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ABSTRACT

Cognitive factors related to problem solving have been explored, but affective factors also play an important role in the teaching of mathematical problem solving. This paper outlines the theories of George Mandler and Bernard Weiner, providing a useful background for research related to affect and problem solving. Data related to the two theories are presented. Current issues on affect and problem solving are addressed: the role of belief systems in problem solving; the nature of metacognitive processes; and the influence of particular instructional settings, especially the role of technology in the teaching of problem solving. Finally, some methodological considerations for future research are presented. (MNS)

Affect and Problem Solving:

Two Theoretical Perspectives

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Abstract

Affective factors play an important role in the teaching of mathematical problem solving. This paper outlines two theories (of George Mandler and Bernard Weiner) that form a useful background for research related to affect and problem solving. Data related to the two theories are presented, and suggestions are made for ways to strengthen research on affective issues in mathematical problem solving.

This paper was prepared for presentation at the symposium on "Affective Influences on the Problem-Solving Processes of Mathematics Learners" at the Annual Meeting of the American Educational Research Association, Washington, DC, April 1987.

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Affect and Problem Solving: Two Theoretical Perspectives

Research on mathematical problem solving has made substantial progress in analyzing the cognitive processes that are important in the performance of mathematics learners. More recently researchers have turned their attention to the role of affect in the achievement patterns of mathematics students on problem solving tasks. The purpose of this paper is to present two theoretical perspectives that can be used to guide research on affect, especially affective influences that are related to the teaching of mathematical problem solving.

This paper serves as an introduction to the symposium on the topic of "Affective Influences on the Problem-Solving Processes of Mathematics Learners". The other papers in the symposium present data from studies of mathematical problem solving that provide evidence related to the strengths and weaknesses of the theories presented here.

Affect: An Underrepresented Theme

Research on mathematical problem solving has generally concentrated on cognitive issues, such as issues of knowledge organization and retrieval. Silver (1985) points out that this emphasis on cognition has made affect an underrepresented theme in current research on problem solving. In the area of cognitive science, Norman (1981) notes that most cognitive theorists prefer to ignore the affective domain; he lists emotion as one of the twelve issues that cognitive science needs to address. Although most would agree with Simon's (1982) view that affect is generally more complex and difficult to analyze than cognition, the importance of affective issues in the learning of mathematical problem solving makes this effort appropriate.

Researchers who focus on the cognitive side of mathematics learning often note the role of affect, but fail to treat it in any depth. Nesher (1986), for example, begins her recent review of research on mathematics learning by noting how failure and phobia influence students from an early age. Similarly, in a thoughtful exposition of a constructivist perspective on mathematics learning, von Glasersfeld (1987) notes how students who achieve a major reorganization of their ideas report feelings of great satisfaction. Neither of these authors, however, provides a theoretical framework that would encourage the incorporation of these important affective factors into their cognitive theoretical positions.

One purpose of this paper is to encourage those who do research on mathematical problem solving to incorporate affective concerns into their (primarily) cognitive research. A variety of theoretical perspectives have been proposed for research on affect. Two of the most useful for research on mathematical problem solving are the theories of Mandler (1984) and Weiner (1982).

Mandler's View

Mandler (1984, 1985) approaches research on affect from the perspective of cognitive science. His view is that most emotions arise from the interruption of plans or planned behavior. In Mandler's terms, these plans result from the activation of a schema. The schema produces an action sequence, and if the action sequence cannot be completed, the result is the physiological arousal of the student. The physiological arousal is then interpreted as surprise, frustration, or some other emotion.

In mathematics education, problems are usually defined as those tasks where some sort of blockage or interruption occurs. The student either does not have a routine way of solving the problem, or the routine solutions that the student attempts all lead to errors. As a result, the kind of problem solving that is attempted by mathematics students results in exactly the kind of interruption that Mandler has analyzed in his theory of emotion.

Researchers who gather detailed interview data on mathematics learners often observe exactly the kinds of blockages that form the core of Mandler's theory. For example, Cobb (1985) discusses the role that affect can play in the development of early number concepts. He compares the learning of two students who differ in confidence, persistence, and their tendency to get upset when they are unable to solve problems. Confrey (1984) comments on the confusion and frustration that young women often experience in a summer program on problem solving, and notes how these intense affective experiences are related to the students' development of a new set of beliefs about mathematics. Ginsburg and Allardice (1984) document the intense feelings of frustration and sadness that low achievers express about their difficulties in learning mathematics. Wagner, Rachlin, and Jensen (1984) report how the algebra students that they interviewed would sometimes get upset and lose control of their problem-solving processes, and start groping wildly for any response that would get them past the blockage that faced them. Each of these studies provides useful information on how interruptions and blockages can produce negative feelings about mathematics.

Although Mandler's theory tends to apply most naturally to the negative emotions that result from the interruption, he notes that the effects of an unexpected event can be positive as well as negative. He does not, however, give a special place in his theory to the kind of positive affective response that von Glasersfeld (1987) describes as the satisfaction of reorganizing one's ideas. Similar positive responses are documented by Lawler (1981), who notes the surprise and shocked look that can accompany that moment of insight when a child sees the connection between two schemas that were previously unconnected. Although it would be possible to interpret these reorganizations as responses to interruptions, the nature of such an "internal" interruption seems quite different from the type of blockage that causes problem solvers to be frustrated. Rather than defining interruption so broadly, it may be more useful to think of these reorganizations as constructions that generate a positive emotion that can occur independently of any interruption.

Another source of positive emotion in problem solving is the joy of making conjectures (Brown & Walter, 1983). The satisfactions that come from creating your own problems are intrinsic, and can result from activities that seem quite unrelated to interruptions of on-going schemas. Creative activities of all kinds, including those that involve esthetic judgments, seem to be important sources of positive feelings; see Mandler (1982) for further discussion of how these ideas fit his theory.

The joy that is a part of the "Aha!" experience in mathematical problem solving has been documented by many writers (e.g., Mason, Burton, & Stacey, 1982), but this positive emotion seems to result indirectly from exactly the kind of interruption that Mandler emphasizes. The experience of most problem solvers suggests that the "Aha!" experience is not an interruption directly, but the release from an interruption or blockage that may have been the source of a considerable amount of frustration.

In summary, Mandler's (1984) ideas on interruption and blockage fit very well with much of the research on problem solving. It is not so clear that the theory fits the constructivist emphasis on the satisfaction that comes from reorganizing ideas to achieve a new understanding; however, even Piaget's writings on affect (Piaget, 1981) say little about this affective side of the process of developing new schemas.

Weiner's View

Weiner (1979, 1982) is a leading theorist in the area of causal attributions. In his view, students attribute their successes and failures to causes that vary in their locus (internal or external to the individual), their stability, and whether or not the cause can be controlled. (See Figure 1.) For example, a student who fails to solve a mathematics problem could say that the problem was too hard, a cause that is external, stable, and uncontrollable by the student. A student who succeeds in solving a problem might attribute that success to hard work, a cause that is internal, unstable, and controllable.

A number of researchers have applied these ideas in mathematics education (Reyes, 1984), especially in the context of research on gender-related differences in mathematics achievement (Fennema & Peterson, 1985). For example, Fennema (1982) reports that males are more likely than females to attribute their success in mathematics to ability, and females are more likely than males to attribute their failures to lack of ability. In addition, females tend to attribute their successes to extra effort more than males do, and males tend to attribute their failures to lack of effort more than females do. Since gender-related differences in mathematics achievement tend to occur in high-level cognitive tasks, Weiner's approach to affective issues appears to be especially important for research on mathematical problem solving.

Fennema and Peterson (1985) have addressed these gender-related differences in high-level cognitive tasks by developing a model of autonomous learning behavior (ALB). This model does a good job of describing the characteristics that make up good problem solvers. The ALB model provides a useful guide for research on gender-related differences (Tittle, 1986), and it also has important implications for the teaching of mathematical problem solving. The ALB model (Fennema & Peterson, 1985) suggests that gender-related differences in complex cognitive tasks (such as mathematical problem solving) result from differences between girls and boys in autonomous learning behaviors. These behaviors, in turn, are a consequence of internal motivational beliefs as well as societal influences, including particularly the society of the classroom. Their discussion of internal motivational beliefs centers on Weiner's ideas about causal attributions, especially on how differences in causal attributions are related to differences in confidence. In their analysis of societal influences, Fennema and Peterson (1985) pay particular attention to the nature of teacher-pupil interactions in the classroom. Although their analysis concentrates on differences between the treatment of girls and boys in the classroom, the nature of teacher-pupil interactions in classrooms can often lead to beliefs about mathematics that work against problem-solving performance for both sexes. In this area, Fennema and Peterson's work has much in common with Schoenfeld's (1985) discussion of beliefs about mathematics, how those beliefs are often unintended consequences of the way mathematics is taught, and how those beliefs can interfere with performance in mathematical problem solving.

In summary, research on causal attributions has produced a large amount of data that is relevant to affective issues in problem solving. The research provides useful background for the current concerns regarding beliefs about mathematics and their influence on problem-solving performance. This research area also provides interesting data and hypotheses about how differences in problem-solving achievement that are related to gender and race have developed (Fennema & Peterson, 1985; Matthews, 1984).

Current Issues in Research on Problem Solving

There are a number of important issues related to affect and problem solving need to be addressed from the framework of one or both of these theories if we are to make problem-solving instruction as effective as it needs to be. Three of the most important are the role of belief systems in problem solving, the nature of metacognitive processes, and the influence of particular instructional settings, especially the role of technology in the teaching of problem solving.

Belief Systems

Students' affective reactions are influenced by their beliefs about mathematics and problem solving. Silver (1985) suggests that the investigation of belief systems provides a promising new direction in research on mathematical problem solving. Schoenfeld (1985) has also emphasized the role of beliefs, especially beliefs about mathematics, as a major influence on problem-solving performance. Although a solid theoretical framework for research on belief systems in mathematical problem solving is still missing, the work on causal attributions provides useful information on how beliefs about self can influence a student's problem-solving performance. What remains to be constructed is a framework that will tie together beliefs about self and beliefs about mathematics so that instruction will be able to encourage beliefs that help rather than hinder good problem-solving performance.

From a cognitive science perspective, Mandler would see beliefs as part of the cognitive evaluation of the interruptions that occur during problem solving. Cognitive science has little to say about how such beliefs develop. However, in one interesting paper, D'Andrade (1981) takes an anthropological perspective in his discussion of the role of beliefs and affect in learning. From this perspective, beliefs and affective responses develop out of a process of guided discovery, where the guidance comes from the general cultural milieu. Providing students with appropriate problem-solving experiences so that they can develop facilitating beliefs about themselves and about mathematics will be a challenge for both teachers and curriculum developers.

Metacognition

The role of metacognition in mathematical problem solving has received increased attention in recent years. Garofalo and Lester (1985) have analyzed the various conceptions of metacognitive processes, both in terms of the individual's knowledge of cognition and regulation of cognition. The work on causal attributions tends to emphasize the development of confidence, a factor that is closely tied to the individual's knowledge of their own cognition. On the other hand. Mandler's information-processing perspective is more closely tied to notions of limited processing capacity and its consequences for the individuals' ability to regulate their cognitive processes. In brief, when students are blocked as they try to solve a problem, their limited processing capacity may be taken over by the need to deal with the frustration. As a result, they may not have the capacity to consider alternative problem solving strategies, and just continue to repeat the same unsuccessful strategy.

For further elaboration on this point, see McLeod (in press). Further investigation of both aspects of metacognition are needed.

Instructional Environment

New software and other technological developments can have a profound effect on the environment within which mathematical learning and problem solving occur. For an example, see the recent description of the Geometric Supposer (Yerushalmy & Houde, 1986). The relationships between new technologies (like the Geometric Supposer) and affective influences in the classroom form an important new research area. These technologies have an impact on the specific ("local") environment within which the individual student solves problems independently, as well as on the more global classroom environment where implementation of the new technology occurs. Mandler's theoretical position seems particularly appropriate for the local environment where the individual is working independently. Weiner's ideas have generally been applied more to the large-group setting of the classroom. In the larger environment of the classroom, the belief systems of both teachers and students will be challenged by the new technologies (properly implemented), and as a result, the influence of affect will be particularly important. For example, using the computer as a tool for teaching raises questions about authority in the classroom that will be very important to the feelings of many teachers. Neither of the theories under discussion has much to contribute to these kinds of classroom issues.

Future Research: Some Methodological Considerations

Mandler and Weiner each provide a framework for research on affect in mathematical problem solving, but they approach the task in quite different ways. Mandler's work comes out of the tradition of cognitive science, with its emphasis on observations of individuals, rather than judgments about groups. The main emphasis is on a microanalytic approach to building a theory that explains the development of the individual in detail. The methods that are used include individual interviews and protocol analysis, with a long-term plan for using the theory to provide a conceptual basis for computer simulations of learners.

Weiner's theory, on the other hand, is used more frequently to discuss differences between groups. Data are usually gathered through the use of questionnaires, and are analyzed using the usual statistical techniques.

The applications of the theory will also include models, but the model is frequently a statistical one, using path analysis or similar strategies (Reyes, 1984).

Both sets of methods are useful for the study of affect and problem solving. However, researchers who approach the area from the point of view of research on problem solving (with its emphasis on cognitive science methodology) will probably feel more comfortable with Mandler's ideas. And other researchers, who approach the area from the point of view of affect (and this includes most work on gender-related differences) will probably continue to use the methods of Weiner. Based on the discussion presented in this paper, I hope that both sets of people will realize that they are working on the same problems and that mutual cooperation and support are not only possible, but also desirable.

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Some Causes of Success and Failure

	Internal		External	
	Stable	Unstable	Stable	Unstable
Uncontrollable	Ability	Mood	Task Difficulty	Luck
Controllable	Typical Effort	Immediate Effort	Teacher Bias	Unusual Help From Others

Adapted from Weiner, 1979, p. 7.