DOCUMENT RESUME

ED 271 394 SO 017 450

AUTHOR	Stevenson, Harold W.
TITLE	An Analysis of Japanese and American Textbooks in Mathematics.
SPONS AGENCY	Office of Educational Research and Improvement (ED), Washington, DC.
PUB DATE	Oct 85
GRANT	NIE-P-85-3030; NIE-P-85-3031
NOTE	181p.; For other studies in this series on education in Japan, see SO 017 338 and SO 017 443-460.
PUB TYPE	Reports - Descriptive (141)
EDRS PRICE	MF01/PC08 Plus Postage.
DESCRIPTORS	Comparative Education; Cross Cultural Studies; *Elementary School Mathematics; Elementary Secondary Education; Foreign Countries; Mathematical Concepts; Mathematics Curriculum; *Mathematics Education; *Secondary School Mathematics; *Textbook Content; *Textbook Evaluation; Textbook Selection; Textbook Standards
IDENTIFIERS	*Japan; *United States; United States Study of Education in Japan

ABSTRACT

A two part examination of the evidence concerning one possible basis for the poor mathematical performance of United States students, the content of the textbooks used in their classes, is presented. The results of a detailed analysis of Japanese and U.S. mathematics textbooks for grades 1 through 12 are reported. Two international studies (1964 and 1981) and the Dallas Times Herald study are used as the basis for comparison. The first part of the research study contains a discussion of elementary school children in Japan and the United States, similarities and differences in Japanese and U.S. elementary school textbooks, and the differences between the two sets of textbooks when concepts and skills are introduced. The study concludes that material contained in the Japanese elementary school textbooks is somewhat more advanced than in the U.S. texts and concepts and skills are likely to appear earlier in Japanese textbooks. The second part of the study discusses an analysis of secondary school textbooks in the United States and Japan. The paper concludes that U.S. textbooks may form an obstacle to the learning of mathematics because they are long, wordy, and repetitive. Two extensive appendices (over 70% of the document) contain exhaustive summaries of the contents of the Japanese and U.S. elementary and secondary mathematics textbooks chosen for the study, arranged by concept and skill. (RSL)

ED271394

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Preface

This paper was prepared under the auspices of the National Center for Educational Statistics and the U.S. Study of Education in Japan, National Institute of Education, Washington, D. C.

October 22, 1985

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The work on which this publication was based was pursuant to Contracts No. NIE P-85-3030 and NIE P-85-3031 of the National Institute of Education. It does not, however, necessarily reflect the views of that agency.

An Analysis of Japanese and American Textbooks in Mathematics

Mathematics is an important academic discipline in its own right, and plays a critical role in nearly all forms of scientific inquiry. In a scientific era it is important, therefore, that a nation's citizens possess a solid understanding of the fundamentals of mathematics. Not only is such knowledge important to those who pursue careers in which mathematics is a basic tool, but also to those who seek to understand the operations of modern science and commerce. It is of serious concern, therefore, when the fundamentals of mathematics are not being acquired effectively by a nation's children. A lack of knowledge of the concepts and operations of mathematics seriously impedes their ability to adapt to a world in which mathematics plays an increasingly important role. Such is the case with America's children and youth. They consistently have demonstrated a level of understanding of mathematics that is below that of students in other industrialized nations.

Explaining the remarkably poor performance of American students will undoubtedly be a long and complex task. Large differences exist among countries in the activities of students at home and at school. It is not the purpose of this report, however, to discuss the relation of these differences to performance in mathematics. This report seeks to examine the evidence concerning one possible basis of the poor performance of American students: the content of the textbooks that are used in their classes. Results of a detailed analysis of mathematics textbooks used in Japan and in the United

States from the first through the twelfth grades is reported. Before describing this analysis, it is helpful to discuss the international studies of mathematics achievement that led to the current national concern.

First International Study

The inadequate performance of American children in mathematics first received dramatic emphasis in the results of the international comparison of mathematics achievement conducted in 1964 (Husen, 1967). This study involved thirteen-year-olds and pre-university students from 12 countries, including Japan and the United States. We will concentrate here on the comparisons involving these two countries.

Detailed tests of mathematical concepts and operations were given to four groups of adolescents: (a) thirteen-year-olds from 381 schools in the United States and from 210 schools in Japan; (b) all adolescents who were enrolled in the classes where thirteen-year-olds were enrolled; (c) students in their pre-university year who were taking mathematics in 149 schools in the United States and 91 schools in Japan; and (d) students in their pre-university year who were not taking courses in mathematics in 155 schools in the United States and 349 schools in Japan.

On none of the 16 tests did the American students obtain scores comparable to those of their Japanese counterparts. Total scores of the American children were consistently below the average for the 12 countries; scores of the Japanese children were consistently above the average. For example, among thirteen-year-olds, the average z score for the American children was -.25 and for the Japanese children, .76; among the pre-university students taking mathematics courses, the corresponding z scores were -.90 and .38; and of the pre-university group not taking mathematics courses the z scores were -.90 and .34.[1] Average scores for the American and Japanese groups differed, and scores for the top students in each country differed. For example, of the top 4% of the terminal students in math classes, Japanese students solved an average of 43.9 problems and the American students, 33.0. Perhaps the most devastating comparisons were those involving the percentage of students who received scores at various percentiles derived from all countries. It would be expected that 75% of the students would be at or above the 25th percentile, 50% at or above the 50th percentile, and so forth. Of pre-university students taking mathematics courses, 82% of the Japanese, but only 36% of the Americans obtained scores that were at or above the 25th percentile for all students taking the test. At the lower percentiles, therefore, the American students were below the Japanese students but did not perform especially badly in comparison to all the other students. However, 63% of the Japanese students, but only 18% of the American students obtained scores above the 50th percentile; 43% of the Japanese students, but only 9% of the American students obtained scores above the 75th percentile. Thus, American students fell further and further behind the students of other countries as the overall percentiles increased.

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1. A z score represents the degree of departure in standard deviation units of an individual score in a distribution with a mean of 0. The standard deviation for each of the four sets of comparison groups was computed from a distribution of scores for participating students from all countries.

Rural students in both countries fell behind city students, but whether the students were from rural areas, towns or cities, the Japanese children excelled. Similarly, children from families of the lowest occupational group in each country received lower scores than children from families in the highest occupational group; again, however, the differences always favored the Japanese students. In fact, students from the highest occupational groups in the United States received scores very similar to those of Japanese children from the lowest occupational groups. The respective scores, in terms of percentage of items passed, were 26.8 and 27.7. (Japanese children from the highest occupational groups passed 41% of the items; those from the lowest American occupational groups passed 15% of the items.)

Efforts were made to categorize the problems in different ways, but whatever categories were used, the results were the same. Scores for the American students were as depressed for items involving routine application of what had been learned as for items requiring applications of knowledge in solving novel problems. Differences were just as great between the American and Japanese students for verbal problems as for problems involving only computation. Mathematical skills of American adolescents simply were not competitive with those of their Japanese peers.

Second International Study

Only preliminary results of a second international comparison of mathematics achievement of eighth graders and high school seniors are available (Crosswhite, Dossey, Swafford, McNight, & Cooney, 1985), but the results are very similar to those of the first study. Students from 20 countries at the eighth grade and from 15 countries at the twelfth grade participated in this study. The students were tested in 1981, and nearly 12,000 American students participated. Among eighth graders, Japanese students received the highest average scores on all five of the subtests given: arithmetic, algebra, geometry, statistics, and measurement. The status of American students ranged from 18th (in measurement) to 8th (in statistics). Among high school seniors, the Japanese students ranked second on each of the seven tests: sets and relations, number systems, geometry, algebra, probability and statistics, and elementary functions/calculus. Again, the American students performed more poorly on all of the tests than did the Japanese students. The American students ranked 14th in algebra, and 12th in all other areas except sets and relations, where they were 10th. In general, the scores of the Japanese students were from 10 to 20 percentage points above those of their American peers.

As in the earlier study, the top American students were not competitive with top students from other countries. For example, of the students in precalculus courses, American students tended to be at or below the 25th percentile for all countries. Among those who were enrolled in calculus classes, a group that contains approximately 2 to 3% of all American high school students, performance was at the average level for students from all the participating countries.

Some of the items in the 1981 test were the same as those that had been used in the 1964 test. Among the 36 items common to the tests given at the eighth grade, the American students showed a modest decline in performance.

American students in 1981 passed 3% fewer of the items than had the American students in 1964. The decline was greater for problems involving comprehension and application (-4%) than for computational problems (-2%). At the twelfth grade, there was a slight improvement during the 17 years: American students solved 6% more of the problems in 1981 than the American students had solved in 1964. More thorough presentations of the results will be made in 1985, but they will offer little basis for satisfaction to American parents and educators.

The Dallas Times Herald Study

A study of achievement in mathematics and science among twelve-year-olds was commissioned by the Dallas Times Herald in 1983. American children received the lowest average scores among the children from the eight countries in which children were tested. The other countries included Australia, Canada, England, France, Japan, Sweden, and Switzerland. Japanese children received the highest scores. Japanese children were able to answer an average of 50.2% of the questions asked, while their American age-mates were able to answer only 25.3%.

In summary, it appears clear that Japanese junior high and high school students are among the world's most outstanding performers in tests of mathematics achievement. American students are not. In fact, American students seldom perform at a level comparable to that of average students in other industrialized countries.

Elementary School Children

The poor academic performance of American children in mathematics does not begin in junior high school. Even during kindergarten, American children perform significantly more poorly than Japanese children, and differences increase between the first and fifth grades (Stevenson, Lee, & Stigler, 1985). The latter study, conducted in Japan, Taiwan, and the United States, included 288 kindergarten, 240 first- and 240 fifth-grade children in each country. The great disparity found in mathematics scores was in marked contrast with the cross-national differences in reading ability. American children received significantly lower scores on the reading tests than the Chinese children, but their reading scores were not lower than those of the Japanese children. It is in mathematics, therefore, that the poor performance of American children is most pronounced.

Elementary School Textbooks

One of the many questions that arise in attempting to interpret the preceding results is whether the content of the mathematics textbooks used in the two countries is comparable. If it is not, the better performance of Japanese children may be attributed, in part, to more comprehensive coverage of mathematical concepts, or to the introduction of concepts and operations at earlier periods in Japanese than in American textbooks. These questions can be answered through detailed analyses of the content of mathematics textbooks in the two countries.

The analysis of textbooks used in elementary schools in Japan and in the United States were completed in the spring of 1985 for another research project. The procedure was based on a still earlier analysis of elementary school textbooks (Stigler, Lee, Lucker, & Stevenson, 1982). The purpose of both studies was to provide information about the mathematics curricula that would be necessary for constructing valid cross-cultural tests of mathematics achievement.

One series of elementary mathematics textbooks was selected from the United States, Japan, Taiwan, and the People's Republic of China for our analyses. The Japanese textbook series selected was *Atarashii sansu*, (*New mathematics*), which is published by Shoseki Kabushiki Kaisha. The American series was the 1985 edition of *Holt Mathematics*. Both are among the most popular elementary textbook series used in each country. Only the results for the Japanese and American textbooks will be presented here.

The first question faced in analyzing the textbooks is how to define mathematical concepts and skills. We have sought to define concepts and skills at intermediate levels of complexity. For example, if a lesson in the text discussed how to write the numerals from one to ten, we chose not to consider each numeral as a separate concept. Nor did we consider this lesson as part of a more global concept, writing numerals. This meant, therefore, that writing numerals from one to ten was entered in our list of concepts and skills, as were such concepts and skills as writing numerals from 11 to 20, writing and counting to fifty, writing large numbers, writing numerals for word names, each of which constituted a discrete section in the text. Our level of classification was determined, therefore, by the intent of the lesson as well as by its content. We believed there was little utility in making the concepts and skills so global that elementary school mathematics could be described by a relatively small number, or in making them so specific that thousands of concepts and skills would have to be included in a description of the textbooks. The level of analysis that we used is illustrated in the complete lists of the concepts and skills that appear in Appendix A.

Each textbook series was examined page by page by native speakers of each language in order to obtain the lists of the concepts and skills. The three examiners worked closely with each other in making these lists in order to obtain maximal agreement as to the level of complexity of the concepts and skills that should be included. An effort was made to be as specific as possible in describing the concepts and skills and by providing examples of the types of problems that illustrated these concepts and skills. Concepts and skills were listed for each curriculum according to the grade and semester in which they first appeared.

A master list was then created. Before this could be done, it was necessary to organize each of the individual lists. As a preliminary step, all concepts and skills were categorized by topic. The topic divisions were as follows: whole number addition, subtraction, multiplication, and division; mixed calculation; number; fractions; decimals; percentages; geometry; measurement; and problem solving. Within each topic the concepts and skills were listed in order of appearance within each of the curricula. If a grade level was not indicated in the list for a concept or skill for one of the countries, a second search was made through the textbooks to see if the skill or concept truly was missing or merely had been overlooked. Statements that a concept or skill was missing are made, therefore, with a high degree of confidence.

General Impressions

Many differences between the the textbooks and the curricula of the two countries are readily apparent to anyone who spends a few hours reading the books. Before presenting the quantitative results of the analyses, some of these general impressions will be described.

One of the most obvious differences in the textbooks of Japan and the United States is in the appearance of the books and the general way the material was presented. The American textbooks are colorful, complete with color photographs at the beginning of each chapter and colored illustrations and figures on nearly every page. Japanese textbooks, in contrast, are smaller, sho... paperback books with closely printed problems and text, and few illustrations.

A guiding principle underlying the presentation of material in the American textbooks seems to be the importance of repetition and review. The textbooks tend to present problems at each grade level that are related to many different topics, such as geometry, addition, and measurement, and to repeat the discussion with some elaboration at later grades. The Japanese textbooks, in contrast, seem to be developed on the assumption that knowledge should be cumulative from semester to semester; if the concept or skill is taught well the first time, it is unnecessary to repeat the discussion at a later time.

A related feature of Japanese textbooks is that they often appear to be less explicit in their presentations than the American textbooks. For example, the Japanese textbooks never had addition problems with two three-digit numbers involving carrying, although later problems clearly assumed this ability. Because mathematics occupies more time in the Japanese than in American classrooms, Japanese textbook writers may place more reliance on the teacher to assist children with discussion and elaboration of the content of the lesson than is the case in the United States. Another possibility is that Japanese textbook writers seek to engage the child's active participation in the development of concepts to a greater degree than American writers believe is necessary or possible. If all the steps in the development of a concept or skill are presented, children simply must follow the successive steps in the development of the argument, and will not encounter gaps that the children must figure out for themselves.

The textbook analysis suggested that Japanese children are taught to place more emphasis on accuracy than are American children, but American children are given more practice in estimation. Whereas Japanese children were being taught to carry out the division of a fraction such as 1/3 to many decimal places, American children were being taught to estimate how many balls would be contained in a colored photograph of a box of balls.

Although some concepts may be introduced earlier in American textbooks, they seem not to be as sophisticated as those appearing in the Japanese textbooks. For example, probability concepts are introduced in the second semester of the fourth grade in American textbooks, and not until the sixth grade in Japanese textbooks. But when the concepts are introduced in the Japanese textbooks, inferential skills are immediately discussed.

Other, more specific kinds of overall impressions emerged:

1. In geometry problems, the Japanese textbooks presented more work with three-dimensional figures than in American textbooks. Similarly, Japanese textbooks placed more emphasis was placed on the concept of symmetry than American textbooks.

2. There appeared to be more frequent examples of the conversion of numbers, shapes, and other concepts in the Japanese than in the American textbooks. For example, conversion from decimals to fractions was not presented in American textbooks until the second half of fifth grade, but occurred earlier in Japan. Japanese textbooks much more frequently illustrated how shapes can be converted into other shapes or constructed out of various materials, such as ribbons and matches.

3. Japanese books placed greater stress on proportions and ratios than did the American books. This may be related to the fact that more emphasis was placed on "mixed" calculation in Japan than in the United States. Problems such as "4 x 6 - 3" were common in Japanese textbooks, but were nearly non-existent in American textbooks.

4. Large place values were emphasized in Japan. In fact, children were taught place values into the thousand trillions.

5. More emphasis was placed in Japan than in the United States on reading graphs, charts, and tables, and being able to infer information from these presentations.

These are general impressions. More precise information is contained in the tabulation of the various concepts. An overall table, illustrating when each skill and concept first appeared in the textbooks, can be found in Appendix A.

Differences in the Elementary School Curricula

Our lists contained a total of 497 different concepts and skills that appeared in the American and Japanese textbooks (see Table 1). All were not taught in each country, nor were they taught at the same time. Some concepts and skills appeared in the Japanese, but not in the American textbooks, and some appeared in the American, but not in the Japanese textbooks. Nevertheless, 411 of the concepts appeared in the Japanese textbooks, and 408 in the American textbooks. Thus, nearly identical percentages of the total concepts were represented in the textbooks of each country: 82.7% and 82.1%, respectively. As a consequence, only 17.9% of the concepts appeared solely in the Japanese textbooks, and only 17.3% appeared solely in the American textbooks. Generally, then, children in neither country were placed at a disadvantage by having fewer concepts and skills introduced in their texts.

Insert Table 1 about here

There was a strong tendency found for the concepts to be introduced earlier in the Japanese than in the American textbooks. This was true in 30.4% of the cases. Concepts and skills were presented earlier in the American textbooks in only 18.5% of the cases.

Concepts appeared two or more semesters earlier in the Japanese than in the American textbooks in 38% of the cases (see Table 2). The following topics appeared earlier in the Japanese than in the American textbooks: addition, subtraction, decimals, dimensions of solids, decimals and fractions, time, money, relation of circle and sphere, triangle, discussion of the square, rectangle, polygon, rhombus, parallelogram, and calculation of areas. All of these differences were quite large; for example, 64% of the 21 concepts related to subtraction were taught earlier in Japan or appeared only in the Japanese textbooks, as contrasted with only 14% in the American textbooks. Only 20% of the concepts appeared two or more semesters earlier in the American than in the Japanese textbooks. The only topics which appeared earlier in the American than in the Japanese textbooks were ratio and proportion, problem solving, fractions, and weight.

Insert Table 2 about here

The differences in the percentage of concepts and skills appearing first in either the Japanese or American textbooks was small for several topics, including the general concept of number, multiplication, division, mixed addition, subtraction, multiplication, and division, tables and charts, volume and capacity. For the remaining topics, only a few concepts and skills appeared in the textbooks. These included mixed addition and subtraction, mixed multiplication and division, and the use of coordinates.

Topics Covered

The number of concepts included in each of the major categories of concepts included in the elementary school textbooks appear in Table 3. The distributions are comparable for most of the categories, but there are noteworthy discrepancies in several cases. For example, the categories in which there was a difference between the two countries of four or more concepts or skills differed in the two sets of textbooks. Japanese textbooks contained four categories in which there was this degree of departure from the American textbooks: Mixed addition, subtraction, multiplication, and division; three-dimensional solids; data, tables, and charts; volume and capacity. Discrepancies of the same magnitude favored the American textbooks for the following four categories: concept of number, addition, length, and time. Some differential emphasis exists, therefore, in some of the central concepts and skills included in the two sets of textbooks.

Conclusion

We conclude from these analyses that the curricula contained in the elementary school textbooks are somewhat more advanced in Japan than in the United States. Concepts and skills are likely to appear earlier in Japanese textbooks, thereby giving the Japanese children a greater opportunity during the elementary school years to practice these skills and use these concepts than is possible for the American children. A relatively small number of concepts and skills are introduced during the same semester in the two countries, and neither country introduces a notably greater number of concepts and skills than the other. There are differences in the emphases given to various topics in the two countries, but more notable than the differences are the similarities in the number of mathematical concepts and skills presented to the children through these elementary school textbooks.

Secondary School Textbooks

The study of the mathematics textbooks written for students from the seventh through the twelfth grades in Japan and the United States consisted of several phases. The first phase involved selecting the books that were to be analyzed.

Selecting the American Mathematics Textbooks

The decision was made to analyse the three most popular textbooks currently being used in each country at each grade. This decision was easily implemented in Japan, but selecting the most popular textbooks in American junior high and secondary schools proved to be an extremely complicated problem. For example, a call to the National Association of State Boards of Education indicated that they have no statistics related to this question. The lack of a standard curriculum and the independence of school districts have permitted the proliferation of textbook series by publishing companies. There are perhaps scores of textbooks for each topic covered from the seventh through the twelfth grades. Moreover, within each grade there is a large number of topics that can be covered, ranging from consumer mathematics to calculus. After an extensive search, we found that there are no current, comprehensive, national statistics concerning the the frequency with which various textbooks are used in the United States.

Illinois study. A list of frequently used textbooks was complied by Dr. Ken Travers of the University of Illinois in 1981-82, but this list includes only the textbooks used in the eighth and twelfth grades. Travers obtained his data from a national probability sample of mathematics classes in these two grades. The rank orders of the five most popular eighth grade textbooks were:

1. Scott-Foresman: *Mathematics around us*
2. Houghton-Mifflin: *Modern school mathematics*
3. Holt: *School mathematics*
4. Heath: *School mathematics*
5. Silver-Burdett: *Mathematics for mastery*

The most popular textbooks at twelfth grade included:

1. Harcourt-Brace-Jovanovich: *Advanced mathematics*
2. Houghton-Mifflin: *Modern trigonometry*

3. Houghton-Mifflin: Modern introductory analysis

National statistics. Although there is little information about the textbooks that are used, there are data from the National Center for Educational Statistics that indicate the classes taken in a national sample of students who graduated from public and private schools. Information about the courses in which the students were enrolled in 1982 was collected from the time they were in their sophomore year. The courses and the percentage of students enrolled in the courses were as follows:

General mathematics:	2.5%
Prealgebra:	14.7%
Algebra I:	63.2%
Algebra II:	31.2%
Advanced algebra	8.0%
Geometry:	48.2%
Trigonometry:	7.4%
Calculus:	5.6%
Other advanced courses:	13.4%
Unified mathematics (algebra, geometry, calculus)	1.4%
Statistics	1.2%

(The percentage of students enrolled in pre-algebra classes may be an underestimate, since many students may have taken this course in the eighth grade.)

Publishers.

The obvious source of information about the use of textbooks within the United States is the publishers of mathematics textbooks. This is not an easy source of information. Because of the financial implications of such statistics for their company, publishers generally are not interested in providing information about the frequency with which various books are used.

However, in an effort to obtain as much information as possible, national offices of the following publishing companies were called: Harcourt-Brace-Jovanovich, Houghton-Mifflin, D. C. Heath, Addison-Wesley, Scott-Foresman, Holt-Rinehart-Winston, and Merrill. These are the major publishers of junior high and high school textbooks in mathematics. In our attempts to obtain the desired information from these publishers, it was necessary to call many of their offices throughout the United States.

In our telephone calls, the purposes of the study were outlined to a representative of the company. The representative was then asked which were their best selling books for the average level mathematics classes and which books currently were being used the most frequently in classrooms. An effort also was made to see if they would be willing to compare the sales of their books with those of other companies. Varying degrees of willingness to answer these questions were encountered. Some representatives claimed they had no information about sales and would not answer the questions; others were very cooperative and provided information about their sales and those of other companies.

After obtaining as much information from the publishers as possible, we put together a list of what appeared to be the five most popular textbooks at each grade level. The validity of these lists was checked with the representatives who had been the most responsive to our requests for information. On the basis of their feedback, three of the five texts at each grade level were picked for analysis. As a final check of the validity of these judgements the opinions of several leading mathematics educators were obtained.

Textbooks Selected

Textbooks series in the United States typically are divided into three series: the series for elementary schools, junior high schools, and high schools. There are series, however, for grades 1 through 8 and grades 9 through 12. Schools tend to adopt a series for one of these segments. Moreover, students tend to enroll in a sequence of classes that includes Algebra I, Geometry, Algebra II, and some type of twelfth-grade mathematics. We decided, therefore, to select the series that were most popular for grades 7 and 8, and for grades 9 through 12. The textbooks we chose for analysis were:

Grades 7 and 8:
- Heath: Heath mathematics, 1985
  - Seventh grade: 13 chapters; 404 pages
  - Eighth grade: 13 chapters; 404 pages
- Harcourt-Brace-Jovanovich: Mathematics today, 1985
  - Seventh grade: 15 chapters; 440 pages
  - Eighth grade: 15 chapters; 438 pages
- Houghton-Mifflin: Mathematics structure and methods, 1985
  - Seventh grade: 12 chapters; 437 pages
  - Eighth grade: 12 chapters; 465 pages

Grade 9:
- Harcourt-Brace-Jovanovich: HBJ Algebra I, 1983
  - 14 chapters; 534 pages
- Houghton-Mifflin: Algebra structure and methods, 1984
  - 12 chapters; 484 pages
- Merrill: Merrill Algebra I, 1983
  - 16 chapters; 512 pages

Grade 10:
- Harcourt-Brace-Jovanovich: HBJ Geometry, 1984
  - 16 chapters; 626 pages
- Houghton-Mifflin: Geometry, 1985
  - 12 chapters; 543 pages
- Merrill Merrill geometry, 1984
  - 14 chapters; 470 pages

Grade 11:

Harcourt-Brace-Jovanovich: HBJ Algebra II with trigonometry, 1983
16 chapters; 626 pages

Houghton-Mifflin: Algebra and trigonometry structure and method: Book II, 1984
16 chapters; 637 pages

Merrill: Merrill Algebra II with trigonometry, 1983
17 chapters; 571 pages

Grade 12:

Harcourt-Brace-Jovanovich: HBJ Advanced mathematics, 1984
14 chapters; 856 pages

Houghton-Mifflin: Modern introductory analysis, 1984
15 chapters; 700 pages

Merrill: Advanced mathematics concepts, 1981
15 chapters; 564 pages

Selecting the Japanese Mathematics Textbooks

Obtaining information about the most commonly used Japanese textbooks was relatively simple. Although textbooks are published by a number of companies, all texts must adhere to the national curriculum in mathematics established by the National Ministry of Education (Monbusho). Information is compiled by the Japanese Publishers' Union (Nihon Shuppan Kyokai) about the yearly sales of textbooks in Japan. They were able to provide us with information about which textbooks were used most frequently in mathematics classes in Japan from grades 7 through 12. Once this information was obtained, the three most popular textbooks at each grade were purchased and were available to us for analysis within a very short period. These textbooks were published by four Japanese companies: Tokyo Shoseki Kabushiki Kaisha, Keirinkan, and Gakkotosho Kabushiki Kaisha at grades 7, 8 and 9; at grades 10 to 12 the Gakkotosho series is replaced by the series published by

Suken Shuppan Kabushiki Kaisha.

The textbooks used in the analyses for Japan were the following:

Grade 7:
Tokyo Shoseki: Atarashii sugaku
(New mathematics), 1985
7 chapters; 183 pages
Keirinkan: Kaitei sugaku
(Mathematics, revised edition), 1984
7 chapters; 175 pages
Gakkotosho: Chugakko sugaku
(Middle school mathematics), 1985
7 chapters; 172 pages

Grade 8:
Tokyo Shoseki: Atarashii sugaku (New mathematics), 1985
8 chapters; 192 pages
Keirinkan: Kaitei sugaku
(Mathematics, revised edition), 1984
8 chapters; 199 pages
Gakkotosho: Chugakko sugaku
(Middle school mathematics), 1985
8 chapters; 196 pages

Grade 9:
Tokyo Shoseki: Atarashii sugaku (New mathematics), 1985
7 chapters; 187 pages
Keirinkan: Kaitei sugaku
(Mathematics, revised edition), 1984
8 chapters; 192 pages
Gakkotosho: Chugakko sugaku
(Middle school mathematics), 1985
7 chapters; 188 pages

Grade 10:
Tokyo Shoseki: Sugaku I (Mathematics I), 1985
5 chapters; 230 pages
Keirinkan: Sugaku I (Mathematics I), 1984
7 chapters; 216 pages
Suken Shuppan: Sugaku I (Mathematics I), 1985
7 chapters; 223 pages

Grade 11:
Tokyo Shoseki: Daisu. Kika (Algebra. Geometry), 1985
4 chapters; 158 pages
Kisokaiseki (Fundamental calculus), 1985
4 chapters; 162 pages
Keirinkan: Daisu.Kika (Algebra.Geometry), 1985
4 chapters; 144 pages
Kisokaiseki (Fundamental calculus), 1984

5 chapters; 144 pages

Suken Shuppan: Daisu.Kika (Algebra. Geometry), 1985

5 chapters; 166 pages

Kisokaiseki (Fundamental calculus), 1985

5 chapters; 167 pages

Grade 12:

Tokyo Shoseki: Kakuritsu. Tokei (Probability. Statistics), 1985

4 chapters; 148 pages

Keirinkan: Kakuritsu.Tokei (Probability. Statistics), 1984

5 chapters; 120 pages

Sukenshuppan: Kakuritsu. Tokei (Probability. Statistics), 1985

4 chapters; 166 pages

Tokyo Shoseki: Bibun. Sekibun Integral and Differntial Calculus, 1985

3 chapters; 174 pages

Keirinkan: Bibun. Sekibun Integral and Differential Calculus, 1985

6 chapters; 160 pages

Sukenshuppan: Bibun. Sekibun Integral and Differential Calculus, 1985

6 chapters; 191 pages

Procedure for Analysis

Groups of native speakers of Japanese and English undertook the analyses of the textbooks. There were four coders of the Japanese books, all of whom were graduate students at the University of Michigan. They were enrolled in political science, aeronautical engineering, linguistics, and psychology, and were familiar with the mathematics contained in the textbooks they were assigned. Five American students coded the American textbooks. Three were graduate students in psychology, one was entering law school, and one was a major in Japanese studies. Again, all were familiar with the material contained in the textbooks they were assigned.

One person analyzed all three textbooks for each grade or semester. This was an efficient procedure; after completing a detailed analysis of one textbook, it was much easier to analyze the second and third textbooks than to become familiar with the materials appearing at another grade or semester. The advanced books were assigned to the individuals who had enrolled in the greatest number of courses in mathematics.

The coders were given examples of concepts that would be coded. Individuals who had participated in coding the elementary school textbooks assisted the coders in defining the concepts that would be coded. The coders were told that when they were in doubt about a concept, it should be coded. We believed that it would be easier later to compensate for overinclusiveness of concepts than for an overly restrictive approach.

The coders listed each concept on a separate card, along with an example of the concept from the textbook. The grade, semester, and page number at which the concept appeared were also recorded. This master set of cards for each grade constituted the basic data for the study.

In some ways it was easier to develop the master list of concepts for the secondary school textbooks than it had been for the elementary school textbooks. It was sometimes difficult to assess just what category of information the writers of the elementary school textbooks were attempting to impart. The purposes of a section or a chapter typically are explicitly described in the secondary school textbooks.

After the master list had been compiled, all of the cards were reviewed by one person, a graduate student in psychology. The purpose of this review was to integrate the concepts by eliminating redundant entries, and by deciding whether the concepts were represented at an appropriate level of generality and at a similar level across the textbooks from the two countries. It would be nearly impossible for more than one person to function effectively in doing this for all of the textbooks.

In order to obtain a manageable list, it was necessary to organize the concepts into general categories. To do this, narrow, specific concepts were subsumed under more general categories. Several examples may clarify this process: when limits in a function were represented in terms of limits approaching from the right hand and from the left hand, the two concepts were combined as limits of a function; the matrix expression of a linear transformation was subsumed under "linear transformation;" the reciprocal of a complex binomial (a + bi) was subsumed under "reciprocals." Variations or applications of the same concept to different content areas were included under the more basic concepts.

A new master set of cards was then constructed. Each card contained the concept, a brief explanation of the concept, and one or more examples of the concept. In addition, the grade(s) and semester(s) at which the concept appeared were represented in a matrix defined by grade and publisher. If it was not clear to the reviewer whether the same concept defined by two different coders represented the same mathematical idea, the pages of the textbooks on which the concept appeared were consulted. It was not difficult to decide whether or not the two or more concepts matched, once the concept was located in the textbook within the context of the lesson. This type of check was performed for approximately one-third of the cards.

At times it appeared unreasonable that certain concepts would appear in the textbooks of one country but not in the textbooks of the other country. In these cases, the original books were reviewed to ascertain whether the concept was present but had been missed by the original coder. It was necessary to re-check approximately one-fourth of the cards to be confident that the concept had not been missed. Concepts were located in approximately half of these cases, but in the other half of the cases we decided that the concept was truly missing.

A second type of checking was necessary for some of the basic concepts. It was possible that these concepts had been taught before the seventh grade, and thus would not be found in the junior high and high school textbooks. One of the coders of the elementary school textbooks reviewed all of these cases and identified those concepts that had appeared in the elementary school textbooks.

The final list of concepts, separated by major categories, appears in Appendix B. This list contains a total of 485 concepts, a number very close to the 497 concepts and skills derived from analyses of the elementary school textbooks.

General Impressions

It is immediately evident in looking at the textbooks that the American textbooks are much longer than those published in Japan. The average number of pages in the American textbooks was 540. No American textbook had fewer than 400 pages, and the longest book had 856 pages. Japanese textbooks were vastly shorter. None had more than 230 pages, and the average length was 178 pages. Japanese texts tend to be tersely written, while the American textbooks contain long, sometimes repetitive presentations of the basic arguments.

As was the case with the elementary school textbooks, the American textbooks are colorful and have numerous illustrations. Illustrations in the Japanese texts tend to depict only the central mathematical concepts. Seldom do the Japanese textbooks contain information that is not necessary for the development of the concepts under consideration.

American textbooks are much more concrete in their mode of presentation of information than the Japanese textbooks. American writers often attempt to engage the readers' interest by describing practical problems, discussing how the information can be used in everyday situations, and presenting biographical information about eminent mathematicians and scientists. Problems are often personalized and placed in a particular setting. Japanese textbooks tend to be more abstract; there is little effort to place the problems in concrete, everyday settings. Japanese textbooks present the essence of the lesson, with the expectation that the information will be elaborated and supplemented with other materials when it is presented in class by the teacher. American textbooks appear to be written so that understanding the content of the lesson is less dependent upon what happens in mathematics classes.

Problems presented in the Japanese textbooks tend to be more complex than those in the American textbooks. Although simple problems are presented at the beginning of each exercise, problems rapidly become difficult in the

Japanese textbooks. Here, for example, are three word problems, the first from a Japanese seventh-grade text, the second from a Japanese tenth-grade text; and the third from a Japanese eleventh-grade text:

> Between 7 and 10 o'clock, Bus A leaves every 10 minutes, bus B leaves every 15 minutes, and bus C leaves every 12 minutes. At 7:05 the three buses left at the same time. What will the time be when they next leave together?
>
> Points A and B are located on the side of a hill. The elevation of Point A is 160 meters above that of Point D. The angle formed by a line drawn from A to B and a line, AC, parallel to the ground is 21 degrees. What is the altitude of Point B?
>
> There are 100 g of salt water in each of two vessels. The density of these are x% in A, and y% in B. Now, 30 g of the salt water are taken out from each vessel, and transferred to the other vessel. The density of these then becomes x'% in A, and y'% in B. Show that the mapping (x,y) -(x'y)' is a linear transformation.

Examples from the American textbooks are much less complex. As a result, the standard of performance is necessarily lower among American than among Japanese children. It appears that the questions in American textbooks are written at a level of difficulty such that all children should be able to solve them. This is not the case in Japan; problems are included that will not necessarily be solved by every child.

In the American textbooks there are drills, exercises, and examples. Answers to many of the problems can be found in the back of the book. A much smaller number of problems follows each of the Japanese lessons, and in only a few of the books is it possible for children to check the accuracy of their answers. As a consequence, Japanese children cannot adopt a routine, mechanical approach to the solution of problems, but often must evaluate the correctness of their solution themselves.

As was the case with the elementary school textbooks, many of the elementary steps in the development of concepts are omitted in the Japanese textbooks. American textbooks tend to present the information step-by-step and all details are specified. For example, the American textbook might contain the statement that one of the sides of a triangle is shorter than the other two sides. An explicit description of such a fundamental fact would not tend to appear in Japanese textbooks; the children would be expected to make this common sense deduction themselves, or it will be pointed out casually by the teacher.

One of the major impressions one gains in examining the two sets of textbooks, therefore, is the effort of American textbook writers to relate mathematics to the everyday world. Explanations are much longer and problems are simpler and more repetitive than are those found in the Japanese textbooks. Japanese textbook writers, on the other hand, appear to strive to present the information in brief discussions which emphasize the abstract nature of mathematical concepts and engage the child's active participation in the derivation and development of the mathematical concepts.

Results of the Analyses

Analyses of the mathematical concepts in the textbooks from grades 7 through 12 followed the same general pattern used in the analyses of the elementary school textbooks. However, since three books rather than a single book for each grade were analyzed in each country, consideration can be given to the degree to which the concepts are represented in each of the textbooks at a grade. If each concept appeared in each of the textbooks, there should be 1455 entries in our total list of concepts: 3 times the total number of 485 concepts. The total number of entries was 1240, indicating that some concepts appeared in only a single text. We will return to this point later.

When concepts were introduced. Each concept was categorized according to when it was first introduced in the textbooks of the two countries. The results of this analysis appear in Table 4.

Insert Table 4 about here

Only 11% of the concepts were introduced in the same semester in the Japanese and American textbooks. Concepts appeared earlier in one country than in the other 57% of the time. The difference favored the Japanese children. Concepts appeared earlier in Japanese than in American textbooks 34% of the time, and appeared earlier in the American texts 23% of the time. There was not a strong tendency for more concepts to be introduced in the textbooks of one country, rather than in the other. Fifteen percent appeared only in the Japanese textbooks, and 17% appeared only in the American textbooks. It is clear from this analysis that differences in performance in mathematics cannot be attributed to the introduction of a greater number of concepts in Japanese textbooks.

There is a second way to approach the calculation of the percentages presented in Table 4. Rather than use the number of concepts as the variable, the number of times the concepts appear in the three textbooks in each country can also be counted. If each concept appeared in all three textbooks at a grade, the proportions calculated in the two ways would be the same. Both numerator and denominator would simply be three times larger than the numbers of concepts involved. Differences in the two sets of proportions reflect the degree to which different concepts appear in all three textbooks at a grade.

It is apparent from the data in Table 5 that the proportions obtained for Japan by this second method are similar to those found with the first method. A large number of concepts appeared in all three textbooks. However, the number appearing in all three American textbooks was much smaller. The percentage of times a concept was repeated in all three textbooks was determined by dividing the number of concepts appearing in all three textbooks by the total number of concepts. (The data for calculus were omitted, thereby reducing the total number of concepts from 485 to 437. The calculus data are omitted because no American textbook was devoted solely to calculus.) The resulting percentages were 64% for Japanese textbooks and 37% for American textbooks.

Insert Table 5 about here

This large difference in the frequency with which concepts appear in all three textbooks at each grade appears to be directly attributable to the fact that there is a national curriculum in Japan that defines the curriculum and directs when concepts are to be introduced. All textbook writers in Japan are strongly influenced by the content and order of material described in the national curriculum. In the United States, the order and time for introducing concepts are more a matter of individual choice of the teacher and individual textbook writers.

Time of introduction of concepts. How much earlier are concepts introduced in Japan than in the United States and vice versa? Of the 276 concepts introduced earlier in one country than in the other, 41% appeared two or more semesters earlier in the Japanese textbooks than in the American textbooks and 27% appeared two or more semesters earlier in the American textbooks (see Table 6). Thus, the advantage in favor of the Japanese students is often a year or more in length. Earlier introduction of concepts may allow greater opportunities for practice, thereby translating this advantage in time into higher levels of performance.

Insert Table 6 about here

Topics covered in elementary school. Some of the concepts introduced in the secondary school textbooks in one country did not appear in the textbooks of the other country. To assess whether these concepts had already appeared in the elementary school textbooks of each country, a search was made of the elementary school textbooks. An overlap of 34 concepts was discovered. That is, 34 concepts discussed in secondary school textbooks had been discussed earlier in the textbooks of elementary school. Of these 34 concepts, 32 were included in the Japanese elementary school textbooks, and 19 in the American elementary school textbooks.

Two concepts appeared in the American, but not in the Japanese elementary school textbooks, but 15 concepts found in Japanese elementary school textbooks had not appeared in the American elementary school textbooks. (Seventeen concepts appeared in both sets of elementary school textbooks.) Thus, concepts that appeared in American secondary textbooks were more likely to have appeared in Japanese elementary school textbooks than was the opposite case.

Concepts in three categories were introduced in the Japanese, but not in the American elementary school textbooks. These dealt with measurement, decimals, and probability. Concepts in the following categories were included in the elementary school textbooks of both countries: addition and subtraction; percentages, proportions, and ratios; fractions; and factoring.

Representation of concepts. The total number of 485 concepts have been divided in Table 7 into 19 major categories according to the number of concepts in each category. Generally, concepts are represented with equal frequency in the two sets of textbooks. There are areas, however, in which greater emphasis is given to a category in one country than in the other.

The difference between the textbooks of the two countries in the number of concepts appearing in each category was determined. A difference of 3 or more concepts was considered to constitute greater emphasis on a category of mathematics in one country than in the other. With this criterion, American textbooks gave greater emphasis to fractions, percentages and ratios, addition and subtraction, measurement, decimals, general geometry, and geometric triangles and angles than did the Japanese textbooks. Japanese textbooks, on the other hand, gave greater emphasis to graphing functions, three dimensional figures, equations and sets, probability and statistics, and calculus. When a differential emphasis was given to the categories of concepts in the two countries, emphasis appears to be placed on more advanced topics in Japan than in the United States.

Repetition of concepts. The American curriculum in mathematics is often described as a spiral in which topics reappear in successive years at more and more advanced levels. Although it was impossible to evaluate the level of sophistication with which each concept was introduced, it was possible to assess how frequently the textbooks returned to particular concepts after they had been introduced in an earlier year. The frequencies are summarized in Table 8.

Insert Table 8 about here

The description of the American spiral curriculum was supported in this analysis. A great majority of the concepts (72%) were repeated at least once in the American textbooks after their initial introduction. Nearly a quarter (24%) were repeated twice, and 9.9% were repeated three times. These values are in striking contrast with those obtained from the Japanese texts. There was some tendency to return to a topic once; this occurred for 38% of the concepts. Very few (6%) were repeated more often.

The philosophy underlying Japanese textbooks appears to be that the student should master the concept in its initial presentation or with one repetition at most. This is not the guiding philosophy of American authors of mathematics textbooks. It is apparently assumed that repetition facilitates learning. In view of the American students' performance, the alternative hypotheses that repetition may also lead to boredom and to a failure to master the concept at any time must also be entertained.

Conclusions

We end this analysis with some general conclusions about the Japanese and American textbooks. We are led, after spending months reviewing the mathematics textbooks of the two countries, to conclude that American textbooks may form an obstacle to the learning of mathematics by American students. The books are long, wordy, and repetitive. There is basis for the statement often made by teachers of mathematics that there is no possibility of completing the coverage of the textbooks within a year. Overwhelmed by the length of the texts and aware that the concepts will be encountered by students again at a later grade, most teachers may exert little effort to cover the material contained in elementary and secondary textbooks.

The American textbooks do not make high demands on the students in terms of level of difficulty of the concepts presented nor in the necessity for independent thinking on the part of the student in solving problems. Expectations appear to be very different in Japan. Textbooks appear to be written with the expectation that students can master the contents of the textbooks within the school year. The textbooks are short, definitive, and conducive to intensive study. Teachers obviously are expected to supplement the materials contained in the textbooks in several ways: through elucidation of the content of the textbook and through offering the children opportunities for practice and drill through supplementary problems and exercises. American textbooks are self-contained and require little supplementation by teachers.

Although efforts are made by American textbook writers to make the material meaningful, the result often seems to be that of distracting the students from appreciating the necessity for abstract representation within mathematics. Problems within each lesson tend to follow common patterns of solution and answers are readily available to the students. The problems presented in Japanese textbooks are more varied, both in level of difficulty and in mode of solution. Our impression is that Japanese children appreciate the practical applications of mathematics, but that these applications are emphasized by the teacher rather than in the textbooks.

Japanese textbooks appear to be more advanced than the American textbooks in the level of the concepts presented. The number of concepts is no greater in the Japanese than in the American textbooks, but in both elementary and secondary school concepts tended to be introduced earlier in the Japanese textbooks. Japanese children are expected to master complex concepts more rapidly during the first 12 years of schooling than American children.

Improved performance in mathematics by American children will depend upon altering many current practices in American elementary and secondary schools. One productive source of ideas about these modifications can be obtained through the analysis of the curricula and textbooks of other countries. Continued analyses of Japanese textbooks in mathematics may be particularly productive as a source of ideas in view of the outstanding success of Japanese students in mathematics. We must keep in mind that many of the innovations in teaching mathematics in Japan followed the development of the "new mathematics" curriculum in the United States. The ways in which Japanese educators have modified and are utilizing ideas from this curriculum should be particularly informative to American mathematics educators.

Table 1

Time At Which Concepts and Skills Were Introduced in Japanese and American Elementary School Textbooks in Mathematics

	N	%
Japan earlier than U.S.	151	30.4
U.S. earlier than Japan	92	18.5
Same time	79	15.9
Japan, but not U.S.	89	17.9
U.S., but not Japan	86	17.3

Table 2

Separation (in Semesters) between Time Topics Were Introduced in the Japanese and American Elementary School Curricula

	Number of Semesters							
	1		2		3		>3	
	N	%	N	%	N	%	N	%
Japan Earlier	60	25	36	15	24	10	31	13
U.S. Earlier	32	13	36	15	12	5	12	5

Table 3

Distribution of Number of Concepts and Skills in Japanese and American Elementary School Mathematics Textbooks

Concept or skill	Japan	U.S.
Addition	26	35
Subtraction	21	18
Multiplication	20	22
Division	25	27
Mixed addition and subtraction	3	2
Mixed multiplication and division	1	0
Mixed addition, subtraction, multiplication, and division	8	4
Concept of number	35	39
Factoring	5	7
Ratio and proportion	11	7
Fractions	35	33
Decimals	27	25
Three-dimensional solids	15	6
Coordinates	1	4
Length	10	15
Integers, decimals and fractions	11	8
Data, tables, charts	20	14
Time	14	24
Money	6	7
Problem solving	15	18
General Concepts	23	21
Circle and sphere	8	6

Table 3, continued

Concept or skill	Japan	U.S.
Triangle	8	10
Square, rectangle, parallelogram, rhombus, polygon	13	11
Symmetry	4	5
Area	13	10
Volume/capacity	19	15
Weight	11	12
Other	3	3

Table 4

Time At Which Concepts Were Introduced in Japanese and American Secondary School Textbooks in Mathematics

	Concepts		Concepts in Three Textbooks	
	N	%	N	%
Japan earlier than U.S.	163	34	411	33
U.S. earlier than Japan	113	23	204	17
Same time	53	11	244	20
Japan, but not U.S.	73	15	163	13
U.S., but not Japan	83	17	218	18
	485	100	1240	101

Table 5

Number of Concepts Appearing in All Three Mathematics Textbooks at Each Grade of Secondary School

	Japan	U.S.
Japan earlier than U.S.	106	58
U.S. earlier than Japan	89	32
Same time	40	18
Japan, but not U.S.	44	
U.S., but not Japan		54
	279	162

Table 6

Separation (in Semesters) between Time Topics Were Introduced in the Japanese and American Secondary School Curricula

	Number of Semesters							
	1		2		3		>3	
	N	%	N	%	N	%	N	%
Japan Earlier	50	18	58	21	13	5	42	15
U.S. Earlier	40	15	40	15	6	2	27	10

Table 7

Frequency with Which Concepts Are Represented in Each Major Category of Japanese and American Secondary School Textbooks

Concept or Skill	Japan	U.S.
Multiplication/Division	12	14
Fractions	11	19
Irrational Numbers	15	1[illegible]
Graphing Functions	44	41
Factoring	27	29
Inequalities	8	8
Percentages/Ratios	4	12
Equations	2	1
Circles 3-Dimensional	27	24
Equations, Sets	27	24
Addition/Subtraction	12	21
Measurement	4	13
Problem Solving	16	15
Decimals	4	13
Probability/Statistics	46	32
Exponential/Polynomials	22	22
General Geometry	31	35
Geometric Triangles and Angles	23	27
Calculus	48	28
	485	485

Table 8

Frequency With Which Concepts Are Repeated in Japanese and American Textbooks

	Times Repeated				
	1	2	3	4	5
Japan	189	26	3	1	
U.S.	347	115	48	7	1

APPENDIX A

<u>Summary of Contents in Primary School Math Texts</u>

Code is as follows:

JP=Japan US=United States

10=First Year, First Semester
15=First Year, Second Semester
20=Second Year, First Semester, etc.

General Concept of Number

	JP	US
Counting to 10	10	10
Concept of °more/less'	10	10
One to one matching	10	10
Writing numerals from 1 to 10	10	10
(Rule of using) ordinal mumber	10	--
°0' appear	10	10
Counting to 20	10	10
Writing 11 to 20	10	10
Concept of place value (10's & 1's)	15	10
Counting by 2's: 2, 4, 6,...20.	10	20
Counting and writing to 38 (possibly more)	10	10
Decomposition in 10's 30=10+10+10	10	--
°One more' 5 6	10	10
°One less' 6 5	--	10
Write & count to 50	15	10
Counting & writing to 100	15	15
Counting by 10°s to 100	15	10
Comparison within 100, (no < or > sign)	15	15
Counting by 5's to 100	15	10
<,> sign introduced	20	20
Read and write # < 1000, & expanded notation	20	25
Place value (hundreds)	20	25
Compariaon of #s within 999	20	25
Give numerals to word names seventeen=17	--	20
Count, read, write, & compare up to 10,000	30	40
Place value (10 thousands) °Wang'	30	40
Count by 100's to 900	20	25
Read & write large number 1,000,000,000,000	40	50
°About' number introduced	40	30
Round in/off introduced	40	30
Meaning of number line	15	10
Write numerals to 4-dig for word names	--	30
Write numerals to 4-dig for expanded notation	50	30

3456=3x1000 + 4x100 + 5x10 + 6		
Round whole numbers to nearest 10,000	40	30
Estimate sums & differences of whole numbers & money amount	--	30
Place value thousand trillion	40	--
Expanded notation of large number a trillion	--	60
Roman numerals	--	40
Numeral comparisons up to 99,999	40	40
Odd & even	50	20
Negative integers on number line -5, -1<0,-5<-1	--	65
Rounding # through 100 trillions	40	60

Ratio & Proportion

Ratio & °value of ratio': meaning, read, & write 2 to 6 = 1/3	60	45
Equivalency between ratios 20:30 = 4:6 = 2:3	60	45
Ratio application 3 : 10 = () : 100	60	65
Ratio using fraction : fraction	60	--
Ratio using decimal / fraction	60	--
Direct proportion, concept of a = b x c relation between a,b &/or a,c	60	65
Express direct proportion relations in equations & figures on coordinate	60	55
Inverse proportion, concept of a = b x c relation between b & c	60	65
Express direct proportion relations in equations & figures on coordinate	60	55
Inverse proportion, concept of a = b x c relation between b & c	60	--
Express inverse proportion relation in equations & figures on coordinate	60	--

Addition

1 dig + 1 dig sum < 11	10	10
1 dig + 1 dig sum < 19	15	25
2 dig + 1 dig no carrying 15 + 3	15	15
2 dig + 1 dig with carrying 25 + 7	20	25
2 dig + 2 dig no carrying 23 + 15	15	15
2 dig + 2 dig with carrying 56 + 26	20	25
3 dig + 3 dig with carrying	25	30
4 dig + 4 dig	25	40
5 dig + 5 dig	30	40
6 dig + 6 dig	30	50
Decomposition : 5 can be decomposed to 2 & 3	10	--
Composition : put 3 and 5 together, makes 8	10	10
°+' & °=' introduced	10	10
Term: addend	--	40
Term: sum	40	10
Commutative property of +	20	10
Solve picture problems with + sentence	10	10
+ with 0 (3 + 0)	15	10
Methods of mental calculation; within 20 addition : 8 + 5 = 13 ; 8 + 2+3 = 13	15	--
Addition puzzle solving 2 4 1 x 3 x 1 4 5 5 – 10	--	15
Vertical addition, 2 dig, carrying	20	25
Vertical addition, 2 dig, no carrying	20	10
Check by order property	--	25
Complete addition table (within 18)	15	25
Add up to 5 1-dig number sum < 10	--	25
Addition · sum < 100, 3 2-dig 21 + 35 + 27	20	30
Add 3 2-dig number, no carrying	20	25
Associative property of 0	20	30
Add with renaming 3 1-dig	15	40
Add with renaming 4 1-dig	--	40
Add with renaming 5 1-dig	--	40

Add with renaming 3 2-dig	20	40
Add with renaming 3 3-dig	--	40
Add with renaming 3 4-dig	--	50
Add with renaming 3 5-dig	--	50
Add with renaming 3 6-dig	--	50
Adding with like & unlike signs -3 + 5 = 2 & subtract by adding opposite	--	65

Subtraction

1 dig - 1 dig	10	10
2 dig - 1 dig minuend < 20 no borrowing	15	15
2 dig - 1 dig minuend < 20 with borrowing	20	25
2 dig - 1 dig no borrowing	15	15
2 dig - 1 dig with borrowing	20	25
3 dig - 1 dig no borrowing	20	25
3 dig - 1 dig with borrowing	20	30
4 dig - 1 dig	25	30
5 dig - 2 dig	30	40
6 dig - 3 dig	30	50
1 dig - 1 dig - 1 dig	15	--
'-' introduced	10	10
Term: difference	40	10
Solve picture problems with - sentence	10	10
- with 0 (3 - 0)	15	10
Methods of mental calculation 2dig - 1dig Subtraction : 15 - 8 = 7 ; 15 - 5-3 = 7 minuend <= 18	15	--
Vertical subtraction, 2 digits, borrowing	20	25
The sequence of two subtrahend can be changed (13 - 5 - 3 = 13 - 3 - 5)	20	--
Vertical subtraction, 2 digits, no borrowing	20	10
Subtraction check by addition	20	30
Subtract with 0's 3-dig 400 - 254	25	30

Multiplication

'x' sign introduced	25	25
Concept of multiplication	25	25
Multiplication (0 in the middle or at the end) 108 x 9	30	35
Multiplication : with 0's at the ends of multiplicend and multiplier	50	45
Multiply in vertical form	30	25
Terms: product	40	30
Multiples find multiples for 4 4, 8, 12	25	40
Introduce multiplication table	25	30
Commutative property of x	30	25
Associative property of x	35	30
Multiply 3 1-dig using associative property (2 x 2) x 4 = 2 x (2 x 4)	35	30
Multiplication facts : up to 5	25	25
Multiplication facts . up to 9	25	30
Multiplication fact : about 0	30	25
1 dig x 1 dig	25	25
2 dig x 1 dig	30	35
3 dig x 1 dig	30	35
4 dig x 1 dig	--	45
5 dig x 1 dig	--	50
2 dig x 2 dig	35	45
3 dig x 3 dig	40	50
1 dig x 1 dig x 1 dig	35	30

Division

Division : 1 dig / 1 dig	30	25
Division : 2 dig / 1 dig	30	35
Division : 3 dig / 1 dig	35	35
Division : 4 dig / 1 dig	35	40
Division : 5 dig / 1 dig	35	45

Division : 6 dig / 1 dig	35	45
Division : 1 dig / 1 dig ...R	30	35
Division : 2 dig / 2 dig	40	45
Division : 2 dig / 2 dig ...R	40	45
Division : 3 dig / 3 dig	40	--
Division : 3 dig / 3 dig ...R	40	--
Find quotient on multiplication table	--	30
Concept of equal division	30	15
Divide facts through 25 / 5	30	25
Introduce division : 6 / 3 = 2	30	25
Relations between division and multiplication	30	30
Relations between division and subtraction	--	30
Read and write horizontal equations (division)	35	25
Division : a / a, a / 1	30	25
Division : 0 / a	35	25
Term: quotient	40	30
Term: remainder	30	30
Terms: dividend, divisor	--	30
Division check : multiplication	30	35
Division check : multiplication and addition	30	35
Remainder < divisor	30	35
Division : 0's in quotient 1506 / 3 = 502	35	45
Division : with 0's at the end of dividend & divisor	50	40

Factor, G.C.F., & L.C.M.

Term: factor	--	30
Find factors for a number 6 : 1, 2, 3	50	40
Meaning of common factors, G.C.F.	50	55
Find out common factors, G.C.F.	50	55
Meaning of prime number	--	50
Meaning of multiple, common multiple, & L.C.M.	50	50
Find out multiples and L.C.M.	50	50

Mixed + & -

3 1-dig	within 10	7 + 1 - 4	15	--
1 2-dig, & 2 1-dig	within 20	12 - 5 + 7	15	--
Transformations, horizontal equations to vertical for + & -			20	10
Reciprocit property of natural number calculation a + b = c, so c - a = b, c - b = a			--	10

Mixed x & /

Mixed calculation x & /		32 / 4 x 6	40	--

Mixed +, -, x, & /

Four rules calulation +, -, x, & /	40	--
Write arithmetic sentence with + & x	25	30
Distributive property of x & / over + & - (a + b) x c = ac + bc	45	--
() introduced	40	30
Use () in mixed calculation	40	--
Rough estimate the results (#s > 1000) of +, -, x, & /	50	50
Calculation (+, -, x, & /) of rounded integers	50	50
Closure property of integer calculation +, -, & x (/)	60	--

Fraction

Know units in fraction (1/2 1/3, 1/4, ...)	35	25
Calculation (+ & -) of fraction same denom	35	35
Recognize half, third, fourth of a region Tell number of same-size parts that make a whole	35	15
Concept of fraction	35	25
Read & write real fraction : denominator < 100	35	35
Terms: numerator, fraction, denominator	35	35
Comparison between 2 real fractions, same denom	35	35
Comparison between 2 mixed fractions, same denom	45	45
Comparison between 2 fractions, same numerator	45	35
Fraction equivalents 1/2 = 2/4 = 3/6 = 4/8 ...	55	35
Terms: proper fraction, improper fraction, & mixed fraction	45	45
Use number line to express fractions, comparison	45	45
Conversion between improper fraction and mixed fraction	45	45
Fraction = 1 5/5 = 1	45	45
Sub improper/mixed fraction 2 1/5 - 3/5, same denominator	45	45
Add improper/mixed fraction 2 1/5 + 3/5, same denominator	45	45
Write a division for a fraction 2/6 = 2 / 6	-?	45
Fraction reduction and enlargement	55	55
Meaning and methods of fraction commensuration 1/2, 1/3 become 3/6, 2/6	55	55
Rounding and renaming mixed numbers 2 3/4 = 3	--	55
Fraction comparison different denominators	55	55
Fraction addition different denominators	55	45
Fraction subtraction different denominators	55	45
Multiplication fraction x integer	55	55
Multiplication fractions x mixed numbers	55	55
a / b = a/b divison & fraction	55	35
Fraction / integer	55	45
Mixed calculation of fraction +, -, x, & /	60	--

Fraction x fraction, meaning & calculation	60	55
Integer x fraction, meaning & calculation	60	55
Relations between multipicand, multiplier, & product multiplier is a fraction	60	--
Closure property of fraction calculation +, -, x, & /	60	--
Shortcut to multiplying fractions 5/1 x 3/10 = 1/1 x 3/2 = 3/2 = 1 1/2	60	65
Fraction / fraction, meaning & calculation	60	65
Integer / fraction, meaning & calculation	60	65
Dividing fraction & mixed # 1 2/3 / 7/8	55	65
Relations between dividend, divisor, & quotient divisor = fraction	60	--

Decimals

Read, write, of tenth place decimal 0.2, 1.6	35	35
Compare tenth place decimal 3.4 ; 3.6	35	55
Terms: decimal, integer, decimal point, tenth place	35	35
Calculation (+ & -) of decimals tenth	35	35
Decimal notation through hundredths	45	35
Decimal, thousandth 2.465	45	55
Decimal comparison through thousandths	45	55
Division : decimal / integer (1.08 / 12 = 0.7)	45	45
Division : integer / integer = decimal (30 / 4 =7.5)	45	60
Add/sub calculation decimals (hundredth +/- hundredth)	45	35
Decimals calculation thousandth +/- thousandth	45	55
Commutative & associative properties of °x'	50	--
The distributive property of °x' over °+' are valid in decimal calculation.		
Expanded notation with decimal : 1253.7 = 1 x 1000 + 2 x 100 + 5 x 10 + 3 + 7 x .1	50	--
Sophisticated with the shifting of decimal point after the # has been multiplied by 1000, 100, or 10	50	60

Sophisticated with the shifting of decimal point after the # has been multiplied by 1/10, 1/100, 1/1000	50	--
Equal decimals .72 = .720 & comparison	--	55
Decimals on number line or scale thousandth	45	55
Rounding decimals thousandth	--	55
Estimate sums of decimals	--	55
Multiplication decimal x decimal 4.38 x 2.05	50	65
Relations between multiplicand, multiplier, and product	50	--
when multiplier < 1 product < multiplicand		
multiplier = 1 product = multiplicand		
multiplier > 1 product > multiplicand		
multiplier is a decimal		
Decimal multiplication tenth x tenth	50	55
Multiplication : decimal x integer (0.34 x 17)	45	45
Multiplication : integer x decimal	50	45
Division : integer / decimal	50	45
Division : decimal / decimal	50	60
Relations between dividend, divisor, & quotient	50	--
when divisor < 1 dividend < quotient		
divisor = 1 dividend = quotient		
divisor > 1 dividend > quotient		
divisor = decimal		
Multiplying through 3-place decimals .017 x .234	--	60
Annexing 0s to the dividend 96 / .08 becomes 9600 / .08	50	60
Closure property of decimal calculation +, -, x, (/)	60	--
Decimals through 10 thousandth, place value comparing, & equal decimals	50	60

Integer, Decimal & Fraction

Relations between decimal & fraction .001 = 1/1000	40	55
Conversion fraction to decimal 1/4 = 0.25	50	55
Percentage, meaning & method to find out 1% = 1/100	55	65

Problems about discount	--	65
Mixed calculation of integer, decimal, & fraction	60	--
Law of calculation (commutative, associative, & distributive) can also apply to mixed calculation	60	--
Relations & conversion between interger, decimal, & fraction	??	65
Conversion from fraction to decimal infinite decimal 1/3 m=0.333...m=0.33 m according to the need of accuracy	50	60
The usage of integer, fraction, & decimal e.g. zip code in integers, population density in decimals, hours in fraction	60	--
Estimate using integer, fraction or decimal e.g. estimate amount of water using fraction...	60	--
Relation between integer & fraction : 25 = 25/1 Define integer in terms of fraction & its application 35 '/. 24 x 6 '/. 5 = 35/24 x 6/5 = 7/4	60	45
Mark integer, fraction, or decimal #s on number line	60	55
Mixed calculation x & / , among decimal, fraction, & integer	60	--

Data, Table, & Chart

Read bar-graph (p. 69)	30	15
Making a data table	30	30
Read a picture graph	--	10
Data table (use o or x to mark a record table and transform it into a data table)	--	35
Read bar graph : statistic figure	30	35
Data table, transform to bar graph	30	55
Data transformation with <, >, & =	30	--
Meaning of average	55	45
Method of average a set of data; application	55	45
Map reading	--	40
Method of data process raw data - classify -	40	--

data table - data chart Read and draw broken line graph	40	55
Read the frequencies on a bar graph with a fixed group range	65	--
Methods of data classification, tabulation, graphing -- provided a set of data with the range of each group: classify the data into groups make a data table with frequencies forward the data table to a bar graph	65	65
Draw broken line on a co-ordinate, predict something in the future by reading the graph	60	--
Read & draw percentage on data table or circle	55	--
Percentage represented on a filled bar	55	65
Read & draw to scale (reduce or enlarge size)	60	--
Know the correspondence of sides & angles between true shape & map figure	60	--
Application of reduced & enlarged drawing in a map A is two times bigger than B	60	--
Observe the distribution of cases in the graph to see how each case is distributed	65	--
Convert data tables into appropriate types of graph	65	--
Making tables, bar graphs, line graph	65	65
Making pictographs	--	65

Time

Monthly calendar, weekday names	--	20
Concept of month, week, and day	--	10
Concept of year	--	20
Time to the half hour: two o'clock; nine thirty	10	15
Read time on a digital clock (10:00)	--	10
Tell time: o'clock, min. (7:12)	20	25
Name of the months in a year, # of days in in each month	20	20

Be able to use monthly calendar	--	10
Time reading & calculation (+/-)in a certain hour	20	35
Time calculation 10:30 + 40 min. when is it?	30	35
Time calculation within a day	45	45
Time facts: 1 min. = 60 sec.	30	45
Time facts: 1 day = 24 hr.	20	45
Time facts: 1 hr. = 60 min.	20	35
Tell time & use column notation (5:00) to 5 min. int.	20	20
Ordinal position of the days of the week	--	25
Tell time before the hour	--	30
Date writing 12/13/82	--	30
Facts calculation about calendar, yr. mo. & day	--	45
AM. PM. identification	20	40
Time conversion: hours, minutes---minutes (2 hr 50 min = 170 min)	45	45
24 hour system (1 pm = 13 o'clock)	45	--
Decade, century introduced	--	55
Interpret & make a calendar page (for a month)	--	10
Relations among distant, time, & speed	55	60

Money

Native currency introduced	15	25
Give value of a set of money, count it (penny, nickle, dime, quarter in U.S.)	15	10
Give an amount of money, pay a bill	15	15
Solve money problem (+/-) 3c + 5c = 8c	15	10
Money system, +, -, calculation (1, 5, 10, 50, 100)	15	25
Make $ change using subtraction (& addition) model	15	25
Write word names for money amount	--	40

Others

Read thermometer Celcius (as a number line)	--	15
Temperature to nearest degree, measure	--	35
Application of 'average'	55	45
- population density population / area		
- density g/cm**3		
- rate km/hr, m/min., m/sec. ...		
- find x from a set of data and its average		
- production rate (per hour)		

Problem Solving

Complete patterns in sequence (alternate shapes, colors)	--	15
Estimation, solve problem Give a picture, guess how many toys and then count them	--	15
Solve word problems using map Give a simplified map, calculate distance from a to b	--	25
Method to make formula Apples bought - unknown = apples left	30	30
Unknown number in (+or-) equation, and solve it	30	10
Use chart to solve word problems e.g., use circle graph to answer what is the most popular TV program	55	30
Tell if answer to a word problem is true or not	--	35
Multiple choice of a problem sentence (+/-)	--	35
Identify extra information in word problems	--	35
Solve word problems, two steps	15	35
Make formula for word problems all symbols ( )	45	--

Find unknown # from x, or / equation A/5=35 A=?	45	--
Relations between two addends A+B=constant A increases while B decreases e.g. day + night = 24 hr.	45	--
Give a formula, make a table to show the relations between them. e.g. older 14 15 16, younger 10 11 12, o - y = 4	45	--
Basic concepts of probability	60	45
Induce the probability from the outcome of trials	60	45
Problems of prediction	60	55
Determine the probability of an event (1/n)	--	45
Problem solving: methods of tabulating the known facts, find out its regulations, and solve the problems.	65	65
Problem solving method of thinking, analyzing, and simplifying arithmetic problems (explain & check the answer)	65	--
Finding possible combinations	65	--
Concept of function (informal)	65	--
Making up a word problem from a set of data	--	65
Interpreting remainders	??	65
Estimate sums & differences with fractions	--	65

Others

Group 3-D shapes, classify	10	--
Abacus (1)	30	--

General Concepts

Making shapes with rt-triangles & el-triangle	15	--
Use 2 right-isosceles triangles making	15	--

square, triangle, parallelogram		
Use 3 rt-triangle making different shapes	15	--
Use a ribbon, bars, or matches making different shapes	15	--
Right angle = 90 degree	40	55
Method of drawing angle > 180 degree	40	55
Calculation of angles 180 - 60	40	55
Degree of 2, 3, & 4 rt-angle = 180, 270, 360		
Draw right angle	25	45
Features of triangle, square, rectangle, & drawing these figures	25	25
Draw straight line by a thread, rule, or triangle rule.	25	55
Introduce apex, side, of triangle & quadrangle	25	25
Identify pts. inside, outside, on geometric figures	--	25
Comparisons between angles	35	45
Draw line, & line segments	25	55
Concepts : apex, side, angle, rt-angle	25	55
Definition : obtuse angle, acute angle	--	55
Identify line & line segments	25	35
Angle : meaning of, draw	40	45
Use protractor to measure degree	40	55
Perpendicular lines, meaning & drawing	40	45
Parallel lines, meaning & drawing	40	45
Intersecting lines	40	45
Line between any 2 pts. is the shortest line	--	45
Congruent figures rectangle, trapezoid	50	55
Numerical relations betweeen apexs and sides	65	--
Copying angle using protractor	40	65

Circle/Sphere

Introduce : radius & diameter of a circle	30	35
Introduce : center of a circle	30	55
Introduce : circumference	30	65
Know : diameter = 2 x radius, circumference = diameter x 3	30	65
Introduce : sphere	30	25
Introduce : radius & diameter of sphere	30	--

Compass : draw circles	30	55
Meaning of 360 degrees and the angle at the center of circle	55	--

Triangle

Identify and find perimeter of triangles	--	35
Know : equilateral triangle, isosceles triangle	35	65
Draw : equilateral triangle, isosceles triangle	35	65
Conditions for congruent triangle	50	55
Definition and relations of : obtuse triangle, right triangle, acute triangle, equilateral triangle, isosceles triangle	35	55
The sum of the degree of interior angles equals 180 (triangle)	50	65
Know : different triangles can have same area given same base x height	--	55
Know : if two triangles have same base (or height), but one has 2 (3, 4 ...) times the base (height) of the other's, it has 2 (3, 4 ...) times the area of the other	50	65
Drawing congruent triangle (w/protractor & ruler)	50	65
Conditions for similar triangles	60	65

Square, Rectangle, Parallelogram, Rhombus, & Polygon

Recognize 2-D shapes: square, rectangle	10	15
Define square, rectangle	25	10
Perimeter calculation of squares & rectangles	--	35
Parallelogram, meaning & drawing	40	55
Trapezoid, meaning & drawing	40	55

Rhombus, meaning & drawing	40	55
Meaning and property of diagonal	40	55
The sufficient conditions for parallelogram, rhombus & rectangle	65	55
Relations between square, parallelogram, rhombus & rectangle	65	55
Meaning and property of regular polygons	55	55
Know how to draw regular polygons (use ruler & compass)	55	--
Draw parallelograms, quadrangles (use ruler & compass)	50	--
(Regular) polygons introduced	50	55
Degree of the angles in different polygons	55	--

Concept of Shape Symmetry

Identify & make symmetrical shapes (folding)	--	15
Identify symmetrical figures	--	25
Know the meaning of point symmetry on polygons	65	55
Know the meaning of line symmetry on polygons	65	65
Find out symmetric shapes, and the # of symmetric lines on the shapes	65	55
Line symmetric shapes : two shapes are congruent	65	--

3-Dim Solids

Introduce cube, rectangular parallelopiped, apexes, sides, faces	20	25
Unfold boxes, cube & RP	45	65
Rectangular parallelopiped (RP), know constructive features	20	55
Constructive elements of cube and RP	45	55

Relations between faces or sides (in RP)	45	--
Identify cones, cylinder	65	--
Locate objects using 3-D	35	--
Recognize triangular prisms	65	55
Prism, # of faces, apexes, sides & their relations	65	55
Pyramid, # of faces, apexes, sides & their relations	65	--
Unfolding shapes of pyramid and prism	65	--
Circular cylinder & circular cone: their constructive elements and unfolded shapes	65	--
Find out the formula for the # of sides (of base), faces, apexes on different pyramids and prisms	65	--
Calculate the length of sides on cylinder & cone	65	--
Identify pyramid or prism from the top or side	65	--

Co-ordinate

Identifying ordered pairs	--	45
Row and column as a two dimensional co-ordinate	--	45
Graph coordinate points with integer pairs (+ numbers only)	60	65
Graph coordinate points with integer pairs (+ and - numbers)	--	65

Length

Concept of length, comparision (no units)	15	15
Native units of length	--	35
Measure of nearest inch	--	15
Measure to nearest quarter inch	--	35
Measure length with ruler	--	35

Methods of measuring length of objects (in xx cm and xx mm)	20	50
Conversion : 5 cm + 3 mm = 5.3 cm	35	55
Read & draw a line of specific length (use ruler)	20	35
Length calculation +/-	20	35
Concept of length, units of length (cm, mm)	20	15
Length "m"	25	25
1 km = 1000 m	25	40
Length estimation (eye measuring)	25	25
Length estimation in inches, ft., yds., & mile	--	45
Estimate, add and subtract with metric units	25	65

Area

Concept of area, comparision (no units)	15	15
Meaning of area	45	35
Comparision of the surface area of shapes	45	65
Surface area is measurable, can be expressed by units of area e.g. cm**2, m**2; 1 (m**2) = 1000000 (cm**2)	45	45
1 km**2 = 1,000,000 m**2	45	--
Know and use : 1 a = 10m x 10m; 1ha = 100m x 100m; 1km**2 = 1km x 1km = 100ha = 10000a	65	--
Can understand and use area formula for square rectangle to calculate area of complicated shapes	45	35
Know : the rectangle area formula is still valid in case where lengths of the sides of rectangle are in decimals	50	--
Area formula for parallelogram, trapezoid, and its application	50	--
Area formula for rhombus and its application	50	65
Area formula for triangle and its application	50	65
"Pi" = 3.14, meaning, method to find out, relations between "pi", circumference, radius, and area of circle	55	65
Calculate surface area of pyramid	--	65
Calculate surface area of compound solids	50	65

Volume/Capacity

Concept of capacity (water) no units	15	15
Measure to nearest cup, pint, quart	--	15
Concept of volume capacity, comparision (how many cups of water)	15	15
l, dl introduced, conversion, use them to measure...	20	15
Add/sub calculation of capacity	20	35
Find area or volume by counting square or cubic units	50	35
Meaning of volume	50	35
Volume formula of cube and rectangular parallelopiped	50	55
Volume of compound solid, counting	50	35
Volume of compound solid, calculating	50	--
Content capacity and solid volume, concept & calculation	50	55
Volume of objects is measurable, units of volume cm**3, m**3	50	35
1 m**3 = 1,000,000 cm**3	50	--
1 m**3 (kl) = 1000 l	50	55
1 l = 1000 cc (ml)	50	--
1 dl = 100cc	50	--
1 cc = 1 ml = 1 cm**3	50	--
Find volume & area in decimals 28cm x 4.2m	50	65
Difference estimation of capacity	--	40
Capacity estimation: cup, pt., qt., gal.	--	45
Show relation between wt. & vol. of water	65	--
Methods of measuring the vol. of a stone	65	--

Weight

Measure to nearest pound	--	15
Weight : measure, convert, calculate (+/-), with units	35	25
Concept of weight	35	15
Read wt. on scale to nearest g or kg, oz or lb	35	25
Estimation of weight	--	40
Informal units of weight	--	15
Concept of g	35	25
Units kg (choose appropriate instument)	35	15
Units ton	65	40
1 ton = 1000 kg	65	--
1 kg = 1000 g	35	35
1000 mg = 1 g	65	--
Read, measuring objects (ex.: 2kg and 300g)	35	35
Concept of gross weight, net weight, and container's weight	35	--
Weight estimation oz., lb., and ton	--	45

APPENDIX B

Summary of Concepts in Secondary School Math Texts

Code is as follows:

DCH=D.C. Heath

HBJ=Harcourt, Brace and Jovanovich

HM=Houghton, Mifflin

Mer=Merrill

G=Gakkotosho

K=Keirinkan

T=Tokyo Shoseki

S=Suken Shuppan

7.0=Seventh Year, First Semester
7.5=Seventh Year, Second Semester, etc.

	DCM	HBJ	HM	MER	G	K	T	S
ADDITION/SUBTRACTION								
Distributive Property	-	8.5	8.0	9.0	8.0	7.0	8.0	10.0
a x (b + c) = (a x b) + (a x c)								
9 (w - 20) = (9 x 35) - (9 x 20)								
Distributive Property:								
Used to add like terms	-	9.0	9.0	9.0	8.0	8.0	8.0	-
add:								
-5x + a + 7x + -4a								
(-5x + 7x) + (a + -4a)								
(-5 + 7)x + (1 + -4)a								
2x + -3a								
Like terms: Definition	-	9.0	9.0	9.0	-	7.0	7.0	-
Expression containing 2 or more								
terms which can be simplified.								
5x and 3x								
2xy and 7xy								
Subtracting positive and								
negative numbers	7.5	7.5	7.5	9.0	-	7.0	7.0	-
12.4 - (-8) = 12.4+8								

	DCM	HBJ	HM	MER	G	K	T	S
Adding positive and negative numbers	7.5	7.5	7.5	9.0	7.0	7.0	7.0	-
Rules for-								
Sum 2 positive #s								
Sum 2 negative #s								
Sum 1 positive, 1 negative #								
Checking subtraction using addition	-	7.0	-	-	-	-	-	-
758 632								
− 126 + 126								
632 758								
"Carrying" subtotals in addition	-	7.0	-	-	-	-	-	-
3.7 9 5								
+ 6.3 5 2								
1 0.1 4 7								
"Borrowing" in subtraction	-	7.0	-	-	-	-	-	-
7 13 10								
6.~~8~~ ~~4~~ 0								
− 2.5 9 5								
4.2 4 5								
Inverse operations	-	-	8.0	-	-	-	-	-

	DCM	HBJ	HM	MER	G	K	T	S
-adding and subtracting same numbers								
Rounding whole numbers if digit < 5 -down if digit >= 5 -up 7,850 to 8,000	7.0	7.0	7.0	-	-	-	-	-
Order of operations 18 - (15 + 6) ÷ 3 36 ÷ 2 + 4 x (4 - 2)	7.0	7.0	7.0	9.0	-	8.0	-	-
Operations: meanings in words "+" is addition "-" is subtraction "x" is multiplication "÷" is division	7.0	7.0	7.0	9.0	-	-	-	-
Identity elements 0: and 1 for addition, subtraction, multiplication, division 4.53 + 0 = 4.53 29.7 x 1 = 29.7 a x 0 = 0	7.0	7.0	7.0	9.0	-	-	-	-
Additive inverse or								

	DCM	HBJ	HM	MER	G	K	T	S
<u>"opposites"</u> -sum must be zero -additive inverse of 3 is -3 -for every number a: a + -a = 0	-	9.0	9.0	9.0	-	-	-	-
<u>Properties of equality</u> -reflexive a = a -symmetric if a = b, then b = a -transitive if a = b, b = c then a = c -substitution if a = b, then a may be replaced by b	-	10.0	9.0	9.0	-	-	-	-
<u>Grouping symbols</u> -parenthesis -brackets -fraction bars	-	9.0	9.0	9.0	7.0	7.0	-	-
<u>Associate property: addition and multiplication</u> (a + b) + c = a + (b + c) (a x b) x c = a x (b x c)	7.0	7.0	7.0	9.0	7.0	7.0	7.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
<u>Commutative property:</u> <u>addition and multiplication</u> $a + b = b + a$ $ab = ba$	7.0	7.0	7.0	9.0	7.0	7.0	7.0	10.0
<u>Addition on the number line</u> -translation of number line addition to problem	-	9.0	9.0	9.0	8.0	8.0	8.0	-
<u>Properties of equality:</u> <u>addition and multiplication</u> if $a + c = b + c$, then $a = b$ if $ac = bc$ $(c \neq 0)$, then $a = b$	-	9.0	8.0	9.0	7.0	7.0	7.0	-
<u>Division Rule</u> for all #'s a and b $(b \neq 0)$: $a \div b - a/b = a(1/b)$	-	-	9.0	9.0	8.0	8.0	8.0	-
<u>MEASUREMENT</u>								
<u>Mass:</u> Customary units -Pounds, ounces, tons	7.0	7.5	8.0	-	-	-	-	-
<u>Significant Digits</u> -Measuring accuracy	-	-	-	9.5	7.5	7.5	-	-

	DCM	HBJ	HM	MER	G	K	T	S
<u>Liquid Volume</u> Customary units -gallon, quart, pint, etc.	7.0	7.5	-	9.0	-	-	-	-
<u>Length</u> -Customary units -Foot, inch, mile, yard	7.0	7.5	-	-	-	-	-	-
<u>Liquid volume metric units</u> 1 ml = 1 cc	-	-	-	-	-	-	-	-
<u>Weight or mass: metric units</u> 1500 mg = _?_ g 426.5 g = _?_ kg	7.0	7.5	7.5	9.0	-	-	-	-
<u>Measuring length:</u> <u>metric units</u> km ---> mm -changing units within the metric system	7.0	7.5	7.0	9.0	-	-	-	-
<u>Temperature: celsius</u> <u>thermometer</u>	7.0	7.5	-	-	-	-	-	-
<u>Mass and density</u> mass = density x volume	-	-	8.5	-	-	-	-	-

	DCM	HBJ	HM	MER	G	K	I	S
<u>Error of measurement</u> -greatest possible error is 0.5 units of measurement	8.0	8.5	-	-	-	-	-	-
<u>Velocity</u> $h = vt - 16t^2$ h = height v = initial velocity t = time	-	9.5	9.0	9.0	-	-	-	-
<u>Distance formula</u> d = rt distance = rate x time	7.0	7.0	9.0	9.0	-	-	-	-
<u>Temperature</u> formula for conversion -celsius to fahrenheit and -fahrenheit to celsius	8.5	-	-	-	-	-	-	-
<u>DECIMALS</u>								
<u>Dividing by a decimal</u>	7.0	7.0	7.0	-	-	-	-	-
<u>Multiplying by a decimal</u>	7.0	7.0	7.0	-	-	-	-	-

	DCM	HBJ	HM	MER	G	K	T	S
4.7 x 1.8 = ?								
6.54 x 42.1 = ?								
<u>Adding and Subtracting decimals: lining up decimal points</u>	7.0	7.0	7.0	-	-	-	-	-
5.94 + 3.8 = ?								
5.9384 + 2.6578 = ?								
4.87 4.87 3.2 → 3.20 6 6.00 —— —— ? 14.07								
<u>Dividing decimals: writing zeros in the dividend</u>	7.0	7.0	-	-	-	-	-	-
<u>Multiplying decimals: inserting zeros in the product to locate decimal point</u>	7.0	7.0	-	-	-	-	-	-
0.16 (2) x 0.3 (1) ———— 0.048 (3) places								

	DCM	HBJ	HM	MER	G	K	T	S
<u>Divide decimal by a whole number</u> $183\overline{)26.8}$	7.0	7.0	7.0	-	-	-	-	-
<u>Decimal system</u> -place values: tenths, hundredths, etc. 16.9 = (1 x 10) + 6 + (9 x .1) write in expanded form 960,125	7.0	7.0	7.0	-	-	-	-	-
<u>Rounding used to estimate products (decimals)</u> 32.9 x 8.7 $\approx$ 33 x 9	8.5	7.0	7.0	-	-	-	-	-
<u>Rounding decimals</u> -round up if last digit greater than five 0.0615 -round to nearest thousandth	7.0	7.0	7.0	-	-	-	-	-
<u>Multiplying by 0.1, 0.01, 0.001: moving decimal points</u>	7.0	7.0	-	-	9.0	9.0	9.0	-
<u>Multiplying by 10, 100,</u>								

	DCM	HBJ	HM	MER	G	K	T	S
<u>1000: decimal points</u> 10 x 46.83 100 x 46.83 1000 x 46.830	7.0	7.0	7.0	-	9.0	9.0	9.0	-
<u>Changing decimals to fractions</u> 0.75 = 75/100 = 3/4	7.0	7.5	7.0	-	9.0	9.0	9.0	-
<u>Fractions into decimals</u> -fractions represented as decimals -repeating decimals a/b = a ÷ b	7.0	7.5	7.0	-	8.5	8.5	8.5	-

<u>MULTIPLICATION/DIVISION</u>

	DCM	HBJ	HM	MER	G	K	T	S
<u>Expanded form</u> Whole numbers 40,000 + 1,000 + 700 + 8 Write in expanded form: 47, 423, 192	7.0	7.0	7.0	-	-	-	-	-
<u>Symbols for multiplication and division</u> -using a/b for -using a(b) for <u>x</u>	-	-	-	-	-	7.0	-	-

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	DCM	HBJ	HM	MER	G	K	T	S
-rewrite the following expression without using x or -a ÷ b - c x 4								
<u>Coefficient: Definition</u> -the numerical part of a term -name the coefficient of these terms $19g^2h$	-	-	9.0	9.0	7.0	7.0	7.0	-
<u>Definition of "term"</u> -a number, variable, product, quotient $5x^2$, ab/4, y/x	-	9.0	9.0	9.0	7.0	7.0	7.0	10.0
<u>Dividing positive and</u> <u>negative numbers</u> (-3.06) ÷ (0.9)=?	7.5	7.5	7.5	-	-	7.0	-	-
<u>Multiplying positive and</u> <u>negative numbers</u> -Rules for -multiplying 2 pos. #'s -multiplying -1 x # -multiplying neg #'s	7.5	7.5	7.5	9.0	-	7.0	7.0	-

	DCM	HBJ	HM	MER	G	K	T	S
5.32 (-1) =? -9.2 (-3.1)=?								
Multiplicative Inverse Property -for every nonzero number a: $1/a\ (a) = 1$	-	-	9.0	9.0	-	-	-	-
Inverse operations -multiplication and division $5 \times 8 = 40$ $40 \div 8 = 5$	-	7.0	7.0	9.0	-	-	-	-
Multiplicative property of -1 -product of # and -1 is its additive inverse $-1\ (3) = -3$	-	-	9.0	9.0	7.0	7.0	7.0	-
Division terms -divident -divisor -quotient	-	9.0	9.0	9.0	7.0	7.0	7.0	-
Division Property of equality if $a = b$ then $a/c = b/c\ (c \neq 0)$	-	-	-	9.0	7.0	7.0	7.0	-

	DCM	HBJ	HM	MER	G	K	T	S
Properties of equality -multiplication and division if a x c = b x c ($c \neq 0$) then a = b then a/c = b/c ($c \neq 0$) then a = b	-	9.0	8.0	9.0	7.0	7.0	7.0	-
Reciprocals find the reciprocal number of -3/5	7.0	7.5	7.5	9.5	7.0	7.0	7.0	-
Multiplication terms -product -factor	-	9.0	9.0	9.0	7.0	7.0	7.0	-
PERCENTAGES/PROPORTIONS/RATIOS								
Interest -principal, rate -interest = p x r x time	7.5	7.5	7.5	9.0	-	-	-	-
Finding percent increase or decrease -original price: \$19.00 -new price: \$14.25	8.5	7.5	7.5	9.0	-	-	-	-

$= \frac{\text{amount change}}{\text{original amount}}$

	DCM	HBJ	HM	MER	G	K	T	S
Rates as ratios -rates are ratios comparing quantities of different kinds d = rt distance - rate x time	7.5	7.5	7.5	-	-	-	-	-
Estimating with percentages 65 2% ≈ 2/3	8.5	-	-	-	-	-	-	-
Find percent by equivalent decimal 25% of 9.560=?0.25 x 9.560=?	7.5	7.5	7.5	-	-	-	-	-
Percentages, fractions and decimals: conversions 30% = 30/100 = 0.30	7.5	7.5	7.5	-	-	-	-	-
Percentage: definition -"percent" means "per hundred"	7.5	7.5	7.5	9.0	-	-	-	-
Percentages: solving using proportions 9 is 15% of 60 9 = 15/100 x 60 9/60 = 15/100	7.5	7.5	8.5	-	-	-	-	-

	DCM	HBJ	HM	MER	G	K	T	S
Ratio -a comparison of #'s expressed as a/b ($b \neq 0$)	7.5	7.5	7.5	9.0	8.0	8.0	8.5	10.0
Proportion -proportion is a statement of equality of two ratios 2 : 5 = 6 : 15	7.5	7.5	7.5	9.0	8.0	8.0	8.5	10.0
Means-Extremes Property of proportion -if a/b = c/d then ad = bc	-	9.5	9.5	9.0	8.0	8.0	8.5	-
Finding percentages of numbers 38% of 50 =? 38/100 x 50 = 19 50 is what % of 60 50/60 = r/100 r = 83 1/2	7.5	7.5	7.5	9.0	8.0	-	-	-
FRACTIONS								
Algebraic fractions Solving Equations using LCD Solve: 3/2x - 2x/ x + 1 = -2	-	9.5	9.5	9.5	8.0	8.0	8.0	-

	DCM	HBJ	HM	MER	G	K	T	S
$2x\ (x + 1)(3/2x - 2x/x + 1) =$ $-2(2x)(x+1)$ $3\ (x + 1) -2x\ (2x) = -4\ x^2\ \ -4\ x$ $3x + 3 -4x\ \ = -4x^2\ \ - 4x$ $7x =$ $x = -3/7$								
<u>Rational Numbers: Definition</u> If a, b are integers, and $b \neq 0$, then a/b is a rational number	8.5	7.5	8.0	-	9.0	9.0	9.0	10.0
<u>Complex fractions</u> $\frac{2/3}{5/6} =$ $2/3 \div 5/6 = 2/3 \times 6/5$	7.0	-	11.0	11.5	-	-	-	-
<u>Finding fractions for mixed numbers</u> $2\ 1/4 = 2 + 1/4 = 8/4 + 1/4 = 9/4$	7.0	7.0	7.0	-	-	-	-	-
<u>Fractions: terms</u> - numerator - denominator	7.0	7.0	7.0	-	-	-	-	-
<u>Finding mixed numbers for</u>								

	DCM	HBJ	HM	MER	G	K	T	S
<u>fractions</u> 1 1/4 = 1 + 1/4 = 5/4 15/4 → $4\overline{)15}$ with quotient 3, $\frac{12}{3}$ → 3 3/4	7.0	7.0	7.0	-	-	-	-	-
<u>Mixed numbers</u> whole number and a fraction 1 1/4 7 15/27 = 7 5/9	8.0	8.0	8.0	-	-	-	-	-
<u>Equivalent fractions and "lowest terms"</u> $\frac{a}{b} = \frac{a \times c}{b \times c}$ $\frac{5}{6} = \frac{10}{12}$	7.0	7.0	7.0	-	-	-	-	-
<u>Dividing fractions by whole numbers</u> 3/4 ÷ 6 = ? 3/4 ÷ 6/1 = 3/4 x 1/6	-	7.5	-	-	-	-	-	-
<u>Operations with algebraic fractions</u> cancelling factors	-	9.5	9.0	9.5	-	10.0	10.0	10.0
<u>Operations with mixed numbers</u> -addition, subtraction, multip-	7.0	7.0	7.5	-	-	-	-	-

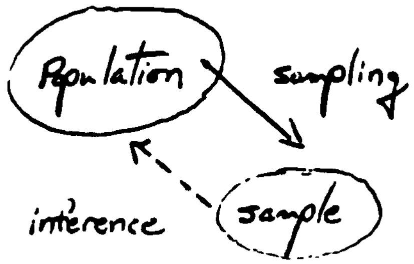
	DCM	HBJ	HM	MER	G	K	T	S
lication using LCD 5/8 x 16 = ? 1 1/2 + 2 1/4 = ?								
<u>Least common denominator</u> 1/6, 3/4, = x/12	7.0	7.0	7.0	-	7.0	7.0	7.0	-
<u>Adding and subtracting fractions</u> a/c + b/c = a + b/c LCD before operation	7.0	7.0	7.5	9.0	8.0	8.0	8.0	-
<u>Fractions: zero in denominator</u> - a ÷ b = a/b -b cannot be zero, -cannot divide by zero	-	7.0	7.0	-	9.0	9.0	9.0	-
<u>Dividing fractions (use reciprocal)</u> a/b : c/d = a/b x d/c	..0	7.5	7.5	9.5	8.0	8.0	8.0	10.0
<u>Multiplying fractions</u> -multiply both numerators and denominators a/b x c/d = ac/bd	7.0	7.5	7.5	-	8.0	8.0	8.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
<u>Multiplying fractions with common factors</u> -cancelling factors $\frac{5}{\cancel{8}_4} \times \frac{\cancel{2}^1}{3} = \frac{5}{12}$	7.0	7.5	7.5	-	8.0	8.0	8.0	-
<u>Proper vs Improper fractions</u> -proper: numerator $<$ denominator -improper: numerator $\geq$ denominator	-	-	7.0	-	-	9.0	9.0	-
<u>Simplifying algebraic fractions containing variables</u> -cannot assign values such that denominator is zero	-	9.0	9.5	9.5	8.0	8.0	8.0	-
<u>Comparison property of rational numbers</u> -if $a/b < c/d$, then $ad < bc$ -if $ad < bc$, then $a/b < c/d$	-	-	9.5	9.0	8.0	8.0	8.0	-

PROBABILITY AND STATISTICS

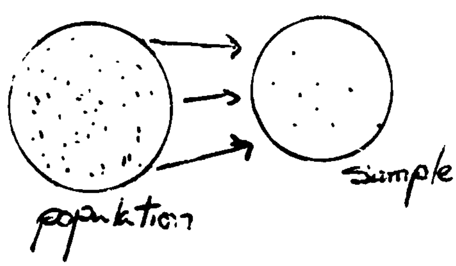
	DCM	HBJ	HM	MER	G	K	T	S
<u>Statistical inference</u> -from sample to population	-	-	-	-	-	12.5	12.5	12.5

	DCM	HBJ	HM	MER	G	K	T	S



	DCM	HBJ	HM	MER	G	K	T	S
<u>Sampling</u>	-	-	-	-	9.5	9.5	9.5	12.5

Sample vs Population



	DCM	HBJ	HM	MER	G	K	T	S
<u>Sampling methods:</u>	7.5	8.5	8.5	–	-	12.5	12.5	12.5

-Random sample

<u>Probability</u>

	DCM	HBJ	HM	MER	G	K	T	S
<u>of equally likely outcomes</u>	7.5	7.5	7.5	-	9.5	9.5	9.5	12.5

$$P(E) = \frac{\text{\# outcomes } E}{\text{total outcomes}}$$

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	DCM	HBJ	HM	MER	G	K	T	S
<u>Mutually Exclusive Events</u>	-	12.5	7.5	12.5	-	12.0	12.0	12.0

	DCM	HBJ	HM	MER	G	K	T	S
p(A or B) = p(A) + p(B) if A & B are mutually exclusive								
<u>Standard Deviation</u> $S=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 f_i}$	-	-	11.5	11.5	-	12.0	12.0	12.0
<u>Normal Distribution</u> if probability of x is binomial B (np), as n becomes large the prob. dist. approaches a normal curve	-	-	11.5	11.5	-	12.5	12.5	12.5
<u>Mean and Variance of Binomial Distribution</u> E(x)=np given that V(x)=npq q=1-p	-	-	-	-	-	12.5	12.5	12.5
<u>Binomial Coefficient</u> nCr= $\frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$	-	-	-	-	-	-	11.0	-
<u>Hypothesis testing</u>	-	-	-	-	-	12.5	12.5	12.5

	DCM	HBJ	HM	MER	G	K	T	S
-null hypothesis, rejecting								
-significance levels								
-critical region (of distribution)								
<u>Confidence interval</u>	-	-	-	-	-	12.5	12.5	12.5
$\bar{x} - \frac{2\sigma}{\sqrt{n}} < \mu < \bar{x} + \frac{2\sigma}{\sqrt{n}}$								
<u>Variance</u>	-	-	-	-	-	12.0	12.0	12.0
-deviation from the mean								
<u>Variables: continuous vs discrete</u>	-	-	-	-	-	12.0	12.0	12.0
e.g. height, weight vs. number of heads in coin toss								
<u>Probability distribution of a random variable</u>	-	-	12.5	-	-	12.5	12.5	12.5
e.g. tossing a coin twice								
$S = \{HH, HT, TH, TT\}$								
<u>Outcomes: simultaneous events</u>	-	-	-	-	-	12.0	12.0	12.0

	DCM	HBJ	HM	MER	G	K	T	S
-how many different outcomes from several types of events that can occur simultaneously in k ways m + n - k								
<u>Statistical measures: (Central tendency)</u> -median -mean -mode	7.5	7.5	7.5	11.5	8.5	8.5	8.5	12.0
<u>Mean of frequency distribution</u> If a variable scores are x_1, x_2, xk, and their frequencies are f_1, f_2, fk, respectively, then $\bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i f_i$ given that: $n = \sum_{i=1}^{k} f_i$	-	-	-	-	-	12.0	12.0	12.0
<u>Symbols x and f</u>	-	-	-	-	-	12.0	12.0	12.0

X=variable

	DCM	HBJ	HM	MER	G	K	T	S
f=frequency								
Properties of summation	-	-	-	-	-	12.0	12.0	12.0
Sample Space of Events	-	-	12.5	-	-	12.0	12.0	12.0
Calculation formulas for variance and standard deviation	-	-	-	-	-	12.0	12.0	12.0

f=frequency

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + \dots}{n}$$

Properties of summation

$$\sum_{i=1}^{n} c = nc$$

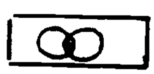
$$\sum_{i=1}^{n} c x_i = c \sum_{i=1}^{n} x_i$$

Sample Space of Events

$$S = \{e_1, e_2, \dots e_n\}$$

with subsets

- union 

intersection 

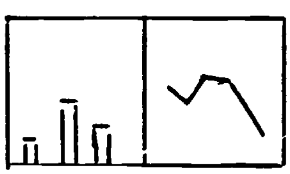
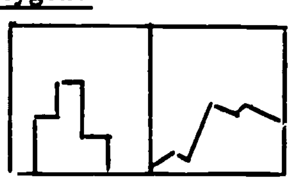
null set 

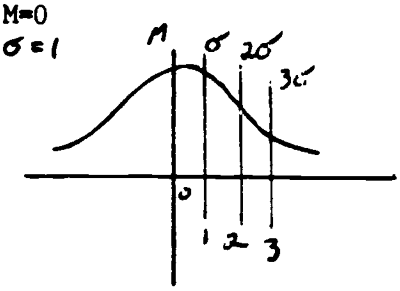
Calculation formulas for variance and standard deviation

$$Var = \frac{1}{n}\sum_{i=1}^{k} x_i^2 f_i - \bar{x}^2$$

$$SD = \sqrt{\frac{1}{n}\sum_{i=1}^{k} x_i^2 f_i - \bar{x}^2}$$

	DCM	HBJ	HM	MER	G	K	T	S
<u>Line Graphs</u> -data represented by points on coordinate axes connected by segments	8.5	8.5	8.5	-	8.5	8.5	8.5	-
<u>Multiple Permutations</u> If choosing r ways out of n elements, then there are n ways of arranging them -Find the number of 3 digit integers you can make using 1, 2, 3, 4 $4^3 = 64$ integers	-	12.5	12.5	12.5	-	12.0	12.0	12.0
<u>Standardized Scores</u> If $X_i = \overline{X}$, $S_x = c$, then $\overline{y} = 0$, $S_y = 1$ (mean = 0, S.D. =1)	-	-	-	-	-	12.0	12.0	12.0
<u>Relative Frequency Distribution</u> -divide class frequency by total N	-	-	-	-	-	12.0	12.0	12.0

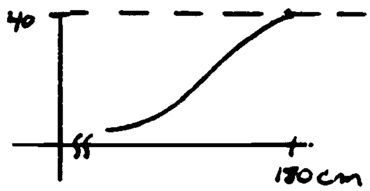
	DCM	HBJ	HM	MER	G	K	T	S
<u>Class Frequency Distribution</u>	-	-	-	-	-	12.0	12.0	12.0
1) representative value								
-a center score								
(max + min value) ÷ 2								
2) class interval								
(max - min) = I								
<u>Frequency Distribution</u>	7.5	7.5	7.5	-	8.5	8.5	8.5	12.0
-group data into intervals								
<u>Bar graphs and broken-line graphs</u>	7.5	7.5	7.5	-	-	12.0	12.0	12.0
								
<u>Histograms and frequency polygons</u>	8.5	-	7.5	-	8.5	8.5	8.5	12.0
								
-graphs of frequency distributions								
<u>Bar Graphs:</u>	8.5	8.5	8.5	-	8.5	8.5	8.5	-

	DCM	HBJ	HM	MER	G	K	T	S
Data is numerical facts								
Standard Normal Distribution M=0 σ=1 	-	-	-	-	-	8.5	8.5	8.5
Expected value of random variable Ev(x) = 5(2/6) + 10 (3/6) + 15(1/6) = 9 1/6	-	12.5	8.5	-	-	12.5	12.5	12.5
Finding probability of different outcomes	-	-	-	-	9.5	9.5	9.5	-
Probability 1.00 = prob. of an event that definitely happens 0.00 = prob. of an event that definitely doesn't happen	-	11.5	11.5	9.5	11.5	9.5	9.5	-
Binomial Distribution -of binomial events e.g., prob. distribution for #1 appearing in 5 die tosses	-	-	12.5	-	-	12.5	12.5	12.5

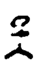
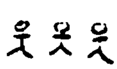
	DCM	HBJ	HM	MER	G	K	T	S
n independent trials $q = 1 - p$ $P(X=x) = {}_{n}C_{x}\, P^{x} q^{n-x}$ $(X = 0, 1, 2, \ldots, n)$ Given that x is the number of times E occurs								
<u>Statistical measures:</u> <u>Variability</u> Range	7.5	7.5	7.5	11.5	8.5	8.5	8.5	12.0
<u>Outcomes: sequential events</u> -how many different outcomes from several types of events? -m x n different outcomes	7.5	7.5	8.5	11.5	-	12.0	12.0	12.0
<u>Cumulative frequency distribution and curve</u>	-	-	-	-	-	12.0	12.0	12.0

168	24
172	33
176	36
180	40
total	40

Cumulative frequency curve



	DCM	HBJ	HM	MER	G	K	T	S
<u>Dependent events:</u> <u>conditional probability</u> p(A and B) = p(A) x p(B/A) marble drawing w/out replacement	8.5	8.5	8.5	-	-	12.0	12.0	12.0
<u>Independent vs. dependent</u> <u>events</u> independent- tossing a large and small disc simultaneously dependent- draw one card from a deck = T_1 draw another card from small deck = T_2. T_1 and T_2 are not independent	-	12.5	-	-	-	12.0	12.0	12.0
<u>Sample space</u> finite vs. infinite	-	-	-	-	-	-	-	12.0
<u>Non-mutually exclusive</u> <u>events</u> p(A or B) = p(A) + p(B) -p(A and B)	-	-	8.5	-	-	12.0	12.0	12.0

	DCM	HBJ	HM	MER	G	K	T	S
<u>Probability: use of AND and OR</u>	-	7.5	-	-	-	-	-	-
<u>Odds</u> odds in favor p/(1 - p) odds against (1 - p)/p	8.5	8.5	7.5	11.5	-	-	-	-
<u>Independent events</u> p(A and B) = p(A) x p(B) if A, B independent	7.5	7.5	8.5	12.5	-	-	-	-
<u>Pictographs</u> each  stands for 20 people 1st grade  2nd grade  3rd grade 	-	7.5	7.5	-	-	-	-	-
<u>Circle graphs</u> -multiply % by 360° to draw angles on graph	7.5	7.5	7.5	-	-	-	-	-
<u>Permutations</u> -arrangements of a group of	-	8.5	8.5	11.5	-	12.0	12.0	12.0

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	DCM	HBJ	HM	MER	G	K	T	S
things in a particular order 4 things 4 at a time $_4P_4 = 4 \times 3 \times 2 \times 1 = 24$ $_nP_r = n(n - 1)(n = 2)$ $\ldots(n - r + 1)$								
<u>Combinations</u> -a group of objects selected without considering order $_nC_r = \frac{_nP_r}{_nP_r}$	-	8.5	8.5	11.5	-	12.0	12.0	12.0
<u>Predictions</u> -predict the expected outcomes	-	8.5	-	-	9.5	9.5	9.5	-
<u>Conditional Probability</u> P(B/A) = P(A intersect B)/P(A)	-	12.5	12.5	12.5	-	-	-	-
<u>Estimating Probabilities</u> -using past probability or "history"	-	-	8.5	-	9.5	-	-	-
<u>EXPONENTS, POLYNOMIALS</u>								
<u>Polynomial</u> -expression that can be written	-	9.5	9.0	9.0	8.0	8.0	8.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
as a sum of monomials								
<u>Multiplying polynomials</u>	-	9.5	9.0	9.0	9.0	9.0	9.0	-
-using the distributive property								
$(2x + 3)(5x + 8) = 1$								
$2x(5x + 8) + 3(5x + 8) = 1$								
<u>Finding the sums of polynomials</u>	-	9.5	9.0	9.0	8.0	8.0	8.0	10.0
-using properties of addition and multiplication								
add $(3x + 2y) + (8x + 3y)$								
$=(3x + 8x) + (2y + 3y)$								
-associative, commutative								
$=x(3 + 8) + y(2 + 3)$								
-distributive								
$=11x + 5y$								
<u>Monomials</u>	-	9.5	9.0	9.0	8.0	8.0	8.0	10.0
-a number, variable, or product of #'s and variables.								
$-9, y, 7a, 3y^3, \ldots$								
-constant monomial								
-no variables								
$5/3, 7, \ldots$								

	DCM	HBJ	HM	MER	G	K	T	S
<u>Squares of a sum or differences</u>	-	9.5	9.0	9.0	9.0	9.0	9.0	-
$(a + b)^2 =$ $a^2 + 2ab + b$								
<u>Multiplying a polynomial by a monomial</u>	-	9.5	9.0	9.0	8.0	8.0	8.0	-
-using distributive property $2m^2(5m^2 - 7m + 8) =$ $2m^2(5m^2) - 2m^2(7m)$ $+ 2m^2(8) =$								
<u>Division of polynomials by synthetic substitution</u>	-	-	-	-	-	10.0	10.0	10.0
$(a_0x^3 + a_1x^2 + a_2x + a_3) \div (x - \alpha)$ $\begin{array}{c\|cccc} \alpha & a_0 & a_1 & a_2 & a_3 \\ & & \alpha b_0 & \alpha b_1 & \alpha b_2 \\ \hline & b_0 & b_1 & b_2 & \boxed{R} \end{array}$								
<u>Multiplying and dividing by powers of 10</u>	8.0	8.5	-	-	.	-	-	-
$10^{7} \times 10^2 = ?$								
<u>Fractional exponents</u>	-	11.5	11.5	11.5	-	10.0	10.0	10.0
$a^r = a^{\frac{m}{n}} = \sqrt[m]{a^n}$								
<u>Power of an exponent</u>	-	9.5	9.0	9.0	-	10.0	10.0	10.0
$(a^m)^n = a^{mn}$								

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	DCM	HBJ	HM	MER	G	K	T	S
<u>Negative integers as exponents</u> $a^{-m} = 1/a^{m}$ $(a \neq 0)$	-	9.5	8.0	9.0	-	10.0	10.0	10.0
<u>Exponents</u> definition: $a(a = 0)$ $a^{0} = 1$	7.0	7.0	7.0	9.0	8.0	7.0	8.0	10.0
<u>Power of a product</u> $(ab)^{m} = a^{m} b^{m}$	-	9.5	9.0	9.0	8.0	8.0	8.0	10.0
<u>Dividing polynomials: long division</u>	-	9.5	9.0	9.0	-	10.0	10.0	10.0
<u>Dividing exponents</u> $a^{m} / a^{n} = a^{m-n}$ $(a \neq 0)$	-	9.5	9.0	9.0	-	10.0	10.0	10.0
<u>Subtracting polynomials using additive inverses</u> $(5x^{2} - 2x + 7)$ $- (2x^{2} + 4x - 3)$ $=(5x^{2} - 2x + 7)$ $+ -1(2x^{2} + 4x - 3)$ $=5x^{2} - 2x + 7$ $- 2x^{2} - 4x + 3$	-	9.5	9.0	9.0	8.0	8.0	8.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
$=3x^2 - 6x + 10$								
<u>Adding polynomials using column form</u> $3y^2 + 5y - 6$ $+7y^2 \quad - 9$ $10y^2 + 5y - 15$	-	9.5	9.0	9.0	8.0	8.0	8.0	10.0
<u>Multiplying exponents</u> $(a^m)(a^n) = (a)^{m+n} / (a \neq 0)$	7.0	7.0	7.0	9.0	-	7.0	10.0	10.0
<u>FOIL rule</u> -multiplying polynomials (binomials) First, Outside, Inside, Last $(3x - 5)(5x + 2) =$ $3x(5x) + 3x(2) - 5(5x) - 5(2)$	-	9.5	9.0	9.0	9.0	9.0	9.0	10.0
<u>Product of a sum and difference (polynomials)</u> $(a + b)(a - b) =$ $a^2 - b^2$	-	9.5	9.0	9.0	9.0	9.0	9.0	10.0
<u>Column form of multiplication for polynomials</u>	-	9.5	9.0	9.0	9.0	9.0	9.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S

$$
\begin{array}{r}
5x^2 - 3x + 7 \\
\times \quad 4x + 7 \\
\hline
20x^3 - 12x^2 + 28x \\
+ 35x^2 - 21x + 49
\end{array}
$$

	DCM	HBJ	HM	MER	G	K	T	S
<u>Dividing polynomials - using long division</u>	-	9.5	9.0	9.5	-	10.0	10.0	10.0

$$
\begin{array}{r|l}
 & x - 12 \\
\cline{2-2}
x+2 & x^2 - 10x - 23 \\
 & \underline{x^2 + 2x} \\
 & -12x - 23 \\
 & \underline{-12x - 24} \\
 & -1
\end{array}
$$

	DCM	HBJ	HM	MER	G	K	T	S
<u>Degree of a monomial</u>	-	9.5	9.0	9.0	-	10.0	10.0	10.0

-degree is the sum of the exponents on the variables

<u>FACTORING</u>

	DCM	HBJ	HM	MER	G	K	T	S
<u>The quadratic formula</u>	-	9.5	9.5	9.5	9.0	9.0	9.0	10.0
<u>Formulas for factoring polynomials</u>	-	-	-	-	9.0	9.0	9.0	10.0

1) $x^2 + (a + b)x + ab$
$=(x + a)(x + b)$

2) $x^2 + 2ax + a^2 = (x + a)^2$

	DCM	HBJ	HM	MER	G	K	T	S
Multiplication shortcut -treating monimials as polynomials 99×99 $=(99)^2$ $=(100-1)^2$ $=10{,}000 - 200 + 1$ $=9801$	-	-	-	-	9.0	9.0	9.0	-
Solving quadratic equations by factoring -factor, then solve $a^2 + 16a + 64 = 0$ $(a + 8)(a + 8) = 0$ $a = -8$ -8	-	9.5	9.0	9.0	9.0	9.0	9.0	-
Factoring third degree polynomials $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$ $x^3 + a^3 = (x + a)(x^2 - ax - a^2)$ $(x + a)^3 =$ $x^3 + 3ax^2 + 3a^2 + a^3$	-	-	12.0	-	-	10.0	10.0	10.0
Factoring differences of squares $a^2 - b^2 = (a - b)(a - b)$	-	9.5	9.0	9.0	-	10.0	10.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
$(3x)^2 - (10y)^2$ $= (3x - 10y)(3x + 10y)$								
<u>Inverted short division</u> -to pick out prime factors 2 | 42 3 | 21 7 2, 3, 7 prime factors of 42	-	-	7.0	-	-	7.0	-	-
<u>Pascal's triangle</u> 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1	-	-	12.0	-	-	12.0	11.0	12.0
<u>Factorials</u> $4! = 4 \times 3 \times 2 \times 1 = 24$ $= n \times (n - 1) \times \ldots \times (1)$	-	-	7.0	11.5	-	12.0	11.0	12.0
<u>Factoring using the distributive property</u> -write polynomials in factored	-	9.5	9.0	9.0	9.0	9.0	9.0	10.0

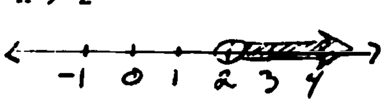
	DCM	HBJ	HM	MER	G	K	T	S
form $12a^5 b + 8a^3 - 24a^3 c$ (use GCF:) $4a^3(3a^2 b) + 4a^2(2) -$ $=4a^3(3a^2 b + 2 - 6c)$								
<u>Factoring trinomials into binomial factors</u> $x^2 + bx + c$	-	9.5	9.0	9.0	9.0	9.0	9.0	10.0
<u>Binomial Theorem</u>	-	11.5	11.5	11.5	-	12.0	11.0	12.0
<u>Squaring Trinomials: treating trinomials as binomials</u>	-	-	-	-	9.0	9.0	9.0	-
<u>4th Power Polynomials</u> $(a + b)^4 =$ $a^4 + 4a^3 b + 6a^2 b +$ $4ab^3 + 6^4$	-	-	12.0	-	-	12.0	12.0	12.0
<u>Perfect cube polynomials</u> $(a + b)^3 =$ $a^3 + 3a^2 b + 3ab^2 + b^3$	-	-	12.0	-	-	10.0	10.0	10.0
<u>Perfect square trinomial</u> $(a + b)^2 = a^2 + 2ab + b^2$	-	9.5	9.0	9.0	-	10.0	10.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
factor: $9x^2 - 12xy + 4y^2$								
Multiples -a multiple of a whole number is the product of that number and any whole number	7.0	7.0	7.0	-	7.0	-	-	-
Even and Odd numbers -divisible by 2?	-	7.0	7.0	-	-	-	-	-
Binary Number System 11101 $=1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times$ $2^1 + 1 = 29$	-	9.5	9.0	9.0	-	-	-	-
Prime and composite numbers -prime = whole # with only 2 factors -itself and 1 -composite = not prime	7.0	7.0	7.0	9.0	-	-	-	-
Prime factorization method	-	8.0	8.0	9.0	8.0	8.0	8.0	-
Greatest common factors GCF of 24 and 18 is 6	7.0	7.0	7.0	9.0	7.0	7.0	7.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
<u>Factoring by grouping</u> -group terms in pairs, factor a monomial from each group	-	9.5	9.0	9.0	9.0	9.0	9.0	-
<u>Zero product property: solving equations</u> If ab = 0, then a = 0 or b = 0	-	9.5	9.0	9.0	9.0	9.0	9.0	-
<u>Least common multiple</u> -product of GCF and prime factors not common to both numbers	7.0	7.0	7.0	9.0	7.0	7.0	7.0	10.0
<u>Factors</u> -a number is divisible by its factors 16 = 1 x 16 2 x 8 4 x 4 1, 2, 4, 8, 16 are factors of 16	7.0	7.0	7.0	9.0	7.0	7.0	7.0	10.0
<u>Factoring ax + bx + c</u> -multiply outer terms -find factors whose sum = middle term	-	9.5	9.0	9.0	9.0	9.0	9.0	-

	DCM	HBJ	HM	MER	G	K	T	S
<u>Roots of quadratic equations</u> -solutions are called "roots" which are 2 x-intercepts of the parabola	-	9.5	9.5	9.5	9.0	9.0	9.0	10.0
<u>The discriminant</u> b - 4ac is the discriminant -tells how many real roots for the equation -if: b -4ac > 0, 2 roots = 0, 1 root < 0, 0 roots	-	9.5	9.5	9.5	9.0	9.0	9.0	10.0
<u>Completing the square:</u> <u>quadratics</u> $x^2 - 36 = 0$ $x^2 = 36$ $(x) = 6$ $x = \pm\ 6$ -works if expression is a perfect square	-	9.5	9.5	9.5	9.0	9.0	9.0	-
<u>Fundamental theorem of algebra</u>	-	-	12.0	12.0	-	-	-	-
<u>INEQUALITIES</u>								
<u>Properties of inequality</u>	-	-	12.0	-	8.0	8.0	8.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
1) if $a < b$, then $a + c < b + c$ $a - c < b - c$								
<u>Inequalities</u> greater than $>$ less than $<$ $38 < 45$ $(-15) > (-17)$	7.0	7.0	8.0	9.0	7.0	8.0	8.0	-
<u>Compound Sentences</u> combinations of two inequalities $5 < 6$ $6 < 8$ thus $5 < 6 < 8$	8.0	9.0	8.0	9.0	8.0	8.0	8.0	-
<u>Converting word problems into compound inequalities</u> $x + 2 < 7$ $3x > -6$ $(-2 < x < 5)$	-	-	-	-	8.0	8.0	8.0	-
<u>Graphing inequalities on number line</u> $x > 2$	8.5	7.0	9.5	9.0	8.0	8.0	8.0	-



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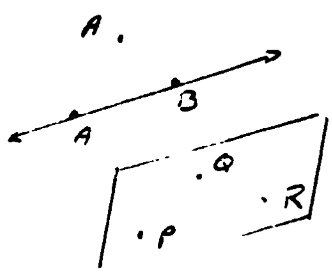
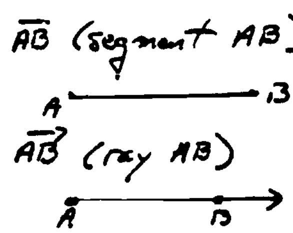
	DCM	HBJ	HM	MER	G	K	T	S
<u>Inequalities: solving by several transformation</u> $4r + 6 + 2r - 18 < -8$ $r < -2$ (division and addition)	-	8.0	8.0	-	8.0	8.0	8.0	-
<u>Absolute value as a compound sentence</u> solve $\|2x + 1\| \leq 7$: $2x + 1 > -7$ and $2x + 1 < 7$ $x \geq -4$ and $X \leq 3$	-	9.0	9.5	9.0	-	-	-	-
<u>Transitive property of order</u> if $a < b$ and $b < c$, then $a < c$ if $a > b$ and $b > c$, then $a > c$	-	-	12.0	9.0	-	10.0	10.0	10.0
<u>Equivalent inequalities</u> -solving inequalities for variables by making two sides equivalent	-	7.0	7.0	9.0	8.0	8.0	8.0	-
<u>GENERAL GEOMETRY</u>								
<u>Trapezoid</u> -a quadrilateral with only	7.5	7.5	7.0	10.0	8.5	8.5	8.5	-

	DCM	HBJ	HM	MER	G	K	T	S
one pair of parallel sides								
<u>Types of Parallelograms</u>	7.5	7.5	7.0	10.0	8.5	8.5	8.5	-
-rectangle								
-square								
-rhombus								
<u>Parallel planes</u>	-	-	7.0	-	-	7.5	-	-
-no intersection								
<u>Points, lines, planes in space</u>	7.5	7.5	7.0	10.0	7.5	7.5	7.5	-
0, 1, and 2 dimensions respectively								



	DCM	HBJ	HM	MER	G	K	T	S
<u>Line Segments and Rays</u>	7.5	7.5	7.0	10.0	7.5	7.5	7.5	-

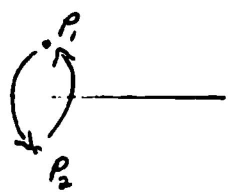


	DCM	HBJ	HM	MER	G	K	T	S
<u>Quadrilaterals:</u>	-	8.0	-	8.5	8.5	8.5	-	-
-sum of measures of interior angles is 360°								

	DCM	HRJ	HM	MER	G	K	T	S
Polygons finding perimeters	7.5	7.5	7.0	9.0	7.5	7.5	7.5	-
Reduced drawing	-	-	-	-	9.5	9.5	9.5	-

	DCM	HRJ	HM	MER	G	K	T	S
Inverse transformations	-	-	10.5	-	-	11.0	11.0	11.0

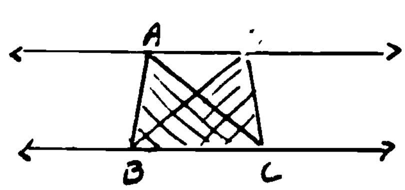
-opposite direction of mapping



P1 → P2 . . . inverse is

P2 → P1

	DCM	HRJ	HM	MER	G	K	T	S
Parallel Lines and Area	-	-	-	-	8.5	8.5	8.5	-



areas of _BAC and _BDC are equal

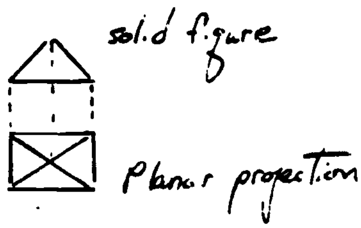
	DCM	HRJ	HM	MER	G	K	T	S
Polygons: sum of angles	-	10.0	10.0	10.0	8.5	8.5	8.5	-

sum of interior ∠'s is 180°x(n-2)

(n=#sides)

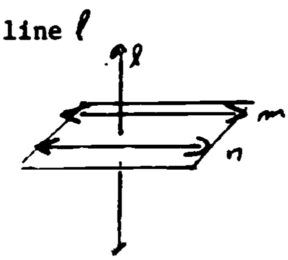
sum exterior ∠'s: 360° (convex)

Topic	DCM	HBJ	HM	MER	G	K	T	S
Projection and an unfolded figure	-	-	-	-	-	7.5	-	-



Topic	DCM	HBJ	HM	MER	G	K	T	S
Orthogonal lines	-	-	-	-	-	11.0	11.0	11.0

lines m and n are $\parallel$, and $\perp$ to the same line ℓ



Topic	DCM	HBJ	HM	MER	G	K	T	S
Geometric construction	7.5	7.5	7.0	10.5	-	7.5	-	-

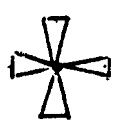
-using compass and straightedge

-draw an obtuse angle and construct a congruent angle

Topic	DCM	HBJ	HM	MER	G	K	T	S
Areas of symmetric figures	-	-	7.5	-	-	-	-	-

(point symmetry)

A = 4 x 1/2 b



Topic	DCM	HBJ	HM	MER	G	K	T	S
Coincident lines and collinear points	-	-	12.0	-	7.5	7.5	7.5	-

-include the same set of points

	DCM	HBJ	HM	MER	G	K	T	S
-same line passes through them								

	DCM	HBJ	HM	MER	G	K	T	S
<u>Geometric mean</u>	-	-	10.0	10.0	10.5	-	-	-

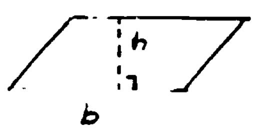
if a/x = x/b and x is positive

then x is the geometric mean

between a and b

	DCM	HBJ	HM	MER	G	K	T	S
<u>Area of parallelogram</u>	7.5	7.5	7.5	9.0	-	-	-	-

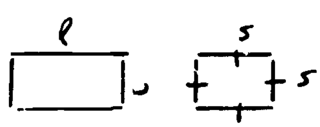
Area = base x height



	DCM	HBJ	HM	MER	G	K	T	S
<u>Area of rectangles and squares</u>	7.5	7.5	7.5	9.0	-	-	-	-

rectangle area = length x width

square area = side x side



	DCM	HBJ	HM	MER	G	K	T	S
<u>Area of trapezoids</u>	7.5	7.5	7.5	9.0	-	-	-	-

Area = 1/2 h(a + b)

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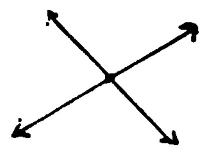
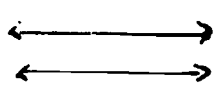
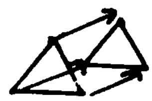
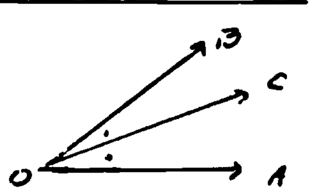
	DCM	HBJ	HM	MER	G	K	T	S
Skew lines -two non-parallel lines that do not intersect	-	-	7.0	-	-	-	-	-
Symbol "is congruent to" " ≅ "	-	7.5	7.0	10.0	-	-	-	-
Postulates in geometry -rules that tell how different sets of points are related	-	10.0	10.0	10.0	-	-	-	-
Planes: intersection planes intersect at a line	-	10.0	10.0	10.0	7.5	7.5	7.5	-
Congruent polygons -if vertices of one match vertices of the other so that corresponding sides and angles are congruent	7.0	7.0	7.0	-	8.5	8.5	8.5	-
Regular polygons -all sides and angles are congruent	7.5	7.5	7.0	10.0	7.5	7.5	7.5	-
Polygons: names and terms	7.5	7.5	7.0	10.0	7.5	7.5	7.5	-

	DCM	HBJ	HM	MER	G	K	T	S
-pentagon -hexagon -decagon -octogon -vertices and sides								
<u>Lines: intersection</u> -lines intersect at a point 	-	10.0	-	-	8.0	8.0	8.0	-
<u>Parallel lines: definition</u> -coplanar lines that do not intersect 	8.5	7.5	8.0	10.0	7.5	7.5	7.5	-
<u>Apothems and area of a regular hexagon</u> -apothem is a segment equal to the radius of an inscribed circle Area = 1/2 (apothem)(perimeter)	-	10.5	10.5	10.5	7.5	7.5	7.5	-
<u>Translation or "slide"</u> Sliding planar figure to new position	-	7.5	7.0	10.5	7.5	7.5	7.5	-



	DCM	HBJ	HM	MER	G	K	T	S
<u>Parallelogram</u> -a quadrilateral with both pairs of opposite sides parallel	-	10.0	10.0	10.0	8.5	8.5	8.5	-
<u>Bisector of a segment</u> -a segment, line, or plane intersecting segment at mid-point	7.5	7.5	7.0	10.0	7.5	7.5	7.5	-
<u>Rotation</u> -geometric figure is "turned" to a new position	-	7.5	7.0	10.5	8.5	8.5	8.5	-
<u>Dilation</u> -enlarging or reducing a figure without changing its shape -contraction -expansion	-	10.5	10.5	10.5	8.5	-	8.5	-
<u>Symmetric figures</u> -line of symmetry	7.5	7.5	8.5	10.5	8.5	8.5	8.5	-

	DCM	HBJ	HM	MER	G	K	T	S
-point of symmetry								
Similarity	7.5	7.5	10.0	10.5	8.5	8.5	8.5	-
-figures with the same shape								
-corresponding angles of polygons are congruent								
-ratios of lengths of sides equal								
Perpendicular lines	7.5	7.5	7.0	10.0	7.5	7.5	-	-
-intersect to form 4 right angles								
Theoreum	-	10.0	10.0	10.0	8.5	8.5	8.5	-
-statements that must be proven before accepted								
GEOMETRY-ANGLES, TRIANGLES								
Bisector of an angle	7.5	7.5	7.0	10.0	7.5	7.5	-	-



a ray that divides an angle into 2 equal angles

Triangles: sum of angles

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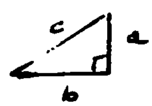
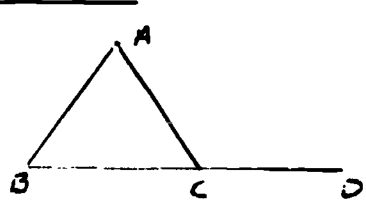
	DCM	HBJ	HM	MER	G	K	T	S
<u>is 180°</u> -what are the measures of isoceles right triangle? $x + x + 90 = 180$ $2x = 90$ $x = 45$	-	9.5	9.5	9.5	8.5	8.5	8.5	-
<u>Pythagorean theoreum</u> -hypotenuse, legs $c^2 = a^2 + b^2$ 	8.5	8.5	8.5	9.5	9.5	9.5	9.5	-
<u>Circumcenter of a triangle</u> -perpendicular bisectors of the sides of a triangle intersect at a point equidistant from the verticies -point is the circumcenter	-	10.5	10.5	-	-	10.0	10.0	10.0
<u>Exterior and Interior Angels of triangles</u>  angle A, angle B are interior angles of $\triangle$ ABC angle ACD is exterior	-	-	10.0	10.0	8.5	8.5	8.5	-

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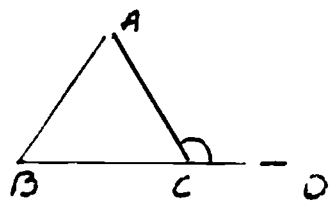
	DCM	HBJ	HM	MER	G	K	T	S
<u>Geometric proofs</u>	-	10.0	10.0	10.0	8.5	8.5	8.5	-

Why is this true?

angle A + angle B = angle ACD

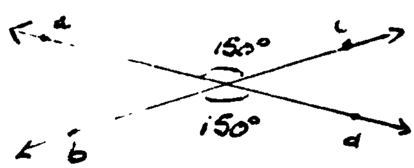


	DCM	HBJ	HM	MER	G	K	T	S
<u>Acute and Obtuse Angles</u>	-	-	-	-	8.5	8.5	8.5	-

acute: $0° < x < 90°$

obtuse: $90° < x < 180°$

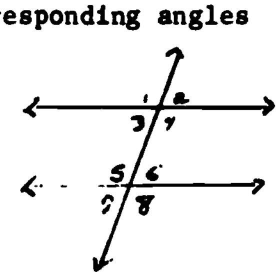
	DCM	HBJ	HM	MER	G	K	T	S
<u>Vertical Angles: are congruent</u>	-	-	-	-	8.5	8.5	8.5	-



	DCM	HBJ	HM	MER	G	K	T	S
<u>Parallel Lines: transversals</u>	8.5	7.5	8.0	10.0	8.5	7.5	8.5	-

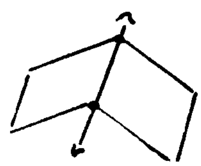
-interior/exterior angle's

-corresponding angles

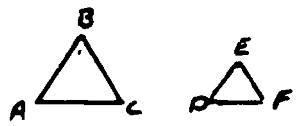


	DCM	HBJ	HM	MER	G	K	T	S
<u>Congruent triangles</u>	8.5	8.0	8.0	10.0	8.5	7.5	8.5	-

Tests for congruence in tri-

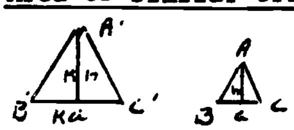
	DCM	HBJ	HM	MER	G	K	T	S
angles SAS side . angle . side ASA angle . side . angle SSS side . side . side								
<u>Dihedral Angle</u>  -consists of two half-planes with a common edge -each half-plane is a face	-	10.0	10.0	10.0	-	11.0	11.0	11.0
<u>Triangles: Acute and Obtuse</u> -acute angle triangle -obtuse angle triangle	-	10.5	10.5	10.5	8.5	8.5	8.5	-
<u>Triangles: definition and sides</u> -vertices, sides -scalene, isosceles, equilateral	7.5	7.5	7.0	10.0	8.5	8.5	8.5	-
<u>Similar triangles</u> -proportional sides, equal angles	8.5	8.5	8.5	9.5	8.5	8.5	8.5	-

	DCM	HBJ	HM	MER	G	K	T	S



$\Delta\ ABC \sim \Delta\ DEF$

	DCM	HBJ	HM	MER	G	K	T	S
<u>Area of similar triangles</u>	-	-	-	-	9.5	9.5	9.5	-



$$S = \text{Area} = \tfrac{1}{2}ah$$

$$\text{Area}' = \tfrac{1}{2}Ka\ Kh = \tfrac{1}{2}ah \cdot K^2$$

$$= K^2 S$$

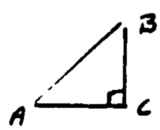
$$\therefore\ S'/S = K^2$$

	DCM	HBJ	HM	MER	G	K	T	S
<u>Orthocenter - Centroid</u> -altitudes of triangle inter- sect at the ORTHOCENTER -medians of the triangle intersect at the CENTROID	-	10.5	-	-	-	-	-	-
<u>Triangle inequality theorem</u> -sum of any 2 sides is greater than remaining side	-	10.0	7.0	10.0	-	-	-	-
<u>Adjacent angles</u> -common vertex, non-overlapping	8.5	7.5	8.0	10.0	-	-	-	-
<u>Solving right triangles</u> -using table of trigenometric ratios	-	8.5	8.5	9.5	-	-	-	-

	DCM	HBJ	HM	MER	G	K	T	S
Isosceles right triangle -hypotenuse = (leg) 2	-	10.5	8.5	-	-	-	-	-
Special Angles -complementary angles, sum is 90° -supplementary angles, sum is 180°	8.5	7.5	7.0	9 5	-	-	-	-
Angles: types -actute, obtuse, and right angles	7.5	7.5	7.0	10.0	8.5	8.5	8.5	-
Area of triangles 1/2 base x height	7.5	7.5	7.5	9.0	8.5	8.5	9.5	-
Area of triangles: using trigonometric functions Area = 1/2 (side 1) (side 2) sin (included angle)	-	12.0	11.5	12.0	-	10.5	10.5	10.5
Equilateral triangles: are equiangular	-	10.0	10.0	10.0	8.5	8.5	8.5	-
Triangle rules	-	10.0	7.0	10.0	8.5	8.5	8.5	-

	DCM	HBJ	HM	MER	G	K	T	S
-largest side is opposite largest angle -shortest side is opposite smallest angle -two angles congruent only if sides opposite them are congruent								
<u>Right triangles: 30° and 60°</u> -in any 30° - 60° right triangle, the measures of the side opposite the 30° is 1/2 the measure of the hypotenuse	-	10.5	10.0	9.5	9.5	9.5	9.5	-
<u>Trigonometric ratios</u> sine, cosine, tangent	8.5	8.5	8.5	9.5	-	10.5	10.5	10.5
<u>Trigonometric ratios: right triangles</u>	-	10.5	10.0	10.5	-	10.5	10.5	10.5

-sine (A) = $\frac{BC}{AB}$ —> diagram



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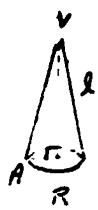
CIRCLES AND 3-D

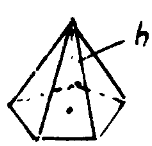
	DCM	HBJ	HM	MER	G	K	T	S
Surface area of right circular cones	-	10.5	10.5	10.5	9.5	9.5	9.5	-

Area = (pi) $r\ell$ + (pi)r^2



	DCM	HBJ	HM	MER	G	K	T	S
Volume of a right pyramid	8.0	8.5	8.5	10.5	7.5	7.5	9.5	-

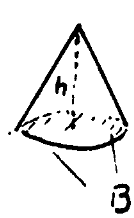
Volume = 1/3 $\begin{Bmatrix}\text{area}\\\text{base}\end{Bmatrix}$ h --> diagram



	DCM	HBJ	HM	MER	G	K	T	S
Volume of right circular cones	8.0	8.5	8.5	10.5	7.5	7.5	9.5	-

Volume = 1/3 (area) h --> diagram

base

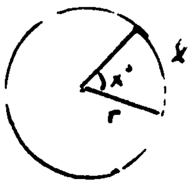


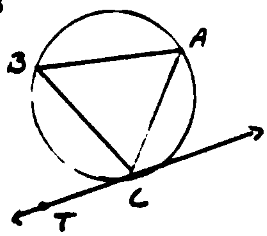
	DCM	HBJ	HM	MER	G	K	T	S
Circles: terms	8.0	7.5	7.0	10.5	9.5	9.5	7.5	10.0

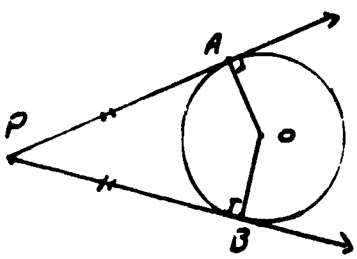
-radius

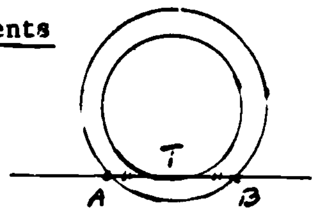
-chord

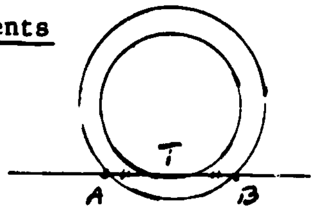
	DCM	HBJ	HM	MER	G	K	T	S
-diameter -arcs -A set of points that are the same distance from 0 is a circle								
<u>Area of "fan-shape" (Japan) or sector (U.S.)</u> = 2(pi)r x X/360 Area = (pi)r xX/360	-	10.5	10.5	10.5	-	7.5	-	-
<u>"Circles" Inscribed polygons and angles</u> ∠ TCB = ∠ CAB	-	-	-	-	9.5	9.5	9.5	-
<u>Circles: length of a tangent</u> PB = PA	-	10.5	10.5	10.5	9.5	9.5	9.5	-







	DCM	HBJ	HM	MER	G	K	T	S
Circles: -concentric circles and tangents AT = BT 	-	-	-	-	9.5	9.5	9.5	-
Circles: -tangents line with one point of intersection -segment to center of circle is perpendicular to tangent	-	10.5	10.5	10.5	9.5	9.5	9.5	10.0
Arcs and central angle x = central angle, AB = arc	-	10.5	10.5	10.5	9.5	7.5	9.5	-
Secant of a circle -a line intersecting circle at two points	-	10.5	10.5	10.5	-	10.0	10.0	10.0
Area of circle Area = (pi)r^2	7.5	7.5	7.5	10.5	9.0	7.5	9.0	-
Circles: Inscribed and central angles A = inscribed X = central	-	10.5	10.5	10.5	9.5	9.5	9.5	-

	DCM	HBJ	HM	MER	G	K	T	S
<u>Circles: -concentric circles and tangents</u> AT = BT 	-	-	-	-	9.5	9.5	9.5	-
<u>Circles:</u> -tangents line with one point of intersection -segment to center of circle is perpendicular to tangent	-	10.5	10.5	10.5	9.5	9.5	9.5	10.0
<u>Arcs and central angle</u> x = central angle, s = arc	-	10.5	10.5	10.5	9.5	7.5	9.5	-
<u>Secant of a circle</u> -a line intersecting circle at two points	-	10.5	10.5	10.5	-	10.0	10.0	10.0
<u>Area of circle</u> Area = (pi)r^2	7.5	7.5	7.5	10.5	9.0	7.5	9.0	-
<u>Circles: Inscribed and central angles</u> A = inscribed X = central	-	10.5	10.5	10.5	9.5	9.5	9.5	-

	DCM	HBJ	HM	MER	G	K	T	S

$A = 1/2\ (\angle X)$

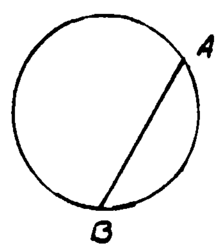
$C = 1/2\ (\angle Y)$

$A = 1/2\ (\quad \widehat{BD})$

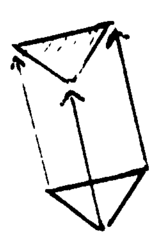
$X = \widehat{BD}$

	DCM	HBJ	HM	MER	G	K	T	S
Chords of a circle	-	-	-	-	9.5	7.5	9.5	-

$\overline{AB}$ is a chord (defines $\widehat{AB}$ arc)



	DCM	HBJ	HM	MER	G	K	T	S
Moving planar figures in 3-D space	-	-	-	-	-	7.5	-	-



	DCM	HBJ	HM	MER	G	K	T	S
Polyhedron	-	7.5	7.5	-	-	7.5	-	-

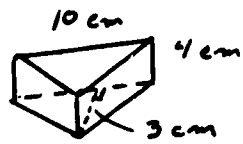
-a solid figure with flat faces

-prisms

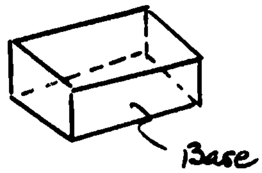
-pyramids

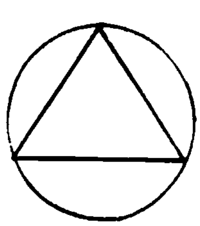
	DCM	HBJ	HM	MER	G	K	T	S
Volume of Polyhedrons	7.5	7.5	7.5	-	7.5	7.5	9.5	-

Find the volume --> diagram



	DCM	HBJ	HM	MER	G	K	T	S
<u>Volume and surface areas of spheres</u> Area = 4 (pi) r^2 Volume = 4/3 (pi) r^3	8.0	10.5	8.5	10.5	7.5	7.5	7.5	-
<u>Pyramid</u> -all faces except the base meet at the vertex and are triangles	-	10.5	10.5	10.5	7.5	7.5	7.5	-
<u>Finding circumference of circles</u> C = 2(pi)r -find circumference of circle with radius of 4 cm	7.5	7.5	7.0	10.5	7.5	7.5	7.5	-
<u>Prisms: definition</u> -bases - 2 faces formed by congruent polygons in parallel planes lateral faces - formed by parallelograms —> diagram	-	10.5	10.5	10.5	7.5	7.5	7.5	-



	DCM	HBJ	HM	MER	G	K	T	S
Sphere: definition -set of all points a given distance from a given point in a three-space	8.0	10.5	8.5	10.5	7.5	7.5	7.5	-
Semi-circle -created by secant line through the center $\overset{\frown}{XSY}$ and $\overset{\frown}{XRY}$ = semicircles	-	10.5	10.5	10.5	7.5	7.5	7.5	-
Limit: defined in geometric terms for circumference and area of circles, pi is used in approximating the limit	-	10.5	10.5	10.5	-	-	-	-
Inscribed polygon -all vertices on the circle 	-	10.5	7.0	10.5	7.0	7.0	7.0	-
Surface areas of prisms and cylinders	7.5	7.5	7.5	10.5	7.5	7.5	7.5	-

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	DCM	HBJ	HM	MER	G	K	T	S
right cylinders -Area = 2(pi) rh + 2(pi) r^2								
<u>Volume of cylinders</u> V= (pi) r^2 h	8.0	7.5	7.5	10.5	7.5	7.5	7.5	-
<u>Cylinder</u> -bases = 2 congruent circles in parallel planes -axis -right cylinder = axis perpendicular to bases	-	10.5	10.5	10.5	7.5	7.5	7.5	-
<u>EQUATIONS</u>								
<u>Equations in two variables</u> -choose a value for x then solve for y x + y = 5	8.5	8.5	8.5	-	7.0	7.0	7.0	-
<u>Sum and Product of roots</u> <u>(quadratic)</u> $ax^2 + bx + c = 0$	-	9.5	9.5	9.5	-	10.0	10.0	10.0

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	DCM	HBJ	HM	MER	G	K	T	S
-coefficient of x is negative sum of roots								
-constant term is product of roots								
Solve for variable using inverse	-	8.0	8.0	9.0	7.0	7.0	7.0	-
$y - 12 = 18$								
$y - 12 + 12 = 18 + 12$								
$y = 30$								
IRRATIONAL NUMBERS								
Simplify radicals	-	9.5	9.5	9.5	-	10.0	10.0	10.0
-simplest term: radicand has no perfect square factors other than 1								
-no radical is a fraction								
-no fraction has a radical in denominator								
Radicals: terms	-	9.5	9.5	9.5	9.0	9.0	9.0	-
$\sqrt{\ }$ is a radical sign								
$\sqrt{X}$, X is a radicand								
if $x^2 = y$, then x is a square root of y								

	DCM	HBJ	HM	MER	G	K	T	S
<u>Product property of square roots</u> $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$	-	9.5	9.5	9.5	9.0	9.0	9.0	10.0
<u>Real numbers</u> -are rational and irrational numbers together, the entire number line	-	9.0	9.0	9.5	9.0	10.0	9.0	10.0
<u>Approximating square roots</u> -divide-and-average method	8.5	8.5	8.5	9.5	-	9.0	9.0	-
<u>Using square root table: interpolation</u>	-	8.5	8.5	-	9.0	9.0	9.0	-
<u>Conjugates</u> binomials in the form: $x - \sqrt{y}$ and $x + \sqrt{y}$ product is rational	-	9.5	9.5	9.5	-	-	-	-
<u>Radical Equations</u> -variables in the radicand, square both sides $\sqrt{3y - 5} - 4 = 0$ $(3y - 5)^2 = (4)^2$	-	9.5	9.5	9.5	9.0	9.0	9.0	-

	DCM	HBJ	HM	MER	G	K	T	S
$3y - 5 = 16$								
<u>Finding square roots from perfect squares</u> $\sqrt{40} = ?$, $36 < 40 < 49$, $6 < \quad 40 < 7$	8.5	8.5	8.5	-	9.0	9.0	9.0	-
<u>Evaluating Radical Expressions</u> -use product and quotient properties and use tables	-	9.5	9.5	9.5	9.0	9.0	9.0	-
<u>Distributive Property applied to radicals</u> simplify: $9\sqrt{7} - 4\sqrt{7} =$ $(9 - 4)\sqrt{7} = 5\sqrt{7}$	-	9.5	9.5	9.5	9.0	9.0	9.0	-
<u>Rationalizing the denominator of radical expressions</u> -multiply by one simplify: $\frac{\sqrt{32}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{96}}{3} = \frac{4\sqrt{6}}{3}$	-	9.5	9.5	9.5	-	10.0	10.0	10.0

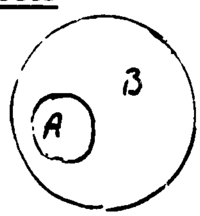
	DCM	HBJ	HM	MER	G	K	T	S
Irrational Numbers -square roots of #'s that are not perfect squares -neither terminating nor repeating decimal -can't be expressed as a/b	-	9.0	9.5	9.5	9.0	9.0	9.0	10.0
Quotient Property of square roots $a/b = \sqrt{a}/\sqrt{b}$	-	9.5	9.5	9.5	-	10.0	10.0	10.0
Property of Order square roots if a, b > o and a < b, then $\sqrt{a} < \sqrt{b}$	-	-	-	-	9.0	9.0	9.0	-
Estimating Square Roots	-	-	-	-	9.0	9.0	9.0	-

EQUATION, WHOLE NUMBERS, SETS

	DCM	HBJ	HM	MER	G	K	T	S
<u>Solving Equations:</u>	-	-	-	-	9.0	9.0	9.0	-
<u>square root of both sides</u> $(x + a)^2 = b$ $(x - 3)^2 = 16$ $\sqrt{(x - 3)^2} = \sqrt{16}$ $x - 3 = \pm 4$ $x - 3 = 4$ or $x - 3 = -4$ $x = 7$ or $x = -1$								
<u>Sets: union and intersection</u> $R \cap S$ = intersection $R \cup S$ = union	-	-	12.0	-	-	10.0	10.0	10.0
<u>Set: definition and elements</u> [3, 6, 9, 12, 15, 18] elements 3, 6, . . .	-	-	12.0	9.0	7.0	7.0	7.0	10.0
<u>Evaluating expressions with variables</u> $a = 5$ evaluate $a \times a$	-	8.0	8.0	9.0	8.0	8.0	8.0	-
<u>Place value: whole numbers</u>	7.0	7.0	7.0	-	-	-	-	-

	DCM	HBJ	HM	MER	G	K	T	S
tens, hundreds, etc.								
<u>Positive and Negative numbers</u> -write following #'s with a(+) or a(-) 1) Number that is 12 smaller than 0 2) Number that is 9 larger than 0	7.5	7.5	7.5	9.0	7.0	7.0	7.0	-
<u>Integers</u> -opposites, number line $-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$	7.5	7.5	7.5	9.0	-	10.0	10.0	10.0
<u>Natural numbers</u> 1, 2, 3, 4, . . .	-	-	12.0	-	7.0	7.0	7.0	-
<u>Equivalent equations</u> -simplifying numerical expressions and variable expressions $y - 8 + 3 = 24 \div 6$ $y = ?$	7.5	7.5	7.5	9.0	7.0	7.0	7.0	-
<u>Number line</u>	8.5	-	7.0	-	-	10.0	10.0	10.0

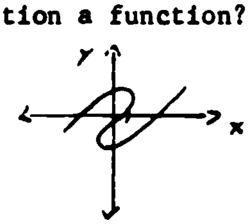
	DCM	HBJ	HM	MER	G	K	T	S
-the greater the number, the further to the right it is								
<u>Open equation</u> -numerical expression with 1 or more variables $x + 6 = 13$ $x + 0.6 = 4800$	7.0	7.0	9.0	9.0	7.0	7.0	7.0	-
<u>Solving equations with addition and subtraction</u> $n - 3 = 24$ $n - 8 + 8 = 24 + 8$ $n = 32$	7.0	7.0	7.0	11.0	7.0	7.0	7.0	-
<u>Solving equations with multiplication and division</u> $13y = 195$ $13y \times 1/13 = 195 \times 1/13$ $y = 195/13 = 15$	7.0	7.0	7.0	-	8.0	8.0	8.0	-
<u>Whole numbers</u> 0, 1, 2, 3, 4, 5, . . .	-	7.0	7.0	-	-	10.0	10.0	10.0
<u>Number line of whole numbers</u>	8.0	-	8.0	9.0	-	10.0	10.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
<u>Subsets</u>  if $x \leqq A$, then $x \in B$	-	-	-	-	-	10.0	10.0	10.0
<u>Sets: Correspondence</u> X=[A, B, C, D, E] Y=[0, 1, 2, 3, 4] -elements in set X correspond to those in set Y	-	-	-	-	9.0	9.0	9.0	-
<u>Steps for solving an equation</u> 1. transform fractions to integers, distribute to eliminate parenthesis 2. transfer terms 3. combine like terms to get "ax = b" 4. solve x by division: x = b/a	-	-	-	-	7.0	7.0	7.0	-
<u>Identity equation</u> -equation that is true for any value of variables	-	-	9.0	9.0	-	10.0	10.0	10.0

	DCM	HBJ	HM	MER	G	K	T	S
$3(x + 1) - 5 = 3x - 2$ $3x = 3x$ identi-y								
Absolute Value -distance from zero (e.g. on a number line)	8.0	7.5	7.5	9.0	7.0	7.0	7.0	10.0
Transference of terms -from one side to the other in an equation $7x = 24 + 4x$ $7x - 4x = 24$	-	-	-	-	7.0	7.0	7.0	-
Simplifying expressions -no like terms remaining $\frac{(y^2-15)3^2}{3^3-(2\times 12)} = ?$			8.0	9.0	8.0	8.0	8.0	
Simple Equation -all variables raised to the 1st power $ax = b$	-	-	-	-	8.0	7.0	8.0	-
Solving equations with								

	DCM	HBJ	HM	MER	G	K	T	S
<u>variables on both sides</u> solve: 3n + 2 = 5n - 7 3n = 5n - 9 -2n = -9 n = 9/2	-	9.0	9.0	9.0	7.0	7.0	7.0	-
<u>Using several transformations</u> -to solve an equation 13 + f - 8f + 25 x 5 5(y - 1) + 8 = -9	8.5	8.0	8.0	9.0	-	7.0	-	-
<u>Periods</u> grouping digits in 3's with commas 1,256,394,000	7.0	7.0	-	-	-	-	-	-
<u>Equations</u> a "number sentence" with an = sign 10 x 10 x 10 = 1,000	-	7.0	7.0	9.0	-	7.0	-	-

	DCM	HBJ	HM	MER	G	K	T	S
Replacement set -intended values, a solution makes an open sentence true x + 6 = 13 replacement set for x is (5, 6, 7) solution is 7	-	-	7.0	9.0	-	-	-	-
Set symbols $\in$ element of $\emptyset$ null set	-	11.0	11.0	9.0	-	10.0	10.0	10.0
Venu diagrams of sets	-	8.5	12.0	-	8.5	8.0	8.5	10.0
GRAPHING FUNCTIONS								
Functions -a relation in which each element of the domain is paired with exactly one element of the range -vertical line test Is this relation a function? (no)	-	9.0	9.5	9.5	7.5	7.5	7.5	10.5



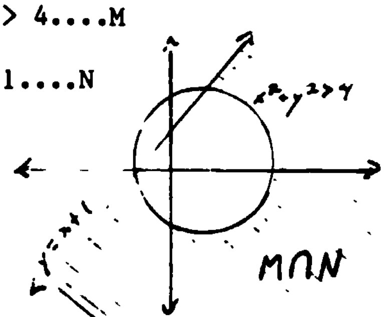
	DCM	HBJ	HM	MER	G	K	T	S
Midpoint of a line segment (coordinate plane)	-	10.5	10.5	9.5	-	10.0	10.0	10.0

x_1, x_2 midpoint is $(x_1 + x_2)/2 = P$

Domain of Linear Inequalities (coordinate plane)	-	-	-	11.0	-	10.0	10.0	10.0

$x^2 + y^2 > 4$....M

$y < x + 1$....N



Domain of a function	-	-	-	-	-	10.5	10.5	10.5

$[(x, y): y = f(x), x \in \text{domain}]$

Function $y = /x/$

if $x \geq 0$ ... $y = x$

if $x < 0$ ... $y = -x$

Half Plane and Boundary line: inequalities	-	9.0	9.5	9.5	-	10.0	10.0	10.0

	DCM	HBJ	HM	MER	G	Y	T	S
<u>Writing equations of lines</u> -using all info about a line to write the equation	-	11.0	9.5	9.5	-	11.0	11.0	11.0
<u>Distance in three space</u> $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ X $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (Z_2 - Z_1)^2}$	-	-	-	10.5	-	11.0	11.0	11.0
<u>Distance formula: (coordinate plane)</u> two points (x_1, y_1) & (x_2, y_2) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	-	9.5	9.5	9.5	-	10.0	10.0	10.0
<u>Inverse variation (vs. direct variation)</u> $y = a/x$ (indirect) $y = ax$ (direct)	-	11.5	11.5	11.5	9.0	9.0	9.0	-
<u>Linear Transformations</u> -mapping specified by equations of the form: $x_1 = a_1x + a_2y$ $y_1 = b_1x + b_2y$	-	12.5	12.5	-	-	11.5	11.5	11.5
<u>Imaginary Numbers</u>	-	11.5	11.0	11.0	-	10.0	10.0	10.0

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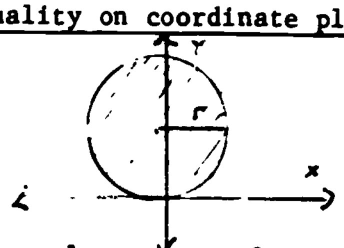
$i \times i = -1$

i.e. $(i)^2 = -1$

and $ai = ai$

	DCM	HBJ	HM	MER	G	K	T	S
Area inside of circle: inequality on coordinate plane	-	-	-	-	-	10.0	10.0	10.0



$(x - a)^2 + (y - b)^2 < r^2$

Equation of a circle	-	10.5	10.5	10.5	-	10.0	10.0	10.0

$(x - h)^2 + (y - k)^2 = r^2$

Linear transformation of figures	-	-	-	-	-	11.5	-	-

-symmetrical displacement

$x' = x$

$y = -y$

-enlargement

$x' = 2x$

$y' = 2y$

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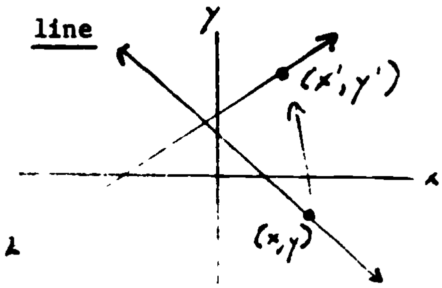
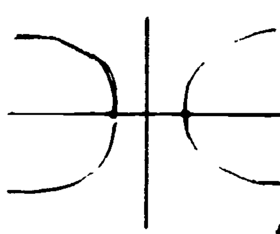
	DCM	HBJ	HM	MER	G	K	T	S
<u>Linear transformation of a line</u> 	-	-	-	-	-	11.5	11.5	11.5
<u>Linear Equation</u> ax + by = c where (a and b) $\neq$ 0 graph is a straight line	-	9.0	9.5	9.5	-	10.0	10.0	10.0
<u>Equation of Hyperbola</u>  hyperbola = $[(x,y): \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1]$ where $b^2 = c^2 - a^2$	-	11.5	11.5	11.0	-	11.5	11.5	11.5
<u>Equation of Ellipses (co-ordinate plane):</u>	-	11.5	11.5	11.0	-	11.5	11.5	11.5

	DCM	HBJ	HM	MER	G	K	T	S
ellipse = $\{(x,y): \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ where $b^2 = a^2 - c^2$								
<u>Conic Section</u> -is the graph of an equation in the form $Ax^2 + Bxy + Cy^2$ $+ Dx + Ey + F = 0$	-	-	11.5	11.0	-	11.5	11.5	11.5
<u>Inverse Matrix</u> A^{-1} is the inverse of A such that $A \times A^{-1}$ = identity matrix	-	11.5	12.5	12.0	-	11.0	11.0	11.0
<u>Graphing linear equations</u> -using slope and intercept information graph $3x - 2y = 12$ using x and y intercepts	-	9.0	9.5	9.5	8.0	8.0	8.0	10.0
<u>Matrix: systems of equations in matrix form</u> $\left.\begin{array}{l} ax + by = p \\ cx + dy = q \end{array}\right\} \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$ $A \times X = P$	-	11.5	11.5	11.0	-	11.0	11.0	11.0

	DCM	HBJ	HM	MER	G	K	T	S
Matrix	-	11.5	11.5	11.0	-	11.0	11.0	11.0

rectangular arrangement of #'s

$$\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$$

	DCM	HBJ	HM	MER	G	K	T	S
Matrix: addition and subtraction	-	12.5	11.5	12.0	-	11.0	11.0	11.0

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$= \begin{bmatrix} a + p & b + q \\ c + r & d + s \end{bmatrix}$$

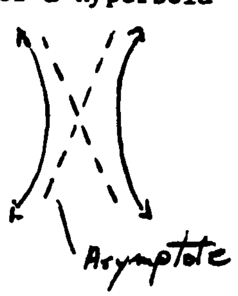
	DCM	HBJ	HM	MER	G	K	T	S
Matrix: row vector and column vector	-	12.5	-	-	-	11.0	11.0	11.0

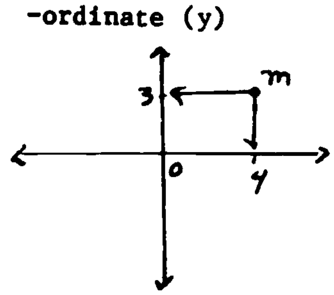
$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & & \\ a_3 & & \end{bmatrix} \rightarrow \text{row}$$

column

	DCM	HBJ	HM	MER	G	K	T	S
Asymptote	-	-	12.5	-	-	11.5	11.5	11.5

example of a hyperbola



	DCM	HBJ	HM	MER	C	K	T	S
<u>Domain and Range (coordinate plane)</u> domain = set of all x components range = set of all y components	-	9.0	9.5	9.5	-	11.5	11.5	11.5
<u>Relation between circle and ellipse</u> $x^2/a^2 + y^2/b^2 = 1$ $x^2 + y^2 = a^2$	-	-	-	-	-	-	11.5	-
<u>The coordinate plane</u> -x and y axes, origin -coordinates of m -ordered pair (x,y) -abcissa (x) -ordinate (y)  	7.5	7.5	7.5	9.5	8.0	7.5	8.0	10.0
<u>Products of Matrixes</u> if A = (k x ℓ) and B = (ℓ x m)	-	12.5	11.5	12.0	-	11.0	11.0	11.0

	DCM	HBJ	HM	MER	G	K	T	S
<u>Intercept</u> -definition of x and y intercept	-	9.0	9.5	9.5	7.0	7.0	7.0	-
<u>Graphing equations in the coordinate plane</u>	7.5	7.5	7.5	9.5	7.0	7.0	7.0	-
<u>Complex numbers: factoring using imaginary numbers</u> $x^4 - 16 =$ $(x - 2i)(x + 2i)(x - 2)(x + 2)$	-	11.5	11.0	11.0	-	10.0	10.0	10.0
<u>Determinant of a matrix (2 x 2)</u> $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ determinant = delta = $a_1 b_2 - a_2 b_1$	-	11.0	11.5	11.0	-	-	-	-
<u>Operations with complex numbers (ordered pairs)</u> addition subtraction multiplication $(a, b) - (c, d) = (a - c, b - d)$	-	-	12.0	-	-	-	-	-

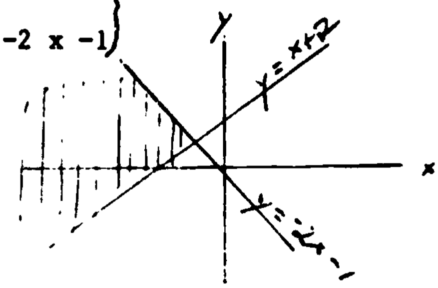
	DCM	HBJ	HM	MER	G	K	T	S
<u>Determining slope given 2 points</u> $m = (y_2 - y_1)/(x_2 - x_1)$	-	9.0	9.5	9.5	8.0	8.0	8.0	10.5
<u>Rotation in the coordinate plane</u> -function which maps each point (x,y) to a point $(x \cos\phi - y \sin\phi, x \sin\phi + y \cos\phi)$	-	-	12.5	-	7.5	7.5	7.5	-
<u>Linear Relation</u> -equation that can be written in form $Ax + By = C$ $(A, B \neq 0)$	-	9.0	9.5	9.5	7.5	7.5	7.5	10.0
<u>Solving systems of equations</u> -using elimination of variables or substitution	-	9.0	9.5	9.5	8.0	8.0	8.0	-
<u>Systems of equations (vector equations)</u> e.g. $2x - 3y = 4$ e.g. $5x - 7y = {}^-2$	-	-	12.0	-	-	-	-	-

	DCM	HBJ	HM	MER	G	K	T	S
Graphing systems of inequalities	-	9.0	9.5	9.5	-	10.0	10.0	10.0
Synthetic substitution	-	11.0	11.0	11.0	-	10.0	10.0	10.0
Equation of a Parabola	-	-	12.5	-	-	10.5	10.5	10.5
Vectors: scalar multiplication	-	-	12.0	-	-	11.0	11.0	11.0
Equation of a sphere	-	-	-	10.5	-	11.0	11.0	11.0

Graphing systems of inequalities

$y > x + 2$

$y < -2x - 1$



Synthetic substitution

If $F(y) = 2y4 + 17y - 9y^{2} + 3$

find $F(-3)$

Equation of a Parabola

Parabola = $[(x,y) . x^2 = 4py]$

Vectors: scalar multiplication

v=vector (a,b)

r=scalar

rv=(ra, rb)

rv=scalar multiple of v

Equation of a sphere

$x^2 + y^2 + z^2 = r^2$

center at (0, 0, 0)

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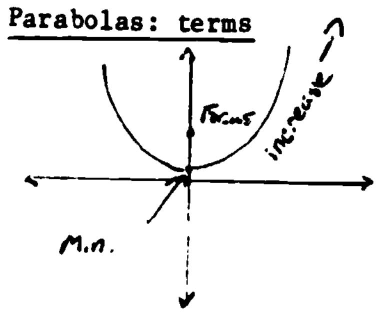
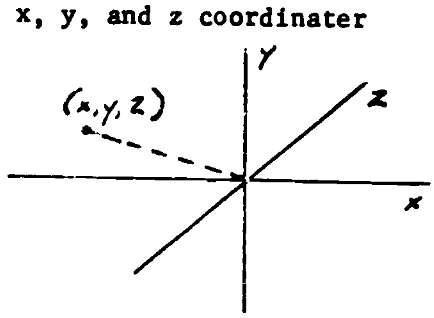
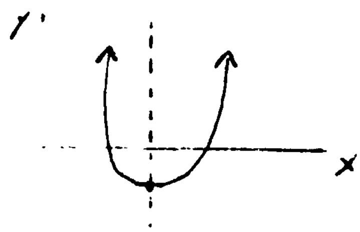
	DCM	HBJ	HM	MER	G	K	T	S
<u>Vector Addition</u> -the sum of two ordered pairs is a vector sum (resultant) $(-v_1 , -v_2)$ v is the additive inverse ot (v_1 , v_2)	-	12.5	12.0	-	-	11.0	11.0	11.0
<u>Parabolas: terms</u>  	-	11.5	11.0	11.0	9.0	9.0	9.0	10.5
<u>Normal vector</u> -any nonzero vector perpen- dicular to the direction vector of a line	-	-	12.0	-	-	11.0	11.0	11.0
<u>Vectors in a three-space</u> x, y, and z coordinater  	-	-	12.5	12.0	-	11.0	11.0	11.0
<u>Graphing systems of equations</u> <u>(coordinate plane)</u>	-	8.5	8.5	9.5	8.0	8.0	8.0	-

	DCM	HBJ	HM	MER	G	K	T	S
-find point of intersection								
<u>Graphing quadratic functions</u> $y = ax^2 + bx + c$ 	-	8.5	9.5	9.5	9.0	9.0	9.0	10.5
<u>Step functions</u> $0 \leq x < 1 \ldots y = 0$ $1 \leq x < 2 \ldots y = 1$ $2 \leq x < 3 \ldots y = 2$	-	11.0	11.0	11.0	9.0	9.0	9.0	-
<u>Finding equations from relations</u>	-	-	-	9.5	8.0	8.0	8.0	-
<u>Translations and transformation on the coordinate plane</u> -function which maps each pt. (x,y) to the point (x+r, y+s) where (r,s) is a vector	-	12.5	12.5	12.5	-	10.5	10.5	10.5

	DCM	HBJ	HM	MER	G	K	T	S
Slope	-	9.0	9.5	9.5	9.0	9.0	9.0	-
Equations of tangent lines	-	-	-	-	-	11.5	11.5	11.5
Equation of a plane	-	12.5	-	-	-	11.0	11.0	11.0
Perpendicular Lines	-	11.0	11.0	9.5	-	10.0	10.0	10.0

Slope

$\frac{\text{Change in } y}{\text{Change in } x}$

Equations of tangent lines

-ellipse:

$$\frac{x_1 x}{a^2} + \frac{y_1 y}{b^2} = 1$$

-hyperbola:

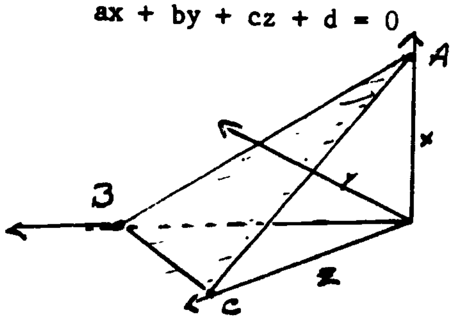
$$\frac{x_1 x}{a^2} - \frac{y_1 y}{b^2} = 1$$

parabola:

$y_1 \; y = 2p\,(x+x_1)$

Equation of a plane

$ax + by + cz + d = 0$

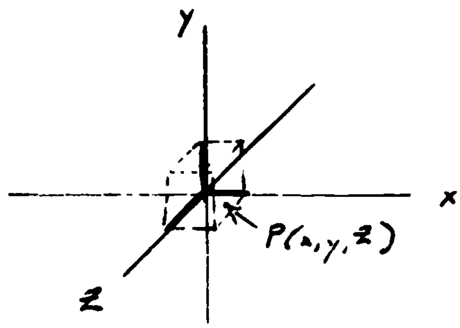


Perpendicular Lines

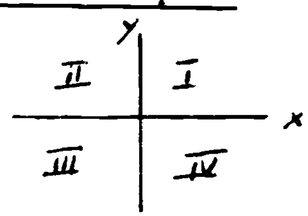
-if product of slopes is -1.

Vertical lines are ⊥ to horiz. lines (in a plane)

	DCM	HBJ	HM	MER	G	K	T	S
Relations -sets of ordered pairs (2, 2), (-2, 3), (0, -1)	-	8.5	8.5	9.5	9.0	9.0	9.0	-
Slope-intercept form of linear equation $ax + by = c$ $y = -a/bx + c/b$ $y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{\text{intercept}}$	-	9.0	9.5	9.5	-	11.0	11.0	11.0
Slope of linear equation $ax + by = c$ slope $= -a/b$	-	11.0	11.0	9.5	-	10.0	10.0	10.0
Vectors on the coordinate plane -any ordered pair of real #'s is a vector	-	11.5	11.5	12.0	-	11.0	11.0	11.0
Coordinates in space x, y, and z axes	-	11.5	-	10.5	-	11.0	11.0	11.0



	DCM	HBJ	HM	MER	G	K	T	S
<u>Parallel lines (coordinate plane):</u> -2 lines with same slope are parallel (all vertical lines are ∥)	-	9.0	9.5	9.5	-	10.0	10.0	10.0
<u>Quadrants of the coordinate plane</u>	-	9.0	9.5	9.5	-	7.5	10.0	10.0



<u>PROBLEM SOLVING</u>

	DCM	HBJ	HM	MER	G	K	T	S
<u>Systems of equations in word problems</u>	-	9.0	9.5	9.5	8.0	8.0	8.0	-
<u>Problem solving</u> -transforming word problems into functions $y = ax + b$ ($a \neq c$)	-	-	-	-	8.0	8.0	8.0	-
<u>Transform word into equations</u> -solve equation and check results	-	8.0	8.0	9.0	7.0	7.0	7.0	-

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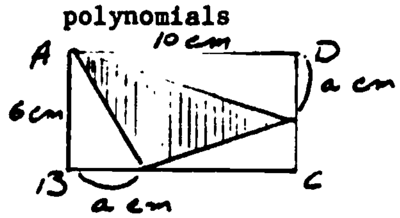
	DCM	HBJ	HM	MER	G	K	T	S
using word problem Write an equation for: A big brocher has "a" yen and a little brother has "b" yen. If the big brother gives "c" yen to his little brother, the amount of money they have will be equal. (Japan) a - c = b + c								
<u>Inverse Varification</u> Described by y = k/x (k ≠ o) (or xy = k)	-	9.5	9.5	9.5	7.5	7.5	7.5	-
<u>Direct Variation</u> -described by y = kx (k ≠ o) k is the constant of variation	-	9.0	9.5	9.5	7.5	7.5	7.5	-
<u>Problem Solving</u> -geometry problems with polynomials	-	-	-	-	9.0	9.0	9.0	-



1) what is the area of △ABP, △PCQ, △QDR?

	DCM	HBJ	HM	MER	G	K	T	S
2) what is the area of $\triangle$APQ? (Japan)								
<u>Formulas: definition and use</u> -equation that states a rule for relationship between two quantities A = l x w	-	-	-	9.0	8.0	8.0	8.0	-
<u>Word problems with LCM</u> -Bus A leaves every 10 min., bus B leaves every 15 min., and bus C leaves every 12 min. between 7 and 10 o'clock. At 7:05, the three busses left at the same time. What time will they leave to- gether next? (Japan)	-	-	-	-	7.0	7.0	-	-
<u>Problem solving</u> Word problem involving linear transformation	-	-	-	-	-	11.5	-	-
<u>Patterns of Inference</u> 1. Principle of inference or detachment	-	-	12.0	-	-	-	-	-

	DCM	HBJ	HM	MER	G	K	T	S
2. Contraposition or negative inference 3. Disjunctive inference 4. Equivalence inference 5. Syllogism 6. Substitution axioms = statements assumed to be true								
<u>Transform equations into word problems</u> -state word problem that can be represented by a given equation	-	-	8.0	9.0	-	-	-	-
<u>Using formulas: solving for one variable</u> solve for N S = (N/2)(A + t) N = 2S/(A + t)	-	-	-	9.5	7.0	7.0	7.0	-
<u>Equations and inequalities from word sentences</u> -twice a number is less than or equal to 14 (2 x ≤ 14)	8.0	11.0	8.0	9.0	7.0	7.0	7.0	-

	DCM	HBJ	HM	MER	G	K	T	S
Expression for word phrase "increased by" (+) "product" (x)	-	9.0	8.0	9.0	7.0	7.0	7.0	-
Plan for solving problems -sort out and organize facts -what is asked for? -what facts are given? -enough information? -too much information?	7.0	7.0	7.0	9.0	7.0	7.0	7.0	-
Solving and checking word problems	8.0	-	8.0	-	7.0	7.0	7.0	-
Solving problems using graphs -find slope of line from word problems	8.5	-	8.5	-	8.0	8.0	8.0	-
Problem solving: -word problems converted to inequalities -"at least" ($\geq$) -"at most" ($\leq$) -"between" ($a < x < b$)	-	9.0	9.5	9.0	8.0	8.0	8.0	-
Problem solving: quadratics	-	9.5	9.5	9.5	9.0	9.0	9.0	-

	DCM	HBJ	HM	MER	G	K	T	S
-decide which method is best to solve								
CALCULUS								
Maximum-Minimum word problems	-	-	12.0	-	-	-	-	11.5
Trigonometric functions (on triangle)	-	11.5	11.5	11.5	-	10.5	10.5	10.5
Definitive Integral	-	-	-	-	-	-	-	11.5
$\int_a^b F(x)dx=F(b)-F(a)$								
a=lower limit								
b=upper limit								
Limits of constant and entire functions	-	-	12.0	-	-	-	-	11.5
-constant function								
if f(x) =c, then $\lim_{x\to h} (c) = c$								
-entire function								
if f(x) is entire function then								
$\lim_{x\to a} f(x) = f(a)$								
Limit of a function	-	12.0	12.0	12.5	-	-	-	11.5
$\lim_{x\to a} f(x)$								

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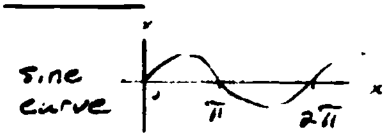
	DCM	HBJ	HM	MER	G	K	T	S
<u>Function f(t)dt</u>	-	-	-	-	-	-	-	11.5
the indefinite integral of f(x)								
<u>Application of Integral to</u>								
<u>Volume</u>	-	-	-	-	-	-	-	11.5
<u>Volume of a body of rotation</u>	-	12.5	-	-	-	-	-	11.5
<u>Arithmatic Progression</u>	-	11.5	11.5	11.5	-	-	-	11.0
$a_{n+1} = a_n + d$								
d=common difference								
sum of arithmatic progression								
$S_n = n(a + \ell)/2$, a= 1st term,								
ℓ = last term								
<u>Differentiating</u>	-	-	-	-	-	-	-	11.5
=obtaining f'(x) from f(x)								
=f'(x), y', dy/dx, d/dx f(x)								
<u>Common Logarithm</u>	-	-	12.5	12.5	-	-	-	11.0
$\log_{10} N$ n = r								

	DCM	HBJ	HM	MER	G	K	T	S
<u>Degree system</u>	-	-	12.5	-	-	-	-	11.0
-degree								
-minute								
-second								
<u>Summation Notation</u>	-	-	-	11.5	-	-	-	11.0
sigma: Σ								
<u>Quadrants</u>	-	-	12.5	-	-	-	-	11.0
<u>Radian system</u>	-	11.5	11.5	11.5	-	-	-	11.0
<u>Graphs of trigonometric functions</u>	-	11.5	11.5	11.5	-	-	-	11.0
<u>Antilogarithm</u>	-	-	-	11.5	-	-	-	-
log x = N								
x is the antilog of N								
<u>Geometric Application of Derivative</u>	-	-	12.0	-	-	-	-	11.5
<u>Even and odd functions</u>	-	-	-	-	-	-	-	11.0



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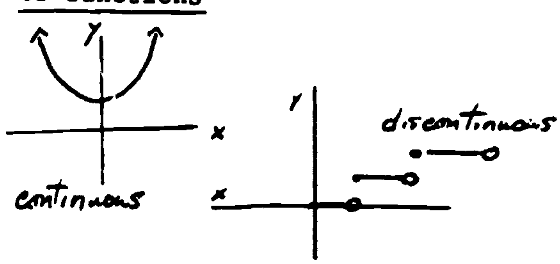
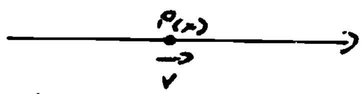
	DCM	HBJ	HM	MER	G	K	T	S
even : f(-x)=f(x) odd : f(-x)=-f(x)								
<u>Simple Oscillation</u>	-	-	-	-	-	11.0	-	12.5
<u>Convergence and Divergence of Infinite Progression</u> convergence - progression has constant limit divergence - limit of progression is either +<u>∞</u>, -<u>∞</u> or has not limit	-	-	-	-	-	-	-	12.0
<u>Conversions of repeating decimal to a fraction</u>	-	-	-	-	-	-	-	12.0
<u>Continuity vs incontinuity of functions</u>	-	-	-	-	-	-	-	12.0



	DCM	HBJ	HM	MER	G	K	T	S
<u>Rational vs Irrational functions</u> rational function - irrational function -	-	-	-	-	-	-	-	12.0

	DCM	HBJ	HM	MER	G	K	T	S
<u>Derivative</u> $f'(x) = \lim_{h \to 0}$ (increment of y' increment of x)	-	12.5	12.0	12.5	-	-	-	11.5
<u>Differentiability</u> if y=f(x) is differentiable at any point a within an interval, y=f(x) is "differentiable at the interval".	-	-	-	-	-	-	-	12.0
<u>Composite function</u> y=f(g(x))	-	-	-	-	-	-	-	12.0
<u>The number "e"</u> -an irrational number e=2.718. . .	-	12.5	-	12.5	-	-	-	12.0
<u>Natural Logarithm</u> $\log_e x$ log(e)=1	-	-	-	12.5	-	-	-	12.0
<u>Higher-Order Derivatives</u> second order dirivative	-	-	-	-	-	-	-	12.0

	DCM	HBJ	HM	MER	G	K	T	S
y'', $f''(x)$, $d^2 y/dx$, $d^2 /dx^2 f(x)$								
<u>Cycloid</u>	-	-	-	-	-	-	-	12.0
<u>Velocity and Location</u>  $\frac{dx}{dt} = v$ $x = \int f'(t)dt$ $= F(t) + C$	-	-	-	-	-	-	-	11.5
<u>Sequence and series</u> -finite and infinite sequence: 3, 7, 11, 15 series: 3 + 7 + 11 + 15	-	11.5	11.5	11.5	-	-	-	-
<u>Logarithm</u> $\log_x N=y$ $x^y =N$	-	-	11.5	11.5	-	-	-	11.0
<u>Progressions and sum of progression</u> $a_n = 4n - 3$ first term a = 4 x 1 - 3 = 1 second term a = 4 x 2 - 3 = 5	-	-	12.0	-	-	-	-	11.0

	DCM	HBJ	HM	MER	G	K	T	S
third term a = 4 x 3 - 3 = 9								
1, 5, 9, ...								
<u>Exponential functions</u>	-	-	-	-	-	-	-	11.0
square root								
cube root								
nth root								
<u>Harmonic Progression</u>	-	-	-	-	-	-	-	11.0
1, 1/2, 1/3, 1/4, . . .								
<u>Geometric Progression</u>	-	11.5	11.5	11.5	-	-	-	11.0
$\frac{a_{n+1}}{a_n} = r$, $r \neq 0$								
r = common ratio								
<u>Longarithmic function</u>	-	11.5	11.5	11.5	-	-	-	11.0
$a^r = R$								
$r = \log_a R$								
<u>Trigonometric functions on unit circle</u>	-	11.5	11.5	11.5	-	10.5	10.5	10.5
<u>Local minimum and local maximum (local extrema)</u>	-	-	12.0	-	-	-	-	11.5
-points at which								
f'(a) = 0 and there is a								

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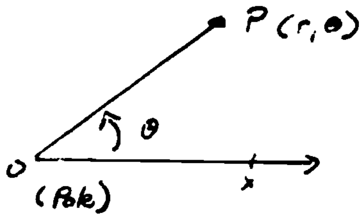
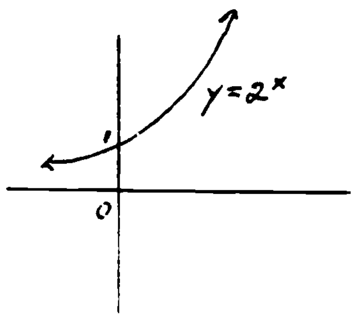
	DCM	HBJ	HM	MER	G	K	T	S
change of sign of f'(x)								
Polar coordinates r and θ 	-	12.0	12.5	12.5	-	11.0	-	-
Angular Velocity	-	-	-	-	-	-	-	12.5
Application of derivative velocity (speed) acceleration	-	-	12.5	12.5	-	-	-	11.5
Maximum and minimum values of a function (on a closed interval) -vs local max and min] -when y=f(x) on a closed internal [a, b] local minimum or maximum may or may not be the minimum or maximum of the function	-	12.0	12.0	11.5	-	-	-	11.5
Definitive Integral and limit of a series	-	-	-	-	-	-	-	12.5
Exponential function	-	-	12.5	12.5	-	-	-	11.0

	DCM	HBJ	HM	MER	G	K	T	S
								
<u>Radical signs and exponents</u>	-	-	-	-	-	-	-	11.0
<u>Indefinite integral</u> f(x) dx -constant of integration "C"	-	12.5	-	-	-	-	-	11.5
<u>Definite integral and area</u>	-	12.5	-	-	-	-	-	11.5
<u>Length of a curve</u> -length of curve between A(g(α), h(α)) and B(g(β), h(β)) is s	-	-	-	-	-	-	-	12.5
<u>Differential equation</u> $\frac{dx}{dt} = a$ $\frac{d^2y}{dt^2} = -g$	-	-	-	-	-	-	-	12.5