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ABSTRACT

This project was based on recent interview research which suggested that in choosing operations for story problems in mathematics many students are guided by computational considerations rather than meanings for the operations. This project aimed to refine the catalogue of strategies used by students and to design and test instructional materials intended to foster a concept-based approach to story problems among sixth graders. In the first year of the project (1985-86) students were interviewed, textbooks analyzed, a teaching experiment was conducted, and materials were developed and refined. During the second year materials were tried out, further textbook analysis was conducted and additional student interviews were conducted. The major sections included in this report are: (1) the catalogue of strategies used by students in solving story problems; (2) what textbooks offer; (3) the teaching experiment with sixth graders; (4) the development of the supplementary materials; (5) the try-out of the materials; and (6) the spin-off studies. Four appendixes contain: a list of presentations and publications based on the project; the Story Problem Supplement; and reports to participating teachers of grades 8 and 6 concerning results of the experiment. (PK)

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The administrators and teachers of the Illinois and California schools and classrooms involved in the study were promised anonymity so I cannot name them. Their assistance and interest were, of course, vital to the project and must be acknowledged.

Dr. Raymond Hannapel of the National Science Foundation must be thanked for his calm, competent aid in expediting administrative matters for the project.

My colleague and wife, Judy, has always been willing to interrupt her many tasks to offer her valued counsel, optimism, and support.




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## PROJECT OVERVIEW

The basis for the project lay in recent interview work which suggested that in choosing operations for story problems in mathematics many students are guided by computational considerations rather than meanings for the operations. The project aimed to refine the catalogue of strategies used by students and to design and test instructional materials intended to foster a concept-based approach to story problems among sixth graders.

**Year 1 (1985-86).** Interviews of sixth graders supplemented earlier work to give a catalogue of strategies used by students. The analysis of textbooks was begun to see what conceptual background one could expect from the usual curriculum. A teaching experiment with a small group of sixth graders provided the basis for materials development; the materials were refined with the aid of a sixth grade teacher during summer, 1986. A supplementary study, prompted by a consultant (B. Greer) and relevant literature, examined whether U.S. ninth graders in algebra classes showed evidence of one particular strategy.

**Year 2 (1986-87).** The materials developed during Year 1 were tried out in schools in Illinois and California. Further textbook analysis showed that some with recent copyrights are giving more attention to language for the operations that earlier books did. Since whole numbers were the focus of Year 1 work, during Year 2 interviews with eighth graders were carried out to see whether students use concepts of operations with rational numbers in a variety of forms. Another spin-off study (in connection with D. McLeod's project) explored affect associated with the solution of story problems.

These parts of the project are addressed in the following sections of this report:

- The Catalogue of Strategies Used by Students in Solving Story Problems
- What Textbooks Offer
- The Teaching Experiment with Sixth Graders
- The Development of the Supplementary Materials
- The Try-out of the Materials
- The Spin-off Studies
  - Nonconservation of Operation in American Algebra Students
  - Searching for Affect in the Solution of Story Problems in Mathematics

Some concluding remarks close this report.

## THE CATALOGUE OF STRATEGIES USED BY STUDENTS IN SOLVING STORY PROBLEMS

Interviews of sixth and eighth graders confirmed earlier indications (Sowder, Threadgill-Sowder, Moyer, & Moyer, 1983) that many students use disappointing strategies in approaching story problems. Although each strategy listed was observed during the interviews of this project and the earlier studies, researchers have commented on various ones (e.g., Greer, 1987, Greer & Mangan, 1984; Kalmykova, 1975; Lester & Garofalo, 1982, 1987; Noddings, 1983; Sherrill, 1983; Sowder, Threadgill-Sowder, Moyer, & Moyer, 1983; Stevenson, 1925; Trafton, 1984). The list does show the variety of ways in which students may be attempting to reach solutions to story problems.

Twenty-nine classrooms of students were first given paper-pencil tests of story problems (see Appendices III and IV for item summaries). Results on these tests, along with consultation with the teachers involved, were the basis for selection of students to be interviewed. Interviews in the earlier



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work had shown that users of the most immature strategies were unlikely to give revealing interviews. Hence in later work we did not interview students whose written work suggested that they might be users of such strategies (e.g., students who always added). Thus the bulk of the more-than-70 interviews (collectively, across this project and the earlier ones) were conducted with average and above-average students.

Here is the catalogue of the strategies, roughly in ascending order of sophistication, with a few comments (adapted from "Aspects of Solving Routine Story Problems;" see Appendix I).

1. Find the numbers and add (or multiply or subtract...; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "...if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The first four strategies, of course, are not taught and would be amusing if they were not used by real learners. Although most often the students who use these strategies are not successful students, there are exceptions. One seventh grader in a gifted program used strategy 4, her computational facility made the strategy feasible with one-step problems.

Strategies 4-6 do involve at least a bit of number sense, a minimal semantic processing, and a very minimal awareness of the meanings for the operations. The key-word strategy (#5) unfortunately is occasionally taught by well-meaning teachers who are not aware of its defects. Sherrill (1983) noted a "pervasive" use of this key-word strategy in an interview study, and Nesher and Teubal (1975) found that even primary school children were using key words. Note that although strategies 4 and 6 may be effective with some whole numbers, they are less likely to be successful when large whole numbers, fractions, or decimals for which the student has less number sense are involved.

All but the last strategy (#7) are extremely difficult to apply to multistep problems. Hence there are implications for National Assessment of Educational Progress findings (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980; Carpenter, Matthews, Lindquist, & Silver, 1984; NAEP, 1983): The relatively good performance on one-step problems in NAEP testings may be specious, and we have a possible explanation for the relatively poor performance on more complicated problems – reliance on these immature strategies.

What was most disappointing was the rarity of the meaning-based strategy 7. In the interviews, even students who made correct choices of operations rarely could give any justification for their choice of operation. "I just know" was a common explanation, as was a recital of why the other operations would not give correct solutions (usually based on the anticipated size of the answer). Zweng, in her report of her interview work on problem-solving with elementary school students (1979), noted that the then-typical curriculum did not provide the students with language with which to describe generically some common applications of the operations. This lack is being addressed in some current text series, and it will be interesting to see whether this attention has a positive effect on the strategies used by students.



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Although the overall findings from the interviews are dismaying, there is a positive note. Many students "shift gears" when confronted with a multi-step problem and, apparently realizing that the immature strategies are inadequate, adopt some other strategies. What these strategies are, and whether the students indeed fall back on a meaning-based strategy, has been difficult to ascertain. There would seem to be profit in including many more multi-step problems in the curriculum.

[See Appendix I for citations of project presentations or publications stemming from the work on strategies.]

## WHAT TEXTBOOKS OFFER

Even without documentation it is fairly safe to assert that in most elementary classrooms the mathematics textbook "carries" the curriculum. The degree to which textbooks, then, provide work with the meanings for the operations is of direct pertinence to a study of student strategies. Little or no provision for conceptual underpinnings could result in the adoption of immature strategies by default.

While it is difficult to generalize about all text series, the picture appears to be improving slightly. In preparing the proposal for this project, I reviewed three text series that were fairly representative of those on the market. In one series, 9 pages of the grade 3 text, with roughly 56 demonstrated exercises and student exercises, could be said to give some conceptual background for multiplication, in the sense that it was possible to use given drawings to determine products. For division, the counts were 8 pages and 39 exercises. Very little additional work on concept development for multiplication and division, however, appeared in grades 4-6! Only two pages of the grade 4 text would allow the reader to attach meaning to multiplication by giving drawings which could show a meaning of multiplication, with about the same in the grade 5 and 6 texts. There was even less work for a concept of division in the grade 4-6 texts. Although one might claim that the many story problems in the texts no doubt added to the students' concepts, it seems as likely that, without being forearmed with sufficient conceptual background, students would flounder and by default resort to immature strategies.

The slightly positive side of the picture is that some – but not all – recent text series are providing somewhat more conceptual work, at least in grades 3-6. A small amount of picture-based work is often included in grades beyond grade 3, perhaps even so far as with 1 digit by 2 or 3 digit multiplication. There are also occasional calls for students to make up story problems. Some texts also provide verbal support for the operations (e.g., putting together equal sets means you should multiply). Still missing seem to be many exercises calling for the student to make a drawing for a given fact, for example. With a few exceptions the "translations" practiced most often originate in pictures or stories and end in symbols, rather than the reverse (e.g., make a drawing to show 2 x 4, or make up a story problem for 144 ÷ 18). And, conceptual work for multiplication and division of fractions and decimals is often skimpy or, if given, narrow in scope (e.g., only an area interpretation for the multiplication of fractions).

## THE TEACHING EXPERIMENT WITH SIXTH GRADERS

A teaching experiment was carried out to study the feasibility of different ways to add meaning to the operations and to serve as a basis for the materials development planned. In September, 1985, I met with consultant B. Greer (a psychologist who has worked with story problems) to review his work and that of other Europeans (e.g., Bell, Fischbein, & Greer, 1984; Ekenstam & Greger, 1983; Fischbein, Deri, Nello, & Marino, 1985) and to seek his advice on the teaching experiment.

Over a nine-week period in spring, 1986, I worked with a small group of seven sixth grade students (an eighth student joined the group late, on request). Work was scheduled for the last period of the day, was supplementary to the usual mathematics class, and did not require homework, so I had



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a fair monitoring of the work the students did. A group test given to all of the sixth graders, along with consultation with the teachers, was used to identify students who could likely profit from the work. That is, neither mainstreamed students nor the highest-scoring students were considered for the project. All the students invited to participate (with parent consent) agreed to do so: scheduling difficulties for one student resulted in his joining the group late. I audiotaped the sessions and wrote a debriefing log after each session. Teachers were given a weekly written synopsis of the work we had done in the small group.

The sessions varied in format, but each one focused on some aspect of meanings for the operations. For examples, students wrote story problems for given expressions like 150 ÷ 25, worked with concrete materials (counters, money, bags of Tootsie rolls, discount store advertisements) to illustrate given expressions, and filled out worksheets. Discussion and defense or explanation of answers was common, in an effort both to have the students verbalize and to provide feedback to me on what was guiding their thinking. Calculators were used to carry out any computations.

The following were the types of situations emphasized during the instruction. A chalkboard summary and later, their own reference sheet, for these links were frequently used by the students.

Addition – known amounts put together
Subtraction – a known amount taken away from a known amount
– two amounts compared
Multiplication – several amounts of the same size put together
– "combinations" (= Cartesian product)
– finding a part of an amount
Division – how many 2s in 8?
– if 8 are put equally into 2 amounts, how many are in each?

There was not time for attention to missing addend subtraction and scaling multiplication.

Cartesian product multiplication was understandable to these students. On the other hand, making diagrams and matching given diagrams with numerical expressions were more difficult for them than I expected. For simple story problems, students saw no value in making diagrams (nonroutine problems were used, therefore, in the materials developed).

Although the intent of the teaching experiment was to try different teaching techniques without concern for a cumulative effect, a small summative evaluation was carried out by re-administering the paper-pencil story problem test used as the group pretest (with an added multipart problem). Performance on the pretest items improved an average of 17.8% (to 66.4% from 48.6%); without a control group it is difficult to gauge whether this improvement was due to the small group work.

## THE DEVELOPMENT OF THE SUPPLEMENTARY MATERIALS

I met with consultants R. Brannan and D. Jungst in May, 1986, to review the teaching experiment, to react to psychologist B. Greer's comments, and to plan the development phase of the project. The advice received there and the teaching experiment experience were utilized during the development of the preliminary materials during early summer, 1986. The reactions, direct assistance, and further advice of C. Meister (an elementary teacher) to the preliminary materials were used to refine them, and to guide the development of the Teacher Commentaries to accompany each sample lesson. The try-out form of the materials was labelled **Story Problem Supplement (SPS)**. The SPS is included as Appendix II.



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## THE TRY-OUT OF THE MATERIALS

Try-outs of the SPS took place in schools in Illinois and in California. As a result of a presentation on the project theme at the annual meeting of the Illinois Council of Teachers of Mathematics (see Appendix I), several teachers expressed a desire to use any materials developed. These teachers were teaching primarily fourth grade classes by the time the SPS was developed. Since the target audience was grade six students, the results from the Illinois schools were not analyzed. The pretest and posttest results for the students in these 12 classes were sent to the teachers, however. The Illinois teachers whose students showed the greatest improvement tended to be enthusiastic about the materials. While this fact is encouraging, it must be noted that perhaps the teachers' enthusiasm for story problems, rather than the SPS, was the element responsible for the students' performance.

The main try-out of the SPS took place in the San Diego City Schools. Dr. Vance Mills, the district's mathematics and science coordinator, and Mrs. Corky Gray of his office were remarkably cooperative and helpful in locating a mix of schools with teachers willing to help in the evaluation. Fourteen sixth grade teachers (and one fifth grade teacher, whose class results are not included in the analysis) in 13 schools agreed to work with the project. Pairs of schools could be identified as roughly equivalent in socio-economic status; two of the teachers were in the same school. The SPS was made available to seven of the teachers (the experimental classes), with the other seven classes serving as a control group.

Control group teachers were asked to teach as they ordinarily would, whereas experimental group teachers were invited to use their choices from the SPS. I sent a mid-year memo to remind the teachers of the SPS and to offer assistance if they wanted any. Usage varied considerably, quantitatively from virtually all of the materials to only about 20% of the lessons and no doubt with equal variance qualitatively. Hence, the try-out can be considered a "weak" test of the materials, as opposed to a "strong" laboratory-like test, in which teachers would use prescribed parts of the materials in a prescribed sequence and a prescribed manner.

In each class, a pretest was given in October, 1986, and repeated as a posttest in April, 1987 (the pretest could not be scheduled within a reasonable time frame in one control class). These tests served to judge the equivalence of the experimental and control groups, to give a rough measure of the efficacy of the SPS, and to identify (with teacher help) students for the follow-up interviews. Results on the pretest and posttest are given by item in Appendix IV. These overall results and the scores of a teacher's students were given to the teacher as a report of the project; each control group teacher was also given a copy of the SPS.

Table 1 on the next page gives class means for the control and experimental classes on both the pretest and the posttest for students taking both tests. (Recall that one control class did not take the pretest.) One immediately notes the variety of levels among the classes and the perhaps disappointing overall improvement in performance on story problems over the course of nearly a school year.

The dubious equivalence of the control group and the experimental group also stands out; an analysis of variance of pretest scores verified that the two group means were different (observed $F_{1,297} = 7.6$, $p = 0.006$). Hence, to adjust for this initial lack of equivalence, an analysis of covariance of the posttest scores, with pretest score as covariate, was carried out (using the class as the unit) The results of this analysis are given in Table 2 (next page), along with the observed and adjusted means. Homogeneity of regression was verified (observed $F = 0.03$, $p = 0.86$). The results enable one to reject the hypothesis of equality of posttest performance, adjusted for pretest performance, between the control and the experimental groups. Hence, insofar as this statistical adjustment equates the two groups (a moot assertion, given the multitude of factors that enter into schooling), the experimental group did outperform the control group.

TABLE 1

Pretest and Posttest Means for Control and Experimental Classes
(Maximum score = 17)

| Class | n | Pretest | Posttest |
|---|---|---|---|
| Control | | | |
| Class 31 | 25 | 6.8 | 8.9 |
| Class 32 | 16 | 5.3 | 6.4 |
| Class 33 | 30 | 9.6 | 10.1 |
| Class 34 | 17 | 5.4 | 7.6 |
| Class 35 | 24 | 5.6 | 6.1 |
| Class 36 | 14 | 5.8 | 6.2 |
| All control | 126 | 6.7 | 7.9 |
| Experimental | | | |
| Class 40 | 25 | 11.0 | 12.6 |
| Class 41 | 17 | 6.4 | 7.5 |
| Class 42 | 24 | 7.5 | 10.3 |
| Class 43 | 24 | 5.8 | 9.9 |
| Class 44 | 33 | 11.0 | 13.4 |
| Class 45 | 26 | 9.4 | 11.5 |
| Class 46 | 24 | 2.8 | 4.9 |
| All experimental | 173 | 8.0 | 10.3 |
| All students | 299 | 7.4 | 9.3 |

TABLE 2

Analysis of Covariance of Posttest Scores,
with Pretest as Covariate, Class as Unit

| Source | df | Mean Square | F (probability) |
|---|---|---|---|
| Within | 10 | 0.81 | |
| Regression | 1 | 57.55 | 70.67 (0) |
| Constant | 1 | 6.38 | 7.83 (0.019) |
| Treatment | 1 | 4.86 | 5.96 (0.035) |

| Group (n) | Observed mean | Adjusted mean |
|---|---|---|
| Control (6) | 7.54 | 8.18 |
| Experimental (7) | 10.02 | 9.46 |

Although one can view the statistical analyses of the written tests as encouraging, the interview studies have shown that such data are not trustworthy if one's concern is how the students arrive at their answers. Good scores on one-step story problems with whole numbers can result from the use of the immature strategies. (It should be pointed out that the tests used here did contain multistep problems and problems with irrelevant data – see Appendix IV for the items.) A better test of whether the SPS was effective in getting students to use a concept-driven approach lay in the interviews.

As usual, performance on the posttest and teacher consultation were used to identify students for interviews. Since the premise of the project was that the use of a concept-driven strategy would give better performance, students who had shown a marked improvement over the year were singled out for consideration. For practical reasons, only 16 students could be interviewed (roughly one per classroom involved in the project). Interviews took place in May and June, 1987, with the posttest problems providing the primary basis for the interview, although as time permitted interviewees were asked about other problems. Students were told they had been picked because they had improved so much during the year and that we wanted to try to find out how they did story problems so we could perhaps better help other students.

There are occasions in the interviews when students seemed to be using conceptual understanding of the operations as a guide. Here are some examples:

-"Cause they sold this much and had to throw away this much, and so, you'd probably add those together."
-"...see, I figured that if he had a 200 inch board, and he has a piece that is 36 inches left, you just subtract that..."
-"...so you have to, either add 36 twelve times or you can times 12 times 36."
-Interviewer: "How come divide?"
"You need to see how many, um, how many 36 cents you can get into 6 dollars."
-Interviewer: "How come divide?"
"Divide 90 by 6 to find out how many times 6 goes into 90."

These positive signs are balanced by less encouraging evidence:

-Interviewer: "Now what made you think of this (subtraction)?"
"Well, nothing else would work. Adding wouldn't work. Multiplying wouldn't work. So, and dividing wouldn't work, so right away that only left one thing....
-Interviewer: "Why did you think of divide?"
"That's usually what I think of when I see a big number and a one-digit number. I just try to divide."

With only 16 interviews it is difficult to arrive at definite conclusions, of course. My impression from the interviews is that, despite some encouraging signs, a decided emphasis on meaning-driven approaches will be necessary to help students avoid relying on the immature strategies, ideally starting much earlier than grade six – certainly by grade three, when multiplication and division usually receive significant attention. Since the immature strategies are often successful in one-step story problems with whole numbers, they may become habituated and their success may work against the development of a meaning-driven strategy.

## THE SPIN-OFF STUDIES

Two studies not in the original proposal arose naturally during the course of the project. Perhaps students naturally come to adopt concept-driven strategies as they progress through the curriculum and gain more experience in mathematics. If that were the case, then younger students' use of the immature



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strategies would perhaps be not quite so worrisome. European work, however, has suggested that is not the case. Surprisingly, students who correctly decide that a particular operation will give the solution to a problem involving whole numbers may choose a different operation when the same problem is given with fractions or decimals less than 1 in place of the whole numbers. For example, consider "A pound of cheese costs $2.46. How much will 3 pounds of the cheese cost?" A student who correctly chooses to multiply on that problem may choose a different operation when the "3 pounds" is replaced with "0.82 pounds"! Such students are apparently guided by this (unintended) precept from their whole-number work: Multiplication makes bigger, division makes smaller (MMBDMS, per Greer & Mangan, 1986), and can be labelled "nonconservers of operation" (Greer & Mangan, 1984). Since MMBDMS is associated with immature strategy 6, nonconservers are unlikely to be using a meaning-based strategy. The first spin-off study looked for evidence of nonconservation among the students in ten algebra classes. As many as 30% of the students showed regular evidence of nonconservation, with up to 46.6% showing at least occasional lapses. Hence, there is no automatic "cure" for the use of the immature strategies in the usual K-8 curriculum. See the "Nonconservation of Operation in American Algebra Students" citation in Appendix I for a fuller report of this study

The second spin-off study was devoted to the difficult area of affect. There is no doubt that story problems carry a strong affective dimension, often negative, for many students. D. McLeod's NSF project called for other, on-going NSF projects on problem solving to attempt to incorporate investigations of affect into their project work, so it was natural to review transcripts for evidence of affect and to attempt, in later interviews, to identify elements of affect entering the solution of story problems. This effort was notably unsuccessful, perhaps because the earlier interviews were not concerned with affect and the later interviews were of eighth graders, for whom story problems may be so familiar as to not give clear-cut evidence of affect. Somewhat more successfully, a semantic differential instrument was devised and used with preservice elementary teachers in an effort to distinguish different degrees of affect within pairs of problems of the type used to test for nonconservation. See the "Searching for Affect in the Solution of Story Problems in Mathematics" citation in Appendix I for a fuller report of this study.

## CONCLUDING REMARKS

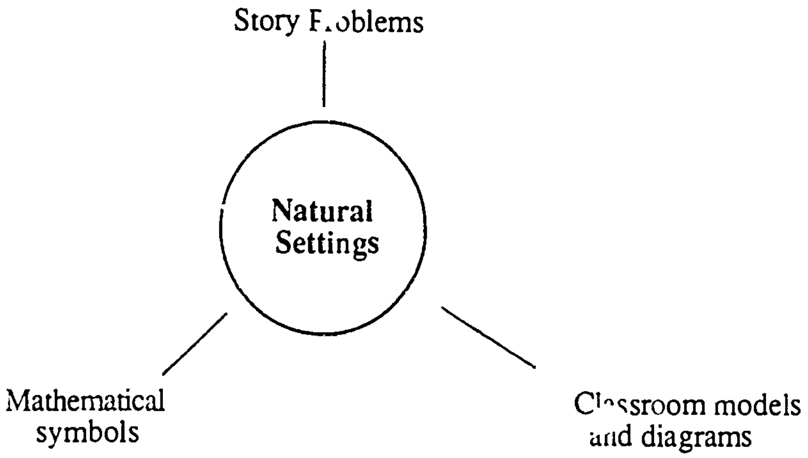
1. The **existence** of the immature strategies should receive wider attention in preservice teacher preparation. Teachers must be aware of the weaknesses of the immature strategies, so that they do not encourage or even teach them. Stronger instruction in using a meaning-based strategy for solving story problems is likely necessary for preservice teachers; Mangan (1986) found that changing a multiplier from a whole number to a decimal less than 1 resulted in a decrease on the order of 40% in a sample of teachers-in-training!

2. The **prevalence** of the immature strategies demands a curricular response. The limited, although statistically significant, success of the supplementary materials suggests that, if meaning rather than computation is to guide our students in their approaches to story problems, earlier and more extensive attention must be given to meanings for the operations and to the utility of these meanings in choosing operations for story problem. Otherwise, students may use the immature strategies by default.

The inclusion in the curriculum of many more multi-step problems, of problems involving irrelevant information, and of problems using "key words" in misleading ways is a relatively easy approach toward thwarting the success of the immature strategies – and hence dimming their attractiveness.

3. During the interviews, some students who were otherwise giving unenlightened performances gave the impression that, on encountering a multi-step problem, they abandoned the immature strategies. A one-step problem involves a decision about four operations; MMBDMS quickly reduces that number to two. But a multi-step problem is, a priori, much more complex; there are 4 x 3 = 12 possible orderings of different operations as possible solution paths, obviously a much more difficult task even if deciding which operands go where is ignored. If indeed students can "rise to the occasion" and call on cognitive resources not usually used, perhaps their use of the immature strategies, while disappointing, is not so troublesome. Whether this "gear-shifting" does indeed exist, what provokes it, and whether many students use the immature strategies only as courses of least resistance are questions to be examined.

4. When concrete materials of some sort were used during the teaching experiment, the students noticeably "perked up." Hands-on work rather than sole reliance on paper and pencil and symbolic procedures has been advocated for years; the teaching experiment supports that advocacy and suggests that motivation and perhaps meaning can be added to a lesson by the use of concrete materials. Indeed, a greater centrality of natural settings of all sorts in mathematics lessons would appear to diminish the gap between students' ability to compute and their ability to use the operations in the settings which story problems represent.




5. Why do students seem untroubled by their use of the immature strategies? It may be that they are completely unaware that more reliable means are available, which is the tack this project has pursued. The work of Dweck and her colleagues (1987) is provocative in that it suggests another possible reason: Some students just want to get answers, others want to understand (this is a paraphrase). Some students have surmounted a deficient curriculum and approach problems as we might wish they would. How are they different — is it in their different orientation toward learning? While the current project has focused on a curricular approach, it may be that an additional, broader concern will also need to be addressed: How can we get all students to want to understand?

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# APPENDIX I

## Presentations and Publications Based on the Project

### Presentations

(1985, October). Strategies Students Use with Story Problems. Presentation at the annual meeting of the Illinois Council of Teachers of Mathematics, Normal, IL.

(1986, April). Students' Strategies with Story Problems in Grades 6 and 7. Presentation at the annual meeting of the National Council of Teachers of Mathematics, Washington, D.C.

(1987, February). Concepts Needed for Solving Story Problems. Presentation at the annual meeting of the Greater San Diego Council of Teachers of Mathematics, San Diego, CA.

(1987, April). Enriching Middle Grade Students' Problem Solving Approaches. Presentation at the annual meeting of the National Council of Teachers of Mathematics, Anaheim, CA.

### Presentations appearing in Proceedings

(1986, September). Non-conservation of Operation in American Algebra Students. In Proceedings of the Eighth Annual Meeting of PME-NA. East Lansing, MI.

(1986, July). Strategies Students Use in Solving Problems. In Proceedings of the Tenth International Conference of PME. London, England.

### Publications to appear

To appear. Searching for affect in the solution of story problems in mathematics. In D. McLeod (Ed.), Affect in Mathematical Problem Solving: A New Perspective. New York: Springer-Verlag.

To appear. Aspects of solving routine story problems. In R. Charles & E. Silver (Eds.), The Research Agenda Project: The Teaching and Evaluation of Problem-Solving. Reston, VA: National Council of Teachers of Mathematics and L. Erlbaum Associates.

In preparation. A Missing Link in Children's Solutions of Story Problems (tentative title). Prepared for a special issue of the Journal of Mathematical Behavior.



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# APPENDIX II

The Story Problem Supplement (SPS)



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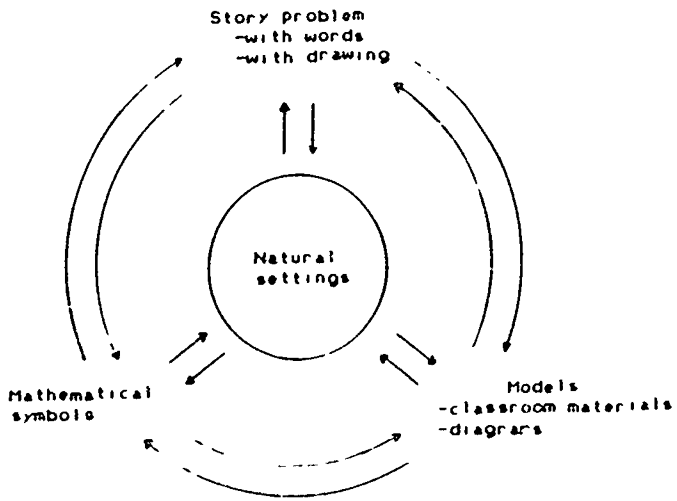
STORY PROBLEM SUPPLEMENT

(SPS)







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STORY PROBLEM SUPPLEMENT

TABLE OF CONTENTS

PROLOGUE FOR TEACHERS



STUDENT MATERIALS AND TEACHER COMMENTARIES

An indented title indicates dependence on the page listed before it.
* denotes materials which should be used early on to facilitate the use of other supplement pages.





ERIC

STORY PROBLEM SUPPLEMENT
(SPS)

Prologue for Teachers

## Why was SPS created?

Story Problem Supplement was motivated by some interviews of children while they were solving story problems. It became apparent that many students were using immature strategies which, although they might give success with simple one-step story problems, would fail them with more complicated story problems.

Some of these immature strategies are listed here:

1. Find the numbers and add
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you what to do
4. Try all (addition, subtraction,...) and choose the answer that is most reasonable.
5. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication, and choose the more reasonable answer. If smaller, try both subtraction and division, and choose the more reasonable.
6. Look for isolated "key" words to tell what operation to use.

Lacking was a strong use of meanings for the operations--a concept driven approach. 12 x 16 merely signalled a computational procedure. There was little evidence that the children understood that 12 x 16 "fit" situations in which, say, 12 groups of 16 each were totalled.

Thus, it appeared that students could profit from supplementary work with an emphasis on different facets of the operations.

## What is in SPS?

The bulk of the supplement is in student pages. These student pages are samples which could be used with the students and which might give teachers ideas of other things to try, in an effort to encourage a more thoughtful, concept based approach to story problems. In some cases, a page might be duplicated for use with the children, singly or in pairs or small groups; in other cases, a page might be used for a five or ten minute teacher-led discussion.

The reverse side of each student page contains a teacher commentary, with information about using the student page and with answers. On page iii is a guide indicating when the student pages might be used in the typical curriculum.

## How are the SPS materials supposed to help?

Story problems are included in the curriculum in the hope that they will enable students to function when they encounter such problems in natural settings (as opposed to the story problem form). A teacher or a student could also model a story problem with a diagram or with classroom materials such as counters. Eventually one gets a description in the form of mathematical symbols. There are, then, these ways of representing a problem based on a natural setting:

A story problem
-Mathematical symbols
Models (diagrams, classroom materials)

A story problem representation is a translation from a natural setting. The mathematical symbols for a story problem can be thought of as a translation from the story problem to the mathematical symbols. In making a diagram for a problem, one has translated from the story problem form to a model form. The SPS materials are based on these representations, and linking these representation by translating among them (Figure 1). Psychological theory indicates that a facility in translating among these representations gives evidence of understanding the mathematics underlying the problem.

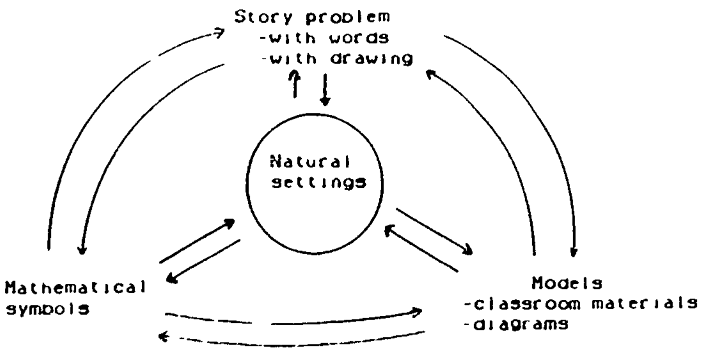


Figure 1 Translations among representations

What meanings or uses for the operations are covered in SPS?

SPS does not cover every use for every operation. Since the target students are in upper elementary, most of the work on meanings treats multiplication and divison. Most students at these grades already seem comfortable with addition and subtraction uses, so there are no pages devoted solely to addition or subtraction.

Uses for the operations which can reasonably come up in the upper elementary grades are given in Figure 2. All of these (except missing addend subtraction) are covered in the student pages. The ones marked with "*" are regarded as basic. Sixth





graders, for example, are likely to have encountered all of the starred meanings earlier.

Addition
* -Put together known amounts to find a total.

Subtraction
* -Take-away: one amount is taken from another known amount.
* -Comparison: two amounts are compared (how many more or less).
-Missing addend: one addend and the total are known in an addition setting

Multiplication
* -Repeated addition. several amounts, all the same, are totalled
* -Cartesian product. "combinations" of choices are counted
Part of an amount: a fractional part of an amount is sought.
-Scaling: an exact enlargement of a figure is made.

Division
* -Repeated subtraction. how many amounts of a given size are in a given amount? (Sometimes called measurement or quotitive division)
* -Sharing equally. how much is each part when a known amount is split equally among a known number of parts? (Also called partitive division)

Figure 2. Some meanings for the operations.

Is work with "natural settings" included?

The school, of course, is a natural setting for some problems. Purchases at the school store or lunch room, planning for outings, sharing materials, choosing teams or class officers.... Many teachers draw on these settings as they arise for mathematics problems. But unfortunately the classroom is limited in the natural settings which can be represented there. Thus, SPS is also limited in its suggestions for using natural settings. But there are occasional student pages which lend themselves to using natural materials (e.g., "Let's Shop").

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## When and how should SPS materials or ideas be used?

SPS materials should lend themselves to a rather flexible insertion into the mathematics time. The materials are designed for "plug-in" use rather than "tack-on" use. Such plans as the Missouri Mathematics Program call for daily attention to problem solving for part of the mathematics time. To work toward such a plan, some pages give the basis for a short oral review of the operations (e.g., "Choosing Operations Review") or can be drawn on frequently for a daily story problem (e.g., "Story Problem Bank").

Some pages have prerequisites or build on earlier student pages, as suggested by the table of contents and the usage guide. Many pages could be used virtually any time. Rather than take the pages in order, a teacher is more likely to choose from the different kinds of translations (see Figure 1).

## How can I tell whether my students benefit from SPS?

Most curricula give attention only to the translation from story problem to mathematical symbols. With SPS, a teacher might also naturally look for different sorts of translating skills: for example, can a student make up a story problem, from either an expression like 15 x 57 or from a diagram? In addition, a teacher might look for evidence of a concept driven approach to story problems, rather than reliance on the immature strategies.



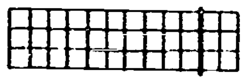
## USAGE GUIDE

An indented title indicates dependence on the page listed before it.
* denotes materials which should be used early on to facilitate the use of other supplement pages.

## COUNTING SQUARES

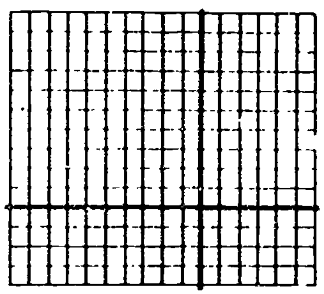
NAME ____________

3 x 12 = 36 TELLS YOU THAT 3 ROWS OF 12 SQUARES EACH GIVE 36 SQUARES.

3 ROWS  36 SQUARES IN ALL

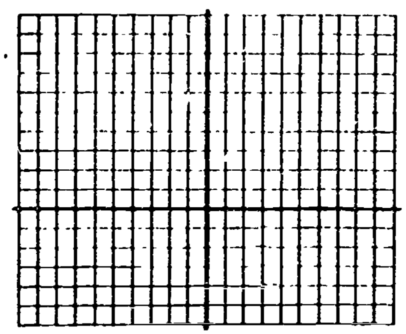
12 SQUARES IN EACH ROW

LIGHTLY SHADE THE GRIDS TO SHOW EACH OF THESE. THEN USE THE GRID TO GIVE THE ANSWER. DO NOT CALCULATE!

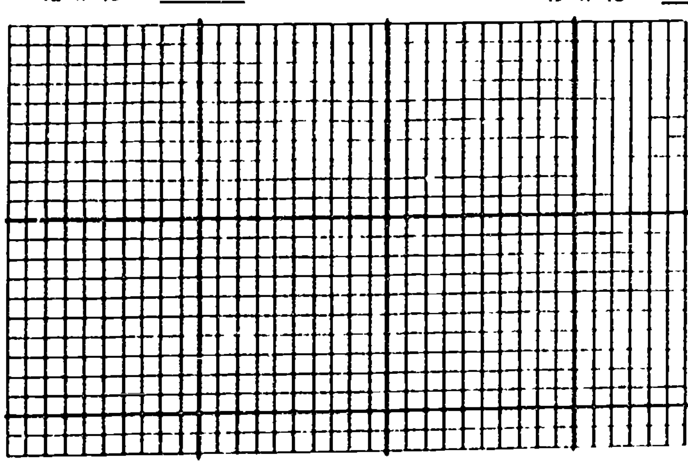
1. 

12 ROWS, WITH 13 IN EACH ROW

12 x 13 = ______

2. 

15 x 18 = ____

3. 

21 x 32 = ________





__Main Objective__ The student can show a given multiplication expression on a grid.

__When to use__ Early in the year.

__Suggested use__

Review arrays (equal rows of squares, dots, Xs,...) as one way to illustrate multiplication. The usual convention is that the first factor tells the number of rows (rows are sideways), and the second factor gives the number in each row. Arrays can be regarded as a special model for the repeated-addition view of multiplication, but it has carry-over to area and situations where the factors are not whole numbers too.

Showing the expressions should be fairly easy, but using the grids to find the products is likely to be new and may cause some puzzlement. Advise the students to look for "big" pieces they know, like a 10 by 10 piece that is 100, or like rows of 10 as in the example problem.

As a follow-up, you might ask how one would show 100 x 100 (or other larger numbers) on a grid, or something like 3.5 x 12 on a grid (3 and a half rows with 12 in each row).

__Answers__

1. 156: 1 hundred, 2 strips of ten, 3 more strips of ten, plus 6
2. 270: 1 hundred, 5 strips of ten, 8 more strips of ten, plus 40
3. 672: 6 hundreds, 3 strips of ten, 4 more strips of ten, plus 2

__Notes__

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:







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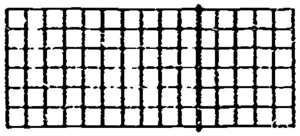
HOW MANY SQUARES? NAME ____________________

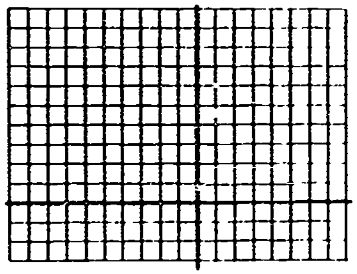
HOW MANY SMALL SQUARES ARE INSIDE EACH RECTANGLE? WHAT IS THE BEST WAY TO FIND OUT?

1.

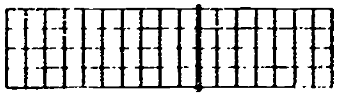


____ SMALL SQUARES

2.



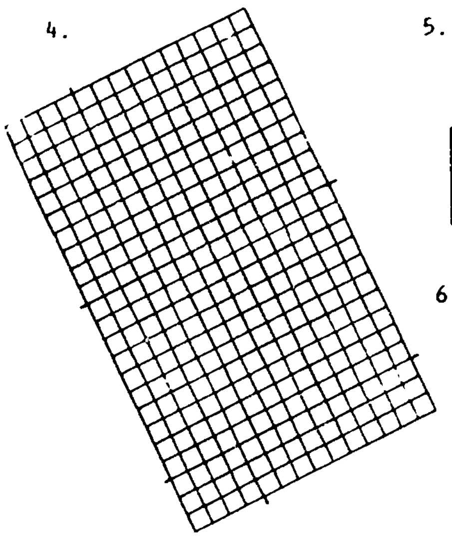
3.



_____ SMALL SQUARES

_____ SMALL SQUARES

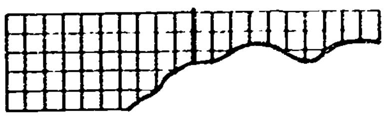
4.



RECTANGLE? ____

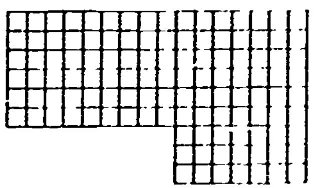
_____ SMALL SQUARES

5. PART OF JUAN'S RECTANGLE WAS TORN OFF. HOW MANY SMALL SQUARES DID HE HAVE AT THE START?



_____ SMALL SQUARES

6.



____ SMALL SQUARES





__Main Objective__ The tudent can apply multiplication to an array on a grid.

__When to use__ Early in the year; good __before__ area work is done.

__Suggested use__

If you use #1 as an example, very little additional direction should be necessary since many curricula include work with grids and multiplication in earlier grades. You may choose to have your students "brainstorm" for things that come in equal rows (e.g., items on grocery shelves, square tiles on the floor, the squares or rectangles in fluorescent light covers, holes in screen wire, bleacher seats, seats in classroom rows, spaces in parking lots,...). You may also wish to discuss how this relates to area (the length and width tell how many are in each row and how many rows there are).

Problem #6 might deserve some follow-up discussion, since it can be solved two ways (a horizontal cut or a vertical cut). Students need frequent reminders that many problems can be solved in more than one way.

__Answers__

1. 90 (6 x 15 is fastest way)
2. 234
3. 68
4. 322
5. 109
6. 117 (using a "cut" to get either 6x16 and 3x7 pieces or 6x9 and 9x7 pieces)

__Notes__

Additional activities (with natural settings or as extensions):

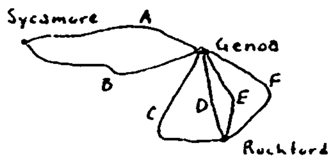
Trouble spots:

Other:

NAME ____________________

## MAKING CHOICES

YOU KNOW 2 WAYS TO GO FROM SYCAMORE TO GENOA AND 4 WAYS TO GO FROM GENOA TO ROCKFORD. IN HOW MANY DIFFERENT WAYS CAN YOU GO FROM SYCAMORE TO ROCKFORD, BY GOING THROUGH GENOA?




WAYS
AC
AD
AE
AF
BC
BD
BE
BF

$2 \times 4 = 8$

MULTIPLICATION CAN TELL YOU HOW MANY COMBINATIONS OF CHOICES YOU CAN MAKE.

1. LES HAS A BLUE, A RED, A GREEN, AND A WHITE SHIRT. LES HAS JEANS, BROWN CORDS, AND DRESS-UP PANTS. (A) HOW MANY PIECES OF CLOTHING IS THAT? (B) HOW MANY DIFFERENT OUTFITS DOES LES HAVE, IF ALL THE COLORS LOOK ALL RIGHT TOGETHER?

   (A) _____ (B) _____

2. LES HAS TWO SWEATERS. HOW MANY DIFFERENT SWEATER-SHIRT-PANTS OUTFITS DOES LES HAVE?

   _____

3. AN ICE-CREAM STORE HAS 26 FLAVORS OF ICE CREAM. YOU CAN GET A REGULAR CONE OR A SUGAR CONE. HOW MANY DIFFERENT KINDS OF SINGLE-DIP ICE-CREAM CONES CAN YOU GET?

   _____

4. ONE AFTERNOON THEY SELL $51.94 WORTH OF SINGLE-DIP CONES. IF ONE CONE COSTS 53 CENTS, HOW MANY CONES DID THEY SELL?

   _____

5. YOU HAVE TWO KINDS OF BREAD: WHITE AND WHEAT. YOU HAVE PEANUT BUTTER, TUNA SALAD, CHEESE, AND LUNCH-MEAT. HOW MANY DIFFERENT KINDS OF SANDWICH CAN YOU MAKE, IF YOU USE JUST ONE KIND OF BREAD AND ONE KIND OF "INSIDE" ON A SANDWICH?

   _____

6. MAKE UP A SANDWICH PROBLEM LIKE NUMBER 5.





Main Objective: The student can identify selected cartesian product situations as uses of multiplication.

Materials needed: Access to a calculator
(Optional) Cutouts of costumes--say, 3 shirts, differently colored, 2 kinds of trousers.

When to use: Before the textbook treatment of cartesian product multiplication.

Suggested use

Before showing the student page, introduce the lesson with a problem like "You have 3 different shirts and 2 different kinds of trousers or slacks. How many different outfits do you have?" Act out the six outfits with the cutouts or chalkboard drawings. If your textbook will be using tree diagrams, you might use that sort of recording scheme. The example on the student page uses pairs of letters.

Then ask, "Is there an easy way to get 6, just using information in the problem?" Students will suggest multiplication, of course, you can emphasize that this kind of situation gives another use for multiplication. The language used on the student page is "making choices," you will probably choose to use whatever language is in your text ("cartesian product" is not usually used with children).

You might also do the example problem from the student page before passing out the page. Students will readily multiply, but believing that there are eight may take several examples. In checking the problems, you should ask for a "proof" for one or two in the way of a list like that in the example (or a tree diagram, if you are using them). You might also add another town with 2 roads to the example, to show that the choices can be extended. Problem #2 calls for this extension. Problem #4 is a division problem to keep the children honest.

Answers

1. (A) 5 (B) 12 (This one could be "proved" by making a list.)
2. 24
3. 52
4. 98
5. 8 (This one doesn't take long to "prove.")
6. Answers will vary. The intent is that cartesian product be involved, but some children may just write a story about sandwiches.

Notes
Additional activities (with natural settings or as extensions).

Trouble spots

Other:



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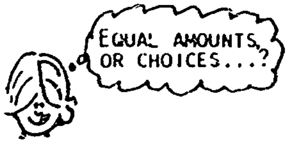



NAME ________________

## TWO USES FOR TIMES

REMEMBER:

> I. MULTIPLICATION CAN TELL YOU HOW MANY THERE ARE WHEN SEVERAL AMOUNTS OF THE SAME SIZE ARE PUT TOGETHER.
>
> II. MULTIPLICATION CAN TELL YOU HOW MANY COMBINATIONS OF CHOICES THERE ARE.

WRITE I OR II IN THE BLANK TO SHOW WHICH USE OF MULTIPLICATION THE PROBLEM WOULD NEED.

___ 1. A BIG CAFETERIA HAS 6 KINDS OF SANDWICHES, 8 KINDS OF DRINKS, AND 4 KINDS OF FRUIT FOR LUNCH. YOU WANT ONE OF EACH: SANDWICH, DRINK, AND FRUIT. YOU HAVE $5. IN HOW MANY WAYS CAN YOU CHOOSE LUNCH?

___ 2. IN ONE SCHOOL, THERE ARE SIX SIXTH-GRADE AND FIVE SEVENTH-GRADE CLASSROOMS. EACH CLASSROOM CAN HOLD 32 DESKS. HOW MANY SIXTH-GRADERS CAN THE SCHOOL TAKE?

___ 3. THE STUDENT COUNCIL IS GOING TO ELECT A PRESIDENT AND A TREASURER. THE STUDENT COUNCIL HAS 12 MEMBERS. HOW MANY WAYS IS IT POSSIBLE FOR THE ELECTION TO TURN OUT?

___ 4. A SCHOOL ALLOWS $5.25 FOR SUPPLIES THAT WILL BE USED WITH EACH STUDENT. HOW MUCH DOES THE SCHOOL ALLOW IN ALL, IF THERE ARE 570 STUDENTS IN THE SCHOOL?

___ 5. A FARMER HAS A WAGON THAT HOLDS 95 BUSHELS. THE FARMER HAULS 12 FULL LOADS OF CORN FROM A FIELD 3 MILES AWAY. HOW MANY BUSHELS OF CORN DID THE FARMER GET FROM THE FIELD?

___ 6. HOW FAR DID THE FARMER DRIVE TO GET ALL THAT CORN HAULED IN?

___ 7. JUST USING FIRST INITIAL AND LAST INITIAL, HOW MANY WAYS CAN PEOPLE'S INITIALS BE WRITTEN?





**Main Objective** The student can distinguish between repeated-addition and cartesian-product situations.

**When to use** Any time after cartesian product multiplication has come up.

**Suggested use**
Some of the problems contain irrelevant information. You may wish to have your students also record what numbers they would multiply.
This lesson lends itself to the use of an overhead transparency and a whole-class model. Write the two uses on the chalkboard, and have the children hold up one or two fingers as you expose each problem. This gives quicker feedback than a worksheet approach.

**Answers**
1. II (6x8x4. Ask about the irrelevant $5.)
2. I (6x32. Ask about the 5 seventh-grade rooms.)
3. II (12x11. This is a hard point. After one officer is chosen, there are only 11 students left for the other office. You might use the names of a small number of your students if the students do not seem to see this.)
4. I (570x5.25)
5. I (12x95)
6. I (12x3x2, or 12x6. Most students will not think of the 2 because they do not think of a roundtrip.)
7. II (26x26. Some students will write 2x26, and might benefit from looking at the problem if just 4 letters, say ABCD, were used.)

**Notes**

Additional activities (with natural settings or as extension):

Trouble spots:

Other:

NAME ____________

## HOW MUCH IS 0.75 OF AN AMOUNT?

A. HOW MUCH IS $\frac{1}{2}$ OF 24? _____

HOW MUCH IS 0.5 OF 24? _____

USE A CALCULATOR TO GET 0.5 x 24: _____

B. HOW MUCH IS $\frac{1}{4}$ OF 8? _____

HOW MUCH IS 0.25 OF 8? _____

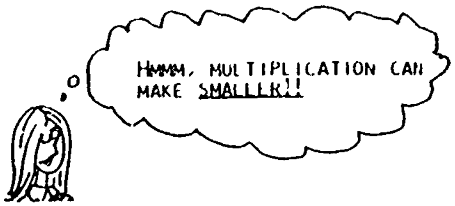
USE A CALCULATOR TO GET 0.25 x 8: _____

> MULTIPLICATION CAN TELL YOU HOW MUCH PART OF AN AMOUNT IS.

1. YOU CAN FIND $\frac{3}{8}$ OF 112 BY CALCULATING $\frac{3}{8}$ x ____________
2. YOU CAN FIND 0.75 OF 392 BY CALCULATING ____________
3. YOU CAN FIND $\frac{3}{5}$ OF $62\frac{1}{2}$ BY CALCULATING ____________
4. YOU CAN FIND 0.6 OF 43.5 BY CALCULATING ____________
5. MULTIPLICATION CAN TELL YOU ____________

6. CIRCLE THE CORRECT CHOICE. DO NOT USE A CALCULATOR.

| | | | | |
|---|---|---|---|---|
| A. 0.39 x 565 | EQUALS | IS LESS THAN | IS GREATER THAN | 565 |
| B 3.24 x 565 | EQUALS | IS LESS THAN | IS GREATER THAN | 565 |
| C. 0.97 x 565 | EQUALS | IS LESS THAN | IS GREATER THAN | 565 |
| D. 1.00 x 565 | EQUALS | IS LESS THAN | IS GREATER THAN | 565 |
| E. 0.62 x 0.85 | EQUALS | IS LESS THAN | IS GREATER THAN | 0.85 |








Main Objective The student can give a third use of multiplication--finding part of an amount.

Materials needed Calculators

When to use After decimals have been reviewed; can be used before decimal or fraction multiplication.

Suggested use

This is the first student page on this third use of multiplication. Virtually all of the students' earlier work with whole-number supports a "multiplication makes bigger" frame. Since this lesson goes against that frame, it may not register with the students and should get frequent follow-up.

Before presenting the page, spend some time on recognizing which decimals and fractions are less than 1. It will also help some students to review that 1/2 = 0.5 and 1/4 = 0.25, since these equivalences are taken for granted.

Do examples A and B as whole-class work. Point out that the answer is less than 24 (or 8, for B) since we are finding just a part of 24 (c. 8). If you are keeping a chart of uses of the operations, this new use could be entered now.

Some follow-up is essential since this new idea is inconsistent with the old, entrenched "multiplication makes bigger" idea. A few exercises like those in #6 or like those in "Which Is Biggest? Which Is Smallest?" could be used on several occasions. Story problems like "Your dad buys 0.83 gallons of gas for the lawnmower. One gallon costs $1.039. How much does the gas for the mower cost?" could also be used, since they involve this part-of-an-amount use of multiplication. Students often reason correctly that the answer will be less than $1.039 but conclude incorrectly that subtraction or division are the only operations that could work. "Buddy Problems" contains more story problems using a number less than 1.

Answers

A. 12, 12, 12
B. 2, 2, 2
1. 3/8 x 112
2. 0.75 x 392
3. 3/5 x 62 1/2
4. 0.6 x 43.5
5. ...how much part of an amount is.
6. A. is less than B. is greater than C. is less than
D. equals E. is less than

Notes

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:



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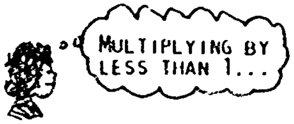
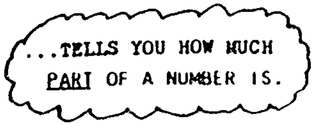



NAME ________________

## WHICH IS BIGGEST? WHICH IS SMALLEST?

CIRCLE THE BIGGEST ONE IN EACH ROW. BOX THE SMALLEST ONE IN EACH ROW.

| | | | |
|---|---|---|---|
| 1. | 56 | 0.285 x 56 | 1.003 x 56 |
| 2. | 153.2 | $\frac{3}{4}$ x 153.2 | $1\frac{1}{2}$ x 153.2 |
| 3. | 0.928 | 0.8 x 0.928 | 0.4 x 0.928 |
| 4. | $\frac{7}{8}$ | $\frac{3}{4}$ x $\frac{7}{8}$ | $2\frac{1}{4}$ x $\frac{7}{8}$ |
| 5. | 10.000 | $1\frac{1}{2}$ x 10.000 | 2.01 x 10.000 |
| 6. | 92 | 0.9999 x 92 | 1.00001 x 92 |
| 7. | 24 | $\frac{2}{1}$ x 24 | $\frac{3}{2}$ x 24 |
| 8. | 68.5 | 1.3 x 68.5 | 0.425 x 68.5 |










Main Objective The student can apply the part-of-an-amount use of multiplication

Materials needed Calculators

When to use After introductory work on the part-of-an-amount use of multiplication (e g , "How Much is 0 75 of an Amount?")

Suggested use

Before the lesson, review the uses of multiplication, especially the finding-part-of-an-amount one (e.g., What does 2/3 of 168 tell you?). Also review ordering decimals, since this is not an easy skill for students and it will be needed in believing the answers to the exercises.

Depending on how earlier work with this idea has gone, you might choose either to use the page as a whole-class, exercise by exercise discussion, or as an individual worksheet. It may help the students to confirm that indeed multiplication can give a smaller number by using calculators on the ones with decimals. Summarize the point at the end: Multiplication by a number less than one tells you what part of the number is.

Answers

| | Biggest | Smallest | |
|---|---|---|---|
| 1. | 1.003 x 56 | 0 285 x 56 | (These could be confirmed by calculator computations.) |
| 2 | 1 1/2 x 153.2 | 3/4 x 153 2 | |
| 3 | 0 928 | 0 4 x 0 928 | (Confirm by calculator) |
| 4 | 2 1/4 x 7/8 | 3/4 x 7/8 | |
| 5 | 2 01 x 10.000 | 10,000 | |
| 6 | 1 00001 x 92 | 0 9999 x 92 | (Confirm by calculator) |
| 7 | 2/1 x 24 | 24 | |
| 8 | 1 3 x 68 5 | 0 425 x 68 5 | (Confirm by calculator) |

Notes

Additional activities (with natural settings or as extensions):

Trouble spots.

Other.

NAME ______________

## BUDDY PROBLEMS

WRITE IN THE BLANK WHETHER YOU SHOULD ADD, SUB RACT, MULTIPLY, OR DIVIDE TO FIGURE OUT THE ANSWER.

1. A. A WOMAN BUYS 5 GALLONS OF GASOLINE. IT COSTS $1.039 FOR ONE GALLON. HOW MUCH DOES THE GASOLINE COST HER? ____

   B. A WOMAN BUYS 0.95 GALLONS OF GASOLINE. IT COSTS $1.039 FOR ONE GALLON. HOW MUCH DOES THE GASOLINE COST HER. ____

2. A. A RUNNER RAN 10 MILES ONE DAY, AND TWICE AS FAR THE SECOND DAY. HOW FAR DID THE RUNNER RUN ON THE SECOND DAY? ____

   B. A RUNNER RAN 10 MILES ONE DAY, AND 2/3 AS FAR THE SECOND DAY. HOW FAR DID THE RUNNER RUN ON THE SECOND DAY? ____

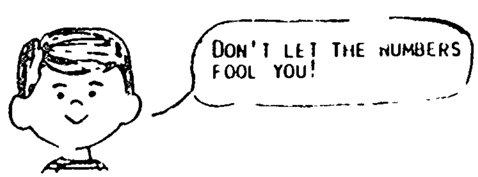
3. A BOY HAD TWO CANS FOR GASOLINE. HE PUT 0.73 GALLONS IN ONE CAN AND ?.5 GALLONS IN THE OTHER CAN. HOW MUCH GASOLINE DID HE GET? ____

4. A. IN ONE CITY THE SALES TAX IS 0.08 OF THE AMOUNT YOU BUY. IF YOU BUY $2.39 WORTH OF THINGS, HOW MUCH WILL THE SALES TAX BE? ____

   B. HOW MUCH WILL YOUR TOTAL BILL BE? ____

5. A JOGGER RAN 4.7 MILES ON MONDAY AND 6.2 MILES ON TUESDAY. HOW MUCH FARTHER DID THE JOGGER RUN ON TUESDAY? ____

6. ONE POUND OF HAMBURGER COSTS $1.68. YOU BUY 0.65 POUND. HOW MUCH DO YOU PAY? ____




40





Main Objective: The student can recognize part-of-an-amount multiplication situations.

When to use: After "How Much Is 0.75 of an Amount?" and/or "Which Is Biggest? Which Is Smallest?"

Suggested use
Review that multiplication can be used to tell you how much a fractional part of an amount is. Work problems #1 and #2 as a group, emphasizing that since the B parts involve part of the amount, multiplication is still the correct operation. As before, the idea of using multiplication to make a number smaller needs much reinforcement since it is counter to earlier experiences with multiplication.
Problems #3 through #6 involve a mix of operations.

Answers
1. A. multiply
   B. multiply
2. A. multiply
   B. multiply
3. add
4. multiply (since the tax is part of the amount), add
5. subtract
6. multiply (since you are buying only part of a pound)

Notes
Additional activities (with natural settings or as extensions):

Trouble spots:

Other:



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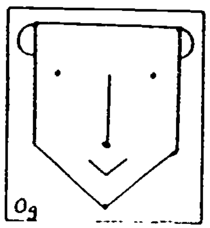
NAME ____________

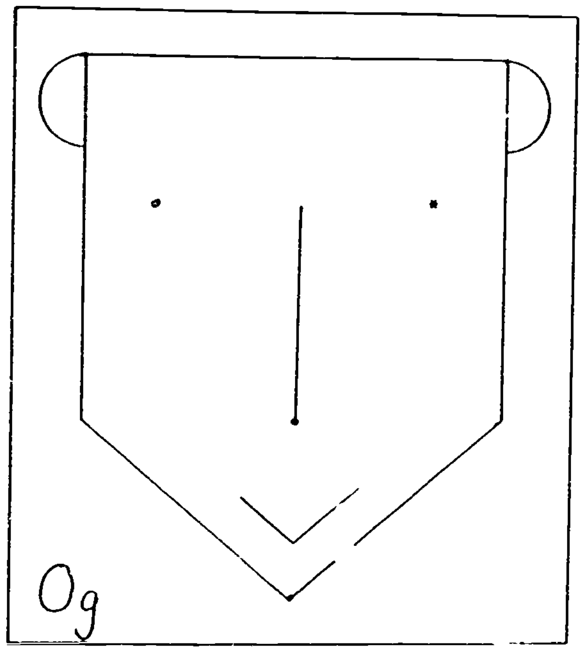
## ENLARGEMENTS

HOW ARE LENGTHS RELATED IN EXACT ENLARGEMENTS, AS IN PHOTOGRAPHS? MEASURE AND USE THE TABLE BELOW TO FIND OUT. USE CENTIMETERS.

OLD PICTURE




NEW PICTURE




| | NEW | OLD | NEW + OLD | NEW − OLD | NEW x OLD | NEW ÷ OLD |
|---|---|---|---|---|---|---|
| 1. DISTANCE BETWEEN EYES | | | | | | |
| 2. LENGTH OF TOP OF HEAD | | | | | | |
| 3. DISTANCE FROM TOP OF EAR TO CHIN | | | | | | |
| 4. DISTANCE FROM EYE TO TOP OF HEAD | | | | | | |

WHAT DO YOU NOTICE?





<u>Main Objective</u> The student experiences another use of multiplication and division, scaling (enlargements).

<u>Materials needed</u> Rulers with centimeter scales
(Optional) A photograph and an enlargement of the photograph

<u>When to use</u> Any time; fits nicely when school pictures are taken

<u>Suggested use</u>

If you have a photograph and an enlargment of it, show them to the students and ask if they think mathematics has anything to say about enlargments. That is the topic of the page, and gives another use of multiplication and division--in working with enlargments (or "shrinks," although these are not covered here).

An answer to "What do you notice?" at the bottom might be "The new distance divided by the old distance is always 3." To get multiplication explicitly involved, as if one could say "The new distance is 3 times the old distance." The latter form is useful in answering questions like "If Og's arm had been 12 centimeters long in the small picture, how long would Og's arm have been in the enlargement?" Be sure that students realize that with other pictures, the number probably will not be 3.

Be somewhat careful of language here. Everyday language like "twice as big" for a picture probably refers to <u>lengths</u> being twice as big. Since widths are also twice as big, areas are <u>four</u> times as big! Say lengths or distances are two times as big" rather than "the picture is two times as big."

Follow-ups could lie in measurements of pictures and their enlargements to find the number involved (often called the scaling factor), or asking what might happen if all the lengths were 1/2 or 1/3 as big (the picture "shrinks").

<u>Answers</u>

(All in centimeters.)

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. | 6 | 2 | 8 | 4 | 12 | 3 |
| 2. | 9 | 3 | 12 | 6 | 27 | 3 |
| 3 | 12 | 4 | 16 | 8 | 48 | 3 |
| 4. | 3 | 1 | 4 | 2 | 3 | 3 |

What do you notice? New distance divided by old distance is always 3.

<u>Notes</u>

Additional activities (with natural settings or as extensions):

Trouble r s:

Other:

NAME ______________________

## DIVIDING PAPER TWO WAYS

NEEDED: 6 STRIPS OF PAPER 24 CENTIMETERS LONG.

1. A. MARK OFF PIECES 4 CM LONG. HOW MANY PIECES DO YOU GET? ___

   B. FOLD ANOTHER STRIP INTO 4 EQUAL PIECES. HOW LONG IS EACH PIECE? ___

   EACH WAY SHOWS THAT 24 ÷ ___ = ___

2. A. MARK OFF PIECES 2 CM LONG ON ANOTHER PIECE. HOW MANY? ___

   B. FOLD THE STRIP INTO 2 EQUAL PIECES. HOW LONG IS EACH PIECE? ___

   EACH WAY SHOWS THAT 24 ÷ ___ = ___

3. MAKE UP TWO DIFFERENT PROBLEMS LIKE THE ONES ABOVE FOR 24 ÷ 8 = 3.

   A.

   B.

4. MAKE UP TWO DIFFERENT PROBLEMS FOR 24 ÷ 3 = 8.

   A.

   B.

5. MARK OFF PIECES 1.5 CM LONG. HOW MANY ARE THERE? ___

   24 ÷ ___ = ___

6. MARK OFF PIECES 0.5 CM LONG. HOW MANY ARE THERE? ___

   24 ÷ ___ - ___

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<u>Main Objective</u> The student experiences both repeated-subtraction and sharing equally division.

<u>Materials needed</u> Narrow strips of paper cut long-ways from regular sheets of paper, 6 strips for each student (plus some extras). Rulers with metric scale.

<u>When to use</u> After students have had some metric experience.

<u>Suggested use</u>

The page gives some practice with two meanings for division, without introducing any language for them. Language used in the teacher commentaries is "repeated subtraction" for the how-many-2s-in-8 type, and "sharing equally" for the how-many-in-each-when-8-are-split into-2-equal-parts type.

If you are keeping a classroom chart (e.g., "Chart for Uses of +, -, x, and ÷"), you may wish to refer to it, either to review the two uses or to motivate additional entries.

First have the stu~ ts measure off 24-cm lengths on the strips and tear them off (6 to 8 should be enough). Since the page involves making measurements, you may need to review how to use the type of ruler your students have.

Problems #5 and #6 are included to give the students experience with the repeated-subtraction meaning for division even though they do not know how to carry out the calculations.

Follow-up questions could center on larger numbers; e.g., "What would be two ways of showing 120 ÷ 8 with paper?"

<u>Answers</u>

1. A. 6 B. 6 cm 24 ÷ <u>4</u> = <u>6</u>
2. A. 12 B. 12 cm 24 - <u>2</u> = <u>12</u>
3. (samples, any order is all right, but both types should be covered)
   A. Mark off pieces 8 cm long. How many are there?
   B. Fold the strip into 8 equal pieces. How long is each piece?
4. (samples, any order)
   A. Mark off pieces 3 cm long. How many are there?
   B. Fold the strip into 3 equal pieces. How long is each piece?
5. 16 24 - <u>1.5</u> = <u>16</u>
6. 48 although with this much measuring, some students will be off
   24 ÷ <u>0.5</u> = <u>48</u> Point out that the answer is bigger than either 24 or 0.5. This can happen when you are dividing by a number less than 1

<u>Notes</u>

Additional activities (with natural settings or as extensions).

Trouble spots

Other.

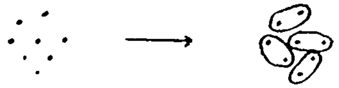
NAME ______________________

# ONE THING DIVISION CAN TELL

EXAMPLE

8 ÷ 2 CAN TELL HOW MANY 2S ARE IN 8.

DRAWING 8 AT THE START THEN SEE HOW MANY 2S



THERE ARE 4.
8 ÷ 2 = 4

USE THE EXAMPLE TO HELP YOU FILL IN THE BLANKS. USE A CALCULATOR IF YOU NEED TO.

1. 20 ÷ 4 CAN TELL ______________________________

   SHOW THAT THERE ARE ___ 4S IN 20.

   . . . . . . . . . .
   . . . . . . . . . .

2. ____________ CAN TELL HOW MANY 3S ARE IN 18.

   MAKE A DRAWING FOR THIS ONE.

3. ____________ CAN TELL HOW MANY 64S ARE IN 2048.

   HOW MANY 64S ARE IN 2048? _____

4. HOW MANY 117S ARE IN 6786? _____

5. HOW MANY $1.29S ARE IN $7.74? _____

6. HOW MANY 0.035S ARE IN 12.25? _____

7. A TAPE COSTS $6.98. YOUR FAMILY SPENT $55.84 ON TAPES AND $39.90 ON RECORDS LAST YEAR. HOW MANY TAPES DID YOUR FAMILY BUY? ____





Main Objective The student can link repeated-subtracti.n phrasings and division expressions.

Materials needed Calculators
(Optional) Counters for demonstration on overhead

Suggested use

Review these two forms for a given division expressions: 8 ÷ 2 and 2)8. The first is the better one here, since calculators can be used.

Ask a rhetorical, "What does 5280 ÷ 45 mean?" With two or three examples like 6 ÷ 3 or 8 ÷ 4 or 12 ÷ 2, ask how one would act them out with counters (or Xs on a chalkboard). Then use an example or two with larger numbers--e.g., 232 ÷ 8 or 465 ÷ 15; these would not be acted out but are good since the students do not already know the answers, as they do on the easy ones. The meaning featured in the lesson is, 6 ÷ 3 can tell how many 3s are in 6. (Sharing equally may come up if the children do the demonstration; praise this as a way that will get more attention later.)

Discuss the example, and "brainstorm" for examples of situations where repeated-subtraction division might come up (e.g., how many cartons are needed for 24 empty bottles, how many 25 cent cookies can one buy with $1,...).

If only one calculator is available, the page might best be done as a teacher-led lesson. Problem #7 involves a piece of irrelevant information.

A possible extension is to ask for a story problem for, say, 360 ÷ 24 that involves how many 24s in 360. If the students are not experienced at making up story problems, you might provide a setting: "I have 360 sheets of paper in a drawer.. "

Answers
1. 5, 5, drawing should show 5 sets of 4 dots.
2. 18 ÷ 3, drawing should show 6 sets of 3 (dots, probably).
3. 2048 ÷ 64, 32
4. 58
5. 6
6. 350 Note with the students that the answer to a division can be larger than either number if you are asking how many tiny amounts (less than 1) are in some number. This is likely a new idea for them.
7. 8 Ask about the $39.90 (extra information)

Notes
Additional activities (with natural settings or as extensions):

Trouble spots:

Other:

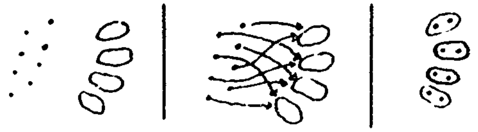
NAME ____________

# DIVISION CAN ALSO TELL...

REMEMBER: 150 ÷ 25 CAN TELL HOW MANY 25S ARE IN 150.

| DIVISION CAN ALSO TELL YOU HOW MANY EACH WILL GET WHEN SOME AMOUNT IS SPLIT UP EVENLY. |
|---|

EXAMPLE 8 ÷ 4 CAN TELL YOU HOW MANY EACH OF 4 WILL GET WHEN 8 ARE SHARED FAIRLY.



EACH WILL GET ___

SO 8 ÷ 4 = ___

1. DRAW ARROWS TO SHOW THAT 12 ÷ 3 TELLS YOU HOW MANY EACH WILL GET WHEN 12 ARE PASSED OUT EQUALLY TO 3 PEOPLE.

PUT NUMBERS IN THE BLANKS.

2. 224 ÷ 7 CAN TELL YOU HOW MANY EACH PERSON WILL GET IF ___ PEOPLE SHARE ___ THINGS EQUALLY.

3. 1088 ÷ 32 CAN TELL YOU HOW MANY EACH BOX WILL GET WHEN _____ ARE SPLIT UP EQUALLY INTO _____ BOXES.

4. $12.50 ÷ 5 CAN TELL YOU HOW MUCH MONEY EACH PERSON WILL GET WHEN ___ PEOPLE SHARE _____ EQUALLY.

5. ANN, BETTY, CASS, DIANA, ELLEN, AND FRAN WANT TO SELL 96 BOXES OF GIRL SCOUT COOKIES. IF EACH ONE SELLS THE SAME NUMBER OF BOXES, HOW MANY BOXES WILL EACH GIRL SELL? ______

6. JORGE AND KEN WANT TO SELL 48 TICKETS IN ALL. THEY HAVE ALREADY SOLD 16 TICKETS. HOW MANY DO THEY STILL HAVE TO SELL? ______





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Main Objective: The student can link sharing-equally phrasings and situations with division expressions.

Materials needed: Some amount of real or play money (or some classroom material that could be shared)
Calculator for teacher

When to use: After repeated-subtraction meaning for division (e.g., "One Thing Division Can Tell") has been practiced.

Suggested use

Show some money, say $7.95. Say something like "Suppose I am a rich person with $7.95. Pretend that I am going to share it with (2 students), so we all three have the same amount of money. How much will each of us get? How would you figure that out?" Repeat with 4 students and you, then with some number that does not give an exact number of cents (use calculator throughout). Tell that division can help when a known amount is put into a certain number of equal parts. Ask your students for other examples where an amount is shared or split up equally among several people or places.

Discuss the example so the sequence of drawings is clear.

Problem #6, a comparison subtraction, is to guard against "They're all division."

As a follow-up, give and contrast the two uses with, e.g., 524 ÷ 8 (how many 8s in 524, and how many each gets if 524 are shared equally among 8).

Answers
1. Drawing should suggest 4 dots "go" to each ring.
2. 7, 224
3. 1088, 32 (Some many incorrectly write the reverse order by imitating #2.)
4. 5, $12.50
5. 16 (Note that the divisor, 6, is not written in the story.)
6. 42

Notes
Additional activities (with natural settings or as extensions):

Trouble spots:

Other:

NAME ________

## TWO USES FOR DIVISION

REMEMBER:

| |
|---|
| I. DIVISION CAN TELL YOU HOW MANY AMOUNTS OF A CERTAIN SIZE ARE IN SOME AMOUNT. |
| II. DIVISION CAN TELL YOU HOW MUCH EACH GETS IF A CERTAIN AMOUNT IS SHARED EQUALLY. |

WRITE I OR II IN THE BLANK TO SHOW WHICH USE OF DIVISION THE PROBLEM WOULD NEED.

___ 1. CANDY IS ON SALE FOR $0.88 A BOX. HOW MANY BOXES COULD YOU BUY WITH $7? (DO NOT COUNT TAX.)

___ 2. A BIG BOX OF FANCY CHOCOLATES HAS 72 PIECES IN IT. HOW MANY SHOULD EACH PERSON IN A FAMILY OF SIX EAT, TO BE FAIR?

___ 3. THE AMOUNTS IN ONE SOUP RECIPE MAKE ENOUGH FOR 8 SERVINGS. TO MAKE ENOUGH FOR 40 SERVINGS, HOW MANY TIMES SHOULD THE AMOUNTS IN THE RECIPE BE USED?

___ 4. AT HALLOWEEN, ONE PERSON GAVE 3 SUCKERS TO EVERY TRICK-OR-TREATER. SHE GAVE AWAY 54 SUCKERS. HOW MANY TRICK-OR-TREATERS CAME TO HER HOUSE?

___ 5. ONE HUNDRED EIGHTY-FOUR SIXTH GRADERS ARE GOING ON A FIELD TRIP. THERE ARE FOUR BUSES TO TAKE THEM. HOW MANY SHOULD RIDE ON EACH BUS, TO KEEP THE LOADS EVEN?

___ 6. A RICH MAN DECIDES TO SHARE HIS WEALTH BY GIVING $500 TO EACH POOR PERSON IN ONE SCHOOL IF HE OR SHE GOES TO COLLEGE. HOW MANY PEOPLE CAN HE GIVE MONEY TO, IF HE HAS $180,000?

___ 7. JANE'S FAVORITE SONG TAKES 2 1/2 MINUTES TO PLAY. HOW MANY TIMES CAN SHE LISTEN TO THE SONG IN HALF AN HOUR?

___ 8. A PACKAGE OF HAMBURGER WEIGHS 1.35 POUNDS. A MOTHER MAKES 5 EQUAL HAMBURGER PATTIES OUT OF IT. HOW MUCH WILL EACH ONE WEIGH?





<u>Main Objective</u> The student can distinguish between repeated-subtraction and sharing division situations.

<u>Materials needed</u> (Optional) Response cards labelled I and II

<u>When to use</u> Any time after both meanings for division have been introduced.

<u>Suggested use</u>

Since some of these discriminations are difficult, this page might be done a whole-class discussion mode, perhaps from an overhead transparency. First review the two ways of thinking about division, perhaps acting each out with counters or chalkboard Xs (say, for 16 ÷ 2).

As you do each problem, consider having the children write both the ÷ and the $\overline{)\quad}$ forms so they are reminded of their differences. If you write the material in the box on the board, you can ask for a whole-class response by having the children hold up the correct response card.

Advise the students to think how they would act the situations out. Problems #4 and #6 are very tricky, since they carry a sharing idea but are in fact repeated subtraction situations. You might again ask the students to suggest situations for each kind of division.

<u>Answers</u>

1. I
2. II
3. I (want to know, how many 8s in 40)
4. I
5. II
6. I
7. I (how many 2 1/2s in 30; tell the children that we do not use the ) notation with fractions)
8. II

<u>Notes</u>

Additional activities (with natural settings or as extensions):

Trouble spots:

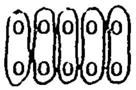
Other:

NAME ___________

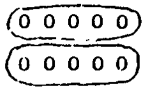
## DIVISION DIAGRAMS

THERE ARE TWO WAYS OF THINKING ABOUT DIVISION. PUT THE LETTER OF THE CORRECT DIAGRAM IN THE BLANKS.

$10 \div 5$

A. 

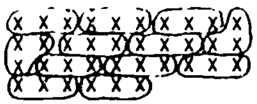
____ 1. HOW MANY 5S ARE IN 10?

B. 

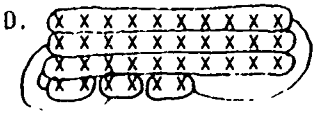
____ 2 IF 10 ARE PUT INTO 5 EQUAL PARTS, HOW MANY ARE IN EACH PART?

---

$36 \div 3$

C. 

____ 3. HOW MANY 3S ARE IN 36?

D. 

____ 4. IF 36 ARE PUT INTO 3 EQUAL AMOUNTS, HOW MANY ARE IN EACH AMOUNT?

---

$34 \div 2$

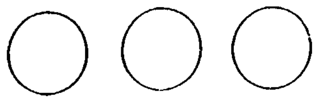
5. IN THE DRAWING TO THE RIGHT, SHOW HOW MANY 2S ARE IN 34.

X X X X X X X X X X
X X X X X X X X X X
X X X X X X X X X X
X X X X

---

$3 \div \frac{1}{2}$

6. IN THE DRAWING TO THE RIGHT, SHOW HOW MANY $\frac{1}{2}$ S ARE IN 3.







**Main Objective** The student has diagram experience with two interpretations for division.

**When to use** Early in the year, right before or with the first division work

**Suggested use**

This page should probably be done as a teacher-led discussion, since students may not have formally encountered both uses for division and since their work with diagrams may have been minimal.

Problem #6 is included to show that this meaning also transfers to division by a fraction. Point out that the answer (6) is greater than either 3 or 1/2; this sort of thing can happen because of the question: how many 1/2s are in 3?

**Answers**

1. B
2. A
3. C
4. D
5. Diagram should show 17 groups of 2 each.
6. Diagram should show that there are 6 halves in 3.

**Notes**

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:

NAME ______________

## ADD, SUBTRACT, MULTIPLY, OR DIVIDE?

READ THE EXAMPLE AND THINK ABOUT IT. THEN FILL IN THE BLANK WITH ADD, SUBTRACT, MULTIPLY, OR DIVIDE.

1. EXAMPLE: JANE HAS 6 TAPES. SUE HAS 8. HOW MANY TAPES DO THEY HAVE TOGETHER?

   *WHEN YOU WANT TO D OUT HOW MANY IN ALL, YOU ____________.

2. EXAMPLE: L HAD 2 DOLLA. THEN HE SPENT 65 CENTS. HOW MUCH DID HE HAVE LEFT?

   *WH ONE AMOUNT IS TAKEN AWAY FROM ANOTHER, YOU ____________.

3. EXAMPLE. BILL HAD 2 DOLLARS. CARL HAD 65 CENTS HOW MUCH MORE DID BILL HAVE THAN CARL?

   *WHEN YOU WANT TO COMPARE T AMOUNT , ONE WAY IS TO ____________.

4. EXAMPLE: WALTER PAYTON SCORED 36 TOUCHDOWNS. A TOUCHDOWN IS WORTH 6 POINTS. HOW MANY POINTS DID PAYTON SCORE?

   *TO FIND OUT HOW MANY THERE AR. TOGETHER WHEN ALL AMOUNTS ARE THE SAME, YOU CAN ____________.

5. EXAMPLE: THE PTA HAD 75 OATMEAL COOKIES AT A BAKE SALE. THEY PUT 3 COOKIES IN EACH PLASTIC BAG. HOW MANY BAGS DID THEY GET?

   *TO FIND OUT HOW MANY AMOUNTS OF THE SAME SIZE THERE ARE WHEN YOU KNOW THE TOTAL, YOU ____________.

6. EXAMPLE: THE PTA HAD 75 OATMEAL COOKIES AT A BAKE SALE. THEY PUT THE SAME NUMBER OF COOKIES AT EACH OF 3 TABLES. HOW MANY COOKIES WERE AT EACH TABLE?

   *WHEN AN AMOUNT IS SHARED OR PASSED EQUALLY TO SEVERAL PLACES, YOU ____________.





**Main Objective** The student can choose the correct operation, given a story problem or a descript n of a situation.

**When to use** Any time.

**Suggested use**

Perhaps with a question like "How do you decide what to do when you have a story problem?" introduce the idea that thinking about the kind of situation involved in the story problem is the best wa to decide.

. is page reviews the kinds of situations that the students have likely dealt with, so individual work llowed by discussion could serve as a diagnostic. Note that #4 could be done by addition (adding 36 sixes) but that multiplication is far faster.

To encourage the students to think about uses for the operations, ask them for reasons like the starred statements whenever story problems are discussed. "Choosing Operations Review" and "Chart for Uses..." might be useful in this respect. Student response cards with the signs for the operations could be used in group settings to get feedback on all the students.

As another extension, you could ask, as you encounter them, whether answers to the ed statements would change if the numbers were frac cimals. Children often errone ly think the oper tion doe end on the number rather than the situation. " d, Subtract, Multiply, or Divide? 2" asks the students to make up a story problem from a given general statement like the starred statements here, and could be used fairly soon after this sheet.

**Answers**

1. add
2. subtract
3. subtract
4. multiply
5. divide
6. divide

**Notes**

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:



ERIC

NAME ______________

## CHART FOR USES OF +, , X, AND ÷

| KIND OF SITUATION | DIAGRAM | USUALLY YOU |
|---|---|---|
| A. | | ADD |
| A. | | |
| B. | | SUBTRACT |
| A. | | |
| B. | | MULTIPLY |
| C. | | |
| A. | | DIVIDE |
| B. | | |

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<u>Main Objective</u> The student can eventually link descriptions of situations and their operations.

<u>When .o use</u> Introduce early in the year, even though the chart will not be completely filled in until later.

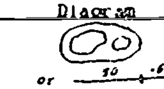
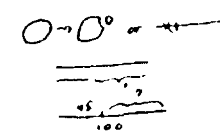
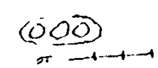
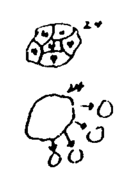
<u>Suggested use</u>

The page serves as a model for a poster you might keep on the wall. As the uses and meanings for the operations are reviewed or introduced, you could tape up new phrasings and diagrams for the operations. So that the students do <u>not</u> adopt the chart language :s key words, it is a good idea to vary the language frequently.

You might also provide a copy for each child for use with homework.

Periodically or once you are confident that the children have the content of the final chart mastered, you might ιemove it from the wall.

Herɔ is one form a final chart might take:

| Kind of situation | Diagram | Usually you |
|---|---|---|
| A. Put known amounts together to find total |  | Add |
| A. One amount is taken from another<br>B. Two amounts are compared<br>(C. Missing addends) |  | Subtract |
| A. Several amounts, all the same, are totalled (repeated addition)<br>B. Counting combinations of choices<br>C. Finding a part of an amount<br>D. Enlargements |  | Multiply |
| A. Finding how many amounts are in a total (repeated subtraction)<br>B. Finding each share when a total is shared equally (sharing equally) |  | Division |

NAME ______________

## PROBLEMS WITH MORE THAN ONE STEP

MAKE A DIAGRAM IF IT WILL HELP YOU. ONE OF THE PROBLEMS TAKES ONLY ONE STEP.

1. TOM RAN 2320 FEET FROM JOE'S HOUSE TO BILL'S HOUSE. THEN HE RAN 4 MORE BLOCKS. EACH BLOCK WAS 1050 FEET LONG. WHAT TOTAL DISTANCE DID HE RUN?

2. THERE WERE 224 STUDENTS AT THE SCHOOL PLAY. ONE HUNDRED FIFTY-SIX OF THEM SAT ON THE BLEACHERS. THE REST SAT ON BENCHES. EACH BENCH HAD 6 STUDENTS. HOW MANY BENCHES WERE USED?

3. JAN WAS READING A BOOK THAT HAD 152 PAGES. SHE READ 20 PAGES ON MONDAY AND 32 PAGES ON TUESDAY. HOW MANY PAGES DID SHE READ THOSE TWO DAYS?

4. A POST 12 METERS LONG IS POUNDED INTO THE BOTTOM OF A RIVER. 2.5 METERS OF THE POST ARE IN THE GROUND UNDER THE RIVER. 0.5 METER STICKS OUT OF THE WATER. HOW DEEP IS THE RIVER AT THAT POINT?

5. A CARPENTER HAS A BOARD 200 INCHES LONG AND 12 INCHES WIDE. HE MAKES 4 IDENTICAL SHELVES AND STILL HAS A PIECE OF BOARD 36 INCHES LONG LEFT OVER. HOW LONG IS EACH SHELF?

6. A CASE OF BOTTLES OF ORANGE DRINK COSTS $11.50. THERE ARE 12 BOTTLES IN THE CASE. THERE IS ALSO A 15 CENT DEPOSIT FOR EACH BOTTLE IN THE CASE. WHAT WILL BE THE TOTAL COST FOR 12 BOTTLES?





<u>Main Objective</u> The student can solve multi-step problems.

<u>When to use</u> Fairly early in the year

<u>Suggested use</u>

The immature strategies mentioned in the prologue often break down completely with multi-step problems. On the other hand, some students can "shift gears" and use much more thought on multi-step problems. Thus, this page can serve as a diagnostic: which students <u>can</u> shift gears, and which can't? There will be frustration, so if you can monitor the work well enough, you may allow pair or small group work after some individual work. The word "deposit" (as in bottle deposit) appears in #6 and is not always familiar to the students.

USING MORE MULTI-STEP PROBLEMS MAY BE THE BEST THING WE CAN DO TO DISCOURAGE THE IMMATURE STRATEGIES. Frequently have a multi-step problem as the "problem of the day." Several are included in "Story Problem Bank."

<u>Answers</u>

1. 5520 feet (more than a mile)
2. 12 (benches)
3. 52 (pages) (This is the one-step problem! It has extra information.)
4. 9 meters (This is difficult without a diagram.)
5. 41 inches (The 12 inches wide is attractive, especially if the students have done area recently.)
6. $13.30

<u>Notes</u>

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:



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## GIVING REASONS 1

NAME ____________

DIRECTIONS READ THE INFORMATION BELOW AND THEN ANSWER AT LEAST THREE OF THE QUESTIONS. SHOW YOUR WORK AND GIVE YOUR REASON FOR DECIDING TO ADD, SUBTRACT, MULTIPLY, OR DIVIDE. YOU HAVE TO DO TWO STEPS IN ONE OF THE PROBLEMS.

HERE ARE THE SUPPLIES A TEACHER HAD FOR 32 STUDENTS. THERE WERE 25 PACKAGES OF WHITE PAPER AND 20 PACKAGES OF YELLOW PAPER. ONE PACKAGE OF PAPER HAS 500 SHEETS IN IT. THERE WERE 24 PADS OF ART PAPER. EACH PAD OF ART PAPER HAD 36 SHEETS. THERE WERE EQUAL AMOUNTS OF BLACK, RED, GREEN, BLUE, PINK, AND ORANGE PAPER IN EACH PAD OF ART PAPER.

1. HOW MANY MC PACKAGES OF WHITE PAPER WERE THERE THAN OF YELLOW PAPER?

WORK I ____________ BECAUSE ________________

2. HOW MANY PIECES OF BLUE ART PAPER ARE THERE IN ONE PAD?

WORK I ____________ BECAUSE ________________

3. HOW MANY PACKAGES OF PAPER DID THE TEACHER HAVE?

WORK I ____________ BECAUSE ________________

4. HOW MANY SHEETS OF YELLOW PAPER DID THE TEACHER HAVE?

WORK I ____________ BECAUSE ________________

5. IF THE TEACHER PASSED OUT ALL OF THE ART PAPER, HOW MANY PIECES WOULD EACH OF THE STUDENTS GET?

WORK I ________________________





Main Objective The student can give reasons for choices of operations with story problems.

Materials needed (Optional) A ream of paper
(Optional) Calculators

When to use Any time after language for all the operations has been discussed.

Suggested use

Review the uses for the operations if they have not been discussed recently (e.g., through the "Chart for Uses. ." if you are using it).

Pass out or display the sheet. Illustrate an example (e.g., How many sheets of white paper did the teacher have?) so the students see how the blanks are to be filled (I multiplied because there were 25 amounts of paper with 500 in each amount ).

Assign pairs of students to work together on the sheet. You may choose to have the students work 2 of the first 4 and the last one, a multistep problem.

Having all the information at the top is more difficult for the students to deal with, but it is much more realistic than the three-sentence form for a story problem.

As an additional exercise for fast-finishers, you could ask that they make up another problem on this theme for the bulletin board. "Giving Reasons 2" repeats this sort of lesson.

Answers

1. 25 - 20 = 5. I subtracted because I wanted to compare two known amounts.
2. 36 ÷ 6 = 6. I divided because the 36 pieces were in 6 equal amounts.
3. 25 + 20 = 45. I added because I wanted to find out how much there was in all.
4. 20 x 500 = 10,000. I multiplied because there were 20 equal amounts and I wanted to know how many there were in all.
5. 24 x 36 = 864, 864 ÷ 32 = 27. I multiplied because I needed to know how many pieces of art paper there were in all, and then I divided because the paper was shared equally.

Notes

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:





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NUMBERLESS PROBLEMS NAME ____________

YOU NEED TO KNOW ONLY SOME OF THE MISSING NUMBERS TO ANSWER THE QUESTIONS BELOW. MAKE UP YOUR OWN NUMBERS AND PUT THEM IN THE BLANKS JUST FOR THE NUMBERS YOU NEED TO KNOW. THEN ANSWER THE QUESTION.

1. LAST YEAR ARNIE MISSED ____ DAYS OF SCHOOL. CLINT MISSED ____ DAYS. MICHAEL MISSED ____ DAYS. HOW MANY DAYS DID ARNIE AND MICHAEL MISS IN ALL?

   WORK:

2. ANNIE HAD $______. THE ANNIE BOUGHT A RECORD FOR $______ AND A PENCIL FOR ______. HOW MUCH DID ANNIE SPEND?

   WORK.

3. CLARE RODE HER BIKE FOR ____ HOURS. EACH HOUR SHE WENT ____ MILES. SHE RESTED FOR ____ MINUTES. HOW FAR DID SHE RIDE?

   WORK:

4. A TOUCHDOWN IN FOOTBALL IS WORTH 6 POINTS, AND A FIELD GOAL IS WORTH 3 POINTS. IN __ GAMES, YOU MADE ____ TOUCHDOWNS AND __ FIELD GOALS IN ALL. HOW MANY POINTS DID YOU SCORE IN THESE GAMES?

   WORK.

5. YOU HAD ____ CENTS. YOU BOUGHT AN APPLE FOR ____ CENTS AND A CARTON OF MILK FOR ____ CENTS. THEN YOU HAD ____ CENTS LEFT. HOW MUCH DID YOU SPEND?

   WORK:





__Main Objective__ The student can identify the information needed to solve a story problem.

__When to use__ Any time

__Suggested use__

Work #1 together so the students will understand that all the blanks do not have to be filled. Emphasize that careful reading of the question is necessary to know what information you need to know. Problem #5 allows two different choices of blanks. This fact could come out in the discussion.

__Answers__

(Numerical answers will vary.)

Need to know:

1. number of days Arnie missed; number of days Michael missed; add these
2. how much the record cost; how much the pencil cost; add these
3. how many hours Clare rode; how many miles she went each hour; multiply
4. number of touchdowns; number of field goals; # of TDs x 6, plus # of FGs x 3.
5. __Either__ how much the apple cost and how much the milk cost (then add), __or__ how many cents at the start and at the end (then subtract)

__Notes__

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:



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## CHOOSING OPERATIONS REVIEW

WHAT WOULD YOU DO (ADD, SUBTRACT, MULTIPLY, DIVIDE)...

1. WHEN ONE AMOUNT IS TAKEN AWAY FROM ANOTHER?
2. IF A LENGTH OF ROPE IS CUT INTO SMALLER PIECES OF EQUAL LENGTHS?
3. WHEN YOU COMBINE 16 AMOUNTS OF EQUAL SIZE?
4. WHEN YOU HAVE 5 EQUAL GROUPS OF 38 EACH?
5. WHEN YOU JOIN $276 TO ANOTHER AMOUNT?
6. IF AN AMOUNT IS SPLIT EQUALLY TO SEVERAL PLACES?
7. IF YOU COMBINED 148 GROUPS OF 8?
8. WHEN YOU COMPARE THE HEIGHT OF YOURSELF AND A FRIEND?
9. WHEN YOU TAKE AN AMOUNT FROM $20?
10. WHEN YOU SHARE 425 EQUALLY AMONG 25 PEOPLE?
11. IF ONE AMOUNT IS COMPARED TO ANOTHER AMOUNT?
12. WHEN YOU JOIN THREE AMOUNTS OF DIFFERENT SIZES?
13. IF YOU WANT TO SEE HOW MANY 16S ARE IN SOME AMOUNT?
14. IF YOU PASS OUT SEVERAL PIECES OF CANDY TO THREE FRIENDS?
15. WHEN YOU INCREASE AN AMOUNT BY 1658?
16. WHEN YOU REMOVE A CERTAIN AMOUNT FROM AN AMOUNT?
17. IF YOU HAVE SEVERAL AMOUNTS OF THE SAME SIZE AND YOU WANT TO FIND OUT HOW MANY THERE ARE ALL TOGETHER?
18. IF AN AMOUNT IS SHARED EQUALLY AMONG SEVERAL CONTAINERS?
19. WHEN YOU JOIN CERTAIN GROUPS OF EQUAL SIZE?
20. WHEN YOU WANT TO FIND OUT HOW MANY IN ALL?
21. IF YOU WANT TO COMPARE THE WEIGHT OF TWO ITEMS?
22. IF YOU PUT 5 BOXES WITH 36 PENCILS IN EACH IN ONE CONTAINER?
23. WHEN YOU SEPARATE 392 INTO 7 GROUPS OF EQUAL AMOUNTS?
24. IF 2286 ITEMS ARE PUT INTO GROUPS OF NINE?
25. IF 2286 IS COMBINED WITH 9?





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26. IF 2286 IS SEPARATED EQUALLY INTO 9 CONTAINERS?
27. IF 9 IS REMOVED FROM 2286?
28. IF 9 IS COMPARED TO 2286?
29. IF 9 GROUPS OF 2286 ARE JOINED?
30. IF 2286 GROUPS OF 9 ARE PUT TOGETHER?
31. WHEN YOU WANT TO FIND OUT HOW MANY IN ALL?
32. WHEN 48 IS REMOVED FROM 1467?
33. WHEN ONE DISTANCE IS COMPARED TO ANOTHER?
34. WHEN TWO DIFFERENT AMOUNTS ARE JOINED TO 365?
35. IF 17 GROUPS OF EQUAL AMOUNTS ARE MADE FROM 7769?
36. IF YOU WANT TO FIND ALL POSSIBLE COMBINATIONS WHEN YOU ARE GIVEN CHOICES?
37. WHEN AN AMOUNT IS TAKEN AWAY FROM ANOTHER?
38. WHEN 67 IS TAKEN FROM 456?
39. IF YOU WANT TO FIND OUT HOW MANY THERE ARE ALL TOGETHER WHEN SEVERAL KNOWN AMOUNTS ARE THE SAME?
40. IF YOU WANT TO SEE HOW MANY 4S ARE IN 2276?
41. WHEN YOU WANT TO FIND OUT THE TOTAL OF ITEMS AND THERE ARE 28 ITEMS EACH IN A CERTAIN NUMBER OF GROUPS?
42. IF YOU COMBINE 6 AND 7?
43. IF YOU COMBINE 6 GROUPS OF SEVEN?
44. IF YOU REMOVE AN AMOUNT FROM ANOTHER?
45. IF YOU WANT TO FIND THE NUMBER OF CHOICES YOU HAVE FOR DIFFERENT OUTFITS WHEN YOU HAVE 4 SHIRTS, 2 TIES, AND 7 PANTS?
46. WHEN YOU WANT TO SEE HOW MANY 7S ARE IN 497?
47. WHEN SEVERAL AMOUNTS ARE THE SAME AND YOU WANT TO FIND OUT THE TOTAL?
48. WHEN A KNOWN AMOUNT IS SHARED EQUALLY TO SEVERAL PLACES?
49. IF YOU WANT TO FIND OUT HOW MANY AMOUNTS OF THE SAME SIZE THERE ARE IN A GIVEN AMOUNT?
50. WHEN YOU WANT TO COMPARE 5 AND 6?

51. WHEN 127.8 IS DECREASED BY SOME KNOWN AMOUNT?

52. IF YOU PUT TOGETHER 23 AMOUNTS OF THE SAME KNOWN SIZE?

53. IF YOU WANT TO SEE HOW MANY 17S THERE ARE IN SOME AMOUNT?

54. IF YOU TAKE AWAY A KNOWN AMOUNT FROM 615.2?

55. WHEN YOU WANT TO COMPARE 76.2 WITH SOME OTHER AMOUNT?

56. IF YOU COMBINE 53.34 AND SOME OTHER KNOWN AMOUNT?

57. IF YOU POUR 14 CANS OF THE SAME SIZE INTO A BUCKET AND YOU WANT TO KNOW HOW MUCH YOU HAVE?

58. IF YOU WANT TO FIND THE DIFFERENCE BETWEEN $11.89 AND SOME OTHER AMOUNT?

59. IF YOU HAVE 10 METERS OF ROPE AND YOU WANT TO KNOW HOW MANY PIECES 2.5 METERS LONG YOU CAN GET?

60. IF YOU HAVE SHREDDED WHEAT CEREAL AND OAT CEREAL, AND YOU CAN HAVE A BANANA OR PEACHES OR BERRIES ON YOUR CEREAL, AND YOU WANT TO KNOW HOW MANY WAYS YOU CAN CHOOSE BREAKFAST?

61. IF YOU WANT TO KNOW HOW MUCH 2/3 OF AN AMOUNT IS?

62. IF YOU WANT TO KNOW HOW MANY 2/3S ARE IN 4 1/2?

63. IF YOU WANT TO PAY 0.06 OF AN AMOUNT FOR TAX?

64. IF YOU WANT TO KNOW HOW MANY 0.06S ARE IN 1.02?

65. IF YOU WANT TO FIND THE DIFFERENCE BETWEEN 0.06 AND 1.2?





**Main Objective** The objective will vary depending on how you use the material.

**Materials needed** (Optional) Student response cards with the operation symbols on them

**When to use** Any time, to review uses of operations

**Some suggested uses**

As with "Story Problem Bank," these questions can be used in a variety of ways: as a bank to select certain types of situations, as a source for a short oral review, as short quiz material.... Since one of the four operations is the response in each case, you might wish to use student response cards.

Some of the phrases are *not* complete in themselves, leaving the sought quantity implied. For example, "...when you have 5 equal groups of 38 each?" (#4) only implies that you want to know how many in all. If this abbreviated version is not clear to your students, you might make explicit the quantity wanted.

**Answers**

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. - | 2. ÷ | 3. x | 4. x | 5. + | 6. ÷ | 7. x |
| 8. - | 9. - | 10. ÷ | 11. - | 12. + | 13. ÷ | 14. ÷ |
| 15. + | 16 - | 17. x | 18. ÷ | 19 x | 20. + (or x, if equal) | |
| 21. - | 22. x | 23. ÷ | 24. ÷ | 25. + | 26. ÷ | 27. - |
| 28. - | 29. x | 30. x | 31. + (or x, if equal) | | | 32. - |
| 33. - | 34. + | 35. ÷ | 36. x | 37. | 38. - | 39. x |
| 40. ÷ | 41. x | 42. + | 43. x | 44. - | 45. x | 46. ÷ |
| 47. x | 48. ÷ | 49. ÷ | 50. - | 51. - | 52. x | 53. ÷ |
| 54. - | 55. - | 56. + | 57. x | 58. - | 59. ÷ | 60. x |
| 61. x | 62. ÷ | 63. x | 64. ÷ | 65. - | | |

**Notes**

Additional activities:

Trouble spots:

Other:

## STORY PROBLEM BANK

1. MARY BOUGHT SOME SHORTS FOR $17, A PAIR OF SHOES FOR $28, AND A SHIRT FOR $9. HOW MUCH DID SHE SPEND FOR THE SHOES AND THE SHIRT?

2. WILBUR HAD 138 MODEL AIRPLANES. HE GAVE ORVILLE 54 OF THEM. HOW MANY DID WILBUR HAVE THEN?

3. LOU BOUGHT 48 CANS OF DRI'K. THEY ARE PACKED IN CASES OF 12. HOW MANY CASES DID LOU E ?

4. GINNY DELIVERS 84 MORNING PAPERS AND 68 EVENING PAPERS. LAST NIGHT SHE COLLECTED $148 FOR HER PAPER ROUTE. HOW MANY PAPERS DOES SHE DELIVER EACH DAY?

5. ON SATURDAY THE RESTAURANT SERVED 106 PEOPLE FOR LUNCH AND 248 PEOPLE FOR DINNER. HOW MANY MORE PEOPLE WERE SERVED FOR DINNER?

6. FELIPE HAS 25 QUARTERS. HOW MANY NICKELS CAN FELIPE GET FOR HIS 25 QUARTERS?

7. MARTIN HAS TWENTY COINS TO PUT INTO HIS BOOK. F . COINS GO ON EACH PAGE. HOW MANY PAGES WILL HE FILL?

8. THE RECIPE FOR FRUIT SALAD CALLS FOR 4 APPLES. H MANY APPLES ARE NEEDED FOR 3 BATCHES OF THE SALAD?

9. MRS. GARCIA DROVE 350 KILOMETERS EACH DAY. SHE TRAVELED FOR 15 DAYS. HOW FAR DID SHE RAVEL DURING HER TRIP?

10. PETE SOLD 49 ITEMS ON MONDAY, 56 ITEMS ON T SDAY, AND 14 ON WEDNESDAY. HOW MANY MORE ITEMS DID HE S L ON MONDAY THAN ON WEDNESDAY?

11. FIFTY-FOUR STUDENTS ATTENDED THE CLASS PICNIC. THERE WERE NINE PICNIC TABLES. THE SAME NUMBER OF DENTS SAT AT EACH TABLE. HOW MANY STUDENTS SAT AT EACH TABLE?

12. THERE ARE SEVEN PACKAGE OF CUPCAKES WITH 4 CUPCAKES IN A PACKAGE. HOW MANY CUPCAKES ARE THERE?

13. TED WENT TO DAY CAMP FOR SIX WEEKS, AND HE ATTENDED 5 DAYS A WEEK. HOW MANY DAYS DID HE SPEND AT CAMP?

14. RAQUEL SPENT $36 FOR SOME TICKETS. EACH TICKET COST $4. HOW MANY TICKETS DID SHE BUY?

15. THERE WERE 4 SHIPS LEAVING IN THE NEXT HOUR. EACH SHIP HAD A CREW OF 276. HOW MANY CREW MEMBERS WERE LEAVING?

16. ROSA WALKED FOR 26 MINUTES. SHE SWAM FOR 45 MINUTES. HOW MUCH LONGER THAN AN HOUR DID SHE EXERCISE?



17. IN ONE GAME EACH TIME YOU PASS HOME YOU COLLECT $400. HOW MUCH DO YOU COLLECT BY PASSING HOME 12 TIMES?

18. FOR AN AFTER-SCHOOL SNACK LORI COULD HAVE AN APPLE, A TANGERINE, OR A RANGE. SHE COULD HAVE A GLASS OF JUICE OR MILK FOR A DRINK. HOW MANY DIFFERENT CHOICES DID SHE HAVE FOR A SNACK AND DRINK?

19. SAM BUYS ORANGES BY THE CRATE. EACH CRATE COSTS $13.98. HOW MUCH WILL 17 CRATES COST?

20. SCOTT WENT TO SCHOOL FOR 186 DAYS LAST YEAR. HOW MANY DAYS D RING THE YEAR WAS HE NOT IN SCHOOL?

21. RAMON HAD 28 PIECES OF BUBBLE GUM. HE SPLIT THEM EQUALLY AMONG 4 FRIENDS. HOW MANY TREATS DID EACH FRIEND GET?

22. CHRIS EARNS $13 A WEEK ON HIS PAPER ROUTE. HE IS SAVING TO BUY A SKATE BOARD FOR $59. HOW MUC ILL HE EARN IN 5 WEEKS?

23. 20 BEADS ALL GETHER. 4 S GS HOW MANY BEADS ON EACH STRING?

24. WINTER VACATION LASTED 3 WEEKS. HOW MANY DAYS WAS THIS?

25. RENT FOR OUR APARTMENT IS $450 A MONTH. WHAT DOES IT OST TO RENT FOR THE YEAR?

26. MICHELLE HAS THREE SHIRTS, TWO PAIRS OF SLACKS, AND TWO SWEATERS. HOW MANY DIFFERENT WAYS CAN SHE DRESS?

27. LOU HAD 375 BASEBALL CARDS. MINDY HAD 148 CARDS. HOW MANY MORE DID LOU HAVE?

28. SENA SLEEPS 8 HOURS EACH NIGHT. HOW MANY HOURS EACH DAY IS SENA AWAKE?

29. BRENNAN HAD 500 MILLILITERS OF MILK. HE USED 212 MILLILITERS IN A RECIPE. HOW MUCH MILK DOES BRENNAN NOW HAVE?

30. SUMMER VACATION LASTED 91 DAYS. HOW MANY WEEKS IS THIS?

31. MARIA SWIMS 35 MINUTES EACH DAY. HOW MANY MINUTES DID SHE SWIM DURING THE LAST 2 WEEKS?

32. THE BICYCLE SHOP HAS 21 TIRE PUMPS. THEY SELL ALL BUT 4 OF THEM FOR A TOTAL OF $255. WHAT IS THE AVERAGE PRICE OF A TIRE PUMP?

33. TERRY PRACTICES PIANO 45 MINUTES EACH DAY. HOW MANY MINUTES DOES HE PRACTI A WEEK?

34. A MATH TEXTBOOK COSTS $7.50. THE WORKBOOK THAT GOES WITH THE TEX COSTS $2.69. THE PRINCIPAL ORDERED OF EACH. WHAT IS THE TAL COST?

35. MARILYN SOLD 27 RAFFLE TICKETS AT $2.50 A TICKET. HOW MUCH DID SHE COLLECT FOR THE RAFFLE?

36. FRAN COMPLETED THE BICYCLE RACE IN 3 HOURS. HER TOTAL DISTANCE WAS 33.6 KILOMETERS. HOW FAR DID SHE TRAVEL IN ONE HOUR?

37. THE ART SHOW WAS PRESENTED ON MONDAY AND TUESDAY. 78 PEOPLE SAW THE SHOW ON MONDAY. 216 PEOPLE SAW THE SHOW IN ALL. HOW MANY PEOPLE SAW THE SHOW ON TUESDAY?

38. THE BASEBALL TEAM'S NAME WILL BE TWO WORDS. THE FIRST WORD WILL BE RED, MIGHTY, OR HARD. THE SECOND WORD WILL BE HITTERS OR KNOCKERS. HOW MANY DIFFERENT NAME COMBINATIONS WILL THERE BE FOR THE TEAM TO CHOOSE?

39. MANUEL BUYS FOUR NOTEBOOKS AT 89 CENTS EACH AND 6 PENCILS AT 15 CENTS EACH. HOW MUCH DID HE SPEND?

40. A LIBRARY HAS 7 SHELVES OF BIOGRAPHIES. EACH SHELF HOLDS 45 BOOKS. HOW MANY BIOGRAPHIES ARE IN THE LIBRARY?

41. JENNY COLLECTED STICKERS. EACH STICKER BOOK CONTAINS 35 STICKERS. HOW MANY STICKERS ARE NEEDED TO FILL 27 BOOKLETS?

42. MELISSA MADE 9 CAKES FOR THE STUDENT COUNCIL BAKE SALE. SHE SOLD THEM FOR $2.50 EACH. EACH CAKE COST $1.08 FOR INGREDIENTS. HOW MUCH MONEY DID STUDENT COUNCIL MAKE ON EACH CAKE AFTER THEY PAID MELISSA FOR THE INGREDIENTS?

43. IN THE MORNING THERE WERE 9,578 BOOKS IN THE LIBRARY. AT THE END OF THE DAY THERE WERE 7,989 BOOKS IN THE LIBRARY. HOW MANY WERE CHECKED OUT THAT DAY?

44. MILDRED RIDES 10 KILOMETERS EVERY WEEK TO DELIVER PAPERS. HOW MANY KILOMETERS DOES SHE RIDE IN A YEAR?

45. TYRONE ORDERED 4 MICROSCOPES FROM THE SCIENCE CATALOG. THE TOTAL COST OF SHIPPING EACH MICROSCOPE IS $1.65. HOW MUCH WILL IT COST TO SHIP ALL 4 MICROSCOPES?

46. TWO TEACHERS AND TWO PARENTS ARE GOING WITH THE CLASS TO THE AMUSEMENT PARK. ADMISSION FOR AN ADULT IS $13.75. HOW MUCH DOES IT COST FOR THE 4 ADULTS?

47. AN AVERAGE PAGE HAS 385 WORDS ON IT. HOW MANY WORDS ARE ON 35 PAGES?

48. TAMMY'S DAD TOOK US TO A BASEBALL GAME. HE SPENT $4.00 ON PARKING, $15.00 ON TICKETS, AND $17.78 ON FOOD. HOW MUCH DID HE SPEND?

49. STUDENT COUNCIL CHARGED $0.25 ADMISSION TO THE MOVIE FESTIVAL. 98 STUDENTS ATTENDED. HOW MUCH MONEY DID STUDENT COUNCIL EARN?





50. TOM COLLECTED $3.10 FROM EACH OF THE 24 CUSTOMERS ON HIS PAPER ROUTE. HE ALSO EARNED $13.75 IN TIPS. HOW MUCH DID HE COLLECT IN ALL?

51. THERE ARE 864 BOOKS ON THE LIBRARY SHELVES. 148 BOOKS ARE REFERENCE BOOKS AND 120 BOOKS ARE FICTION. HOW MANY BOOKS ARE NOT REFERENCE OR FICTION BOOKS?

52. THE FOOTBALL STADIUM HOLDS A TOTAL OF 896 PEOPLE. THERE ARE 8 SECTIONS OF THE SAME SIZE. HOW MANY PEOPLE CAN BE SEATED IN EACH SECTION?

53. EACH STUFFED ANIMAL COSTS $5.89. WILLY BOUGHT 3 ANIMALS. HOW MUCH DID HE SPEND?

54. 27 BENCHES. 3 PEOPLE TO A BENCH. HOW MANY PEOPLE?

55. EVAN ENTERED A BICYCLE RACE. HE TRAVELED 6.9 KILOMETERS AN HOUR. HOW FAR DID HE TRAVEL IN 4 HOURS?

56. ROBERTO TRAVELED 95 METERS PER MINUTE ON HIS BICYCLE. HOW FAR DOES HE TRAVEL IN 15 MINUTES?

57. TWO LITERS OF DRINK COST $1.78. FOUR LITERS OF DRINK COST $3.40. FIND EACH UNIT COST. WHICH IS A BETTER BUY? WHAT IS THE DIFFERENCE BETWEEN THE UNIT PRICES?

58. THE SCHOOL SUPPLY STORE HAD 60 PAIRS OF SCHOOL SHOELACES ON SALE FOR $1.50 EACH. AFTER THE SALE THERE WERE 36 PAIRS LEFT. HOW MUCH MONEY WAS COLLECTED FOR THE LACES THAT WERE SOLD?

59. IN ART JASON MADE A SET OF MATCHING PLACE MATS. HE USED 64 GREEN PLASTIC STRIPS AND 80 YELLOW PLASTIC STRIPS. EACH PLACE MAT NEEDED 18 STRIPS. HOW MANY MATS DID JASON MAKE?

60. RAMON HAS $68. SALLY HAS $49. SALLY SPENDS $13. HOW MUCH MORE MONEY DOES RAMON HAVE THAN SALLY DOES NOW?

61. A RECIPE FOR PARTY PUNCH CALLS FOR 6 CUPS OF CRANBERRY JUICE AND 8 CUPS OF GINGER ALE. HOW MANY CUPS OF LIQUID WOULD IT TAKE TO MAKE 9 RECIPES?

62. THE PTA ORDERED 18 BOXES OF PUNCH. IN EACH BOX THERE WERE 32 PLASTIC BOTTLES OF PUNCH. DURING THE FUN FAIR 267 BOTTLES WERE USED. HOW MANY WERE LEFT?

63. SUPPOSE YOU TAKE 16 BREATHS PER MINUTE. HOW MANY MINUTES HAVE PASSED IF YOU HAVE TAKEN 208 BREATHS?

64. THE WALL PLAQUE FOR THE SPEECH CONTEST COST $59. THE TROPHY FOR THE FIRST PLACE WINNER WAS $75.65. HOW MUCH WAS SPENT ON THE TWO ITEMS?

65. IN THE SUPERMARKET THERE ARE 10 CHECKOUT COUNTERS. TWO COUNTERS TAKE 10 ITEMS OR LESS. THREE TAKE 20 ITEMS OR LESS. HOW MANY TAKE MORE THAN 20 ITEMS?





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66. WILDWOOD SCHOOL RECEIVED 210 BANNERS. EACH OF THE 14 CLASSROOMS WILL RECEIVE THE SAME NUMBER OF BANNERS. HOW MANY BANNERS WILL EACH CLASS RECEIVE?

67. A SET OF MODEL CARS COSTS $10.44. THERE ARE 12 MODELS IN THE SET. HOW MUCH DOES EACH MODEL CAR COST?

68. 12 DIVISION PROBLEMS. 18 MULTIPLICATION PROBLEMS. 12 MORE DECIMAL PROBLEMS. 126 MINUTES SPENT SOLVING THE PROBLEMS. ON THE AVERAGE HOW MANY MINUTES WERE SPENT ON EACH PROBLEM?

69. MS. ROBERTS PURCHASED 12 PAINT SETS. HOW MUCH CHANGE DID SHE GET FROM $50.00 IF THE SETS COST $3.75 EACH?

70. POSTER COLORS COME IN RED, BLUE, OR GREEN. POSTER LETTERING COMES IN WHITE, YELLOW, OR BLACK. FIND ALL THE DIFFERENT POSSIBLE COMBINATIONS.

71. THERE ARE 114 PACKAGES OF PAPER IN THE STORE ROOM. 78 PACKAGES ARE WHITE AND 27 ARE BLUE. HOW MANY ARE NOT WHITE OR BLUE?

72. IN ONE WEEK 24 STUDENTS COLLECTED 504 CANS OF CANNED GOODS FOR THE SCHOOL DRIVE. PRETEND EACH STUDENT GATHERED THE SAME AMOUNT. HOW MANY CANS DID EACH STUDENT GATHER?

73. THE STUDENTS HAVE 19 BOXES OF T-SHIRTS WITH 25 SHIRTS IN EACH. HOW MANY T-SHIRTS DO THE STUDENTS HAVE?

74. THERE WERE 430 PEOPLE AT THE MUSICAL PERFORMANCE GIVEN BY THE FIFTH AND SIXTH GRADE STUDENTS. 295 OF THEM SAT IN CHAIRS. THE REST SAT ON BENCHES. EACH BENCH HOLDS 15 PEOPLE. HOW MANY BENCHES WERE NEEDED?

75. BILL HAS 4 SHORTS, 3 SHIRTS, AND 2 SWEATERS. HOW MANY DIFFERENT OUTFITS CAN HE WEAR?

76. RHONDA WANTS TO BUY A NEW BICYCLE. THERE ARE 5 DIFFERENT COLORS, 4 DIFFERENT STYLES, AND 3 DIFFERENT SPEEDS (1-SPEED, 3-SPEED, 10-SPEED). HOW MANY CHOICES DOES SHE HAVE?

77. TONY WANTS TO BUY A CAMERA THAT COSTS $112.50. TONY CAN SAVE $2.50 A WEEK. HOW MANY WEEKS WILL IT TAKE TO SAVE ENOUGH MONEY?

78. MRS. JOHNSON ORDERED 24 BOXES OF COMPUTER DISKS. THE TOTAL COST WAS $456. HOW MUCH WAS EACH BOX?

79. SOME COMPUTER PRINTERS CAN PRINT 15,000 LINES A MINUTE. HOW MANY LINES A SECOND IS THIS?

80. A DICTIONARY COSTS $22.14. THERE ARE 35 DICTIONARIES ON ORDER. WHAT WILL BE THE TOTAL COST OF THESE BOOKS?

81. EACH PAGE OF A PHOTO ALBUM HOLDS 6 PHOTOS. HOW MANY PHOTOS WILL BE NEEDED TO FILL 48 PAGES?



82. A PHOTOGRAPHER TOOK 972 PICTURES. THERE ARE 36 PICTURES ON A ROLL OF FILM. HOW MANY ROLLS OF FILM DID THE PHOTOGRAPHER USE?

83. THE BASKETBALL TEAM SPENT $108.48 FOR BALLS, $268 FOR WARM-UP SUITS, AND $58.79 FOR TWO BACKBOARDS. HOW MUCH CHANGE DID THE TEAM GET BACK FROM $500.00?

84. MATT'S FAMILY PAID $349 FOR A VIDEO RECORDER. BRAD'S FAMILY PAID $489 FOR A RECORDER. HOW MUCH MORE DID BRAD'S FAMILY SPEND?

85. THERE ARE 41,520 TOOTHBRUSHES TO BE PUT IN BOXES. 120 TOOTHBRUSHES ARE PACKED IN EACH BOX. HOW MANY BOXES ARE NEEDED?

86. SANDY BUYS A GLOVE FOR $17.85 AND AN UNIFORM FOR $34.29. HOW MUCH CHANGE WILL SHE RECEIVE FROM $100?

87. THE STUDENT COUNCIL HAD $1286 TO SPEND ON VIDEO EQUIPMENT. THEY SPENT $575.85 ON A VIDEO RECORDER AND $214.27 ON A TELEVISION, AND THEY BOUGHT 5 VIDEO RECORDER TAPES AT $4.89 EACH. HOW MUCH MONEY DOES THE STUDENT COUNCIL HAVE NOW?

88. SARAH ENTERED THE BIKE-A-THON. SHE RECEIVED A PLEDGE OF 25 CENTS A MILE FROM HER FATHER AND A PLEDGE OF 10 CENTS A MILE FROM HER BROTHER. SARAH RODE 22 MILES. HOW MUCH MONEY DID SHE COLLECT FOR THE BIKE-A-THON?

89. JONATHON HAD A PIECE OF ROPE 48 METERS LONG. IF HE CUT THE LONG PIECE INTO SMALLER PIECES 3 METERS LONG AND SOLD THEM AS JUMP ROPES AT 59 CENTS EACH, HOW MUCH MONEY WILL HE MAKE?

90. THE SIXTH GRADE DONATED 446 CANS OF FOOD TO THE FOOD DRIVE. THE FIFTH GRADE DONATED 259 AND THE FOURTH GRADE DONATED 379 CANS. THESE CANNED GOODS WERE SHARED EQUALLY AMONG THE 4 AREA FOOD PANTRIES. HOW MANY CANS DID EACH ONE RECEIVE?

73

Teacher Commentary    Story Problem Bank

<u>Main Objective</u>  The objective will depend on how the bank is used.

<u>When to use</u>  Whenever a selection of story problems is needed

<u>Some suggested uses</u>

The bank could be used as a source of problems of particular types--e.g., you might want to pick out multiplication problems to give the students to tell what kind of multiplication use they illustrate. Or you might be looking for some multi-step problems to use as class openers on certain days. You might be planning a mixed operations list of story problems, and could draw from the ones in the bank...

Whenever your students make up story problems, or you find additional lists of problems, add them to the bank!

<u>Answers</u>  ms = multi-step  irr = irrelevant information

| | | | |
|---|---|---|---|
| 1. $37 + irr | 2. 84 - | 3. 4 ÷ | 4. 152 + irr |
| 5. 142 - | 6. 126 x | 7. 4 ÷ | 8. 12 x |
| 9. 5250 km x | 10. 35 - irr | 11. 6 ÷ | 12. 26 x |
| 13. 30 x | 14. 9 ÷ | 15. 1104 x | 16. 11 ms + - |
| 17. $4800 x | 18. 6 x | 19. $237.66 x | 20. 179 - |
| 21. 7 ÷ | 22. $65 x irr | 23. 5 ÷ | 24. 21 x |
| 25. $5400 x | 26. 12 x | 27. 227 - | 28. 16 - |
| 29. 288 mL - | 30. 13 ÷ | 31. 490 ms x x | 32. 15 ms - ÷ |
| 33. 315 x | 34. $1457.17 ms +x, or xx+ | | 35. $67.50 x |
| 36. 11.2 ÷ | 37. 138 - | 38. 6 x | 39. $4.46 ms xx+ |
| 40. 315 x | 41. 945 x | 42. $12.78 ms xx- or -x | |
| 43. 1609 - | 44. 520 x | 45. $6.60 x | 46. $55 x |
| 47. 13,475 x | 48. $36.78 + | 49. $24.50 x | 50 $88 15 ms x+ |
| 51. 696 ms +- or -- | 52. 112 ÷ | 53. $17.67 x | 54. 81 x |
| 55. 27.6 km x | 56. 1425 m x | 57. $0.04 ms ÷-- | |
| 58. $36.00 ms -x | 59. 8 ms + ÷ | 60. $32 ms - - | 61. 126 ms +x or -x+ |
| 62. 309 ms x- | 63. 13 ÷ | 64. $134.65 + | 65. 5 ms - - or + - |
| 66. 15 ÷ | 67. $0.87 ÷ | 68. 3 ms + - | 69. $5 ms x - |
| 70. 9 x | 71. 9 ms + - or - - | | 72. 21 ÷ |
| 73. 475 x | 74. 9 ms - ÷ | 75. 24 x | 76. 60 x |
| 77. 45 ÷ | 78. 419 ÷ | 79. 250 ÷ | 80. $774 90 x |
| 81. 288 x | 82. 27 ÷ | 83. $64.73 ms + - or - - - | |
| 84. $140 | 85. 346 ÷ | 86. $47.86 ms + - or - - | |
| 87. $471.43 ms x+- or --x- | | 88. $7.70 ms xx+ or +x | |
| 89. $9.44 ms ÷x | 90. 271 ms + ÷ | | |

74



(Page 48 is blank to keep the student pages starting on odd pages.)

75



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NAME ____________

## 240 BOXES

PUT +, -, X, OR ÷ IN THE BLANK TO TELL WHAT YOU WOULD DO TO ANSWER THE QUESTION.

1. 240 BOXES, WITH 16 CANS IN EACH BOX. HOW MANY CANS ARE THERE IN ALL? ______
2. 240 BOXES, WITH 16 BOXES PUT ON EACH TRUCK. HOW MANY TRUCKS ARE NEEDED IN ALL? ______
3. 240 BOXES, WITH 16 OTHER BOXES HIDDEN. HOW MANY BOXES ARE THERE IN ALL? ______
4. 240 BOXES. THEN 16 OF THEM ARE HAULED AWAY. HOW MANY BOXES ARE THERE NOW? ______
5. 240 BOXES, ALL TO BE PUT ON 16 SHELVES. HOW MANY BOXES WILL BE ON A SHELF, IF EACH SHELF GETS THE SAME NUMBER? ______
6. 240 BOXES. THERE ARE PLACES FOR 16 OF THEM. HOW MANY BOXES DO NOT HAVE PLACES? ______
7. WRITE YOUR OWN PROBLEM ABOUT 240 BOXES.







<u>Main Objective</u> The student can select the appropriate operation for a given story problem.

<u>When to use</u> Any time after basic meanings for all the operations have been reviewed

<u>Suggested use</u>

Sometimes students look at the numbers in a story problem and look at their relative sizes for cues as to what operation to use. To show that this is <u>not</u> at all a good method, this page uses the same numbers, 240 and 16, in each problem. <u>Thinking</u> is the best policy!

If the students have had quite a bit of experience with the meanings of the operations, this could serve as an individual worksheet, with some sharing of the story problems written for #7. (You could add the story problems to "Story Problem Bank.")

If the students are not so experienced, using a transparency and basing a discussion on the problems, one by one, might be most profitable.

As a follow-up, you might have the students write story problems using the different operations but the same numbers. You could use "easy" number like 12 and 4 for some students, and larger numbers like 144 and 18 with others.

<u>Answers</u>

1. x
2. ÷
3. +
4. -
5. ÷
6. -
7. Problems will vary. Collect for later use

<u>Notes</u>

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:

GIVING REASONS 2 NAME ________________

DIRECTIONS READ THE INFORMATION BELOW AND THEN ANSWER AT LEAST THREE OF THE QUESTIONS. SHOW YOUR WORK AND GIVE YOUR REASON FOR DECIDING TO ADD, SUBTRACT, MULTIPLY, OR DIVIDE.

A CAFETERIA SERVED 18 FOOTBALL PLAYERS LUNCHES ONE WEEK, MONDAY THROUGH FRIDAY. THE PLAYERS ATE 270 HOT DOGS AND 288 HAMBURGERS. EACH SANDWICH HAS 240 CALORIES. THEY DRANK 360 DRINKS; 250 WERE MILK AND THE REST WERE JUICE DRINKS.

1. HOW MANY HOT DOGS DID THEY EAT EACH DAY, IF THEY ATE THE SAME NUMBER EACH DAY?

WORK

I ________ BECAUSE ________________________

________________________________________

2. HOW MANY FEWER HOT DOGS DID THEY EAT THAN HAMBURGERS?

WORK

I ________ BECAUSE ________________________

________________________________________

3. HOW MANY CALORIES WERE IN THE HAMBURGERS ALL TOGETHER?

WORK

I ________ BECAUSE ________________________

________________________________________

4. IF THE PLAYERS DRANK 26 JUICE DRINKS ON MONDAY, HOW MANY JUICE DRINKS DID THEY DRINK THE REST OF THE WEEK?

WORK

I ________ BECAUSE ________________________

AND I ________ BECAUSE ________________________

5. HOW MANY SANDWICHES DID EACH PLAYER EAT, IF THEY ALL ATE THE SAME NUMBER?

WORK

I ________________________________________

________________________________________

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Main Objective The student can give reasons for choices of operations with story problems.

Materials needed (Optional) Calculators

When to use Any time after "Giving Reasons 1".

Suggested use

Depending on how "Giving Reasons 1" went, you might treat this page as an individual-student activity or as a work-in-pairs one. Have someone read the story, and remark that the players had two kinds of sandwiches (some students may doubt that "hot dog" and "hamburger" mean sandwiches).

The first three problems are one-step ones; the last two are two-step ones. The directions call for the students to work three; you may choose to have them work 2 of the first three and #4, or some other selection.

Answers

1. 270 ÷ 5 = 54. I divided because the total amount was split up evenly among the 5 days.
2. 288 - 270 = 18. I subtracted because I wanted to compare the two amounts.
3. 288 x 240 = 69,120. I multiplied because I wanted to know how many in all and each hamburger had the same amount.
4. 360 - 250 = 110. 110 - 26 = 84. I subtracted the first time because part of the total was 250, and the second time because Monday took away 26 of the 110 juice drinks.
5. 270 + 288 = 558. 558 ÷ 18 = 31. I added to find the total number of sandwiches, and then divided because each player ate the same number. (Another way, (270 ÷ 18) + (288 ÷ 18) = 31, assumes that each player ate the same number of each kind of sandwich.)

Notes

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:

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NAME ____________________

## SPACE VISITORS

SPACE VISITORS FROM ANOTHER PLANET NEED SOME HELP WITH THEIR MATHEMATICAL THINKING. PUT +, -, X, OR ÷ IN THE BLANKS TO TELL THE VISITORS WHAT TO DO.

1. TO COMBINE [illegible] ZORKS AND [illegible] ZORKS: ________
2. TO FIND HOW MANY OXYGEN MASKS ARE IN [illegible] BOXES, EACH WITH [illegible] OXYGEN MASKS: ________
3. TO FIND HOW MANY BOXFULS OF FILM CARTRIDGES THERE ARE IF THERE ARE [illegible] FILM CARTRIDGES AND [illegible] FILM CARTRIDGES IN EACH BOX: ________
4. TO FIND HOW MANY OXYGEN MASKS EACH SPACE VISITOR GETS IF THEY HAVE [illegible] OXYGEN MASKS FOR [illegible] VISITORS: ________
5. TO FIND HOW MANY SNIPPLES THERE ARE AFTER [illegible] SNIPPLES ARE EATEN FROM A BOX OF [illegible] SNIPPLES: ________
6. TO FIND HOW MANY SPACE SUITS ARE NEEDED IF THERE ARE [illegible] VISITORS BUT ONLY [illegible] SPACE SUITS (EACH VISITOR NEEDS ONE SPACE SUIT): ________
7. TO FIND OUT HOW MANY WAYS THEY CAN GO FROM EARTH TO PLANET MOOK AND THEN TO PLANET ZYOR, IF THERE ARE [illegible] WAYS FROM EARTH TO MOOK AND [illegible] WAYS FROM MOOK TO ZYOR: ________
8. TO FIND OUT HOW MANY SPACE VISITORS THERE ARE IF THEY ARE SITTING IN [illegible] ROWS WITH [illegible] VISITORS IN A ROW: ________
9. MAKE UP YOUR OWN PROBLEM ABOUT THE SPACE VISITORS.





Main Objective The student can choose the correct operation for a story problem

When to use Any time after work with uses and meanings for all the operations.

Suggested use

This page is a review page, so an individual worksheet lesson could be used There are a few nonsense words in the problems (#1 zork, #5 snipple) You may want some sharing of the story problems made up in #9

Answers

1. +
2. x
3. ÷
4. -
5. -
6. -
7. x
8. x
9. Problems will vary

Notes

Additional activities (with natural settings or as extensions).

Trouble spots

Other



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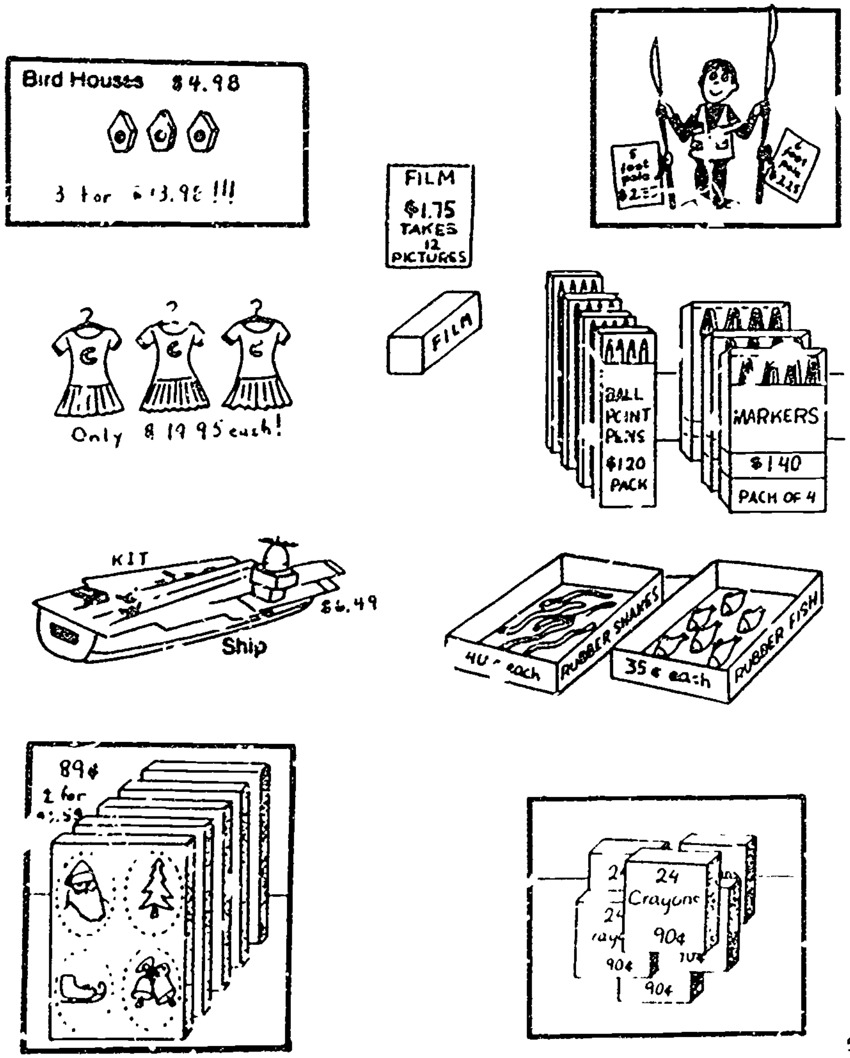
# LET'S SHOP






**Main Objective** The student can make up story problems for given operations from a fairly natural setting.

**Materials needed**

1. Cards or small pieces of paper with one of "add," "subtract," "multiply," or "divide" written on each.
2. (Best) Enough discount story ads or newspaper ads so that there is one for every two students. Otherwise, the reverse page can serve as an advertisement.
3. Calculators

**When to use** Any time after meanings for all the operations have been discussed.

**Suggested use**

Announce that today we are going to plan our shopping. Give each child a card with an operation named on it, and give pairs of children an advertisement. It is best if the pair sharing an advertisement have cards with _different_ operations. Each student is to make up a story problem using information from the advertisement and requiring the operation on his/her card. You may want the student to show a solution for his/her problem also. (Students may want to trade cards, so plan whether you want to let them or not

After each student has made up a shopping problem, have the pair make up a story problem that requires _both_ operations. Problems can be shared, but often students know what operations were on their neighbors' cards.

The story problems can be collected and used later for a worksheet of story problems (see "Shopping Problems" for an example), or added to "Story Problem Bank."

Collect the cards with the operations on them since this sort of activity is easy to repeat with different advertisements (or a transparency of newspaper advertisement). You may choose to limit the cards to multiplication and division, or have more multiplication and division cards than addition and subtraction ones.

**Notes**

Additional activities (with natural settings or as extensions):

Trouble spots:

Other: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NAME ________________

## SHOPPING PROBLEMS

FIGURE OUT ANSWERS TO THESE SHOPPING PROBLEMS THAT SOME STUDENTS MADE UP. YOU MAY USE A CALCULATOR.

1. OUR FAMILY WAS REDOING OUR BATHROOM. MY MOM WANTED 2 NEW BATH TOWELS ($5.14), NEW DRAPES ($21.97) AND 3 BOTTLES OF VANISH ($2.97). HOW MUCH WOULD EVERYTHING COST COMBINED?
2. IF YOU GET TWO JARS OF PEANUTS FOR $1.97, HOW MUCH WOULD EACH JAR COST?
3. HOW MUCH MONEY WOULD PEANUTS ($1.97) AND CHOCOLATE-COVERED CHERRIES ($.88) COST?
4. SARAH BOUGHT 2 JARS OF PEANUTS. YOU CAN BUY 2 JARS FOR $1.97. SARAH HAD $5.00. HOW MUCH CHANGE WILL SHE GET BACK?
5. IF I HAD FIVE DOLLARS, AND I BOUGHT CHOCOLATE-COVERED CHERRIES AT 88 CENTS A BOX AND A BOX OF SANDWICH BAGS AT 58 CENTS A BOX, HOW MUCH WOULD I HAVE LEFT?
6. I BOUGHT A PAIR OF SHOES FOR SHIRLEY, JENNY, KRISTA, AND KAREN. EACH PAIR COST $14.90. HOW MUCH MONEY DID I SPEND?
7. I WENT TO THE K-MART. MY FRIENDS AND I HAD $10.00. I BOUGHT 4 CANS OF PEANUTS AND THE COST WAS $1.97 EACH. HOW MUCH CHANGE WOULD WE GET BACK?
8. GINA BOUGHT 2 TOWELS FOR $5.14 AND NEW DRAPES FOR $21.97 AND 3 BOTTLES OF VANISH FOR $2.97. IRLNE BOUGHT 2 JARS OF PEANUTS FOR $3.94. HOW MUCH WILL WE HAVE TO PAY IN ALL? IF EACH ITEM WAS THE SAME PRICE, WHAT WILL EACH ITEM COST? (THERE ARE 8 ITEMS.)
9. WE BOUGHT 4 BOXES OF CHOCOLATE-COVERED CHERRIES FOR $.88 A BOX. WE ALSO BOUGHT 1 JAR OF PEANUTS FOR $1.97 AND 2 BAGS OF POPCORN FOR $1. HOW MUCH MONEY DID WE SPEND? HOW MANY ITEMS DID WE BUY?








Main Objective   The student can select the correct operation in a mixed operations list of story problems.

Materials needed   Calculators

When to use   Any time

Suggested use
This list of shopping problems could serve in lieu of a selection of the problems written during "Let's Shop." These were written by students, and in a case or two (#9 for example) may not be completely clear (did 2 bags cost a total of $1 or did each bag cost $1?). These can be used to point out the desirability, and even the necessity, of clear writing.

Answers
1. $30.08
2. 99 cents ($.985)
3. $2.85
4. $3.03
5. $3.54
6. $59.60
7. $2.12
8. $34.02; $4.2525 (meaning?)
9. $7.49, 7 items

Notes
Additional activities (with natural settings or as extensions).

Trouble sp

Other:

NAME ____________

## ADD, SUBTRACT, MULTIPLY, OR DIVIDE 2

FILL IN THE BLANK WITH ADD, SUBTRACT, MULTIPLY, OR DIVIDE. WRITE A STORY PROBLEM TO SHOW EACH STATEMENT.

1. WHEN YOU WANT TO COMPARE TWO AMOUNTS, ONE WAY IS TO ________.

   STORY PROBLEM:

2. WHEN YOU WANT TO FIND OUT HOW MANY IN ALL, YOU ________.

   STORY PROBLEM:

3. TO FIND OUT HOW MANY AMOUNTS OF THE SAME SIZE THERE ARE IN A KNOWN TOTAL, YOU ________.

   STORY PROBLEM.

4. TO FIND OUT HOW MANY THERE ARE ALL TOGETHER WHEN ALL THE AMOUNTS ARE THE SAME, YOU CAN ________.

   STORY PROBLEM:

5. WHEN AN AMOUNT IS SHARED OR PASSED EQUALLY TO SEVERAL PLACES YOU ________.

   STORY PROBLEM:







**Main Objective** The student can write story problems, given general descriptions of situations.

**When to use** Soon after the first "Add, Subtract, Multiply, or Divide?"

**Suggested use**

If your students have not had much experience at making up story problems, you may wish to have pairs of students work together. Some class discussion ("What operation would be used?") of a few "favorite" story problems is worthwhile. Collecting the problems and reading them may give you an idea of which situations need more attention.

First efforts are often not extremely imaginative (you may get lots of problems about apple). Typically, inexperienced students make these errors forgetting to write a question; writing a take-away subtraction story when a comparison subtraction is involved (#1); mixing up the two kinds of division situations, repeated subtraction (#3) and sharing equally (#5).

As a follow-up, you might cut out and post several of the story problems, (a) as challenges to others or (b) grouped by operation so that the students see a variety of situations for addition, subtraction, etc. As an extension for experienced students, suggest that they inlude extra (irrelevant) information in their problems.

**Answers**

1. subtract (The stories should involve a contrast of two distinct amounts, as opposed to the removal of a subset of a set in take-away subtraction.)
2. add
3. divide (Repeated subtraction should make sense for the story.)
4. multiply
5. divide (Sharing equally should make sense for the story.)

**Notes**

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:





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NAME ________ ______

## HEADLINES!!

MAKE UP A STORY PROBLEM FOR EACH HEADLINE!

1. $10,000 + $20,000!

2. $50 - $36.42!

3. 58 x $0.49!

4. $100 ÷ 4!

5. (3 x 35) + 45 !

88






Main Objective: The student can translate from a given mathematical expression to an appropriate story problem.

When to use: Any time, adapts to make a good "filler" or a "do while the teacher is getting something ready" activity

Suggested use

You might have groups of five write problems for the five "headlines", critique them within their group, and share with other groups. "Headlines!" might encourage a more imaginative approach. As the children gain more experience, the story problems they write should become somewhat more elaborate and creative; they often involve irrelevant information and multiple steps.

The first four problems contain a money cue. You might prefer to use others (60 x 55 miles!!) or omit any cues (as in #5).

Collect the problems for your problem bank.

Notes

Additional activities (with natural settings or as extensions).

Trouble spots

Other

89

NAME ________________________

## ASKING THE QUESTION

WRITE A QUESTION THAT THE PERSON MIGHT HAVE BEEN ANSWERING.

1. DELORES SAW SOME EARRINGS THAT COST $2.98 FOR A PAIR. SHE SAW SOME BRACELETS THAT COST $1.29 EACH.

   AMY'S WORK: WHAT QUESTION DID AMY ANSWER?

   $2.98
   + 1.29
   $4.27 Ans

   DIONNE'S WORK: QUESTION

   $2.98 $8.94
   × 3 + 1.29
   $8.94 $10.23
   Ans.

2. ONE STORE HAS BOOKS ABOUT SPORTS FOR $0.89 EACH. IT ALSO HAS GOOD BALLS FOR $6.98 EACH.

   PAT'S WORK. WHAT QUESTION DID PAT ANSWER?

   $10.00
   - 6.98
   $3.02 Ans.

   DAVE'S WORK: QUESTION:

   $.89 $1.78
   × 2 + 6.98
   $1.78 $8.76

   GREG'S WORK (ON A CALCULATOR). QUESTION?

   10.00 ÷ .89 = 11.235955
   11 books Ans





<u>Main Objective</u> The student can ask a question appropriate for a given calculation in a given context.

<u>When to use</u> Any time; calculations are already done.

<u>Suggested use</u>

Writing questions may be a new experience for the students, so you may choose to use this page as a whole-class discussion, with some time for individuals to think or or write down, their questions. Since the work is likely to be new, money contexts are used. As an example for problem #1, you might write the calculations, 2 x 1.29 = 2.58 and 2.58 + 2.98 = 5.56, and ask what question you might have had in mind (How much would 2 bracelets and a pair of earrings cost?). Some students may need assurance that it is all right to include another sentence if they can't see how to write just a question (Delores bought 2 bracelets and a pair of earrings. How much did she spend?).

Joke(?): On this page the answer is a question.

Follow-up can be fit into many situations, even outside of math time, where some information invites asking a quantitative question. "What's the Question?" is a page for which students are to write the question, given the calculation and the context

<u>Answers</u>

(Possible questions)

1. Amy's work: How much did she pay for a pair of earrings and a bracelet?
   Dionne's work: How much did she pay for 3 pairs of earrings and a bracelet?
2. Pat's work: If you buy a ball, how much change do you get from $10?
   Dave's work: How much would you pay for 2 books and a ball?
   Greg's work: How many books can you buy for $10?

<u>Notes</u>

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:



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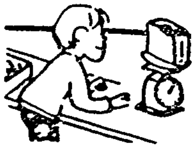

NAME ______________

## WHAT'S THE QUESTION?

> DALE'S FAMILY HAS 5 RABBITS. ONE RABBIT EATS 2 POUNDS OF FOOD EACH WEEK. THE FOOD COSTS 49 CENTS A POUND. IT TAKES 12 FEET OF CHICKEN WIRE TO MAKE A RABBIT PEN. DALE TAKES CARE OF THE RABBITS FOR 3 HOURS EVERY WEEK.

1. 5 x 2 = 10. THE ANSWER IS 10 POUNDS.

   WHAT IS THE QUESTION? ______________________________

2. 5 x 2 = 10, AND 52 x 10 = 520. THE ANSWER IS 520 POUNDS A YEAR.

   WHAT IS THE QUESTION? ______________________________

3. 520 x 0.49 = 254.8. THE ANSWER IS $254.80 A YEAR.

   WHAT IS THE QUESTION? ______________________________

4. 3 x 12 = 36. THE ANSWER IS 36 FEET OF WIRE.

   WHAT IS THE QUESTION? ______________________________

5. 52 x 3 = 156. THE ANSWER IS 156 HOURS A YEAR.

   WHAT IS THE QUESTION? ______________________________

6. 156 ÷ 24 = 6.5. THE ANSWER IS 6.5 DAYS A YEAR.

   WHAT IS THE QUESTION? ______________________________

92





**Main Objective** The student can write a question for given story problem work.

**When to use** After earlier work with writing questions for story problems (e.g., "Asking the Question") and with dealing with "Hidden Information."

**Suggested use**

Have someone read the information in the box. Write on the board, "2 x 49 = 98. The answer is 98 cents." Tell the students you wrote that because you were thinking of a question about the story. What do they think it was? (E.g., How much would the food for one cost each week?)

Since this is a more difficult version of writing questions for a story, you may want students to work in pairs or small groups. Some of the problems involve "hidden" information: how many weeks in a year (#3), how many hours in a day (#6), and in #4 the student's question will have to introduce the 3 (e.g., how much wire will it take for 3 cages?)

**Answers**

Possibilities:

1. How many pounds of food do the rabbits eat each week?
2. How many pounds of food do they eat each year?
3. How much does the food for a year cost?
4. How much chicken wire does it take for 3 cages?
5. How many hours does Dale work in a year?
6. How many 24-hour days does Dale work in a year?

**Notes**

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:

63



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NAME ____________________

## HIDDEN INFORMATION

SOMETIMES IN A STORY PROBLEM YOU HAVE TO USE INFORMATION THAT IS NOT WRITTEN DOWN. WRITE DOWN THE "HIDDEN" INFORMATION FOR EACH PROBLEM. THEN ANSWER THE QUESTION.

1. THERE ARE 60 MINUTES IN ONE HOUR. HOW MANY MINUTES ARE IN A DAY?

   HIDDEN INFORMATION: ____________________
   WORK:

2. THE COLLEGE BASKETBALL PLAYER WAS 84 INCHES TALL. HOW MANY FEET TALL WAS THE PLAYER?

   HIDDEN INFORMATION ____________________
   WORK:

3. DANA BOUGHT 2 POUNDS OF GUM DROPS, AND SHONDRA BOUGHT 10 OUNCES OF GUM DROPS. HOW MANY OUNCES OF GUM DROPS DID DANA AND SHONDRA BUY TOGETHER?

   HIDDEN INFORMATION: ____________________
   WORK.

4. A MILE IS 5280 FEET. HOW MANY YARDS IS THAT?

   HIDDEN INFORMATION: ____________________
   WORK.

5. THE PTA COLORED 25 DOZEN EGGS FOR THE EGG HUNT. HOW MANY EGGS DID THEY COLOR?

   HIDDEN INFORMATION. ____________________
   WORK





Main Objective The student can call on hidden information for use in story problems

When to use Any time, especially useful before first encounters with story problems needing hidden information

Suggested use
Since the point of this page is to alert the students to the fact that information not appearing in a story problem may be essential for its solution, you might choose to have this as a class discussion, and skip the solutions to the problems.
An excellent follow-up is to "brainstorm" for additional hidden information that might be useful in problems. Relationships among units of measurement (as in #1, #2, #3, #4) give one source - 2000 pounds in a ton, 100 centimeters in a meter and other metric relationships, 365 days in a year, . Some everyday terms (as "dozen" in #5) may not be familiar to all the students: How many in a duet, trio, pair, quartet, quintet, sextet, octet gross (a dozen dozen, or 144), score (as in "Four score"), ream (as a ream of paper, 500 sheets) Some general-information facts could also be useful Temperature of boiling or freezing water in Fahrenheit and Celsius, federal speed limit in miles per hour or kilometers per hour, normal body temperature, important dates..

Answers
1 24 hours in a day 24 x 60 = 1440, 1440 minutes in a day.
2. 12 inches in a foot 84 - 12 = 7, 7 feet tall.
3 16 ounces in a pound (2x16) + 10 = 42, 42 ounces
4 3 feet in a yard 5280 ÷ 3 = 1760, 1760 yards in a mile.
 2 in a dozen 25 x 12 = 300, 300 eggs

Notes
Additional activities (with natural settings or as extensions)

Trouble spots

Other

3589 GLASSES IN A DAY?!

NAME ______

## REASONABLE NUMBERS

PUT THE NAMES OF THINGS OR NUMBERS IN THE BLANKS. USE REASONABLE NUMBERS. THEN ANSWER THE QUESTION.

1. YOU WANT 5 ______. ONE OF THEM COSTS ______. HOW MUCH MONEY DO YOU NEED?
WORK:

2. YOU DRINK ____ GLASSES OF MILK OR ______ OR ______ EVERY DAY. HOW MANY GLASSES DO YOU DRINK IN A YEAR?
WORK:

3. TISHA AND JAMES' FATHER DRIVES A ______ AT HIS WORK. HE DRIVES ______ MILES IN A YEAR. HOW MANY MILES WOULD THAT BE EACH MONTH?
WORK:

4. YOU HAVE ______ FOR RIDES AT ______. EACH RIDE COSTS ______. HOW MANY RIDES CAN YOU TAKE?
WORK:

5. THERE ARE ____ STUDENTS IN ONE CLASSROOM. PRETEND THAT EACH STUDENT WEIGHS ____ POUNDS. HOW MANY POUNDS WOULD THE STUDENTS WEIGH ALL TOGETHER?
WORK:

96





Main Objective  The student can supply reasonable numbers into familiar story problem contexts

When to use  Any time after "Hidden Information"

Suggested use
Problem #2 uses background information not given explicitly in the problem but needed for the solution ("hidden" information): there are 365 days in a year.
Some sharing might be fun, e.g., in #1 hearing what other students wanted to buy could be interesting.
One thing to look for in the students' responses is the reasonableness of their numbers. No one (probably) drinks 20 glasses of liquid in a day, for example (#2).

Answers
(Numerical answers depend on the students' choices.)
1. Operation: multiplication.
2. Operation: multiplication, 365 × ___
3. Operation: division, ___ ÷ 12
4. Operation: division, #1 ÷ #2
5. Operation: multiplication, #1 × #2

Notes
Additional activities (with natural settings or as extensions):

Trouble spots:

Other



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NAME ______________

## USING EASY NUMBERS

READ THIS:

HOW MUCH TIME WILL IT TAKE A PERSON TO BIKE 11.25 MILES, IF THE PERSON CAN BIKE 7.5 MILES IN ONE HOUR? (ASSUME THAT THE PERSON DOES NOT GET TIRED AND CAN KEEP UP THE SAME SPEED.)

SOMETIMES A STORY PROBLEM SEEMS HARD BECAUSE IT HAS UNFAMILIAR NUMBERS IN IT. ONE WAY TO SOLVE A PROBLEM LIKE THAT IS TO

1. USE FAMILIAR NUMBERS THAT ARE EASY TO RELATE,
2. SEE WHAT YOU WOULD DO WITH THESE "EASY" NUMBERS, AND
3. DO THE SAME THING WITH THE NUMBERS IN THE STARTING PROBLEM.

EXAMPLE TRY EASY NUMBERS IN THIS COPY OF THE PROBLEM ABOVE:

HOW MUCH TIME WILL IT TAKE A PERSON TO BIKE ~~11.25~~ 12 MILES, IF THE PERSON CAN BIKE ~~7.5~~ 6 MILES IN ONE HOUR?

IT WOULD TAKE THE PERSON ____ HOURS. YOU CAN __ (+,-,X,÷) TO GET THE ANSWER.

DO THE SAME THING WITH THE NUMBERS THAT WERE MARKED OUT: ______.

THE ANSWER TO THE QUESTION IS ______________________.

TRY EASY NUMBERS AND USE A CALCULATOR TO HELP YOU FIGURE THESE OUT:

1. GASOLINE COSTS $0.989 A GALLON AT ONE STATION. HOW MUCH DO 15.6 GALLONS COST AT THAT PRICE?
2. A PACKAGE OF ONE KIND OF MEAT WEIGHS 1.79 POUNDS AND COSTS $3.01. HOW MUCH WOULD ONE POUND OF THE MEAT COST?
3. ONE KIND OF GERM IS 0.02 CENTIMETERS ACROSS. HOW MANY OF THE GERMS WOULD BE IN A LINE 2.54 CENTIMETERS LONG?
4. ONE PERSON CAN JOG 5.5 MILES EACH HOUR. IF THE PERSON DOES NOT GET TIRED, HOW MANY MILES CAN THE PERSON GO IN 2.5 HOURS?





**Main Objective** The student can substitute easier numbers in story problems.

**Materials needed** Calculators

**When to use** Any time story problems with very large numbers, decimals, or fractions will be coming up.

**Suggested use**

Story problems with less familiar numbers--large numbers, decimals, fractions, mixed numbers--sometimes cause students problems. Often this trouble is caused by the students' reliance on the immature strategies cited in the prologue. But such numbers can cause even adults to stop and think. Substituting "easy" numbers is a good approach to such problems.

The top half of the page should be used as an example. The exercises might be done by pairs, since thinking of "easy" numbers is a new task for students.

Recall "easy numbers" whenever a student is stuck on a story problem in the future, and the difficulty may be the numbers involved.

**Answers**

Example: 2, divide, 1.5, 1.5 hours

1. Possible easy numbers: $1 or $2, 15 or 16 (gallons)
   $15.43 (Students may need help interpreting the 15.4284 that a calculator will give.)
2. Possible easy numbers: 2 (pounds), 3 (dollars)
   $1.69 (Again, the calculator's 1.6815642 may puzzle students.)
3. Possible easy numbers: 1 or 2 (cm), 3 (cm)
   127 (Students may doubt this answer, since they expect division to give a smaller number. #3 wants to know how many 0.02s are in 2.54, so 127 is correct.)
4. Possible easy numbers: 5 (miles each hour), 2 or 3 (hours)
   13.75 miles

**Notes**

Additional activities (with natural settings or as extensions):

Trouble spots:

Other:



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NAME ____________________

## KEY WORDS CAN MISLEAD YOU

FINDING A KEY WORD IS NOT ENOUGH. YOU HAVE TO READ THE WHOLE PROBLEM AND THINK ABOUT IT. TELL HOW THE UNDERLINED KEY WORDS COULD MISLEAD YOU.

1. DALE SPENT $1.25. THEN DALE HAD 55 CENTS. HOW MUCH DID DALE HAVE AT THE START?

2. EACH CLASSROOM AT ONE SCHOOL HAS 32 CHILDREN. THE SCHOOL HAS 12 CLASSROOMS. HOW MANY CHILDREN ARE AT THE SCHOOL ALL TOGETHER?

3. BEN DIVIDED UP HIS PIECES OF CANDY EVENLY WITH JOSE AND CLEVELAND. EACH BOY GOT 15 PIECES OF CANDY. HOW MANY PIECES DID BEN START WITH?

4. FLO HAS 3 TIMES AS MUCH MONEY AS LACY DOES. FLO HAS 84 CENTS. HOW MUCH DOES LACY HAVE?

5. EACH PERSON AT THE PARTY GOT 2 BALLOONS, A HAT, AND 2 BOXES OF RAISINS. HOW MANY THINGS DID EACH PERSON GET?

6. MANNY'S MOTHER BOUGHT SOME THINGS AT THE GROCERY STORY. SHE GAVE THE CLERK $10 AND GOT $1.27 IN CHANGE. IN ALL, HOW MUCH DID SHE SPEND AT THE STORE?

100





Main Objective The student can reject an automatic response to key words

When to use Any time, but especially if your students seem to rely on single key words in story problems

Suggested use

Sometimes teachers point out the importance of key words in understanding what is involved in a story problem. But then students abuse this advice by skimming only for key words without any sort of thoughtful reading. Although such a practice can give success on some one-step story problems, it is much more difficult to apply to multi-step problems or problems with irrelevant information.

If your students need a warning about reliance on sole key words, this page might be useful. In each case the underlined word is often a key word for an operation which is not appropriate for the problem's solution. The page might best be handled in a whole-class discussion.

Answers

1. "Spent" might suggest subtraction, but addition is the correct operation.
2. "All together" might suggest addition of 32 and 12, but multiplication is correct.
3. "Divided" naturally suggests division, but multiplication (3 x 15) is correct.
4. "Times as much" naturally suggests multiplication, but division is correct.
5. "Each" might suggest multiplication or division, but here addition is all that is needed.
6. "In all" might suggest addition, but subtraction is the correct choice.

Notes

Additional activities (with natural settings or as extensions):

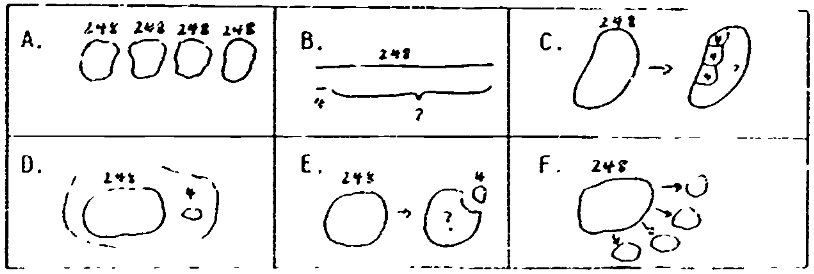
Trouble spots:

Other:

NAME ______________

## SIMPLIFIED DRAWINGS, OR DIAGRAMS

SOMETIMES IT HELPS TO THINK OF A STORY PROBLEM WITH A SIMPLIFIED DRAWING, OR DIAGRAM.




IN THE BLANK, PUT THE LETTER OF THE BEST DRAWING FOR THE PROBLEM.

___ 1. 248 + 4

___ 2. 248 - 4 (TAKE AWAY)

___ 3. 248 - 4 (COMPARISON)

___ 4. 4 x 248

___ 5. 248 ÷ 4 (HOW MANY 4S IN 248?)

___ 6. 248 ÷ 4 (SHARING)

MAKE A SIMPLIFIED DRAWING, OR DIAGRAM, FOR EACH OF THESE.

7. 3 x 186 8. 62.3 - 49.8 (COMPARISON) 9. 600 ÷ 4 (SHARING)





Main Objective The students gains more experience at linking diagrams and mathematical expressions.

When to use After you have modelled some story problems with diagrams.

Suggested use

Before showing the student page, ask the students how you could make a simple drawing for a problem like "There were 4 rats. 3 were caught. How many were still free?" Use dots for the rats to make the point that a simplified drawing does not have to look like a rat. Then ask about a simple drawing like "There were 1000 rats. 568 were caught. How many were still free?" It would clearly take too much time to draw even 1000 dots, so you can introduce the further simplification of using a labelled "blob" as in the student page. It is probably a good idea to introduce the word "diagram" for the dot sketches and for the blob drawings. ("Drawing" seems to suggest something closer to a photograph.)

This type of work is likely quite new to the students, so you choose to do the first six problems individually as a whole-class discussion. Make the sketches on the chalkboard and then ask, one at a time, about #1, #2, etc., recording the mathematical expression by the eventual choice.

The last three problems involve *making* the diagram instead of just choosing one.

Answers

1 D
2 E, where the arrow suggests something happening later
3 B, since comparison involves two separate amounts.
4 A
5 C
6 F
7-9 Drawings will vary. the best answers can be presumed to be similar to those in the earlier part of the lesson. The important feature is, does the drawing communicate the action or the relationship?

Notes

Additional activities (with natural settings or as extension)

Trouble spots.

Other

NAME ________________

## CHOOSING DIAGRAMS

IN EACH BLANK, PUT THE LETTER OF THE BEST DRAWING FOR THE PROBLEM.

____ 1. 120 + 15

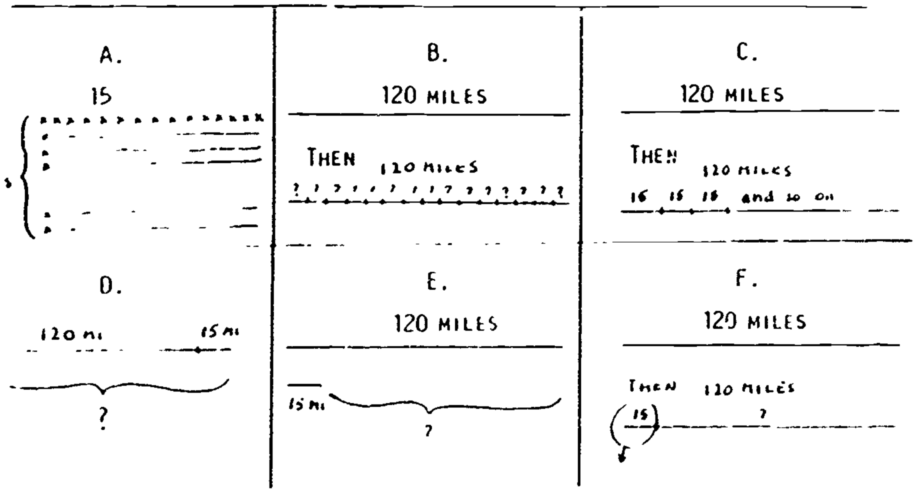
____ 2. 120 - 15 (TAKE-AWAY)

____ 3. 120 - 15 (COMPARISON)

____ 4. 120 x 15

____ 5. 120 ÷ 15 (HOW MANY 15S IN 120?)

____ 6. 120 ÷ 15 (SHARING)

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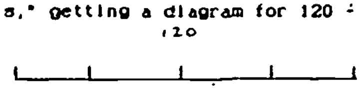
__Main Objective__ The student has more experience in linking diagrams and mathematical expressions.

__When to use__ After some work with diagrams (say, "Simplified Drawings, or Diagrams").

__Suggested use__

Before the students see the student page, do some preliminary work to help them distinguish between B and C and between E and F. Draw a segment labelled "120" on the chalkboard and ask the children what you have done as you mark off 4 equal pieces on the segment, putting a question mark on each piece ("divided 120 into 4 equal parts," getting a diagram for 120 ÷ 4).




Leave that drawing on the board, and again draw a segment for 120. This time mark off several little pieces labelled "4" and then write "and so on." What does this diagram suggest?--how many 4s are in 120, or 120 ÷ 4 also. Contrast the two diagrams, and how the two uses for division give different diagrams.

Similarly, use two drawings like E and F (with, say, 108 and 24) to show that take-away and comparison subtractions give different diagrams.

The diagrams here are based on line segments. Such diagrams are more common than "blobs," even with quantities that are not really lengths (like money). In later grades, diagrams are most frequently based on line segments.

__Answers__

1. D
2. F
3. E
4. A
5. C
6. B

__Notes__

Additional activities (with natural settings or as extensions):

Trouble spots:

105

Other:

NAME _______________

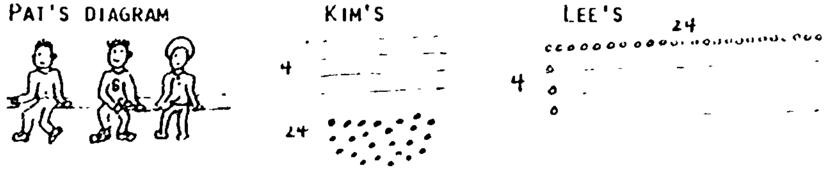
## HOW'S MY DRAWING?

> A DIAGRAM IS A <u>SIMPLIFIED</u> DRAWING.
>
> A DIAGRAM FOR A PROBLEM SHOULD GIVE THE INFORMATION NEEDED, BUT NOT TAKE TOO MUCH TIME TO DRAW.

WHAT ARE SOME GOOD AND BAD POINTS OF PAT'S, KIM'S, AND LEE'S DIAGRAMS?

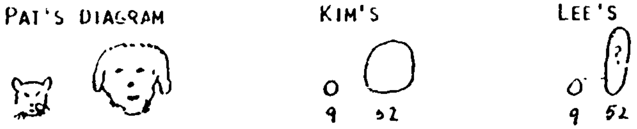
1. THERE WERE 4 ROWS OF SIXTH-GRADERS SITTING IN EVEN ROWS ON THE BLEACHERS. EACH ROW HAD 24 STUDENTS. HOW MANY SIXTH-GRADERS WERE THERE?

PAT'S DIAGRAM KIM'S LEE'S




2. JO'S CAT WEIGHS 9 POUNDS. LES' DOG WEIGHS 52 POUNDS. HOW MUCH MORE DOES THE DOG WEIGH THAN THE CAT?

PAT'S DIAGRAM KIM'S LEE'S

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Teacher Commentary How's My Drawing?

<u>Main Objective</u> The student can give some criteria for evaluating drawings for story problems

<u>When to use</u> In the early stages of work with diagrams

<u>Suggested use</u>
A whole-class discussion of a transparency of the page might be a good way to bring out the points. Have someone read the material in the box, then the first story, and then ask for opinions about Pat's diagram. Continue with the other diagrams, and with the second problem. Use the term "diagram" often.

<u>Answers</u>

1. Pat's diagram is too detailed and does not even include the numerical information. It would take too long to draw.
   Kim's diagram just shows 4 rows and 24 students, <u>not</u> that there are 24 students in every row.
   Lee's drawing is probably the best of the three. It shows 4 rows and suggests that there are 24 in each row. Lee probably did not draw 24 in every row to save time.

2. Pat's diagram is again too detailed and omits the numerical information.
   Kim's diagram is all right but there is nothing in the diagram to suggest what's going on (that the two amounts are being compared).
   Lee's drawing is again probably the best, showing the amounts and how they might be related.

<u>Notes</u>
Additional activities (with natural settings or as extensions).

Trouble spots

Other

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ERIC

NAME ______________________

## PROBLEMS --> DIAGRAMS 1

MAKE A DIAGRAM FOR EACH PROBLEM.

1. FRANCEY WAS FASTER THAN GIL. HENRY WAS SLOWER THAN GIL. FRANCEY WAS SLOWER THAN IAN. WHO CAME IN NEXT-TO-LAST?

2. A FAMILY HAD 400 MILES TO GO ON A TRIP. THEN THEY DROVE 5 HOURS AT A SPEED OF 55 MILES EACH HOUR. HOW FAR DID THEY STILL HAVE TO GO?

3. THERE WERE 77 CARS PARKED IN A PARKING LOT. THEY WERE IN 3 ROWS. TWO ROWS HAD THE SAME NUMBER OF CARS. THE OTHER ROW HAD 2 MORE CARS THAN EITHER OF THEM. HOW MANY CARS WERE IN EACH ROW?

4. RALPH HAD TWO ROLLS OF ROPE. ONE WAS 50 FEET LONG AND THE OTHER WAS 25 FEET LONG. HE NEEDED TWO PIECES OF ROPE EACH 24 FEET LONG TO MAKE A ROPE LADDER. HE NEEDED A PIECE 28 FEET LONG TO MAKE A TIRE SWING IN A TREE. DID HE HAVE ENOUGH ROPE.

108





<u>Main Objective</u> The student has experience at making a diagram for a story problem.

<u>When to use</u> After the students have seen several diagrams and evaluated some (as in "How's My Drawing?").

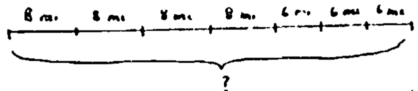
<u>Suggested use</u>

Review that diagrams are simplified drawings, that they should show the important information and how it is related, and that they should not take too long to draw.

These problems are difficult since students are reluctant to make diagrams for problems they can easily see how to solve. Accordingly, you might show an example before they start working.

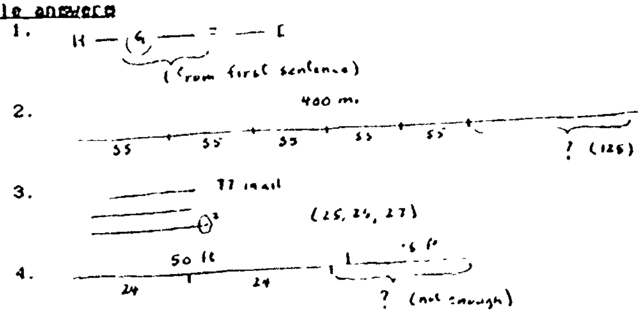
Example: (Some student) rode his/her bike 4 hours at a speed of 8 miles an hour, rested, and then rode another 3 hours at a speed of 6 miles an hour. How far did (the student) ride in all? (Remark: You may have to review what "8 miles an hour" means.)

One possible diagram:




A whole-group lesson might be your best choice here, since the students probably have not had much diagram-drawing experience.

<u>Possible answers</u>




<u>Notes</u>

Additional activities (with natural settings or as extensions):

Trouble spots: 109



Other:

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NAME ________________

## PROBLEMS --> DIAGRAMS 2

MAKE A DIAGRAM FOR EACH PROBLEM.

1. AT 8:00 IN THE MORNING, IT WAS 37 DEGREES. DURING THE DAY, IT WARMED UP BY 18 DEGREES. THAT NIGHT THE TEMPERATURE FELL 43 DEGREES. WHAT WAS THE TEMPERATURE THEN?

2. THERE WERE 54 STUDENTS SITTING ON THE BLEACHERS IN 3 ROWS. THE LONGEST ROW HAD 5 MORE STUDENTS IN IT THAN THE SHORTEST ROW DID. THE OTHER ROW HAD 1 MORE STUDENT IN IT THAN THE SHORTEST ROW DID. HOW MANY STUDENTS WERE SITTING IN EACH ROW?

3. DAVE THREW THE BALL 8 FEET FARTHER THAN EDUARDO. FLIP THREW IT 10 FEET LESS THAN EDUARDO. FLIP'S LITTLE SISTER THREW IT 20 FEET. DAVE THREW IT 106 FEET FARTHER THAN FLIP'S LITTLE SISTER. HOW MUCH FARTHER DID FLIP THROW IT THAN HIS LITTLE SISTER?





Main Objective The student has experience at making a diagram for a story problem

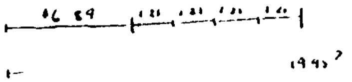
When to use After experience like "Problems --> Diagrams 1"

Suggested use

Review that diagrams are simplified drawings, that they should show the important information and how it is related, and that they should not take too long to draw.

An example may be needed, if it has been some time since the students have had diagram work. Here is an example. (Someone) needed $19.95 to buy some records on sale. (His/her) parents said it was all right if (someone) had the money. (Someone) had $6.89 saved up. (Someone) baby sat for 4 hours, and the people paid (him/her) $1.25 an hour. Did (someone) have enough? If not, how much more did (she/he) need?

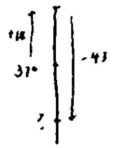
One possible diagram, developed in steps, is this:



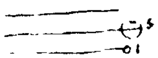

You may decide to have small groups or pairs of students work on these problems.

Possible answers

1 →




2

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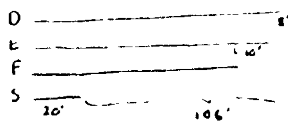



So
D 20 + 106 = 126'
E 126 - 8 = 118'
F 118 - 10 = 108'
F-S 108 - 20 = 88'

Notes

Additional activities (with natural settings or as extensions).

Trouble spots.

Other:

NAME ______________

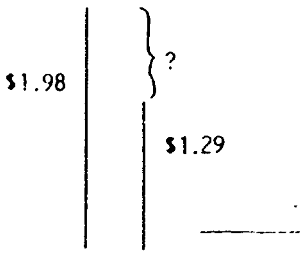
## FROM DIAGRAMS TO NUMBERS

WRITE DOWN WHAT THE DIAGRAM TELLS YOU.

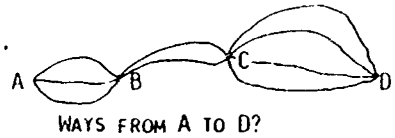
EXAMPLE


156 - 48

1.




2.

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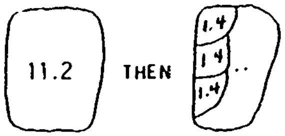


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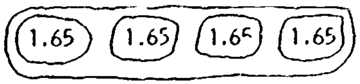


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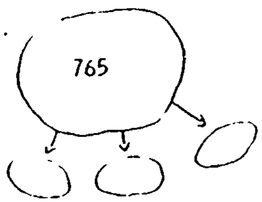
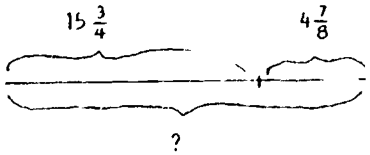



6.

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Main Objective The student can write a mathematical expression for a given diagram.

When to use After some work with diagrams and with meanings for all the operations

Suggested use
This page might best be done in pairs or small groups. If you have a chart of uses of the operations, suggest that the students might want to refer to it

Answers
1. 1 98 - 1 29 (comparison subtraction)
2 3 x 2 x 4 (cartesian product multiplication)
3. 11 2 ÷ 1 4 (repeated subtraction division)
4 4 x 1 65 (repeated addition multiplication)
5 765 ÷ 3 (sharing equally division)
6. 15 3/4 + 4 7/8

Notes
Additional activities (with natural settings or as extensions):

Trouble spots

Other.

113

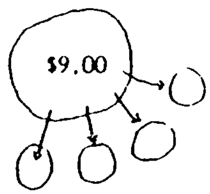
NAME ____________

## STORY PROBLEMS FROM DIAGRAMS

WRITE A STORY PROBLEM THAT THE DIAGRAM MAKES YOU THINK OF.

1. ________426 MILES__________

__142 MILES

2. 


3. | 56 | 56 | 56 | 56 | 56 | 56 |





Main Objective: The student can make up a story problem that fits a given diagram

When to use: Any time after some work on diagrams and after meanings for operations have been covered

Suggested use

Writing a story problem for a diagram is fairly open-ended. Units (miles or dollars) are given in the first two problems to suggest something of a context for the students. If you are maintaining a chart of uses with diagrams, you might review that before the page.

If some students really seem stuck, you might quietly suggest a context to them. #1--a hiking trip or distances between towns or a story about giants. #2--sharing money or buying 4 of something or hiding money. #3--buying 7 of something or going on a trip or taking 56 cm steps.

Set a tone for friendly critiquing and share some of the problems, to see whether there is a "fit" between diagram and story. This sort of follow-up gives further practice at translations between diagrams and story problems.

Collect the problems to add to your problem bank.

Intended interpretations

The diagram is intended to suggest:

1. a comparison subtraction (some students may think of a multiplication comparison [how many times as far as] even though that type of comparison is not used in this supplement)
2. a sharing equally division
3. a repeated-addition multiplication

Notes

Additional activities (with natural settings or as extensions):

Trouble spots.

Other

# Appendix III

## Reports to the Grade 8 Teachers

Cover letter
Commented Item Report



ERIC Full Text Provided by ERIC

December 1986

To: (Grade 8 Teachers involved)

From: Larry Sowder

Re: Item-by-item report for the Word Problem Thinking Test

You will recall that earlier work has shown that younger students frequently use "immature" strategies in solving word problems. Unfortunately, such limited "rules" as "times make bigger, division makes smaller" do give correct answers on many one-step whole-number problems. But life in junior high is tougher.

The Word Problem Thinking Test gives some information on how eighth graders might be thinking. (The interviews should give more information.) Keep in mind my biases: One of the principal reasons for mathematics in the curriculum is to enable students to function in natural settings which (many) word problems represent; hence, in a calculator age we do not need a curriculum which yields students who can compute but don't know when to use what computation.

1. There were two forms of the test, labelled A and B here. On the copies you have, * on the cover = form A, and the unmarked cover = form B.

2. Fractions on the tests were written in the bar form. They're in the / form here for ease of typing.

3. Scoring was based on choice of correct operations. Hence, transcription errors or errors in translating (e.g., 60% = 60, or 2/3 = 2.3) were ignored.

4. There were 9 classes tested, a total of 255 students with a 128-127 split between forms A and B. Some of the percents do not represent percents of the total taking the form: in a few cases I collected the tests before the students had had time to work on all of the problems; these were ignored in the tabulation.

5. As you can imagine, the nine classes were quite different. Six of them seemed to be about the same caliber; they are lumped together as "Subset 1" in the report. The other three classes performed noticeably better; they make up "Subset 2."

Word Problem Thinking Test
Item-by-Item Report

(Problems 1 enable a contrast of performance of decimals vs fractions, although the metric system might cloud any claims. Problems 13 involve the same contrast.)

1A. One kind of germ is 0.015 centimeters long. How many of these germs would it take to make a line 0.9 centimeters long?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 25.0 | 8.9 | 51.0 | |
| Percent reversals | 22.7 | 24.1 | 20.4 | |
| Percent choosing x | 23.4 | 29.1 | 14.3 | |
| Percent choosing - | 9.4 | 13.9 | 2.0 | |

1B. One kind of germ is 3/200 inch long. How many of these germs would it take to make a line 9/10 inch long?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct | 18.9 | 14.5 | 25.5 | |
| Percent reversals | 22.0 | 14.5 | 33.3 | |
| Percent choosing x | 22.0 | 25.0 | 17.6 | |
| Percent choosing - | 2.4 | 2.6 | 2.0 | |

Comment: The reversal error (interchanging dividend and divisor) is very common, for either fractions or decimals. It is disappointing that so many students chose multiplication. What meaning do they have for multiplication of fractions? Or are they choosing multiplication because they "know" the answers is lots, and multiplication is the tool for getting lots?

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(Problems 2 also allow a [clouded] contrast of mixed numeral and decimal performance.)

2A. A post 12 feet long is pounded into the bottom of a river. 2 1/4 feet of the post are in the ground under the river. 1 1/2 feet stick out of the water. How deep is the river at that point?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct + - | 30.5 | 26.6 | 36.7 | |
| Percent correct - - | 21.1 | 8.9 | 40.8 | |
| Percent choosing only one of + - | 25.0 | 35.5 | 8.2 | |



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2B. A post 12 meters long is pounded into the bottom of a river. 2.25 meters of the post are in the ground under the river. 1.5 meters stick out of the water. How deep is the river at that point?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct + - | 22.0 | 13.2 | 35.3 | |
| Percent correct - - | 13.4 | 6.6 | 23.5 | |
| Percent choosing only one of + - | 22.8 | 28.9 | 13.7 | |

Comment: Is the B version more difficult because of the metric measurements, or the decimals (or both)? Only a few students made a drawing for this problem.
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(A contrast of "easy" fractions vs "hard" fractions was built into #3.)

3A. At one school 3/4 of all the eighth graders went to one game. Two-thirds of those who went to the game travelled by car. What part of all the eighth graders travelled by car to the game?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | 11.7 | 6.3 | 20.4 | |
| Percent choosing - | 54.7 | 62.0 | 42.9 | |
| Percent choosing div | 7.8 | 3.8 | 14.3 | |

3B. At one school 2/5 of all the eighth graders went to one game. Four-sevenths of those who went to the game travelled by car. What part of all the eighth graders travelled by car to the game?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | 4.7 | 2.6 | 7.8 | |
| Percent choosing - | 44.9 | 43.4 | 47.1 | |
| Percent choosing div | 17.3 | 15.8 | 19.6 | |

Comment: Not their finest hour. Multiplication of fractions is usually introduced in sixth grade, so they can't claim newness. Does the "what part" really suggest subtraction, or have the students been doing more + - of fractions at this time of eighth grade? Perhaps the interviews will give some more information. The "hard" fractions in B had only a little effect.
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(Problems 4 were for two things: does length <u>times</u> width overwhelm and do decimals < 1 present more difficulty than decimals > 1? Other work says yes to both.)

4A. A small computer piece is shaped like a rectangle which is 0.25 centimeters long. Its area is 0.15 square centimeters. How wide is the piece?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 7.8 | 1.3 | 18.4 | |
| Percent reversals | 7.8 | 3.8 | 14.3 | |
| Percent choosing x | 27.3 | 22.8 | 34.7 | |
| Percent choosing + | 22.7 | 34.2 | 4.1 | |
| Percent choosing - | 16.4 | 21.5 | 8.2 | |

4B. A small computer piece is shaped like a rectangle which is 2.5 centimeters long. Its area is 15 square centimeters. How wide is the piece?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 18.9 | 3.9 | 41.2 | |
| Percent reversals | 4.7 | 3.9 | 5.9 | |
| Percent choosing x | 29.9 | 28.9 | 31.4 | |
| Percent choosing + | 19.7 | 30.3 | 3.9 | |
| Percent choosing - | 5.5 | 6.6 | 3.9 | |

Comment: Problems 21 are set up similarly, with the width rather than the length given (fishing expedition).

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(Problems 5 were to see how well, if covered, the minimal work in earlier grades on Cartesian-product multiplication is being grasped. The A problem is almost certain to be familiar if Cartesian-product multiplication has come up.)

5A. One ice-cream store has 6 kinds of toppings, 18 kinds of ice cream, and 2 kinds of cones. How many different kinds of single-scoop cones with a topping could you order?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | 20.3 | 8.9 | 38.8 | |
| Percent choosing + | 18.0 | 26.6 | 4.1 | |
| Percent omitting | 10.2 | 10.1 | 10.2 | |



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5B. Eight students are running for student council president, 4 other students are running for vice-president, and 6 others are running for treasurer. In how many ways could the election turn out?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | 6.3 | 2.6 | 11.8 | |
| Percent choosing + | 28.3 | 26.3 | 31.4 | |
| Percent omitting | 25.2 | 28.9 | 19.6 | |

Comment: The difference in Subset 2's performances on the two problems is striking.
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(Problems 6, 7, & 8 focus on "non-conservation of operation," the willingness of students to change their choices of operations when only the numbers in the problem are changed. These problems involve division; Problems 14-16 involve multiplication.)

6-7-8A. Three people bought different kinds of candy in a store.

(a) Mr. Black paid $8.64 for 3 pounds of plain fudge. How much does one pound cost?

(b) Mrs. Carter paid $2.60 for 0.65 pounds of nutty fudge. How much does one pound cost?

(c) Mrs. Dean paid $4.32 for 3/4 pound of fancy fudge. How much does one pound cost?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | (a) 89.1 | 82.2 | 100.0 | |
| (Figures include | (b) 64.1 | 64.6 | 63.2 | |
| reversals.) | (c) 53.5 | 51.3 | 57.2 | |

6-7-8B. Three people kept track of how much gas they used.
(a) Mr. Black's car went 240 miles on 15 gallons of gas. How many miles would the car go on one gallon of gas?
(b) Mr. Cortez' boat went 12 miles on 0.6 gallons of gas. How many miles would the boat go on one gallon of gas?
(c) Miss Dean's moped went 48 miles on 3/8 gallon of gas. How many miles would the moped go on one gallon of gas?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | (a) 77.9 | 68.4 | 92.1 | |
| (Figures include | (b) 58.3 | 52.6 | 64.7 | |
| reversals.) | (c) 37.3 | 28.0 | 51.0 | |
| Percent of all students doing all three probs and showing non-con | 34.8 | 33.3 | 36.8 | |

Comment: Two surprises in the A problems--the decrease from (a) to (b) is usually about 40% but Subset 1 kept that from happening, and the "weaker" performance of the "stronger" Subset 2. Stronger problem solvers do usually exhibit greater flexibility, which might work against them here.

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(Another decimal vs fraction contrast, and an intent to see whether distance = rate <u>times</u> time dominates.)

9A. An explorer travelled through 1 1/8 miles of forest in 3/4 hour. What was the explorer's average speed (in miles per hour) through the forest?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 28.3 | 17.9 | 44.9 | |
| Percent reversals | 3.1 | 3.8 | 2.0 | |
| Percent choosing x | 18.9 | 17.9 | 20.4 | |
| Percent choosing + | 11.8 | 19.2 | 0 | |
| Percent omitting | 16.5 | 19.2 | 12.2 | |

9B. An explorer travelled through 1.2 miles of forest in 0.75 hour. What was the explorer's average speed (in miles per hour) through the forest?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 19.8 | 13.3 | 29.4 | |
| Percent reversals | 20.6 | 25.3 | 13.7 | |
| Percent choosing x | 27.0 | 21.3 | 35.3 | |
| Percent choosing + | 5.6 | 9.3 | 0 | |
| Percent omitting | 15.9 | 18.7 | 11.8 | |

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(10A and 10B differ on two counts--context [allowance vs baseball cards] and numeral form [symbols vs words]. 10A is a modification of a problem used by Marshall in an earlier study.)

10A. Janet spends 2/3 of her allowance on school lunches and 1/6 on other food. What part of her allowance is left?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct | 15.0 | 2.6 | 34.7 | |
| Percent only + | 20.5 | 20.5 | 20.4 | |
| Percent only - | 44.1 | 48.7 | 36.7 | |

10B. Two-thirds of Pete's baseball cards are about Padres' players and one-sixth of them are about Angels' players. What part of his cards are about players on other teams besides the Padres and the Angels?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct | 14.3 | 6.7 | 25.5 | |
| Percent only + | 26.2 | 21.3 | 33.3 | |
| Percent only - | 26.2 | 26.7 | 25.5 | |

Comment: "Left" seems to be a persuasive key word in 10A! A few students did try a solution with a pie drawing (I counted these as correct).

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(Problems 11 were included as fillers and because performance by ninth graders in an earlier study was puzzling: about 1/5 of them chose division.)

11A. Les had 20 books to read. Les has read 2 1/4 of them. How many books does Les still have to read?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (-) | 77.8 | 71.4 | 87.8 | |
| Percent reversals | 4.0 | 5.2 | 2.0 | |
| Percent choosing div | 5.6 | 6.5 | 4.1 | |

11B. Les had 20 books to read. Les has read 2 1/4 of the books. How many books does Les still have to read?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (-) | 65.6 | 52.7 | 84.3 | |
| Percent reversals | 4.0 | 5.4 | 2.0 | |
| Percent choosing div | 11.2 | 12.2 | 9.8 | |

Comment: Any ideas on why the B version was harder for Subset 1? The B version was included since someone looking at the earlier results thought that the B version would be easier. I hope to interview at least one student who chose division.

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(Problems 12 yield easily to proportions, a topic that some of the students had had but one that others may not have had. Variations of each of these problems have been used in other studies with younger students.)

12A. A parking lot can hold 24 buses. If 8 cars can be parked in the space used by 3 buses, how many cars can be parked in the parking lot?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct | 30.2 | 11.7 | 59.2 | |
| Percent choosing an incorrect x only | 26.2 | 31.2 | 18.4 | |

12B. To make 6 popcorn balls, you need 8 cups of popcorn. Pam made 24 popcorn balls. How much popcorn did she use?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct | 36.6 | 18.1 | 62.7 | |
| Percent choosing an incorrect x only | 19.5 | 25.0 | 11.8 | |

Comment: Students using a proportion were not tabulated separately.
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(Problems 13 again compare performance, fraction vs decimal. Students having fractions in Problem 1 had decimals in Problem 13 and vice versa. The students should be all warmed up by now.)

13A. How many 1/8 ounce doses of medicine are in 2/3 ounces of the medicine?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 29.4 | 26.0 | 34.7 | |
| Percent reversals | 20.6 | 13.0 | 32.7 | |
| Percent choosing x | 7.9 | 9.1 | 6.1 | |
| Percent choosing - | 20.6 | 23.4 | 16.3 | |

13B. How many 0.2 gram doses of medicine are in 0.75 grams of the medicine?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 61.0 | 43.1 | 85.3 | |
| Percent reversals | 6.5 | 9.7 | 2.0 | |
| Percent choosing x | 8.9 | 11.1 | 5.9 | |
| Percent choosing - | 8.9 | 2.5 | 3.9 | |

Comment: Comparisons with Problems 1 are interesting.
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(Problems 14, 15, & 16 bring up non-conservation again. Problems 6-8 involve division; these involve multiplication.)

14-15-16A. Some people bought gasoline that cost $0.819 for a gallon.
(a) Mrs. Brown bought 9.1 gallons for her car. How much did she pay?

(b) Mr. Cruz bought 0.39 gallon for a weed trimmer. How much did he pay?

(c) Mrs. Davis bought 2/3 gallon for a lawnmower. How much did she pay?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | (a) 64.5 | 48.7 | 89.6 | |
| | (b) 56.6 | 39.2 | 83.3 | |
| | (c) 51.6 | 31.1 | 83.3 | |

14-15-16B. Some people bought cheese that cost $2.55 for a pound.

(a) Mrs. Cruz bought 5.1 pounds. How much did she pay?

(b) Mrs. Duarte bought 0.85 pounds. How much did she pay?

(c) Mr. Egan bought 3/5 pound. How much did he pay?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | (a) 75.4 | 63.4 | 92.2 | |
| | (b) 55.7 | 42.3 | 74.5 | |
| | (c) 56.6 | 40.8 | 78.4 | |
| Percent of all students doing all three and showing non-con | 23.5 | 29.8 | 16.7 | |

Comment: Good news--better than students in the rest of the world on conservation! Bad news--Subset 1 performance on the (a) problems. Will they be intelligent consumers?

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(Problems 17 and 18 are unfair for most students. They probably have had little or no work with multiplication in scaling. And for the A versions, they may have minimal work with percent in grade 7. Scoring was very liberal with respect to translation of the 60%.)

17-18A. One kind of copy machine can reduce to 60% of the beginning size.
17A. If a building in a drawing is 24 centimeters tall, how tall is the building in a reduced copy of the drawing?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | 31.9 | 22.1 | 45.8 | |
| Percent choosing - | 21.6 | 33.8 | 4.2 | |
| Percent choosing div | 25.0 | 17.6 | 35.4 | |

Comment: "I want something smaller than 24" and the erroneous feeling that only subtraction and division can make smaller may account for these data.

18A. If a building is 18 centimeters wide in the <u>reduced</u> copy, how wide was it in the drawing at the beginning?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 11.3 | 3.0 | 22.9 | |
| Percent reversals | 3.5 | 4.5 | 2.1 | |
| Percent choosing x | 33.9 | 22.4 | 50.0 | |
| Percent choosing - | 13.4 | 22.4 | 2.1 | |
| Percent choosing + | 6.1 | 9.0 | 2.1 | |

Comment: Similarly, the realization that something greater than 18 was needed and the erroneous "times makes bigger" may have affected the choice.

17-18B. One kind of copy machine can reduce to 2/3 of the beginning size.
17B. If a building in a drawing is 24 centimeters tall, how tall is the building in a reduced copy of the drawing?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | 10.8 | 11.4 | 10.0 | |
| Percent choosing - | 27.5 | 32.9 | 20.0 | |
| Percent choosing div | 40.0 | 25.7 | 60.0 | |

Comment: As for 17A.

18B. If a building is 18 centimeters wide in the <u>reduced</u> copy, how wide was it in the drawing at the beginning

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 7.5 | 7.1 | 8.0 | |
| Percent reversals | 1.7 | 1.4 | 2.0 | |
| Percent choosing x | 38.3 | 18.6 | 66.0 | |
| Percent choosing - | 10.0 | 17.1 | 0.0 | |
| Percent choosing + | 14.2 | 12.9 | 16.0 | |

Comment: As for 17B.

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(Variations of Problem 19A have been used in other studies. It was included here as filler and because it is a two-step problem of some difficulty and includes attractive irrelevant information. 19B was included to see whether a repeated-subtraction division would be easier than the sharing division of 19A.)

19A. A carpenter has a board 100 inches long and 8 inches wide. He makes 4 identical shelves and still has a piece of board 10 inches long left over. How long is each shelf?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (- div) | 16.5 | 6.0 | 31.2 | |
| Percent omitting | 20.9 | 23.9 | 16.7 | |
| % using irrel info | 41.7 | 46.3 | 35.4 | |

19B. A carpenter has a board 100 inches long and 8 inches wide. He makes some shelves that are 22 1/2 inches long, and still has a piece of board 10 inches long left over. How many shelves did he make?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (- div) | 4.2 | 1.4 | 8.2 | |
| Percent doing 100 divided by 22 1/2 | 29.2 | 15.5 | 49.0 | |
| Percent omitting | 17.5 | 26.8 | 4.1 | |
| % using irrel info | 15.0 | 22.5 | 4.1 | |

Comment: Isn't the percent using the irrelevant 8 inches in 19A phenomenal? And only about a third of those using it seemed to have area or perimeter in mind. As I recall, <u>no one</u> made a drawing for either problem, even among those with nonroutine problem solving experience. 19B wasn't as informative as I had hoped, since the single division could give the solution if the calculation were interpreted correctly (as you probably know, Dr. Silver has done some work showing that correct interpretations are not to be assumed).

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(Problems 20 involve percent, so the possible unfairness noted earlier applies here. The two problems involve numbers of different magnitudes to see how that might affect choices. Again, scoring treated abuses of 6% liberally--e.g., 6% = 0.6 was not marked down if the student indicated multiplication.)

20A. The bill for a meal at a restaurant was $2.90, plus sales tax. The sales tax is 6%. How much does the sales tax add to the bill?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | 51.7 | 41.2 | 66.7 | |
| Percent choosing + | 14.7 | 20.6 | 6.2 | |
| Percent choosing div | 15.5 | 13.2 | 18.2 | |
| Percent omitting | 8.6 | 13.2 | 2.1 | |

20B. The bills for all the meals at one restaurant added up to $12,185.80, plus sales tax. The sales tax is 6%. How much does the sales tax add to the total?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (x) | 57.8 | 43.3 | 77.6 | |
| Percent choosing + | 7.8 | 11.9 | 2.0 | |
| Percent choosing div | 12.9 | 11.9 | 14.3 | |
| Percent omitting | 12.9 | 20.9 | 2.0 | |

Comment: Why division? Possibly the sense that the sales tax will be an amount smaller than the bill, and division makes amounts smaller.

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(Problems 21 are companions to Problems 4. They enable a contrast of performance with decimals > 1 and decimals < 1, and an examination of a possible over-influence of A = lw.)

21A. A rectangular strip of cloth is 4.5 meters wide. Its area is 1.8 square meters. How long is the strip?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 6.1 | 0.0 | 14.9 | |
| Percent reversals | 11.4 | 1.5 | 25.5 | |
| Percent choosing x | 32.5 | 34.3 | 29.8 | |
| Percent choosing + | 17.5 | 28.4 | 2.1 | |
| Percent choosing - | 9.6 | 11.9 | 6.4 | |



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21B. A rectangular strip of cloth is 0.45 meter wide. Its area is 0.18 square meter. How long is the strip?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 8.0 | 4.6 | 12.5 | |
| Percent reversals | 18.6 | 10.8 | 29.2 | |
| Percent choosing x | 28.3 | 24.6 | 33.3 | |
| Percent choosing + | 19.5 | 27.7 | 8.3 | |
| Percent choosing - | 7.1 | 10.8 | 2.1 | |

Comment: The "stronger" Subset 2 students reverse a lot. Are they thinking incorrectly, or are the reversals a result of the two division forms? (Teachers have commented that the fraction form for division helps students avoid reversals.) The "times" in the area formula does seem to influence students.

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(Problems 22 allow a study of fraction vs decimal again, and like Problems 9, of whether the "times" in d = rt has undue influence. They also include attractive irrelevant information.)

22A. During part of a 80-mile trip, a hiker walked through 20 miles of hills at a speed of 0.8 miles an hour. How much time did it take him to walk through the hills?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 10.6 | 1.51 | 23.4 | |
| Percent choosing x | 24.9 | 12.1 | 42.6 | |
| Percent using irr info | 32.7 | 43.9 | 17 0 | |
| Percent omitting | 18.6 | 27.3 | 6.4 | |

22B. During part of an 80-mile trip, a hiker walked through 20 miles of hills at a speed of 4/5 miles an hour. How much time did it take him to walk through the hills?

| | All classes | Subset 1 | Subset 2 | Your class |
|---|---|---|---|---|
| Percent correct (div) | 8.0 | 4.6 | 12.8 | |
| Percent choosing x | 25.9 | 10.8 | 46.8 | |
| Percent using irr info | 38.4 | 43.1 | 31.9 | |
| Percent omitting | 19.6 | 30.8 | 4.3 | |

Comment: It looks as though students could use more exposure to problems with extra information especially when it is "attractive." It is interesting that the "stronger" Subset 2 students seem to be more influenced by the multiplication in d = rt than do the Subset 1 students.

## Appendix IV

### Reports to the Grade 6 Teachers

Cover letter
Summary
Strategies Students Use in Solving Story Problems
Report by Item

SDSU

DEPARTMENT OF MATHEMATICAL SCIENCES
COLLEGE OF SCIENCES
SAN DIEGO STATE UNIVERSITY
SAN DIEGO CA 92182

(619) 265-6191

August, 1987

Dear (class code )--

Last year you helped in the evaluation of some supplementary materials for story problems. The enclosed represent some of the results, the ones I thought you would find of greatest interest:

1. A summary of the findings.
2. An overview of the strategies that students use, with some quotes from the interviews.
3. Story problems written by your students, for 3x6 (#19) and 42÷3 (#20). Besides being of interest to you, these might serve as examples, both good and bad, when you have new students write story problems.
4. An item-by-item report for the posttest, with the percent correct for all students and the percent correct for such students from your class only. The whole group of students represents quite a range, from GATE students to less talented or motivated students, but the results may give you an idea as to how your students stacked up against some other sixth graders in San Diego.

I hope the enclosures are of as much interest to you as they are to me. If there is other information on the results that you would like, let me know. Let me mention again that all the results in reports to others are coded, so that schools, students, and teachers cannot be identified.

Thank you so much for your cooperation and assistance last year. Best wishes for another good school year.

Sincerely,

Larry Sowder

# SUMMARY

1. **Students who used the supplementary materials performed better statistically than those who did not.** ("Truth-in-research-reporting" cautions: The difference was not earth-shaking; getting a quite substantial improvement probably takes more than a year. Also, whether the better performance is due to the materials could be argued; perhaps just the additional attention--in quantity or in quality--to problem solving was the reason.) You are, of course, free to use anything from the materials you wish. No doubt <u>how</u> you use supplements is as important as what sort of supplement you use. Those of you using the new books will find that they have some of the things used in the materials--for example, guidance on choosing the operation in a story problem.

2. **The strategies students use will quite often break down when the problems become even slightly more complicated.** For example, many students seem to use a strategy in which their judgment of the size of the answer, rather than their thinking about what sort of situations the operations fit, dictates what operation they choose. This strategy is quite effective on one-step story problems with familiar-in-size whole numbers; it is quite difficult to carry out with multistep problems, or problems with large whole numbers or fractions or decimals for which the student may have little "feel." In particular, the "multiplication makes bigger, division makes smaller" idea from whole-number work can **misguide** a student when numbers less than 1 are involved.

   The thrust of the project materials was to emphasize the uses of the operations, thereby giving the students a reliable method to use in choosing the operations for story problems. To keep students from adopting, or continuing to use, the less mature strategies, one might try a combination of several things: a. Routinely have students give reasons for their choices of operations ("I used division because it wants to know how many 3.5s are in 1260" rather than "I divided because I wanted a smaller number"). b. Include more multistep problems and problems with irrelevant information in the curriculum, since such problems work against the success of some of the immature strategies. c. As a reminder that each of the operations fits certain types of situations, have students make up story problems for given computational expressions (this also serves as a diagnostic). d. Even sixth graders can profit from work with diagrams or concrete materials in connection with the uses of the operations.

# STRATEGIES STUDENTS USE IN SOLVING STORY PROBLEMS

Here is a list of the most common strategies that have been observed as students solve story problems during interviews. Since in most cases last year the interviews were with students who had shown a good improvement from the fall test, no interviewee used either of the first two strategies. These two strategies are included since they have been observed with other sixth graders. Excerpts from interviews with sixth graders illustrate the other strategies.

**Strategy 1.** Find the numbers and add. (Besides these "automatic adders" there are also occasional "automatic multipliers," etc.)

**Strategy 2.** Guess at the operation to be used. (What has been practiced recently in class often seems to have an influence on the choice.)

**Strategy 3.** Look at the numbers; they will "tell" you what to do.

Example: (S = student; I = interviewer)
I: Why did you think of divide?
S: That's usually what I think of when I see a big number and a one-digit number. I just try to divide.

**Strategy 4.** Try all the operations, +, -, x, ÷, and choose the answer that is most reasonable.

Example:
I: How did you figure it out...?
S: Well, I did all the possible ways. I added, I multiplied, I subtracted, and then I got to division, I found out that was the best way.
I: How did you know it was the best way?
S: Because some of them came out too much or not much.
I: You went ahead and figured out the answers completely?
S: Yes.

Another example:
I: Now what made you think of this (subtraction)?
S: Well, nothing else would work. Adding wouldn't work. Multiplying wouldn't work. So, and dividing wouldn't work, so right away that only left one thing....(It is not clear that S actually performed the rejected calculations. Our most striking example has been an Illinois seventh grader <u>in a gifted program</u> who used this strategy, carrying out the calculations and then making her choice.)

**Strategy 5.** Look for isolated "key words" to tell what operation to use. (As you know, the spirit of "key words"--think about the situation--is all right, but students abuse the advice by looking only for the key words.)

Example: [Correct operation: multiplication, 12 x 36]
S: ...so you plus 'em up. [12 + 36]
I: How do you know?
S: 'In all.'



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**Strategy 6.** Decide whether the answer should be larger or smaller than a particular given number. If larger, consider both + and x, and choose the more reasonable. If smaller, consider both - and ÷, and choose the more reasonable.

Example:
S: Well, I would think that you have to subtract, because, er, it'd either be a subtract or, um, division, and then one that sounds right would be the subtraction.

**Strategy 7.** Choose the operation which fits the story.

Example:
S: Cause they sold this much and had to throw away this much, and so, you'd probably add those together.

Example:
S: ...see, I figured that if he had a 200 inch board, and he has a piece that is 36 inches left, you just subtract that...

Example:
S: ..so you have to, either add 36 twelve times or you can times 12 times 36.

Example:
I: How come divide?
S: You need to see how many, um, how many 36 cents you can get into 6 dollars.

Example:
I: How come divide?
S: Divide 90 by 6 to find out how many times 6 goes into 90.

# REPORT BY ITEM
## All Classes, Pretest and Posttest
## (299 Students Taking Both)

(Note: The Comments here were retained from a report-to-teachers form, which included the posttest results for a given teacher's class instead of the pretest results.)

<u>ONE-STEP MULTIPLICATION ITEMS</u> (See also #16 and #17.)

1. A store has 12 big boxes of cat food. There are 36 cans of cat food in each box. How many cans of cat food are there in all?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 65.9 | 73.2 |
| Incorrect + | 25.4 | 14.4 |
| Incorrect - | 0.7 | 0.3 |
| Incorrect ÷ | 6.4 | 10.0 |
| Incorrectly checked "missing information" | 1.3 | 0.3 |
| Other ("don't know," omit,...) | 0.7 | 1.7 |

**Comment:** The students (about 1 in 7) who chose addition may be misusing such "key words" as "in all." Such misuses are the reason that the use of a key word approach is <u>not</u> recommended. The 10% of the students who chose division may be users of the "let the numbers tell you what to do" strategy or are focussing on the "each" in the problem statement.

7. A bag of snack food has 4 vitamins and weighs 228 grams. How many grams of snack food are in 6 bags?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 60.5 | 64.5 |
| Incorrect + | 2.3 | 3.3 |
| Incorrect - | 0.3 | 0.3 |
| Incorrect ÷ | 6.7 | 5.0 |
| Incorrectly checked "missing information" | 5.4 | 5.0 |
| Used irrelevant information | 21.7 | 15.4 |
| Other ("don't know," omit,...) | 3.0 | 6.4 |

**Comment:** It is disappointing that nearly one in six students used the irrelevant 4 from the problem. Are we giving them too few story problems with irrelevant information?

10. A store has notebooks in these colors: red, blue, green, yellow, brown, or black. A notebook can have paper with holes or without holes. How many different kinds of notebooks could you buy at this store?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 20.7 | 37.1 |
| Incorrect + | 22.7 | 23.1 |
| Incorrect x | 0.3 | 0.7 |
| Incorrect ÷ | 1.0 | 0.0 |
| Incorrectly checked "missing information" | 25.4 | 17.1 |
| Out of time | 0.7 | 0.7 |
| Other ("don't know," omit,...) | 29.1 | 21.4 |

**Comment:** This item involves a different use of multiplication, in counting "combinations" (the Cartesian product view). Your text may not have given any attention to this use of multiplication (some do in the form of tree diagrams), so you can judge whether this was fair for your students. It was encouraging that the experimental students, perhaps because of their use of the relevant pages from the experimental materials, did substantially better on this item than the overall figure. Sixth graders can handle this type of multiplication (some literal interpreters of Piaget might assert that only older children can). Note that more than one in six students checked "needed information is missing"; is this because no numerals appear in the problem statement?

## ONE-STEP DIVISION ITEMS

2. A parking lot has 54 cars in all. The cars are in 3 equal rows. How many cars are in each row?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 71.2 | 88.3 |
| Incorrect + | 4.7 | 2.7 |
| Incorrect - | 0.7 | 0.0 |
| Incorrect | 15.1 | 6.4 |
| Incorrectly checked "missing information" | 5.4 | 2.0 |
| Other ("don't know," omit,...) | 3.0 | 0.7 |

**Comment:** Good performance. Students during the interviews, however, did not seem able to give any generic description of why division should be used here (such as, "all the cars have been put into 3 equal amounts").



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6. A wall is 90 inches high. A worker is putting tiles in rows across the wall. Each tile is 6 inches high and costs 96 cents. How many rows of tiles will it take to cover the wall?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 39.5 | 47.5 |
| Incorrect + | 2.7 | 1.3 |
| Incorrect x | 22.4 | 15.7 |
| Incorrectly checked "missing information" | 17.1 | 20.1 |
| Used irrelevant information | 9.7 | 5.4 |
| Other ("don't know," omit,...) | 8.7 | 10.0 |

**Comment:** From the interviews it was clear that some students thought the total number of tiles to cover the whole wall was sought, and they realized that the length of the wall was needed to do that. So perhaps the performance here is somewhat better than the figures might suggest. Relatively few students "bit" on the irrelevant 96 cents. But why did almost one-sixth of the students want to do 6 x 90? Do questions about walls usually involve area, in their experience? Note that #6 involves the measurement use of division (how many 6s in 90?) rather than the partitive use, as in #2 (how many in each, if 54 are split equally among 3?)

12. One piece of paper is 12 inches wide. It has a total of 96 smiley faces in equal lines on it. There are 6 lines of faces. How many smiley faces are in each line?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 42.1 | 51.2 |
| Incorrect + | 2.0 | 1.3 |
| Incorrect - | 2.0 | 0.3 |
| Incorrect x | 13.4 | 7.7 |
| Incorrectly checked "missing information" | 5.7 | 7.4 |
| Used irrelevant information | 22.7 | 17.1 |
| Out of time | 0.3 | 1.3 |
| Other ("don't know," omit,...) | 11.7 | 13.7 |

**Comment:** Again, a disappointing number of students used the irrelevant number, 12.

15 A package of 12 stickers costs 36 cents. How many packages can you buy for $6.00?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 40.5 | 51.8 |
| Incorrect + | 3.3 | 0.7 |
| Incorrect - | 2.3 | 1.7 |
| Incorrect x | 8.7 | 4.7 |
| Incorrect ÷ | 0.3 | 0.7 |
| Incorrectly checked "missing information" | 4.0 | 2.3 |
| Used irrelevant information | 31.8 | 29.8 |
| Out of time | 1.0 | 1.7 |
| Other ("don't know," omit,...) | 8.0 | 6.7 |



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**Comment:** The extraordinary percent using the irrelevant information, 12 stickers, is a puzzle. Surely the context, buying items, is familiar to the students. The four one-step division items show no particular pattern when performances on the measurement pair (#6 and #15) and on the partitive pair (#2 and #12) are compared, except that irrelevant information <u>may</u> be quite seductive and that context is influential (as in contexts where area might be pertinent).

## <u>TWO-STEP ITEMS</u>

4. An empty truck weighs 9600 pounds. The truck has 8 cars on it. Each car weighs 3100 pounds. What is the total weight of the truck and cars?

| | Pretest percents | Posttest percents |
|---|---|---|
| Both steps (x, then +) correct | 43.1 | 60.5 |
| Incorrect + as first step | 33.8 | 18.7 |
| Only first step correct | 8.4 | 7.7 |
| Incorrectly checked "missing information" | 1.3 | 1.0 |
| For 2nd step, other (usually omitted) | 52.2 | 35.8 |

(Percents represent the two steps, so do not sum to 100.)

**Comment:** Many curricula do not give a lot of work with multistep problems, so performance here may be understandable. Students who started the problem with 9600 + 3100 stopped there. Some of these were interviewed and almost always gave a corrected solution when the "8 cars" was pointed out.

8. A carpenter has a board 200 inches long. He makes 4 identical shelves and still has a piece of board 36 inches long left over. How long is each shelf?

| | Pretest percents | Posttest percents |
|---|---|---|
| Both steps (-, then ÷) correct | 9.4 | 21.4 |
| Incorrect ÷ as first step | 36.1 | 37.8 |
| First step only correct | 8.7 | 6.0 |
| Incorrectly checked "missing information" | 9.0 | 10.0 |
| For 2nd step, other (usually omitted) | 75.9 | 63.2 |

(Percents represent the two steps, so do not sum to 100.)

**Comment:** The most common error was to choose only 200 ÷ 4, ignoring the (relevant) 36 inches left over. Although only about one in five were completely successful on the problem, that is about twice as many as were successful on the pretest in the fall. Variants of this problem have been used in other research studies and are quite difficult. From the interviews, "missing information" students wanted to know the width of the board; students who were asked to make a drawing for the problem recognized that 200 ÷ 4 was incorrect and then succeeded in solving the problem, although occasionally needing prompting.

11. To make 6 popcorn balls, you need 8 cups of popcorn. How much popcorn would you need to make 24 popcorn balls?

| | Pretest percents | Posttest percents |
|---|---|---|
| Both steps (÷ x) correct | 16.1 | 27.4 |
| Incorrect x as first step | 34.8 | 30.4 |
| Incorrect ÷ as first step | 16.1 | 11.0 |
| First step only correct | 5.3 | 4.7 |
| Incorrectly checked "missing information" | 5.4 | 4.3 |
| For second step, other (usually omitted) | 71.9 | 59.9 |

(Percents represent the two steps, so do not sum to 100.)

**Comment:** This problem was on the test just-to-see: A problem like this is usually encountered in the curriculum only when ratios and proportions are treated, even though it is do-able without proportions. (The supplementary materials do not give extra work on proportions.) The common first step of 8 x 24 was curious, and defended during interviews by "you would need more popcorn." A drawing for the problem usually helped an interviewee (there is software under development which provides drawings for similar problems).

13. A roll of film costs $1.80. You can take 12 pictures with a roll. Then it costs 35 cents to get each picture developed. What will be the cost for film and development for 12 pictures?

| | Pretest percents | Posttest percents |
|---|---|---|
| Both steps (x, then +) correct | 20.7 | 31.1 |
| Incorrect + as first step | 30.1 | 23.7 |
| First step only correct | 26.1 | 30.1 |
| For 2nd step, other (usually omitted) | 74.2 | 63.5 |

**Comment:** During interviews, students who had chosen only 12 x 35 as a solution most often gave a corrected solution when the question was emphasized.

## MULTIPLICATION INVOLVING DECIMALS (One Step)

16. One kind of cheese costs $2.60 a pound. A package of the cheese weighs 4 pounds. How much does this package of cheese cost?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 66.9 | 72.6 |
| Incorrect + | 4.7 | 3.3 |
| Incorrect - | 1.7 | 0.3 |
| Incorrect ÷ | 8.4 | 6.7 |
| Incorrectly checked "missing information" | 13.0 | 9.7 |
| Out of time | 1.7 | 1.3 |
| Other ("don't know," omit,...) | 3.7 | 6.0 |

**Comment:** This problem was included because of #17; this pair represent an exploration of a phenomenon that you may have noticed even with older students (and researchers have found fairly prevalent even among adults!): Students who correctly choose multiplication on #16 may then choose some other operation on #17, even though the context is exactly the same. All that is changed is the amount of cheese bought, from 4 pounds to 0.65 pounds. Such students realize that less than $2.60 is needed in #17, but are guided by the "multiplication makes bigger, division makes smaller" principle that holds only for whole numbers (ignoring 0 and 1, which rarely appear in story problems.) Hence, they opt for division or subtraction. Such behavior is understandable with sixth graders, whose multiplication experience has been predominantly with whole numbers. The supplementary materials included work on this topic, so #16 and #17 were included. Lessons for us are (1) not to emphasize or encourage a "multiplication makes bigger, etc." approach and (2) when multiplication with factors less than 1 is introduced and practiced, to emphasize that such multiplications can tell us how much part of an amount is. For example, 0.9 x 12.3 can tell us how much 0.9 of 12.3 is.

None of the students interviewed had chosen "needed information missing" so I have no explanation for why about a tenth of the students chose that option. It should be mentioned that 66.9% of the students were successful on #16 at the time of the pretest.

17. One kind of cheese costs $2.60 a pound. A package of the cheese weighs 0.65 pound. How much does this package of cheese cost?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 29.1 | 39.8 |
| Incorrect + | 10.4 | 7.0 |
| Incorrect - | 5.0 | 3.3 |
| Incorrect x | 0.7 | 0.3 |
| Incorrect ÷ | 18.4 | 17.1 |
| Incorrectly checked "missing information" | 15.1 | 13.4 |
| Out of time | 2.0 | 1.3 |
| Other ("don't know," omit,...) | 19.4 | 17.7 |

**Comment:** (See the comment after #16 if you skipped it.) Note the drop from 72.6% in #16 to 39.8% here, and the 10+% increase in choices of division. Students during interviews rightly rejected subtraction because "subtracting pounds from money doesn't make sense." Since many of the students may not have encountered multiplication with a decimal less than 1, the increase in "I don't know" choices is understandable. Again, the interviews gave no insight into the one-in-eight choices of "needed information is missing."



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FILLER ITEMS (to avoid a multiplication/division set)

3. Mr. Jones lost 15 pounds on a 2-month diet. Now he weighs 217 pounds. How much did he weigh before the diet?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 66.6 | 75.6 |
| Incorrect - | 13.7 | 5.7 |
| Incorrect x | 2.0 | 1.3 |
| Incorrect ÷ | 2.3 | 1.7 |
| Incorrectly checked "missing information" | 6.7 | 7.7 |
| Used irrelevant information | 5.4 | 3.3 |
| Other ("don't know," omit,...) | 3.3 | 4.7 |

**Comment:** This filler item was included to see whether what is regarded as a "subtracting" situation, losing weight, would sway the students toward a choice of subtraction.

9. A store bought some bananas in 24-pound boxes. It sold 189 pounds of them and had to throw away 27 pounds. How many pounds did the store buy?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 23.1 | 32.1 |
| Incorrect - | 8.7 | 6.7 |
| Incorrect x | 1.7 | 1.3 |
| Incorrect ÷ | 1.0 | 1.7 |
| Incorrectly checked "missing information" | 18.1 | 21.1 |
| Used irrelevant information | 34.1 | 22.1 |
| Out of time | 0.0 | 1.0 |
| Other ("don't know," omit,...) | 13.4 | 14.0 |

**Comment.** Again, this filler was included to see whether a "throw away" context would lead students to choose subtraction incorrectly. Note that the irrelevant information here is much more attractive than the irrelevant information in #3. Performance is actually "better" than the numbers indicate since some students correctly used the irrelevant 24 in answering a different question: How many <u>boxes</u> did the store buy? During an interview, one student defended the choice of "needed information is missing" by saying that the problem did not say how many boxes had been bought, thus showing a mind-set on an alternate solution. Another student said that how many pounds the store bought (which is the question of the problem) was missing.

14. There were 270 students at the first game. That was 15 more students than there were at the second game, and 18 more than at the third game. How many students were at the second game?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 29.1 | 47.8 |
| Incorrect + | 35.5 | 28.4 |
| Incorrect - | 0.3 | 0.7 |
| Incorrect ÷ | 1.7 | 0.7 |
| Incorrectly checked "missing information" | 4.0 | 6.4 |
| Used irrelevant information | 16.7 | 8.4 |
| Out of time | 0.7 | 1.7 |
| Other ("don't know," omit,...) | 9.4 | 6.0 |

**Comment:** "More than" can signal addition to students who rely solely on key-words, so this filler item was included.

5. Lee read 45 pages on Monday, 33 pages on Tuesday, and 36 pages on Wednesday. How many more pages does Lee have to read to finish the book?

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct (Needed information is missing) | 76.6 | 83.9 |
| Incorrect + | 18.4 | 9.7 |
| Incorrect - | 2.3 | 2.3 |
| Incorrect x | 0.0 | 0.7 |
| Incorrect ÷ | 0.0 | 0.7 |
| Other ("don't know," omit,...) | 2.7 | 2.7 |

**Comment:** This is the only item on the test for which "needed information is missing" was the correct choice. Even though such an option is probably a new experience for students, they performed well, with the 9.7% choosing addition perhaps answering the question that would be expected: How many pages did Lee read in all on those days?

## DIAGRAM

18. Circle the best drawing for showing 8 x 4:

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct (8 segments with top labelled 4) | 38.5 | 51.2 |
| Choice A (adjacent segments, one 8, one 4) | 12.7 | 10.7 |
| Choice B (segment marked 8 over one marked 4) | 21.1 | 16.1 |
| Choice D (segment marked 8 cut into 4 pieces) | 8.0 | 9.4 |
| Choice E (segment marked 8 with 4 arrow to left) | 8.0 | 7.4 |
| None or no answer | 11.7 | 3.3 |

**Comment:** Diagrams and drawings are so helpful in problem solving and in later mathematics that the supplementary material contains work with them. Typically text treatments give no student exercises in which the student is required to make a diagram or drawing. Choice B may have been relatively popular since it is the closest to the algorithm form; similarly, choice A is closest to the horizontal form 8 x 4.

STUDENT-WRITTEN PROBLEMS

19. Make up a story problem that would need 3 x 6 to figure it out.

| | Pretest percents | Posttest percents |
|---|---|---|
| Correct | 44.1 | 59.5 |
| Other (incorrect, omit,...) | 55.9 | 40.5 |

20. Make up a story problem that would need 42 ÷ 3 to figure it out.

| | Pretest percents | Posttest percents |
|---|---|---|
| Total correct | 35.5 | 50.4 |
| Correct (partitive) | 29.8 | 43.1 |
| Correct (measurement) | 5.4 | 7.0 |
| Other correct (e.g., area) | 0.3 | 0.3 |
| Incorrect | 64.5 | 49.5 |

**Comment:** Knowing that other researchers have found an even lower percent able to write a story problem for a computation form makes the figures for #19 and #20 a little encouraging (and up about 15% in each case from the fall pretest). Even so, that 2 out of 5 sixth grade students cannot write such a story problem makes one wonder what their understandings of the multiplication and division concepts are. It was also disappointing to find that many of the correct problems were unimaginative and stereotypic; perhaps instructions like "Make up an interesting story problem..." or "Make up a different kind of story problem..." might have provoked a little more creativity rather than a reliance on the "safe" textbook kind.