DOCUMENT RESUME

ED 273 494 SE 047 198

AUTHOR	Suydam, Marilyn N., Ed.; Kasten, Margaret L., Ed.
TITLE	Investigations in Mathematics Education. Volume 19, Number 1.
INSTITUTION	Ohio State Univ., Columbus. Center for Science and Mathematics Education.
PUB DATE	86
NOTE	73p.
AVAILABLE FROM	SMEAC Information Reference Center, The Ohio State Univ., 1200 Chambers Rd., 3rd Floor, Columbus, OH 43212 (U.S. subscription, $8.00, $2.75 single copy).
PUB TYPE	Information Analyses (070) — Reports - Research/Technical (143)
JOURNAL CIT	Investigations in Mathematics Education; v19 n1 Win 1986
EDRS PRICE	MF01/PC03 Plus Postage.
DESCRIPTORS	Academic Ability; *Cognitive Processes; Computation; Computer Managed Instruction; Educational Research; Elementary Secondary Education; Estimation (Mathematics); Fractions; Lesson Plans; *Mathematics Achievement; *Mathematics Education; Mathematics Instruction; Matrices; *Problem Solving; *Research Methodology; Student Behavior; Time on Task
IDENTIFIERS	*Mathematics Education Research

ABSTRACT

This issue of the journal contains abstracts and critiques of 11 research reports. Three of the reports concern problem solving; the remainder pertain to the following topics: the impact of secondary schooling and secondary mathematics on student mathematical behavior; figural matrices; computer-video instruction in mathematics; equivalent fractions; counting and numerical estimation; cognition and time on task; effects of lesson format on the acquisition of mathematical concepts; and the influence of class ability level on student achievemet and classroom behavior. Mathematics education research reports and articles listed in "Resources in Education" (RIE) and "Current Issues to Journals in Education" (CIJE) for July-September 1985 are also listed. (MNS)

MATHEMATICS EDUCATION INFORMATION REPORT

INVESTIGATIONS IN MATHEMATICS EDUCATION

Volume 19, Number 1 - Winter 1986

THE ERIC SCIENCE, MATHEMATICS AND ENVIRONMENTAL EDUCATION CLEARINGHOUSE
in cooperation with
Center for Science and Mathematics Education
The Ohio State University

INVESTIGATIONS IN MATHEMATICS EDUCATION

Published quarterly by

The Center for Science and Mathematics Education
The Ohio State University
1945 North High Street
Columbus, Ohio 43210

With the cooperation of the ERIC® Clearinghouse for Science, Mathematics and Environmental Education

Volume 19, Number 1 - Winter 1986

Subscription Price: $8.00 per year. Single Copy Price: $2.75
$9.00 for Canadian mailings and $11.00 for foreign mailings.

INVESTIGATIONS IN MATHEMATICS EDUCATION

Winter 1986

Bell, A.; Fischbein, E.; and Greer, B. CHOICE OF OPERATION IN VERBAL ARITHMETIC PROBLEMS: THE EFFECTS OF NUMBER SIZE, PROBLEM STRUCTURE AND CONTEXT. Educational Studies in Mathematics 15: 129-147; May 1984.

Abstract and comments prepared for I.M.E. by GERALD A. GOLDIN, Rutgers University.

1. Purpose

The authors of this study set out to investigate how children understand certain applications of multiplication and division. They seek to observe the effects of problem structure, problem context, and numerical characteristics on the difficulty of choosing a correct operation, and to determine the nature of the interactions between these task variables and mathematical misconceptions on the part of the students.

2. Rationale

Errors in the choice of operation in problem solving often stem from mathematical misconceptions; for example, the belief that "multiplication always makes bigger" (Bell et al., 1981; Hart, 1981). While the use of calculators in the classroom may help eliminate some misconceptions by allowing less attention to be paid to computation and more to size relationships, the question remains as to how the meanings that students can give to arithmetic operations--i.e., the types of "structure" they can interpret as embodying an operation--influence the difficulties observed in choice of operation. In contrast to Vergnaud (1980) . present authors stress differences in context as well as differences in mathematical structure in their classifications of problems, as context familiarity (for example) may affect problem difficulty.

3. Research Design and Procedures

Thirty 12- and 13-year-old students of slightly above average ability, from a fairly traditional mathematics background, were given two kinds of tasks: (i) to write down the calculations required to solve verbal problems in applied arithmetic (multiplication and division), and (ii) to make up stories to fit given arithmetic calculations.

(i) Seven multiplication problems and 12 division problems were included in the study (one problem was excluded due to an error in presentation). The problems were varied with respect to the kinds of positive numbers used in combination (an integer times an integer, an integer times a decimal less than 1, etc.), and with respect to the type of "structure" being embodied (repeated addition [sets], repeated addition [measure], rate structure, measure conversion, and enlargement, for multiplication problems; partition, fractional partition, quotation, rate [partition type, speed = distance/time], rate [quotation type, time = distance/speed], and measure conversion, for division problems). The problem contexts also varied from problem to problem. Tables of the problems are given in the paper; the following item is representative:

> "In the school kitchen the cooks use 0.62 kg of flour to make one tray of doughnuts. How much flour will it take to make 27 trays?"
>
> Calculation: 0.62 x 27
> Type of structure: Repeated addition (measure)

(ii) Two multiplication story-writing items (0.51 x 33, and 10.5 x 0.71), and 8 division story-writing items with whole numbers and decimals in various positions (for example, 12.7 ÷ 9 and 4 ÷ 24) were employed.

Each child responded to 10 verbal problems, five story items, 10 more verbal problems, and five additional story items. Interviews were then conducted with selected students using some of the test problems, to investigate their thinking processes further.

The tabulated data for each problem include the number of students who solved the problem correctly, the number who did not respond, and the number who made certain kinds of errors (using division on the multiplication problems, using multiplication or reversing the numbers on the division problems, etc.). The responses to the story items are classified according to the correctness of the correspondence between the story written and the calculation given, and the type of embodiment employed in the story. The interview data are discussed qualitatively.

4. Findings

The multiplication problems ranged in difficulty from 30 correct answers on the repeated addition problem with whole numbers to three correct answers on the enlargement problem which involved one decimal. The division problems ranged from 27 correct answers on the partition problem with whole numbers to three correct answers on a rate problem [quotation type] involving two decimals. On the multiplication story items, repeated addition embodiments occurred most frequently among the "correct" stories. On the division story items, those which could be embodied by partition or fractional partition were answered correctly with the greatest frequency. The interview data included expressions of doubt about order in division problems ["You can read it two ways--divided into, or divided by."], as well as qualitatively correct reasoning from which the students were unable to draw qualitatively correct relationships. Of course many more details are described in the paper.

5. Interpretations

Among the sources of problem difficulty identified by the authors are numerical factors (multiplication by a decimal less than one, divisor larger than the dividend, etc.) which they note are consistent with previously identified common misconceptions. The story-writing tasks are used to assist in interpreting the effects of problem structure. For multiplication problems, "repeated addition" is deemed the most accessible structure, while for division problems a hierarchy of accessibility is identified, with "partition" the most accessible and "fractional quotation" and "rate" embodiments the least. Complicated interactions are noted, however, between the contextual differences among the problems and the effects of numerical and structural factors, which the authors note may qualify some of their conclusions.

Among the educational implications drawn from the study are the potential value of calculators in widening children's experience with decimal numbers, the importance of instruction aimed directly at correcting certain common misconceptions, and the need to teach the different multiplicative <u>structures</u> (as opposed to multiplication and division <u>algorithms</u>) in a variety of contexts, with emphasis on the invariance of operational structure over different types and sizes of numbers.

<u>Abstractor's Comments</u>

I find myself in agreement with the majority of the conclusions drawn in this study, but in doubt that most of those conclusions in fact follow from the experiments described. In the construction of the problems themselves, no attempt was made to let some task characteristics vary systematically while holding others constant (cf. Goldin and Caldwell, 1984). The numerical variables, the contexts, and the structural characteristics, as well as the verbal syntax, <u>all</u> change from one problem to the next. Thus the authors' conjectures about the sources of difficulty, while consistent with the data, cannot be regarded (even qualitatively) as confirmed. Furthermore, tests of statistical significance were not performed, so that one cannot draw inferences from the data in this experiment for a larger population of students. The authors state, for example, that the difference in difficulty between the two multiplication story items (21 correct _vs._ 15 correct) "confirms" that "multiplication of a decimal less than 1 is straightforward if the situation can be seen as one of repeated addition" (p. 143). But one must first ask if the difference is statistically significant, using a two-tailed test (since no advance hypothesis as to the direction of the difference in difficulty was made). The authors have tried to achieve a balance, I think, between detailed clinical data about individual children and population data about the effects of task characteristics; but for the above reasons I found the conclusions drawn from the population data unconvincing.

It was also unclear to me how some of the story items were scored. In response to the division item 4 ÷ 24, one child wrote,

> "Mary has 4 lb. apples. She has got to share them between her and her 23 friends. How many apples do they get each? (12 apples to a lb.)"

I was unable to identify a classification for this response with which I could agree, from those provided in the table.

It should also be mentioned that the term "problem structure" as used in this paper refers to conceptual embodiments, and thus differs considerably from the use of that term by other researchers. For example, Kulm (1984) might consider such embodiments to be "content" rather than "structure."

Having noted some limitations of the study, let me conclude by expressing admiration for much that is here. The analysis of multiplicative "structures" and their incorporation into verbal problems is extremely useful. Many interesting responses to the story-writing items are presented and discussed. The conjectures concerning student misconceptions and their interaction with numerical, contextual, and structural variables are insightful, and the authors suggest several interesting questions for further investigation. This paper rewards careful reading.

References

Bell, A., Swan, M., and Taylor, G. (1981). Choice of operations in verbal problems with decimal numbers. Educational Studies in Mathematics, 12, 399-420.

Goldin, G. A. and Caldwell, J. H. (1984). Syntax, content, and context variables examined in a research study. In G. A. Goldin and C. E. McClintock (eds.), Task Variables in Mathematical Problem Solving, Philadelphia: Franklin Institute Press (presently Hillsdale, N.J.: Erlbaum), 235-276.

Hart, K. M., ed. (1981). Children's Understanding of Mathematics, London: John Murray.

Kulm, G. (1984). The classification of problem-solving research variables. In G. A. Goldin and C. E. McClintock (eds.), Task Variables in Mathematical Problem Solving, Philadelphia: Franklin Institute Press (presently Hillsdale, N.J.: Erlbaum), 1-21.

Vergnaud, G. (1980). Didactics and acquisition of "multiplicative structures" in secondary schools. In W. F. Archenhold, A. Orton, R. H. Driver, and C. Wood-Robinson (eds.), Cognitive Development Research in Science and Mathematics, University of Leeds.

Bell, Elizabeth S. and Bell, Ronald, N. WRITING AND MATHEMATICAL PROBLEM SOLVING: ARGUMENTS IN FAVOR OF SYNTHESIS. School Science and Mathematics 85: 210-221; March 1985.

Abstract and comments prepared for I.M.E. by RICHARD E. MAYER, University of California, Santa Barbara.

1. Purpose

The purpose of this study was to determine the effectiveness of integrating instruction in writing and mathematical problem solving.

2. Rationale

The rationale is that the "public dissatisfaction with the educational system's ability to deal with basic instruction" (p. 211) demands innovative approaches. As pointed out by the authors, one innovation suggested by the NCTM's Agenda for Action for the 1980's is that "relationships among all the basic skills should be explored and taught across the curriculum" (p. 211). More specifically, this study is based on the premise that there are similarities between the problem-solving skills involved in expository writing and the problem-solving skills involved in solving mathematics problems.

3. Research Design and Procedures

One ninth-grade general math class (experimental group) spent four weeks learning "problem solving skills by a method which combines traditional math techniques with a structured expository writing component" (p. 214). Activities included solving an algebra story problem and writing a paragraph to describe one's problem-solving process for the problem; writing a paragraph to explain why one method is preferable to another; writing a paragraph to describe how to solve a specific word problem; writing a paragraph to explain what is confusing about a problem; writing a paragraph about how to solve a problem that skills numbers. Students were expected to learn how to explain a process including "sequencing or chronological organization skills" (p. 216) and how to prove an argument including "ability to state a thesis and support it with specific information" (p. 216). In contrast, another ninth-grade general math class from the same school (control group) was taught mathematical problem solving "by using only the traditional math methods" (p. 214).

4. Findings

On a problem-solving pretest, the experimental group scored 24.3 and the control group scored 20.2 out of 40 possible points, with this difference significant at $p < .05$. On a problem-solving posttest, the experimental group scored 26.4 and the control group scored 20.6, with this difference significant at $p < .01$.

5. Interpretations

The authors interpret these results as follows: "Pretests given both groups before the study began showed that at the .01 level, there was no significant difference in problem-solving skills of the groups of ninth-grade students. However, four weeks later...the results of the posttest showed that the difference between the two groups was now significant at the .01 level. This indicates that the writing component the experimental group underwent positively affected its progress in math problem solving."

Abstractor's Comments

My comments concern the instructional method suggested by the authors, the theoretical basis for the instructional method, and the empirical support for the instructional method.

First, the authors suggest an extremely interesting procedure for integrating expository writing and mathematical problem-solving skills. I am intrigued by the idea, partly because it is consistent with previous research on "cognitive process instruction." In particular, the instructional method suggested by the authors could serve to help students describe their own problem-solving processes--a skill which has been a component in several successful mathematical problem-solving programs (Mayer, 1983).

Second, the authors do not tightly connect their instructional procedure with any precise theory of problem solving. For example, it would be useful to perform a task analysis in order to specify which components of expository writing are similar to which components of mathematical problem solving. This research should make better use of cognitive psychology techniques for analyzing students' knowledge, including knowledge about problem-solving strategies (Mayer, 1981).

Third, my most serious reservation concerns the authors' interpretation of the results. Dealing with pretest-to-posttest gains is a tricky business that calls for a conservative approach. Unfortunately, we begin with the experimental group performing somewhat better than the control group on the pretest. It is somewhat misleading to argue that a difference of 4.1 between the two groups is not significant on the pretest but a difference of 5.8 between the groups is significant on the posttest. In fact, both differences produce t-values that are significant at the .05 level, and the authors present no evidence that there is a significant difference between 4.1 and 5.8. Instead, a more careful approach would be to match subjects in the two groups based on pretest score (so that both groups have the same mean pretest score) and to conduct a t-test on the difference between the groups on posttest score; an alternative is to compare gain scores between groups (2.1 for the experimental group versus .4 for the control group) using a t-test or by looking for a group by test interaction in an analysis of variance. Based on the author's conscientious presentation of group means, standard deviations and sample sizes, it appears from Table 1 that the gain score for the experimental group is probably not significantly greater than the gain score for the control group.

In summary, the authors present an intriguing idea for integrating writing and mathematics instruction. Unfortunately, the pilot study used to justify the approach is not convincing. Large scale, methodologically sound studies are needed before accepting the authors' "arguments in favor of synthesis."

References

Mayer, R. E. (1981). The Promise of Cognitive Psychology. New York: Freeman.

Mayer, R. E. (1983). Thinking, Problem Solving, Cognition. New York: Freeman.

Clarke, David. THE IMPACT OF SECONDARY SCHOOLING AND SECONDARY MATHEMATICS ON STUDENT MATHEMATICAL BEHAVIOUR. Educational Studies in Mathematics 16: 231-257; August 1985.

Abstract and comments prepared for I.M.E. by ROBERT C. CLARK, Florida State University, Tallahassee.

1. Purpose

The purpose of this study was to investigate the changes in students' mathematical behavior affected by the transition to secondary school in order to determine significant variables and create a comprehensive descriptive framework.

2. Rationale

Observation has long demonstrated significant discontinuities in student behavior and achievement in mathematics during the transition between elementary and secondary school. Some students who have been successful in elementary school mathematics are much less successful in secondary mathematics.

Previous works have identified expectations, attributions of ability, criticism of significant others, sex typing, mathematical ability, and mathematical understanding as variables which affect a student's mathematical behavior. The study observed the effects of these variables, and attempted to organize the variables into a descriptive framework.

3. Research Design and Procedures

The design is in the form of a case study of ten students over a three-year period covering elementary school (grade 6) and two years of secondary school (grades 7 and 8). Data collection techniques included clinical interviews, classroom observations, formal testing, questionnaires, and teacher diaries. The author's model of mathematical behavior consisted of the following elements:

a. Mathematical Ability

   i. Observed ability - Demonstrable skills, as displayed on tests and interviews
   ii. Perceptions of ability - As viewed by peers, teachers, parents, and self

b. Understanding of Mathematics - Types and levels of understanding were identified in the following areas:

   i. Fractions
   ii. Decimals and place-value
   iii. Proportion
   iv. General understanding

c. Mathematical Performance - As measured by:

   i. Achievement tests
   ii. Task persistence
   iii. Demonstrated error patterns

d. Conception of Mathematics

   i. Nature of mathematics - What is mathematics?
   ii. Competence - Who is good at it?
   iii. Teaching - How is mathematics taught?
   iv. Attitudes - How do you feel about it?

e. Self-Concept

   i. Attributions
   ii. Expectations
   iii. Self-esteem
   iv. Significant others

f. Individual Student Classroom Practices - Interaction between the student's personal attributes and the environment.

g. Practices of the Learning Environment

   i. School
   ii. Peer group
   iii. Home

The study attempts to demonstrate that "sufficiently clear relationships exist between elements for inferences to be made about likely changes to a student's mathematical behaviour arising from initial change in a single element" (p. 234).

4. Findings

Because this was a case study, the results are reported as comparisons and contrasts between the behaviors of two of the ten students at both the elementary and secondary school levels. The idosyncratic reports of the experiences of the two students help to explain the mathematical behavior of students in this age range.

The two students had similar achievement scores in elementary school and both were successful in elementary school mathematics. At the secondary level achievement scores were still equivalent, but one student had a successful mathematical experience while the other had a "destructive and personally-restrictive" (p. 255) experience.

This study makes it clear that teachers must consider much more than a student's test scores in planning for learning. The wide differences in behavior between the two students cannot be attributed to demonstrable ability, as the two demonstrated essentially the same ability level at both the beginning and end of the three-year study. Hence, the other variables in the model must have critical importance in explaining these differences.

5. Interpretations

The author reaches the following conclusions:

a. "...greater recognition (should) be given to the importance of the social context in which mathematics is learnt..." (p. 255).

b. "...the impact of secondary mathematics on a student's mathematical behaviour may be determined during the first year of secondary school through the evolution of community opinions which the child comes to share" (p. 255).

c. "...the variety in student background, the differences in personality, and the evident diversity in students' responses to the same mathematics classroom, do not suggest that a single teaching style or administrative structure would be likely to meet the needs of children commencing secondary mathematics" (p. 255).

Clearly, the author finds the model to be helpful in describing changes in a student's mathematical behavior. This case study forms the basis for a research study being conducted by the author in 36 Victoria, Australia classrooms.

Abstractor's Comments

I found the individual case study reports very helpful in adding to my understanding of the mathematical development and behavior of children in the age range studied. These reports are an important aspect of this study and should be read by any educator working with students in this age range. Clearly there are environmental variables, which the teacher may either control or influence that strongly affect students' attitudes towards mathematics, their success, and their willingness to continue study. These variables go beyond choosing the appropriate content and method of presentation. I infer that to ignore the social context of learning will deprive significant numbers of students of the possibility of success and continued study in mathematics.

A model is an external manifestation of the way an individual internally organizes information. I had difficulty describing the author's model because it does not conform with the way I organize information in this area. I had no difficulty with the particular variables, just the way they were organized. The model obviously does conform to the way the author organized information.

Many professionals would not consider this case study to be "real" research and many journals would not publish such works. No attempt is made at measurement (other than to report achievement test scores) and no statistics are reported. However, the case study is the most appropriate way to approach a new area of research. The researcher must identify the variables and develop an intuitive knowledge of their relationships. Almost all studies in the behavioral sciences should begin this way. If journals are unwilling to publish such studies, then there will be little justification for building the necessary foundation for later study. I look forward to the report of the author's follow-up study.

Foorman, Barbara R.; Sadowski, Barbara R.; and Basen, Jeffry A. CHILDREN'S SOLUTIONS FOR FIGURAL MATRICES: DEVELOPMENTAL DIFFERENCES IN STRATEGIES AND EFFECTS OF MATRIX CHARACTERISTICS. Journal of Experimental Child Psychology 39: 107-130; February 1985.

Abstract and comments prepared for I.M.E. by LARRY SOWDER, San Diego State University.

1. Purpose

The purposes of this investigation were further to "examine developmental differences in processes and strategies involved in geometric matrix solution and to pursue the relationship between strategy differences and item complexity" (p. 108).

2. Rationale

Earlier work had pointed to developmental increases in latencies or performance as task complexity increased (with Raven's Standard Progressive Matrices and schematic analogies). Possible explanations lay in an increase in working memory and in Sternberg's reasoning framework. The first experiment gathered further evidence on the developmental growth of strategies, with working memory and spatial reasoning in mind. The second experiment focussed on dimensions of the matrix problems in an effort to seek further clarification of the pertinence of the dimensions to performance.

3. Research Design and Procedures

Ninety students from grades 2, 5, and 8 were given these tests for Experiment 1: Figural Intersection Test (as a measure of spatial reasoning), an oral Backward Digit Span (as a measure of working memory), Raven's Standard Progressive Matrices, A-E, and an experimental analogue of the Raven's called the geometric matrices (Stone and Day). The latter matrices are derived in nine forms by using 1, 2, or 3 types of figures crossed with 0, 1, or 2 transformations of the figures (figure added, figure expanded in size, figure rotated in the matrix). Students were to tell whether the last entry for each task was correct or incorrect. Six items with correct last entries ("true" matrices) and six with incorrect last entries ("false" matrices) were given for each of the nine forms. The task could be described as a discovery of a principle task, given no information about the universe. Students were given training sessions with the matrix tasks, which were administered via slides with a timing mechanism. Both correctness of results and time for response were measured.

Experiment 2 focussed more narrowly on the tasks, using computer-presented geometric matrices but with only a single transformation for the false matrices. It was designed to investigate some counter-intuitive findings in Experiment 1, where additional complexity on false matrices did not always result in longer response times. Thirty fifth-graders were given the Figural Intersection Test, the Raven's, and the Children's Embedded Figures Test, as well as the geometric matrices.

4. Findings

In Experiment 1, the second graders gave significantly fewer correct responses than the fifth or eighth graders, and the fifth graders responded significantly more slowly than the other students. As the items became more complex, the response times of the older students also increased, but the response times of the second graders levelled out or were even shorter than those for the easier tasks.

In Experiment 2, performances were higher than those of Experiment 1. Type of transformation, when two or three figures were involved, give different latencies and also interacted with the cause of falseness and the number of elements. The Raven's and the Embedded Figures tests gave the highest correlations with number correct on the geometry matrices.

5. Interpretations

"These data support the hypothesis of developmental differences in accuracy and latency to solution on figural matrices" (p. 117), with the second graders apparently using a more limited strategy, perhaps due to working-memory limitations (although regression analysis with the working-memory test score at each grade level explained at most 12% of the variance in response time). Performance on some items suggested that eighth graders may use a "see what's happening" strategy, explaining the result that false matrices involving no transformation took longer than items with one transformation.

Experiment 2 suggests that strategy use differs across the types of transformation in the geometric matrices. Circular transformations appear to be more difficult than the addition or expansion transformations.

Abstractor's Comments

Although the finding that task variables can play a surprising role in performance on ostensibly comparable tasks is no surprise to mathematics educators, the methodology of this study is of great interest. How does one find out what is going on inside a black box? Inferring the use of strategies from a study of performance, coupled with independent measures of selected cognitive processes, involves a well-designed set of tasks and a careful, painstaking analysis. Such characterize this study.

A few points deserve mention. Finding external measures that indeed tap, preferably solely, the cognitive process(es) desired is difficult. I was surprised, for example, to find an oral backward digits test used when none of the tasks were oral. And no evidence was presented that the Figural Intersection Test is a good measure of "spatial reasoning." In view of one of the transformations in the matrices, wouldn't a measure of spatial orientation have been potentially useful? Second, the important role of affect was illustrated by the better performance on the tasks in Experiment 2, attributed by the writers to the greater motivation of the microcomputer presentation (p. 122). Third, since the puzzling results in the first experiment were with eighth graders, why were fifth graders used in the second experiment? Finally, given the spirit of the times, it was surprising to find that no attempt was made to determine what strategies were used by interviewing a few students. Any such strategies uncovered could be surprising and a comforting supplement to, or confirmation of, any posited by the investigators, however plausible these might be.

Henderson, Ronald W.; Landesman, Edward M.; and Kachuck, Iris. COMPUTER-VIDEO INSTRUCTION IN MATHEMATICS: FIELD TEST OF AN INTERACTIVE APPROACH. Journal for Research in Mathematics Education 16: 207-224; May 1985.

Abstract and comments prepared for I.M.E. by ARTHUR F. COXFORD, University of Michigan.

1. Purpose

The purpose of the project was to develop and field test an interactive, computer-video-delivered set of mathematics instructional modules for learners who had histories of difficulty with school mathematics.

2. Rationale

Television has been demonstrated to be an effective tool to deliver instruction to learners at all ages. Similarly, computer-assisted instruction has been shown to be effective. Yet these individual media have weaknesses. Television is not interactive while computer-based instruction is; computers are not versatile in presenting graphics displays while television is. Linking the two electronically allows the strengths of each to be used in a coordinated manner for instructional purposes.

Much of the instruction presented technologically via television or computer is observational in nature. In such instruction, research has indicated that the materials must pay attention to attentional, retentional, and motivational processes. Thus the scripts that were prepared consciously built in features which addressed attentional, retentional, and motivational concerns.

3. Research Design and Procedures

Materials development: The topics chosen for inclusion in the instructional package were related to common fractions. These topics were chosen after consultation with classroom practitioners who suggested they were troublesome for lower-achieving youth. The desired skills were task analyzed in the fashion of Gagné to develop instructional sequences. The results were three modules dealing with factors, primes, and multiples and six modules dealing with fraction concepts and operations. Each of the modules was scripted, edited, story-boarded, and cast with attention to motivational, attentional, and retentional concerns relevant to secondary youth who have experienced difficulty in mathematics. The materials were specifically designed to teach skills related to the topics chosen and reflected that orientation.

Evaluation instruments were prepared to assess the skill developed by the two sequences of modules. The factors and primes test was made up of 32 recognition and 32 constructed response items. The fractions test was composed of 12 recognition and 48 constructed response items. The two types of items were used "...because evidence indicates that these response forms may have differential influence on achievement...". The Spearman-Brown split half reliability of each test was .96.

A Likert-type response format School Learning Questionnaire was developed to assess to what students attributed success and failure in mathematics and other school subjects. Eight subscales were included to attribute success or failure in the task to the following causes: ability, effort, chance, and task or situational factors. The first two were classified as internal causes, the latter two were external causes.

Field Trials: Full-scale field trials were not possible. Two field trials were designed. The first used only the factors and primes modules and occurred with 45 students and a comparison group of 43. The second field trial used all nine modules, but was run with only 8 to 11 volunteer students from a summer school program. No comparison group was available here.

The procedure was the same in each field trial. The students were introduced to the computer and video display, and they then worked through the module by interacting with the module through the computer key board. An observer placed away from the student observed the student-module interaction throughout.

Procedures: All students were given the factors and primes test as a pretest. They then completed the module and were asked questions about how they did, what they thought of the delivery system, whether they would like to use the system again, whether they liked the tape, whether they learned, and whether they would like to do more. For field trial 1 the procedure took three months to complete for all 45 students, then the posttest was given to all students. For field trial 2 the posttest was given as soon as the 11 students finished, the pretest on fractions was given, the modules done, and the posttest given. Finally, the School Learning Questionnaire was given to all participants at the end of each trial.

4. Findings

The findings are derived, in the main, from field trial 1 because of the possiblity of random assignment to treatment and the availability of a comparison group. The posttest scores for factors and primes on both the recognition and the constructed response items were significantly higher for the computer-video instructed or treatment group. The gains for both males and females were greater for treatment students than for comparison students. A factor analysis of the

School Learning Questionnaire produced four factors accounting for 65% of the variance. These factors were named Mathematics Attributions, Non-Mathematics Attributions, Effort Attributions, and Ability Attributions for Success. Using correlational techniques, it was reported that Effort Attributions was significantly correlated with treatment group, Mathematics Attributions was significantly correlated with total score, and Mathematics Attributions and Effort Attributions accounted for 12.35% of the variance of the total score. The verbal responses recorded at the end of the session were uniformly positive.

The second field trial was minimally reported. It showed the treatment group making significant achievement gains on all measures.

5. Interpretations

The achievement fostered by the video-computer treatment was substantial. This suggests that the modules were effective with secondary school students who had experienced difficulty with mathematics. Additionally, there was some support for the prediction that exposure to the modules would help students recognize that it was possible for them to learn mathematical skills. There was also evidence supporting the notion that students perform differentially to constructed response and recognition items and "...that these differences are related to differences in the sources to which students attribute their success or failure." Thus both kinds of test items are needed if relations between attribution and performance are to be clarified.

Abstractor's Comments

The development of the instructional materials as reported here was careful and thorough. The authors' attention to detail was meticulous in both the report and in the development. It is unfortunate that a full-fledged field trial of the complete set of modules was not possible, for adequate data on the nine-module sequence would be useful. In particular, would the positive expressions about the material continue when used for a long period? Certainly even excellent teachers become less positively thought of as time passes. Would the same occur for this medium?

The need for three months for 45 students to complete three relatively short modules is another concern. The obvious issue is whether the achievement scores adequately represent the performance when some participants had to wait three months to perform. The second issue is that if it takes three months to pass 45 students through three modules, is the procedure time and cost-efficient? In order for it to be done in a few days, the amount of equipment would be multiplied several times. Would this be out of the question for districts financially? Since cost needs to be considered by schools, could continuing research deal with more group instructional uses of the same media? The positive features reported here could be multiplied tenfold if replicable with group settings. This seems worth a try practically as well as theoretically.

Hunting, Robert P. UNDERSTANDING EQUIVALENT FRACTIONS. Journal of Science and Mathematics Education in Southeast Asia 7: 26-33; July 1984.

Abstract and comments prepared for I.M.E. by IPKE WACHSMUTH, Universität Osnabrück, West Germany.

1. Purpose

The study was designed to get information on students' thought processes as they attempted solutions to a variety of fraction equivalence problems. Particular questions raised were:

1. What types of strategies do students display in the context of being directed to complete an equivalence expression where the denominator is given?
2. What types of strategies do students display in providing justifications for the validity of equivalence expressions using discrete quantity representations? (p. 27)

2. Rationale

"Knowledge of fraction equivalence is necessary for a mature understanding of rational number" (p. 26). Previous research has shown that many problems 10- to 14-year-olds have with fractions are related to fraction equivalence. If students' fraction computations are to become less dependent on rote algorithms and if their view of mathematics is to be based on more than narrowly applicable procedures, "then we need to teach our students meaningful bases for thinking about fraction equivalence. Observation of student problem solving behavior is a sensible source of such information" (p. 26).

3. Research Design and Procedures

Subjects were nine fourth-grade, 10 sixth-grade, and 10 eighth-grade average to above-average mathematics students from two elementary and one middle school in Georgia. In videotaped clinical one-on-one interviews they were given six tasks in fixed order, one at a time, in which they were shown a written fraction and asked to complete an equivalent fraction whose denominator was given. Problems given included 1/2 = /4; 1/4 = /8; 1/4 = /12; 2/6 = /3; 6/15 = /5; and 3/12 = /8. Subjects' explanations were recorded. Then subjects were to verify the equivalences they stated using a small number of counters (four) and also a larger number of counters (twelve).

When no spontaneous solution was reached by a subject, assistance was offered by the investigator by suggesting that representations of the fractions be constructed using the material. When a student had difficulties using the material for a verification, diagrams could be drawn instead. Not in all cases was the full sequence of problems asked. Year 6 and 8 students were not given the problem 1/4 = /12 but were to give an argument about the number of fractions in the equivalence class of 1/4.

Transcripts of the verbal and some non-verbal transactions between subject and investigator served as the basis of evaluations.

4. Findings

Results are presented in two portions, (a) subjects' production behavior: a classification of production strategies displayed and explained by students in completing the equivalence expressions; (b) subjects' verification behavior: an analysis of students' material-based verifications for the first problem, 1/2 = /4.

(a) As for the production behavior, the following seven strategy types were identified: (1) Common factorisation; (2) cross-multiplication; (3) recalled knowledge; (4) invented algorithm; (5) use of ratios; (6) intermediate fraction; and (7) guess and see (i.e., the subject offered a solution in the expectation of receiving further information from the investigator). Occurrences of these strategies are presented in three problem-by-subject tables, one each for Year 4, Year 6, and Year 8. No information as to whether responses were correct or incorrect is included. Only spontaneous responses were categorized; some cases where Year 4 students successfully reached an answer only upon being given the suggestion to use materials were excluded among production strategies.

The author has summarized information from the tables as follows: "Overall, the most popular strategies were common factorisation (46%), in which the greater denominator was divided by the lesser to obtain the factor with which to multiply the given numerator, and invented algorithm (31%). Frequencies of other strategies were: use of ratios (8%); guess and see (5%), cross-multiplication (4%); intermediate fraction (3%); and recalled knowledge (2%)" (p. 28).

Among the Year 4 students, predominant strategies were "invented algorithm" and "guess and see." Data are sparse for Year 4 students: only three subjects answered and explained at least two problems; in no case were more than the first four problems given. Among both the Year 6 and Year 8 students, predominant strategies were "common factorisation" and "invented algorithm." No significant preference of a certain strategy for a certain problem type is observed. Four of the Year 6 subjects and seven of the Year 8 subjects expressed in some way that the number of fractions in the equivalence class of 1/4 is very large or unbound.

(b) An analysis of students' verification behavior for the first problem is based on selected interview segments (no particular arguments were made with respect to subject data of the different years). These show that in several cases students changed their verification strategies as the numbers of counters were varied. Several students could not successfully demonstrate equivalence between physical representations of two fractions because they were unable to represent individual fractions using the discrete quantities available. Less successful attempts were observed when 12 counters were used than when four counters were used. For example,

- one subject who had successfully adopted a common factorisation strategy when a fourth was represented by one out of four counters was unable to do so when a fourth was to be represented by more than one counter;
- another subject who had verified the equivalence with four counters after using a pie diagram fell back on numerical methods in the context of 12 counters;
- in four cases subjects who had made two groups of six to show 1/2 based on 12 counters partitioned these groups into a group of two and a group of four in order to show 2/4 (presumably corresponding to the digits in 2/4);
- in other cases subjects, when directed to use 12 counters to show fourths, partitioned the set of counters into groups <u>of four</u>.

Overall, discontinuities were observed between strategies students used for producing solutions and for verifying their results based on counters. Even the most successful students used ways of verification which had no relation to their strategy in producing the solution.

5. Interpretations

The author argues that "examples given highlight the dependence of equivalence knowledge on the possession of appropriate action strategies for constructing physical representations for fractions" (p. 32). In other research he found that "students have well-developed operational systems for constructing, defining, and transforming fractional units in non-mathematical contexts (...)" (p. 32). The author concludes that mathematics teachers need to reconsider their methods of teaching fractions in the light of these findings and suggests that students be made to verify their solutions of equivalence problems using physical materials. Also, such settings should be used where the unit fraction is represented by more than one counter "so that literal interpretations of fractions, like, for example, one-fourth means 'one out of four things,' can be broadened and extended" (p. 32).

Abstractor's Comments

I don't think that students' equivalence knowledge depended much on the possession of appropriate action strategies for constructing physical representations for fractions. Other than mention of several Year 4 students who had difficulties with the first problem but were successful when using the materials (cf. p. 27), there is no support for that statement. Could it be the case that the author has made this statement under a tacit assumption, namely, that support of meaning (e.g., through physical representations) will improve students' ability to produce computational solutions? This issue I have often discussed with North American mathematics educators. I think Hunting's study gives us a fine basis from which to continue this discussion.

We would learn little about students' "understanding" of fraction equivalence in computational settings if most subjects had used production strategies that were dependent on rote algorithms, such as cross-multiplication. And we would learn little about students' different "understandings" of fraction equivalence in computational vs. meaning-based settings if the author had not observed the different production and verification behaviors.

Inspection of subjects' production strategies reveals that in fact many response explanations are categorized as based on having thought about it instead of having applied a rote procedure. For example, the second-most frequent strategy (31%) was "invented algorithm." Whether or not it was successful, isn't invention a subject's own way to make sense of the task? The most frequent strategy, "common factorisation" (46%), is based on the notion that a/b and c/d are equivalent if a/b can be expressed as nc/nd or the other way round. Isn't that understanding, at least in the sense of realizing the numerical relationship between equivalent fractions? On the other hand, cross-multiplication, though being the most universal algorithm for this set of problems, was used in only 4% of the strategies classified, which is surprising, as the author mentions himself. Of course, one would have to know more about the preceding instruction in order to make an adequate judgment.

Most tasks were such that one denominator was a multiple of the other. The only task where this was not the case was 3/12 = /8. It is even more interesting here to what extent solutions were based on rote methods. Only information on Year 6 and Year 8 students is available: Three Year 6 students gave responses, two were "invented algorithm" and one was "cross-multiplication." Eight of the Year 8 students responded, among them five invented an algorithm, one subject's answer was based on an intermediate fraction to compare both others with, and again only one used cross-multiplication. Note that the most prominent strategy, finding a common factor, does not apply in this case. (One response classified as such is inconsistent with the description given of this strategy type.) So again we get the feeling that in many cases subjects' responses involved at least an attempt to "understand."

The second topic researched, students' verification strategies in the context of physical materials, constitutes an attempt to observe students' understanding of fraction equivalence in the sense of interpreting an equivalence expression by a physical representation.

What I felt was missing is an argument as to why discrete representations were used and why they should be preferable over others. For one thing, to use any concrete representation purposefully and successfully requires agreements between teacher and student about how to interpret and use them. Clearly, there are differences in the "handiness" of different types of representations (which is not to say the handier one is naturally preferable). Continuous representations (e.g., area models) seem easier than discrete ones insofar as the unit is not problematic, since no number of counting items may conflict with the numbers present in a fraction symbol.

From my experience I believe that, in particular, the successful use of a discrete representation (e.g., counters) involves some agreement, if not training, in terms of the following:

- How many counters to choose as the unit so that both denominators can be expressed based on an appropriate grouping?
- How to arrange the counters (i.e., in an array or in piles) so that it is possible to see how they represent the fractions in question?

As the author states, "a number of students were prevented from successfully demonstrating equivalence between physical representations of two fractions because they could not represent individual fractions using the discrete quantities available" (p. 32). I asked myself, does a subject's failure in using these physical materials tell more about a lack of understanding of fraction equivalence or, rather, about an inability to use such material to express the relationship in question?

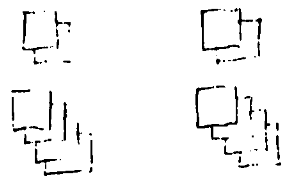
I feel one problem here is the subject's understanding of unit in the context of the discrete physical representation of fractions. I will argue that the investigator may be implicitly assuming a system of reference in which the whole unit is equalized with a set of counters, while the subject is only able to interpret the materials in this way when explicit indication is given of this fact. To support this argument, I quote an interview segment from the article (pp. 30-31) where a Year 6 student, CC (11;8), was asked to show the equivalence of one-half and two-fourths with 12 counters. I'll just underline what I think are the critical parts. (So far, the subject has made two piles of six counters each.)

I: "So. How are two-fourths and one-half equivalent?"
CC: (Silence.)
I: "Well, you've made me one-half of them, right?"
CC: "Uh-huh. There's one-half, right there."
I: "Now how can that be two-fourths?"

CC: (Silence.)

I: "Can you make those squares into fourths for me?"

CC: (Makes the following arrangement:)



I: "How is that fourths?"

CC: "Let's see, that would be..." (pause).

I: "What does fourths mean?"

CC: "Fourths means like, um, in altogether there's four, but then there would be how many out of that four?"

I: "O.K. Well. Show me one-fourth of all those squares."

CC: "One-fourth of all?"

I: Uh-huh."

CC: "Here's one-fourth of all the squares (holds up three counters)."

(...) (Upon this, the subject shows six counters for two-fourths and assents that it is the same as one-half.)

What we see here is what I meant to be particular of discrete representations: observation of the unit becomes critical; there is no implicit unit (like the circular whole in a pie) which lets the material support the student's doing, even without the student becoming actually aware of unit. That does not mean at all that using discrete representations is irrelevant for the issue of understanding equivalent fractions. Rather, discrete representations seem especially important since they require an explicit consideration of unit, much more than, say, area models. For this reason, they would help to develop understanding of unit, an understanding not necessary for computations with equivalent fractions but very necessary when equivalent fraction problems are embedded in situations.

Even when subjects are able to use physical materials successfully, would it necessarily guide their behavior in producing solutions? As the author states, "even the most successful students interviewed adopted different procedures for obtaining solutions and verifying their results" (p. 32). Apparently, the possession of appropriate action strategies for constructing physical representations was not so important for producing solutions. So when discontinuities were observed between subjects' production and verification behavior, just mention of it is not enough. What needs discussion here are the questions: what is the explanation? and what are the implications?

I feel the whole idea of using physical representations has less to do with producing solutions, avoiding errors in constructing the solution, etc., but has more to do with being able to model situations such that mathematics can be applied, to verify solutions obtained algorithmically, or to embed mechanical algorithmical doing in the context of meaning where necessary. Not necessarily ought verifications conform exactly with the way a solution is achieved.

To me, the study raises important further questions on the role of material representations meant to support meaning. The author's remark that students have been found to have well-developed operational systems in non-mathematical contexts, "including mechanisms for physically partitioning discrete quantities into equal shares, numerical procedures for anticipating fractional unit sizes, (...)" (p. 32), is certainly a challenge for teachers to be taken up in the mathematics classroom. I suppose, activities where students model such non-mathematical situations with parsimonious physical material like counters will help them to become proficient in the use of it and organize their thought on the basis of a relatively abstract model. In this way, an understanding of equivalent fractions should be achieved which exceeds an ability to complete equivalence expressions possibly based on physical representations. I doubt that in the course of a computation involving equivalent fractions students will, by themselves, go back to counters.

Newman, Richard S. and Berger, Carl F. CHILDREN'S NUMERICAL ESTIMATION: FLEXIBILITY IN THE USE OF COUNTING. Journal of Educational Psychology 76: 55-64; January 1984.

Abstract and comments prepared for I.M.E. by PAUL R. TRAFTON, National College of Education.

1. Purpose

This study examined the relationship between counting skills and the use of strategies, and performance on a numerical estimation task in order to gain further evidence on the role of counting in the development of mathematical understanding.

2. Rationale

The role of counting in learning mathematical content and the development of mathematical reasoning has received much attention in recent years. It has been suggested that automaticity of basic processes is required if learners are to develop more advanced number skills, understandings, and strategies.

Numerical estimation was viewed as a productive area to further investigate how counting skill affects performance. Numerical estimation tasks involve subitizing and counting, as well as higher-order thinking skills. The authors note that the "inherent subjectivity and novelty in numerical estimation tasks...would seem to provide an opportunity to explore how children apply their basic numerical skills to new situations."

3. Research Design and Procedures

Sixty-one randomly selected kindergarten, first- and third-grade children were involved in the study, completing an estimation task and a counting skills test.

The estimation task was a darts game displayed on a computer monitor. An unmarked, vertical line, with endpoints of 1 and 23, was shown at the right, together with a "balloon" next to the line. After a subject guessed the ordinal position of the balloon, a dart was shot across the screen. If the first guess missed the target, additional attempts were made until the balloon was hit. There were 21 randomly presented target positions, one for each point from 2 to 22. The target positions were classified in the analyses by range; small (from 2 to 8), medium (from 9 to 15), and large (from 16 to 22).

After the 21 trials, subjects were given three more trials (one at each range) and asked to describe their thinking for their first guess.

Several days later, the children were administered a counting skills inventory consisting of three forward-counting tasks (by 2, 3, and 5) and three backward-counting tasks (by 1, 2, and 5).

4. Findings

a. Estimation Accuracy. Data are reported for the mean number of misses per trial and the mean absolute deviation of the initial estimate from the target. The ANOVA analyses for both sets of data showed a highly significant difference ($p < .001$) for age, position, and age x position interaction. Both younger groups made more errors in the medium and large target ranges, although there were no significant differences between the large and medium ranges for the mean number of misses. All groups did well in the small range, and third graders had a high level of performance in all ranges.

b. Counting Skill. There were significant differences in forward and backward counting by the three age levels, with the greatest differences occurring for the large counting interval. There was a significant, moderately strong correlation (-.490), between overall counting performance and number of estimation errors, indicating a relationship between counting skill and accuracy in estimating, regardless of age. Further analysis revealed that counting skill in both directions was significantly related to accuracy at all three target ranges.

c. Counting Strategies. Student responses about their use of counting strategies ranged from guessing to flexible approaches including the use of an intermediate reference point for middle range positions. The responses were used to classify students into four levels of strategy use. Older children reported using more sophisticated approaches than younger children (11 of the 12 children at level 4 were third graders, and 20 of the 23 third graders were at level 3 or level 4). There was also a significant relationship between flexibility of strategy usage and accuracy is estimating, particularly in the middle and large ranges.

In summary, the results showed developmental differences in estimation accuracy, counting fluency, and strategy-use sophistication. Third graders estimated equally well at both ends of the line, and were able to make use of fractional parts when a point was in the middle range (i.e., for a point in the middle range, these students were more apt to select 10 or 11 as a starting point).

5. Interpretations

The findings clearly show an increasing tendency to count strategically when estimating as age increases from 6 to 9. The ability to utilize appropriate counting strategies enabled the older students to minimize the number of steps and maximize accuracy, that is, the mastery of counting allowed it to be applied more flexibly. They further state, "Counting perhaps exemplifies skill learning pulling along behind it a variety of more complex skills and processes." This position is suggested to be consistent with the view that "cognitive development is a process of hierarchial skill integration," rather than a Piagetian model of development.

Abstractor's Comments

This is a well-designed and carefully reported study. It was conducted to provide additional data on questions that are central to the information processing position on the nature of mathematical reasoning and performance. The findings provide additional support for the importance of counting skill, although they do not eliminate the possibility of additional explanations, beyond counting proficiency, for the stronger performance of the older children. Two or three additional years of instruction for the third-grade children likely provided them with a better sense of how to handle such a task, as well as mathematical knowledge and insight that made the task easier in other ways. For example, work with measurement facilitates establishing a scale and judging distances between intervals that are greater than one.

The position that well-mastered skills can facilitate learning other mathematical content and reasoning is an interesting one. It is an important question that should receive more attention in research.

Although numerical estimation was only a setting for the central questions of the study, some information was provided that is useful for those with an interest in the topic. First, all groups performed extremely well in the small target range (2-8), with even the kindergarten students only having a mean of 1 error before hitting the target. The poorest performance was by kindergarten children in the large target range (16-22). Even here the mean number of errors was about 2.5, with a standard deviation of 1.7. The mean deviation for the initial guess by these children was about 4.2, with a standard deviation of 3.6, suggesting relatively good skill in making adjustments in the initial estimates.

It would be interesting in additional research to present situations with a starting point other than 1, and to use a variety of scales (from 1 to 23, from 1 to 40, etc.). Such variations could provide useful information on children's thinking and performance on this type of numerical estimation task.

Perhaps one important measure of a good study is its ability to stimulate the thinking of those who read it. By that criteria, this was an excellent study for this reviewer.

Owen, Elizabeth and Sweller, John. WHAT DO STUDENTS LEARN WHILE SOLVING MATHEMATICS PROBLEMS? Journal of Educational Psychology 77: 272-284; June 1985.

Abstract and comments prepared for I.M.E. by JOHN ENGELHARDT, Southern Oregon State College.

1. Purpose

This study was designed to investigate whether use of a means-ends strategy (working backwards from the goal to the given and reversing the argument) impedes problem-solving achievement in mathematics.

2. Rationale

Previous research has established that expert problem solvers classify problems on the basis of underlying structure while novices tend to focus more on contextual details of the problems. Experts also tend to use a forward-thinking strategy while novices often display a backwards technique, linking the goal statement with antecedents until arriving at the given conditions. The researchers assert that the ability to classify problems correctly allows the expert to use a forward strategy, while the novice's inability forces him to use a means-ends approach.

Research indicates that teaching the ability to classify correctly is not necessarily part of instruction. Additionally there is evidence that using a means-ends approach results in minimal learning. The present study attempted to prevent the use of means-ends analysis by reducing the goal specificity, i.e., by replacing conventional problem statements searching for a specific unknown with ones requiring the calculation of as many unknowns as possible. If means-ends analysis could be prevented, the researchers were interested in detecting any differences in problem-solving achievement.

3. Research Design and Procedures

The investigators conducted three separate experiments. The mathematical content for all three centered on trigonometric ratios of sine, cosine, and tangent with subjects asked to solve routine one-triangle and two-triangle problems. Each experiment included a brief period of instruction designed to familiarize students with the trigonometric ratios. Each experiment also contained an acquisition phase in which subjects solved problems that were either goal specific (specific unknown) or goal modified (many unknowns).

Experiment #1 randomly assigned 20 tenth-grade students to either goal-specific or goal-modified groups. These students had been exposed to trigonometric ratios in the ninth grade. Instruction was followed by pretesting and an acquisition phase of five minutes duration during which subjects solved problems orally, stating the necessary equations for calculating an unknown. Posttesting on conventionally stated one-triangle and two-triangle problems followed.

The second experiment involved 22 ninth-grade subjects who had no previous exposure to trigonometry. They were randomly assigned to either goal-specific or goal-modified groups, given some instruction in the trigonometric ratios, and asked to solve as many problems as possible during the 20-minute acquisition phase. The problems (16 total) were identical to those used in the first experiment. Following this, subjects were posttested on one-triangle and two-triangle problems and a nested two-triangle problem.

The final experiment was to determine if reducing goal-specificity aided in solving a structurally different trigonometric problem. Twenty ninth-grade students with no previous trigonometry exposure were randomly assigned to either a goal or no-goal group. Subjects were given instruction in the principles of trigonometry and in the use of a scientific calculator. During the 30-minute acquisition period students were asked to solve problems in writing, with all calculations done by machine. Following this phase students were given a structurally different trigonometry problem to solve.

4. Findings

Students were classified as employing a means-ends or backwards strategy if they mentioned the problem goal before specifying any sides that would be calculated. A forward strategy was recorded if subjects mentioned a trigonometric equation of a side before mentioning a goal. In experiment 1 all subjects used a means-ends strategy on the pretest and posttest.

Errors were divided into two cases: fundamental and trigonometric. Fundamental errors indicated a lack of understanding of trigonometric ratios while a trigonometric error indicated an inability to use the trigonometric ratio correctly. No significant differences (.05 level throughout) were recorded between groups on the pretest. The no-goal group did have significantly fewer errors and a lower error rate (number of errors divided by the number of sides calculated) on acquisition problems than did the goal group. ANCOVA indicated significantly fewer errors were made by the no-goal group on one-triangle problems but not on two-triangle problems.

Analysis of experiment 2 acquisition problems indicated no significant differences in total errors or fundamental errors, though the error means were lower for the no-goal group. The error rate was significantly lower for the no-goal group. Posttest data indicated significantly fewer fundamental errors on the two-triangle problems. There was no significant difference in total errors.

Results of the third experiment indicated the error rate of the no-goal group was significantly lower on the acquisition problems than that of the goal group. Performance by the no-goal group on the transfer problem was significantly better than that of the goal group.

5. Interpretations

The experiments indicated that reducing goal specificity in trigonometry problems can enhance problem-solving achievement. After initial instruction, allowing students some practice in solving for a variety of sides rather than a specific unknown can help them perform better on subsequent conventional problems of the same type as well as on structurally different problems. When prevented from using means-ends analysis, students tend to perform better and acquire some classification expertise as evidenced by performance on the transfer problem.

The researchers conjectured that since means-ends analysis demands that the problem solver keep a number of items in mind simultaneously (problem goal, current state, relationship of goal to current state, subgoal stack, etc.) he or she has less capacity for knowledge acquisition. Means-ends may be an efficient strategy, but it may interfere with gaining expertise in developing a strategy to classify problems. The researchers suggest that there may be a direct relationship between the efficiency of a problem-solving method and interference with knowledge acquisition.

Several practical implications for mathematics educators are evident, the most obvious being that at early stages of teaching new principles, reducing goal specificity may help in improving student understanding.

Abstractor's Comments

As a mathematics teacher and teacher educator I was impressed with this study. The analysis was sound, and the information gleaned from the research is quite relevant. Implications for classroom instruction are quite apparent. The research brings out the need to prioritize objectives of instruction. If one is interested in an efficient problem-solving strategy, means-ends may be an appropriate choice. But at the early stages of instruction this method may indeed retard understanding of new principles. There are tradeoffs in developing mathematical expertise, and this study clearly points out a situation that merits attention.

Peterson, Penelope L.; Swing, Susan R.; Stark, Kevin D.; and Waas, Gregory A. STUDENTS' COGNITIONS AND TIME ON TASK DURING MATHEMATICS INSTRUCTION. American Educational Research Journal 21: 487-515; Fall 1984.

Abstract and comments prepared for I.M.E. by JAMES BIERDEN, Rhode Island College.

1. Purpose

This study addressed the following questions: 1) What cognitive processes do students report engaging in during mathematics instruction? 2) What affective thoughts do students report having during mathematics instruction? 3) How are student's aptitudes, including ability and attitudes, related to their reported cognitions and affective thoughts during mathematics instruction? 4) How are students' reports of their cognitions and affective thoughts during mathematics instruction related to later achievement and attitudes?

2. Rationale

This study is an extension of previous studies by the major author[1,2] on students' cognitive processes during instruction in mathematics. These studies have used interviews, stimulated-recall, video taping, and observer judgment of students, and related the observed or recalled cognitive processes to the students' ability, achievement, and attitudes. The present study investigated the generalizability of these earlier findings by examining similar variables in a natural classroom setting.

3. Research Design and Procedures

Participants in the study were 29 white and 9 minority students from two fifth-grade classes in an urban elementary school.

Pretest measures included the mathematics concepts subtest from the STEP, the reading vocabulary subtest from the CAT, and a 15-item questionnaire on attitude towards mathematics developed by the principal author. Posttest measures included a Cognitive Process Questionnaire designed as an objective measure of some of the cognitive processes that the students were expected to report during an interview. A Motivational Self-Thoughts Questionnaire was developed to assess whether students encouraged or discouraged themselves and to evaluate their capabilities, performance on, and interest in the mathematics task. An achievement test assessed learning of the basic concepts and skills taught. The attitude toward mathematics scale was administered again as a posttest. The curriculum for the study consisted of a unit on measurement taught in 9 one-hour sessions.

Student behavior was coded during each class, using an observation system adapted from previous research, to provide a measure of students' overt attention. After each lesson, students were interviewed using a stimulated-recall procedure. Students' responses to the stimulated-recall interview were audio-taped, transcribed and then coded into six major categories.

The teacher taught the same measurement lesson to both classes during successive periods. In general, each lesson included review, development, controlled practice, and seat work. The behavior of students who were to be interviewed that day were recorded and coded for consecutive 20-second intervals. Each day 4 to 6 students were video-taped during the lesson. Following the lesson, the video-taped students were interviewed using the stimulated-recall procedure. All students were interviewed once on one of the first four days of instruction and once on one of the last five days of instruction.

The article presents the means and standard deviations of students' scores on the aptitude measures, observations of students behavior, Cognitive Processes and Motivational Thoughts Questionnaires, and student outcome measures. To examine the relationships among these variables, Kendall's Tau coefficients were computed. (This nonparametric measure of correlation was used because scores on the categories from the stimulated recall interview were not normally distributed.) Only those cognitive processes and affective categories that were significantly related to student outcomes were selected for further analysis. The means, medians, standard deviations, range of scores, and generalizability coefficients for categories from the stimulated-recall interviews that met these criteria were reported. The Kendall coefficients for only those categories were presented. The significance of the Kendall coefficients was tested at the .05 level using a one-tailed test of significance because hypotheses about the direction of the correlation had been formed based on the previous studies.

4. Findings and Interpretations

The article discusses results according to each of the six categories of students' responses from the stimulated-recall interview. Because of the length and diversity of this part of the article, only some of the results discussed will be given.

Attending. The findings suggest that important mediating processes necessary for learning may involve more than simply 'attending' to the lesson, and that cognitive processing is an important companion to time on task. The data also suggest that observation of overt behavior during classroom instruction may be inadequate as a measure of student attention.

Understanding. "The results show that students who provided a good explanation of what they did not understand tended to have higher achievement scores." In addition, the study concludes that "students with positive attitudes may persist longer in the face of poor understanding than their counterparts with negative attitudes. Such persistence would increase their chance for eventual understanding."

5. Implications

a. The main results of the study showing that ability and achievement were significantly related to students' reports of their thoughts during classroom instruction -- including their reports of attending, understanding, and engagement in various specific cognitive processes -- support the results of previous studies.

b. Students' reports of understanding and cognitive processes during classroom instruction may be more reliable and valid indicators of learning than observers' judgments of students' attention.

c. The quality of time that students spend attending to the academic task may be as important, or possibly even more important, than the quantity of that time.

One implication of the present research presented in the article is that there are certain cognitive processes that students report using to learn from classroom instruction and these cognitive processes are related to students' ability and subsequent achievement.

Abstractor's Comments

Following the purposes of Investigations in Mathematics Education, the abstractor looked at this article from three points of view: 1) the research itself, 2) the report of the research as seen in the article, and 3) the potential use of the research as clues for the teaching and learning of mathematics.

1. Since this study was part of ongoing research done by the principal author, a number of problems that beset educational research had already been taken care of. The conceptualization and the focus of the research had been refined. Care was taken in the design and the development of specific measures for the study. Proper statistics

were used. For example, the Kendall Tau correlation was used in place of the more standard Spearman Rho correlation because the assumption of normality was not present. The significance of the correlations was tested using a one-tailed test because the researchers could make hypotheses about the directions of the correlations from the results of previous studies. The results of the study, although not completely shown in this abstract, follow from the statistics that were involved. Care was obviously taken, not only to observe what students were doing but then also to ask them what they were doing. As the results indicate, the students' own analysis of their thoughts and activities were more highly correlated to achievement and attitudinal measures. In general, this study points up positive aspects of such follow-up studies and continuing research problems.

2. In general, this abstractor thought there were too many measures - both in terms of measureable constructs and actual measurements taken - defined specifically for this study. The numbers of definitions and reported results made following the article very difficult. In particular, the description of the coding, and the attendant definitions necessary to understand the coding, at times were barriers to a clear understanding of the study from the article. Quite simply put, the definitions and results could have been presented in a better format. Although I am not familiar with the requirements of the particular journal in which the study was reported, it seems that the results could have been laid out in a more visually accessible format. The fact that the major correlation table fills two pages and is still not sufficient to include all the data is evidence of this fault in the article. In the estimation of this abstractor, readers with less incentive to complete the article could be easily bogged down and therefore frustrated by the way the report was put together.

Students' reports of cognitive processes. Students' reported use of two specific cognitive processes, "trying to understand the teacher/problem (level 2)" and "student checks answers", were significantly related to students' seatwork scores and achievement test scores.

Students' reports of teaching processes. "The most consistent significant correlations with students' achievement occurred not with students' reports of specific teaching processes but rather with students' lack of a report of a teaching process that helped them learn." This led the researchers to conclude: "Perhaps as a prerequisite for engaging in cognitive processes for learning from teaching, a student must first be able to differentiate the important teaching process in order to then engage in the intended cognitive process in response to the teaching process."

Mathematics-related content of students' reports of processes. Students who mentioned a mathematics concept in their reports of cognitive processes, either explicitly or implicitly, tended to do better on seatwork and the posttest of achievement. In addition, students' reports of mathematics concepts were significantly related to mathematics ability as measured by the STEP Test.

Students' affective thoughts. The results show that three categories of affective thoughts from the stimulated recall interview were significantly related to students' achievement and attitudes: motivational self-thoughts, negative evaluation of self, and wanting to get the task done. In a corresponding result, scores on a negative sub-scale of the Motivational Self-Thoughts Questionnaire were significantly and negatively related to scores on three of four seat-work scores, both achievement scores, scores on the attitude pretest and posttest, and scores on the STEP and vocabulary tests.

Even having said this, this abstractor would have liked to see one more category of data: the variables that did not correlate with student observations according to the methodology developed by the researchers. The article reports how many variables were left out of the final analysis and their general categories, but specifics may have been important here for a more complete understanding of what transpired.

3. This study gives some significant clues for teaching and learning mathematics. The methodology of the study made it possible to tie observed students' behavior and later confirmation by the students to specific pieces of mathematics content as well as specific teaching techniques. Since this aspect of the study was so well done, it is no wonder that the observations of what was happening in the class took second place to the students telling what they were doing during the lesson. As the study rightly points out, in many instances the students could not verbalize the activities that were taking place or the concepts that were important because they had no previous experience with any verbalization of teaching strategies. However, even given this handicap, the researchers were able to find significant results in terms of relationships between teaching and learning variables. Although it would be foolhardy for teachers to try to replicate this experiment in their own classrooms, the study contains guidelines for the kinds of questions which teachers should ask students about what they are thinking at various times during the lessons and how they are responding to the teachers' stimuli. This type of communication can only help to improve the classroom climate for learning.

Footnotes

[1]Peterson, P. L., Swing, S. R. (1982). Beyond time on task: Students' reports of their thought processes during classroom instruction. Elementary School Journal, 82, 481-491.

[2]Peterson, P. L., Swing, S. R., Braverman, M. T., Buss, R. (1982). Students' aptitudes and their reports of cognitive processes during direct instruction. Journal of Educational Psychology, 74, 535-547.

Sindelar, Paul T.; Gartland, Deborah; and Wilson, Richard J. THE EFFECTS OF LESSON FORMAT ON THE ACQUISITION OF MATHEMATICAL CONCEPTS BY FOURTH GRADERS. Journal of Educational Research 78: 40-44; September/October 1984.

Abstract and comments prepared for I.M.E. by LAURIE HART REYES, University of Georgia.

1. Purpose

The purpose of this study was to examine the relationship between the ratio of teacher-led time to student seatwork time and student achievement. It was hypothesized that the more time students spent during a lesson in teacher-led activities, the greater their achievement would be.

2. Rationale

This experiment attempts to bring together two results from research on teaching. First, several studies have found a positive correlation between student engaged time and student achievement. And second, engaged time has been shown to be higher during teacher-led instruction than during independent seatwork. This study is the third in a series of experiments done by Sindelar and his colleagues to assess the relative effects of teacher-led instruction and seatwork on student achievement.

3. Research Design and Procedures

Fourth-grade students in groups of three were taught 15- or 30-minute lessons on exponential notation by teachers who were university students in special education. The subjects for the study were 108 children from four elementary schools (27 students from each school) in rural Pennsylvania. A lesson consisted of one teacher presenting material to a group of three children. An experimental design with four treatment conditions was used. The two independent variables were teacher and lesson format. The dependent variable was the score on the posttest over the material taught.

The lessons taught by the teachers in all four conditions covered five objectives:

(1) identifying the base and exponent of a term written in exponential notation;
(2) renaming a term written in exponential notation as a series of factors;
(3) renaming a series of factors in exponential notation;
(4) renaming a product of two terms with a common base; and
(5) renaming the product of two terms with a common exponent. (p. 41).

The four treatment conditions were structured with differing ratios of teacher instruction and seatwork. Condition A was 100% teacher-led instruction and consisted of 15 minutes of instruction with no followup seatwork. In Condition B, a 7.5-minute teacher-led lesson was followed by 22.5 minutes of seatwork. This second condition was 25% teacher instruction and 75% seatwork. In Condition C, there were 15 minutes of instruction led by the teacher and 15 minutes of seatwork. This condition was 50% teacher instruction and 25% seatwork, and included 22.5 minutes of teacher instruction followed by 7.5 minutes of seatwork. Thus, the total length of lesson for the various conditions was 15 minutes for Condition A and 30 minutes for Conditions B, C, and D.

In all four conditions, the teachers were carefully trained to follow a scripted presentation. Each teacher had to meet time and accuracy criteria before being allowed to teach groups of children. For each objective, the teacher presented the rule for solving the problem and modeled the solution to a sample problem. The differences in amount of teacher-led instruction time were produced by using different numbers of additional examples beyond the sample problem. In each condition where there was independent seatwork (i.e., Conditions B, C, and D), the teacher "circulated", responded to student questions, and praised students for effort. Following completion of the instructional activities, all of the children in all the conditions completed the same posttest. The posttest contained 64 problems, 16 for each of the four instructional objectives.

Engaged time data were collected in each lesson format. A momentary time sampling procedure was used in which the observer coded behavior as on-task or off-task every 10 seconds. One child was observed at a time and the three students were observed in a predetermined order. The timing of the observations was determined by a tape recording. For Conditions B, C, and D, observations were done in both the teacher instruction and seatwork portions of the lesson. The reliability of the engaged time observations was assessed, and the interobserver agreement was over 90% for all conditions in both teacher instruction and seatwork, except for the teacher instruction segment of Condition B where the agreement was 83%.

A hierarchical design with teachers nested within conditions was used to analyze the posttest means, with student as the unit of analysis. After the omnibus F-test, tests for simple effects were conducted using the sequential Newman-Keuls test. Post hoc analyses of the engaged time data and worksheet data were conducted using Scheffe multiple comparison tests.

4. Findings

The analysis of variance on the posttest means indicated a significant effect for lesson format $F(3,96) = 5.95$, $p < .001$, and a nonsignificant effect for teachers, $F(8,96) = 1.58$, $p < .14$. The followup tests for simple effects showed that the mean score for

Condition D was greater than the mean scores for Conditions C and A, and that the mean score for Condition B was greater than the mean score for Condition A. The Scheffe contrasts produced no significant differences in mean percentage of time on task by condition among either the teacher-led instruction or the seatwork means.

5. Interpretations

The authors concluded that the results did not support the hypothesized relationship between length of time in teacher-led activities and student achievement. Lesson format did affect student achievement, but not in the way the authors expected. One explanation for the obtained results is that achievement is related to the amount of sustained time students spend in a single instructional activity. However, this explanation does not account for all the data from earlier studies by Sindelar and colleagues.

Abstractor's Comments

This study has many strengths. It was carefully designed and implemented. The researchers did an excellent job of designing treatment conditions which differed only in the desired ways. Other indications of good research technique are found in the care with which teachers were trained and in the checks of interobserver agreement on the engaged time observations. Another strength of the study is that it is the third in a series of experiments conducted by the first author and his colleagues. This kind of sustained inquiry into a topic usually produces better quality research and more useful results than a single study.

The study is very tightly designed, but I think there are problems with external validity. The researchers studied groups of three students, apparently assuming that a small group would respond to different ratios of teacher instruction to seatwork in similar ways as would a larger group. The class organization for fourth-grade mathematics instruction is typically whole class or large group. It is unlikely that a class with three students and one teacher, who is not the regular teacher, would work in the same way as a larger group with a regular teacher. I doubt the results of the study would generalize to a regular classroom situation.

This problem of an atypical setting also seems to interact with the researchers' hypothesis. One of the reasons nonteacher-led activities often result in lower engaged time is that the teacher is not able to monitor the activities of each child. It would be clear to each child in a group of three students that the teacher would be able to closely monitor all student behavior. Groups of three would probably lead to a greater increase in engaged time during seatwork activities than during teacher-led instruction, because, even in a larger group, students perceive that the teacher can monitor their behavior during whole-class instruction. With the teacher circulating through a group of three, as in this study, students probably felt that they were under constant supervision. Support for this point is found in the engaged time data. Contrary to the typical classroom situation, in this study students were on-task a greater percentage of the time during seatwork than during teacher-led instruction. Though using larger groups would increase the expense of this study, larger groups would enable the researchers to test their hypothesis in a more realistic setting.

Another criticism of the study has to do with the title. I find the title misleading. The study does not appear to deal with acquisition of "mathematical concepts." The lessons which were taught covered five specific skills related to exponential notation. The detailed description of the various lessons indicated that students were given a rule and then told how to manipulate the symbols. There appears to have been no attempt to explain the underlying concepts. This is more a study of acquisition of skills than it is of acquisition of concepts.

Another weakness of this study is the inclusion of only lower cognitive level tasks in the lessons. Though no single study can answer all our questions, the results of this study would be of greater interest to mathematics educators if the lessons had included some higher cognitive level tasks such as applications of exponential notation in solving problems. Even without adding these tasks to the lessons, the study could have been strengthened by including some transfer items on the posttest and/or testing for retention one or two weeks after the posttest.

Veldman, Donald J. and Sanford, Julie P. THE INFLUENCE OF CLASS ABILITY LEVEL ON STUDENT ACHIEVEMENT AND CLASSROOM BEHAVIOUR. American Educational Research Journal 21: 629-644; Fall 1984.

Abstract and comments prepared for I.M.E. by IAN D. BEATTIE, University of British Columbia.

1. Purpose

The purpose of this study was to investigate the relationship between class composition and student behaviour and achievement. Specifically, the authors set out to determine whether: (a) classroom behaviour and achievement levels are systematically different among classes of higher and lower ability, (b) classroom behaviours of higher and lower ability students differ within classes, and (c) whether the behaviours and achievement of higher and lower ability students differ with class ability level.

2. Rationale

The authors assert that aptitude-treatment interaction studies have shown that, within classes, the effectiveness of instructional methods varies with groups of students of different abilities. They cite studies which concluded that both high and low ability students made greater achievement gains in classes which included more high aptitude students, and that teachers who taught both average and low ability classes were less effective with the low ability classes. The paucity of research in this area indicates a need to study class composition as a context variable affecting instructional processes and outcomes.

3. Research Design and Procedures

The study utilized data from the Texas Junior High School Study (1978) which provi'ed measures of class composition (class mean entering ability level), student ability level (scores on the California Achievement Test), student achievement (scores on specially constructed tests), classrom behaviours (frequency counts and proportions of 25 classroom process variables), and student behaviours (observer ratings of 25 behaviour characteristics of target students). The data were derived from 58 mathematics and 78 English classes from grades 7 and 8 in nine junior high schools in one city. From these classes individual data were available on 10-12 randomly (within sex) selected target students per class, resulting in totals of approximately 500 students in mathematics classes and 650 students in English classes.

The data were analysed by means of linear regression models using class means or individual student scores where appropriate as the unit of analysis.

4. Findings

Predictably, high correlations were found between class ability level and class achievement ($r = .93$ and $r = .95$), and within classes high ability students achieved better than low ability students ($p < .001$). With regard to achievement, there was a significant interaction between student ability and class ability [increments in R^2 were .01 ($p < .001$) for English and .02 ($p < .0006$) for Mathematics], indicating that both higher and lower ability students do better in higher ability classes, with the difference being greater for the lower ability students.

With regard to student behaviours, significant ($p < .05$) relationships were found between observer rating variables for class and student-within-class ability levels in both mathematics and English classes. Higher ability classes and higher ability students within classes were associated with more positive behaviours and with fewer behaviour problems. Significant interactions between class and student ability levels were found for three observer rating variables, showing the differences in behaviour were greater for the lower ability students than for the higher ability students.

With regard to classroom process variables, significant ($p < .05$) results were found for a number of variables suggesting that higher ability classes were associated with better learning environments.

5. Interpretations

The authors conclude that better learning environments are associated with classes of higher ability, that both higher and lower ability students achieve better in higher ability classes and that behaviour and achievement differences are greater for lower ability students than for higher ability students. They conclude that achievement gains can not be attributed to the hypothesis that fewer management problems allow teachers to spend more time with lower ability students and suggest that the gains may be due to teachers having more time for active instruction in small or large groups. They also suggest that lower ability students may be more reactive to class norms than higher ability students or that teachers' norms for behaviour may differ in higher and lower ability class.

The authors explain the small R^2 increments by saying that any one variable can be expected to contribute only a small amount to a measure that is affected by so many variables.

Abstractor's Comments

The question of whether there is a relationship between class composition and achievement is, of course, of considerable interest to educators, but the answer is elusive. While there does appear to be a relationship between class ability level and student behaviour and achievement, the reader is left with many questions and reservations about the study.

1. The rationale for the study repeatedly mentioned class composition but, as the authors point out in their discussion, this study addressed only the question of ability level, not the mix of students in a class.
2. While ostensibly investigating both ability and behaviour, the bulk of the study dealt with classroom behaviours which were analysed in some detail. Yet no information was given as to how observers scored variables such as extraversion, calmness, motivation, etc., or what the criteria for choosing the variables were. There was no similar breakdown of mathematics achievement even though a number of different units of work must have been completed.
3. There was no indication as to the ratio of high and low ability students in the target group.
4. The analysis is inadequate. There was no rationale for using a linear model as opposed to a curvilinear model, no mention of whether the basic assumptions of a regression model were met, no cross-validation, no check for overlap of variables, and no proper test for interaction.
5. The reporting of the results included only R^2 gains. Gains need to be put in practical as well as statistical significance and the variance accounted for by each variable is generally reported. The actual results are weak and gains in R^2 only 1% and 2%.
6. Although the authors properly conclude in the discussion

section that they found an association between class ability level and achievement and class ability level and learning environment, they repeatedly implied or made specific attribution of causality. This is inappropriate.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
July - September 1985

EJ 314 808 Goodlad, Sinclair. Putting Science into Context. Educational Research, v27 n1, 61-67, February 1985.

EJ 314 977 Hackett, Gail. Role of Mathematics Self-Efficacy in the Choice of Math-Related Majors of College Women and Men: A Path Analysis. Journal of Counseling Psychology, v32 n1, 47-56, January 1985.

EJ 315 876 Zehavi, Nurit; And Others. Qualitative Evaluation of Mathematical Activity and its Relation to Effective Guidance. Educational Studies in Mathematics, v16 n1, 27-40, February 1985.

EJ 315 877 Rabijewska, Barbara; Trad, Mieczyslaw. A Mathematical Camp for Bright Pupils. Educational Studies in Mathematics, v16 n1, 41-57, February 1985.

EJ 315 878 Fidelman, Uri. Hemispheric Basis for Schools in Mathematics. Educational Studies in Mathematics, v16 n1, 59-74, February 1985.

EJ 315 879 Streefland, Leen. Search for the Roots of Ratio: Some Thoughts on the Long Term Learning Process (Towards...A Theory) II: The Outline of the Long Term Learning Process. Educational Studies in Mathematics v16 n1, 75-94, February 1985.

EJ 315 908 Pedersen, Katherine; And Others. Attitudes and Career Interests of Junior High School Mathematics Students: Implications for the Classroom. Arithmetic Teacher, v32 n7, 45-49, March 1985.

EJ 315 910 Suydam, Marilyn N. Research Report: Improving Multiplication Skills. Arithmetic Teacher, v32 n7, 52, March 1985.

EJ 315 918 Bell, Elizabeth S.; Bell, Ronald N. Writing and Mathematical Problem Solving: Arguments in Favor of Synthesis, School Science and Mathematics, v85 n3, 210-21, March 1985.

EJ 316 064 Ortiz, Ellen; Kourilsky, Marilyn. The Mini-Society and Mathematical Reasoning: An Explanatory Study. Social Studies Review, v24 n2, 37-45, Spring 1985.

EJ 316 170 Elawar, Maria Cardelli; Corno, Lyn. A Factorial Experiment in Teachers' Written Feedback on Student Homework: Changing Teacher Behavior a Little Rather Than a Lot. Journal of Educational Psychology, v77 n2, 162-73, April 1985.

EJ 316 459 Newville, Lynne; Hamm, Norman H. Content Validity of the WRAT and Grade Level Achievement of Primary Grade Students. Journal of School Psychology, v23 n1, 91-93, Spring 1985.

EJ 317 205 Dalton, David W.; Hannafin, Michael J. Examining the Effects of Varied Computer-Based Reinforcement on Self-Esteem and Achievement: An Exploratory Study. AEDS Journal, v18 n3, 177-82, Spring 1985.

EJ 317 359 Baroody, Arthur J. Mastery of Basic Number Combinations: Internalization of Relationships or Facts? Journal for Research in Mathematics Education, v16 n2, 83-98, March 1985.

EJ 317 360 Ashcraft, Mark H. Is It Farfetched That Some of Us Remember Our Arithmetic Facts? Journal for Research in Mathematics Education, v16 n2, 99-105, March 1985.

EJ 317 361 Rubenstein, Rheta N. Computational Estimation and Related Mathematical Skills. Journal for Research in Mathematics Education, v16 n2, 106-19, March 1985.

EJ 317 362 Behr, Merlyn J; And Others. Construct a Sum: A Measure of Children's Understanding of Fraction Size. Journal for Research in Mathematics Education, v16 n2, 120-31, March 1985.

EJ 317 363 Ferrandez-Reinisch, Anne-Marie. The Acquisition of Inverse Proportionality: A Training Experiment. Journal for Research in Mathematics Education, v16 n2, 132-40, March 1985.

EJ 317 371 Rubba, Peter A.; Becker, Jerry P. Qualities Secondary School Principals Examine When Hiring Mathematics and Science Teachers. School Science and Mathematics, v85 n4, 271-78, April 1985.

EJ 317 375 Sadowski, Barbara. Sentence-Solving Strategies of Elementary School Children. School Science and Mathematics, v85 n4, 317-29, April 1985.

EJ 317 427 Baker, Dale R. Predictive Value of Attitude, Cognitive Ability, and Personality to Science Achievement in the Middle School. Journal of Research in Science Teaching, v22 n2, 103-13, February 1985.

EJ 317 642 Grougeon, Deborah. CEEB SAT Mathematic Scores and Their Correlation With College Performance in Math. Educational Research Quarterly, v9 n2, 8-11, 1984-85.

EJ 317 705 Pigott, H. Edmund; And Others. A Student-Administered Group-Oreinted Contingency Intervention: Its Efficacy in a Regular Classroom. Child and Family Behavior Therapy, v6 n4, 41-55, Winter 1984.

EJ 317 793 Greenan, James P. The Development and Validation of Generalizable Mathematics Skills Assessment Instruments. Journal of Vocational Education Research, v9 n3, 14-30, Summer 1984.

EJ 318 919 McKnight, Curtis C.; And Others. Twelfth-Grade Mathematics in U.S. High Schools: A Report from the Second International Mathematics Study. Mathematics Teacher, v78 n4, 292-300, 270, April 1985.

EJ 318 944 David, Edward E., Jr. The Federal Support of Mathematics. Scientific American, v252 n5, 45-51, May 1985.

EJ 318 968 McKnight, Curtis C.; And Others. Eighth-Grade Mathematics in U.S. Schools: A Report from the Second International Mathematics Study. Arithmetic Teacher, v32 n8, 20-26, April 1985.

EJ 318 972 Suydam, Marilyn N. Research Report: Individualized or Cooperative Learning. Arithmetic Teacher, v32 n8, 39, April 1985.

EJ 319 022 Bower, B. The Left Hand of Math and Verbal Talent. Science News, v127 n17, 263, April 27, 1985.

EJ 319 039 Schein, Martin W. Student Achievement as a Measure of Teaching Effectiveness. Journal of College Science Teaching, v14 n6, 471-74, May 1985.

EJ 319 293 Smith, Lyle R. Presentational Behaviors and Student Achievement in Mathematics. Journal of Educational Research, v78 n5, 292-98, May-June 1985.

EJ 319 417 Graybeal, Sheila S.; Stodolsky, Susan S. Peer Work Groups in Elementary Schools. American Journal of Education, v93 n3, 409-28, May 1985.

MATHEMATICS EDUCATION STUDIES REPORTED IN
RESOURCES IN EDUCATION
July - September 1985

ED 254 393 Wilson, Sandra Meacham; And Others. A Study of Math Achievement in Elementary Schools in the State of Washington. 101p. MF01/PC05 Plus Postage. Available from EDRS.

ED 254 395 Smith, Lyle R. Verbal Clarifying Behaviors, Student Participation, and Student Attitudes in Mathematics. 16p. MF01/PC01 Plus Postage. Available from EDRS.

ED 254 397 Schunk, Dale H.; Hanson, Antoinette R. Influence of Peer Models on Children's Self-Efficacy. 30p. MF01/PC02 Plus Postage. Available from EDRS.

ED 254 398 Schunk, Dale H.; Gunn, Trisha P. Strategy and Attributional Effects on Children's Self-Efficacy and Skills. 28p. MF01/PC02 Plus Postage. Available from EDRS.

ED 254 399 Johnson, Lizbeth Champagne. The Effects of the "Groups of Four" Program on Student Achievement. 30p. MF01 Plus Postage. PC Not Available from EDRS.

ED 254 400 Raizen, Senta A. The Quantity and Quality of Teachers of Mathematics and Science, Grades 1 to 12. 41p. MF01/PC02 Plus Postage. Available from EDRS.

ED 254 409 Wavering, Michael J. The Logical Reasoning Necessary to Make Line Graphs. 29p. MF01/PC02 Plus Postage. Available from EDRS.

ED 254 410 Berlin, Donna F.; White, Arthur L. Computer Simulations and the Transition from Concrete Manipulation of Objects to Abstract Thinking. 8p. MF01 Plus Postage. PC Not Available from EDRS.

ED 254 417 Gourgey, Annette F. The Relationship of Misconceptions about Math and Mathematical Self-Concept to Math Anxiety and Statistics Performance. 35p. MF01/PC02 Plus Postage. Available from EDRS.

ED 254 418 Bromme, Rainer; Juhl, Katharina. Students' "Understanding" of Tasks in the View of Mathematics Teachers. Occasional Paper 58. 22p. MF01 Plus Postage. PC Not Available from EDRS.

ED 254 419 Hart, Lynn C. Factors Impeding the Formation of a Useful Representation in Mathematical Problem Solving. 22p. MF01/PC01 Plus Postage. Available from EDRS.

ED 254 556 Wolfe, Mary L. Development and Validation of a Pre-Post Instruction Course Evaluation Questionnaire: A Pilot Study. 13p. MF01/PC01 Plus Postage. Available from EDRS.

ED 255 156 Suddick, David E.; Collins, Burton A. A Longitudinal Study of the English Usage and Algebra Basic Skills Testing Remediation Paradigm for Older, Masters' Level Students. 7p. MF01/PC01 Plus Postage. Available from EDRS.

ED 255 195 Campbell, Donald L.; And Others. Effectiveness and Achievement: A Preliminary Comparison of Computer Assisted and Print Drill Performance Levels. 18p. MF01/PC01 Plus Postage. Available from EDRS.

ED 255 198 Hayes, LaMarian. 1984 Summer Instructional Television Evaluation. 13p. MF01/PC01 Plus Postage. Available from EDRS.

ED 255 356 Suydam, Marilyn N. Assessing Achievement Across the States: Mathematical Strengths and Weaknesses. 228p. MF01/PC10 Plus Postage. Available from EDRS.

ED 255 360 Dillashaw, F. Gerald; Bell, Samuel R. Learning Outcomes of Computer Programming Instruction for Middle-Grades Students: A Pilot Study. 11p. MF01/PC01 Plus Postage. Available from EDRS.

ED 255 369 Suydam, Marilyn N., Ed.; Kasten, Margaret L., Ed. Investigations in Mathematics Education, Volume 17, Number 4. 74p. MF01/PC03 Plus Postage. Available from EDRS.

ED 255 370 Suydam, Marilyn N. Research on Mathematics Education Reported in 1984. 90p. MF01/PC04 Plus Postage. Available from EDRS.

ED 255 376 Yackel, Erna; Wheatley, Grayson H. Characteristics of Problem Representation Indicative of Understanding in Mathematics Problem Solving. 43p. MF01/PC02 Plus Postage. Available from EDRS.

ED 255 377 Wepner, Gabriella. Assessment of Mathematics Remediation at Ramapo College of New Jersey. 24p. MF01/PC01 Plus Postage. Available from EDRS.

ED 255 378 De Corte, Erik; Verschaffel, Lieven. An Empirical Validation of Computer Models of Children's Word Problem Solving. 29p. MF01/PC02 Plus Postage. Available from EDRS.

ED 255 383 Marsh, Jeffrey Flake; Anderson, Norman D. An Assessment of the Quantitative Skills of Students Taking Introductory College Biology Courses. 9p. MF01/PC01 Plus Postage. Available from EDRS.

ED 255 392 Lampert, Magdalene. Understanding, Doing, and Teaching Multiplication. 82p. MF01/PC04 Plus Postage. Available from EDRS.

ED 255 395 Jesunathadas, Joseph; Saunders, Walter L. The Effect of Task Content upon Proportional Reasoning. 12p. MF01 Plus Postage. PC Not Available from EDRS.

ED 255 579 Chatman, Steven P. The Relationship Between Response Pattern Aberrance and Course Performance in Math Placement. MF01/PC02 Plus Postage. Available from EDRS.

ED 255 684 Carlson, Janet. Defining Math and Science Competencies in Vocational Programs. Final Report. 162p. MF01/PC07 Plus Postage. Available from EDRS.

ED 256 302 Albright, Michael J. Student Incentive Preferences in a Competency-Based Remedial Algebra Course. 18p. MF01/PC01 Plus Postage. Available from EDRS.

ED 256 308 Dalton, David W.; Hannafin, Michael. Examining the Effects of Varied Computer-Based Reinforcement on Self-Esteem and Achievement: An Exploratory Study. 23p. MF01/PC01 Plus Postage. Available from EDRS.

ED 256 315 Goetzfried, Leslie; Hannafin, Michael. The Effect of Adaptive, Advisement, and Linear CAI Control Strategies on the Learning of Mathematics Rules. 28p. MF01/PC02 Plus Postage. Available from EDRS.

ED 256 439 Schonberger, Ann K. Developmental Mathematics in College: What the Research Is and Why There Isn't More. 22p. MF01/PC01 Plus Postage. Available from EDRS.

ED 256 452 Belcher, Marcia. The General Education Mathematics Curriculum and the CLAST. Research Report No. 85-12. 32p. MF01/PC02 Plus Postage. Available from EDRS.

ED 256 568 Shemesh, Michal; Lararowitz, Reuven. Formal Reasoning Skills of Secondary School Students as Related to Gender, Age, School Type and Learning Abilities. 22p. MF01/PC01 Plus Postage. Available from EDRS.

ED 256 863 Epstein, Joyce L. Effects of Teacher Practices of Parent Involvement Change in Student Achievement in Reading and Math. 22p. MF01/PC01 Plus Postage. Available from EDRS.