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ABSTRACT

Numerous studies indicate that performance in solving single step multiplicative word problems is influenced by both problem structure and the types of numbers involved in the problem. For example, including numbers less than one often increases the difficulty of a problem. What remains unclear is how problem structure and number type interact in influencing difficulty. This study systematically investigated these two factors with math-proficient and remedial college students. Findings indicate that it is the role played by fractions in word problems that influences difficulty rather than simply the number of fractions in the problem. Pedagogical strategies for improving students' ability to perceive and represent multiplicative word problems are discussed. (Author)

Understanding of Multiplicative Contexts Involving Fractions

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Abstract

Numerous studies indicate that performance in solving single step multiplicative word problems is influenced by both problem structure and the types of numbers involved in the problem. For example, including numbers less than one often increases the difficulty of a problem. What remains unclear is how problem structure and number type interact in influencing difficulty. This study systematically investigates these two factors with math-proficient and remedial college students. Findings indicate that it is the role played by fractions in word problems that influences difficulty rather than simply the number of fractions in the problem. Pedagogical strategies for improving students' ability to perceive and represent multiplicative word problems are discussed.

Understanding Multiplicative Contexts Involving Fractions

In the minds of many children, the concepts of 'fraction' and 'multiplication' mix in a word problem like oil and vinegar--if a fraction is involved, multiplication cannot be involved. This perception is illustrated by Greer (1987), who showed that children often identify multiplication as the appropriate operation to solve a problem when they are prevented from seeing the numbers involved, yet frequently change their minds when they are shown the numbers and see a multiplier that is less than one. The difficulty is not that children believe that the computation cannot be done, or that they are unable to perform the computation. Results from the National Assessment of Educational Progress (Carpenter, Corbitt, Kepner, Lindquist and Reyes 1980) indicate that computational abilities with fractions are far better than ability to solve word problems involving fractions. The source of the difficulty lies in child's lack of understanding of the different ways that multiplication can be embodied in a word problem.

To date, most comprehensive studies of arithmetic problem solving have focused on single-step addition and subtraction word problems. These studies have shown that children's ability to solve these problems is influenced by the structure of the problem (Briars and Larkin, 1984; Carpenter and Moser, 1983; Riley, Greeno, and Heller, 1983). Problem structure has also been shown to be an important factor in multiplication and division problems (Bell, Fischbein, and Greer, 1984; Bell, Swan, and Taylor, 1981; Nesher, 1987). Further, the selection of an operation for solving multiplicative problems is also influenced by the types of numbers (whole numbers versus fractions or decimals) in the problem (Bell, Fischbein, and Greer, 1984; Bell, Swan and Taylor, 1981; Carpenter, Corbitt, Kepner, Lindquist, and Reyes, 1981;

Fischbein, Deri, Nello, and Marino, 1985; Greer, 1987; Hardiman, 1985). Children have been observed to choose different operations for identically structured problems that differ only in the types of numbers involved, even when the problems are juxtaposed and attention is drawn to their similarity (Ekenstam and Greger, 1983).

This raises two related questions: "What influences a problem solver's perception of what operation is needed to solve multiplicative problems?" and "What meanings do problem solvers attach to operations?" Sowder (1986) has suggested that rather than having cohesive concepts for operations, middle school students of average ability may merely possess collections of immature strategies. Some of these strategies, such as looking for key words or deciding whether the answer should be larger or smaller than the given numbers, may lead to a correct interpretation in a majority of cases. However, many difficulties may result from inappropriate generalizations of strategies that are useful in limited situations (Bell, Fischbein, and Greer; 1984). Clearly, such findings suggest the need to develop a more complete picture of how and why students incorrectly interpret multiplicative word problems. Hence, the goal of this study is to investigate students' conceptions of multiplication in simple word problems, and determine when and why these conceptions fail.

The Nature of Multiplicative Relations

Multiplicative relationships, by definition, tend to be much more complex than additive relationships. In additive situations, the quantities involved must be like quantities before it makes sense to add or subtract them. The result is always the same kind of quantity. In contrast, multiplicative situations always involve different kinds of quantities. Even if it is the case that the two quantities being multiplied are alike, which is unusual, the resultant quantity always differs in kind from either of the two multiplied numbers. For example, (length) x (length) = (area), (amount) x (unit price) = (total price), (# of marbles Mary has) x (# of marbles Bill has for each marble that Mary has) = (# of marbles Bill has).

The kinds of quantities used in multiplicative problems fall into two different categories: extensive and intensive. Extensive quantities denote a set of objects or measurements, such as "number of marbles" or "number of miles"; intensive quantities denote a map between two extensive quantities, such as "price per quantity" (Shalin and Bee, 1985). Some systems of classification include a third category consisting of a dimensionless multiplicative factor (Nesher, 1987; Shalin and Bee, 1985). However, following the approach taken by Schwartz (see Kaput, 1985), we find it more useful to consider such quantities as "intensive quantities in disguise." To consider the number "3" in the statement "Bill had 3 times as many marbles as Mary" a dimensionless factor is incomplete and possibly misleading. The quantity expressed is intensive, namely "Bill had 3 marbles for each of the marbles Mary had."

Extensive and intensive quantities can be combined in three distinct ways: $E_1 \times I = E_2$, $E_1 \times E_2 = E_3$ and $I_1 \times I_2 = I_3$. To Schwartz (1976, 1981), these three combinations define three categories of word problems: 1) E x I problems, many of which can be thought of as repeated addition, are the most common in school mathematics textbooks, 2) E x E problems, often referred to as Cartesian multiplication problems, are used to determine quantities such as number of combinations (see the third problem in Table 1 for an example) and area, and 3) I x I problems are commonly used in science to convert units or create new intensive quantities.

To other researchers (Nesher, 1987; Shalin and Bee, 1985; Usiskin and Bell, 1983), the primary distinctions among problems are based on textual considerations. They define two different types of multiplication by an intensive quantity, which we present here in Nesher's (1977) terms. "Mapping rule" problems involve an explicitly stated mapping, or intensive relationship, between two extensives (for example, number of gallons x dollars per gallon = number of dollars). "Multiplicative compare" problems relate the number of objects of a start set to the number of objects of the referent set via a scalar, or "intensive in disguise," quantity (for example, number of gallons bought x 2 gallons used for every 3 gallons bought = number of gallons left). The third category used by these researchers is Cartesian multiplication (or $E_1 \times E_2 = E_3$ problems).

Four Types of Multiplicative Situations

We would argue that both the type of quantity (e.g. extensive vs intensive) and the textual aspects (e.g. whether the intensive quantity is explicitly stated or stated as a scalar factor) in a problem should be considered in devising a problem classification scheme, since both have been shown to influence problem solving. Consideration of these two factors yields four basic types of multiplicative word problems: Compute, Compare, Combine, and Convert. Table 1 provides examples of these four types of problems which we discuss further below.

Problems classified as Compute are of the $E_1 \times I = E_2$ type and essentially correspond to Nesher's "mapping rule" problems. In Compute problems, one must determine a total number of units of type E_2, given a number of units of type E_1 and an explicitly stated intensive quantity, I, relating E_1 and E_2. Common examples of explicitly stated intensive quantities relating E_1 and E_2 include unit price and speed.

Compute problems can be further understood by drawing a distinction between the roles played by the two multiplied numbers. By the role played by the two numbers we mean which quantity in a multiplication problem acts as the operator, or multiplier, and which acts as the operand, or multiplicand. In Compute problems, E_1 serves the role of multiplier since it "operates" on the intensive quantity. For example, to compute the total price of 3 loaves of bread costing 69 cents per loaf, one takes the unit price, I, and multiplies by the number of units, E_1; therefore E_1 is the multiplier and I is the multiplicand. The easiest Compute problems are those where E_1 is a whole number, because then the intensive quantity, or multiplicand, can be repeatedly added to obtain the answer. We refer to the strategy of solving problems with repeated addition as the Count strategy, since it is derived from arithmetic counting. The Count strategy is not applicable for Compute problems when E_1 is a fractional quantity.

Our Compare category corresponds to Nesher's "multiplicative compare" category, in that Compare problems involve a size comparison between a start set, E_1, and a referent set, E_2. However, we take the scalar size factor relating E_1 and E_2 to be an implicitly defined intensive quantity. Compare problems can involve independent start and referent sets, like "Bill's marbles" and "Mary's marbles," or a single set that somehow changes over time, like "the number of marbles Mary started with" and "the number of marbles Mary lost."

Compare problems differ from Compute problems in which quantity takes on the role of multiplier and which takes on the role of multiplicand. In Compare problems, it is the intensive quantity that operates on, or multiplies E_1. To say "Bill has 3 times as many marbles as Mary" implies you take three sets of the number of marbles possessed by Mary. So the intensive quantity, "Bill has 3 marbles for each of Mary's marbles," is the multiplier, while the extensive quantity, "Mary has 20 marbles," is the multiplicand. This means that Compare problems in which the intensive quantity is a whole number can be solved using the Count strategy.

Our Combine category is the commonly used category of Cartesian multiplication. Two extensive quantities, whose units may or may not be alike, combine to give a third extensive quantity having new units. This class of problems includes problems in which an area is computed from two lengths, and "number of combinations" problems, such as the example given in Table 1. Finally, Convert problems are I x I problems, where the goal is generally, but not necessarily, to convert units or express a new relationship. For neither Combine nor Convert problems can we say that one of the quantities is obviously the multiplier and the other the multiplicand; the distinctions are not quite so applicable to these two categories.

The descriptions of these four categories make it clear that certain problems are solvable using the concrete additive Count strategy, while others are not, the key feature being whether the multiplier is a whole number. This suggests that the presence of a fraction in a multiplication word problem may not necessarily lead to difficulties in setting it up. Indeed, two identically structured word problems may not be perceived as requiring the same operation for solution if the role of operator is played by a whole number in one of the problems and by a fraction in the other (Bell, Swan, and Taylor, 1981; Greer, 1986, 1987). Greer (1987; Greer and Mohan, 1986) refers to this inability to perceive that multiplication is appropriate in both cases as "nonconservation." Nonconservation is not overcome even when problems with similar contexts, but different types of numbers, are juxtaposed and attention is drawn to the similarity of the situations (Ekenstam and Gregor, 1983). To the person who perceives of multiplication only as repeated addition, the solution strategies involved when the multiplier is a whole number versus when it is a fraction could not possibly be seen as equivalent. In Experiment 1, we offer evidence that students develop alternate incorrect understandings for multiplication problems not solvable by a Count strategy.

Experiment 1

In Experiment 1, we sought to demonstrate that the difficulty of single-step arithmetic problems is influenced by the role played by fractions, and not by their mere presence. This hypothesis led to two predictions: 1) the presence of fractions would have little influence on the level of performance for addition and subtraction problems, but 2) would have considerable influence on the level of difficulty for multiplication/division problems. Additive situations require that the two quantities in the problem be alike, implying that the presence of fractions could influence performance only if the two given elements in the problem play quite different roles. Addends do not usually play drastically different roles, but it is reasonable to expect that filling the roles of minuend and subtrahend with fractions would have a minor influence on performance. In contrast, multiplicative situations require that the quantities in the problem not be alike, making it more likely that the presence of fractions will influence performance. Specifically, multiplication problems with fractional multipliers are likely to be more difficult than those with whole number multipliers because in the former case the Count strategy is not applicable but in the latter case it is.

Accordingly, subjects were asked to set up for solution a variety of arithmetic word problems containing 0, 1, or 2 fractions. This task allowed us to determine whether: 1) the correct operation was chosen, 2) the two numbers and the operation were in the appropriate order with respect to each other (e.g., 8}2 vs 2}8), and 3) an alternative correct method was used to solve the problem. Further, this task allowed us to determine possible ways in which problems could be misunderstood. If the critical determiner of difficulty is merely the number of fractions in a problem, then performance should be adversely affected simply by increasing the number of fractions in a problem, and all single fraction problems should be of equivalent difficulty. However, if it is the role played by a fraction that is critical in determining problem difficulty, there should be differences performance between the single fraction cases: single fraction problems in which the fraction plays the role of operator should be harder than single fraction problems in which the fraction plays the role of operand.

A secondary goal was to study the pervasiveness of Greer's nonconservation phenomenon. More specifically, is inability to perceive that the same operation can be applied to solve two problems having the same internal structure but different types of numbers limited to less mathematically sophisticated students? To investigate this question, we considered students from two populations who were likely to have had distinctly different experiences in mathematics: college students enrolled in introductory physics courses for engineers, and college students enrolled in remedial mathematics courses.

Method

Subjects

Forty-five students enrolled in courses at the University of Massachusetts participated in this set of studies. Fifteen subjects were enrolled in a pre-algebra remedial course; this group will be called the

Remedial group. The remaining 30 subjects were enrolled in the introductory physics course for engineers, a course which requires some mathematical sophistication. These 30 students were divided into two groups of 15 subjects each; subjects in the Good group made not more than two errors in the 32 items solved in the experimental task (explained below), and further, those two errors could not be on problems solvable by applying the same operation; the remaining 15 engineering students comprised the Average group. The 15 subjects in the Good group each made an average of .5 errors, while the 15 subjects in the Average group each made an average of 3.7 errors. The 15 subjects in the Remedial group made 6.3 errors per person. All subjects were paid $6.00 for participating in a session lasting approximately one hour during which Experiments 1 and 2 were administered.

Items

The task for Experiment 1 consisted of 32 one-step arithmetic problems which subjects were to set up for solution. Eight problems could be solved by adding the two numbers in the problem, 8 by subtracting the two numbers in the problems, 8 by multiplying the numbers and 8 by dividing the numbers. Within each set of 8 problems, there were two problems with two whole numbers (denoted by W-W), two problems in which a fraction preceded a whole number (W-F), two problems in which a whole number preceded a fraction (F-W), and two problems with two fractions (F-F). Note that in W-F problems, the whole number was the operand and the fraction was the operator, and vice versa for F-W problems. The problems within each pair were chosen to be somewhat different from each other. The problems and their type classification are listed in Appendix 1.

Procedure

The subjects we' ' in individual sessions. The 32 problems involved in this task were presented in random sequence in a booklet with ample space for writing and making computations. The subjects were told that:

> The attached sheets contain some arithmetic word problems. We would like you to read each problem carefully and set up the solution to the problem. You do not have to carry out the solution to get an actual answer, all you need to do is set it up.
>
> EXAMPLE:
> Harry had 5 dollars in the morning. He spent 2 dollars for lunch. How much money did he have after lunch?
>
> YOUR ANSWER SHOULD BE: 5 - 2
>
> Note that the answer we want you to give is how you would go about getting the answer, and not the answer itself. However, if it would be helpful to actually solve the problem, you may do so.

No time 't was imposed, and all subjects finished the task within 20 minutes.

Results

Several measures indicate that the members of the 3 groups differed in their approaches to the problems. The first such indicator is the difference in overall performance. By group, the mean performance was 98% correct for the Good group, 88% correct for the Average group, and 80% correct for the Remedial group. Given that the Good and Average groups differ by definition, it is not surprising that this difference is significant, F(: `8) = 34.91, p<.0001. Of greater interest is the fact that the mean performance of the Remedial group is significantly lower than that of both the Good group, F(1,28) = 211.31, P<.0001, and the Average group, F(1,28) = 16.85, p=.0003. Within the four sets of eight problems for each operation, there were both similarities and differences in group performance. We discuss these next.

Addition and Subtraction

The best performances were displayed on addition and subtraction items, as expected. Performance on the 8 addition problems for all subjects was nearly perfect (See Table 2). Only one subject responded incorrectly, by making a mistake in the numbers written down after correctly identifying the operation needed. Performance on the 8 subtraction problems was not quite as good as that on addition, $t(44) = 3.48$, $p=.0066$ (corrected for 6 tests), with 28 errors (92% correct). It appears that several subjects, particularly in the Average group, interpreted "How much more?" to mean "How many times more?," because they set up the problem using either multiplication or division. Despite this trend, there were no significant differences in performance on either addition or subtraction problems among the 3 groups.

It seems clear that nearly all college subjects have a reasonable understanding of the types of situations that require addition and subtraction. As predicted, the mere presence of fractions in a problem did not appear to adversely influence understanding. Although it is not apparent that subjects could have correctly computed each answer, they generally recognized when a situation required addition or subtraction and could correctly set up the problem fo lution.

Division

The third best understood operation was division. Mean performance on the 8 division problems, at 83% correct, was significantly lower than that on addition problems, $t(44) = 5.90$, $p<.0001$, but not significantly lower than that on subtraction problems. The mean performance of the Good group, at 98% correct, was higher than that of both the Average group, who got 80% correct, t(28) = 3.29, p=.0081 (corrected for 3 tests), and the Remedial group, who got 72% correct, t(28) = 5.22, p<.0001. The Average and Remedial groups did not differ significantly from each other.

For the types of division problems investigated here, the form of the problem statement seemed to be a more important determiner of performance than the types of numbers involved. The subjects appeared to have a reasonable understanding of division problems that could be fit into a Count scheme. However, they had difficulty with "How many times more?" questions, with most errors committed on this type of problem.

The three groups tended to make different types of errors on the "How many times more?" problems. The members of the Average group appear to have cued on the word "times," leading them to respond with multiplication. Twelve of the 17 errors they made were multiplication responses. In contrast, the errors made by the Good and the Remedial groups suggest that they interpreted "How many times more?" to mean "How much more?". Subtraction was the operation identified in the 3 incorrect responses of the Good group. and in all but one of the 28 incorrect responses of the Remedial group. The size and type of numbers may have helped promote this misinterpretation, since errors were made most often by the Remedial group when both of the numbers were fractions. Performance on F-F problems was significantly lower than that on W-W problems, t(14) = 3.57, p=.0186 (corrected for 6 tests).

For the division problems investigated in Experiment 1, the presence of fractions added complexity. However, the difficulties experienced seemed more related to the interpretation of the situation than to the specific influence of fractions. In Experiment 2, we will show that the role of the fraction can influence division.

Multiplication

Mean overall performance was lowest on the 8 multiplication problems, at 81% correct. This was significantly lower than performance on addition problems, $t(44) = 6.02$, $p<.0001$, but not significantly lower than performance on subtraction or division problems. However, this statement may be misleading, since it definitely was not the case that all subjects had difficulty with the multiplication problems. The performance means of the Good, Average and Remedial groups were 98%, 90% and 55% correct, respectively. The performance of the Remedial group was significantly lower than that of both the Good group, $t(28) = 14.92$, $p<.0001$, and the Average group, $t(28) = 8.65$, $p<.0001$. These differences indicate that members of the Remedial group approached the multiplication problems quite differently than the other two groups.

Examining the performance of the Remedial group by number of fractions in the problem indicates that these subjects see multiplication problems as belonging to at least two, and possibly three, different categories. Ceiling level performance was obtained on problems with whole number operators, i.e., W-W and F-W. In contrast, nearly floor level performance was obtained on problems with fractional operators; the mean performance was 16% correct on W-F problems and 3% correct on F-F problems. Obviously, performance on both W-W and F-W problems was significantly better than performance on W-F problems, $t(14)=10.46$, $p<.0001$ and $t(14)=10.46$, $p<.0001$, and on F-F problems, $t(14) = 29.00$, $p<.0001$ and $t(14) = 29.00$, $p<.0001$. To the Remedial students, multiplication is applicable only to problems in which the operator is a whole number (and thereby the Count strategy applies). Neither the Good, nor the Average group was immune from this misconception; of the 13 errors made on multiplication problems by these subjects, 12 were made on problems in which the fraction played the role of operator.

Problems with fractional operators were viewed as either subtraction or division situations. The choice of subtraction or division seemed to be influenced by the relative size difference between the two numbers. For example, if one number was clearly smaller than the other, as on W-F problems, then the classification tended to be division (21 of 27 errors). However, if the numbers were closer in size, as for F-F problems, then the problem was viewed as requiring subtraction (31 of the 39 errors). No form of addition was ever proposed for solving problems with fractional multipliers.

Discussion

The results of Experiment 1 indicate that ability to correctly identify the operation that should be used to solve a problem is influenced by the presence of fractions only when the numbers in the problem play different roles. The mere presence of fractions in a word problem is not the sole determiner of difficulty.

The simplistic notion that increasing the number of fractions in a problem increases the level of difficulty has been shown to be false, at least for problems that could be solved with the operations of addition and subtraction. For multiplication, this notion may appear to have some validity for Remedial subjects, since the rates of correct solution for problems with 0, 1, or 2 fractions are approximately 100%, 50%, and 0%. However, this conclusion would be misleading; problems with fractional operands are no harder for Remedial college students than problems with whole number operands. It is the type of number filling the operator role that determines difficulty. If the operator is a whole number, then a Count strategy can be used whether the operand is a whole number or a fraction. For a Remedial subject, the applicability of the Count strategy is what determines whether multiplication is appropriate.

This meant there could have been no confusion in Remedial subjects' minds over whether multiplication was appropriate when a fraction played the role of operator--multiplication was probably never even considered as an appropriate operation. Since they were not selecting an operation based on the problem's story line, they had to employ some alternative strategy, such as Sowder's (1986): "Look at the numbers; they will 'tell' you what operation to use." This strategy would yield subtraction when the numbers were approximately the same size, such as 7/8 and 1/4, and division when they were of differing sizes, such as 1500 and 1/4. In fact, the types of errors made suggest that this strategy was probably quite commonly used for fractional multiplier problems. However, in contrast to Sowder's view that students are succeeding with a collection of immature strategies that often yield a correct answer, we would argue that immature strategies are fallback strategies. Students first attempt to fit problems into their conceptual framework for one of the four operations, and consider fallback strategies when a problem fails to fit neatly into the student's conceptual framework. Multiplication problems with fractional multipliers most certainly fail to fit Remedial students' conceptual frameworks, given their limited conceptions of multiplication.

Experiment 2

It is quite probable that Remedial students' underdeveloped conceptual framework for multiplication manifests itself in ways other than failure to recognize problems that are not solvable with a Count strategy as multiplication problems. For example, Remedial students may not believe that a division problem could be solved by initially setting it up as a multiplication problem. On the other hand, possession of a well-developed conceptual framework for multiplication would seem to imply more than simply being able to immediately recognize which operation is appropriate to compute the answer to a problem; it should also imply flexibility in using different operations to appropriately describe a situation. In Experiment 2, we explored in more detail the conceptual framework for multiplication of Good and Remedial subjects with a more challenging task.

The goal of this task was to investigate how subjects structured their approaches to problems and how flexible these approaches were. Hence, after viewing a problem presented on a computer screen and mentally deciding how to set up an equation to solve it, the subject had to determine whether or not a subsequently presented equation could be used to solve the problem (without reference to the problem statement). One of four types of equations would appear: 1) an appropriate equation using a multiplication sign, 2) an appropriate equation using a division sign, 3) an inappropriate equation using a multiplication sign, or 4) an inappropriate equation using a division sign. We expected the form of the equation to have little influence on the correctness of performance of the subjects with a well-developed conceptual framework for multiplication, but considerable influence on those with an underdeveloped framework.

To investigate the differences between a well-developed and an underdeveloped conceptual framework for multiplication, Experiment 2 focused exclusively on $E_1 \times I = E_2$ multiplicative problems where E_1 was always a whole number and I was always a fraction. We included both result-unknown and multiplicand-unknown problems. In result-unknown problems, E_2 was unknown, making these problems solvable by multiplying E_1 and I. In multiplicand-unknown problems, either E_1 or I was missing, depending on which played the role of multiplicand in that problem. These problems were solvable by dividing E_2 by the known multiplier, which was either E_1 or I.

Half of the result-unknown problems were Compute problems solvable by applying the Count strategy (since the multiplier, E_1, was a whole number), while the other half were Compare problems not solvable by the Count strategy (since the multiplier, I, was a fraction). For example:

Compute

The fourth grade math book weighs 7/8 of a pound. A student carried a stack of 12 books to another classroom. How much did the stack weigh?

multiplier:	E_1	= number of books
multiplicand:	I	= weight of each book
result:	E_2	= unknown weight of stack of books

Compare

Jeffrey's mom bought 3 quarts of ice cream for the party. The guests ate 9/10 of the ice cream. How much ice cream did they eat?

multiplier:	I	= amount of ice cream eaten per amount of ice cream bought
multiplicand:	E_1	= amount of ice cream bought
result:	E_2	= unknown amount of ice cream eaten

We expected a replication of the results of Experiment 1, in that all subjects would accept the correct multiplication equation for the Compute problems, but that only Good subjects would accept the correct multiplication equation for the Compare problems. Further, the well-developed conceptual framework for multiplication possessed by the Good subjects should make them better able to distinguishing correct from incorrect equations than Remedial subjects.

The multiplicand-unknown problems were constructed in a similar manner, in that E_1 was always a whole number and I a fraction, but the number playing the role of the multiplicand was always missing. Therefore, in order to insure parallelism among all problems in terms of the number of fractions present, we demanded that the result, E_2, be: a) a fraction when E_1 was a known whole number, and b) a whole number when I was a known fraction. Hence, all problems contained one whole number and one fraction. For example:

Compute

Julia poured 5 cartons of lemonade into a pitcher. There was 3/4 of a quart of lemonade. How much lemonade did one carton contain?

multiplier:	E_1 = number of cartons
multiplicand:	I = unknown amount of lemonade per carton
result:	E_2 = total amount of lemonade

Compare

Peter used 2/3 of the cement he bought in order to build the wall. He used 16 bags of cement. How much cement did he buy?

multiplier:	I = amount of cement used per amount of cement bought
multiplicand:	E_1 = unknown number of bags of cement bought
result:	E_2 = amount of cement used

We hypothesized that all subjects would accept the correct division equation for the Compute problems, since they are solvable by a Count strategy. However, this should be the only situation in which Remedial students are likely to respond correctly. Further, a well-developed conceptual framework for multiplication may suggest to Good subjects that multiplication is a more suitable operation for initially representing multiplicand-unknown Compare problems than is division, even though division is the actual operation needed to solve the problem. In fact, they may be faster at accepting the correct multiplication equation than the correct division equation. If realized, the predicted results would suggest that beginning and remedial students should be taught to do what good students naturally learn to do; that is, to learn to recognize how the multiplicand, multiplier, and result function within a general multiplicative framework.

Method

Subjects

The subjects were the same physics and remedial mathematics students that participated in Experiment 1. The groupings of Good, Average, and Remedial were based on the results of Experiment 1.

Materials

Experiment 2 was composed of 64 one-step multiplicative word problems, each presented with an equation that could possibly be used to solve for the unknown. The subject's task was to determine whether this equation could be used to determine the unknown, and make a binary response.

Each problem could be paired with one of four possible equations: correct multiplication, correct division, incorrect multiplication and incorrect division. For example:

> It takes 2/3 of an hour to spray-paint a car. Harry spray-painted 4 cars on thursday. How long did it take Harry to paint the cars?
>
> 2/3 x 4 = ? (correct multiplication)
> ? ÷ 2/3 = 4 (correct division)
> ? x 2/3 = 4 (incorrect multiplication)
> 2/3 ÷ 4 = ? (incorrect division)

Since it was desirable for the subject to respond to each of the four equations, but not see the same problem context four times, we constructed sets of 4 equivalent word problems with different contexts. The equation was randomly paired with problem context such that within each set of four problems, each equation type appeared only once. In order to increase the pool of items, we made up four such sets of four problems each, for a total of 16 problems. There were 16 such problems for each of the following four problem types for a total of 64 problems: 1) result-unknown Compute problems, 2) multiplicand-unknown Compute problems, 3) result-unknown Compare problems, and 4) multiplicand-unknown Compare problems.

Procedure

The subjects were run individually on an IBM compatible microcomputer. Reaction times from the appearance of the equation on the screen to the subject's response were recorded. No time limit was imposed and all subjects finished this task within 45 minutes.

The subjects were told:

> The problem will appear on the screen. Read the problem carefully and decide how you would set up an equation that could be used to solve the problem. We do not want you to come up with a numerical answer for the problem.
>
> After you have set up an equation in your head, press the spacebar again. At that time the word problem will disappear from the screen and an equation will appear on the screen. The equation will always contain the actual numbers that are in the problem. However, sometimes the equation will be correct and could be used to solve the problem, and sometimes it will be wrong. Your job is to determine whether the equation could be used to solve the problem. Press true if the equation is appropriate for solving the problem or false if it is not.
>
> Note that the equation may not match the equation you have in your head, but it may be true none the less.

Two examples using whole numbers and the appropriate responses were read to the subject. There were two additional practice problems on the computer that the subject could discuss with the experimenter before beginning.

Results

Evidence that the three groups employed different approaches on this task emerged from a 3 (Groups) x 2 (Multiplier type - whole or fraction) x 2

(Unknown - result or multiplicand) x 2 (Correctness of equation) x 2 (Operation in equation - x or ÷) ANOVA of the error data, $F(2,42) = 50.58$, $p<.0001$. Overall, the Good group displayed evidence of solid understanding, performing correctly 91% of the time. As expected, they outperformed the Remedial group, who performed correctly 54% of the time, $F(1,28) = 219.82$, $p<.0001$. Perhaps less anticipated was the degree to which the Good group also outperformed the Average group, who were correct only 72% of the time, $F(1, 28) = 20.29$, $p<.0001$. We will begin our discussion of these data with the Good group, and attempt to develop some understanding of the properties of a well-developed multiplicative conceptual framework. Then we will focus on the limitations of the multiplicative conceptual frameworks of the Average and Remedial groups.

The Good Group: Characteristics of a Well Developed Multiplicative Framework

Performance. The 91% correct overall performance of the Good subjects suggests that their understanding of multiplicative relationships is influenced by problem structure. However, problem structure does not necessarily influence all subjects in the same way. An examination of the errors Good subjects made, combined with an analysis of reaction times, will help to characterize their multiplicative framework.

Two factors significantly influenced the percentage of correct judgments, as can be seen by examining Table 3. The first is whether the result or the multiplicand was unknown, $F(1,14) = 10.84$, $p=.0053$. The subjects were correct 96% of the time on result-unknown problems, but only 86% of the time on multiplicand-unknown problems. Although small, this effect is consistent throughout Good subjects' performance. To explain why the Good subjects performed less well on the multiplicand-unknown items, it is necessary to consider whether the multiplier was a whole number or a fraction.

For multiplicand-unknown problems containing whole number multipliers, a natural assumption is that performance should be relatively good, since they can be solved with a Count strategy. However, this particular set of multiplicand-unknown Compute problems challenges the common erroneous perception that "division must always be of a larger number by a smaller" (Bell et al., 1981; Hart, 1981), since in these problems a fraction should be divided by a whole number. In contrast, the multiplicand-unknown problems with fractional multipliers violate no such assumptions about the relative size of the numbers. Instead, they seem difficult because of the relationship between the quantities, which is not clearly either a quotitive or partitive division relationship. Indeed, these problems seem more readily interpretable as multiplication problems with an unknown multiplicand than as division problems with an unknown result. Therefore, they do not conform to the usual expectations of either multiplication or division.

The second major influence on Good subjects' performance was the operation used in the equation. For all item types, including those that would actually be solved using multiplication and those actually solved using division, correct responses were more frequent when the operation in the equation was multiplication, $F(1,14) = 23.49$, $p=.0003$. For equations having a multiplication sign, 96% of the responses were correct, while only 86% of the responses to equations having a division sign were correct. Clearly, the advantage of multiplication equations should be expected for problems that are actually solved using the operation of multiplication. However, the fact that there is also some advantage for problems that are actually solved by dividing the two numbers supports the notion that a multiplicative approach to non-standard division problems aids understanding. For non-standard division problems, it may be easier to recognize that the situation involves multiplication, and determine which element is missing, rather than recognize division is involved and set up the appropriate division equation.

Reaction Time. The reaction time results both support and add to our description of the Good subjects. Overall, response times to problems with whole number multipliers were faster than those to problems with fractional multipliers, $F(1,14) = 12.36$, $p=.0034$. Responses were also faster for result-unknown problems than for operand-unknown problems, $F(1,14) = 7.83$, $p=.0142$.

Considering only correct responses to correct equations for the remainder of this discussion, Table 4 shows that the expected multiplication form was much faster than the unexpected division form for result-unknown problems, $t(14) = 4.45$, $p=.0006$. Note also that the second fastest responses for result-unknown problems occurred for rejection of the incorrect division equation, that is, the multiplication equation with the wrong sign. Obviously, students recognize these as multiplication problems, but can manipulate the equations when necessary.

For multiplicand-unknown items, the speed of the response was related to the type of multiplier. When the multiplier was a whole number, response to the division equation was faster than that to the multiplication equation, $t(14) = 2.53$, $p=.0241$, implying that subjects had initially prepared a division equation. Interestingly, Table 3 indicates that correct responses were made slightly more often for the multiplication equations. Thus, although a multiplicative approach does not appear to be invoked initially in this case, it is more successful when it is invoked.

When the multiplier was a fraction, the trend for response times was in the opposite direction. Not only were responses to multiplication equations somewhat faster (but not significantly) than responses to division equations, but they were also correct more often. However, in contrast to all other types of problems, the lengthiness of the response times suggests that subjects did not mentally propose either "A x ? = B" or "B } A = ?" for comparison. Instead, it is likely that subjects remembered the essential components of the problem's story line and verified that the equation on the screen could be used to solve the problem. The fact that they were able to verify correct multiplication equations more accurately than correct division equations suggests that the operation actually used to solve the problem is not always the best operation for representing a problem. In this case, a more general multiplicative approach proved superior.

The Remedial Group: Characteristics of an Underdeveloped Multiplicative Framework

The performance of the Remedial subjects suggests that they perceived the problems quite differently from the Good subjects. First, the overall rate of response, at 54% correct, was essentially random. Second, the one main effect which was significant was one that had no influence on the performance of the Good subjects, namely whether the multiplier was a whole number or a fraction, $F(1,14) = 23.02$, $p=.0003$. The rate of correct responses was 64% for whole number multiplier items, versus 44% for fractional multiplier items. This result corroborates the conclusion from Experiment 1 that Remedial subjects view multiplication as appropriate only when the Count strategy can be used, i.e., when the multiplier is a whole number.

Table 3 suggests Remedial subjects' performance is limited in other ways as well. For example, although they recognized that it took more than the mere presence of a multiplication sign for an equation to be appropriate in result-unknown/whole number multiplier problems, they did not believe that any equation with a division sign could be used to find the unknown. In the result-unknown/whole number multiplier items, the 92% rate of correct responses to correct multiplication equations was comparable to that of the other two groups, implying that Remedial subjects could identify Compute problems as multiplication problems. The fact that they seemed to be responding to more than simply the presence of the multiplication sign is indicated by the above-chance performance in correctly rejecting the wrong multiplication equation (67% correct). On the other hand, the presence of a division sign seemed to signal that the equation should be rejected, regardless of whether or not it was correct. Indeed, the low acceptance of the correct division equation (42% correct) and the high rejection of the incorrect one (82% correct), indicates rigidity in representing multiplication problems.

Responses to the fractional multiplier items were close to random, indicating that the Remedial subjects had no idea how to correctly approach these problems. In fact, their only intuition here was incorrect; the one deviation from random performance was in the wrong direction. For result-unknown items, there was a low acceptance rate of the correct multiplication equation (22% correct) and a high rejection rate of the incorrect multiplication equation (73% correct), corroborating the conclusion from Experiment 1 that problems with fractional multipliers are not perceived as multiplication problems.

Average Subjects: Closer to Well-Developed or Underdeveloped Conceptual Framework?

The pattern of performance of the Average subjects indicates that despite their considerably lower level of performance, they perform more like the Good subjects than the Remedial subjects. This is suggested by the fact that there are only 2 interactions involving group when the Average group is compared to the Good group, but 8 interactions involving group when the Average group is compared to the Remedial group. In addition, the same main effects are significant for the Average group as the Good group. For example, whether the unknown was the result or the multiplicand influenced performance, $F(1, 14) = 35.74$, $p<.0001$; 82% of the responses to result-unknown items were correct versus 62% of the responses to multiplicand-unknown items. The operation of the equation also influenced Average subjects' performance, $F(1,14) = 16.45$, $p = .0012$; 78% of the responses to equations with multiplication signs were correct, versus 67% for equations with division signs. This was true for both result-unknown and multiplicand-unknown items.

Basically, the members of the Average group tended to make the same types of errors as the Good group, but made more of them. Unlike the Remedial subjects, they displayed evidence of a reasonable understanding of fractional multipliers. They were also better able than the Remedial group in recognizing that an equation was correct, even when it did not contain the expected operation. Like the Good subjects, they were more likely to correctly determine the appropriateness of a multiplication equation, even for problems that would actually be solved using division. Hence, the performance of the Average group indicates they are closer to a well-developed conceptual framework for multiplication than to an underdeveloped one.

Discussion

The results of Experiment 2 indicate that the multiplicative conceptual frameworks of Good and Remedial students differ in several ways. To begin with, the role played by a fraction in a word problem is critical to Remedial students' understanding, but unimportant to Good students understanding. When a fraction played the role of multiplier, Remedial students rejected multiplication as the appropriate operation for solving the problem.

Experiment 2 also indicates that the Remedial subjects are fairly rigid in their criteria for accepting a correct equation, requiring the presence of the expected operation sign. For problems in which they have strong expectations, there is a strong inclination to reject any equation that seems to contradict those expectations, attesting to their underdeveloped conceptual understanding of the inter relationship between multiplication and division.

The narrow range of items solved correctly by the Remedial subjects coupled with the conformity that these subjects require between a problem's surface structure and the proposed equation contrast sharply with the flexibility in understanding displayed by the Good subjects. The performance of the Good, and to some extent the Average, subjects suggests that they possess a fairly general multiplicative mental framework. Their apparent understanding of the roles (i.e., multiplier, multiplicand and result) played by the quantities in multiplicative problems affords them the flexibility to perceive that a problem can be represented using seemingly different, but mathematically equivalent equation forms.

General Discussion

The two experiments in this study provide further evidence that two factors, namely the types of quantities in a problem (extensive and intensive) and the structure of the text (Compute vs Compare), play a major role in problem comprehension. Since these two factors interact, any model of problem comprehension that fails to involve both factors would yield an incomplete picture.

We defined a preliminary vocabulary of problem types, and provided experimental evidence for two major classes of problems involving the product of an extensive quantity and an intensive quantity. The distinguishing feature of these two classes is whether the extensive quantity or the intensive quantity plays the role of the multiplier in the problem. When the role of the multiplier is played by a whole number, the Count strategy can be used to solve the problem, and all subjects demonstrated success in applying this strategy. However, when the role of multiplier is played by a fraction, the effect is to reduce the quantity multiplied, rather than to increase it, as would be the case in whole-multiplier Count problems. To recognize the similarity between these situations requires a broader understanding of the meaning of multiplication than that displayed by the Remedial subjects.

The Remedial subjects' conceptual framework for multiplication differed from that of the Good subjects in many ways, possibly suggesting that the two groups have different criteria for when to apply their knowledge. We believe the difficulties of the Remedial students are related to their concept of multiplication, not their application of the concept. For both the Good and the Remedial subjects, the primary criterion used for deciding if an operation is appropriate to solve a problem is whether the problem's story line fits their understanding of that operation. Arbitrary strategies are applied when problems do not fit well into the subject's mental representation of any operational class. Since Remedial subjects interpret multiplication to mean repeated addition, it is not surprising that they had difficulty recognizing that an equation with a division sign could be mathematically equivalent to one with a multiplication sign. Their conceptual understanding of division does not involve an inverse relationship with multiplication.

In contrast, the Good subjects appeared to have a much more well-developed conceptual understanding of multiplication and division. They were able to deal equally well with whole number multipliers and fractional multipliers, and they were flexible in perceiving that a problem could be represented via seemingly different yet mathematically equivalent equations. Such understanding would seem to require at least tacit knowledge of the different roles of the quantities involved in a multiplicative situation. It is doubtful that the Good subjects could have performed as well as they did on the difficult fractional-multiplier/multiplicand-unknown problems without such knowledge.

Tacit knowledge is probably quite important in arithmetic in general. Although it may be reasonable to assume that students will develop efficient strategies for addition and subtraction without explicit instruction (Resnick, 1982), it seems less reasonable to assume that all students are as capable of developing such knowledge for multiplication and division. We have already argued that these operations are much more complex than addition and subtraction. What we would like to propose is that students be explicitly taught at some point in time those strategies that are commonly inferred by competent students.

Instead of abandoning the repeated addition Count strategy, as some have suggested (Fischbein et al., 1985), we should use it as a springboard and teach beyond it. The usual pedagogical sequence is to begin with result-unknown problems with whole number multipliers where the Count strategy is applicable and proceed to fractional multipliers. However, this sequence neither encourages a student to develop a sense of the different roles played by the quantities in the problem, nor leaves the student well prepared to understand fractional multipliers.

A more effective sequence might also begin with result-unknown/whole-number-multiplier problems but then move to multiplicand-unknown/whole-number-multiplier problems in an attempt to add flexibility to the student's conceptual understanding. By comparing and contrasting these two classes of problems, both of which are solvable with a Count strategy, students could be guided in understanding that the precise form of an equation is secondary to the appropriate representation of a relationship. Just as important, they could also be shown how it can be easier to perceive and represent some division problems using a multiplication equation. Having grasped a sense of the roles played by the operand, operator and result, students should be better able to cope with the broader meaning of multiplication implied by fractional multiplier problems. Now the effort should focus on conveying that multiplication is not always interpretable as repeated addition. This type of "scaffolding" instructional approach attempts to raise the student's understanding by weaving coherent threads tying various concepts together, rather than by teaching multiplicative concepts in relative isolation.

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Table 1: Four Types of Multiplicative Problems

Compute

Ellen needed to buy 3 pounds of rice.
The rice cost 60 cents per pound.
How much would Ellen have to pay?

Compare

Mary had 20 marbles.
Bill had 3 times as many marbles as Mary did.
How many marbles did Bill have?

Combine

Diane has four different skirts.
She also has 3 different blouses.
How many different outfits can she make with these clothes?

Convert

The bicycle was traveling at 10 miles an hour.
There are 5280 feet in one mile.
How many feet per hour did the bicycle travel?

Table 2: Percent Correct Performance of 3 Groups on Experiment 1

GROUP	OPERATION	NUMBER AND POSITION OF FRACTIONS			
		WW	WF	FW	FF
GOOD	Addition	1.00	1.00	1.00	1.00
	Subtraction	1.00	.93	1.00	.97
	Multiplication	1.00	1.00	1.00	.93
	Division	1.00	.97	1.00	.93
AVERAGE	Addition	1.00	1.00	1.00	1.00
	Subtraction	.93	.80	.77	.87
	Multiplication	.97	.93	.97	.73
	Division	.93	.70	.77	.80
REMEDIAL	Addition	1.00	.97	1.00	1.00
	Subtraction	.97	.93	.93	.97
	Multiplication	1.00	.16	1.00	.03
	Division	.90	.67	.77	.57

Table 3: Percent Correct Performance by 3 Groups on Experiment 2

GROUP	MULTIPLIER TYPE	RESULT UNKNOWN				MULTIPLICAND UNKNOWN			
		CM[1]	IM[2]	CD[3]	ID[4]	CM	IM	CD	ID
GOOD	Whole[5]	1.00	.88	1.00	1.00	.88	.83	.93	.85
	Fractional[6]	1.00	.87	.97	.97	.93	.82	.97	.63
AVERAGE	Whole	.93	.78	.88	.98	.58	.55	.70	.55
	Fractional	.92	.53	.75	.82	.82	.67	.63	.48
REMEDIAL	Whole	.92	.42	.67	.82	.50	.73	.65	.45
	Fractional	.22	.40	.73	.40	.40	.52	.43	.4

1 - Correct Multiplication Equation
2 - Incorrect Multiplication Equation
3 - Correct Division Equation
4 - Incorrect Division Equation
5 - Compute Problems
6 - Compare Problems

Table 4: Reaction Times for Correct Responses of Good Subjects on Experiment 2

(In Seconds)

MULTIPLIER TYPE	RESULT UNKNOWN CM[1]	IM[2]	CD[3]	ID[4]	MULTIPLICAND UNKNOWN CM	IM	CD	ID
Whole[5]	1.53	4.21	3.01	2.58	3.74	1.80	3.50	3.92
Fractional[6]	1.93	4.50	4.42	3.24	3.83	4.41	4.08	5.74

1 - Correct Multiplication Equation
2 - Incorrect Multiplication Equation
3 - Correct Division Equation
4 - Incorrect Division Equation
5 - Compute Problems
6 - Compare Problems

Appendix 1: 32 Items of Experiment 1

Addition

Whole + Whole

Midway through the game, Elaine's bowling score was 75. On her next turn, she scored 16 points. What was Elaine's score then?

Ted sold 15 candy bars on Friday for the school band. Dave sold 8 candy bars. How many candy bars did they sell altogether?

Whole + Fraction

Roger had 7 pounds of weights on his weight set. He added another 1/2 pound to the set. How much weight did he have then?

Maribeth ate 3 pieces of melon for dessert. Her little brother ate 3/4 of a piece of melon. How much melon did the two children eat together?

Fraction + Whole

Luis had 1/3 of a can of oil in the back seat of his car. He bought 6 more cans of oil. How much oil did he have then?

The O'Connors brought 7/8 of a pound of hamburger to the picnic. Their hosts already had 2 pounds of hotdogs. How much meat did they have altogether?

Fraction + Fraction

Franny put 1/4 cup of fancy coffee in the grinder. Then he added 1/3 cup of regular coffee beans. How much coffee did Franny put in the grinder?

The gardener planted 1/3 of a pound of peas in one row. In the next row, she put 3/4 of a pound of green beans. How much seed did she plant?

Subtraction

Whole - Whole

At noon, there were 50 tickets left for the big concert. 37 tickets were sold in the next half hour. How many tickets were left?

Angela has 45 pairs of shoes in her closet. Her husband, Mark, has 13 pairs of shoes. How many more pairs of shoes does Angela have than Mark?

Whole - Fraction

Mike's hair was 3 inches long. He had to get a haircut before his job interview, so he had 3/4 of an inch taken off. How long is Mike's hair now?

Daryl has had her stereo for 8 months. However, she has only had the speakers for 2/3 of a month. How much longer has she had the stereo than the speakers?

Fraction - Whole

The jet was traveling at 9/2 miles above sea level. After it flew over the mountain, it descended 2 miles What was the height if the jet at that time?

The jumbo container holds 9/4 quarts of popcorn. The regular container holds 1 quart of popcorn. How much more popcorn does the jumbo hold than the regular?

Fraction - Fraction

Jacob bought a piece of meat that weighed 7/8 of a pound. He cut off 1/3 of a pound of fat and bone. How much did the piece of meat weigh then?

Janet can hold her breath for 1/2 a minute. Louisa can hold her breath for 2/3 of a minute. How much longer can Louisa hold her breath than Janet?

Multiplication

Whole x Whole

Suzie lined up 4 rows of game pieces on the gameboard. Each row had 5 pieces in it. How many pieces did Suzie line up?

Nancy's mother lives 7 miles away. Her husbands parents live 5 times as far away. How far away do her husband's parents live?

Whole x Fraction

George and Lori received 1500 dollars when they were married. They spent 1/4 of the money during their honeymoon. How much money did they spend?

There are 36 kinds of cereal on the shelf of the grocery store. 2/3 of the cereals contain sugar. How many kinds of cereals contain sugar?

Fraction x Whole

A paper cup holds 2/5 of a pound of sand. Dorothy carried 12 cups of sand to her sand castle. How many pounds of sand did Dorothy carry?

The small tube of skin cream contains 7/16 of an ounce. The larger tube contains 3 times that amount. How much skin cream is in the larger tube?

Fraction x Fraction

The mayonnaise jar was 7/8 full. The picnickers used 1/4 of the mayonnaise on their sandwiches. How much mayonnaise did they eat?

Maura lives 3/4 of a mile away from school. She crosses a big street when she has gone 1/3 of the route. How far is the crossing point from Maura's house?

Division

Whole / Whole

Mrs. Wells made 54 cookies for the bake sale. She packaged them 3 to a bag. How many bags of cookies did she make?

The kiddie pool holds 15 gallons of water. The hot tub holds 45 gallons of water. How many times more water does the hot tub hold than the kiddie pool?

Whole / Fraction

Marie had 6 yards of fabric to make dolls for the Girls' Club. Each doll required 2/3 of a yard of fabric. How many dolls could Marie make?

Pat spent 2 hours waiting for his new license plates. Jean went later in the day and waited 1/4 of an hour. How many times longer did Pat wait than Jean?

Fraction / Whole

The carpenter ·ught a 3/4 pound bag of nails at the hardware store. There were 50 nails in the bag. How much did each nail weigh?

The turtle was 2/5 of the way around the track. The hare had already circled the track 3 times. What fraction of the hare's distance had the turtle gone?

Fraction / Fraction

There is 2/3 of a pound of punch mix in the container. You use 1/12 of a pound of mix to make a quart of punch. How many quarts can you make with one container?

Armand needed 2/3 of a pound of nuts to make the recipe. He had 3/8 of a pound of nuts. What fraction of the amount of nuts needed did Armand have?