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ABSTRACT

This research identified K-6 children's (N=1202) and teachers' (N=54) perceptions of what constitutes doing or using mathematics. Perceptions were documented by a questionnaire which included items reflecting the six major strands of the K-6 syllabi and explicit and implicit use of numbers and operations as both facilitating and distracting elements. Children were given a situation and asked whether the person in the situation was doing or using mathematics. Teachers' perceptions were identified through discussions of questionnaire content and results and taping of classroom discussions with students. The results showed that children's perceptions of the domain of mathematics, although constrained by an emphasis on topics of arithmetic, were reasonably amorphous and fluid and were influenced by developmental factors, explicit numbers and operations, perceived difficulty and activity level of the situation, linguistic factors and experience. (Author)

Kindergarten Through Sixth Grade Students' Concepts of the Domain of Mathematics

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A paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

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This research identified K-6 children's (n = 1202) and teachers' (n = 54) perceptions of what constitutes doing or using mathematics. Perceptions were documented by a questionnaire which included items reflecting the six major strands of the K-6 syllabi and explicit and implicit use of numbers and operations as both facilitating and distracting elements. Children were given a situation and asked whether the person in the situation was doing or using mathematics. Teachers' perceptions were identified through discussions of questionnaire content and results and taping of classroom discussions with students. The results showed that children's perceptions of the domain of mathematics, although constrained by an emphasis on topics of arithmetic, were reasonably amorphous and fluid and were influenced by developmental factors, explicit numbers and operations, perceived difficulty and activity level of the situation, linguistic factors and experience.

Kindergarten Through Sixth Grade Students'
Concepts of the Domain of Mathematics

In the past few years while problem-solving researchers have grown increasingly concerned with noncognitive issues (McLeod, 1985), researchers in the affective domain have become increasingly interested in the more cognitive issues (Armstrong, 1985; Kulm, 1980; Lindquist, Carpenter, Silver & Matthews, 1983). One area of common interest for researchers both in the cognitive and affective domains is the link between concepts or perceptions and attitudes or beliefs. For example, students' or teachers' attitudes toward the utility or difficulty of mathematics might be biased by a perception that mathematics involves only arithmetic operations. This, in turn, may influence how mathematics is taught (Thompson, 1985) or how mathematics is learned or approached (Schoenfeld, 1983). While there has been much research on how concepts and processes are acquired (Ginsburg, 1983; Lesh & Landau, 1983), and some research on teachers' perceptions of the domain of mathematics (Thompson, 1985), there has been less research on the identification of exactly what children see as the domain of mathematics.

Educators concerned with curriculum reform might also be concerned with the effects of the perceptions of the domain of mathematics on what mathematics is actually being taught and learned in elementary classrooms. Curriculum reform in the past 20 years has led to a broadening of the scope of mathematics with the inclusion of topics from probability, statistics, geometry and measurement (Fey, 1979b; NCTM, 1980, 1981). These topics have been successfully placed into both syllabi and textbooks, but educators have questioned how successfully these topics have been included in classrooms and in children's understanding of what is mathematics and where it can be applied (Fey, 1979a; NAEP, 1983). Here, again, both students' and teachers' perceptions of the domain of mathematics influence what does or does not happen in the classroom. Thus, a vital area of research is the investigation of the link between perceptions and other cognitive and affective aspects of mathematics. A first step in that research must be the identification of students' and teachers' perceptions. To that end, this study was designed to identify K-6 children's and teachers' perceptions of what constitutes mathematics.

Method

Subjects

The subjects were 1202 children, and their 54 teachers, in kindergarten through sixth grade from 8 different school districts: one large urban district, three medium to large suburban districts, two small to medium rural districts, one medium rural/suburban district and one small parochial district. Twenty schools in all participated in the study.

Procedure

Children's and teachers' concepts of the domain of mathematics were investigated through the process of administering and discussing a ten-item (grades K-3) or twenty-item (grades 4-6) questionnaire. The children were read a situation and asked to indicate by circling YES or NO whether the protagonist in the situation was doing or using mathematics. Four different questionnaires were constucted in a stratified random manner from a pool of 40 core items (see Table 1). Questionnaire items were judged to be suitable in length, clarity and context through pilot tests with children and reviews by elementary teachers and college mathematics educators. The 40 core items also were reviewed and edited by the 54 teachers involved in the study.

The questionnaire items were designed to reflect the six major content strands of the New York State K-6 syllabus: number and numeration, operations with whole numbers and fractions, probability, statistics, geometry, and measurement. Twelve items included paired explicit (E) and implicit (I) use of cardinal numbers both as faciliating (F) and distracting (D) elements (e.g., Table 1, items B2, B4, B7, and B8). The items also varied from ones where the operational process was clear (e.g., items A8 and D1) to those where it was necessary to infer the mathematical processes involved (e.g., items A6 and B2). Items in which the protagonist clearly was not using or doing math were also included (e.g., items A2, A9 and B5). Classes were assigned questionnaire forms in a stratified random manner to insure that forms A, B, C, and D were spread uniformly across grade levels.

Table 1

FORM A

Did the boy or girl in the sentence use math? Circle YES if the boy or girl used math. Circle NO if the boy or girl did not use math.

A1. Billy put the blocks in piles. Each pile had four blocks. Did Billy use or (or do) math?
A2. Sue rode her bike to school. Did Sue use (or do) math?
A3. Betsy made Valentine cards by cutting out hearts using folded paper. Did Betsy use (or do) math?
A4. Fred watched his pet turtle swim across the turtle dish. Did Fred use (or do) math?
A5. Julie kept track each day of how many miles she rode on her bike. Did Julie use (or do) math?
A6. Ted made half a batch of cookies by using half the amount of each ingredient. Did Ted use (or do) math?
A7. Alan helped his Mom decide how much carpet they should buy for the livingroom. Did Alan use (or do) math?
A8. Linda added 3 and 2 on the calculator and got 5. Did Linda use (or do) math?
A9. Ellen read a book about trees. Did Ellen use (or do) math?
A10. Jim drew a picture to show how many hours he sleeps each night. Did Jim use (or do) math?

FORM B

B1. Alan took out his ruler and measured his desk. Did Alan use (or do) math?
B2E(F). Mary had 3 candy bars and Sally had 2 candy bars. Tim said that they had 5 altogther. Did Tim use (or do)math?
B2I. Mary and Sally had some candy bars. Tim knew how many they had altogether. Did Tim use (or do) math?
B3. Billy looked at the clock to see how long a nap he could take before the soccer game. Did Billy use (or do) math?
B4E(F). Terry went to McDonalds. She paid the lady $1.50 for a hamburger and a coke. Did Terry use (or do) math?
B4I. Terry went to McDonalds. She paid the lady for a hamburger and a coke. Did Terry use (or do) math?
B5. Mark visited the museum to see the dinosaurs. Did Mark use (or do) math?
B6. George said that half of 5 is between 2 and 3. Did George use (or do) math?
B7E(D). Susie ran over to Sally's house to see her first dog. Did Susie use (or do) math?
B7I. Susie ran over to Sally's house to see her dog. Did Susie use (or do) math?

Table 1 -cont.-

FORM B -cont.-

B8E(F). Tommy's lunch bag had 3 brown candies and 5 red candies. He closed his eyes and reached in. Tommy knew that he had a better chance of getting a red candy. Did Tommy use (or do) math?

B8I. Tommy's lunch bag had more red candies than brown candies. He closed his eyes and reached in. Tommy knew that he had a better chance of getting a red candy. Did Tommy use (or do) math?

B9. Chuck noticed that the butterfly had the same pattern on both wings. Did Chuck use (or do) math?

B10. Dave sat in the car and looked out of the window at the rain. Did Dave use (or do) math?

FORM C

C1. Chris cut out shapes of squares and circles and made a design. Did Chris use (or do) math?

C2. Sally played Pacman on her home computer. Did Sally use (or do) math?

C3E(F). Melanie had to tell the teacher which number was greater, 5 or 3. Did Melanie use (or do) math?

C4. Dave played soccer yesterday afternoon. Did Dave use (or do) math?

C5E(F). Anne had 6 stickers to put on three pages, so she put two stickers on each page. Did Anne use (or do) math?

C5I. Anne put the same amount of stickers on each page of her book. Did Anne use (or do) math?

C6. Two girls were given three cookies. Three boys were given six cookies. Vicky said, "That's not fair!" Did Vicky use (or do) math to decide?

C7E(D). George cleaned up room number 7 which was really messy. Did George use (or do) math?

C7I. George cleaned up the room which was really messy. Did George use (or do) math?

C8. Leslie guessed that the penny would come up heads when she tossed it. Did Leslie use (or do) math?

C9. Tom said that 25 cents is the same as a quarter. Did Tom use (or do) math?

C10E(F). Sherrie lifted her little brother. She said that he must weigh about 30 pounds less than she does. Did Sherrie use (or do) math?

C10I. Sherrie lifted her little brother. She said that he weighs less than she does. Did Sherrie use (or do) math?

Table 1 -cont.-

FORM D

D1. Ed added 3 and 1 and got 4. Did Ed use (or do) math?
D2E(F). Jane counted ten trucks that passed by the school. Did Jane use (or do) math?
D2I. Jane counted all of the trucks that passed by the school. Did Jane use (or do) math?
D3. Sam said that half a candy bar is better than a third. Did Sam use (or do) math?
D4. Tina learned about the Pilgrims. Did Tina use (or do) math?
D5E(F). George put all three red marbles in one cup. Then he put both blue marbles in the other cup. Did George use (or do) math?
D5I. George put all of the red marbles in one cup. Then he put all of the blue marbles in the other cup. Did George use (or do) math?
D6. Jack said that he was sure that there was no school tomorrow because it was Saturday. Did Jack use (or do) math?
D7. Fran found all of the squares in the pictures on the page. Did Fran use (or do) math?
D8E(D). Tom watched Smurf cartoons on television for two days in row. Did Tom use (or do) math?
D8I. Tom watched Smurf cartoons on television. Did Tom use (or do) math?
D9. Gerry tried to decide if there was more soda in a can than in a bottle. Did Gerry use (or do) math?
D10E(F). Denise knew that there was a 20% chance that the ballgame would be rained out. Did Denise use (or do) math?
D10I. Denise knew that there was a chance that the ballgame would be rained out. Did Denise use (or do) math?

The questionnaires were administered to the students by their classroom teachers. The teachers were given printed directions on the procedures for the administration of the questionnaires which included instructions on audio-taping the administration; on explaining the YES or NO format of the response sheet to the students, and on the need for emphasizing that the questionnaire items had no right or wrong answer and were assessing only student opinions. The tapes of the administration were checked for teachers' adherence to the directions. Only one discrepancy was found. One sixth grade teacher supplied students with copies of the questionnaire and had the students read the items for themselves rather than reading the items to the students. Since there was little difference in the time taken to complete the questionnaire and little difference on the overall YES/NO responses for that class compared to other sixth-grade classes, the data were retained in the study.

Immediately after the children completed the questionnaire, the teachers conducted an audio-taped discussion in which the children gave their rationale for deciding whether the protagonist in each item was doing or using mathematics. The teachers had been informed that the purposes for the discussion were to document the students' rationales for their answers and to help expand the students' perceptions of the domain of mathematics. For the discussion the teachers again were asked to stress that there were no right or wrong answers. The teacners were also asked not to state what they considered to be the right answer. Rather, after the students had responded, they could suggest rationales for both YES and NO answers.

Data Analysis

The data analysed were of three types: tabulation of students' written YES/NO responses to the questionnaire items, examination of taped student rationales for YES/NO choices, and examination of the teachers' perceptions and reactions.

For each class, students' written YES and NO responses were tabulated on each questionnaire item. For each item, the student responses were matched with the syllabus-specified designation of whether the item involved mathematics. The percent of students agreeing with that designation was recorded for each grade level. Mean agreement across grades K-6 was calculated for each item and the items were ranked by percentage of agreement.

Comments from the transcripts of the taped discussion of the students' rationales for their YES or NO responses were sorted into three types for each item: rationales for responding YES, rationales for responding NO, and irrelevant and seemingly illogical rationales. The comments used in this portion of the analysis included only those given by the students before the teacher gave suggestions or made leading or directive comments about the item.

The comments on why the students thought the protagonist was doing or using math were sorted into categories so that for each of them a relative ranking of most commonly given reasons could be identified. Then these reasons were compared with the syllabus and identification of the mathematics involved in each of them.

The comments on why the student did not think the protagonist was doing or using math were sorted into two groups. The first group was labeled "parroted" responses and consisted of the student essentially saying that the item was not math, with no other rationale given. For example, for item A2, a parroted response would be, "Riding a bike to school isn't math." The second group of students' "no" comments were of two types: those comments clearly identifying what math wasn't (e.g., "using a ruler isn't math") and those comments implying what math was (e.g., "no math was used because there wasn't any addition"). These two types were categorized and tallied in order to identify a relative ranking of these reasons across grades and items.

For both the YES and NO categories exemplars of comments for each category were identified, as well as the irrelevant, illogical responses. Finally, the analysis of the student rationales was examined in conjunction with the ranking of items by percent of agreement with the syllabus-specified designation of whether an item depicted the protagonist using or doing math.

The teachers' perceptions were identified through four indirect means: a) a seminar prior to the administration of the questionnaire during which the teachers suggested changes in the wording of the questionnaire items; b) teachers' taped responses to children's rationale during discussion; c) teacher's comments during a seminar in which results of student data were reported; and d) teachers' summary of the important comments students made during the discussion of the rationales for answers.

Results and Discussion

Percent of Student Agreement by Grade Level

For each of the 40 core items, the student answers on the paper and pencil response sheet were matched against the syllabus-specified designation of whether the protagonist in the item was doing or using math. The percent of students at each grade level agreeing with the syllabus designation are given in Tables 2-5.

The patterns of student responses to many items reflected developmental differences in children's perceptions of the domain of mathematics. On several of the items, kindergarten and first graders responded at a nearly random level (Table 2, A3, A6 and A9; Table 4, C1). But, for the higher grade levels, the response rates on these items generally exhibited a steadily increasing pattern. It appears that the kindergarten and first-grade children often did not understand what was going on and, thus, were randomly choosing YES or NO as a response. Further support for this appears in the data in the tables (Table 2, B1; Table 3, C3 (I) and C9; and Table 4, D1, D3, and D4) and in the anecdotal data from the tapes and meetings with teachers. Many of the irrelevant and illogical rationales came from kindergarten and first-grade students and included such statements as: "he was smart" (C1), "he forgot to do the math" (B5), "she felt like it" (C2), "she liked the number 5" (C3), and "she didn't want any cookies so she did math" (C6).

Table 2
Percent of Students Agreeing with Syllabus Designation of Whether Mathematics was Used: FORM A

Item #	Syll. Desig.	K n=0	1 n=23	2 n=60	3 n=30	4 n=114	5 n=87	6 n=86	Mean Agreement
				GRADE LEVEL					
A1	YES	-	74	53	70	78	77	73	72.1
A2	NO	-	78	88	100	98	99	99	95.9
A3	YES	-	52	7	10	15	24	26	19.9
A4	NO	-	87	85	97	96	99	100	95.4
A5	YES	-	74	37	97	92	99	96	85.5
A6	YES	-	52	45	83	81	90	96	79.3
A7	YES	-	65	47	67	65	86	88	72.0
A8	YES	-	91	90	100	96	94	87	92.7
A9	NO	-	52	83	97	96	100	99	93.1
A10	YES	-	57	45	80	61	92	81	70.8

Table 3
Percent of Students Agreeing with Syllabus Designation of Whether Mathematics was Used: FORM B

		GRADE LEVEL							
Item #	Syll. Desig.	K n=0	1 nE=24 nI=20	2 nE=26 nI=0	3 nE=26 nI=50	4 nE=47 nI=71	5 nE=77 nI=44	6 nE=53 nI=47	Mean Agreement
B1	YES	-	43	19	51	70	87	90	70.2
B2E	YES	-	75	69	92	91	97	100	91.0
B2I	YES	-	65	-	76	97	89	68	82.3
B3	YES	-	23	42	47	66	72	85	63.2
B4E	YES	-	42	62	50	62	60	66	59.1
B4I	YES	-	30	-	64	72	55	72	63.4
B5	NO	-	84	88	97	98	97	100	96.2
B6	YES	-	80	77	93	96	96	96	93.1
B7E	NO	-	79	92	100	100	99	96	96.0
B7I	NO	-	95	-	100	100	100	100	99.6
B8E	YES	-	67	35	58	64	65	66	61.4
B8I	YES	-	45	-	48	70	66	72	62.8
B9	YES	-	50	12	12	14	14	4	14.8
B10	NO	-	89	88	100	96	98	100	96.9

Table 4
Percent of Students Agreeing with Syllabus Designation of Whether Mathematics was Used: FORM C

Item #	Syll. Desig.	K nE=24 nI=21	1 nE=19 nI=19	2 nE=17 nI=23	3 nE=20 nI=23	4 nE=44 nI=47	5 nE=72 nI=63	6 nE=53 nI=47	Mean Agreement
		GRADE LEVEL							
C1	YES	64	43	9	33	11	34	35	31.2
C2	NO	67	78	86	91	90	93	96	88.8
C3E	YES	75	68	94	90	100	88	96	89.6
C3I	YES	48	89	91	100	98	100	98	93.0
C4	NO	64	92	96	98	92	91	95	90.6
C5E	YES	42	37	52	15	91	96	94	75.5
C5I	YES	24	39	43	70	64	83	76	64.4
C6	YES	47	32	73	77	82	80	80	72.8
C7E	NO	37	68	91	100	98	95	98	88.6
C7I	NO	71	84	100	87	94	97	100	93.1
C8	NO	60	57	93	79	91	90	94	85.0
C9	YES	40	70	84	88	80	85	65	74.9
C10E	YES	67	37	76	55	68	76	81	70.1
C10I	YES	29	47	30	61	47	73	57	53.8

Table 5
Percent of Students Agreeing with Syllabus Designation of Whether Mathematics was Used: FORM D

		GRADE LEVEL							
Item #	Syll. Desig.	K n=0	1 nE=17 nI=0	2 nE=25 nI=44	3 nE=28 nI=0	4 nE=50 nI=45	5 nE=52 nI=73	6 nE=38 nI=48	Mean Agreement
D1	YES	-	65	99	100	98	98	100	97.4
D2E	YES	-	18	28	64	80	83	89	69.0
D2I	YES	-	-	50	-	96	88	94	83.1
D3	YES	-	53	26	29	78	90	90	71.2
D4	NO	-	47	94	96	75	98	99	9D.1
D5E	YES	-	47	64	21	54	83	47	56.1
D5I	YES	-	-	52	-	33	26	17	30.9
D6	NO	-	76	94	96	97	97	91	94.3
D7	YES	-	35	23	11	27	33	30	28.0
D8E	NO	-	88	96	89	96	92	87	91.8
D8I	NO	-	-	98	-	100	92	96	95.9
D9	YES	-	65	49	64	73	82	81	72.5
D10E	YES	-	35	16	11	42	54	55	39.5
D10I	YES	-	-	5	-	9	11	4	7.7

Kindergarten and first-grade teachers also reported that their students seemed "lost." Many first-grade teachers, who initially thought their students would have little trouble understanding the items or the task, reported on their summary sheets and in a follow-up workshop that they were surprised by their students' lack of understanding.

A second pattern that emerged was that on items A1, A5, A7, B2, B8, D3 and D9 the second-grade students' percentages were noticeably lower. This occurred across forms and with all ten of the second-grade classrooms. The rationales given by the second-graders offered little explanation other than they often mentioned that a situation <u>is</u> math because counting or addition is involved and that a situation <u>is not</u> math because there is no addition or subtraction. While these comments were not unlike those at other grade levels, it could be that second-grade students cue more on addition than students at other grade levels. This would be consistent with emphasis placed by the syllabus and textbooks on formal addition (the two-digit algorithm) at this grade level. Further differences in the second-grade data occur in items (B4E, C5E, C8, C10E and D5E) on which the second-grade agreement percentages were noticeably higher than grades one or three. Since these mostly involved the explicit use of cardinal numbers, the students appeared to key more on the cardinals than on other aspects of the situations. However, in general, these researchers were unable to determine a clear explanation of the differences present in the second grade data.

A third pattern of responses occurred in the paired items which had explicit and implicit use of cardinal numbers. In general, for the items in which explicit numbers were intended to be faciliting, the explicit forms had higher mean agreements (B2, C5, C10, D5, and D10) and for the items in which the cardinals were intended to be distracting, the implicit forms had higher mean agreement (B7, C7, and D8). For the facilitating items the responses were either consistently higher across all or most grade levels or showed some grade level differences which were generally developmental in nature. Analysis of student rationales indicated that the youngest students tended to cue in on number and counting much more frequently. The differences between C5E and C5I were also made greater by the unintentional removal of the division operation in the implicit form.

The apparent reversal in item B8 for explicit versus implicit was explained through an examination of the student rationales. Although more students identified B8I as mathematics, their responses tended to be more incorrect and included reasons such as, "it's addition," "subtraction," or "division." These results emphasize the necessity for a collection of reasons as well as number counts for such data.

Other differences in explicit and implicit items which were noted in the responses occurred because students often projected beyond the situation described in the item when the numbers or operations were implicit. Sometimes these projections were inappropriate or incorrect, but they did yield more student responses in agreement with the syllabus designation that mathematics was used. For example, many students projected counting into several situations.

Three of the items showed a reversal of the expected effect of the explicit cardinals (B4, C3, and D2), in that the implicit items received a higher percent agreement than the explicit items. Analysis of the student rationales for these items also helped to provide possible explanations. In items C3 and D2, the choice of the small numbers such as 3, 5 and 10, may have affected the results by making the problems seemingly too "easy" to be math. In the rationales for these items, many students made statements such as "you just know." Thus, for item D2, more students may have felt that counting all of the trucks was more mathematical than counting just 10 trucks. One second grade student offered further explanation, saying, "What if she had to count to 100...then it's math." For item C3, comparing two unknown numbers was much more "math-like" than just comparing 5 to 3. For item B4, no similar explanation was found in the data.

Mean Percent of Agreement Across Grades

The questionnaire items were rank ordered by mean percent of student agreement with the syllabus designation of whether the protagonist in the item was doing or using mathematics. The rank order, as well as both the syllabus or expert descriptions of the mathematics involved in each item and the listing of the students' primary rationales for why an item involved mathematics are given in Table 6.

High-rate-of-agreement items. Of the ten items with the highest percent of agreement, nine are items involving no math. Students, therefore, are quite adept at identifying when a situation involves no mathematics. However, since no item had 100% agreement across the grades, there were some students who "read" mathematics into the situations. These students generally saw counting, addition or numbers in the items and concluded that the items were, therefore, mathematics. A few of the students also occasionally associated money, distance or time with a non-math item. For example on item A2, (Sue rode her bike to school), students said it was math because "she could find out how many miles" and it "involved time." On item B5 about a trip to the museum to see dinosaurs, students responded, "He had to pay to get in" and "He might have looked at the clock to see how long he was there."

The only mathematical item that ranked in the top ten on percent of agreement was item D1 about adding 3 and 1 to get 4. Even though 6 of the first-grade students and a few of the second-,fourth-and fifth-grade students responded that the protagonist in the situation wasn't doing or using mathematics, no student offered a rationale for that response. Also, all but one of the rationales for why the item involved mathematics were that it involved "addition, which is math." The one exception was given by a second-grade student who

Table 6
Rank Ordered Mean Percent of Student Agreement with Syllabus Designation of Whether Mathematics was Used: FORMS A-D

% Agree with YES or NO	Item #	Primary syllabus Description(s)	Primary student Description(s)
99.6	B7I	no math	no math
97.4	D1	addition	addition
96.9	B10	no math	no math
96.2	B5	no math	no math
96.0	B7E	no math	no math
95.9	A2	no math	no math
95.9	D8I	no math	no math
95.4	A4	no math	no math
94.4	D6	no math	no math
93.1	C7I	no math	no math
93.1	A9	no math	no math
93.1	B6	estimation,division	numbers addition,division
93.0	C3I	comparing,ordering	comparing,numbers
92.7	A8	calculator skills addition	calculator skills addition
91.8	D8E	no math	no math
91	B2E	addition	addition,counting
90.6	C4	no math	no math,counting
90.1	D4	no math	no math
89.6	C3E	comparing,ordering	comparing
88.8	C2	no math	no math addition,counting
88.6	C7E	no math	no math counting,numbers
85.5	A5	data gathering	counting,adding
85	C8	no math	no math addition, money
83.1	D2I	counting	counting,numbers
82.3	B2I	addition	addition,counting
79.3	A6	division measurement	division measurement
75.5	C5E	division grouping	division counting, no math
74.9	C9	money equality	money,counting no math
72.8	C6	division comparing	comparing,counting addition,no math
72.5	D9	estimating volume,comparing	measuring numbers,no math
72.1	A1	grouping counting	counting,division addition,no math

Table 6 -cont.-

% Agree with YES or NO	Item #	Primary syllabus Description(s)	Primary student Description(s)
72	A7	measuring,area multiplication	measuring figuring out,no math
71.3	B1	measuring	measuring addition,no math
71.2	D3	comparing fractions	fractions comparing,no math
70.8	A10	graphing data gathering	addition graphing,no math
70.1	C10E	estimating,weight comparing	subtraction,estimating no math
69	D2E	counting	counting,no math
64.4	C5I	equality (grouping)	counting,equality division,no math
63.4	B4I	money	money,counting addition,no math
63.2	B3	time subtraction	time,counting no math
62.8	B8I	probability	counting comparing,no math
61.4	B8E	probability	comparing addition,no math
59.1	B4E	money	no math,money counting,addition
56.1	D5E	sorting classifying	no math counting,weight
53.8	C10I	estimating weight,comparing	no math comparing,weight
39.5	D10E	percent probability	no math percent
31.2	C1	geometry	no math geometry,counting
30.9	D5I	sorting classifying	no math counting,addition
28	D7	geometry classifying	no math,counting shapes,addition
19.9	A3	symmetry geometry	no math counting,fractions
14.8	B9	symmetry geometry	no math counting,pattern
7.7	D10I	probability	no math,time

said, "He might have wanted to know something." Although the problem can be readily solved by counting, and counting strategies are some of the most used methods for solving addition problems (Carpenter, Moser & Romberg, 1982), no one said it was counting. In fact, the only two items for which no student offered a counting rationale were this item, D1, and the only other item that used term "added," item A8. This is surprising, given that counting was the most often stated primary rationale for an item being mathematics.

Relationship of rationales to mean agreement. It is important to note at this point that although on several items students had a high percentage of agreement that the items were mathematics, the analysis of the student rationales revealed that, in many cases, the students designated a skill or concept at a lower level than did the experts or the syllabus did. For example, although the syllabus designated item A5, (keeping track) as data gathering or statistics, the students saw counting and adding. This was true not only on items of high agreement, but on most items. Of greater concern, then, are items such as A3 (symmetry in valentines) where only 19.9% of the students agreed that this was mathematics. They agreed, not because they identified the symmetry or geometry involved in the problem, but because they saw counting or fractions. Of equal concern are the items for which students misapplied a mathematics concept or operation. They often incorrectly identified the use of subtraction or division in many items In fact, for every item, there was always at least one student who thought it was subtraction. Similarly, some students thought item C3E (comparing 3 and 5) involved division. These results which identify improper application of subtraction and division to problem situations, are also consistent with the most recent National Mathematics Assessment data (Lindquist, et al., 1983).

Narrow domain of mathematics. As expected, several of the items (B8, D10, C1, D7, A3, and B9) drawn from the less traditionally elementary topic areas of geometry, statistics and probability were not generally accepted as being within the domain of mathematics. Although these items were drawn directly from syllabus examples, students generally did not consider bilateral symmetry, shape identification or probability to be math. Possible explanations of these low rates of identification would include higher proportions of time spent in the classroom on whole numbers and computational skills (Lindquist, et. al., 1983), a lack of appropriate development of language and terminology to match the situation (similar in concept to Zweng, 1979) and lack of consistent identification of these areas as being mathematics, as reported by teachers in this study during a follow-up workshop for showing results. Also, as with the effect of the small cardinals described earlier, students may have seen examples of symmetry and shape identification as being too "automatic" or too "easy" to be mathematics.

Items which involved classification, identification or sorting were correctly identified as mathematics by a relatively low percentage of students (A1, 72.1%; D5E, 56.1%; D5I, 30.9%; and D7, 28%). Based on student comments, responses to item A1 would have been much lower without the explicit cardinal and a projected use of counting. Responses to items D5E and D5I indicated that the students rejected the notion of classification based on color as a mathematical process. Responses to D7 indicated that finding the squares in a picture primarily involved only "seeing," "looking," or "thinking," and no mathematics.

Patterns in students' rationales

Active nature of mathematics. For some students, the nature of the action involved in a situation, that is whether or not it was active or passive, influenced their decision of whether the protagonist was doing or using mathematics. In general, the students perceived mathematics as active. In order to do or use mathematics, a person must "do something." This was most evident in the students' reasons for deciding that no math was involved in a given situation. For example, the students said:

> Just noticing isn't math (B9).
>
> Seeing patterns by looking isn't math (B9).
>
> It's just telling, not doing (C3).

This active attribute was evident for all items except when the use of a tool was involved. The protagonist could be actively using a tool, yet not be doing or using mathematics. The students said that the tool was doing or using the mathematics, not the person. There was ample evidence of this in the rationales, such as:

> The ruler does it for you, really (5th, B1).
>
> He's not using math; he's using a ruler (3rd, B1).
>
> The cash register does it (6th, B4).
>
> She really didn't do math. The calculator told her the answer. She didn't add herself (5th, A8).
>
> It wasn't math because she could have had a scale (4th, C10).

Attributes of mathematics. Analysis of the written and verbal statements made by students established a reasonably clear set of items or activities which students, in general, considered to be mathematics and an equally clear set of concepts and processes which most students did not consider to be a part of the mathematics domain. Alternately, there were several concepts which prompted arguments about their inclusion in the domain.

Overwhelmingly, students saw mathematics when they were able to recognize counting, adding, and numbers. They not only chose these descriptors for situations which they identified as being mathematics, they also consistently used the absence of those as being sufficient explanation for why a situation was not mathematics. One interesting aspect of the data, however, was that while students saw counting as a rationale for the protagonist doing or using mathematics twice as often as addition, they chose the absence of adding most often to be the reason for saying that the protagonist was not doing mathematics. Although the greater ease with which students could project counting into the mathematics items might explain the higher occurance of counting in their "yes" rationales (students saw counting everywhere, for example, "He counted the turtle steps" (A4, 6th)), it does not explain the primary use of the absence of addition for the "not math" rationales.

Other concepts and operations which were often identified as reasons for being mathematical were, in rank order, numbers, measuring, comparing, money, time, subtraction and division. Often students also either parroted the item (for example, "It's math because you're deciding how much carpet"), or they used phrases such as "it's math because you're figuring it out." The frequent use of parroting and imprecise language may be a reflection of the students' lack of knowledge of the appropriate mathematical terminology.

Many of the same mathematical concepts and operations used as reasons for an item being mathematics were also used in the rationales for items not being classified as mathematics. However, it was the absence of these that kept an item from being mathematics. These concepts and operations, in rank order were: addition, counting, subtraction, numbers, and measuring. Primarily, though, students' "no math" rationales included situation specific statements such as "reading a book is not math" or "making half a batch of cookies is not math."

Of particular interest were the terms or concepts which some students used to argue a "pro" case and others used to argue against the presence of math. These included, in rank order: money, counting, time, guessing, measuring, calendar skills, shapes, calculator/computer skills, drawing pictures, dividing up (classifying), using a ruler, patterns, numbers, comparing, chance, percent, making designs, fractions, geometry, equality, adding(1), timing(1), and predicting (1). These items were identified in one of several ways. Most often, the student simply made a statement such as "measuring isn't math," or "shapes aren't math," or "math is not on a clock." Other statements were more like: "He was looking at the clock and could figure out what time it was and then he could add them, but that's not math, he wasn't doing math because he knew he was going to miss the soccer game" (B3, 4th), and "counting isn't math because there's no signs" (D2, 3rd). It is quite possible that, if the students who made the last two statements were directly asked, "Is adding math?", or "Is counting math?" they might say "yes." However, there was something about the particular situation presented to the students which made them decide that what might otherwise be mathematics, was not mathematics in the specific item.

Several of the student responses prompted heated debates among students about whether or not certain concepts were in the domain. These included discussions of measuring (and using a ruler), calculator/computer use, time, money and counting. One of the best exemplars of these debates occurred on item D2 in a fourth grade classroom:

S_1: Yes, counting is math.

S_2: No, counting isn't math because it isn't addition or subtraction.

S_3: It is too, because when you do math that's what the whole thing is...numbers. If you don't know the numbers you aren't going to be able to add or subtract.

S_4: But if you know how to count then you can count to find 3+1...

T: So, counting is adding?

S_4: Yeah.

T: Anyone else?

S_5: No, just counting isn't math because you don't have a plus sign. It's only math because it has numbers, but there's no math problem part like with adding or subtracting.

Other students, especially the younger ones, consistently attempted to use counting as their only gauge for the presence of math, for example "You can't count how much he weighs," (C10, 1st) or "How can you count how many carpets?" [to determine how much carpeting to purchase] (A7, 2nd).

There were two other important sets of reasons which students gave as "no math" rationales. One dealt with the notion that subject areas are mutually exclusive. If a student could identify an item as being art or social studies, then no mathematics could be involved. For example, on item C1, the following exchange occurred in a sixth-grade classroom:

S_1: It was actually a design that she was doing and that's part of art.

T: Part of art? and math and art can't come together?

S_1: Yeah, they can't.

T: Okay.

The second set of reasons was related to the idea of mutually exclusive activities, but was more global. Many students, believed that you cannot do two things at once. For example, you cannot "cut and do math at the same time," "swim and do math at the same time," "read and do math at the same time," or "be in a restaurant and do math at the same time." Some students even directly stated that you "cannot do two things at the same time." Since this occurred primarily with first and second-grade students, this may be a developmental aspect of children's learning.

Product and process. The responses of both students and teachers included the use of labels from both product and process orientations. Each of the 40 items included rationales from students related to mathematics as an object as well as an action. For example, "a ruler is math," and "measuring is math." Since rationale data for each individual student were not available, it is difficult to get a clear indication of whether or not this pattern was consistent within individuals, but the data do reflect the consistent use of both product and process labels across classes and grade levels.

Mathematics is not a static domain. From both the arguments over which concepts or operations were mathematics and the rationales given for the "no math" responses, it became clear that the domain of mathematics is not fixed across grade levels. Once a mathematical operation or concept becomes easy, or becomes automatic, many students, even first grade students, no longer believe it is mathematics. Many students indicated that an item did not involve mathematics because the person could "just look and know" the size or length or relationship or solution. One first grade student explained this by saying, "If he knew already, it wouldn't be math." A sixth grader said, "It's knowledge. You know that, so it's not math. If you're a sixth grader you know" (B6). A fifth grader expressed it best, "You can simply look and know [that 5 is greater than 3]. It wouldn't be math unless you're very young." These results are related to those reported for the Third National Assessment of Mathematics (Lindquist et al., 1983) in which 9-year olds thought learning about money, solving addition problems, and learning how to measure with a ruler were easy. Also in the NAEP data (NAEP, 1983) is the fact that students are highly successful on what they think is easy. Taking all of this together, then, does this mean that mathematics is hard and involves a lot of work? There were many comments to that effect in the student rationales in this study: "it is math because its hard," "it isn't math because it isn't work." What happens, then, to students' attitudes and beliefs about their performance or ability in mathematics if the domain is continually shifting upwards in difficulty? Could this be related to math anxiety and math avoidance? How would this affect students' statements about the domain? Also, does this upward shift in children's perceptions of the domain occur in other subject areas such as science, reading, language arts or social studies?

The role of experience in mathematics. Students' experience also affects students' perceptions of whether or not a particular situation involves mathematics. Often those students having everyday experience with some situations drew from their experience to find reasons why mathematics might not be involved. For example, on the item involving the purchase of carpeting, some of the student rationales for "no math" responses were: "the manager does it," "you can just buy it," "because you get a little bit more and a little bit long," and "you just decide the color." Likewise, students used their experience to find reasons why some items did involve mathematics where "no math" responses were expected. For example, on item D8 about watching Smurf cartoons, some student responses were, "you use math to tell Smurfs apart -- like height." and, "sometimes Papa Smurf does chemicals and how much to put in a pot."

Experience also played a part in students' perception that mathematics has to do with school. For many students, situations can only be mathematical if they occur in school under given conditions. A fifth grader, arguing that counting was not math, said, "You can count before you actually learn how to do math." One fifth-grade teacher tried to convince the students that mathematics did not have to take place only in school:

T: Do you have to go to school to use math?

Ss: Yes.

T: Do you have to learn about math in order to use it?

Ss: (yes and no)

S_1: No, because your parents could tell you.

T: Well, your parents had to go to school. I mean, suppose you were out in the wilderness or suppose you were a prehistoric child. Would you use math?

S_2: No, you wouldn't know about pluses.

T: You could because if you had nuts to share you would divide them equally without learning--not knowing the terms--but just doing it.

S_3: No, because you wouldn't know like 41 times 41.

T: Okay, we know that prehistoric people didn't have a number system. But, I'm saying did they use math?

Ss: No, because they didn't know the numbers.

T: Okay, what I'm asking then, is do we have to know numbers to use math?

Ss: (yes and no)

S_4: Yes, because we need them to divide and stuff.

S_5: Maybe they had different numbers.

T: What about dividing nuts up? I could do it without knowing the numbers. If I gave Tom 4 and Jane 5, I could see Jane would have more. You understand more or less with number. . . and you know our measurement system and how that came about, using the parts of the body.... Let's go on to the next question.

Other similar discussions occurred, with the teachers not much more successful at changing children's beliefs.

Regarding children's beliefs about the link between school and mathematics, from the students' rationales it is clear that several students believe math is something you do in school, need a teacher for, need questions for, need a mathbook and paper and pencil for, can't do with your eyes closed and has to be right.

Developing a Concept of Mathematics

If one examines the results of this study within the framework of models of concept learning (Klausmeier, Ghatala, & Frayer, 1974; Sowder, 1980), treating "the domain of mathematics" as a concept in itself, it is evident that children have gaps in their conceptual development. There are two important gaps which have implications for the instruction of mathematics in the elementary classroom.

First, many students exhibited a lack of knowledge of appropriate labels and mathematical terminology. Students also misused and misapplied terminology. To bridge this gap, teachers can say and write the labels (e.g. symmetry, estimation, measuring) more often and link the mathematical terms to the students' informal descriptions and language (e.g. matching halves, guessing, figuring out).

Second, based on inferences from the teachers' comments, many students did not have experience identifying situations as mathematical or not mathematical, in the sense of learning to distinguish between examples and non-examples. Nor did the students have experience classifying examples with respect to categories of mathematics (e.g. geometry, statistics, and numeration). Thus, to bridge this gap, teachers can provide such experiences whenever possible throughout the school day.

Teacher Perceptions

Teacher's perceptions of the domain of mathematics and of what occurs in the mathematics classroom were examined through informal data collection and analysis. One informal method of data collection was done when the 54 teachers involved in the study were asked to review and edit the 40 core questionnaire items during a workshop session. The major observation was that the teachers fell into two groups regarding the use of the terms "use mathematics" and "do mathematics." The recommendation by the researchers was to follow the reading of each item with the question: Did the person in the story do or use math? Almost all of the teachers thought it unwise to use both the terms "use" and "do," however, they could come to no consensus on which term to use. Although many K-3 teachers favored "do" while many 4-6 teachers favored "use," there was no clear division between primary and intermediate grades. In the end, the forms were printed with both "use" and "do" and the teachers chose during the administration of the questionnaire which term to use. Only six of the teachers used "do." The rest used "use" or both. There was no apparent difference in how the students answered on either the paper and pencil responses or on the taped discussions of rationales.

The second method of informal data collection of teachers' perceptions was to compare the teachers' summaries of student rationales to the overall summary of rationales compiled from the researchers' transcripts of the taped discussions. The teachers were asked to jot down on a summary sheet the most important or most interesting rationales given by their students. Of the 36 teachers who commented as requested, all except two listed comments of significance that clearly matched those identified by the researchers as being important. Several of the teachers provided excellent and detailed summaries and also noted which of the children's responses suprised them the most. Many of the teachers were most surprised by the students' misconceptions and by the students' narrow perception of what was mathematics. One third-grade teacher commented:

> Even though a great deal of time is spent on time, money, and measuring, I was amazed at how entrenched the children were in the idea that "doing" math was adding or counting. They repeatedly used "adding" or "counting" in their comments. Even some of my advanced math students, who I thought would reason out the varied uses of math, did not.

Two important points from these teacher data are: (a) even when teachers' perceptions of the domain of mathematics were narrow or not clearly defined, they had no trouble in noting, without bias, the important aspects of their students' rationales, and (b) clearly, it was important for the teachers to hear the childrens' rationales and be involved in the discussions, for both their own perceptions of mathematics and of what goes on in the mathematics classroom began to change.

The third informal data collection method was a follow-up workshop in which the tabulation and analysis of the students' responses to the YES or NO questionaire were shared with the teachers. During this workshop some teachers expressed a frustration with the ambiguity of the task and with not having the "right" answers. Of these, some thought this was good to know about themselves; others did not think so. Several teachers also noted that their behavior had changed in that they were making increased references to the use of mathematics when it wasn't otherwise explicit. Other teachers said they realized this was the first time they had discussed "things like this" with the students and many teachers appeared to be aware, for the first time, that their students had a much more naive understanding of mathematics than they had expected.

The fourth method of data collection on teacher perceptions was in the teacher responses made during the taped discussions. These data were used in the previous sections of this paper to provide appropriate examples and explanations.

References

Armstrong, J.M. (1985). A national assessment of participation and achievement of women in mathematics. In S.F. Chipman, L.R. Brush & D.M. Wilson (Eds.), Women and mathematics: Balancing the equation (pp.59-94), Hillsdale, NJ: Lawrence Erlbaum.

Carpenter, T.P., Corbitt, M., Kepner, H.S., Lindquist, M.M. & Reys, R.E. (1980). Solving verbal problems: Results and implications from the national assessment. Arithmetic Teacher, 28, 8-12.

Carpenter, T.P., Moser, J.M., & Romberg, T.A. (1982). Addition and subtraction: A cognitive perspective. Hillsdale, NJ: Lawrence Erlbaum.

Fey, J. (1979a). Mathematics teaching today: Perspectives from three national surveys. Arithmetic Teacher, 27(2), 10-14.

Fey, J. (1979b). Mathematics teaching today: Perspectives from three national surveys. Mathematics Teacher, 72, 490-504.

Ginsburg, H.P. (1983). The development of mathematical thinking. New York: Academic Press.

Klausmeier, H.J., Ghatala, E.S., & Frayer, D.A. (1974). Conceptual learning and development. New York: Academic Press.

Kulm, G. (1980). Research on mathematics attitudes. In R.J. Shumway (Ed.), Research in mathematics education (pp.356-387). Reston, Virginia: NCTM.

Lesh, R. & Landau, M. (1983). Acquisition of mathematics concepts and processes. New York: Academic Press.

Lindquist, M.M., Carpenter, T.P., Silver, E.A., & Matthews, W. (1983). The third national mathematics assessment: Results and implications for elementary and middle schools. Arithmetic Teacher, 31 (4), 14-19.

McLeod, D.B. (1985). Affective issues in research on teaching mathematical problem solving. In E. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives, (pp. 267-279). Hillsdale, NJ: Lawrence Erlbaum.

National Assessment of Educational Progress (NAEP). (1983). The third national assessment: Results, trends and issues. Denver, Colorado: NAEP.

National Council of Teachers of Mathematics (NCTM). (1980). An agenda for action: Recommendations for school mathematics for the 1980s. Reston, Virginia: NCTM.

National Council of Teachers of Mathematics (NCTM). (1981). Priorities in school mathematics. Reston, Virginia: NCTM.

Schoenfeld, A.H. (1983). Episodes and executive decisions in mathematical problem solving. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 345-395). New York: Academic Press.

Sowder, L. (1980). Concept and principle learning. In R. J. Shumway (Ed.) Research in mathematics education (pp. 244-285). Reston, Virginia: NCTM.

Thompson, A.G. (1985). Teacher's conceptions of mathematics and the teaching of problem solving. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives (pp.281-294), Hillsdale, NJ: Lawrence Erlbaum.

Zweng, M.J. (1979). The problem of solving story problems. Arithmetic Teacher, 22, 2-3.