the standard geometry syllabus. Many students commented that their classes had dealt with topics in more depth than non-SUPPOSER classes, and some students indicated that while the pace was different, they covered the same material as other classes ("I think you learn pretty much the same. Sometimes we find ourselves ahead of other classes, then they might catch up ...").

The perception that the students in the experimental group covered the standard curriculum is confirmed by the results of geometry midyear and final exams. Teachers reported that on geometry examinations in each school at the middle and end of the school year, students working with the SUPPOSER performed as well as, or better than, their non-SUPPOSER counterparts. From this data we conclude that students in the experimental groups learned at least as much geometry as students in the comparison groups.

In addition, student learning in the experimental groups went well beyond the standard geometry. Students demonstrated understandings, skills, and behaviors that are essential for creating mathematics and for taking an active role in the learning of mathematics. These behaviors were observed in the classroom and in student work, reported by teachers, and confirmed by student performance on posttests.

For example, consider the skills and behavior involved in working with definitions, making conjectures, and devising proofs.

Definitions are the foundation of mathematical knowledge. The guided inquiry approach makes apparent to students that a definition is much more than a statement found in a book. Students in one class generated definitions, presented their definitions to the class, were forced to reconcile the images in their heads with the words that they put down on paper, and modified their definitions based on teacher- and student-supplied counterexamples.

Across all the SUPPOSER classes, we observed, and students and teachers reported, that making conjectures was not a skill or activity that came naturally to students. Early in the year, students were mired in data, worked mostly on a trial and error basis, and were unable to isolate variables. Teachers and students shared with us their frustrations about making conjectures. By the end of the year though, nearly all students were making conjectures on the larger scale projects. The results of the posttest show that given a situation dealing with the particular, SUPPOSER students produced the same level generalizations or higher level generalizations than the comparison group.

When presented with an unfamiliar geometric statement, students were able to consider the statement in terms of their existing knowledge and to generalize from the statement. On one question on the posttest designed to measure student ability to generalize, students in the experimental group were 31 times more likely to produce a higher level generalization when presented with a description of or data from a geometric situation.

There were examples of generalization on the argument/proof test as well. Many SUPPOSER students made arguments based on congruence in quadrilaterals, a property that was not part of their class work, but rather a property that they generalized on their own from congruence in triangles.

Beyond the standard geometry content and the mathematical inquiry skills, there were indications that students were able to make visual generalizations. Students who worked with the SUPPOSER were able to consider geometric shapes and constructions in dynamic terms. We did not investigate this systematically, but base this finding on the observations of research team members and teachers. One teacher reported striking differences in the behavior of SUPPOSER and non - SUPPOSER students working on the same review chapter at the end of the year. "...[S]tudents using the SUPPOSER talked geometry. Their approach was more visual and manipulative ...`flipping,´ `rotating,´ and `zooming´ were frequently used terms." The ability to consider constructions and shapes as dynamic relationships and not as static objects, the ability to consider the transformations of shapes in the abstract in the course of a discussion (not at the computer or with straightedge and compass in hand), and the emphasis on visual manipulation could all be byproducts of working with the SUPPOSER (and its capacities to generate random shapes, to repeat constructions on different shapes, to scale, and to make the construction and manipulation of geometric shapes quick and easy).

The ability to prove generalizations is another essential skill in the making of mathematics. There was some concern in the mathematics education community that the introduction of inductive reasoning and empirical work into geometry instruction would undermine the learning of proof. To the contrary, on the argument/proof test administered at the end of the year, percentages of SUPPOSER and non-SUPPOSER students producing informal and formal proofs were about the same and did not differ significantly in five of six comparisons. On one problem, the chances of a student in one of the experimental classes making a formal proof were five times greater than the chances of a student in one of the comparison classes.

From our observations and review of student work during the year, we see that while SUPPOSER students experienced some trouble with proof, most were able in their year-end projects to produce proofs, often formal proofs, for some of their conjectures. In some cases where students were unable to prove their conjectures, it was because the conjectures went well beyond the standard curriculum and proofs were difficult to devise.

In sum, students who used the SUPPOSER in a guided inquiry approach developed and demonstrated a sensibility and some skills essential for creating mathematics. They learned as much geometry as the comparison group, they significantly outperformed the comparison group in their ability to develop generalizations, and they were equal to and/or somewhat better than the comparison group in their ability to devise informal arguments and traditional formal proofs.

2. LEARNING ABOUT LEARNING

In addition to their mathematics learning, students gained new insights into what it means to be active learners and to be a member of a community of learners.

In a traditional classroom, the primary function of the student is to absorb, not to contribute. This approach of guided inquiry using the SUPPOSER was a departure from previous learning experiences. It was an active process

("... students were thinking, not just responding mechanically, and knew that they could not just sit back and let things go by."). For some students, being creative, active learners was or became second nature; for others there were frustrations and surprises. It caused students to reflect on their roles as students and the role of the teacher in the learning process. They found that being creative and productive learners and the resulting lack of certainty was unfamiliar, difficult, and disconcerting (" ... we are only students, there's lots of stuff that we are not sure of."). One teacher described the end-of-year differences in student behavior: "Students seem more comfortable with the mathematics and less reluctant to explore, to make mistakes."

Cooperation, an essential strategy in a community of learners, is not a hallmark of traditional math instruction. In the beginning of the year, we observed very little cooperation among students. Students did not seem able to work productively with one another or as a class. Some of the initial problems with student collaboration and class discussions may have been the result of a lack of student and teacher experience with a cooperative style of learning. Over the course of the year, we observed the more effective integration of individual and group learning. Discussions became more productive and students seem to take a more vocal and active role. By the end of the year, most students seemed to appreciate the different, yet complementary benefits of working in pairs and as a class.

B. Implementation

Conclusions about implementation include reflections on teaching, curriculum materials and problems, software, hardware, and setting, and our guided inquiry approach.

<u>Implementation</u> - (Research questions)

The SUPPOSER as a software program

- How do students perceive the SUPPOSER? Do they find it difficult to use the SUPPOSER?

  How do teachers perceive and use the SUPPOSER? When do they use it? When do they put it aside?

The guided inquiry approach

What effect does a guided inquiry approach have on the teaching of geometry -- the content, the order in which it is taught, the style in which non-inductive material is taught?

What effect does the guided inquiry approach have on the teacher-student relationship?

Do students' expectations of teacher roles and teachers' expectati ns of student roles change?

Is there a shift in control over the content and process of learning?

Do roles and relationships among students change?

What difficulties do teachers experience?

What difficulties do students experience?

1. TEACHING

The Role and Centrality of the Teacher

The teacher is the key figure in most learning environments. However, the presence of a computer and software often raises a question about the role and the centrality of the teacher.

In this approach there is in fact a significant shift in the teacher's role from what one normally sees in a high school mathematics class. In guided inquiry, the teacher is not the sole source of knowledge, direction, or energy in the classroom. The teacher can shape the overall flow of the curriculum and the learning process, but on a class-to-class basis shares the control with the students. One teacher in reflecting on this shift in control said that it was essential that teachers using this approach be flexible, be able to respond to issues as they arise, and to learn to stand in front of a class and say, "I don't know."

Does asking students to assume more responsibility for their own learning diminish the importance of the teacher in the learning process?

In the extreme, we can restate this as a classical question that has been contemplated in educational applications of computers: Is an intelligent tutor (in lieu of a teacher) for guided inquiry using the SUPPOSER desirable, especially in light of the many pressures and demands that the approach places on teachers? There are occasions when a tutor could provide prompting on heuristics and strategy. For example, one teacher found that student learning was improved by the definition of a series of set inquiry steps -- "1. Give students the problem, 2. ...have pairs of students come up with a paper and pencil diagram, 3. Put a drawing on the screen, 4. Brainstorm," etc. However, we believe that such a tutor is not a replacement for a teacher. We continue to believe that the benefits of the approach and the learning that it produces come not from the student-machine interaction, but from the student-student and student-teacher interactions.

Our response is based on our observations as well as our beliefs. When using the SUPPOSER, teachers had a significant impact on students' mathematical thinking processes, attitudes, and work habits.

On a trivial level, students copied teacher behavior. If a teacher was careful in drawing diagrams, students showed care in their drawings. If the teacher wrote data right on the diagrams, students placed their data right on the diagrams.

On a more significant level, student difficulties with conjecturing and both the negative and the positive ways in which students coped with the difficulties seemed to be a function of teacher style and response. In one class, the observer concluded: "[The teacher] exposed students to effective mathematical thinking skills and problem solving strategies, she focused on logical arguments and visual skills ... They had internalized some of the strategies that had been modelled for them." In contrast, in another class, the observer wrote, " ... students did what they were asked to do, but no more. When asked to put numbers in a table, they complied ..."

One teacher was explicit about being a role model for students. From the outset, he wanted to be sure that students understood that inductive learning was a serious enterprise so he reports that he consciously "distanced himself from this class to make it clear that this was not just playing around." He felt that if his attitude was serious, students would view the work and the process as serious business.

Teachers were also quick to use students as role models as well. In class discussions, they would identify effective and productive student behavior. When work began on year-end projects, we distributed to students a paper that received an A+ in an honors geometry class taught by a non-research site teacher and it set the standard for student work in the students' as well as the teachers' minds.

Working with the SUPPOSER in a guided inquiry approach was a new experience and a new set of behaviors for students. There was no backlog of images to draw on and expectations, as discussed below may have been ambiguous, so modelling became a powerful teaching device. From the student perspective, mimicking may be a safe and an effective strategy when the going gets tough and understanding is elusive.

Adjustments and Demands

With a new pedagogy, a different learning process, and a shift in role come adjustments and new demands. Here are a just a few examples of essential changes.

First there is a need for a shift in attitudes. To succeed, teachers must recognize students as creative and contributing actors in the learning process. They must value and encourage diversity of approach and thought in the development of knowledge.

Related to these attitudes are the skills essential to leading a discussion. With the guided inquiry approach, discussion is a central activity in student learning. Needless to say, discussion is not a keystone of traditional high school mathematics instruction. Most high school math teachers do not have experience as leaders and moderators of student exchange. They may not have the skills to draw on student ideas and findings as a foundation for building understanding among all students in the class.

Another major adjustment is in the areas of expectations about and evaluation of student learning. For one thing, each period may not end with an objective met or an idea wrapped neatly in students' heads. Objectives may be broad and emphasize process as well as outcomes. Many ideas may be legitimate or at least worthy of exploration. And in most cases, there will be more than one correct answer. One teacher voiced her frustration saying that she found it difficult to know what students "would be going away with." Furthermore, it may be difficult to grade or evaluate individual students when they are working in pairs. This same teacher was concerned that to find out what students were "going away with," she would have to spend time (which she did not have) working with students on an individual basis.

To use a guided inquiry approach with the SUPPOSER, teachers have to expand their notions of what students "go away with" to include skills such as collecting data and making conjectures that heretofore had no importance. They need to create or adapt new assessment strategies and techniques.

One productive strategy for assessment is to take the time to work with and observe one set of lab partners while the class is in the laboratory. There are also paper and pencil options such as the pretest and posttest administered in this research to assess students' abilities to generalize and to make arguments. Finally, it is important for teachers to communicate to their students that expectations and outcomes change over the course of the year as students' mathematical knowledge and inductive and deductive skills evolve. What was considered exemplary work in October may be trivial or irrelevant in May.

Finally, expectations of students and evaluation of student learning both influence how teachers assess their own performance. The experimental group teacher using the SUPPOSER for the second year made an interesting observation about teacher expectations. In reflecting on the disappointment she felt about her students' performance, she recognized that "the SUPPOSER provides no opportunity for evasion. In traditional geometry, students can get by with memorizing theorems, axioms, and proofs. With the SUPPOSER, a student cannot fake what he or she knows and it quickly becomes apparent when and where a student is having trouble."

What are the ramifications of these changes for day-to-day teacher preparation?

For one thing, teaching this way with the SUPPOSER requires a large on-going investment of time. How much time?

One teacher suggested that a teacher should spend "one or two hours every night getting organized ..." She found it impossible to plan in advance - "Even if you are prepared, something may happen in class that you did not expect." Another said that it takes "... initially ... ten hours per week on the SUPPOSER, playing with the software, working on the problems, trying to stay ahead of the class. "Teacher reports from the end of the year indicate that these demands decrease dramatically over time. The experience is probably more akin to the initial preparation and ongoing work required to teach a graduate seminar the first time. What does not decrease as the year progresses or the second time out is the amount of time and energy needed to review student work and to respond to and to integrate findings from the lab into classroom lessons.

In short, to be successful in this guided inquiry approach requires a person who knows the subject matter, can function as a leader and manager of a community of learners, who is flexible, and who has time for planning and preparation throughout the year.

2. CURRICULUM MATERIALS AND PROBLEMS

Curriculum materials and problems are a critical component in any instructional process. Materials and problems focus attention and energy and guide students in the application, integration, and extension of knowledge. In this study, we gained new insights into the critical and complex roles that problems and written material play in an "inquiry" approach that draws heavily on an open-ended software environment such as the SUPPOSER.

The SUPPOSER is a tool with a set of capabilities; it has no explicit curriculum content or instructional framework. The utility and power of the SUPPOSER become apparent only in the context of a task or a problem. So the curriculum materials and problems must define the problem and the task for students in terms of the geometric content and provide students with directions to guide their inquiry.

In designing problems, we looked for challenges that would engage students and illuminate topics in the curriculum. We constructed problems that would yield more than a single, easily defined answer, that did not ask students to rephrase or to prove a given assertion, or to reinvent the wheel. We wanted students to be able to pursue or explore a task in more than one direction, but not to be frustrated by the open-ended nature of the problem. The tasks require students to use both inductive and deductive skills. Directions were designed to be explicit about the process for reaching a solution without providing step by step or keystroke by keystroke directions. They suggested approaches and mechanisms for organizing and summarizing results and findings.

In one respect, the content of the problems did not differ at all from those in a textbook. The SUPPOSER problems addressed the same topics in geometry and were composed to allow for easy integration with the textbooks being used in the classes. In another respect, the content was very different. In traditional geometry instruction, definitions and theorems define the curriculum. The presence of a tool such as the SUPPOSER makes other kinds of learning possible and this inquiry must be shaped and supported by the problems. So in addition to geometric content, we must teach inductive skills as well. We chose to let the problems bear this burden and provided no teacher or student materials that dealt explicitly with the nature of induction or inquiry. Teachers indicated that a set of materials or exercises that focused on question posing, drawing inferences, and making generalizations, for example, might be useful and provide a stronger foundation for the approach.

As the three classes worked with the problems, we found that different classes, students, and teachers need different types of instructions and different kinds of problems. Reactions to the problems varied from teacher to teacher and from class to class. Some commented that instructions were not clear enough ("the students had many questions about what they were supposed to do ..."); others said that instructions were not sufficiently detailed or structured; while still others complained that the problems were too structured. Since the problems are the instruments for both the content and the pedagogy, it is not surprising that each teacher feels the need to adapt the materials to his or her ends.

Our findings underlined the centrality of curriculum problems to the success of this approach. Therefore, we are preparing a paper on problem structure and problem posing which begins to illuminate the characteristics of successful problems (see Yerushalmy and Chazan (1987) and Yerushalmy, Chazan, and Gordon (in preparation)). A key element in effective problem design is to differentiate between the definition of the task and the process instructions. The task must be well defined. The objective of the problem must be clear to students without giving away the outcomes. Process instructions must also be well defined. Appropriate strategies for managing and structuring inquiry must also be apparent to students without detailing each step of the way, keystroke by keystroke.

From the more in-depth analysis of problem posing and the structuring of tasks, we draw specific considerations for developing problems that will meet teachers' and students' needs and maximize learning.

3. SOFTWARE, HARDWARE, AND SETTING

Students experienced no problems with the software. They found it easy to learn and easy to use.

Teachers and students used the hardware and software in a variety of configurations. Students worked at the computer individually, in two's, and in three's. Teachers used single computers for demonstration and illustration, as the electronic equivalent of "sending a student or students to the blackboard," and as a reference during classroom discussions. We remain convinced that the most productive configuration for student investigation is two students per computer. Most students who had lab partners agreed that this was a useful arrangement, often matching complementary interests and skills, or strong and weak students.

The most consistent concern voiced by students as well as teachers was how to integrate the various elements and aspects of this approach.

There were difficulties in relating inductive and deductive thinking, class work and lab work, textbook and SUPPOSER problems, homework and school work. For some, the enterprise lacked logic and consistency. They would have preferred to learn geometry one way, or the other (e.g., all classroom or all lab, all text or all SUPPOSER), and found the combination and the back and forth confusing. The security of starting at page one and moving straight through the book was missing.

It is interesting to note that the problem of integration was greatest for the class that met in a standard classroom and worked in a computer lab on another floor, that it was less of a problem for the class whose computer lab was down the hall, and that it was almost no problem in the class where the computers were right in the room. In those classes where the lab was separate, students said that it was difficult to predict when they would be working in the lab. There were times when students appeared in one place only to find that they were scheduled for the other.

4. A GUIDED INQUIRY APPROACH

A guided inquiry approach with the SUPPOSER calls for learning and mastery in three areas: geometry, inductive reasoning, and deductive reasoning. Students need to learn about the properties and relationships among points, lines, and shapes; they need to learn strategies and structures for inquiry such as observation, conjecture-making and testing, data collection and analysis, and generalization; they need to learn about informal and formal arguments and proof; and they need to learn about the relationships among these three areas.

Over the course of the year, we suggested a linear approach for the teaching of inquiry strategies and skills, emphasizing data collection and organization at the outset, then a focus on conjecture-making and finally introducing argument and proof. However, in retrospect, in communicating this approach to teachers and to students, we may have accidentally reinforced the commonly held misconception that inquiry itself proceeds in a linear fashion, i.e. a process in which data is collected and analyzed, a conjecture derived, and a proof devised.

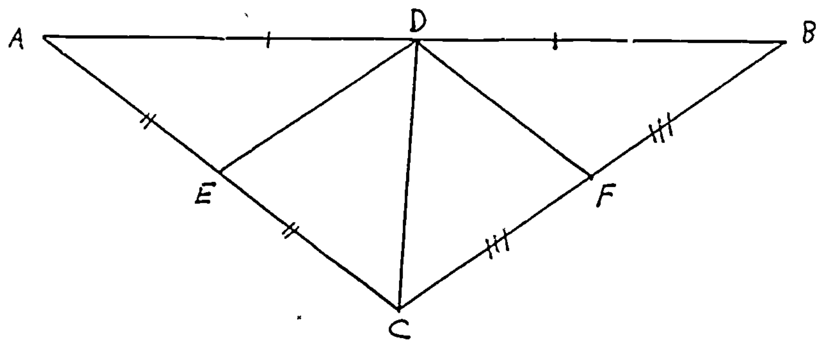
On the whole, this is the concept of inquiry that teachers presented to their students. For example, the linear concept is evident in teachers' instructions to their classes: collect data in the lab, write conjectures as homework. We believe that this characterization of inquiry may have contributed to students' problems with making conjectures.

Our concerns about the linear characterization are informed by critiques from two perspectives.

Philosophical studies about the nature of scientific inquiry suggest that the progression from data to conjecture is incorrect and that the proper order is from the making of an initial conjecture or hypothesis to the collection of data, and then to the refinement of conjecture. In this view, deciding what data to collect grows from an initial conjecture.

Another critique suggests that the linear model obscures the differences between representations in data collection and representations in conjecture and proof. Objects on the screen of the SUPPOSER are specific and represent only themselves. The measurements taken and other observations made are good for that triangle only. To prove a statement about a specific case requires only verification. Understanding is not necessary; a measurement is sufficient.

In contrast, a conjecture applies to a class of objects. It is defined in terms of a specific class of objects. The drawing that accompanies a conjecture is a representation of the class, a schematic drawing, not a representation of the particular object drawn. With appropriate notations, this type of sketch is a symbol.



The language that describes the conjecture is formal (e.g., In any ABC, with a median from vertex C and medians frc the foot of this median in each of the sub-triangles, two pairs of congruent triangles are created.).

Proving a conjecture requires understanding, drawing on and connecting other known facts. Verification is not sufficient.

It is therefore important to recognize that the three parts of the linear progression do not operate at the same level of generalization. There is a leap of generalization from data about a specific case to a conjecture about a class of cases. There is a second leap from conjecture to the understanding of a phenomenon represented in a proof.

From these two critiques, we can gain some insights into the difficulties that students encountered in making conjectures.

For example, we thought that offering students charts and tables that specify which data to collect might help them to make conjectures. In fact they did not. One reason may be that the desired outcome, the conjecture, was of a general level, yet the problem (e.g., find the ...) was posed in terms of data at a specific level.

In contrast, a problem which defines the task as a geometric relationship or concept to be investigated starts out on a general level. Students gather data on the empirical, specific level to inform their inquiry, but find it easier to return to the general level since that is the stated objective of the problem.

In this framework, conjecturing is difficult because of the change in representations. Students must learn to appreciate that they are making a transition in moving from a generally phrased problem to the specific and then back to the general by making a conjecture.

C. Recommendations for the Future

1. THE APPROACH

With these understandings, how can we support and teach a guided inquiry approach with the SUPPOSER more effectively?

An Alternative Characterization of Inquiry

We must avoid characterizing inquiry as a linear process. We must convey to teachers and to students that inquiry is an iterative process between the specific and the general. We should be clear that this involves two transitions, one from the specific to the general, the other from the general to the proof. To emphasize the non-linear nature of inquiry, we might suggest that students conjecture about a relationship before measuring.

A Greater Emphasis on Teaching Inquiry Skills

We must concentrate more teaching time and effort on strategies and structures for inquiry. From the outset, problems should call for all aspects of the inquiry process. They should call for students to be explicit about why they do what they do, what they find, what sense they make of it, and why it is so. They should emphasize generalizations, geometric relationships, collecting data, and asking "why?" from the first day. Problems should always focus on issues that are well-defined, but that can be generalized to other topics (either as part of the problems or by the teacher) and invite further inquiry.

Incorporating a Developmental Scheme for Teaching and Learning About Inquiry

Although a linear characterization of the inquiry process is incorrect, it is essential that we take a linear view of the teaching and learning of the inquiry process. Most students have little experience with learning in an inquiry approach; in fact, most students are made uncomfortable by the new roles and responsibilities that come with being creative and active learners. In planning for future work, we must pay careful attention to the ways in which we introduce and nurture new concepts and skills.

One way to approach this is to think in terms of outcomes and then to consider the most effective routes to the outcomes. Clearly, by year's end we would like students to understand and to be competent in every aspect of the inquiry process and inquiry should be far-ranging and require proof. Students should experience the approach as a whole from the beginning of the year with problems containing some form of conjecture, data collection, generalization, and proof. However, it may be worthwhile to constrain inquiry at times to focus on a specific piece of geometric content or heuristic device. The requirement for proof may evolve from always asking "why," to demanding that conjectures be stated in "if..., then..." form, to requirements for two column or paragraph proofs.

Or consider the order of class activities in this inquiry approach. By the end of the year, we believe that it is desirable for work on problems to begin with individual inquiry (or in pairs) in the lab, to proceed to students writing out conjectures, data, and proof, and to conclude with class discussion. But demanding that it be this way from September without variation is both unrealistic and at times, not desirable. A teacher may decide to alter this process depending on the problem or the skill being taught. Scheduling may be influenced by holidays, pep rallies, or standardized tests. Every week need not be composed of one period of lecture, two periods of lab work, and one or two periods of discussion.

The role and the presence of the teacher will have to vary over the course of the year as well. By the end of the year, the teacher should serve as a resource for inquiry and assume the role as manager of the community of learners. However, at the beginning it may be most effective to work problems as a class, using the SUPPOSER as an electronic blackboard. These early problem sessions can be used to model inquiry techniques. It may then be appropriate to move to discussion of the problem task, twenty minutes of independent work, followed by a review of strategies used and findings.

<u>Preparing Teachers for This Approach</u>

Earlier in the conclusion, we described the new demands and pressures that this approach places on teachers. We wonder whether we are asking too much. There is no question that a guided inquiry approach using the SUPPOSER is difficult and requires hard work. During the year, we watched teachers struggle with many problems and devise creative solutions that were appropriate for their skills and style, their students, and their settings. Despite the difficulties and the obstacles, the performance of students was impressive.

So how can we best prepare and support teachers to use this approach?

First, from the outset we must be explicit with teachers about the philosophical underpinnings of this approach:

(1) Students can contribute actively to the learning of mathematics and the traditional syllabus. Coverage of content need not be sacrificed for student exploration and investigation.

(2) Geometry is not a closed body of knowledge and mathematics is not synonymous with proof. It is essential to understand what it means to learn something and to know something in mathematics.

(3) The primary role of the teacher in this approach is to support and integrate student inquiry and ideas, not to be the repository and transmitter of all knowledge.

Teachers need an opportunity to think about these ideas and to consider them in light of their own classroom practices.

Teachers need a curriculum that parallels the curriculum for students. To teach this approach effectively, a teacher must learn about this type of inquiry and learn it directly. Exemplary teaching behavior must be modelled, but teachers should also experience directly the satisfaction, the excitement, and the frustration of creating mathematics and being a member of a community of active learners.

Teachers need opportunities to try out and practice new teaching behaviors on their own and under observation by supportive colleagues. Finally, they need opportunities to share and to exchange their experiences.

Hardware

Access to hardware is a major factor in the effectiveness of this approach. For future efforts, we recommend strongly that hardware be available for a variety of instructional purposes in a variety of settings.

First and foremost, there should be one location for lab work, discussion, and lecture. We believe physical integration will go a long way toward alleviating confusion about scheduling and the overall structure of the course. It will also reduce the pressure on the teacher to be sure to fill lab sessions with lab work, to bring discussions to a premature close since the next class is scheduled for the lab, or to defer discussions of issues that surface in the lab until the class meets again in a classroom. The presence of the computers in the classroom provides flexibility and opportunities for spontaneous investigation. While conducting the various activities of the approach in a single setting will contribute to the integration of the instructional and learning experience, this alone will not solve the problem of students confusing when and where inductive or deductive behavior is appropriate.

There should be a large monitor or several strategically placed small monitors so that teachers can carry out effective demonstrations or solve problems as a class.

A few machines in study halls or the school library would be an addition welcomed by students. Student investigation is often limited to the length of a class period. Having access to the SUPPOSER during free periods or after school would allow students greater freedom and possibly give teachers the added flexibility to assign SUPPOSER work outside of class time.

Overcoming Isolation

We believe that learning for teachers as well as students is a social process that benefits from exchange and sharing. This year there was only one teacher in each school involved in this study.

Teacher meetings every three weeks or so were designed to help overcome this isolation, but they were insufficient. Initially teachers needed practical and day-to-day help, clarification about expectations, and support. Therefore, at the meetings the communications tended to be teacher to researcher or researcher to teacher, not teacher to teacher. One factor that inhibited teacher to teacher communication was the perception that each class, school, and system were unique. This was true in cases such as lab schedules and course requirements. But from our perspective, what they had in common far outweighed their differences.

Exchange would be more likely, easier, and more productive if more than one teacher in a school were using the approach and the software. We can envision exchanging ideas, experiences, worksheets, and exams; colleagues observing each other's classes; and even team teaching.

By promoting sharing and exchange, we don't mean to imply that all SUPPOSER classes in a school should look the same. We expect that each teacher will adapt the approach, the tools, and the materials to his or her needs and students.

Similarly, students working with the SUPPOSER felt different from their peers. They found it difficult to gauge what they were learning and how much they were learning because they had no basis for comparing their experiences with other geometry students. We know that students this age find it uncomfortable to be different. So having more than one class using the approach in a school would also reduce student isolation.

2. FUTURE RESEARCH

In the course of our research, we identified a number of topics and concerns that are important for the future development and study of this approach, but that we could not pursue within the scope of this study.

Similarity: Teachers had the most difficulty teaching the topic of similarity. The nature of the difficulties and how the software and problems might overcome them is not clear.

Proof and Generalization: One way to focus effort on inquiry skills as suggested above is to design units not on the basis of geometric content (as we did), but rather as occasions for exploring and discussing specific inquiry skills and strategies. Two possibilities are: generalization and proof versus verification.

Small Units: The study this year focussed on a full year intervention. It would be profitable to study the use of smaller units on topics such as similarity, proof, and generalization and the costs and benefits of shorter interventions.

Teacher Change: It is apparent that teachers are the key factor in the effectiveness of the approach. A more detailed study of the changes and the development of teachers implementing this approach would be valuable.

Electronic Networking: We assume that more communication and exchange among teachers will produce more effective implementation of the approach. We wonder whether electronic networking may provide a cost-effective link for connecting teachers.

The Approach in Terms of Student Characteristics: In this study we worked with students of different ability levels, but we did not look specifically at the qualitative and quantitative differences in student abilities. From our observations, from the interviews, and from teacher reports, we know that there are differences among students in understanding, approach to inductive work, preference for visual over numerical data and representations, to name a few. It will be a difficult task, but an important one to design a study to identify the differences among students and to investigate conditions in setting, activities, and materials as they influence student performance.

Transfer, Attitudes, and Retention: We were only able to scratch the surface on these issues and believe that it is important to investigate the longer term effects of this approach on students. We would like to investigate the impact of this approach on student attitudes towards mathematics and mathematics learning, whether and to what extent this style of learning/teaching carries over to other mathematics and non-mathematics learning experiences, and whether and to what extent students retain geometry knowledge at least one year after the experience.

3. A FINAL NOTE

To this point, we have focussed our conclusions on our work with the SUPPOSER this past year in three classrooms. Now we step back from the specifics to reflect on the experience at a more general level.

Having spent a year working with teachers and students, introducing computers and tool software into mathematics instruction, using a strategy of guided inquiry, what is the news?

First of all, there is no question in our minds that the key element in the use of computers and software in instruction is educational philosophy and pedagogy.

The news is not the technology. The computer is a versatile tool that simply processes binary numbers and as such can be a useful instrument in the service of any pedagogical strategy. It is a mirror, reflecting the educational values, objectives, and dreams of the person who holds it.

The news is not really the software. Certainly software may be designed to support a particular pedagogy, and some software will be more congenial to some pedagogies rather than others. But in the end, it is the teacher who defines the student - software interaction. The software is not sufficient.

The news is what happens in the classroom.

In the pedagogy which we call guided inquiry, the teacher introduces content which is illuminated and modified in light of student inquiry and supplemented and complemented by the discussion of student findings. The teacher defines the focus of inquiry by posing a problem for the class. The students working in groups of two or three investigate the problem at the computer. These groups then join together as a class led by the teacher to share their findings and results and to develop a collective understanding of the problem and the issues at hand. The construction of knowledge is enriched and is enforced by a collaborative process among students and between students and teachers.

We have described our findings; here we focus in on one particular effect that took us by surprise.

We were not surprised that the guided inquiry approach required a deeper understanding of content by the teacher. And once we became aware that most teachers and students had little experience with inquiry, we understood the need to address explicitly inquiry skills such as making conjectures and making generalizations. But we did not anticipate that to create a community of learners demands well honed communication skills and new attitudes about the flow of information in the classroom.

A significant example. A guided inquiry approach built around an open-ended software tool calls for a shift in intellectual authority in the classroom. Authority must be shared among teachers and students. There may not be one right answer to a problem, but rather a collection of results some of which may be more right-headed or wrong-headed than others. In the end though, the arbiter is persuasive argument, not an explicit or implicit "Because I say so!"

In this context, communication and discussion are fundamental. Using clear and precise language to communicate mathematical ideas is essential. Proof becomes the language of argument and its utility is evident. In the context of discussion, teachers elicit opinions, acknowledge and value diversity, encourage creativity, build powers of argument, and facilitate resolution of different points of view. Students learn to listen, to respect the contributions of their classmates, and to communicate effectively with one another, not just with or through the teacher.

So what started out as research on the use of the technology to teach mathematics in fact became research on the implementation and the effects of guided inquiry. What began as an examination of the role of the computer and the software became a study of the knowledge, skills, and attitudes of teachers and students and the interactions between teachers and students, and among students. We caution others from the outset to maintain a focus on the pedagogy, the teachers, and the students, and to keep the proper perspective on the computer and the software as important tools, but nonetheless as tools, in service of learning.

APPENDICES

Appendix A

YEAR LONG CURRICULUM PLAN, SELECTED PROBLEMS
FROM PROBLEM SET #1, AND SELECTED YEAR-END PROJECTS

<u>Tentative general plan for the whole school year:</u>

Exercise sets:	Other work:	Weeks:
		1-4
Definitions	Parallel lines	
Computation	SD*	
Data organization		5-6
Parallel	Congruency	7-8
Congruency	SD	
	Proofs	
Inequalities	Proofs	9-10
Convincing arguments	Inequalities	
Tougher problems on triangles: Special points in trinagle. Right triangles: special and trigonometry. Similarity.	The same topics as with the Supposer.	11-18
Quadrilaterals.	The same about triangles as before.	19-20
Loci and constructions.	Quads.	21-2

Not sure yet if Sup-circle will be available	Circles	23–26
Problems involve generalizations between all parts of the curriculum.	Volume Circle computations. Leftovers...	27–32

*Supposer discussion.

Exercise set #1:

Timing:

Four lab periods within two weeks.

The teacher may decide to divide it into two sets and then check it after two periods of work. This way it will be possible to discuss the work from the first two labs.

Content:

-Definition of triangles and elements in the triangle.

-Computation of angles and areas.

-Exterior angles.

-Sum of angles in a triangle.

Teaching goals:

(1) Different ways of organizing data (the worksheets are organized in a certain way but if students identify alternatives they should be considered).

(2) Organizing data so that students can take data from the lab and work on conjectures at home, and discuss their finding in class.

(3) In their work, students may start to think about what constitues a definition. Help them develop criteria for 'good definition', 'better definition' etc.

(4) Making conjectures.

Non Supposer work in parallel:

Parallel lines: computations.

The SUPPOSER lists the following terms under the Draw menu: median, altitude, angle bisector, perpendicular bisector, perpendicular, midsegment, and parallel. Use the SUPPOSER to assign meanings to each of these terms.

A median

___

An altitude

___

An angle bisector

___

A perpendicular bisector

___

A perpendicular

___

A midsegment

___

A parallel

___

The SUPPOSER lists the following names for triangles: right, acute, obtuse, isosceles, and equilateral. Use the SUPPOSER to assign meanings to each of these names.

A right triangle is a triangle

An acute triangle is a triangle

An obtuse triangle is a triangle

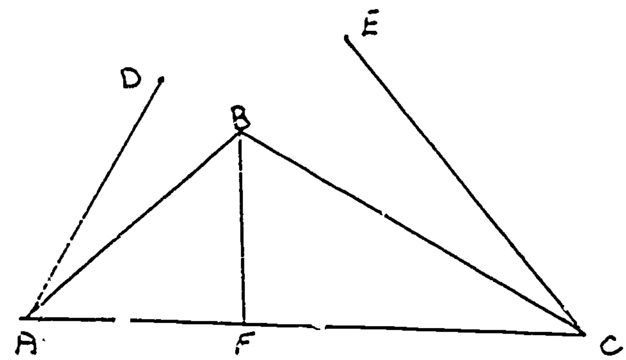
An isosceles triangle is a triangle

An equilateral triangle is a triangle

Use the SUPPOSER to find the sum of the measures of the angles in at least four different triangles. Draw the triangles and indicate their angle measurement.

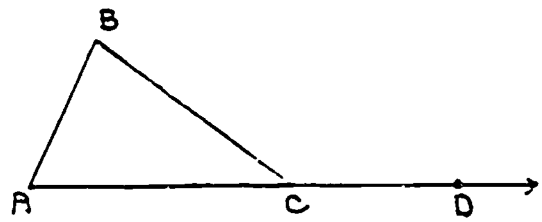
Drawings of $\triangle ABC$	$m\angle ABC$	$m\angle BCA$	$m\angle CAB$	Sum

State a conjecture about the sum of the measures of the angles in any triangle.



$\triangle$ABC is obtuse and $\overline{AD}$, $\overline{BF}$, and $\overline{EC}$ are altitudes. A student claims that there are two angles (neither are right angles) that will always be congruent for any obtuse triangle and its altitudes drawn like those above. Name the two angles that are congruent and write a convincing argument.

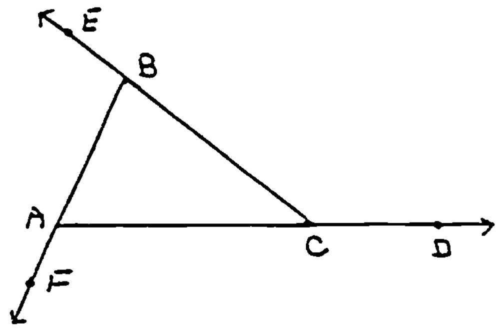
a)



$\angle BCD$ is called an exterior angle for $\triangle ABC$. Conjecture how the measure of exterior $\angle BCD$ is related to the measures of the interior angles of the triangle.

Drawings of $\triangle ABC$	$m\angle BCD$	$m\angle BAC$	$m\angle ACB$	$m\angle CBA$
Acute $\triangle ABC$				
Obtuse $\triangle ABC$				
Right $\triangle ABC$				
Isosceles $\triangle ABC$				
Others				
Conjecture				

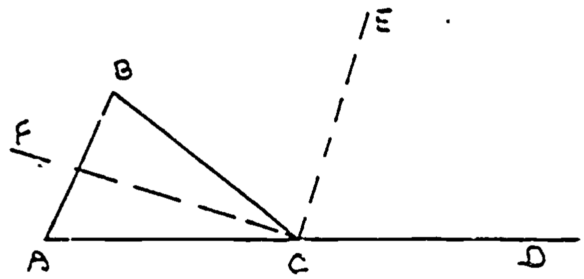
b) Every triangle has three exterior angles. For example, $\angle BCD$, $\angle EBA$, and $\angle FAC$ are exterior angles for $\triangle ABC$.



Do the measures of the three exterior angles for all triangles have anything in common? If so, what?

Drawings and data

c). $\overline{CF}$ is the angle bisector of $\angle ACB$. $\overline{CE}$ is the angle bisector of $\angle BCD$. Are the angle bisectors related? If so, how?



Project #1

Circumscribe ΔABC with a circle (center D). Repeat the construction on different types of triangles and record the diagram in the table below:

Type of triangle	Diagram

Conjecture about possible relationships between:

a) The location of the center D relative to the triangle:

b) The relative size of the circle and the triangle:

c) The properties of point D as a special point of ΔABC:

Project #2

A line segment inside a triangle can divide a triangle into two similar triangles. In some cases these two traingles are also similar to the original triangle; in other cases, the line segment only creates one triangle which is similar to the original triangle.

a) Draw different kinds of line segments (use options of the Draw and Label menu) to create some of the above relationships.

Provide necessary information in the table below; place check marks in the appropriate columns.

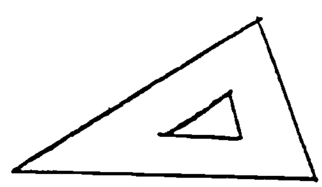
Drawing	Type of Triangle	Type of Triangle	Two similar Δ's	Three similar Δ's

Project #2

a) This problem asks you to describe different methods for drawing a triangle similar to but inside ΔABC such that the two triangles share no points in common. Provide data to verify that your methods work.

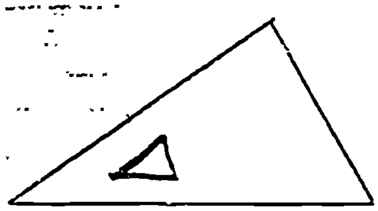
b) Draw the sides of the new triangle inside ΔABC such that they are equidistant from the corresponding sides of ΔABC.

Example:



c) The new triangle is located anywhere inside ΔABC.

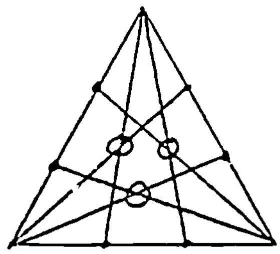
Example:



Project #4

a) Draw an equilateral ΔABC. Subdidvide each side into three equal segments. Connect the vertices of ΔABC to the corresponding points of the subdivisions.

Label the three intersection points circled on the diagram below.



These three intersection points together with the vertices of ΔABC form a star. State conjectures about the relationships among the triangles inside the "star," among the triangles outside the "star," and between the triangles inside and outside the "star." List conjectures about any other relationships that you discover.

b) Do the conjectures you listed for (a) hold true when this construction is repeated on other types of triangles? Which ones hold true for which triangles?

Project #5

a) On an acute triangle, draw three altitudes. Label G as their point of intersection. Now reflect point G in each of the three sides of ΔABC producing points H, I, and J. Draw ΔDEF and ΔHIJ. State as many conjectures as you can about the relationships among the points, segments, angles and triangles. Repeat the construction for other types of triangles?

b) Repeat the procedure, except this time instead of drawing the three altitudes, draw the three perpendicular bisectors.

Appendix B

MIDYEAR TEACHER INTERVIEW SCHEDULE

What has it been like to teach with the SUPPOSER?

How does this relate to your expectations?

What is different? What is harder? What is easier?

What sorts of demands has this approach made on you?

How would you rate the performance of your students? How have your students reacted to the experience?

How can we prepare teachers more effectively to use the SUPPOSER? What kinds of training and support would you recommend for teachers using this approach?

What are your plans for the second semester? How much do you intend to use the SUPPOSER?

What would you do differently next time?

Appendix C

STUDENT INTERVIEW SCHEDULE

Suppose a kid came up to you and said that they were thinking about taking this geometry class next year. They want to know what it is like to learn geometry using the computer and the SUPPOSER.

What would you tell them?

What was hard? What was easy? What was different? What was it like to be taught with the SUPPOSER?

Did the experience change over the course of the year?

What was your experience working in pairs on the computer?

Does the work you do on the computer/SUPPOSER relate to work in class? To your textbook? To homework? How?

Do you think that you learn more or less using the computer and the SUPPOSER?

Ask about work on year end projects: Which project? What have they done? Describe the experience.

Do you like learning this way?

What are your plans for math for next year?

Has this experience had any effect on your thinking or work outside of geometry?

If you were teaching this course next year, what would you do differently?

When do you think geometry was invented? When do you think the last discovery in mathematics was made?

Anything else that you would like to talk about?

Other significant differences between the two groups are presented in Appendix H-3. The results indicate that SUPPOSER students changed more aspects of the given statement in making their generalizations. Their conjectures were more original, and finally, they gave significantly more arguments to support their conjectures. Students in the experimental classes were 15 times more likely to give a supporting argument than students in the comparison classes.

Conclusions

To summarize, at the end of the year there were two important and statistically significant performance differences between the comparison and experimental groups. Most importantly, SUPPOSER students produced higher level generalizations on two out of three questions on the posttest. Second, students in the SUPPOSER classes produced more arguments than the comparison classes on the posttest abstract question, though no arguments were requested.

One of the concerns about teaching with the SUPPOSER originally raised in the research community was that students who study geometry empirically would remain mired in the empirical and would not recognize the need for proof. Our findings do not support this contention. On the contrary, our findings indicate that an empirical approach to geometry may make the need for proof more apparent to students than a purely deductive approach.

In order to examine the issue of proof more closely, we now turn to the results of the argument/proof test which was explicitly designed to study this issue.

2. STUDENT ABILITY TO PRODUCE PROOFS

From the generalization/conjecture posttest, we learned that students who used the SUPPOSER produced more supporting arguments when such arguments were not explicitly requested. However, people have contended that when asked to produce proofs, students who use the SUPPOSER will construct fewer proofs of any kind than their counterparts in traditional geometry classrooms. After all, they reason, students in SUPPOSER classrooms spend a significant portion of the year working empirically with data and doing informal proofs. Therefore, the proofs that they construct are more likely to be informal in nature. At the very least, this argument leads us to expect that the SUPPOSER students will do fewer formal proofs than students from a traditional classroom.

To explore the relationship between proof and empirical work further, we administered an argument/proof test to two of the

comparison classes and to all three of the experimental classes. The objective of the argument/proof test was to determine whether SUPPOSER students would produce fewer proofs, particularly fewer formal proofs, than non-SUPPOSER students.
In this section, we outline the design of the tests, the scoring system, and then present the results of the test.

Test Design

The argument/proof test consisted of three problems, each presenting two true statements from plane geometry accompanied by diagrams (see Appendix I for a copy of this test). The instructions explained that both statements were true and asked students to provide arguments or support for one of the two statements. Students were free to choose whichever statement they found more convincing.

The statements in the first problem were familiar to all students and were a part of the curriculum studied in their classes. In the second problem the statements were generalizations of material studied in class. The statements in the third problem were new material, not covered in the curriculum. The statements were not matched in any other ways.

Scoring of the Test

The argument/proof test was scored on two variables, scored 0 or 1:

FORMAL - whether a formal argument was attempted;
INFORMAL - whether an informal argument was attempted.

In analyzing the results of the argument/proof test, we collected and compared the scores of the responses of the experimental and comparison groups for each of the three questions on each of the three variables. Once again, we used a chi-square analysis to test for statistically significant differences. Specifically, we compared the frequencies of the two variables INFORMAL and FORMAL by experimental and comparison groups for each question on the test.

Results

Summary charts of the data on the number of formal and informal arguments offered by students in the different classes are presented in Figure 2.

Appendix D-1

GENERALIZATION/CONJECTURE PRETEST

INSTRUCTIONS

These exercises and the results will have no bearing on your grade. It is for research purposes only.

1. Please write your name on each piece of paper that you use.

2. There are four problems and you have approximately thirty-five minutes to work on them. Don't feel that you have to complete all four problems. If you only have time for two or three, that is fine.

3. Each problem is on a separate page. If you need additional space to work on a problem, either use the back side of the paper or ask for additional sheets. Just be sure that the number of the problem and you name appears on every sheet.

HAVE FUN!

NAME

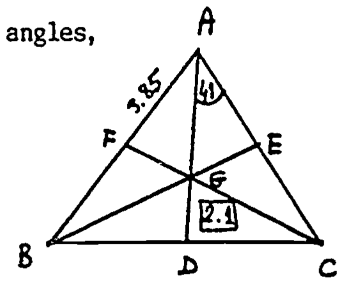
<u>Statement 1:</u>

The numbers on the diagrams below represent the measures of angles, lengths and areas.

For example: The length of $\overline{AF}$ is 3.85

The angle CAD is 41 degrees

The area of triangle CDG is 2.1.



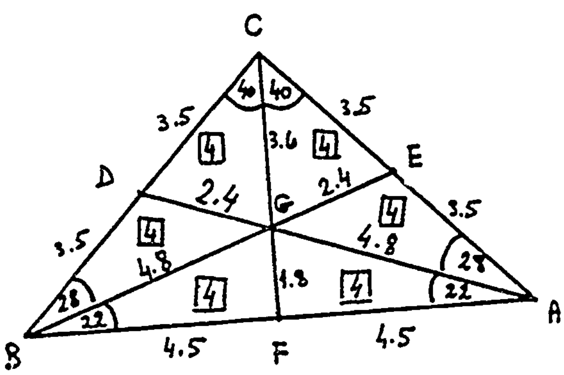


Diagram 1

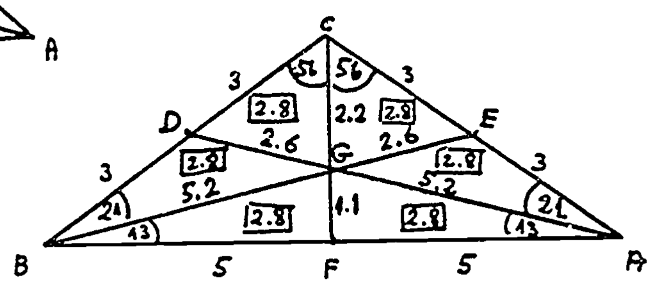


Diagram 2

List as many significant connected statements as you can make.

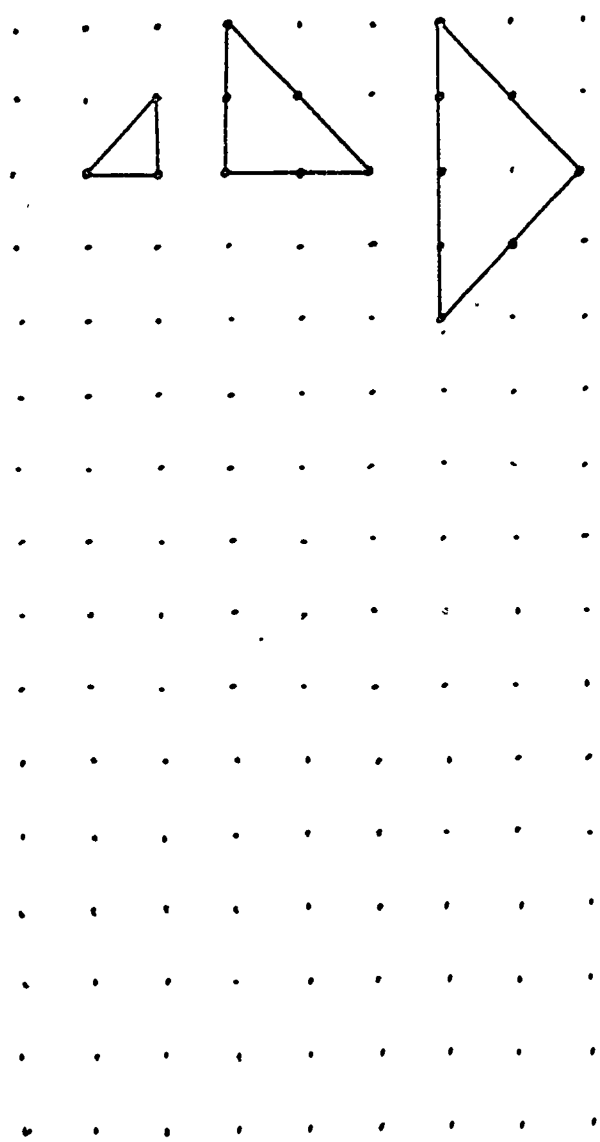
110

NAME

Statement 2:

The right triangles on the grid below have 3, 6 and 8 points on their perimeter.

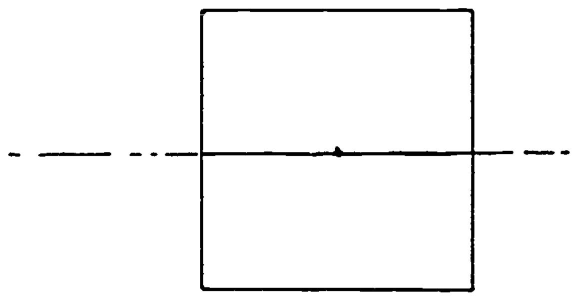
List as many significant connected statements as you can make.



NAME

Statement 3:

A line which passes through the center of a square and is parallel to two of its sides, divides the area of the square into two equal areas.



List as many significant connected statements as you can.

NAME

Statement 4:

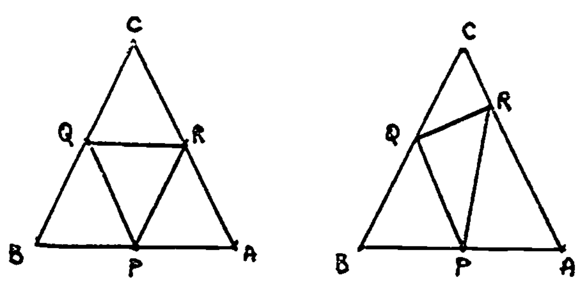
P, Q, R are points on the sides of triangle ABC.

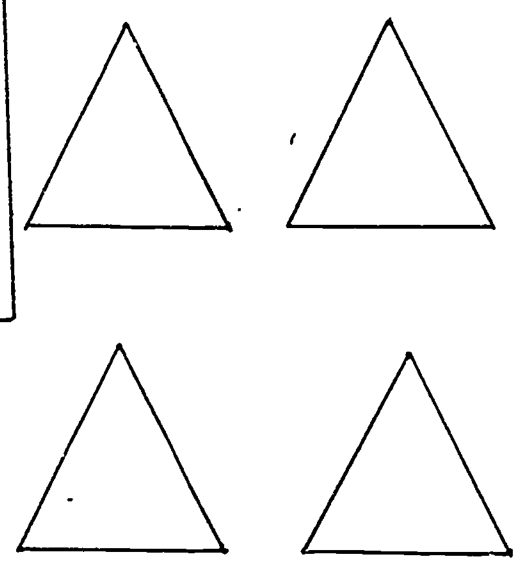
In diagram (1), triangle ABC and triangle PQR are both equilateral.

In diagram (2), triangle ABC is equilateral, triangle PQR is not.

List as many significant connected statements as you can.

Diagram (1) Diagram (2)



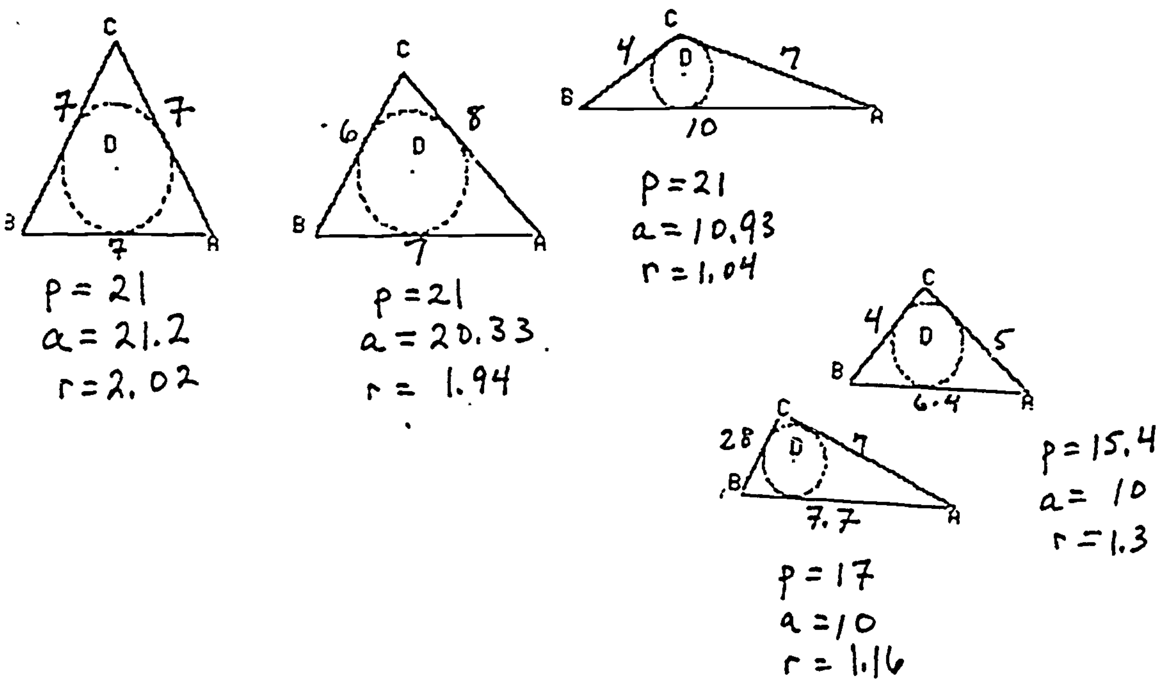


Appendix D-2

GENERALIZATION/CONJECTURE POSTTEST

Name __________

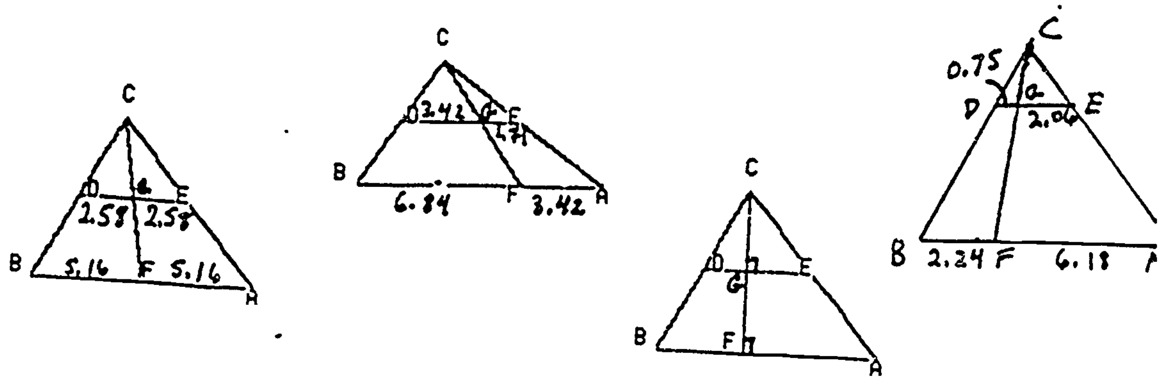
Problem I:
There are five drawings below. On each drawing, you are given the lengths of the sides of the triangle. In addition, underneath each drawing you will find listed the perimeter (p) of the triangle, the area (a) of each triangle and the radius (r) of the inscribed circle.



List as many significant connected statements as you can.

Name _________

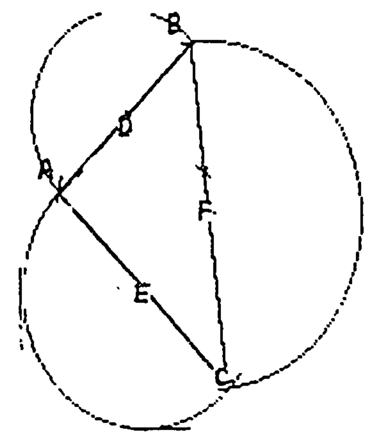
Problem II:
In each of the following triangles the sides BC and AC are divided in the same way. In one case you are given information about angles. In the other three cases, the numbers on the diagrams represent lengths.



List as many significant connected statements as you can.

Name ________

Problem III:
In the diagram below, ABC is a right triangle and D, E and F are the centers of the three half circles which are on the sides.



List as many significant connected statements as you can.

Appendix E-1

SCORING GENERALIZATION/CONJECTURE TESTS

Illustration of Scoring Scheme for Levels of Generalization

In order to clarify the scoring of the levels of generalization of the students´ statements, we will present an ordered list of student generalizations for question one and question three on the posttest. We will point out differences between the levels and the criteria used to score the conjectures.

POSTTEST QUESTION ONE

(Note: These statements are quoted from students´ papers.)

Level 0

"The first triangle is isosceles,the second has lengths that are in order, the third the lengths go by threes."

Level 1

"The perimeters are odd numbers. In the fourth and the fifth triangles the areas are equal but the perimeter is off by about two."

Note: These first two levels are a more advanced version of repeating the given than students wrote for the pretest. In the pretest, they literally listed the numbers that were given in the problem. At the end of the year, they give descriptions of the given. The higher levels are distinguished by use of general language, e.g. "odd numbers".

Level 2

"The closer the inscribed circle is to point C, the smaller the radius, the farther away it is the larger the radius."

Note: It is not clear if this is a general statement or only a description of a phenomenon in the data on the page.

Level 3

"The perimeter of the triangle does not directly affect the area of the triangle."

Note: Again, it is not perfectly clear that this statement is a generalization, however the language is getting more general ("the triangle").

Level 4

"Formula = To find Radius = take the smallest and the second smallest side then divide the product (sum) of side a and side b divided by the largest side so that would show like this side a + side b/side c = radius of circle inscribed in triangle."

Note: Level 4 mentions that the formula works for a "circle inscribed in a triangle". It is clear that we have a generalization.

Level 5

"In a triangle with an inscribed circle the circle size depends on the largest angle."

Note: Level 5 is distinguished by the words "in a triangle", whereas levels 2 and 3 do not explicitly say that their conjecture is true for any triangle. Those conjectures may only be true for the specific cases given in the statement of the problem.

Level 2 is really about angles, though it does not explicitly state that it is. In contrast, level 5 is the same idea stated more clearly. Level 2 also uses the specific letters of the diagrams and does not use the more general "largest angle". In that respect the level 4 conjecture is interesting. It creates a system to label the smallest, medium and largest sides of the triangle.

POSTTEST QUESTION THREE

Level 0

"$\overline{AB}$, $\overline{BC}$, $\overline{AC}$ are diameters."

Note: Simple repetition of the given.

Level 1

"Segment AB appears shortest, segment AC appears middle length, segment BC appears longest."

Note: This statement is dependent completely on the diagram that accompanies the problem.

Level 2

"The semicircle BA and AC, when added up their areas equal to BC."

Note: Again, it is not clear whether the conjectures are truly general. In addition, this conjecture is directly related to the school curriculum and does not change any relationships. It is the `obvious' conjecture for the problem.

Level 3

"Triangle ABC is a right triangle so diameter squared of ⊙D + ⊙E = ⊙F squared."

Note: It is still not clear, but it is more likely that it is general (Triangle ABC "is a right triangle"). The idea is still the `obvious' one for this problem.

Level 4

"If the circles D and E were to be completed the point where they intersect would bisect BC at F."

Note: Notice that the subject of the conjecture is very different from levels 2 and 3. The student has changed the focus of the problem, explored a new facet of the problem and come up with a generalization.

As mentioned before, with an abstract question it is more difficult to determine if the conjectures are really general. Even level 4 on this question does not explicitly use general language. Thus, when comparing the levels of generalization (the variable LEVEL), there is a natural bias in favor of abstract questions. In a problem that is posed with data, it is easy to see when a student is being specific and when they think that a conjecture is true at a more general level. When students are thinking only about the specific case, they tend to refer to the data or to make a statement which points to the case in front of them. "In this triangle, ...", or "In the first three triangles, ...".

When a problem is framed abstractly it is difficult to make this distinction. It is hard to know if the statement is a general statement or if the student for some reason thinks that his statement is only true for the particular triangle pictured. It is also difficult to know if the student even considered this issue. On both the pretest and the posttest, we credited these conjectures as general statements for all classes. Thus, we gave the students the benefit of the doubt on the pretest. On the posttest, after a geometry course, it was more likely that the statements were indeed general ones.

Appendix E-2

SCORING GENERALIZATION/CONJECTURE TESTS

Types of Changes Made in Problem in Order to Make Generalizations

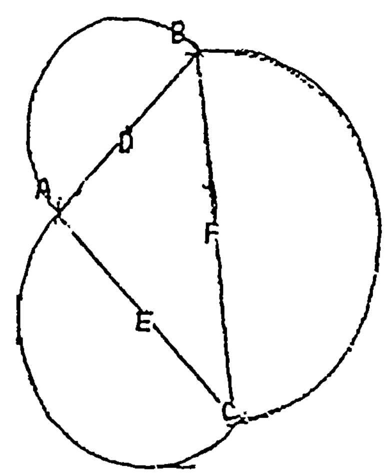
As mentioned in the text, there were three types of changes to the statements given in the problem that we looked for in students papers. Students only made changes that could be classified into two of these types of changes. The third type was found in Dr. Yerushalmy's study, but did not appear in the 1985-86 study.

CHANGE IN OBJECT OF INTEREST

There were many ways in which students changed the object of interest in a problem. Sometimes the change involved adding a construction to the diagram, other times it did not. Problem three in the posttest was a problem that was particularly rich in the amount of changes students made. Instead of describing this category verbally, we will use this problem and the students' work to illustrate this type of change.

The statement of the problem was as follows:

> In the diagram below, ABC is a right triangle and D, E and F are the centers of the three half circles which are on the sides.



> List as many significant connected statements as you can.

The problem was written to have students use the Pythagorean theorem and knowledge about areas. Students changed the focus of the problem to the following objects:

Changes that do not involve additions to the construction

1. Equality of lengths.

   Sample generalizations:

   "Segment BA could equal BF."
   "Hypothesis - AE = BF = BA = FC."

   Note: Mathematically the change of focus can be trivial and based on a non-general assumption.

2. The arclength of the semicircles.

   Sample generalization:

   " $\overset{\frown}{AB} + \overset{\frown}{AC} = \overset{\frown}{BC}$ "

3. The measures of the central angles in the circles as compared to the arcs.

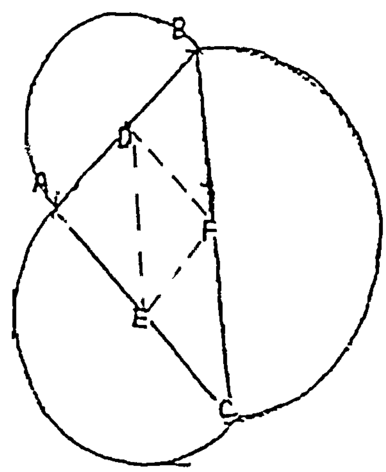
   Sample generalization:

   "$\measuredangle$BAC = twice BC "

   Note: The conjectures are not always correct.

Changes with additions to the construction

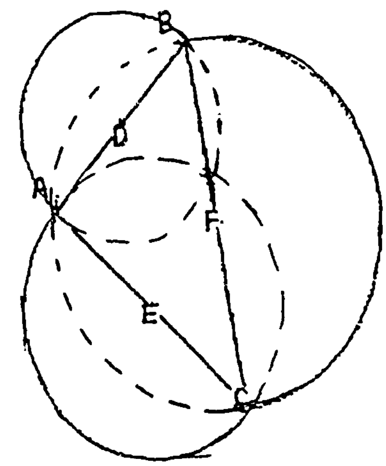
4. The triangle created when D, E and F are connected.



   Sample generalization:

   "DEF is a right triangle."

5. The circles that can be completed from the half-circles in the diagram.



Sample generalization:

"The radius (probably FA) = the median from center point to the opposite side."

Note: The wording of the conjectures is not always perfectly clear. In this example, different circles and their centers are not distinguished by name.

6. The area of circle F in terms of the triangle and the two other circles (see Figure 3 on the next page).

7. The intersection point of circles D and E (see Figure 4).

CHANGE OF GEOMETRIC RELATIONSHIP

This category of change was much less prevalent than the previous category. The notion is that as opposed to changing the focus of the question, one can also change geometrical relationships that are givens in the problem.

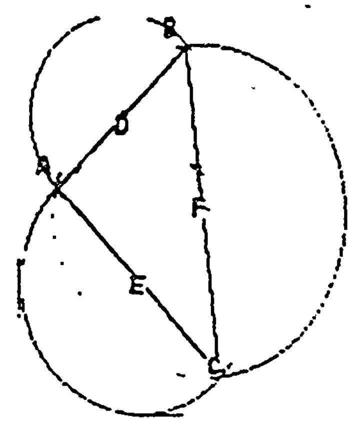
It is important to recognize the difference between these two types of changes. For example, the last example given in the previous section is a change in the object of interest and is not a change of a given relationship. The student thinks that when he completes the circles that the intersection of the circles D and E are the midpoint of BC. He does not alter the problem, rather he uncovers another facet of the problem.

Below is a clear example of a student who is willing to entertain the notion that relationships that appear to be fixed in the

Name *Allan*

Problem III:

In the diagram below, ABC is a right triangle and D, E and F are the centers of the three half circles which are on the sides.

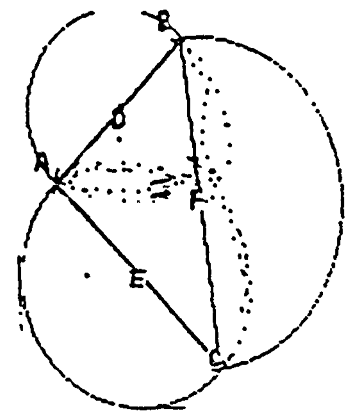


List as many significant connected statements as you can.

- $\overline{bc}$ · diameter of ⊙F
- ◠d + ◠e = ◠F
- $A\Delta abc + \frac{1}{2} A$◠$d + \frac{1}{2} A$◠$e =$ ◠F
-

Name D

Problem III:
In the diagram below, ABC is a right triangle and D, E and F are the centers of the three half circles which are on the sides.



List as many significant connected statements as you can.

$\odot D^2 + \odot E^2 = \odot F^2$

$m\angle A = 90°$

Because $\angle A$ is inscribed on $\odot F$

If ~~the~~ circles where to be D + E completed they point where the.. Intersect would bisect BC at F.

problem can change. We again use problem three from the posttest.

Sample conjecture: "It could be possible for the three circles to be concentric."

The circles as drawn in the diagram as given are clearly not concentric, however this student is able to overcome the relationship suggested in the problem and suggest a new relationship. In this case, she is not correct.

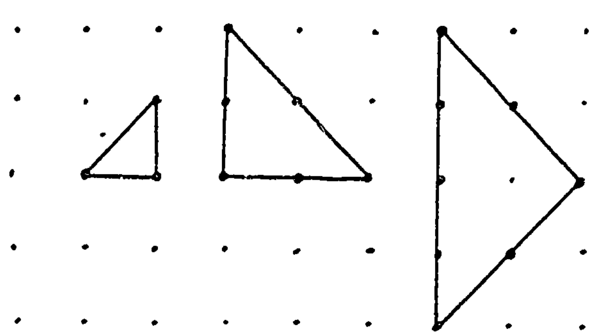
Another type of change in geometrical relationships is illustrated by the third question on the pretest. The statement given was that "A line which passes through the center of a square and is parallel to the two of its sides, divides the area of the square into two equal areas." Here students could alter the relationship between the line and the square and ultimately discover that as long as the line passes through the center of the square that it will divide the area into two shapes of equal area. On the pretest, students did not make this type of change.

CHANGE IN NUMERICAL RELATIONSHIPS

A different sort of change is one that focusses on a numerical aspect of a problem. Students did not exhibit this type of behavior this year, though they did in Dr. Yerushalmy's previous study.

Problem two on the pretest presents an opportunity for this type of change. The right triangles on the grid below have 3, 6 and 8 points on their perimeter. Here a numerical aspect of the problem is easily accessible and readily available to be changed.

Another example can be sketched using the third problem from the pretest. A line which passes through the center of a square and is parallel to two of its sides, divides the area of the square into two equal areas. In this statement, the numerical aspect of the problem is less apparent. However, it is possible to generalize the statement by trying to figure out how many lines in what positions will divide the square into n equal areas for different values of n.



Appendix F

DIFFERENCES IN STUDENT PERFORMANCE ON TWO TYPES OF CONJECTURE PROBLEMS

As noted in the body of the text, there are two types of problems given in the generalization/conjecture tests, problems that are formulated with data and those that are formulated abstractly. In order to examine the issue of the differential performance of the whole sample on the two types of problem formulation, we compared student achievement on questions one and three for both the pretests and posttests.

A. Pretest

In general on the pretest, students produced more high level generalizations on the abstract formulation than on the data formulation. These generalizations involved more changes and were correct a smaller percentage of the time than the generalizations for the data problems. The originality of the generalizations and the number of types of proofs produced were about the same for each type of question.

COMPARISONS OF STUDENT PERFORMANCE ON QUESTIONS ONE AND THREE

Variable	Chi-sq. Stat.	p-value	strength Cramer´s V	 phi
LEVEL (0 vs. 1-5)	7.20	0.003	---	.24
ORIGINALITY (0 vs. 1 vs. 2-3)	1.60	0.451	0.1	---
CORRECT (0-3 vs. 4 vs. 5)	11.23	0.004	0.26	---
ARGUMENTS (0 vs. 1-3)	0.26	0.621	---	0.07
CHANGES (0 vs. 1-3)	3.35	0.067	---	0.15

In our scoring scheme, students who simply repeated the ideas or data that were given by the problem or who stated these givens in a slightly more general form (e.g., noting numerical patterns in some of the given data) would receive low LEVEL scores and very high CORRECT scores. After all, when the information that you state is written directly in front of you, it is not very difficult to be correct. We thought that such statements could be the cause of the tendency for the more general statements on question three being correct less often than the less general statements on problem one.

Therefore, we tried to remove this effect by isolating those students who merely repeated the ideas or data given in the problem from the rest of the students. Then we again examined the differences between the LEVEL and CORRECT scores of the remaining students' conjectures for questions one and three.

After removing the "repeaters," a chi-square analysis revealed that a strong, significant difference remained between the scores of the LEVEL of the rest of the students' conjectures on questions one and three (LEVEL--0 vs. 1-5(collapsed): chi-square = 2.80, Fisher's exact p-value < 0.05, phi = 0.46). The "non-repeaters" also wrote more general statements in response to question three. However, there were no statistically significant differences on CORRECT scores between the students' responses to the two questions (CORRECT--all correct vs. some incorrect: chi-square = 0.002, p-value < 0.965, phi = 0.09). Thus for this sample, the statements that were more general and had a higher LEVEL score were correct as often as statements of a lower LEVEL that did not simply repeat the information conveyed in the problem.

B. Posttest

On the posttest, the chi-square analysis comparing questions one and three yielded the following results for the five important variables.

Variable	Chi-sq. Stat.	p-value	strength Cramer´s V	phi
LEVEL (0-1 vs. 2 vs. 3-5)	4.92	0.085	0.23	---
ORIGINALITY (0-1 vs. 2 vs. 3-5)	4.36	0.112	0.21	---
CORRECT (0-2 vs. 3 vs. 4-5)	6.33	0.042	0.26	---
ARGUMENTS (0 vs. 1-3)	17.50	0.000	---	0.41
CHANGES (0 vs. 1-3)	5.67	0.017	---	0.24
	AFTER REMOVING "REPEATERS"			
CORRECT (0-4 vs. 5)	2.45	0.118	---	0.27
LEVEL (0-1 vs. 2-5)	0.75	0.386	---	0.17

As shown by the table, the difference in LEVEL of the student generalizations between questions one and three was moderately strong and approached statistical significance at the .05 level. Problem three, formulated abstractly, elicited more general statements. There was no statistically significant difference between the questions in ORIGINALITY. On the other hand, there was a strong and significant difference between the questions on the number of ARGUMENTS given by the students to support their generalizations and a moderately strong and significant difference in the CHANGES variable. In both cases, these differences favored question three.

An examination of CORRECT for the complete set of data shows a modest, although statistically significant, difference between the students´ generalizations on the two questions. Students´ generalizations were correct more often on question one, the question for which their statements were not as general. Unfortunately, when one removes those students whose statements merely repeat the given, then for both the LEVEL variable and the correctness measure the difference is no longer statistically significant. Thus we cannot conclude as we did in the pretest that the more general statements were correct as often as the less general ones which did not repeat the given.

C. Conclusions about Problem Formulation

Most students seemed to be less inhibited in their generalizations and experience greater freedom of thought when a question is posed abstractly. Their conjectures are more general and they present more proofs.

One possible explanation for this phenomenon is that the two types of questions elicit different responses. A data problem demands manipulation of the data and reaching conclusions before further generalizations or a convincing argument can be tackled. On an abstract question, there is no need to manipulate the data and one can move immediately to abstract thought and spend the time on making further generalizations. This difference may be exacerbated by a testing situation where time is limited.

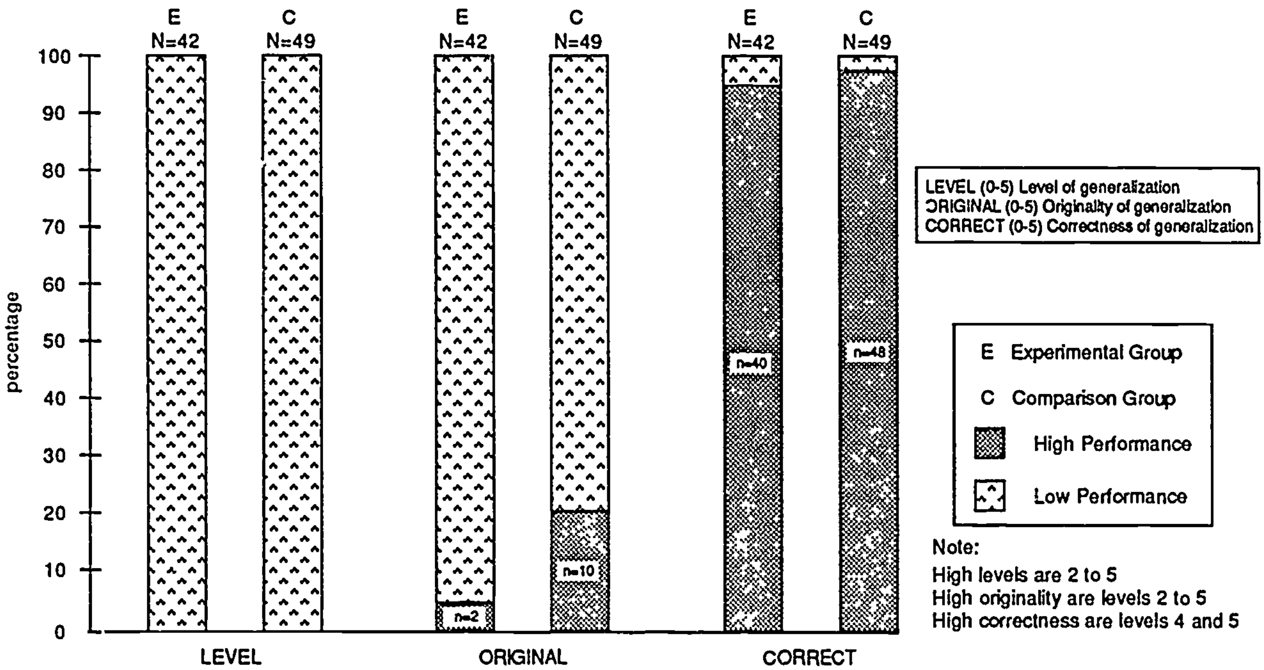
Since the two kinds of formulation call forth different student responses, the different types of questions can be used for different purposes. The data formulation allows the investigator to gauge the level of generalization that students come to when presented with data. It tests student skills in understanding data.

Problems that are formulated abstractly allow the investigator to examine the relationship between conjectures and supporting arguments. Students for whom arguments are an integral part of the process of making conjectures will make more unsolicited arguments on this type of question than students who do not see the connection between proofs and conjectures, or the need for proof.

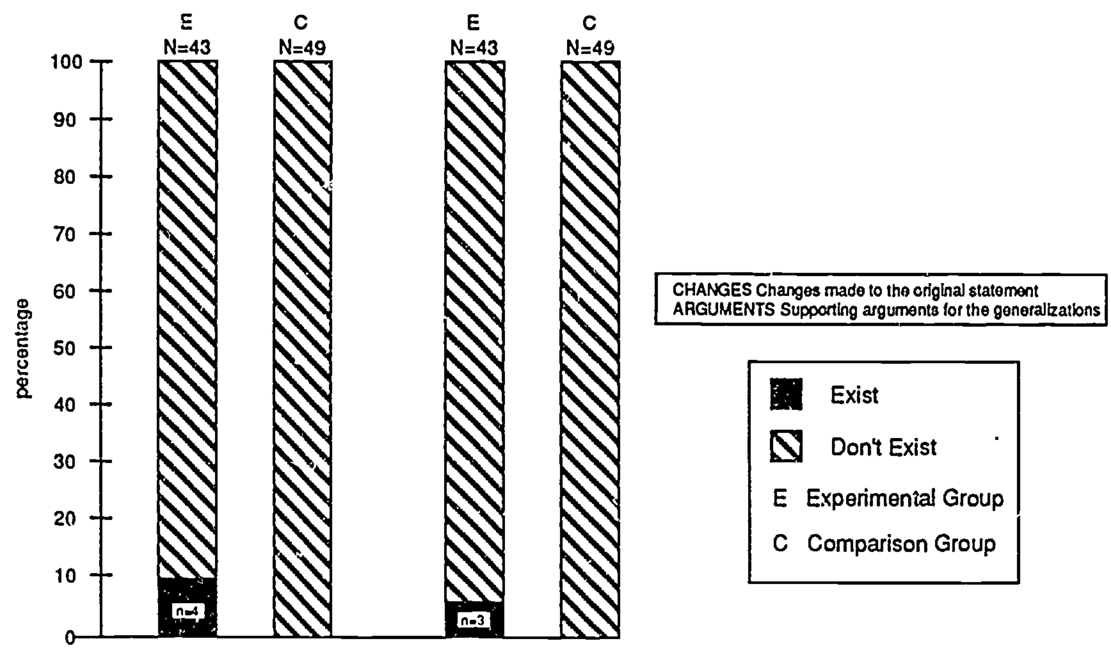
Appendix G-1

GENERALIZATION/CONJECTURE PRETEST COMPARISONS
FREQUENCIES OF SCORES OF STUDENTS IN EXPERIMENTAL AND COMPARISON CLASSES

Question One

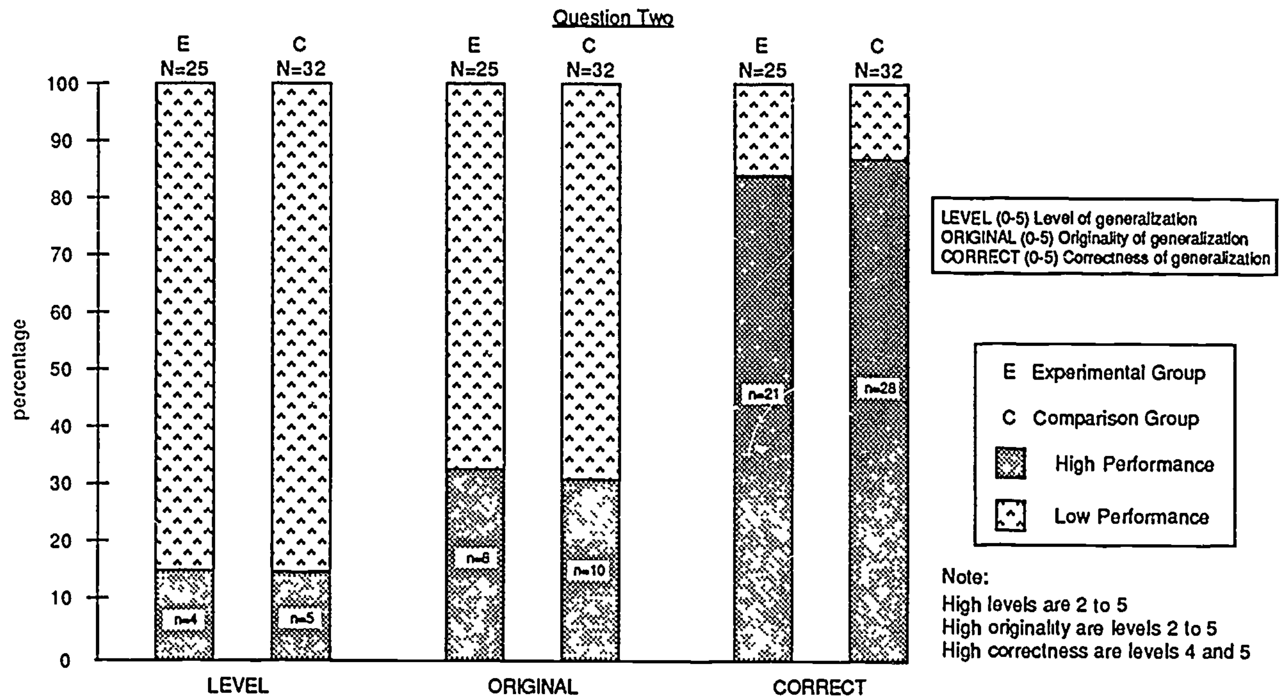


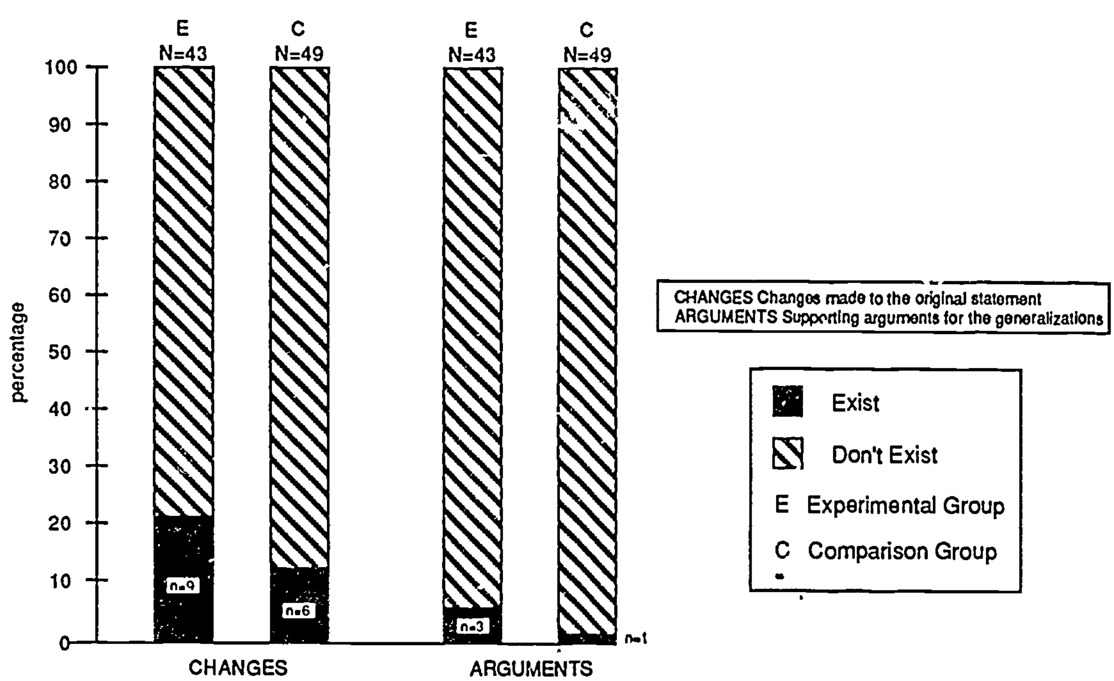
(chi-square=21.80, p<0.00, phi=0.52)



Appendix G-2

GENERALIZATION/CONJECTURE PRETEST COMPARISONS
FREQUENCIES OF SCORES OF STUDENTS IN EXPERIMENTAL AND COMPARISON CLASSES

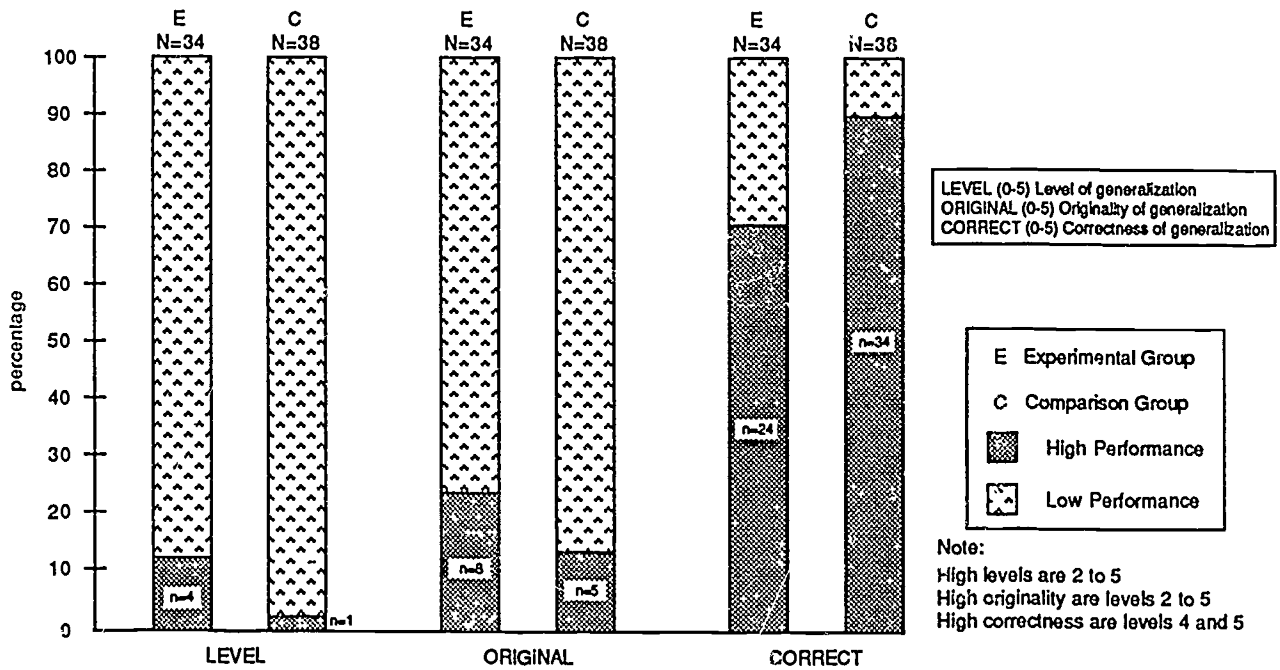


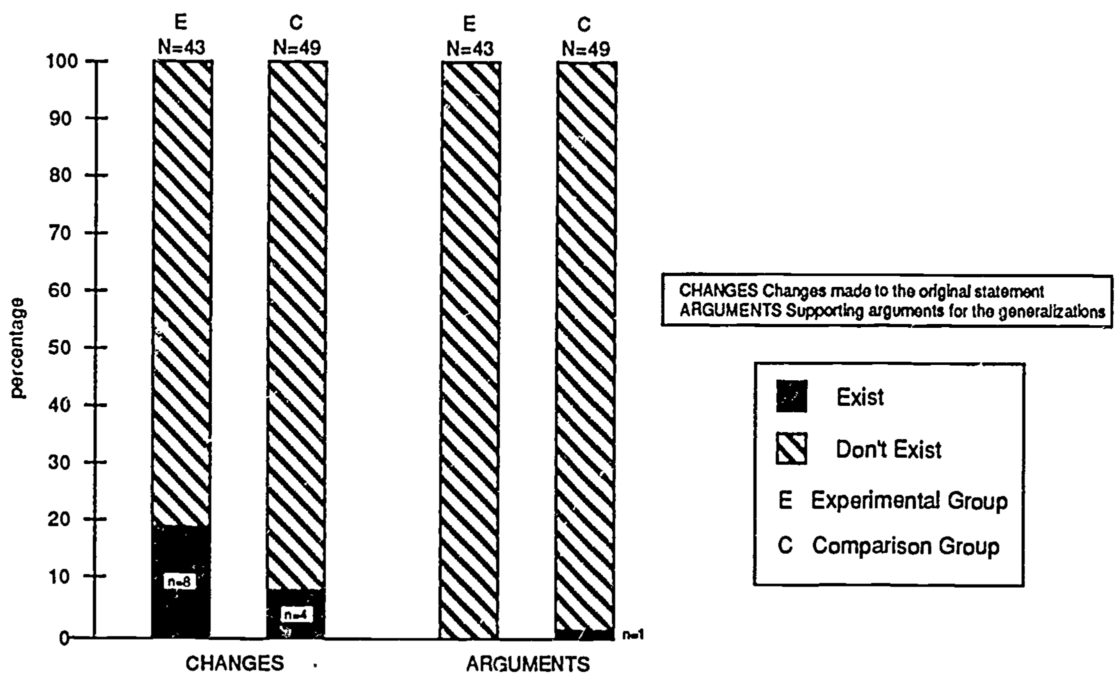


Appendix G-3

GENERALIZATION/CONJECTURE PRETEST COMPARISONS
FREQUENCIES OF SCORES OF STUDENTS IN EXPERIMENTAL AND COMPARISON CLASSES

Question Three

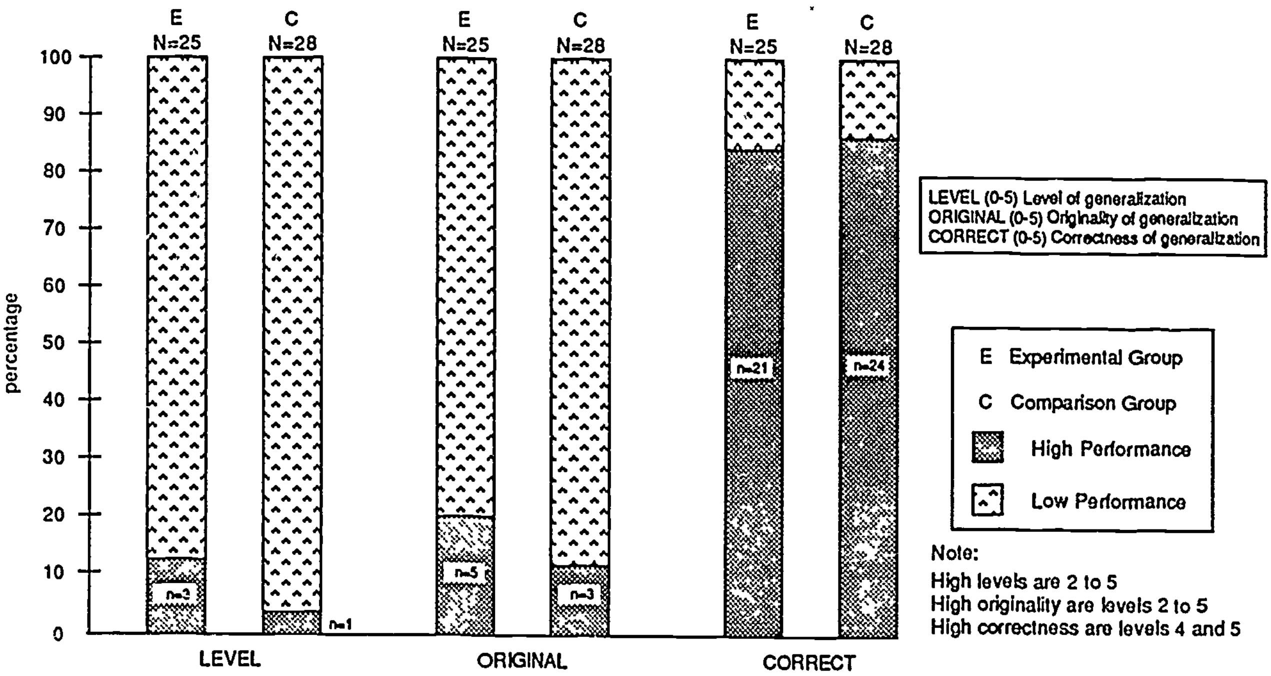


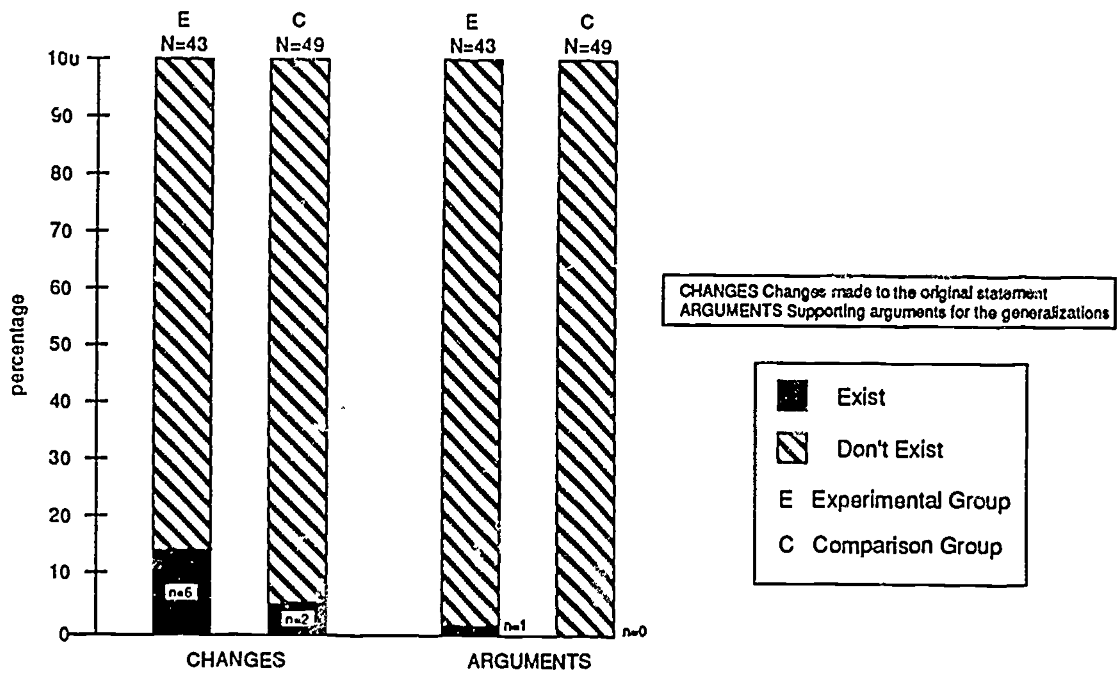


Appendix G-4

GENERALIZATION/CONJECTURE PRETEST COMPARISONS
FREQUENCIES OF SCORES OF STUDENTS IN EXPERIMENTAL AND COMPARISON CLASSES

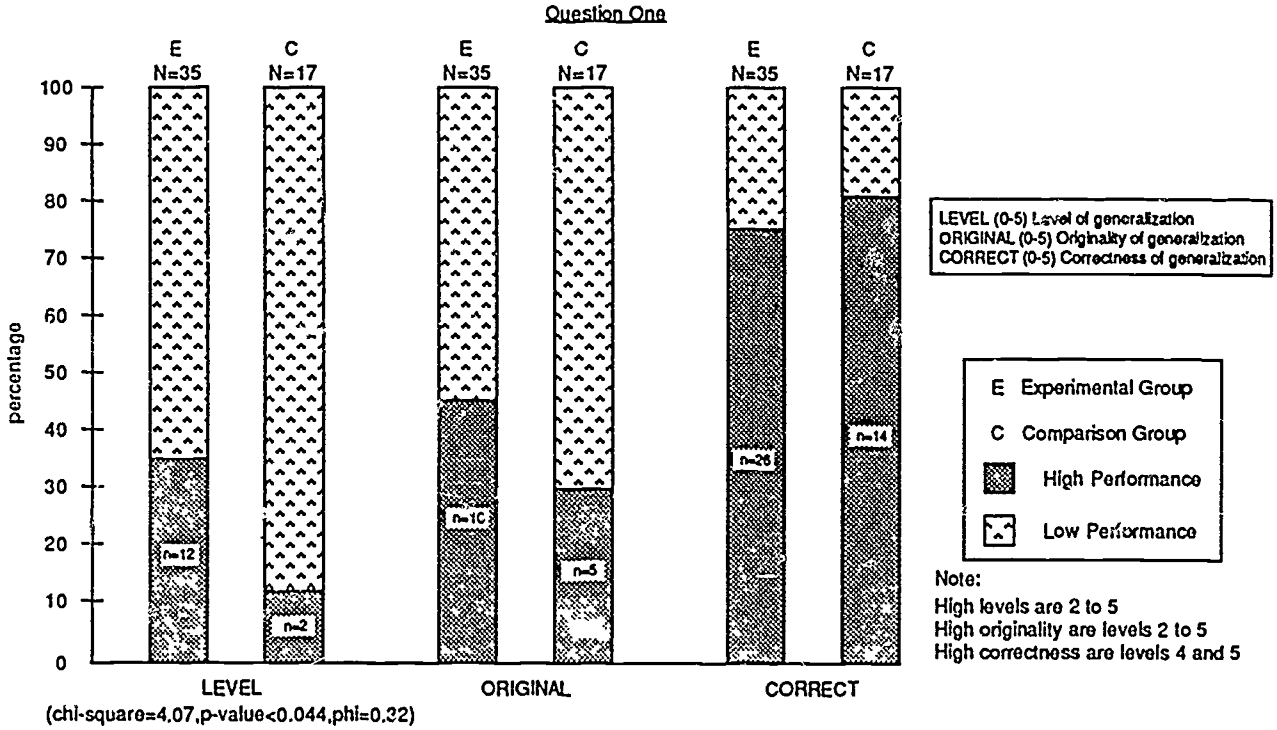
Question Four

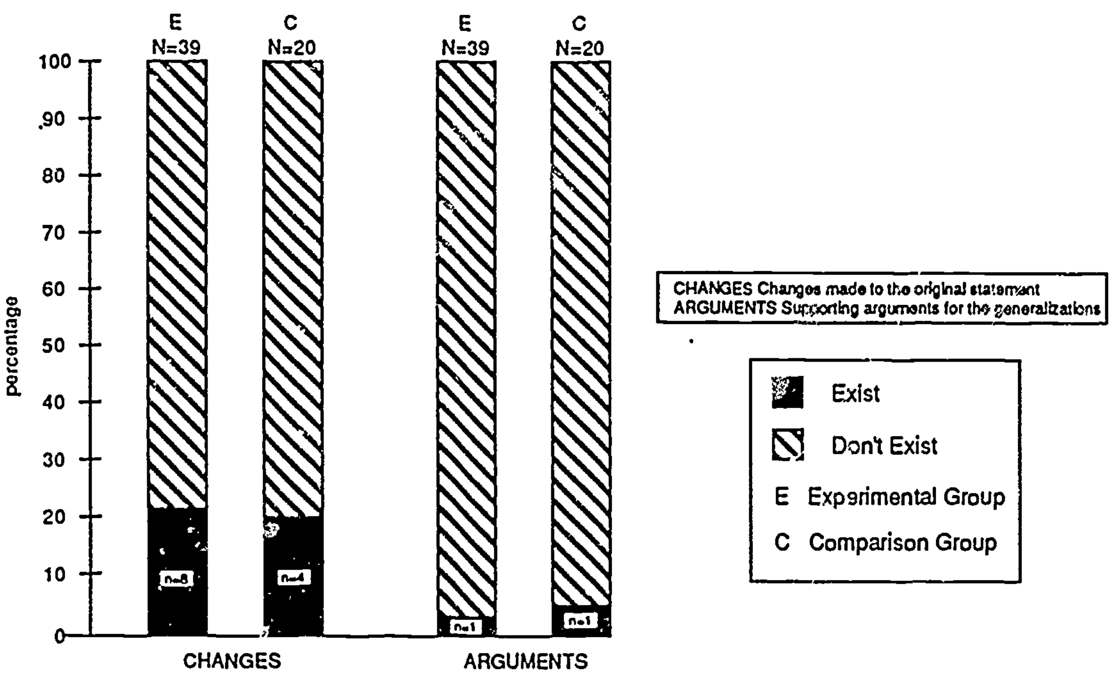




Appendix H-1

GENERALIZATION/CONJECTURE POSTTEST COMPARISONS
FREQUENCIES OF RESPONSES BY VARIABLES

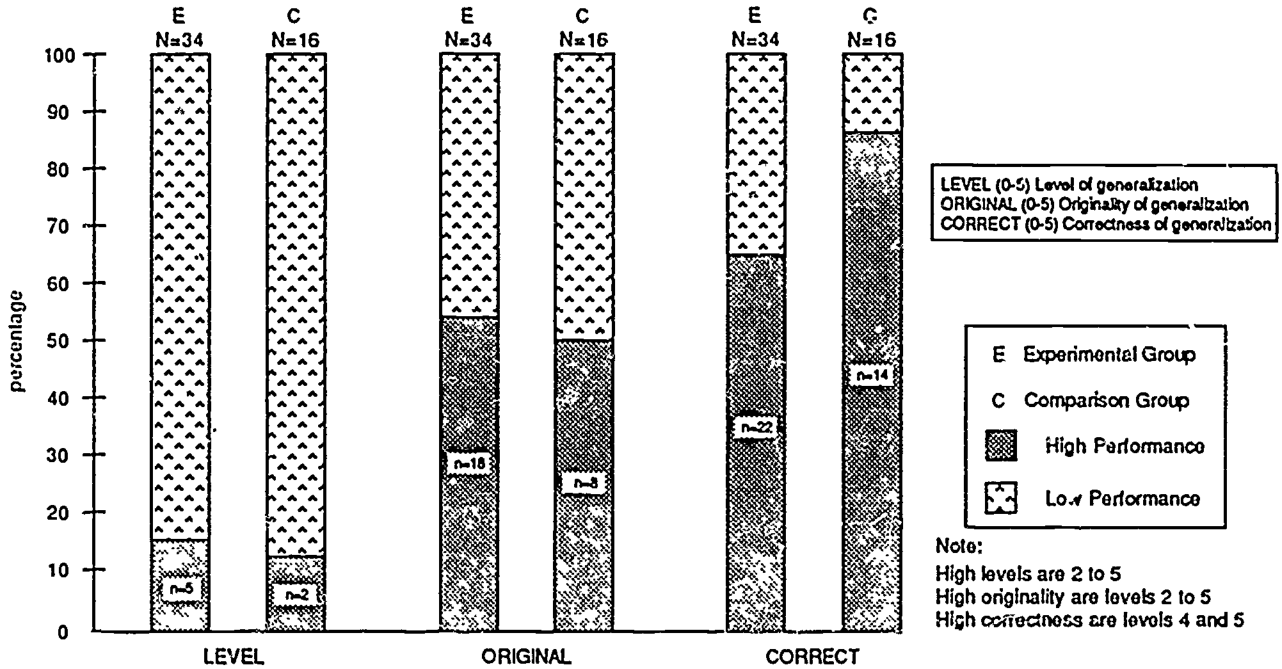


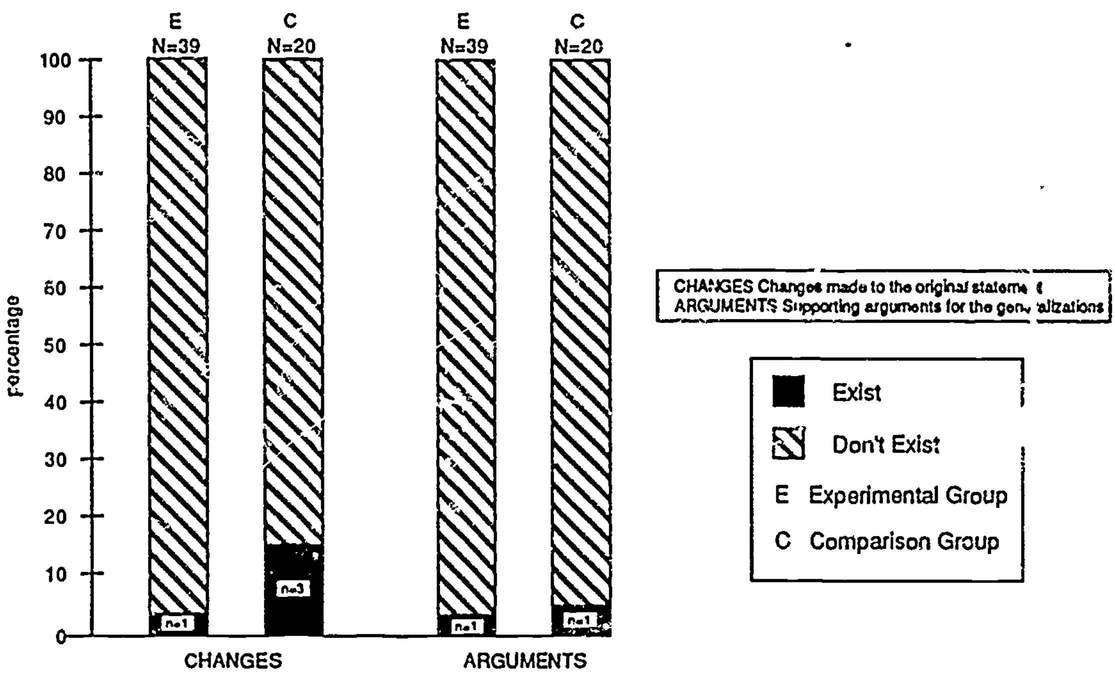


Appendix H-2

GENERALIZATION/CONJECTURE POSTTEST COMPARISONS
FREQUENCIES OF RESPONSES BY VARIABLES

Question Two

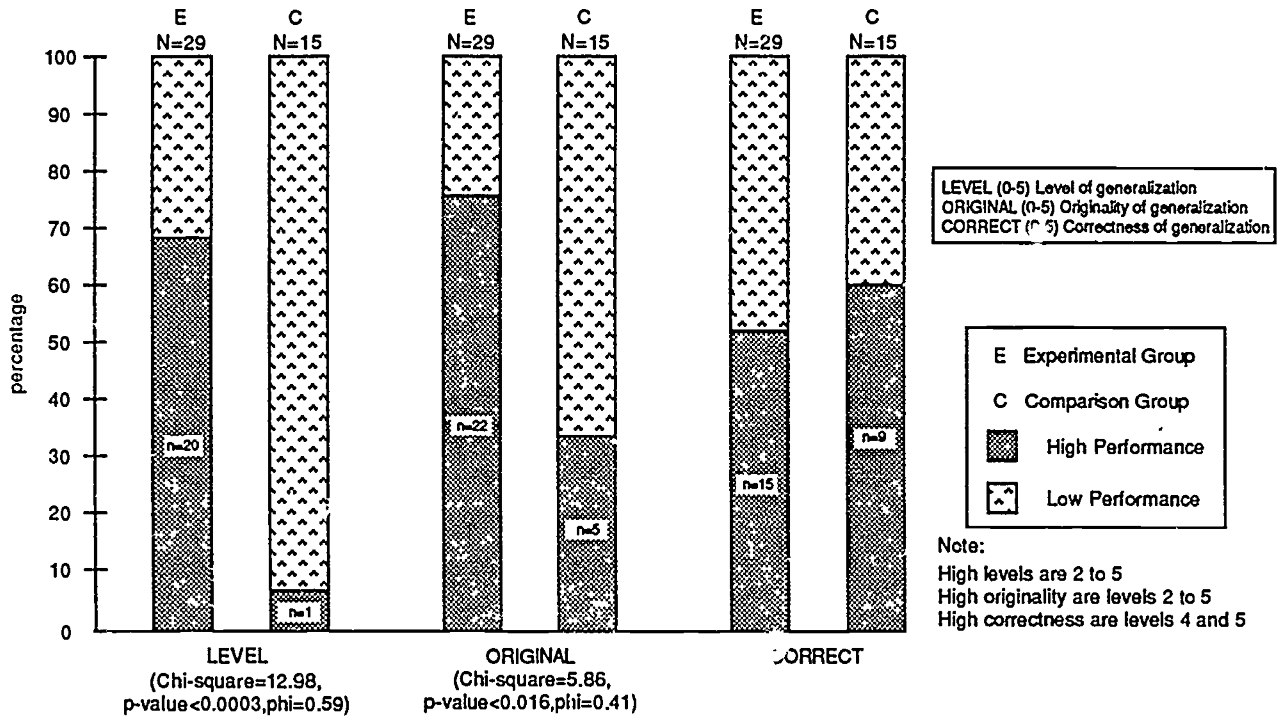


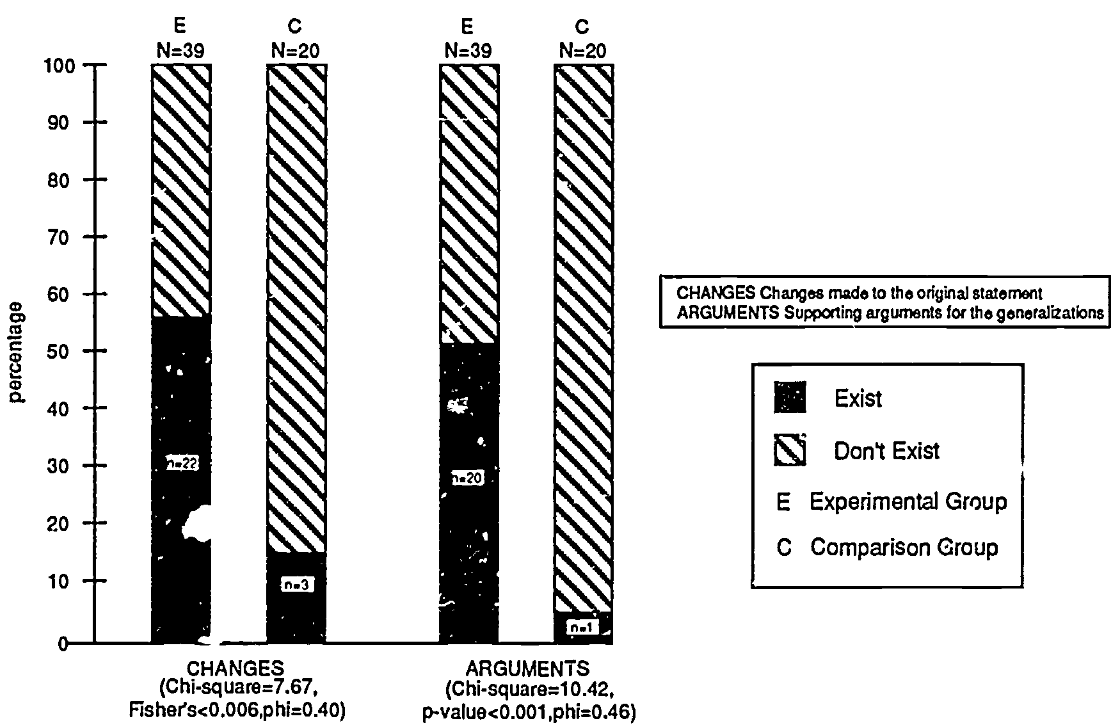


Appendix H-3

GENERALIZATION/CONJECTURE POSTTEST COMPARISONS
FREQUENCIES OF RESPONSES BY VARIABLES

Question Three





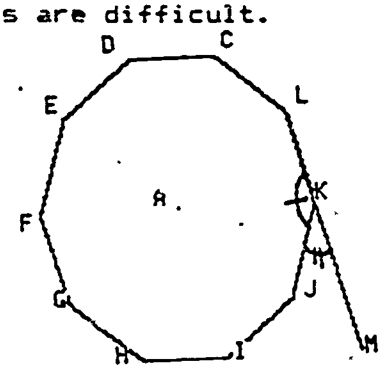
Appendix I

ARGUMENT/PROOF POSTTEST

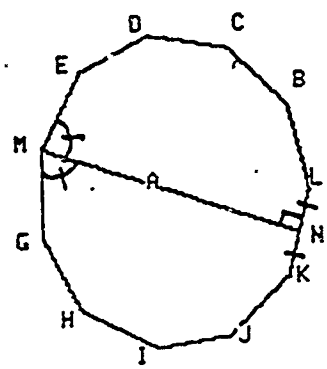
Name _________

Problem I:
There are two true statements below. Please make an argument to show that one of these two statements is true. You may choose either one of the statements.
Please show all of your work. Do not be concerned if you are not able to develop a full-blown proof, these problems are difficult.

(1) In a regular 10-gon each interior angle is four times larger than the adjacent exterior angle.



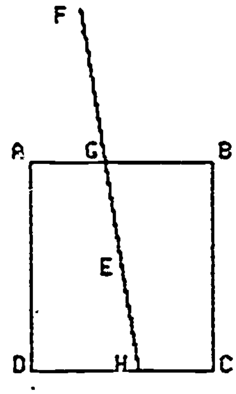
(2) In a~~n~~ regular 11-gon any angle bisector of an interior angle is a perpendicular bisector of the opposite side.



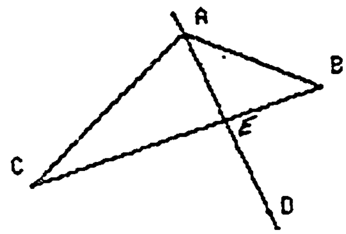
Name _________

Problem II:
There are two true statements below. Please make an argument to show that one of these two statements is true. You may choose either one of the statements.
Please show all of your work. Do not be concerned if you are not able to develop a full-blown proof, these problems are difficult.

(1) In a square any straight line which passes through the center of the square divides the square's area into two equal areas.

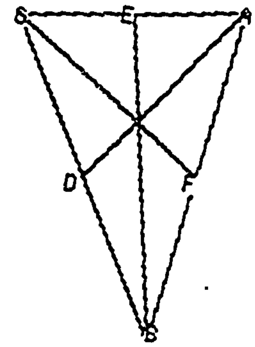


(2) Any straight line that passes through a vertex of a triangle divides the triangle into two triangles whose areas are in the same ratio as the ratio between the two parts of the side of the triangle which is opposite the vertex.



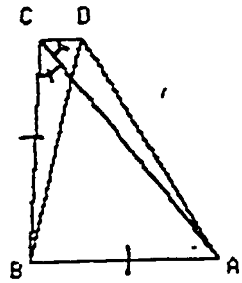
Name ________

Problem III:
There are two true statements below. Please make an argument to show that one of these two statements is true. You may choose either one of the statements.
Please show all of your work. Do not be concerned if you are not able to develop a full-blown proof, these problems are difficult.



(1) In any triangle, the sum of the lengths of the three medians is less than the perimeter.

(2) If a diagonal of any trapezoid is also an angle bisector, then at least one side of the trapezoid is equal to one of the bases of the trapezoid.



SUPPOSER BIBLIOGRAPHY

Chazan, D. "Similarity: Unraveling a conceptual knot with the aid of technology." Proceedings of the Eleventh International Conference: Psychology of Mathematics Education. Eds. Bergeron, Herscovics, Kieran. Montreal, Canada. 2 (1987): 3-10.

Schwartz, J., and M. Yerushalmy. "The Geometric Supposer or the Computer as Intellectual Prosthetic for the Making of Conjectures." The College Mathematics Journal. June, 1985.

Schwartz, J., and M. Yerushalmy. "The Geometric Supposer: Using Microcomputers to Restore Invention to the Learning of Mathematics." In Contributors to Thinking. Eds. Perkins, Lochhead, Butler. Lawrence Ehrlbaum Press. Hillsdale, New Jersey (in press).

Yerushalmy, M. Induction and Generalization: An Experiment in Teaching and Learning High School Geometry. Unpublished Dissertation. Harvard Graduate School of Education (1986).

Yerushalmy, M., D. Chazan, M. Gordon, R. Houde. "Micro-Computer Assisted Instruction in Geometry: A Preliminary Report." In Proceedings of the 8th Annual Meeting PME-NA. Eds. Lappan, Even. East Lansing, MI. (1986): 181-186.

Yerushalmy, M., and D. Chazan. "Effective Problem Posing in an Inquiry Environment: A Case Study Using the Geometric Supposer." Proceedings of the Eleventh International Conference: Psychology of Mathematics Education. Eds. Bergeron, Herscovics, Kieran. Montreal, Canada. 2 (1987): 53-60.

Yerushalmy, M., and R. Houde. "The Geometric Supposer: Promoting Thinking and Learning." Mathematics Teacher, Sept. 1986: 418-428.

Yerushalmy, M., and R. Houde. "Teaching the Expected with the Geometric Supposer and Meeting the Unexpected." In Teaching Mathematics: Strategies that Work. Eds. Driscoll and Confrey. Heinemann Educational Books, Inc. Portsmouth, NH. 1985.

Yerushalmy, M., and R. Houde. Geometry Problems and Projects: Triangles. Sunburst Communications, Inc. Pleasantville, NY (1987).

Yerushalmy, M., and R. Houde. Geometry Problems and Projects: Quadrilaterals. Sunburst Communications, Inc. Pleasantville, NY (in preparation).

Yerushalmy, M., and R. Houde. Geometry Problems and Projects: Circles. Sunburst Communications, Inc. Pleasantville, NY (in preparation).