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ABSTRACT

This paper attempts to show that it is possible to analyze a problem solving protocol in which the subject spontaneously generates a series of analogies. The qualitative physics problem given to the subject describes a situation where that which is unfamiliar is often solved by relating it to several analogous situations that are more familiar. Also described as important in creative problem solving are generating Gedanken experiments, generalizing and specializing, and generating extreme cases. This analysis suggests that three processes are essential in reasoning by analogy: (1) given the initial conception A, the analoguous conception B must "come to mind"; (2) the analogy relation must be "confirmed"; and (3) conception B must be "confirmed." The paper also identifies three major questions to be considered in future research: (1) how do analogous cases come to mind? (2) how are the analogy relations and new conceptions confirmed or disconfirmed? and (3) why do they have explanatory power? (TW)

THE ROLE OF ANALOGY IN SCIENTIFIC THINKING: EXAMPLES FROM A PROBLEM-SOLVING INTERVIEW*

Part I

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ABSTRACT: We attempt to show that it is possible to analyze a problem solving protocol in which the subject spontaneously generates a series of analogies. The qualitative physics problem given to the subject describes a situation that is unfamiliar to him, but he solves it by relating it to several analogous situations that are more familiar. By spontaneously proposing these analogies, he imitates what is, according to Nagel, an important function of science: the process of "making the unfamiliar familiar." He also exhibits other types of behavior that Polya has described as important in creative problem solving: generating Gedanken experiments, generalizing and specializing, and generating extreme cases. Thus, although the student is a freshman who has not taken college science courses, he intuitively uses thought processes similar to those of creative scientists and mathematicians.

He also produces body movements that parallel his arguments, generates personal as well as physical analogies, and generates a special type of analogy called a qualitative interpolation. Several of these observations are accounted for by positing a first-order theory of the cognitive events occuring in the subject, which states that he generates a chain of analogous but tentative conceptions, eventually confirming his conceptions by linking them to a known fact. Our analysis suggests that three processes are essential in reasoning by analogy: (1) given the initial conception A of an incompletely understood situation that the subject is unsure of, the analoguous conception B must "come to mind", (2) the analogy relation must be "confirmed", and (3) conception B must be "confirmed". Attempts at confirmation may suggest a new, more appropriate analogous situation B'. Thus, the paper identifies three major questions to be considered in future research:

How do analogous cases come to mind?
How are the analogy relations and new conceptions confirmed or disconfirmed?
Why do they have explanatory power?

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Part I

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Introduction: Nagel's View of the Role of Analogy in Science

In his classic text, The Structure of Science, Nagel discusses two roles for analogies in science.[1] First, "formal" analogies between abstract theories can play a heuristic role by suggesting directions for theory development. An example would be the definition of the function n^x where it is conjectured that the function may be usefully defined for non-integral values of x by requiring that the rules $n^a \cdot n^b = n^{a+b}$, and $(n^a)^b = n^{a \cdot b}$ still hold. re the more general theory is analogous to the more specific theory. The analogy suggests a plausible direction in which to expand the theory that can then be subjected to more rigorous examination.

Secondly, a very important role of analogy according to Nagel derives from the use of models in science. A scientific model formalizes an analogy between an unfamiliar system and a familiar system. For example, the kinetic theory of gases is built on a model that assumes an analogy between the way the components of a gas behave and the way billiard balls behave.

1. The Structure of Science: Problems in the Logic of Scientific Explanation, New York, 1961, pp. 106-117.

Thinking of the gas molecules in terms of the more familiar case of billiard balls makes the theory more satisfying and comprehensible as an explanation that "makes sense." In Nagel's terms, the model fulfills an important role of scientific explanation in "making the unfamiliar familiar." In this case then there is an analogy between a familiar system (billiard ball behavior) and the hypothesized theoretical mechanism of the system to be explained (gas behavior)[2].

Spontaneous Analogies in a Problem Solving Protocol

The main body of this paper analyzes the transcript of an interview with a college student in which he makes a series of spontaneous analogies while working on a physics problem. Nagel's ideas on the role of analogies in science provide an interesting framework from which to view the way in which this student's analogies help him "make the unfamiliar familiar" as he works through the problem.

The protocol to be discussed, shown on page 4, is the condensed transcript of an interview with Jed about the behavior of a small metal cart being launched by an elastic band. Jed is a freshman at the University of Massachusetts just entering the School of Engineering. He has had a high

2. According to Nagel, scientific models, as analogies, also:

1) Help to articulate newly constructed theories;
2) Suggest key questions for the refinement and extension of theories;
3) Allow the application of theories to concrete physical problems by suggesting points of correspondence between theoretical elements and observable variables;
4) Contribute to the achievement of inclusive systems of explanation by providing links between theories.

school course in physics, but in the interview, he seems to use very little in the way of formal knowledge from a previous course, relying instead on intuitive arguments. In the first part of the interview he launches the cart across the top of a table using the elastic band (as shown in fig. 1) and watches it roll to a stop. He correctly predicts that the cart will attain its maximum speed near the point of release from the band, and that its maximum speed will be lower if an iron weight is placed in the cart, because you're "having a larger mass to pull -- I know in chemistry that a bigger atom goes slower -- if you applied the same amount of force, it would go slower." Also, it won't go as far with the extra mass, "because now it's pushing down on this table -- it would just be weighted down -- it's like a shopping cart - you push a shopping cart a lot further without groceries than you could with it." Thus Jed refers to two preliminary analogies: between the cart with the added weight and "a bigger atom going slower"; and between launching the cart with the added weight and pushing a shopping cart with and without groceries in it. He gives them as part of his explanation for why he thinks the cart will move with reduced speed over a shorter distance when it carries an extra mass.

A more interesting series of analogies appears in the second half of the interview, which is transcribed below. The experimental problem is the same, except that Jed is asked what will happen when the force of gravity is absent. Jed is asked whether there will be a winner in a race between two carts launched sideways from a rocket floating in outer space (as shown in fig. 2). Both are launched with an elastic band of the same strength, but the upper cart is heavier. During the interview, Jed prefers to refer to the two carts being launched from the elastic band as "small rockets". (Theory predicts that the more massive cart's speed should be lower.)

Fig. 1

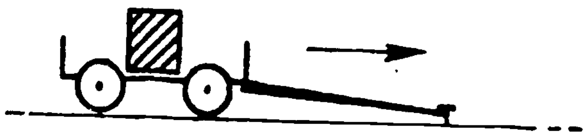
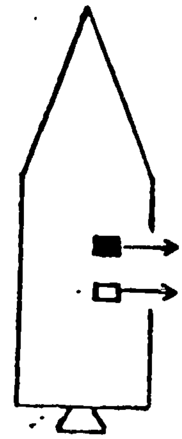


Fig. 2



Jed - "Space Race" Protocol

Section A

1 I: OK. This is the same experiment as the cart [done earlier on the table].

2 S: Yeah.

3 I: And those are two carts there, [inside the rocket in Fig. 2] and one of them we're going to put a weight in, like we did, and we'll shoot them on rubber bands out from this rocket. It's[the rocket is] floating, standing still in space, OK?

19 I: And if I use the same stretch, the question is, who wins the race? Do they both go out at the same speed, or, what would you think?

24 S: Oh. OK, gotcha. Oh, um, one's heavier. Uh, all right, I'll start with the weight one [the upper cart in drawing]. Um, if it's heavier --, uh--, it's heavier -- let's see. What makes heavy is gravity, so heaviness wouldn't matter if you're in space.

25 I: Uh huh.

26 S: a) --'cause there's no gravity. But, I know they try and make the rocket as light as possible, - but that's only to get away from earth, so it doesn't matter. OK, uh, if it doesn't matter how heavy it is -- b) but it does, (nods), 'cause the equation says E goes 1/2 mv^2, so --

27 I: That's the equation?

28 S: Yeah.

29 I: For what?

30 S: Energy. Uh, (Shakes hands up and down 3 times), if you pull this out with a force (points to upper cart and slides finger to right on paper) of, let's say, one, all right, the E = 1 (points to upper cart and slides finger again to right)--no, the E doesn't equal one.

Section B

31 S: Uh, I'd say that -- it would still go slower (points to upper cart and slides finger to right). If you pulled this out with the weight on it, it would go slower than if you pulled this out (points to lower cart and slides finger to right) without the weight.

32 S: Without the weight you'd go faster and with the weight you'd go slower, because I can still think of, uh, you pulling (holds r. hand up and moves it toward himself) something very heavy and pulling something very light. (Repeats hand motion)

33 I: In space?

34 S: All right, in space. I've never been in space. -- Yeah, in space, too, I guess, in space. If you push (moves r. hand away from chest) something heavier than you, you would go back, (moves head and r. hand back) uh, more than it would go that-a-way. (points forward).

35 I: Mmm.

36 S: I'm just guessing. I just think this. I figure if you pushed (moves hand forward) the rocket, you'd go back (moves hand back) more than the rocket would.

38 S: So if, uh, so if you pulled this (slides finger to r. of upper cart) with a heavier [weight in it], it would go slower...

46 S: a) So (looks back at drawing) if you had the rubber band here, (slides finger to right) it would still pull the lighter rocket faster than it would pull the heavier rocket. The heavier rocket would stay slower. b) Oh, I really, I -- the only thing I can think of is that it's still harder to push (moves clenched fist away from chest) that heavy rocket, c) 'cause I could throw a pen (makes flipping motion with hand) out in space, it's really light, and it would go (repeats hand motion) away. I (points thumb back) would go away, too, but it would go away (moves closed hand forward) more than I would (moves hand backward).

66 S: a) Right. Uh, and so the same thing for this one. If, even though there's no weight, but still, I can just think of me trying to hit (punches air with fist) a rocket and trying to make that rocket go away (moves open hand away). I figure if I hit (punches) that rocket, I'd go away more (points back) than the rocket would because it's just so <u>big</u> (spreads hands apart). b) Oh! If, uh, if a meteor comes down and hits the earth, (raises hand and moves it down) the earth is just so big, it's not gonna move out of its orbit. But the meteor sure gets splattered. And the meteor wouldn't -- hm. I wonder if that's the same thing.

67 I: The earth doesn't get pushed?

68 S: No, not by a little thing. (Holds up hand with fingers closed) You'd have to have something bigger (opens fingers as if holding a ball) than the earth or --

69 I: Even though the meteor is weightless?

70 S: (a) Well, oooh, well, the meteor comes down, hits the earth, (raises hand and moves down) the earth just sits there, doesn't, I mean, it's just so little, it doesn't matter about the orbit. (Sweeps hand in circle) It wouldn't knock it [the earth] out of the orbit. (b) But, if that meteor could bounce back, the meteor would bounce right back, (moves hand down, then up over shoulder), if it could, if it didn't splat (moves hand down) (sound effects), it would bounce back. (c) And so, that means that if you try to hit (punches air) something, if I was little (points to self) and I was that meteor and I tried to hit (punches air) that rocket, I'd bounce back (points back) and the rocket wouldn't really move that much. (d) So, that means that, uh, to move a big object (hand opens) is harder than to move a little object, (hand closes) 'cause if I --

71 I: With a rubber band, too.

72 S: With a rubber band, too. No, see, it's the same amount of force, like the force pulling (points to upper cart and slides finger to r.) on this, uh, it's a big object and it's hard to move, so the rubber band, the force of the rubber band would be, uh, like, one, and this is such a hard object to move that it, it would go "slowww," (moves hand away slowly) and it wouldn't go as fast. And if you had a really light thing, it would just (moves hand away quickly) zip along with the rubber band. It would go faster.

Condensed Transcript

Section A 24) S: What makes heavy is gravity, so heaviness wouldn't matter in space.

26a) But I know they try to make the rocket as light as possible -- but that's only to get away from earth.

26b) But it (heaviness) does (matter) because E goes 1/2 mv^2.

Section B 31) It would still go slower... with the weight ...

32) Because you can still think of pulling something very heavy and something very light.

34) If you push something heavier than you, you would go back more than it would go that-a-way.

36) If you pushed the (large) rocket, you'd go back more than the rocket would.

38) If you pulled this (points to upper cart) with a heavier [weight in it] it would go slower.

46a) .. the rubber band ... would still pull the lighter rocket faster.

46b) It's still harder to push that heavy rocket.

46c) I could throw a pen out in space, it's really light, and ... it would go away more than I would.

66a) Even though there's no weight, but still trying to hit a rocket- I'd go away more than the rocket would - it's just so big.

66b) Oh! If a meteor hits the earth, the earth is so big it's not gonna move out of its orbit. But the meteor sure gets splattered! I wonder if that's the same thing.

70b) If that meteor could ... it would bounce right back.

70c) If I was that meteor and I tried to hit the rocket I'd bounce back ... the rocket wouldn't move that much.

70d) That means that to move a big object is harder than to move a little object.

72) With a rubber band too ... it's the same amount of force ... of the rubber band... This is such a hard object to move, that it would go"slowww".

Protocol Summary

The protocol section shown here is the second half of an interview about launching carts. In the first half, where the car was actually launched by Jed from a rubber band attached to a table, his two main conceptions appear to be summarizable as: "The more you stretch the band, the faster the cart will go over the table. The more weight you put in the cart, the slower it will go over the table." In the transcript excerpt just given here from the second half of the interview, Jed was asked about two carts being launched from a spaceship, one with an extra weight in it. His comments here all seem to relate to the question, "Does heaviness matter in space?". Jed makes no clear distinction between the concepts of mass and weight, and indeed whether such a distinction is necessary is very much the issue implied by this question. Nevertheless he does appear to reach a strong conclusion, based on intuitive arguments. He concluded in line 70d that "to move a big object is harder than to move a little object," even in space.

It is convenient to divide the protocol into two parts. In lines 24 through 30 in the first part, there are three basic themes:

1. (Line 24) "What makes 'heavy' is gravity, so heaviness wouldn't matter in space."

2. (26a) "But I know they're trying to make the rocket (a rocket launched from earth) as light as possible -- but that is only to get away from the earth." (He seems to decide that this fact is not relevant).

3. (26b) "But it [heaviness] does [matter] because E goes 1/2 mv^2." (He tries to assign values to the variables in this equation for launching the cart but seems to be unsure of this argument and abandons it.)

With these three preliminary themes, Jed seems to attempt to give reasons, pro and con, for whether "heaviness matters in space" when one is trying to get something moving. In the second section beginning in line 31, he continues to address this question and introduces three more themes.

4. Pulling Objects

(32) "You can think of pulling something very heavy and something very light."

5. Recoil from Pushing

(34) "If you push something heavier than you, you would go back more than it would go that-a-way (forward.)"

6. Meteor

(66b) "If a meteor hits the earth, the earth is so big that it's not going to move out of its orbit."

These three last themes appear in lines 31 through 72. They are not given in a single sequence but appear repeatedly in mixed order. They constitute spontaneous analogies in the sense of being related situations that Jed thinks about to help him solve the original problem. After considering these 3 analogous cases Jed appears to feel confident that he has solved the original problem correctly. Our analysis, therefore, will concentrate on these last three themes.

Figure 3a shows the original problem situation and the three major themes Jed refers to in pictorial form. As he considers the three situations shown on the right, he somehow uses them to help him decide on what will happen in the original problem situation on the left. An initial interpretation, then is the following: Each analogous case is somehow generated from the original problem and used to suggest a prediction for the original problem. He does this for three separate cases, apparently making his prediction stronger with each case. How these analogies are generated and why they have the explaratory value they do for Jed are two major questions raised by the protocol.

Protocol Observations to be Explained

1) Jed's arguments are primarily qualitative with the exception of the equation he mentions in line 26b (which he apparently discards).

2) He refers to a number of related cases -- situations that are not the same as the situation in the given problem. These are analogies in the sense that he seems to believe that some relationship in the related case will also prevail in the original problem.

3) Some of these analogous cases are factual, for example Jed brings in the fact that meteors are destroyed when they hit the earth. Others are Gedanken experiments where he invents a new situation that he has some intuition about and that can tell him something about the original problem.

4) He refers to extreme cases, such as: his statement (32) "You can think of pulling something very heavy and something very light"; and his opposition of the tiny meteor and the enormous earth.

5) The solution as a whole has a "wandering" character -- he seems to explore various aspects of the problem, and these will sometimes remind him suddenly of new situations -- as opposed to solving the problem in a planned way with a series of carefully organized steps.

6) In many of his statements he talks about putting himself in the place of one of the objects. He appears to sort out the way forces are acting by pretending to push or pull on one of the objects himself. These will be called personal analogies; examples are the "Pulling Objects" and "Recoil from Pushing" themes. Non-personal analogies, such as the "Meteor" themes, will be called physical analogies.

7) Many of Jed's statements are accompanied by body motions which parallel the kind of actions he is describing at the time. These occur in both the personal and physical analogies.

8) There are several levels of generality in his statements varying from the very specific: "I could throw a pen out in space and it would go away more than I would [go back]"; to the more general: (70d) "that means to move a big object is harder than to move a little object." There are differences in the level of generality of the sets of objects he refers to as well as differences in the level of generality of the sets of actions that he refers to.

9) A very interesting phenomenon occurs when Jed makes a qualitative interpolation. In line 70c, Jed says, "If I was that meteor, and I tried to hit the rocket, I'd bounce back," This is an intermediate case that can be described in terms of a new combination of two previous themes that Jed has referred to, namely: his previous reference to a meteor hitting the earth and his previous reference to him hitting something heavier than himself and having himself go back more than the heavy object moves forward.

10) The "Bouncing meteor" episode is distinguished by its novelty as a situation. This suggests that it was generated or constructed by Jed -- not simply recalled from previous experiences or factual knowledge.

11) We can symbolize the "two sides" of an analogy by using the letters a and b to denote the two particular situations that are related by analogy for person S.

We can then distinguish the following types of analogies:

I. Provoked Analogies

A) Proposed Analogy	S, the subject, is asked whether situation a is like situation b.
B) Standard Analogy Test Question	S is asked to complete b by filling in y, given u, v, x, and the implied relation:

a		b
u is related to v	is like	x is related to y

II. Spontaneous Analogies

A) Generates an Analogous Case	S considers a and produces b
B) Spontaneous Connection by Analogy	S considers a and b and says they are alike.

We can classify the analogies that Jed makes by using this classification scheme. In terms of the main themes outlined in Figure 3a, we see that Jed's shifts from one theme to another conform to the pattern of spontaneous generation of analogous cases listed under IIA in the chart above. Thus given a situation a Jed generates a new situation b spontaneously as he jumps from an old theme to a new theme. These analogies are more spontaneous than those formulated by a student solving the typical analogy test question in the form shown in IB above, because the test question provokes the analogy by partially describing both a and b, whereas Jed starts only from the original problem situation a and produces related cases on his own.

12) By the end of the interview in lines 62 and 64, Jed states his conclusions with a level of conviction that indicates that his thinking has led him to a conclusion that he is fairly certain of.

Accounting for the Observations

Each of the above observations contains an implied question, namely, how are we to explain the presence of this aspect in Jed's behavior? Eventually we would like to proceed from the observations to a theory describing the cognitive processes going on in Jed that produce the behavior.

Observations 2, 3, and 4 above point to Jed's generation of analogous cases, extreme cases, Gedanken experiments, and to his connection of the problem situation to previously known facts. Jed's arguments here are examples of what Polya (1954) calls plausible reasoning rather than being logical arguments in the strict sense. These characteristics of Jed's behavior are similar to those recognized by Polya as highly important for scientific and mathematical problem solving. When these methods are consciously employed as strategies they are often referred to as heuristics. However, in Jed's case it is not at all clear that he makes a deliberate decision to use any of the strategies that he does use. In his case they seem to be more of a natural way of operating and to be a spontaneous reaction on his part to the dilemmas he finds himself in. We can say that he uses some intuitive heuristics. The fact that Jed has had so little previous training in science suggests that we may be able to study certain types of heuristics in untrained subjects as spontaneous reasoning patterns that reflect certain general cognitive characteristics. This has been one of our motives for studies like the present one.

Fig. 3a. Analogous Situations

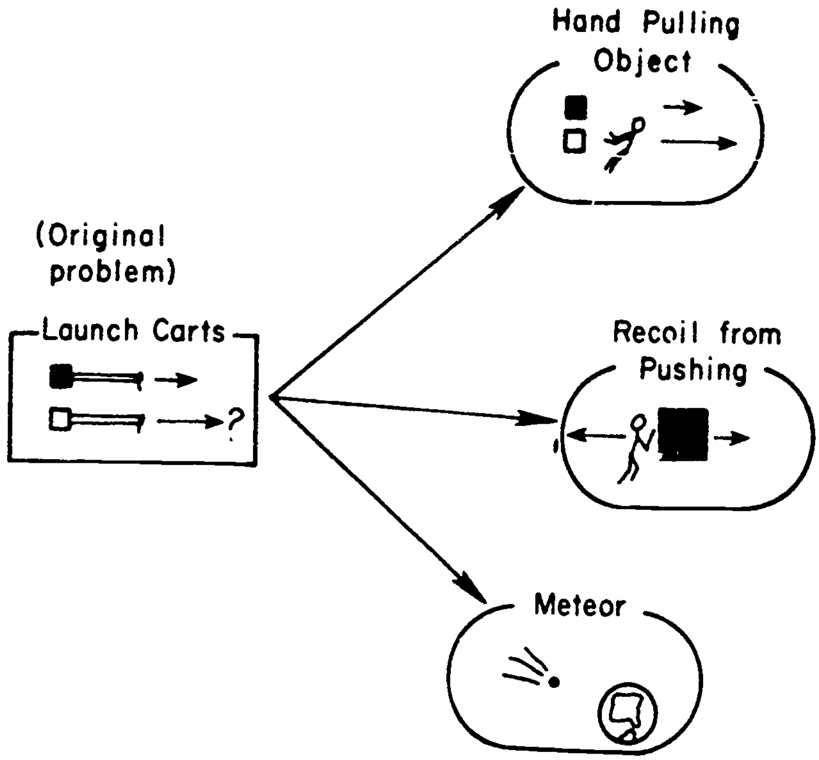
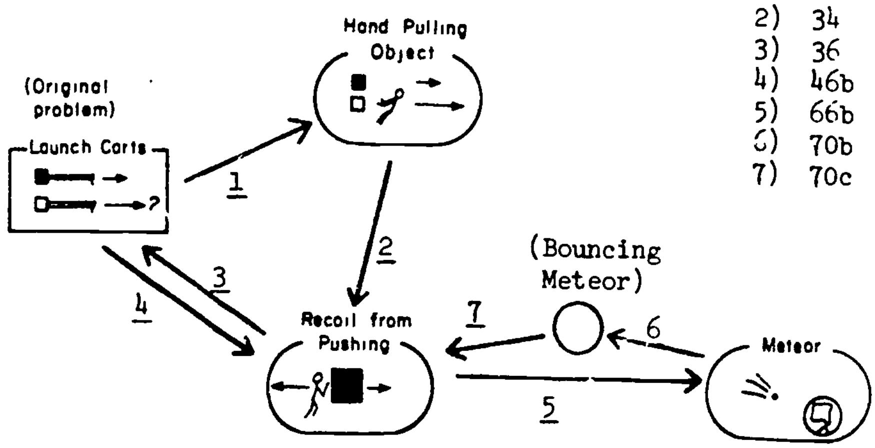


Fig. 3b Analogy Theme Sequence



Line no.	
1)	32
2)	34
3)	36
4)	46b
5)	66b
6)	70b
7)	70c

Fig. 4a
A Confirmed Analogy

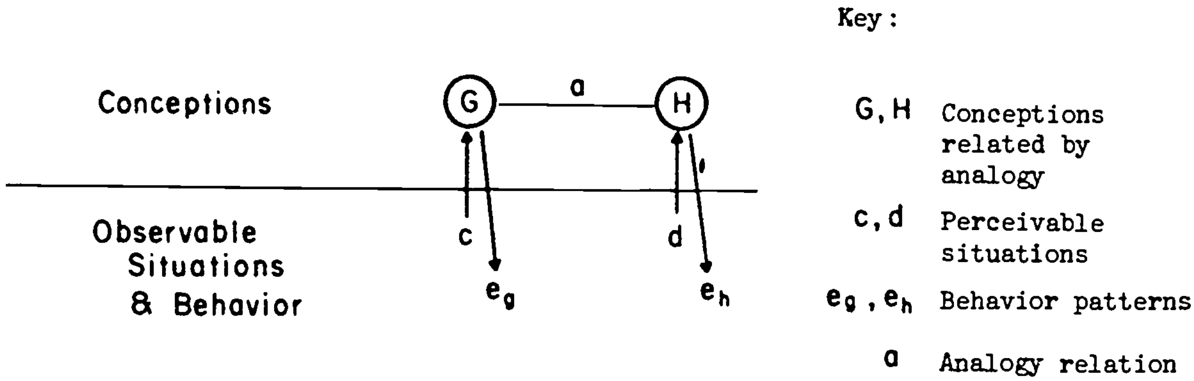


Fig. 4b
Inference by Analogy

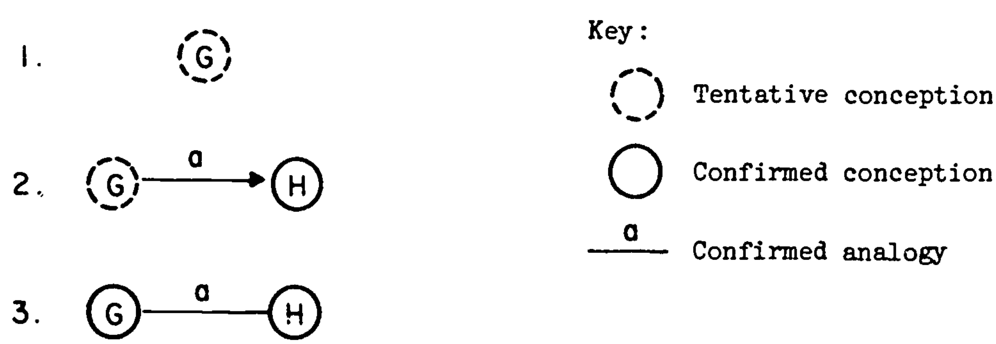


Fig. 3a shows various analogous situations that Jed refers to in lines 24-72 (section B of the transcript). The figure shows pictorially how the cart problem might be related by Jed separately to the three separate analogous cases he discusses. Fig. 3b is a theme sequence diagram summarizing the sequence in which he refers to these themes. The numbers in the theme sequence diagram in Fig. 3b indicate the order in which transitions between themes occur. In contrast to Fig. 3a, this diagram suggests that some of the related cases Jed talks about are <u>secondary analogies</u> in the sense that they are originally generated in relation to a previously generated case, rather than in relation to the original situation. For example, Jed seems to relate the "Meteor" theme directly to the intermediate "Recoil from Pushing" theme, rather than to the original cart launch problem.

Up to now we have been talking primarily about observations of behavior. At this point we would like to say something more definite about the cognitive events occurring in Jed during the interview. In order to do this it will be convenient to develop the following notation for modelling the internal processes associated with analogies:

A <u>confirmed analogy</u> is shown in the daigram in Figure 4 a. <u>G</u> and <u>H</u> are two internal conceptual frameworks of the subject S that for him are related by analogy. <u>c</u> and <u>d</u> are particular observable situations in the external world that S understands in terms of conceptions G and H. e_g and e_h are particular observable behavior patterns (such as physical actions and themes in verbal output) that are produced and controlled by G and H.

An <u>inference by analogy</u> (leading to a confirmed analogy) is shown in the diagram in Fig. 4b. Only internal cognitive events are shown here for clarity. In (1), G enclosed in a dotted region is a tentative or hypothetical conception that S is unsure of.

In (2) the subject begins to think in terms of analogous conception H that is a confirmed conception he feels he understands. He also considers H to be equivalent to G in its salient aspects. In (3), the tentative conception G has become a confirmed conception because of the recognized analogy.

Figure 5a shows a diagram using this notation modelling the way in which several of Jed's conceptual frameworks are connected together by him during the interview. We make the assumption that each of the major themes identified in his output is produced by one of these internal conceptual frameworks. The double arrows signify pairs of conceptions that Jed considers to be analogous.

When conceptions are connected like this, we assume that Jed considers some aspect of the first conception to be similar to some aspect of the second conception-- to "work in the same way." Using this diagram we can make several points:

(1) Numbers on the arrows indicate the hypothesized order in which the analogies form or come to mind for the first time. This is supported by the observed order of first appearance for themes shown in Fig. 5b. Thus Fig. 5b shows an observable pattern in Jed's output which supports the theory of unobservable cognitive events shown in Fig. 5a.

(2) The conceptions shown as regions with a dotted border are tentative conceptions. That is, Jed is unsure of their internal validity. For example, he is unsure that it is harder to pull the heavier cart in space with his hand.

(3) Thus the diagram essentially shows a chain of analogous tentative conceptions formed by Jed. These connect the original cart-launching problem

Fig. 5a
Formation of Analogies between Jed's Conceptions

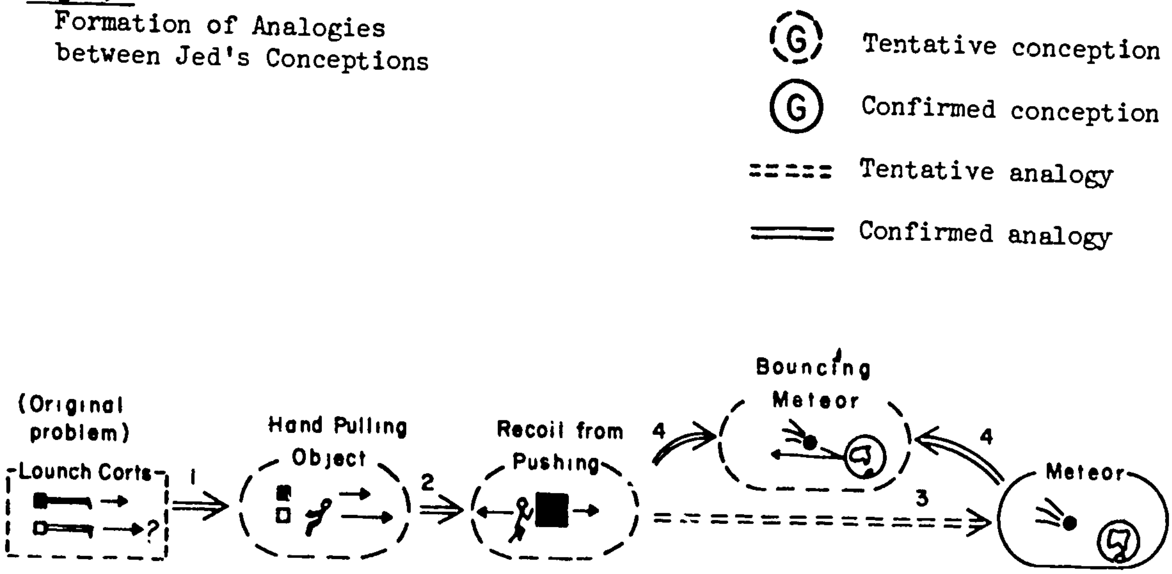
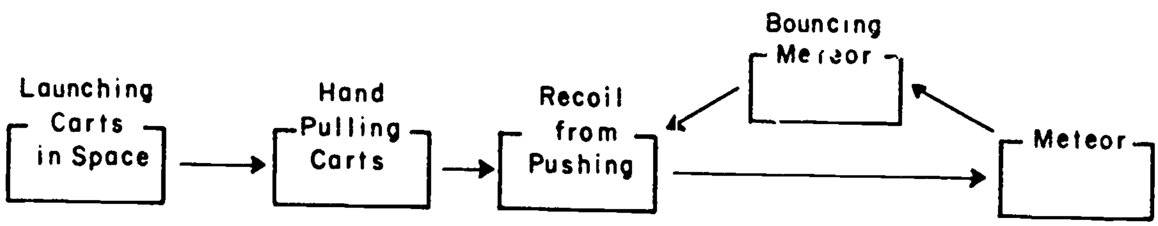


Fig. 5b
Sequence in which Themes are First Mentioned



that he is unsure of to a confirmed conception of meteor behavior that he is sure of. This allows him to make a chain of inferences back to a prediction for the original problem. This final inference chain takes the form: (G)-(H)-(I)-(J)-(K) where the presence of confirmed analogy links and a single confirmed situation creates a dominoe effect to make confirmation of all the other situations plausible. This is true because any pair of the form (J)-(K) leads to the plausible inference that J is confirmed.

(4) Fig. 5a shows clearly how the "Meteor" and "Bouncing Meteor" conceptions play a somewhat different role than the other conceptions. Analogy connection number 3 in the diagram is shown as a dotted line to indicate that it is a tentative analogy between the "Recoil from Push" conception and the "Meteor" conception. We can then distinguish between the formation of tentative analogies that are "leaps", such as the jump to the "Meteor" conception where the connection is initially uncertain, and immediately confirmed analogies, such as 1 and 2 where the analogy relation has been established with confidence immediately. Thus our theory proposes two basic types of analogy formation: (1) a type where the subject "leaps" via a tentative analogy to a more familiar but "distant" situation; and (2) a type where the subject is able to immediately generate a confirmed analogy by modifying aspects of the current problem to produce a second situation for which the similarity in structure is immediately obvious. An example of the second type of analogy formation is Jed's "Hand Pulling a Cart" theme, where he seems to have modified only one aspect of the original situation, namely, he thinks of himself pulling the cart rather than the elastic band. He seems unsure about the question of whether

it would be harder for him to pull a heavier object in space. But he does seem to be sure of the analogy in this case -- that is, he seems to believe that if it is true that it would be harder for him to pull a heavier cart in space, then it would be harder for the elastic band to pull one as well. Presumably, he believes this because he feels that the change from pulling with the band to pulling with the hand does not affect the critical aspects of the experiment. In other words he believes that part of the behavior of the system -- some basic pattern in the events -- is <u>conserved</u> when there is a change in "what's doing the pulling." We will call this second type of analogy formation an <u>immediately confirmed analogy</u> as opposed to a tentatively proposed analogy.

(5) We can now provide a clearer theory for the role of the "Bouncing Meteor" conception that Jed appears to generate. As shown in Fig. 5, the "Bouncing Meteor" conception provides a link by which Jed can convert his tentative analogy between the "Recoil from Pushing" conception and the "Meteor" conception into a chain of two confirmed analogies. This has the effect of linking all of the previous tentative conceptions with the confirmed "Meteor" conception. Thus a <u>qualitative interpolation</u> or <u>bridging analogy</u> like the "Bouncing Meteor" is understood in this theory as reflecting an attempt on the part of the subject to <u>confirm a tentative analogy</u>.

In summary, we have been able to account for several of our observations by positing a first-order theory which states that Jed generates a chain of analogous but tentative conceptions, eventually confirming his conceptions by linking them to known facts. In the end we may come to consider the differences between tentative and confirmed analogies and conceptions to be a matter of degree. But our analysis does suggest that three processes are essential in reasoning by analogy: (1) given the initial conception A, the analogous conception B must "come to mind", (2) the analogy relation must be "confirmed", and (3) conception B must be "confirmed". In addition, attempts at confirmation may suggest a new, more appropriate analogous situation B'. Thus, we can identify three major questions to be considered in future research:

How do analogous cases come to mind?

How are the analogy relations and new conceptions confirmed or disconfirmed?

Why do they have explanatory power?

REFERENCES:

Nagel, E., The Structure of Science: Problems in the Logic of Scientific Explanation, New York: Harcourt, Brace, & World, 1961.

Polya, G., Mathematical Discovery Vol. II. New York: Wiley, 1965.

Polya, G., Mathematics and Plausible Reasoning. (2 Vols.) Princeton: Princeton University Press, 1954.