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ABSTRACT

This issue contains abstracts and critical comments for 11 published reports of research in mathematics education The reports are concerned with: (1) the development of counting strategies; (2) a structural model for mathematics achievement for men and women; (3) teaching children to subtract by counting up; (4) teaching children to add using one-handed finger patterns; (5) effects of lesson organization on achievement; (6) effects of test administration on test performance; (7) problem solving; (8) gender differences in mathematics; (9) teaching division; (10) personalized verbal problems; and (11) the development of children's number concepts. Research references from the Current Index to Journals in Education (CIJE) and Resources in Education (RIE) for October through December, 1986 are also listed. (RH)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Volume 20, Number 2 - Spring 1987

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Spring 1987

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Baroody, Arthur J. THE DEVELOPMENT OF COUNTING STRATEGIES FOR SINGLE-DIGIT ADDITION. Journal for Research in <u>Mathematics Education</u> 18: 141-157; March 1987. Abstracted by MARGARIETE MONTAGUE WHEELER 1
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Fuson, Karen C. and Secada, Walver G. TEACHING CHILDREN TO ADD BY COUNTING-ON WITH ONE-HANDED FINGER PATTERNS. <u>Cognition and Instruction</u> 3: 229-260; Fall 1986. Abstracted by PATRICIA S. WILSON
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Lemoyne, S. and Tremblay, C. ADDITION AND MULTIPLICATION: PROBLEM-SOLVING AND INTERPRETATION OF RELEVANT DATA. <u>Educational Studies in Mathematics</u> 17: 97-123; May 1986. Abstracted by W. GEORGE CATHCART

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Raymond, Cindy L. and Benbow, Camilla Pearson. GENDER DIFFERENCES IN MATHEMATICS: A FUNCTION OF PARENTAL SUPPORT AND STUDENT SEX TYPING? <u>Development Psychology</u> 22: 808-819; November 1986. Abstracted by TRUDY B. CUNNINGHAM
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Mathematics Education Research Studies Reported in <u>Resources in Education</u> October - December 1986



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Baroody, Arthur J. THE DEVELOPMENT OF COUNTING STRATEGIES FOR SINGLE-DIGIT ADDITION. Journal for Research in Mathematics Education 18: 141-157; March 1987.

Abstract and comments prepared for I.M.E. by MARGARNETE MONTAGUE WHEELER, Northern Illinois University, DeKalb.

1. Purpose

Six hypotheses were considered in this study of (a) the learning of a concrete counting strategy for addition, (b) the transition from concrete to mental counting strategies, and (c) the role of the commutativity principle in developing more efficient counting strategies.

- When entering kindergarter, the majority of students already know and use a concrete counting all strategy which involves separate processes for representing each of the addends and determining the sum (CCA).
- The remaining students should be highly ready to learn a CCA strategy and, with minimum adult intervention, should master the strategy.
- Except for children with a low readiness for learning a CCA strategy, children rather quickly abandon a concrete strategy in favor of a mental strategy.
- Strategies that employ objects or fingers are a common transitional step between CCA and mental strategies.
- Children mastering mental addition quickly discover that disregarding addend order saves effort.
- Children inventing strategies that disregard addend order do not know that commuted combinations have the same sum.

2. Rationale

Twenty-one concrete and mental counting strategies specific to young children using objects for counting as related to addend



representation and sum determination were identified through a careful synthesis of the counting and addition literature published since 1972 (only one of the 32 citations was published earlier). Since the development of a CCA strategy cannot be taken for granted among preschool children, the proportion of entering kindergarten children having mastered these strategies and the ease of learning and teaching these strategies become important. Whether a developmental order exists between concrete-counting-all shortcuts (both addends represented) and counting entities strategies (one addend represented) contributes to an understanding of the transition from concrete to mental addition as does an understanding of the relationship between addend order and use of efficient counting strategies.

3. Research Design and Procedures

<u>Student population</u>. From a pool of 27 students attending a half-day kindergarten in a suburban elementary school, 17 students (10 girls and 7 boys) participated in the study.

<u>Task population</u>. Three tasks were used: Initial Addition Task, Addition-Practice Task, and Commutativity Task. The initial addition task consisted of six single-digit addition problems, larger addend first, presented orally and horizontally on cards in the following, fixed order: 5 + 1, 3 + 1, 4 + 2, 3 + 2, 5 + 3, and 4 + 3. To solve the problems the children were first encouraged to use mental strategies (including guessing) or counting. If mental efforts were unsuccessful, children were encouraged to use blocks to solve the problem. Success was defined as at least five correct problems either with or without the use of objects.

The addition-practice task consisted of ten single-digit addition problems, smaller addend first, presented orally and in written horizontal form in a random order: 1 + 3, 1 + 5, 1 + 7, 2 + 3, 2 + 4, 2 + 6, 3 + 4, 3 + 7, 4 + 5, and 4 + 6. The strategy used to solve the last seven problems (those without one as an addend) was defined to be the dominant strategy if a child relied on it more than other strategies and used it on at least three of the seven trials.

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The commutativity task consisted of four problem pairs being randomly presented in oral and written horizontal form. Immediately after the solution to the stimulus problem, the second problem of the pair was written directly beneath the first problem. The child was asked whether the second problem would produce the same answer as the one just computed or sor thing different. The four pairs consisted of two commuted pairs (4 + 5 and 5 + 4; 6 + 4 and 4 + 6) and two noncommuted pairs (3 + 7 and 3 + 4; 5 + 3 acd 2 + 3). Because the use of single-digit addends maximized the chance of success, a second version of the task checked that the child was reasoning from a general principle and not a rule of limited scope. The latter included the original four pairs of problems and four pairs of large number problems: 4 + 16 and 16 + 4, 24 + 5 and $5 \div 24$; 5 + 23 and 2 + 23; 13 + 7 and 13 + 4.

<u>Procedure</u>. The 17 students were individually interviewed 14 times over nine months with the first interview restricted to screening for pre-arithmetic skills and establishing initial addition information. The period between interviews was usually two weeks, with one period of eight weeks. The focus of the 20-30 minute interviews was the initial addition task or the addition-practice task when the criterion for the former task was satisfied.

4. Findings

- Few children "immediately" used a CCA strategy to calculate the sums of symbolically presented problems (3 of 17 children).
- Learning the CCA strategy was difficult for a "sizeable minority" of the students with mumerous, repeated demonstrations required before mastery (5 students required 12 to 21 demonstrations to achieve criterion on the initial addition task).
- Most children relied on CCA strategies and concrete counting shortcuts for extended periods of time (for 6 students the CCA strategy was the dominant strategy at the end of the study; for 4 other students a shortcut to the CCA strategy was dominant).

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- Shortcut variations of the CCA strategy were used infrequently and do not appear to be significant in the development of more sophisticated counting strategies (11 of 17 children never adopted a CCA shortcut).
- Only a few children invented more advanced strategies including mental strategies (10 of 17 children never adopted a mental strategy).
- Mental count-all strategies starting with the larger addend are far more frequent than strategies beginning with the first addend (7 children adopted as a predominant strategy a procedure that disregarded addend order; 4 children exhibited competence on the combined version of the commutativity task).

5. Interpretations

Once again there is evidence that some young children invent advanced strategies to solve an arithmetic task. But the presence of these strategies must not be construed to indicate that the knowledge is systematic. There is also evidence that these inventions represent development, including increased efficiency of the physical count and advancing from count-all starting with the larger addend to count-on from the larger addend. But the resistance of some children to invent increasingly advanced strategies or to advance beyond the slowly learned CCA strategy highlights the diversity among young learners in a school setting.

The difficulty of learning the CCA strategy may be related to whether addition is modeled as a binary concept, the union of two sets, or as a unary concept, a change of state. A better understanding of a child's informal, preschool concept of addition with respect to these models is needed, as is the relationship of the model to directed teaching of the CCA strategy.

The key transitional steps between concrete and mental addition have yet to be identified. The strategies of the seven children



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exhibiting a mental counting strategy or an automated response as the predominant strategy after 13 sessions over an eight-month period is not totally consistent with sequences of strategies previously reported. The preference of some children to count on from the larger addend rather than to count on from the first addend needs to be understood. The importance of some children believing they will get a correct solution, though not necessarily the same solution, regardless of the starting addend, needs to be examined.

Abstractor's Comments

Studies that extend a rich Lody of literature are important and useful. The literature review found in this study is well done. Figure 1 (p. 142) which clarifies the use of objects in counting strategies for sums and distinguishes between concrete and mental counting strategies with respect to whether both addends or one are represented is particularly straightforward and useful. A longitudinal study to chart young children's transition to mental strategies is valuable.

Portions of the report could benefit from additional elaboration. This elaboration would be extremely useful when replication of this study is needed. In particular an onhanced description of the subject pool of students was excluded -- sometimes for specific reasons, such as "did not have parental permission"; sometimes for vague reasons, such as "weak prearithmetic skills". What was the content of the screening instrument and what were the pre-arithmetic skills possessed by the 17 children participating in the study? In a Gagne or an Ausubel sense, what prerequisite skills subordinate to the tasks researched were possessed by the students? Could the subjects orally count to 29 (a sum in the commutativity task)? Could the subjects tead the numbers when the problem was presented in "written horizontal form"? Could the subjects record single-digit sums (necessary in the commutativity task)? Could the subjects count-on from a single-digit number? Could the subjects subsitize sets equal to a single-digit



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addend? The four-line footnote (p. 146) defining "typical children" does not begin to answer these questions.

The interview sessions were "structured" but the dimensions of that structure are unclear. The sessions were "clinical interviews" and also teaching sessions "with at least a modest amount of addition experience". A table specifying the dates and outcome of each interview would have been no more complex than Table 2 (p. 152) and would have clarified the subject-history throughout the 13 sessions.

Concurrent instructional activities needed description. It is not surprising that kindergarten children did not receive formal instruction in admition during the year, but was addition readiness instruction also lacking? What counting activities were part of the kindergarten curriculum?

The narrow scope of the addition practice test was surprising. The potential item pool for the addition practice test contained 36 single-digit items: those without a zero addend and those with the first addend as the smaller addend. Five of the six items from the initial addition task also appear in the ten item addition-practice test. With the same ten problems randomly reordered 13 times, memory and practice are confounding factors and generalizability is restricted.

It is not surprising to this abstractor that so few kindergarten children invented shortcuts to the CCA strategy or developed mental addition strategies in a highly verbal and symbolic setting. Perhaps this work should be considered preliminary to teaching experiments where instruction too has a theoretical base.

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Ethington, Corinna P. and Wolfle, Lee M. A STRUCFURAL MODEL OF MATHEMATICS ACHIEVEMENT FOR MEN AND WOMEN. <u>American Educational</u> <u>Research Journal</u> 23: 65-75; Spring 1986.

Abstract and comments prepared for I.M.E. by JOE GAROFALO, University of Virginia.

1. Purpose

The authors stated that "the purpose of this paper is to address how differences between men and women in mathematics achievement develop" (p. 66).

2. Rationale

Various proposals and hypotheses have been offered to account for the difference between men and women in mathematics achievement. These include: (1) that the difference is due to differential socialization processes, (2) that it is due in part to the pattern of quantitative coursework, and (3) that it is the result of superior male ability. "Despite an extensive body of research designed to explain why this difference develops, no consistent conclusions have been forthcoming" (p. 65).

3. <u>Research Design</u> and Procedures

The authors addressed this issue "by estimating a latent-variable structural equation model of the process of mathematics achievement, including va lables identified by previous studies as being significantly related to mathematics achievement" (p. 66). To test for sex differences they estimated the model separately for men and women, and made comparisons between corresponding parameters.



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The Structural Model. The authors considered the mathematics achievement of high school seniors a function of mathematics and verbal abilities measured when the respondents were sophomores, attitudes toward mathematics, and exposure to mathematics. The mathema i nd verbal abilities were considered exogenous variables, while the attitude and exposure variables were considered endogenous. Positive effects on achievement were expected from all predetermined variables, with mathematics ability and exposure to have the strongest influence.

The Data. "Data for this study were drawn from the first follow-up of High School and Beyond, a nationwide, longitudinal study of high school sophomores and seniors...For this investigation, only the sophomore cohort was used..." (p. 67).

Mathematics ability was measured by a 28-item test taken by the respondents when they were sophomores. Verbal ability was indexed by tests for reading and verbal ability, also taken during the sophomore year. Exposure to mathematics was measured by four variables indicating enrollment in algebra 2, geometry, trigonometry, and calculus. Attitude towards mathematics was indexed by responses to four "statements concerning feelings towards mathematics classes and assignments" (p. 68). Finally, mathematics achievement was measured by a 10-item test taken when the respondents were seniors.

The analyses reported were based on 7,643 men and 8,912 women "who did not have self-reported learning disabilities and who had complete reports for all of the variables used in the analysis" (p. 68).

<u>Methodology</u>. "To determine whether the process of mathematics achievement was the same for men and women, the structural portion of the model was compared across groups" (p. 69). Since several of the variables were highly skewed, the usual approach to comparing such models, using maximum-likelihood methods, was inappropriate. The



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authors used tetrachoric, polyserial, and product-moment correlations to estimate the model separately for men and women using the unweighted least square (ULS) method in LISREL. "An examination of the goodness-of-fit index, root mean square residual and residual matrix for each group gave an indication of the apparent fit of the model for each group" (p. 69). Because methods are not available for testing the significance of changes in ULS measures of goodness-of-fit, the authors used an ad hoc procedure for comparing groups. This procedure, which used ULS estimates of the variances and covariances of the latent factors and maximum-likelihood estimates of the parameters, tested "not for equality between parameter estimates for the full model, but only for those estimates obtained by analyzing the matrices containing the correlations among the latent factors" '... 70).

4. Findings

"Goodness-of-fit indices of .993 and .990 together with root mean square residuals of .045 and .05 for men and women, respectively, indicated a fairly good fit for both men and women" (p. 70).

the structural parameter estimates are shown in the figure below.

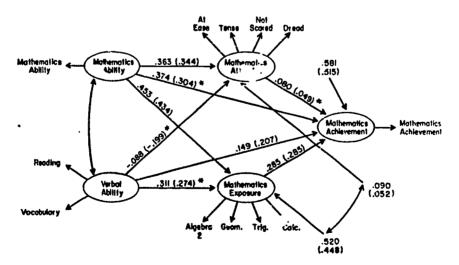


FIGURE 1. Structural equation and measurement models of mathematics achievement.

Note. In each pair of coefficients, values for men are given first and values for women are given second (in parentheses). Pairs of coefficients found to be significantly different are marked ". All coefficients are at least twice their standard errors. The numbers shown by residual error terms are coefficients of determination. (p. 67)



The authors stated:

- Mathematical ability and mathematical exposure were the most influential causes of mathematics achievement.
- (2) Verbal ability had a negative effect on attitude towards math.
- (3) Although all of the effects of predetermined variables on mathematics achievement were significant for both men and women, the parameter estimates were different for the two groups. "As a result of applying equality constraints across groups, four structural coefficients were found to be different. These were the effects of verbal ability on exposure to mathematics, of verbal ability on attitude towards mathematics, of attitudes toward mathematics on mathematics achievement, and of sophomore mathematics ability on senior mathematics achievement" (p. 72).

5. Interpretations

The authors stated they have found that the process of mathematics achievement differs for men and women. Specifically, they found:

- Attitudes towards mathematics are more negatively influenced by verbal abilities for women than for men.
- (2) Attitudes have a significant influence on achievement that is stronger for men than women.
- (3) Men appear to take advantage of prior mathematical abilities to a greater extent than women.

The authors concluded that "it appears that the factors in the model with positive effects of mathematics achievement are stronger for men than for women...In contrast, the factor that had a negative



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effect in the model had a stronger negative effect for women than for men. Differences in mathematical ability and attitudes toward mathematics were less flexible in the process of mathematics achievement among women than among men. On the other hand, among women, differences in verbal ability were more detrimental in their negative effects on mathematics attitudes. Thus, the process of mathematics achievement is more flexible overall for men than for women" (p. 73).

Abstractor's comments

The issue of sex differences in mathematics achievement is an important one which, although it has been the focus of much research and theorizing, is still without any generally agreed-upon resolution. A variety of sociological, psychological, biological, and educational explanations have been offered to account for these differences--each of which has its share of proponents and opponents. The issue is complicated by the large number of variables which might conceivably have some influence.

Ir this study the authors included some of the more "popular" variables---ability, attitude, and exposure---and they did a thoughtful job of analyzing the data. However, several aspects of the study itself and the written report left me disappointed.

First, the authors provided no theoretical justification or discussion of the variables used, or the causal directions specified. While I realize that the variables chosen were somewhat obvious, some discussion of them was warranted, especially in regard to the direction of influence. For example, it seems to me that the relationship between attitude and achievement is not unidirectional.

Second, I found the descriptions and/or the instruments themselves inadequate. How did the mathematics ability test differ from the achievament test? It is difficult to make meaningful



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interpretations without having descriptions of these tests. Also, I believe that both "attitude towards mathematics" and "exposure to mathematics" were measured poorly.

Third, I was unsatisfied with the interpretation and reporting of results. For example, it seems awkward to talk about significant differences in the effect of attitude on achievement with coefficients of .080 and .049. These might be statistically significant, but they are not very significant to me. Also, there was no real discussion of the findings and interpretations.

Fourth, I was very surprised that the authors gave no mention of whether or not they found any sex differences on the mathematics achievement test. After all, the article was about sex differences.

Fifth, I found some of the terminology troublesome. For example, the authors stated that the study addressed how differences "develop". A set of coefficients alone is by no means a description of "development", nor does it show a "process". Also, I did not understand precisely their use of "flexible" in describing the "process".

For all the reasons given above, I do not believe that this article adds much to our understanding of sex differences. It struck me as an article more about methodology than about the development of sex differences.



Fuson, Karen C. TEACHING CHILDREN TO SUBTRACT BY COUNTING UP. Journal for Research in Mathematics Education 17: 172-189; May 1986.

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Abstract and comments prepared for I.M.E. by MARTIN L. JOHNSON, The University of Maryland, College Park.

1. Purpose

The purpose of the study was to determine if first-grade children could learn to subtract by using a counting up strategy.

2. Rationale

Current research indicates that children solve addition and subtraction word problems by using solution strategies that model the semantic structure of the problem. The counting down solution strategy is often used for subtraction but research has shown that this strategy causes considerable difficulty for many children. Assuming that first graders had the operational capacity to use a counting-up strategy for addition word problems, the difficulty experienced with the counting down strategy can be avoided by presenting subtraction in a way that would lead naturally to the use of a simpler counting-up-to procedure.

3. Research Design and Procedures

The subjects were 103 students from five first-grade classes in two small city schools near Chicago. The sample was racially and economically heterogenous with two classes containing children identified as average and below average in first grade mathematics achievement, two classes containing above-average first graders, and one class containing second graders considerably below average in mathematics.

Two meetings were held with the five regular classroom teachers in which they were instructed in how to use the counting up procedure



with finger patterns for each of the subtraction story situations: compare, take-away, and equalize. Lesson plans for the instructional unit and student worksheets were prepared for the teachers. The class sets of student worksheets contained optional worksheets for review of addition as counting on, worksheets with subtraction problems in column form, worksheets with subtraction problems in row form, and one worksheet with both column and row form. The teaching required from 8 to 14 periods of 40 minutes each.

Before the unit began, the students were given a 2-minute subtraction pretest of 20 problems in column form and two 2-minute addition tests of 20 problems each. Two 2-minute subtraction immediate posttests were given at the completion of the unit. The posttests were repeated again one month later. Individual inter iews to determine how well children learned the counting-up procedure were held with 50 children within four school days following the immediate posttest. Two story problems for each of the three kinds of story situations used in the counting up procedure were given in the interview. Two measures of M-space were also given.

4. Findings

- "Overall, the children learned to count up to solve symbolic subtraction problems. They learned to subtract equally well for the column and the row forms of the symbolic subtraction problems" (p. 180).
- (b) "Of the 50 children interviewed, 44 used finger patterns to count up from the smaller to the larger number for the symbolic subtraction problems without having any help with counting up earlier in the interview. Forty-eight of the 50 children interviewed demonstrated the capability of learning to count up for subtraction" (p. 181). "Almost all of the children taught to count up with finger patterns learned to do so" (p. 182).



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(c) The mean percent of story problems of each type was as follows: compare (79%), take-away (76%), equalize (77%). "In each case, all but 4% to 10% of these problems were solved by the use of count-up finger patterns" (p. 183). Overall, the performance of the sample was as good for compare situations as for take-away situations.

5. Interpretations

The author concluded that the counting-up-to instruction using finger patterns was effective in teaching first- and second-grade children to solve both symbolic problems and story problems in subtraction. It is suggested that the procedures taught helped students represent the many different subtraction situations in ways that easily related to their counting up procedures.

Many additional questions were raised from this study. Exactly how the instruction helped the children to use counting up procedures in the solution of story problems, how the counting-up procedures for subtraction relates to or leads into a "thinking strategies" approach, and what conceptual structures are modified or developed as a result of thinking of subtraction as a counting forward procedure are among the many additional directions for future research.

Abstractor's Comments

The author has succeeded in showing that teachers can teach children how to be more effective subtraction problem solvers by using a counting up procedure. It has been shown that first-grade children are capable of learning this counting technique quickly and easily. The study was well designed and skillfully implemented.

As the author pointed out on page 174 of the report, whether first-grade children can learn to subtract by counting up and should they learn to subtract this way are two different questions.



Obviously, counting procedures can be taught and learned as rote procedures with questionable value in improving overall mathematics learning. The author is aware of this possibility and provides an excellent discussion to suggest that something more than rote learning was accomplished in this study. The answer to the second question must include a clear delineation of how knowledge of this specific technique facilitates or inhibits future mathematics learning. The questions raised by the author in the discussion are indeed the important ones if we are to understand the role of strategies such as counting up for subtraction in more advanced whole number computation and problem-solving situations and across other number domains on which subtraction is defined as an operation. Hopefully, future research will be focused in these areas.

Teachers who use the findings of this research should be aware that conceptualizing subtraction as an "adding on" procedure may require a modification of the current subtraction algorithms for students to fully appreciate how the new techniques are useful to them (one such example is the "low stress" procedure of Hutchings, 1976). If the procedures introduced here lead to an efficient, accurate processing of the basic subtraction facts, then this information is of great value to teachers.

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Fuson, Karen C. and Secada, Walter G. TEACHING CHILDREN TO ADD BY COUNTING-ON WITH ONE-HANDED FINGER PATTERNS. <u>Cognition and</u> Instruction 3: 229-260; Fall 1986.

Abstract and comments prepared for I.M.E. by PATRICIA S. WILSON, The University of Georgia, Athens.

1. Purpose

The purpose of this study was to investigate the effectiveness of whole-class instruction teaching addition using one-handed finger patterns. The instruction was conducted by classroom teachers in grades 1 and 2. In addition, the study hoped to identify a distinction between counting-on with objects and the more advanced, abstract task of counting-on without objects.

2. Rationale

Previous research has identified a developmental progression in the solution procedures American children use to solve single-digit addition problems. First, children are able to add two numbers when a set of objects is available for each addend. The two sets are combined and the child counts all of the objects. Students may use their fingers as objects. For example, 8 + 5 would elicit 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13. Next, students learn to count the objects by beginning with the number representing one of the addends and then counting-on using the objects representing the other addend. For example, 8 + 5 would elicit 8, pause, 9, 10, 11, 12, 13.

Researchers investigating the transition from counting-all procedures to counting-on procedures have identified competencies necessary for counting-on, but it was not clear if the competencies were sufficient for counting-on without objects and moving toward more abstract addition problems where objects are not available. When objects are not available, students use sequence counting-on, keeping track of how many objects are represented by the second addend.



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For example, 8 + 5 would elicit 8, pause 9 is one, 10 is two, 11 is three, 12 is four, 13 is five. There was a need to "ttablish instructional methods that could help students progress from counting-all to counting-on including sequence counting-on and abstract symbol addition. The instructional methods needed 1) to be useful for whole-class instruction by regular classroom teachers in grades 1 and 2 and 2) to facilitate meaningful (as contrasted with rote) learning of addition.

3. <u>Research Design</u> and Procedures

The study was based on three experiments involving first- and second-grade students with heterogeneous socioeconomic and cultural backgrounds. Each experiment was motivated and influenced by information from the previous experiment. The first experiment was designed to assess whether teaching finger-patterns could be used in a whole-class situation and whether the learning would transfer to symbolic addition problems where dots were not present. The second experiment was designed to test new instructional methods using one-handed finger-patterns to teach sequence counting-on. The new instruction was taught by regular classroom teachers. The third experiment examined the use of the finger-patterns and sequence counting-on to solve multi-digit addition problems. All experiments assessed transfer and retention.

<u>Experiment 1</u>. A control group (n = 30) received practice on various tests and worksheets. The treatment group (n = 77) received whole-class instruction teaching competencies needed for counting-on. After completing identical worksheets, both groups were given two timed tests involving addition of one-digit numbers. The first test showed addition problems with sets of dots so that students could use counting-all or counting-on with dot procedures to complete the problems. The second test consisted of symbolic addition problems without dots to count. One week later, 18 students from the



ins: uction group were tested in an individual interview for retention and to determine if they were actually counting-on.

<u>Experiment 2</u>. Since Experiment 1 indicated that the identified competencies were not sufficient to help students count-on when objects were not present, new instructional methods were devised.

Five first-grade mathematics classes (n = 106) were given two pretests, one with addition problems accompanied by dots and one with symbolic addition problems. Students were given worksheets and taught competencies for counting-on with dots (similar to instruction in Experiment 1). In addition, students were taught to use one-handed finger patterns to solve symbolic addition problems without dots where students needed to use sequence counting-on.

The finger patterns below are the patterns used in Chisanbop. We use the finger patterns differently from the way in which they are used in Chisanbop. We use then in the way that children spontaneously use fingers on both hands to keep track.

Counting-on begins with saying one addend while the finger pattern hand is up in the air ready to drop down and start finger counting. Then the next counting word is said while making the finger pattern for 1, and then more words are counted up while each new word is matched with the next finger pattern. Counting up stops when the finger pattern for the second addend has been produced.

Problem: 8+5=7



Stop when linger pattern for B is made.

land up in air ready to begin the finger pattern for 1

Children should do the finger patterns with their <u>non-writing</u> hand. To help children remember at which word they should stop, we had them place their pencils to the number word they were counting-on.



Students would lift one hand and say the first addend. Then they would use finger patterns to count and keep track of how many fingers they had counted. Students were also instructed on how to use finger patterns to help them solve subtraction problems by counting-up from the smaller number to the larger number. All instruction was done by classroom teachers. Posttests and interviews were given to assess students' understanding of 1) counting-on with dots, 2) sequence counting-on without dots, and 3) counting at a procedure to solve word problems involving addition and subtraction. Students were tested the day after each instructional segment was completed. They were also tested after 1 month and interviewed after 6 to 8 weeks.

Experiment 3. Since Experiment 3 indicated that the new instructional methods were successful, implications for multi-digit addition were investigated. Six classes of first graders and five classes of second graders participated in the study. Two of the first-grade classes consisted of children considerably below grade level in mathematics. Minor modifications were made in the lessons and worksheets used in Experiment 2. In addition, the second graders and the above-average first graders were taught a unit on place value and multi-digit addition. Posttests, similar to those in Experiment 2, were given after each instructional topic, 1 month later and 2 to 3 months later. Posttests included multi-digit addition problems.

Interviews were conducted with randomly chosen students to teach them and to assess their understanding of using counting procedures co solve subtraction and addition word problems.

4. Findings

Experiment 1. Experiment 1 indicated that it was possible to teach children in whole-class instruction to count-on with dots, but the instruction was not sufficient to teach counting-on without dots.



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Based on the 20-item posttest measuring children's ability to count-on with dots, an ANOVA showed there was significant interaction between the treatment (instruction/practice) and the competence (pretest/posttest). The children with instruction showed considerably more improvement (8.3 to 13.3) than children who only practiced problems (7.6 to 8.7). Based on the 20-item posttest measuring children's ability to count-on without dots (symbolic addition problems), an ANOVA did not show significant interaction between the treatment and the competence. There was a significant main effect for competence, but the improvement was small (6.9 to 7.9).

Interviews indicated that every child had learned to count-on using dots. Students attempted to count-on during the symbolic addition test indicating that they could transfer the procedure to a new task. Although students did count-on during the symbolic addition, their methods of using their fingers in place of the dots were faulty and elow, causing small improvement. Students put down their pencils while using their fingers and had difficulty keeping track.

Experiment 2. Experiment 2 indicated that classroom teachers were able to help children progress from counting-all to counting-on with dots to sequence counting-on with finger patterns.

A correlated <u>t</u> test, comparing the 20-item pretest and posttest scores for counting-on with dots, revealed a large significant difference. The mean test scores changed from 7.8 to 15.0. Based on the symbolic addition tests, one-way ANOVA showed significant differences between the immediate posttests and the pretests as well as the 1-month posttests and the pretests. There was no significant difference between the two posttests.

All of the children interviewed were able to sequence count-on using finger patterns. Even the students who were least successful on the posttest showed that they could count-on with finger patterns.



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Interviews also indicated that students were able to adapt the procedure of counting-on for addition to counting-up for subtraction. Eighty percent of the interviewed students (39 out of 49) were able to spontaneously use counting-on with finger patterns to solve addition ; word problems.

Experiment 3. F iment 3 showed that regardless of ability-level, students were able to improve their addition posttest sources. It also brought into question the idea that finger counting is detrimental to multi-digit addition.

For every class of first and second graders the mean scores for both the counting-on with dots posttest and the symbolic addition posttest were significantly higher after instruction. Four of the eleven classes showed drops on the 1-month posttests following instruction for a subtraction count-up procedure.

Interviews revealed that 89% of the first-grade students used sequence counting procedures to solve story problems. Almost half of the first graders used a counting-up procedure for solving a subtraction problem <u>before</u> instruction for counting-up. A substantial portion of the students understood counting-on well enough to extend the procedure to new problems. Some children clearly experienced interference between counting-on for addition and counting-up for subtraction.

After instruction, most children learned to add very large numbers. On a 10-digit problem, performance increased from an overall pretest mean of 2.4 correct digits to a posttest mean of about 9 correct digits out of 10. Teachers reported children were using counting-on with finger patterns to find the answer for each column.

5. Interpretations

Instruction in counting-on with dots and sequence counting-on with finger patterns can be successfully implemented in the classroom



by teachers. First- and second-grade students, including low-ability first graders, can learn and use both kinds of counting-on in a meaningful, as contrasted with rote, manner.

Experiment 1 demonstrated that teaching counting-on with dots does not necessarily lead to sequence counting-on. The two kinds of counting-on are different. Experiment 2 demonstrated that classroom teachers could effectively teach a one-handed finger pattern for counting-on without dots. Students were able to adapt a counting-on strategy with finger patterns for addition to a counting-up strategy for subtraction. Experiment 3 demonstrated that students could use their counting-on with finger patterns to solve milti-digit addition problems.

The speed with which children learned both counting-on procedures in a whole-class situation suggests that most children have the components necessary for both procedures. The instruction set the goal of not counting all and helped students focus on the counting-on procedures.

While counting-on with finger patterns may be less sophisticated than some methods, it may be more concrete and thus more easily reconstructed when needed. Counting-on with finger patterns is not efficient for all addition tasks, but it is helpful for subtraction and multi-digit addition.

Abstractor's Comments

This study gave a careful, detailed description of student behaviors as students learned to perform addition using dots to count-all and count-on, and finally using finger patterns to count-on without dots. The study did not investigate what children were thinking as they solved the problems or how the procedures related to their concepts of addition, counting, or numbers. A variety of statistical tests and analyses were used to interpret the test scores



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from different experiments. Each experiment contained smaller experiments that dealt with altered populations or different comparisons. Detailed descriptions of the instruments, the populations, data collection procedures, and the data were available in the original article and should be considered carefully before generalizing the results.

The research accomplished the stated goal of demonstrating that classroom teachers could teach finger patterns that would help students perform addition. Conducting a study of instruction by using classroom teachers in a whole-class situation is excellent. The research also established that there are differences between counting-on and sequence counting-on since some primary students who could count-on with dots could not count-on without dots. However, this research does not give us any indication of how students perceive the distinction or what kinds of pictures may exist in children's minds.

The researchers also stated that they hoped the instruction would facilitate meaningful, rather than rote, learning of addition. The study did not show that students had a meaningful concept of addition. There was evidence that the students understood the procedure and how to adapt and modify the procedure to solve addition and even subtraction problems, but it is not clear if they understood the meaning of addition. Students could not understand the meaning of addition if they did not have a functional understanding of numbers, order of numbers, and relationship of numbers. It is doubtful that all first and second graders would have this understanding. In fact, the students seem to have difficulty with the idea that the first group of dots could be represented by the first addend. It was taught in a rote fashion and the students were then able to count-on, but it may have lacked meaning. In experiment 3, it was reported that some of the students could not count out small sets of a specified number of objects, they could not write all the numerals below 10, and they could not begin counting from an arbitrary number in the number-word



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sequence. It is doubtful that these students had a concept of addition even if they could give the correct answer to symbolic addition problems.

The study tested for transfer and for retention. The students were able to transfer the procedure to word problems but this would be a real transfer only if the students perceived the word problems as a different situation. The actual word problems were not reported. The students were able to retain the procedure of counting-on using finger patterns when dots were not available. Students were able to work symbolic addition problems by using the finger pattern procedure. This is quite different from understanding addition. The interviews may have provided more insight into the issue of understanding addition, but the interview questions or responses were not reported. The interviews appeared to focus on whether the students understc...d the procedure and could apply it in different situations.

While the finger pattern procedure may not do any damage, it is expending a critical resource, time. If students do not have a concept of numbers and their relationship, they are not ready to deal with addition. At a higher grade level, or with more advanced students, the finger pattern procedure could be taught and might fit into the child's concepts of numbers, addition, and subtraction. The procedure is efficient and can help bridge the gap from counting concrete objects or dots to sequence counting, but the procedure should not be taught until students have developed firm concepts of numbers, order of numbers, and counting. First and second grade should be spent developing these concepts rather than solving meaningless symbolic addition problems.



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Kallison, James M., Jr. EFFECTS OF LESSON ORGANIZATION ON ACHIEVEMENT. American Educational Research Journal 23: 337-346; Summer 1986.

Abstract and comments prepared for I.M.E. by SAMUAL P. BUCHANAN, University of Central Arkansas.

1. Purpose

This study had two stated purposes. The first purpose was to "identify the low-inference teacher behaviors that comprise lesson organization" (p. 337) and to examine their effect on student achievement. The second purpose stated was to determine the "possible interaction effect of organization and student ability level" (p. 337).

2. Rationale

The two low-inference variables which were identified as constituting lesson organization were: "(a) delivering the learning material in the sequence that best reveals the relationships among the parts, and (b) using verbal statements to accentuate the relationships" (p. 337). These variables were identified in a well-documented review of literature. This review examined the effects of organization on recention of facts delivered in written form, in audiotaped form, or in a lecture format. Also reviewed were studies investigating the effects of statements accentuating the structure of the materials. Again, the methods, with one exception that failed to control teacher behavior variables, were learner-passive in nature. The results of the studies in the review clearly supported that the claim that a strong relationship exists between the organization of material presented to a learner and the level of retention. Also evident from these results was that student achievement can be affected by the use of statements about the structure of the presentation, e.g., transitional statements and preand post-organizers. However, only one of the results was obtained



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from a study using an <u>interactive</u> classroom setting and, as stated previously, it employed a flawed design. The present study was designed to investigate effects of two variables "constituting organization on student achievement in an interactive classroom setting. The first variable is presenting the material in the sequence that best illustrates the relationships among the parts of the lesson. The second is making the organization explicit..." (p. 339).

3. Research Design and Procedures

A group of 66 upper division and graduate university students served as subjects in this study. Each of the four classes containing these students was divided into halves to ensure equal treatment ability means, and the two halves receiving the same treatments were combined and comprised one treatment group.

The topic taught was numeration systems. Four treatments were used to teach this topic. Treatment 1 consisted of a discussion of topics in proper order and with explicit remarks about the organization. Characteristics of Treatment 2 consisted of a manipulated sequence and also explicit remarks about organization. The third treatment included the proper sequence of topics with no explicit remarks on organization. Finally, Treatment 4 consisted of manipulated sequence and no explicit organizational remarks.

The Necessary Arithmetic Operations (NAO) test was used to assess student ability level. An investigator-designed 21-item posttest was used to measure student achievement.

The eight subgroups were taught a twenty-five minute lesson and then tested. Subjects were not allowed to take notes or ask questions. Lessons were videotaped and reviewed to ascertain that the integrity of the treatments had been maintained during the lesson.



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4. Findings

After establishing tri-leveled ability grouping across treatment groups, a 2 (explicit organization) X 2 (sequence) X 3 (ability groups) analysis of variance was performed on the posttest. The comparison of main effect of posttest scores for explicit organization present versus explicit organization absent was significant at the .01 level. The main effect for sequence was not significant. No significant aptitude-treatment interaction was found for either of the two variables.

5. Interpretations

The interpretations of the results of this study indicate that the use of explicit organizational behavior has a positive effect on the achievement of students. The sequence in which the material was presented failed to show any causal effect on student achievement. This failure may be attributable to either the brevity of the treatments or to the sophistication of the subjects. The study failed to show any aptitude-treatment interaction.

From this single-lesson study, the results cannot be generalized beyond the conditions of this experiment. To verify the global effect of organization, additional studies that deal with other topics and subject areas, and that use a variety of teaching styles and subject population, would need to be conducted. (p. 345)

Abstractor's Comments

This study was well conceived and properly conducted. The theoretic foundation was adequate to warrant this investigation. The analysis of data and its interpretation were careful and complete.



The only point of concern is in the use of the phrase "interactive classroom." This definition, it would seem, is stretched to its limit. "To ensure consistency across treatments, subjects were told ... not to ask questions during the lesson. (Subjects did, however, respond to preplanned questions posed by the instructor.)" (p. 343). It appears the investigator has allowed the need to control variables prohibit the conducting of the treatments in a truly interactive setting. The results of such a study might be more easily extrapolated to the teaching population.



Lee, Jo Ann; Moreno, Kathleen E.; and Sympson J. B. THE EFFECTS OF MODE OF TEST ADMINISTRATION ON TEST PERFORMANCE. Educational and Psychological Measurement 46: 467-474; Summer 1986.

Abstract and comments prepared for I.M.E. by LEROY G. CLLLAHAN, State University of New York at Buffalo.

1. Purpose

The study compared performance of subjects on an arithmetic reasoning test when two different modes of administration were used, computerized and paper-and-pencil. Interaction between the modes of administration and ability of the subjects was also examined.

2. Rationale

The increased presence of computers and increase in test administrations in so many segments of society has inevitably resulted in widespread use of computers in test administration. There remains, however, relatively little useful research that sheds light on whether computer administration of tests affect performance. Existing research, though meager, suggests factors of time for testing, test difficulty, and cognitive processes required by the test may contribute to differential performance between computer and paper-and-pencil modes of administration. It was hypothesized that mode should not affect scores with a test requiring mental manipulation of abstract symbols if sufficient time is allowed to complete the test.

3. Research Design and Procedures

Study subjects were 585 military recruits age 18-25. They were randomly placed in two experimental treatments; 300 subjects using a paper-and-pencil mode, 285 a computer mode. Subjects were administered the Experimental Arithmetic Reasoning Test (EXP-AR),



a 30-item word problem test requiring basic algebra and geometry knowledge. Scores on the Arithmetic Reasoning subtest of the Armed Services Vocational Battery (ASVAB-AR) were available for each subject. During the experimental treatment subjects in the computer group each worked at a separate terminal, while those in the paper-and-pencil group were administered the test in subgroups of 4 to 10. Time limits were not imposed in either mode.

Regression analysis was used to perform an analysis of covariance, with EXP-AR the dependent variable, mode of administration the independent variable, and ASVAB-Ak the covariate.

4. Findings

A statistically significant difference in performance was observed between the two groups receiving different modes of administration of the test. The paper-and-pencil group mean on EXP-AR, 19.31 (SE = 5.62), was higher than the computer group mean, 18.27 (SD = 5.81). The test for interaction between mode of administration and ability was not significant. An item analysis was performed to determine if the effect of mode of administration was uniform over all EXP-AR test items. Twenty-one of the 30 items were more difficult in the computer mode of administration.

5. Interpretations

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In considering possible causes of the significant differences that occurred between modes, two were suggested. One attributed the difference to possibly a higher level of anxiety associated with the computer group. Alternatively, the ability to view multiple items in the paper-and-pencil group may have aided performance in that mode.

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Abstractor's Comments

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The "final note" included by the researchers should be underscored. They point out that although a statistically significant difference was found for mode of administration, the practical significance of the difference is probably nil. A difference of about one raw score point on a 30-item test, given the relatively large number of subjects, does not lend much predictive value to the study.

Perhaps the most interesting and potentially insightful aspect of the study was not followed up. That had to do with the inem analysis that was carried out on EXP-AR. Twenty-one of the 30 items were more difficult in the computer mode, tillee were more difficult in the paper-and-pencil mode, and the other six were of approximately equivalent difficulty. It would have been interesting to analyze qualitatively these three subsets of items to see if there might not be structural, contextual, mathematical, or other characteristics that could have been affected by either of the two modes of administration.



Lemoyne, G. and Tremblay, C. ADDITION AND MULTIPLICATION: PROBLEM-SOLVING AND INTERPRETATION OF RELEVANT DATA. <u>Educational</u> Studies in Mathematics 17: 97-123; May 1986.

Abstract and comments prepared for I.M.E. by W. GEORGE CATHCART, University of Alberta.

1. Purpose

The study was designed to develop a "method for intervening on the processing of certain forms of relational expressions often included in addition and multiplication problems" commonly given to upper elementary school grudents.

2. Rationale

While there has been some research on the influence of context and form of a relational expression on problem solving, very little research has suggested any precise methods which would enable context and form of expression to be modified in a rational or systematic way. The ability to place a problem into a category and the learning of specific strategies for that classification probably plays a significant role in problem-solving performance. In developing an interventionist approach, then, the association between previously learned problem-solving strategies and contextual, relational, or lexical data in the problem (a basis for classification) must be emphasized.

3. Research Design and Procedures

A sample of 48 grade 5/6 students was chosen from four classes (N = 115) on the basis of poor problem-solving performance in class. A pretest of 14 problems was administered. These problems were designed using three criteria: type of mathematical situation (additive or multiplicative), field of numerical data, and adherence to various contexts. The sample was then divided into control (N = 19) and experimental (N = 29) groups on the basis of academic level (grade), class group, age, sex, and pretest performance.



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A set of learning exercises was developed and given to the experimental group over a period of three weeks. "The aim of the learning exercises was basically to upgrade skills in the analysis and processing of certain types of expressions very frequently included in the statements of addition and multiplication problems." The exercises were divided into three types and administered to the experimental subjects in small groups of about five students each. "In the first two types of exercise, subjects were asked to process certain expressions in different ways, analyze the results of this operation, identify the processes leading to these results and group together the results that seemed to be generated by similar types of processing." The third type of exercise allowed students to apply, in a familiar context, the procedures developed during the first two sets of exercises.

The 14-item problem solving test used as a pretest was then readministered to both the control and experimental groups immediately after the learning exercises had been completed. The Wilcoxon matched-pairs signed-ranks test was used to compare pre- and posttest scores.

4. Findings

The experimental group obtained significantly better scores on the posttest than on the pretest. This was not true of the control group. About 70% of the experimental subjects scored higher on the posttest than on the pretest. The corresponding figure for the control group was 47%. The progress was most marked on the multiplication component and by the grade 6 students.

On 7 of the 14 problems students were asked to tell what operation might be performed. On the posttest both groups made a more limited choice of operation than they did on the pretest. The numbers of inadequate choices also declined, especially in the experimental group.

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5. Interpretations

Information-processing exercises based on a variety of mathematical situations seem to contribute to more effective problem-solving strategies. This confirms the importance of the interpretation stage in problem solving. Intervention in the interpretation process facilitates a better choice of operation and more attention to all the information contained in the r oblem.

Since grade 6 students derived a greater benefit from the learning exercises than grade 5 students, the effectiveness of the learning exercises may depend on the linguistic and heuristic problem-solving skills of the students. Subjects whose linguistic and heuristic skills are move developed may be better able to recognize and interpret expressions referring to mathematical situations.

Abstractor's Comments

The study reported here has both theoretical significance and practical implications. Mathematics educators, researchers, and classroom teachers are concerned with methods of improving children's problem-solving performance. This study was carefully designed and carried out. The results strengthen our concern about the importance of form of relational expressions in interpreting a problem.

The report does leave the reader with a number of questions and concerns.

 There is not sufficient information about both the design of the learning exercises and the use of the exercises with students to replicate the experiment. The excerpts in the Appendix do not help much.



- 2. The intervention process took place in small groups of about five students. Most teachers still teach mathematics to the whole class. Can the process used here generalize to a whole-class setting?
- 3. In this study a particular intervention process was compared to no intervention (control group). Unless badly designed, any intervention is likely to be better than no intervention. Are there other processes for focusing on relational expressions that would produce better results than the process used here? There is a need to examine alternatives.

Finally, parts of this report were difficult to follow and interpret. This highlights the need for research to be reported in a clear, simple, and concise form.



Raymond, Cindy L. and Benbow, Camilla Pearson. GENDER DIFFERENCES IN MATHEMATICS: A FUNCTION OF PARENTAL SUPPORT AND STUDENT SEX TYPING? Development Psychology 22: 808-819; November 1986.

Abstract and comments prepared for I.M.E. by TRUDY B. CUNNINGHAM, Bucknell University.

1. Purpose

This study attempted to determine whether or not highly gifted 13 year-old students consider the "generally predominant behaviors of the people who major and work" in quantitative and verbal fields to be masculine or feminine and/or perceive that their respective parents or guardians give them differential encouragement to study, enjoy, and accelerate in quantitative and verbal subjects.

2. Rationale

The authors suggested two environmental explanations for the consistent findings that favor males in studies of mathematical ability and achievement. Then they proposed six hypotheses which they investigated for young students who are judged to be exceptionally talented either mathematically or verbally, but not both. The hypotheses included: (1) all mathematically talented (MT) students would report greater encouragement in mathematics than the verbally talented (VT); (2) both male MT and VT groups would report greater encouragement in quantitative subjects than the female groups; (3) all students would report greater paternal involvement in quantitative studies and greater maternal involvement in their verbal studies; (4) all MT students would report more encouragement from fathers and all VT students would report more encouragement from mothers; (b) female MT students ould report greater maternal involvement in quantitative areas than the fem. .e VT, male VT, or male MT students; and (6) all students would characterize quantitative areas as masculine and verbal areas as feminine.

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3. Research Design and Procedures

All 411 students in this study qualified for the Johns Hopkins Talent Search by scoring at or above the 97th percentile on an in-grade achievement test and took the College Board's Scholastic Aptitude Test (SAT) before their thirteenth birthday. The control group consisted of 50 boys and 61 girls who earned a combined score of 540 or less and participated in this study 2.5 years after taking the SAT. The original experimental group included 173 boys and 23 girls who scored above 700M or higher and 44 boys and 48 girls who scored 630V or higher between November 1980 and October 1983. In order to increase the size of the female MT group, 12 girls were added as they were identified. It is known that the average age of the experimental group was 13.7 and assumed by the authors that the students in the control group were either 1' or 15 years old.

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Parent and student questionnaires were sent to 645 students and almost all of the students who responded were included. Only parts of four of the more than 50 questions on the student version of the questionnaire were used in this study. The authors indicate that response rates varied from 57% for the control group, to 78% for the experimental and 92% for the females in the experimental group, primarily because of motivation and followup. Students in the control group were offered five dollars to complete the questionnaires 2.5 years after they took the SAT and were reminded once. The experimental students, all of whom had been followed carefully by the Center for the Advancement of Academically Talented Youth (CTY), were not offered money but were reminded several times with even greater efforts being made to recover questionnaires from every girl in the MT group.

Scales were constructed to make comparisons between students responses to quantitative and vorbal areas and analysis of variance (ANOVA) was the major mode of analysis, although chi-square contingency-cable tests, t tests, Pearson product-moment correlations and effect sizes, internal consistency reliabilities and linear



discriminant function analysis were used freely. The authors: classified mathematics, science, and computer science as quantitative and literature, writing, foreign language, history, and social science as verbal; decided that the social science variable should not be included "due to the fact that the field was not perceived to have as strong a societal sex type"; and omitted the acceleration questions from the calculations because "acceleration would not be as appropriate" for students in the control group as it would be for those in the experimental group.

4. Findings

Many results were listed. Those that relate to the six hyothesis seemed to include:

- (1) Of the nine factors involving parental encouragement only two were significant: paternal support of quantitative (p < .01) and total parental support (p < .05).
- (2) The hypothesis that males, regardless of area or level of act it. would report greater encouragement than fet as not supported by significant findings.
- (3) Likewise, there was no significant support of the hypothesis of greater paternal involvement in quantitative areas and greater maternal support in verbal areas.
- (4) The results in the indestigation of paternal support of mathematically talented and maternal support of verbally talented students were not significant.
- (5) Female MT students did not report significantly higher maternal involvement in quantitative areas.
- (6) Most of the students (59% of the experimental and 72% of the control) did not sex-type subject areas.



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5. Interpretations

The authors conclude that the socialization factors of perceived parental encouragement and sex typing had little influence on the sex difference in mathematical reasoning ability among intellectually gifted students. During the early teens these children did not perceive their parents as providing differential encouragement to males and females. Students were not sex-typing subject areas, but the measure of this variable was inadequate.

There seemed to be no apparent relationship between the student's perception of parental encouragement patterns and his or her sex typing.

Abstractor's Comments

This study is noteworthy because it addresses the question of gender differences in mathematical ability and achievement for the most academically talented youngsters. The article was well-written and to the authors' credit cites several instances where the study could have been better executed. The design suffered from the use of a minute part of a long questionnaire that may not have been well-suited to even verbally talented youngsters and the amount of statistical analysis seemed to overpower the scant data. Long reports of what seemed to be extraneous statistical analysis made it difficult to follow the analysis of the six clearly stated if somewhat complicated hypotheses. Fortunately a well-written discussion helped the reader to find the way back. Several questions come to mind:

(1) Given that both parental and student questionnaires sere used, why not study parental involvement as perceived by the parent or compare the perceptions of students and their parents?



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(2) What was the male-female distribution of the 48 students who qualified as both MT and VT, but were eliminated from the study?

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- (3) Given the magnitude of changes that occur in perceptions of self and society between the ages of 12 and 15, is a control group where the mean age is at least a year higher than the experimental group a productive choice?
- (4) Do errors in computing rate of return suggest that more complicated statistical calculations may be flawed?
- (5) To what extent might the self and parental confidence that is a product of high SAT scores change the perceptions of these junior high school students?
- (6) Is it not likely that the extreme effort to retrieve all questionnaires from the girls in the MT group affected both their responses and the comparability of groups?

This is not a study that could easily be replicated because the sample is so restrictive by definition. Assuming the questionnaires used are still available, it might be of interest to use small sample statistics to test some of the hypotheses raised by this study. Of more value might be a study in which small samples of these students interviewed five or seven years after they first took the SAT to determine how questions of gender affected their mathematical development.



Vest, Floyd. A STUDY OF TEACHING THE MEASUREMENT AND PARTITION CONCEPTS OF DIVISION. Focus on Learning Problems in Mathematics 8: 61-68; Spring 1986.

Abstract and comments prepared for I.M.E. by DONALD J. DESSART, The University of Tennessee, Knoxville.

1. Purpose

The purposes of this study were to determine the extent to which third- and fourth-grade students could distinguish between measurement and partition division word problems on two testings with intervening instruction and to determine their preferences for chese two types of division problems at the times of the two testings.

2. Rationale

Measurement type division word problems pose the question: "How many sets?". For example, determine the number of groups of children, if 12 children are placed into groups of six children each. On the other hand, partition type division word problems pose the question: "How many in each set?". For example, if 12 children are placed into two groups, determine the number of children in each group.

Previous research has studied such questions as: (1) Should children be taught to distinguish the two types of division word problems by identifying the two types of problems?; (2) Which types of division word problems do children prefer?; and (3) Which type of division word problems should be introduced first in a textbook or instructional sequence?. Although the research findings have not been conclusive, it appears that measurement division problems may be easier for children than partition problems and also preferred by them.

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3. Research Design and Procedures

The subjects of the study were two intact classes of third-grade children and two intact classes of fourth-grade children from a metropolitan public school system in the Southwest. The mean IQ's of the third- and fourth-grade children were 101.3 and 105.7, respectively, as measured by The California Mental Measurement Battery. During September and October the children were pretested for their abilities to distinguish measurement and partition problems and their preferences for the two types of problems. The pretesting was followed by three hours of instruction, one hour each day for three days, in distinguishing measurement from partition problems. The instruction was followed by posttesting of their abilities to distinguish these kinds of division problems and their preferences for either of them. The third-grade children had had little or no previous instruction in division, whereas the fourth graders had studied division from a widely used textbook.

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The tiree hours of instructional materials were similar to those used in earlier research by Zweng (1963). The children were taught primarily through verbal instruction that measurement problems ask the question, "How many sets?" and partition problems ask, "How many in each set?"

Three tests were used in the pre- and posttesting. Test 1, which was administered only to the fourth graders, consisted of eight items. Each item was two problems, one measurement and one partition, describing a similar situation. The children were instructed to read the item (both problems) completely and then solve, according to their own preferences, one of the two problems. The items were arranged so that four of the measurement pairs were on the left hand side of the page and four of the partition pairs were also on the left.

Test 2, which was administered to all the children, consisted of eight items. The children were instructed to mark the problems with



"M" if it asked the question, "How many sets?", and mark it "P" if it asked the question, "How many in each set?".

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In test 3, the children were asked their preferences for a problem by checking one of three blanks labeled, "P problem", "M problem", or "No preference".

Three statistical hypotheses were studied:

"Hypothesis 1: At both the pre- and post-administrations of Test 2, the subjects will demonstrate no majority preference for measurement or partition problems.

Hypothesis 2: At both the pre- and post-administrations of Test 2, the subjects will not be able to identify measurement and partition problems more often than by chance.

Hypothesis 3: At the post-administration of Test 3, the subjects will demonstrate equal distributions between checking no preference and checking a preference for measurement or partition problems."

4. Findings

In Test 1, which was administered only to fourth graders, the mean proportion of partition problems attempted on the pretest was .489 and on the posttest was .538. Both of these mean proportions were not significantly different from the hypothesized chance proportion of .5 (p > .05). Consequently, it was concluded that there was no clear preference for either partition or measurement problems by fourth graders on either test.

On Test 2, in which students were to classify eight word problems as measurement or partition, the number of correct classifications could range from zero through eight. For grade three, the mean pretest score was 4.569 and the mean posttest score was 7.431. For



grade four, the mean pretest score was 5.700 and the mean posttest score was 7.440. All four of these means differed significantly from the hypothesized chance mean of four (p < .05). The differences in the pre- and posttest mean scores were also significant for each grade level (p < .05). It was concluded that both third- and fourth-grade students could learn to distinguish satisfactorily measurement from partition problems.

On Test 3, administered to both third and fourth graders, in which children indicated their preferences for one of two problem types or preference for either, 60.6 percent of the third graders, 70 percent of the fourth graders, and a combined percentage of 66.35 percent of the third and fourth graders indicated no preference for either type problem. These results were significant (p < .01), and it was concluded that most children would not prefer either type of problem.

5. Interpretations

The study seemed to indicate that children can successfully distinguish between measurement and partition problems when the only required action was to label a problem, "M" or "2". Furthermore, the childrer did not prefer either type of problem. These findings could lead one to conclude that children should experience both problems at about the same time and that either could be presented first. However, the author cautioned that this may not be true, if more complex actions (such as writing different type division equations) were required.

Since problem colving is a goal of most modern-day mathematical instruction, one might argue that successfully distinguishing between the two types of problems early in children's experiences would add to their storehouse of problem-solving heuristics. Presenting both types early may also discourage a mind set for either problem as the only possible division situation.



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Abstractor's Comments

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This is a most interesting study with several laudable features that I admired very much. The author cogently built his study upon previous investigations. Research in mathematics education cannot advance unless this is done by more researchers. The tests were clean and well constructed; the teaching treatment was clear and well defined.

The author reported that "intact classes" were used in the study. It seems very unlikely that these classes could have been regarded as <u>random</u> samples of any population! "Random sampling" is the very heart of statistical inference; consequently, the use of statistical inference in this study was unwarranted. I sincerely believe that the intact classes could have been treated as <u>populations</u> and that very meaningful descriptive measures could have been reported. In fact, the findings would have changed little except that one could not have generalized the conclusions to a larger population. One can only speculate as to why the author chose the inferential route. One would hope that this "statistical sanctification" wasn't necessary for publication!

The study provided some tasty food for thought and speculation. Some of these are given below.

- The use of a pure control group (no teaching treatment) would have been nice. It could have revealed whether or not students could have learned as much from their own resources between the pre- and posttestings.
- A Solomon Four-Group Design (Campbell and Stanley, . 24) would have been even nicer. It would have evaluated the effects of the pre- and posttestings as learning experiences in themselves.



3. I liked Test 1 as a measure of preference. It conforms nicely to the adage, "Watch what someone does rather than what he or she may say!".

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- 4. The author noted that Test 2 which required students to merely identify problems as "measurement" or "partition" is quite different from the complex problem of writing a division equation. Perhaps, future investigators could combine these features in one test.
- 5. There is little question that students should have an unverbalized awareness that division is the proper operation in both measurement and partition cases. It is not clear that they must intellectualize that difference by labeling problems as "measurement" or "partition". I suspect many of us have operated quite efficiently without knowing this intellectual distinction.

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Wright, Jone P. and Wright, C. Dan. PERSONALIZED VERBAL PROBLEMS: AN APPLICATION OF THE LANGUAGE EXPERIENCE APPROACH. Journal of Educational Research 79: 358-362; July/August 1986.

Abstract and comments prepared for I.M.E. by MICHAEL T. BATTISTA, Kent State University.

1. Purpose

The goal of the study was to determine if "personalizing" word problems affects fourth-graders' performance on the problems.

2. Rationale

The authors cite NAEP results from 1973 and 1978 as indicating that a major weakness of students is "solving everyday type word problems." They assumed that one reasoon for this poor performance is that most of the problems presented to students in the classroom are of a textbook variety that students are not really interested in solving. Research was cited indicating that problem-solving performance is positively correlated with students' familiarity with the problem context. It was hypothesized that the "language experience approach is helpful in mathematical problem solving because children are interested in the problem situation and can relate to the information in the problem." From this somewhat diverse set of ideas, the authors proposed the following research question: "Does the interest or familiarity of the problem setting matter?"

3. Research Design and Procedures

An interest inventory was administered to each of 99 fourth-graders enrolled in rural public schools in Alabama. Each student took two 16-item "personalized" tests that were administered at least six weeks apart. Each test consisted of standard textbook verbal problems and rewritten versions of these standard problems that



took into account the student's responses on the interest inventory. An item and its personalized counterpart always appeared on different Lests. An example of how the authors changed a problem to fit different students' interests follows.

Version 1: Debby's rabbit got into the garden and ate 1/3 of the 18 carrots. How many carrots did the rabbit eat?

Version 2: John's dog got into the chicken and ate 1/3 of the 18 chickens. How many chickens did the dog eat? (This item is reprinted here exactly as it was worded in the original article.)

Four scores were obtained for each student: nonpersonalized answer and rersonalized answer (number of standard and number of personalized word problems answered correctly, respectively); nonpersonalized process and personalized process (number of standard and number of personalized word problems for which the correct arithmetic process was chosen, respectively).

4. Findings

Students' process scores were significantly higher than their answer scores. There was a significant interaction between the answer/process variable and the personalized/nonpersonalized variable. Although no <u>post hoc</u> analyses were performed, the authors reported no differences on personalized and nonpersonalized answers, but slight differences on personalized and nonpersonalized process scores, with students scoring higher on the personalized items. Both answer scores and process scores were positively related to reading and mathematics achievement (as evidenced by an ANOVA).

5. Interpretations

The authors conclude that it is easier for students to decide on the correct arithmetic process, but not to compute the correct answer,



on personalized rather than standard textbook word problems. They conjectured that on personalized problems, "students tended to focus their attention on the problem in ways that were not in effect in the standard textbook problems."

Abstractor's Comments

As can be gleaned from the rationale section of this abstract, the theoretical underpinnings for this study are not clearly delineated. What do the authors mean by the "language experience a proach?" Is it an approach to teaching problem solving or does it mean simply to personalize problem statements? More <u>importantly</u>, why should familiarity with the context of a problem affect students' performance? Although the hypothesis is intuitively appealing, the authors do not provide us with a satisfactory theoretical explanation for why it should be true.

The personalized versions of the word problems were generated by computer. One certainly hopes that the items so generated were more grammatical (and less morbid) than the example given in Version 2 above. Of course, this mistake in grammar was probably a typographical error that appeared only in the description of the research. But because the personalized versions of the problems were generated by computer and this type of mistake is very common in computer-generated materials, the error is unsettling. In any event, more examples of how problems were personalized would have been helpful in evaluating the quality of the computer-generated versions of the problems.

One interesting trend in the data that is not mentioned by the authors is found by subtracting answer scores from process scores. Doing so ind the test that although students chose the correct arithmetic process more successfully on personalized problems, after deciding on the correct process, students made about 64% more computational errors on the personalized problems. This does not seem consistent with the



authors' "attentional" explanation for the differential performance on process scores for personalized and nonpersonalized problems. Indeed, why would students be more focused on personalized problems when they are determining the process but <u>less</u> when they are computing the answer?

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Could it be that the processes used in solving the two types of problems were different? Because the personalized problems were more familiar to the students, these problems may have been easier for students to represent. Thus, on the personalized problems, the students may have been more likely to construct a representation of the problem, then to try to match this representation with previously stored representations for the arithmetic operations. Because the representation and matching operations were explicit, higher process scores resulted. Standard text problems, on the other hand, may have been seen merely as computational exercises "with words." Students may not have attempted to represent such problems explicitly but may have done them more or less algorithmically. Thus, because students' attention on such problems was more focused on calculation, their computations were more accurate.

In conclusion, this study presented some interesting results. But the fact that the difference between the performance on personalized and nonpersonalized problems was only about 22% of a standard deviation and the lack of a theoretical base makes one wonder how the results should be interpreted. The study does, however, raise some interesting questions that future research might attempt to answer.



Yoshida, Hajime and Kuriyama, Kazuhiro. THE NUMBERS 1 TO 5 IN THE DEVELOPMENT OF CHILDREN'S NUMBER CONCEPTS. Journal of Experimental Child Psychology 41: 251-266; 1986.

Abstract and comments prepared for I.M.E. by GRACE M. BURTON, University of North Carolina at Wilmington.

1. Purpose

The study, composed of three related experiments, was designed to answer the question, "What kind of structure of number concept can we assume is present in kindergarten children?" Especially of interest wa the role that the number 5 plays in children's ability to add one- c_{rel} numbers one of which is greater than 5.

2. Rationale

Over the last few years, many investigators have theorized how "udents represent numbers to themselves. The work of Resnick and "burg suggests that young children have not developed a structure of mbers. That of Gelman and Gallistel, however, suggests otherwise. The authors wished to determine what structure, 12 any, exists in young children.

3. Research Design and Procedures

• A series of three experiments was undertaken, each with a different focus. All used Japanese kindergarten children from middle class homes as subjects. The children were individually asked to add (or subtract) one-digit numbers. The authors state that in Japan, children study "the numbers 11 to 20 as composed of 10 plus a single digit" (p. 252) at the beginning of first grade. By the end of first grade children "learn addition and subtraction of numbers below 100 (without carrying or borrowing" (p. 252). They also state that study of the Japanese abacus, in which 5 is introduced as an intermediate grouping unit, does not begin until grade 3.



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Experiment 1.

Subjects were 53 children attending the experimental kindergarten of Miyazaki University. Each of two intervention groups had 19 children, while the control group had 15 children. Each child participated in a warm-up phase in which an oral problem was presented and the child was shown how to solve the problem using his or her fingers. (The sums of 2+1 and 2+2 were to be computed.) For the experiment, 17 addition problems in which at least one term was greater than 5 were presented both orally and in writing.

The first intervention group, called the 5-group, was requested to "resolve" the number 6 into 5 + x, showing 5 fingers on one hand and 1 on the other. An incorrect response resulted in instructor feedback. The subject was then directed a resolve 7 and 8 and then was taught to find the answer to the pr. . .em (sic) 8+9 by the experimenter. She "explained that 8 cou" be resolved into 5 and 3 and 9 into 5 and 4. She added the 5 of the 8 and the 5 of the 9 and let the child cor irm 10 as a result. Next she explained the addition of the remaining numbers, the 3 of the 8 and the 4 of the 9, and asked the child the answer to this addition. Finally, she taught the addition of the 10 and the 7 which had previously been computed" (p. 255).

Two other problems were then demonstrated. rollowing instruction, the student was asked to find the 17 sums without any further intervention or feedback. Responses were timed.

In similar training for the "10-group", children were taught to supplement the first addend to 10, subtract the amount needer for this supplement from the second addend, and then add the remainder of the second addend to 10. The control group received no intervention.



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Experiment 2.

The principle of add tive composition of numbers and the principle of compensation were investigated using 26 children from a private kindergarten as subjects. No materials were used. Each child was asked to resolve 6 into 5 and 1, showing their findings on their fingers. The authors reported that all children understood these directions. For the resolve task, the child was then asked to decompose 7, 8, and 9 in a similar fashion. Following this, he or she was asked to find the supplement (to 10) of 3. Fredback and an additional example were presented if the child failed this practice task. The child was then timed as supplements to 1, 2, 4, 5, 6, 7, and 9 were found.

Experiment 3.

To test whether or not a privileged anchor of 5 is reflected in counting strategies used to solve addition and subtraction problems, 22 children from a private kindergarten were videotaped solving single-digit addition and subtraction examples. (None with differences of 1 were included.) Following a practice session using fingers to find 1+1 and 2-1, the child was presented with 24 addition problems and 35 subtraction exercises, one at a time.

4. Findings

Experiment 1.

All incorrect responses having been eliminated from the study, mean response times were calculated. Four children from the control group were eliminated completely as they could solve none of the problems. The 5-group both answered significantly faster and generated more correct answers than the other two groups. The performance of the 10-group was better than that of the control group, but much weaker than that of the 5-group.

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Experiment 2.

Performance on the resolve task was clearly superior to that on the supplement task. T-tests revealed that children performed both more rapidly and more accurately on the resolve than on the supplement task.

Experiment 3.

Children took between 45 and 90 minutes to complete the 59 examples in sessions limited to 30 minutes per day. No difference was found in mean ratio of correct to incorrect responses between addition and subtraction tasks.

Analysis of the videotapes led to the conclusion that children used four distinct strategies: (1) counting by opening their fingers one by one, (2) directly showing each add nd on their fingers, (3) counting covertly by moving their eyes or head, or (4) arriving at the answer without any external signs of counting. Some children employed more than one strategy. Most used more strategies for subtraction than for addition.

The 218 errors from the 1298 responses were divided into two groups: those relating to 5 and those not relating to 5. Each group was further divided into subtypes.

5. Interpretations

Experiment 1.

The authors concluded that instruction of the formal procedure for addition can be effective and that instruction using 5 as an anchor and relying on addition only is more successful than the , supplemental method.



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Experiment 2.

Children employ the number 5 as a privileged anchor, first learning how to find the supplements for numbers greater than 5 and then learning how to find the supplements of numbers less than 5. Reliance on 5 may be due to the fact that human beings have 5 lingers.

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Experiment 3.

Lack of difference in mean error ratios shows that the differences discovered in experiment 1 were not due to type of operation used but to children's internal representation of number.

Overall Conclusion

Even when children use an external method to keep track of adjends, an internal structure of number exists. In this internal representation 5 plays a major organizing role.

Abstractor's Comments

These researchers have identified an important area of study with respect to children's learning of addition. It is unfortunate that their investigation is less helpful than it might be to contemporary mathematics educators.

I first felt uneasy when I encountered the word "problem" consistently used for examples of the type "2+1 = _____". My concern deepened when I realized their conclusions about how a child conceptualized number were to be based on how fast and accurately the child could finger-count. Indeed, a more appropriate title of the article would be "Differences in Children's Ability to Count on Their Fingers."



Data were examined only from children who already knew how to do the task; children who could not perform on the basis of one or two brief examples (which were on a lower level than the tasks the child would be given) were unfortunately dropped from the study. It might have been instructive to compare the understandings of these children with those of students more successful at finger counting. Where this might 'ave been done, in the examination of errors from experiment 3, the details are too few to be of any help to teachers or researchers.

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Finally, this wordy and rambling article could have profited from careful editing. Those who seek an important study on young children's mental structures or who seek practical advice for the classroom would be well advised to look elsewhere.

Sec. Sec.

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