DOCUMENT RESUME

ED 267 973 SE 046 495

AUTHOR Wearne, Diana; Hiebert, James
TITLE Learning Decimal Numbers: A Study of Knowledge Acquisition. Final Report.
INSTITUTION Delaware Univ., Newark.
SPONS AGENCY National Inst. of Education (ED), Washington, DC.
PUB DATE 28 Jan 86
GRANT NIE-G-83-0054
NOTE 107p.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC05 Plus Postage.
DESCRIPTORS Cognitive Processes; *Decimal Fractions; Educational Research; Elementary Education; *Elementary School Mathematics; Intermediate Grades; Interviews; Learning Activities; *Mathematical Concepts; *Mathematics Instruction; Number Concepts; *Number Systems; *Symbols (Mathematics)
IDENTIFIERS *Mathematics Education Research

ABSTRACT

A study explored the effectiveness of an instructional approach designed to help students establish connections between their understanding of the decimal system and the standard symbols and procedural rules used to solve decimal problems. Ten children each in grades 4 through 6 participated. They were individually interviewed prior to and six weeks after instruction. Group tests containing decimal number items were administered in Fall and Spring to the entire classrooms from which the samples were drawn. It was found that after instruction students could engage in semantic analysis on tasks similar to those covered during instruction and could use their understanding of the symbols to perform novel tasks. The extent to which they engaged in semantic analysis appeared to be tied somewhat to the amount of previous formal instruction in decimals. Students who had already learned a procedure for performing a task were less likely to use semantic meanings for the symbols to solve a problem than students who had not yet learned a procedural rule. Consequently, the effectiveness of instruction designed to create meanings for symbols seems to depend, in part, upon when it is introduced. Appendices contain group activities, test items, and interview questions. (Author/MNS)

FINAL REPORT

GRANT NO. NIE-G-83-0054

LEARNING DECIMAL NUMBERS: A STUDY OF KNOWLEDGE ACQUISITION

DIANA WEARNE AND JAMES HIEBERT
UNIVERSITY OF DELAWARE
NEWARK, DE 19716

JANUARY 28, 1986

The research reported herein was performed pursuant to a grant with the National Institute of Education, U.S. Department of Education. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgement in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official National Institute of Education position or policy.

U. S. DEPARTMENT OF EDUCATION

NATIONAL INSTITUTE OF EDUCATION

Abstract

The purpose of the project was to explore the effectiveness of an instructional approach designed to help students establish connections between their understandings of the decima. stem and the standard symbols and procedural rules that are used to solve decimal problems.

Ten children in each of the grades four, five, and six participated in the instruction. The students were individually interviewed prior to and six weeks after instruction. Group tests containing decimal number items were administered in the Fall and Spring to the entire classroom of students from which the samples were drawn.

It was found that after instruction, students were able to engage in semantic analyses on tasks similar to those covered during instruction and were able to use their understandings of the symbols to perform novel tasks. The extent to which the students engaged in semantic analysis appeared to be tied somewhat to the amount of previous formal instruction in decimals. Students who had already learned a procedure for performing a task were less likely to use semantic meanings for the symbols to solve the problem than students who had not yet learned a procedural rule. Consequently, the effectiveness of instruction designed to create meanings for symbols seems to depend, in part, upon when it is introduced.

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LEARNING DECIMAL NUMBERS: A STUDY OF KNOWLEDGE ACQUISITION

In this report we present the results of a two-year project designed to explore the effectiveness of an instructional approach to the teaching of decimals. The report is divided into three major sections. In the first section we describe the objectives of the project and the rationale for their selection. Also, we present the major conclusions that have emerged from our project. Section two contains the descriptions of the procedures. The third section contains the results. We believe the results support the major conclusions presented in section one.

Section One: Objectives, Rationale, and Primary Conclusions

Objectives

The basic research question investigated was what instructional approaches can be used to help students establish connections between their understandings of the decimal system and the standard symbols and procedural rules that are used to solve decimal problems. A second, and closely allied question, was how do students connect highly specific and previously independent domains of knowledge. The second research question underscores the fact that the project was designed to provide descriptive information on the cognitive processes involved in constructing links between disparate domains of knowledge. Of particular interest is the linking process between symbolic representations and related understandings.

Rationale

Rationale for Selection of Content Topic

The concept of a decimal number emerges from an integration of rationale number concepts and place value concepts with whole numbers. These two domains provide the elementary school mathematical context for the development of decimals. Although at first glance the construction of decimal numbers may appear to be a straightforward extension of whole number and fraction concepts, the genuine integration of all the relevant ideas to form a mature notion of decimal number is significant and complex.

Results of various studies (Hiebert & Wearne, in press-a, 1984; Carpenter, Corbitt, Kepner, Lindquist, & Reyes, 1981) indicate that students experience great difficulty in all but the most routine decimal tasks. Results from National Assessment of Education Progress (Carpenter, et al, 1981) indicate that even 13-year olds lack basic facility with decimals. The NAEP results show that fewer than half of the 13-year olds were able to order decimal numbers, change common fractions expressed in tenths and hundredths to decimals, and less than 40% could write the decimal number equivalent to 1/5.

Further evidence of the lack of relationship between previous knowledge and decimals is provided by Erlwanger (1975) and Ekenstam (1977). These reports suggest that often students see work with decimals as unrelated to previous work, either inside or outside of school. The students construct a separate system of rules to deal with decimals and see little connection between the rules and the basic concepts of a decimal based upon whole number and fraction ideas.

Results of previous work of the authors are consistent with these interpretations and help to illuminate the nature of students' conceptions and misconceptions about decimals. The perception of decimals is dominated by rules about form. It appears that students view written decimals as symbols upon which to perform syntactic maneuvers. The maneuvers are dictated by a very complex set of rules that have been memorized over the course of instruction. Rules are very specific in that slight changes in task format trigger the application of different rules. Errors often result from the application of inappropriate rules. For example, nearly 90% of the seventh- and ninth-graders responded correctly to .87 - .24 but only about two-thirds of them correctly solved .97 - .2. When these same students were asked to write the decimal representation for one shaded part of a region divided into fourths or fifths, nearly one-third of the seventh-graders wrote 1.4 and 1.5, respectively. The numerals probably were generated from the associated fraction, but the written responses show little understanding of decimal fraction equivalents and clearly demonstrate a greater concern for form than for meaning.

The problem becomes urgent when one recognizes the importance of decimal numbers, both from a mathematical and a practical point of view. Mathematically, the decimal system represents a more powerful representation system than those the student has worked with previously. The decimal system culminates previous work with whole number and fractions in that these two systems become subsumed by a single more powerful system. The decimal system also provides new ways of representing a quantitative situation and encourages new insights into the properties of number systems themselves.

Rationale for Instructional Approach

Perhaps the most striking thing about students' knowledge of decimal numbers is the near total separation between their conceptual knowledge of decimals and their procedural knowledge of decimals. Although many students lack certain pieces of conceptual knowledge and procedural knowledge, it is our conclusion that the primary deficit for most students is the absence of connections between aspects of knowledge that they already possess. Many students have acquired some important concepts and have memorized many algorithmic rules. But they have made no connection between them. Many performance errors result from applying symbol manipulation rules that nowhere are connected to the concepts that motivate them or give them meaning.

Students appear to pay attention only to the syntax or surface form of the problem and do not recognize that the rules they memorize are based on reasonable concepts. Such a syntactic approach leads to many predictable kinds of errors. In fact, students' behavior is sufficiently consistent and predictable that it is possible to build a syntactic model that simulates the processes they used and the responses they produce to a wide variety of computation items (Hiebert & Wearne, in press-a).

We regard the separation between conceptual and procedural knowledge as the number one problem in students' understanding of decimal numbers. Memorized rules are not tied to anything that might give them meaning. Without such links, the rules become independent units of information, each applied to a unique class of problems recognized only by features of surface form. Without understanding the rationale for the rules and seeing what they have in common, each rule must be memorized as a new routine.

The instructional approach explored was designed to help students build connections between the various aspects of decimal number form and understanding. The intent was to provide information of a practical nature about the effectiveness of a specific instructional strategy on teaching students about decimal numbers while at the same time contributing to our understanding of cognitive processes that may well have significance for knowledge acquisition and organization in a more general sense. Instruction must be designed to encourage and facilitate the construction of bridges between form and understanding. If students are able to see the links between the form and their own understandings as they are learning a new symbol system, they will be in a better position to learn the algorithmic procedures with meaning and to monitor their own performance in terms of consistency and mathematical reasonableness.

Instruction that is effective in helping students link form and understanding is likely to include two features not found in traditional approaches. First, instruction must attend more carefully to helping students build up a store of understandings that are operational. Instruction must be designed to strengthen the understandings that already exist and to encourage the acquisition of additional understandings that are rich, flexible, and operational.

A second feature of effective instruction is likely to be a special component that helps students make the critical connections between the available understandings and the symbolic form.

Major Conclusions

The instruction was focused directly on helping students assign meaning to decimal notation. The assumption was that if students could connect appropriate referents with the written symbols, then they would be in a position to reflect on these referents when given a decimal task. Such referents would give meaning to the symbols and the students could then engage in semantic analyses. Semantic processes rather than syntactic processes alone could guide their solutions.

We found that students not only were able to engage in semantic analyses on tasks similar to those covered during instruction, but were able to use their understanding of the symbols to perform novel tasks. The extent to which the students engaged in semantic analysis appeared to be tied somewhat to the extent to which they had received previous formal instruction in decimals. Students who had already learned a procedure for performing a task were less likely to provide a semantic rationale and more likely to be rule governed than a student who had not learned a procedure for obtaining a solution to a task.

Section Two: Procedures

The project was divided into three phases. During the first phase, the instructional sequence was developed. The sequence was piloted and revised during the second phase and the third phase consisted of a formal tryout of the instructional sequence. The consultant for the project, Dr. Robbie Case of the Ontario Institute for Studies in Education, met with us prior to the third phase. Dr. Case's interest in applying concepts of cognitive development to the problem of school instruction made him a valuable resource during the planning of the final form of the instruction sequence.

The Instructional Sequence

Charting the effects of instruction on cognitive change requires detailed descriptions of instruction that is designed explicitly to promote such change. The set of cognitive processes that are observed and inferred will be determined in part by the content. The key processes are identified through a combination of rational task analysis and data interpretation. The essential features of instruction are (1) instruction must be limited in scope and clearly focused, and (2) the instructional strategies must be specifiable. Both features support the same goal: to trace the effects of instruction on cognitive change.

By the time students reach high school, many of them have built up a large store of rules that prescribe the manipulations of decimal fraction symbols. Cognitive processes tend to involve searching for rules, selecting them, sequencing them, and executing them. A different type of cognitive process that may contribute to success with decimals is semantic analysis. This approach depends upon the prior or concurrent functioning of a cognitive process that links the written symbol with referents that are meaningful to the learner. If the symbols have no meaning outside of the syntax of decimal fractions, semantic analyses are impossible.

Instruction is designed to produce improved performance through the adoption of different cognitive processes or the more efficient use of existing processes. The cognitive processes targeted for instruction here were those that would help students establish meanings for written decimal symbols and incorporate semantic analyses into their solution procedures.

The instructional sequence developed focused directly on helping students assign meaning to decimal notation. It was conjectured that if students could connect appropriate referents with the written symbol, they would be in a position to reflect upon these referents when given a problem involving decimals. Such referents would give meaning to the symbol, and students could then engage in semantic analyses. Semantic processes, rather than syntactic processes alone, could guide their solutions.

The referents chosen for the instructional sequence were Dienes' or base-ten blocks. The large block was assigned a value of one (see Figure 1) and then an

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Insert Figure 1 about here

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iterative partitioning process was used to create the tenths' and hundredths' blocks. The concreteness and maneuverability of the blocks make them especially useful referents for decimal symbols.

The instruction was designed to create links between the sets of blocks and their written symbol representations and then to use these connections to solve problems. The students were asked to show numbers written symbolically and to write the symbols for numbers shown with blocks. The numbers were limited to the hundredth's place. The students also added and subtracted decimal numbers through hundredths.

The instructional sequence consisted of nine lessons. The first five focused on developing meaning for the symbols and the last four were devoted to addition and subtraction problems. The instruction in these latter four activities encouraged the students to analyze the problem semantically and to operate with the symbols accordingly. Problems were presented with written symbols and the students were asked to determine the answer by using the blocks. Eventually the students were asked to work the problem on paper and to think about the blocks. The sequence of lessons did not include a written relationship between common fractions and decimals, did not involve ordering decimal numbers, did not make use of a number line, and the lessons did not involve an area representation of decimal numbers. These topics were omitted from the instruction, not because they were considered unimportant, but rather to provide transfer measures. The complete set of instruction activities is contained in Appendix A.

The instruction focused on enhancing identifiable cognitive processes--creating meaning for symbol notation and using the meaning to deal with symbol manipulations. Furthermore, the instructional strategies are specifiable. The instructional episodes can be described with some precision. All of this increases the likelihood of linking cognitive change with particular instructional events.

Data Collection Procedures

Sample

The population for the second phase, the piloting of the instructional sequence, consisted of five students from each of the grades four, five, and six. The classroom teacher selected one student who was achieving above average in mathematics, three who were about average in their achievement, and one student who was below average in mathematics achievement.

The population for the third phase consisted of 10 children from each of the grades four, five, and six. Two classrooms at each grade level participated in the study. Each classroom teacher selected five students. As in the pilot phase, one student was above average, three were average, and one student was below average in mathematics achievement.

The two schools involved in the study, one at each of the phases two and three, have a wide range of achievement and socio-economic classes. The schools have been integrated through bussing. The sample of students who participated in the study represented the range in SES, achievement, and racial groups found in the school.

Assessment Tasks

Two different types of assessment tasks were used: group administered tests and individual interviews. The students in the pilot phase of the project received only the interviews. The classrooms from which the students were selected for the third phase received group administered tests in the beginning and at the end of the school year. These group tests consisted of items measuring computation with decimals (all four operations), translation between decimals and common fractions, locating decimals on a number line, using area representation to represent decimals, writing a decimal to correspond with a given area representation, and ordering decimal numbers. The spring test was parallel to the fall test. Copies of the tests may be found in Appendix B.

The second category of assessment tasks were the individually administered interviews. The intent of the interviews was to measure students' use of semantic analysis in dealing with decimal symbols. The students were asked to explain how they found their answers, how they would teach the procedures to friends who had missed class, etc.

Two kinds of tasks were used--direct measures and transfer measures. Direct measures are tasks that assess students' use of the instructed processes in a completely straightforward way. These tasks involve representing written decimal numbers with blocks and conversely, and solving symbolically presented addition and subtraction problems. The computation problems presented were "ragged" decimal problems (e.g., 1.3 + .25) as previous work had shown that such problems discriminate most clearly between students who use semantic analyses and those who recall and execute syntactic rules (Hiebert & Wearne, in press-a, in press-b).

Transfer measures are those that assess the instructed processes in novel contexts. The tasks may involve processes in addition to those instructed, but it is exactly these kinds of tasks that are essential to evaluate the flexibility and robustness of the target processes (Belmont & Butterfield, 1977; Greeno, Riley, & Gelman, 1984). We chose tasks that, based on previous work, either discriminate well between semantic analyzers and syntax rules appliers (in terms of error patterns) or are especially difficult unless on engages in semantic analysis (Hiebert & Wearne, in press-b). Examples of tasks are: representing with blocks a written number with thousandths (instruction dealt only with hundredths, but the "generalizable" partitioning process was demonstrated); choosing the larger of two numbers (e.g., .8 and .34); and writing a given decimal number in common fraction form and conversely. None of these types of problems were included in the instructional activities as mentioned previously under a description of the instructional sequence. A copy of the interviews may be found in Appendix C.

Data Collection

Nine students in grade four and ten students in each of grades five and six participated in the instructional sequence. The fourth grade students had received no prior instruction on decimals, some of the fifth grade students had been introduced to decimals previously, and sixth grade students had been taught to compute with decimals.

Just prior to the instruction, the students were interviewed individually by one of the principal investigators and given the complete set of direct and transfer measures. The interviews were standardized except for follow-up questions that aimed to elicit students' descriptions of how they solved each problem. Questioning continued until the solution process was described completely or until it was clear that no more information was forthcoming. A coding form was used to record responses and the interviews was audio-taped and transcribed for analysis.

Each grade-level group of students was instructed by one of the principal investigators. The nine activities were completed in seven to nine days over a two to three week period, depending upon the group. The style of instruction emphasized students participation, instructor and peer feedback, and instructor monitoring to ensure active attention. About six weeks after instruction, students were interviewed again and presented with a variety of tasks, some of which were alternate forms to those in the pre-instruction interviews. Once again, interviews were audio-taped and transcribed for analysis.

Section Three: Results and Interpretations

Two types of assessments were made during the third phase of the study, group tests and individually administered interviews. The purpose of the group tests was to: (1) provide information about the classes from which the sample of students were drawn, (2) provide information about the sample's understanding of decimals prior to instruction, and (3) provide information to the teacher which would be of some assistance in gaining information about the students' understanding of and ability to use decimal numbers. The Fall and Spring tests were parallel forms of one another. A copy of the test may be found in Appendix B.

The purpose of the individually administered interviews was to measure students' use of semantic analysis to deal with decimal symbols. The students were interviewed prior to the instruction and six weeks after the instruction.

Results of Group Tests

The group tests were given to the entire classroom of students from which the fifth and sixth grade sample was drawn for the third phase of the study. The fourth grade students did not receive these tests as it was determined that none of the students had previously studied nor would they study decimals during the school year. The tests were given in September and May of the school year with the instruction for sixth and fifth graders taking place in October and that for the fourth graders in February.

The group tests consisted of items assessing computation (all four operations), representing a decimal on a number line, representing a decimal using an area model, writing a decimal for a given representation, writing a decimal for a common fraction and conversely, and items related to place value. All of the items were free response. Complete results of the two administrations of the group tests are shown in Table 1.

Insert Table 1 about here

The results of the group tests were similar to those we had obtained with other groups of students on similar items (Hiebert & Wearne, in press-a, Wearne, 1984).

Perhaps the most important interpretation that can be given to the data in the table is that many students have established little or no meaning for the decimal symbols. There are three characteristics of the students' responses that lead one to this conclusion. First, the students appear to manipulate the symbols according to purely syntactic rules. The semantic or conceptual rationale that motivates the symbol manipulation rules apparently was never established or has been separated from the rules. Second, students seem to fall back on the familiar ground of whole numbers when working with decimal numbers. They overgeneralize the relationship between decimal and whole numbers to the extent that the decimal point frequently is ignored. Clearly, the symbol has little conceptual meaning if the decimal point can be ignored when convenient. Third, students seemingly are not aware that their responses should be conceptually reasonable.

Answers are accepted as reasonable if they are produced by following recalled syntactic rules. Students do not look at the question and the answer as a single unit, that is, they do not reflect upon their answers in relation to the question.

The fact that the computation items were differentially difficult for students provides further support for the claim that many students attach little meaning to the decimal symbols. Conceptually all of the addition items, for example, are of equal difficulty. The problem 5.1 + .46 would be no more or less difficult than either 4.6 + 2.3 or 4 + .3 because conceptually one is simply adding like quantities to like quantities (e.g., wholes to wholes, tenths to tenths etc.) and the particular combinations are straightforward. However, from a procedural syntactic point of view, the items are quite different. Some require more rules than others, and some require newly learned rules. For example, a procedural analysis shows that 4.6 + 2.3 is simpler than 4 + .3 for a number of reasons. To solve 4 + .3 at a syntactic level, one must know a rule for inserting a decimal point where none is visually present and must recall the newly learned rule that says to line up decimal points before adding. In contrast, one can get the correct answer for 4.6 + 2.3 by falling back on an old whole number rule of lining up the digits before adding (a rule that would have produced an error for 4 + .3). It turns out that a consideration of procedural complexity alone provides a good indication of the level of difficulty of a particular problem (Hiebert & Wearne, in press-a). Apparently, students compute by applying syntactic rules rather than utilizing conceptual knowledge about the meaning of the symbols.

Common Errors in Computation

The most common error in addition and subtraction of decimal numbers results from treating the symbols as whole numbers rather than as decimal fractions. This is a direct consequence of failing to consider the meaning of the symbols. All of the questions on the tests were presented in a horizontal format. Most students wrote the numbers vertically before adding or subtracting and many students lined up the numerals on the right before calculating the answers. Also, it is apparent from the responses that many who did not rewrite the problem vertically added or subtracted the numbers by combining the two digits farthest to the right, then the next two, etc. Students were more likely to make the right justifying error when the two numbers had the same number of digits. This is startling evident on the item 4 + .3. The most frequently given response was .7 by the fifth grade students and represented 44% of the responses at the sixth grade level. To think that one could obtain .7 when adding .3 to 4 is a clear indication that students are not reflecting upon their answers.

Students perform at a somewhat lower level on the subtraction items than on the addition items. However, the major error remains the same: treating the symbols as whole numbers and ignoring the decimal points. This is evident in the items in which the number of digits to the right of the decimal point were not equal in the two numbers being combined (4.7 - .24, 1.47 - .2, 7 - .15, .86 - .3 and 9.6 - 4).

Correctly setting up a multiplication problem for computation involves exactly the same procedures one would use if the numbers were whole numbers. The only new syntactic procedure the students must apply in multiplying decimal numbers is the rule for placing the decimal point in the answer. The items that were the most difficult for the students were those that demanded the most from this rule. Problems such as .4 x .2 and .05 x .4 were most difficult as the students had to insert a zero to the left of the digit before placing the decimal point in the answer.

Errors in division were similar to those in multiplication. The most difficult items were those requiring an insertion of a zero to the right or left of the decimal point. One of the more difficult problems was 3 - .6. The students tended to divide the larger digit (.6) by the smaller digit (3). The tendency to interpret division as dividing the larger number by the smaller, regardless of the order in which the numbers are written, has been noted in other studies (Hiebert & Wearne, in press-a; Bell, Swan, & Taylor, 1981; Hart, 1981; Bell, Fischbein, & Greer, 1984).

<u>Common Errors in Ordering</u>

A notable example of students neglecting the decimal point and treating the decimal numbers as whole numbers occurs when students are asked to order decimal numbers. Almost one-half of the fifth grade students and two-fifths of the sixth graders thought .42 was greater than .5. When asked to indicate the largest or the smallest of a set of decimal numbers, students once again seemed to disregard the decimal point. In one item they were asked to select the largest of the four numbers .3, .09, .385, and .1814. Only about one-third of either the fifth or sixth grade students at the end of the school year responded correctly to this question. The most frequent response was .1814, the correct response had the numbers been whole numbers.

Students also use whole number rules for zero when ordering decimal numbers. Many believe that if a zero is added to the left of a number, the value of the number is unchanged whereas if a zero is added to the right, the number increases in value. This is apparent in the item in which the students were to circle the larger of the pair .8 and .008 or to indicate if the numbers were equal. Approximately one-half of the fifth graders and one-third of the sixth graders at the end of the school year thought the two numbers were equal.

Common Errors in Representing a Decimal

Students were asked to represent a decimal on a number line and as part of a region. The students were asked to represent a number expressed in tenths on a number line divided into tenths, a number expressed in tenths on a number line divided into fifths, and to represent .5 on a number line divided into units. Students had little difficulty responding correctly when the number line was divided into the same units as the number (approximately three-fourths of the students responded correctly in the Spring testing) but encountered great difficulty if the number line were divided into fifths or units. The students were asked to represent the number .3 on a number line divided into fifths. Only 6% of the fifth graders and 15% of the sixth graders responded correctly to this question. The most common error was counting divisions obtaining a response of .6. Considering a point greater than one-half the distance between 0 and 1 to represent the number .3 indicates the students are not attaching any meaning to the number .3.

Similar difficulties arose when asked to represent a decimal number as part of a region. Those items in which the number of divisions corresponded to the number of places in the number (10 with a number expressed in tenths and 100 with a number expressed in hundredths) were the easiest for the students but if the region was not divided appropriately, the students encountered great difficulty. Although 88% of the students at both grade levels responded correctly when asked to represent .7 on a region divided into tenths, only 58% of the fifth graders and 42% of the sixth graders could correctly represent .4 on a region divided into hundredths on the Spring testing.

Common Errors in Writing a Decimal for a Fraction or Conversely

These items proved to be very difficult for the students. Even though traditional instruction places heavy emphasis on the connection between fractions and decimals (and most of the fifth and all of the sixth grade students had some form of traditional instruction in decimals) the items still proved to be difficult. For example only 40% of the fifth graders and 65% of the sixth graders could write a decimal number for the fraction 3/100 and only 56% of the students at either grade could write the corresponding decimal for 83/100 at the Spring testing. Apparently the students are not connecting these two domains.

Common Errors on Place Value Items

The students were asked to write a number 10 or 100 times as large as a given number or one-tenth and one-hundredths as large as the given number. Only 18% of the students at either grade level could write a number one-tenths as large as .3 at the end of the school year. Only one-third of the sixth graders and one-fourth of the fifth graders could write a number ten times as large as .2 at the Spring testing. The most general approach to finding a number one-tenth as large was to divide the number by 10. Similarly, when asked to find a number 10 or 100 times as large as a given number the most common approach was to actually multiply the number by 10 or 100, indicating an understanding of the question but lack of understanding of base ten notation.

Results of Interviews

The intent of the assessment tasks was to measure students use of semantic analysis to deal with decimal symbols. In other words, we were as interested in the processes students used to solve the tasks as their answers. Consequently, all tasks were administered in individual interviews and students were asked to explain how they found the answers, how to teach the procedures to friends who had missed class, etc.

Two kinds of tasks were used--direct measures and transfer measures. Direct measures are tasks that assess students' use of the instructed processes in a completely straightforward manner. In this case these were tasks that involved representing written decimal numbers with blocks and conversely, and solving symbolically presented addition and subtraction problems. The computation problems were "ragged" decimal problems (e.g., 1.3 + .25) because previous work shows that such problems discriminate most clearly between students who use semantic analyses and those who recall and execute syntactic rules (Hiebert & Wearne, in press-a, in press-b).

Transfer measures are those that assess the instructed processes in novel contexts. The tasks may involve processes in addition to those instructed, but it is exactly these kinds of tasks that are essential to evaluate the flexibility and robustness of the target processes (Belmont & Butterfield, 1977; Greeno, Riley, & Gelman, 1984). We chose tasks that, based on previous work, either discriminate well between semantic analyzers and syntactic rules appliers (in terms of error patterns) or are especially difficult unless one engages in semantic analysis (Hiebert & Wearne, in press-b). Examples of tasks are: representing with blocks a written number with thousandths (instruction dealt only with hundredths but the generalizable partitioning process was demonstrated); choosing the larger of two numbers (e.g., .8 and .34); and writing a given number in common fraction form and conversely. None of these types of problems were included in the instructional activities. The tasks are found in Appendix C.

Data Collection Procedures

Nine students in grade four and ten students in each of grades five and six participated in the special instruction. The students represented a mix of achievement, racial, and gender groups as described earlier. The fourth-grade students had received no prior instruction on decimals, some of the fifth-grade students had been introduced to decimals previously, and sixth-grade students had been taught to compute with decimals.

Just prior to the instruction, the students were interviewed individually by one of the principal investigators and given the full set of direct and transfer measures. The interviews were standardized except for the follow-up questions that aimed to elicit students' descriptions of how they solved each problem. Questioning continued until the solution process was described completely or until it was clear that no more information was forthcoming. Some items asked students to describe to a friend who had missed class how to solve a problem. A coding form was used to record responses, and the interviews were audio-taped and transcribed for analysis.

Each grade-level group of students was instructed by one of the authors. The nine activities were completed in seven to nine days over a two to three-week period, depending on the group. The style of instruction emphasized student participation, instructor and peer feedback, and instructor monitoring to ensure active attention. About six weeks after instruction, students were interviewed again and presented with a variety of tasks, some of which were alternate forms to those in the pre-instruction interview. Once again, interviews were audio-taped and transcribed for analysis.

Results and Interpretation

Comparing performance of students before and after instruction provide an initial gross estimate of the effectiveness of the instructional sequence. Averaged across all students, performance on the three direct measures that were the same in both interviews rose from 25% correct responses before instruction to 89% correct after instruction. On the transfer measures that were common to both interviews, performance rose from 14% correct responses before instruction to 49% correct responses after instruction.

Although the figures on group performance suggest that instruction influenced performance, group data across all items masks the effects that are of most interest. Four considerations, based on the methodology outlined earlier, point to alternative forms of analyses. First, we are interested in changes that are best captured by looking at individuals rather than groups. Second, we are interested in changes in cognitive processes, not just changes in performance. Third, we are interested in changes on different types of tasks. The way in which instruction affects change may differ from task to task. Finally, we are interested in the effects of prior knowledge on changes over instruction. Some students had already received classroom instruction on decimals and those prior experiences may have influenced the effect of the instructional sequence.

Analyses that are sensitive to the four considerations presented above yield the results show in Table 2 and 3. Table 2 presents performance data and Table 3 presents process data.

Insert Tables 2 and 3 here

The tables show the number of students who changed behavior (either in performance or process) over instruction. The first column contains students for whom instruction was counterproductive. We expected zeros in this column. The second column represents students who simply maintained a high level of performance over instruction. The third and fourth columns are of most interest. The third column contains those students for whom instruction was ineffective; the fourth column contains those students for whom instruction was effective, students who improved their performance (Table 2) or who adopted the target cognitive process to complete the task (Table 3).

Results on correct/incorrect performance (Table 2) indicate that on most tasks, 50% or more of the students who could improve their performance did so. Furthermore, improvements were not limited to the items on which they received instruction but extended to the novel transfer items as well. Prior classroom instruction on decimals had no noticeable effect on performance change except on the transfer items of translating between decimals and common fractions. A plausible explanation for the poorer improvement rate on the no-previous-instruction students is that it is likely they had received less classroom instruction on common fractions (in addition to no instruction on decimals). Since many failures on these items resulted from misinterpretation of common fractions, the added classroom instruction on fractions may account for the better improvement rate of the previous-instruction students.

The cognitive process on which instruction focused is semantic analysis, or using the meaning of the written symbols to evaluate the problem and select a solution strategy. Table 3 shows the number of students who adopted this process over instruction and used it to solve each type of task. Evidence for using the meaning of the symbol comes from students' explanations for how they solved each task. Judgments were made by determining whether a student's explanation contained references to the positional values of the digits in the decimal fraction. To assess coder reliability, a second coder was trained and then given randomly selected interview transcripts of one student in each of the grades (4, 5, and 6). Interrater agreement on the process used was 59 agreements out of 60 decisions, or 98.3% agreement. In Table 3, the student is credited with using the target cognitive process if they described using it on at least one of each item type. The items are those shown in Table 2. The transfer block item in Table 3 had no parallel forms in the pre-instruction interview.

The relatively few cases in columns one and two suggest that most students, even those with previous classroom instruction, did not engage in semantic analyses to solve decimal tasks before our instructional sequence. After instruction, 50% or more of the students used such a process on most types of items. Excerpts from transcripts provide some insights into the nature of the cognitive change of some students and the absence of such change in others. Examples of such cases are Amanda and Arthur (not their real names) profiled in Table 4. Amanda is a fourth

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Insert Table 4 about here

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grader who had received no previous instruction in decimals; Arthur is a sixth grader who clearly has been taught some things about decimals. Both students were classified as average students by their teachers. The striking feature of the transcripts is their contrast. Amanda's processes for dealing with decimals change fundamentally; Arthur remains mostly unaffected by our instructional emphasis on semantic analysis.

It is when studying individual student's transcripts, such as Amanda's, that one can trace most directly the effects of particular instructional episodes on subsequent behavior. It is clear that Amanda acquired the meaning for the symbols presented in Activity 4 (refer to Appendix A for the activities) and then applied this meaning in computation situations (Activities 6-9) and on other tasks not instructed. The important point is that the processes Amanda described are attributable to the instructional activities.

Why some students seemed to "benefit" more from instruction than others is not clear, but some differences may be accounted for by the knowledge with which they entered instruction. Not all effects of prior knowledge are in the direction that might be expected, as a further analysis of Table 3 reveals. The proportion of students who changed processes is nearly equivalent between the no-previous-instruction and the previous-instruction groups, but there are two interesting exceptions. On the computation items 50% of the students who had been taught about decimals previously did not change the mental processes used to compute. In contrast, 21% of the students who had not received classroom instruction retained their original processes. This is the only item type for which the no-previous-instruction students clearly were influenced more by instruction than those who had received previous instruction. Based on the interview transcripts, many of the previous-instruction students had learned well-rehearsed routines for computing decimals. As indicated by Arthur's description above, these routines persisted in the face of our instruction. This finding corroborates those of others (Resnick & Omason, in press) and suggests that semantic analyses do not easily penetrate routinized procedures.

The second large difference in processes between the previous-instruction and no-previous-instruction groups occurred on the transfer task of representing a written number with blocks. During the instruction students worked with unit's, tenth's, and hundredth's blocks as referen's for the values of corresponding positions in decimal notation. The transfer tasks asked them to extend the partitioning notion to identify and use the thousandth's block correctly (for 2.503) and to extend the notion further to blocks that did not exist (for 1.0623). These items were not included in the pre-instruction interview so change cannot be measured. However, students who had received previous classroom instruction were more successful than those who had not in dealing with these tasks after our instruction. It is clear from interview transcripts that classroom instruction had, at the least, provided students with the labels for the new positions (e.g., thousandths). This information may have assisted students in transferring the values of decimal notation to create meanings for these positions.

A final question concerns the relationship between changes in cognitive processes and improved performance. Table 5 shows the number of students who employed semantic analyses to generate their answers and the number who did not. It must be remembered that students' descriptions

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Insert Table 5 about here

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provide the only evidence for semantic processes. They may have used the semantic meanings of the symbols and failed to report it. These cases are probably few since the interviewers vigorously questioned students to prevent such errors, but some classification errors may exist.

The data in Table 5 support the following interpretation. If students have learned procedures for generating answers (direct measures-computation), they can perform correctly without calling upon the semantic meanings of the written symbols. However, if they are presented with somewhat novel tasks (transfer measures) for which they have no ready algorithm, they must depend upon semantic analyses and such analyses usually are sufficient for completing these tasks.

Conclusions

Within the past few years, cognitive psychology has begun shifting its attention to the cognitive mechanisms that underlie changes in performance. Over the same time period, instructional science has begun considering the role of student cognitions as intermediary between instruction and learning. The interface of these two converging perspectives is the study of key cognitive processes in particular subject-matter domains, and the study of how such processes change over instruction. The study described in this paper illustrates this approach applied to mathematics, specifically applied to learning about decimal numbers.

The importance of looking at underlying cognitive processes when investigating the effects of instruction on learning is demonstrated in two ways. First, when instruction succeeds in instilling or altering students' processes, such effects often carry over into novel settings. That is, the effects of instruction are felt beyond the tasks directly taught. In this study, many students who acquired semantic meanings for decimal symbols used these meanings to solve problems that were quite different than those instructed. Such findings help to explain improved performance on transfer tasks when it occurs, and also support the notion that effective instruction is instruction that changes deeper cognitive processes not just surface performance.

A second demonstration of the importance of looking at cognitive processes is evident when considering the way in which the effects of instruction are influenced by students' prior knowledge. The findings of this study suggest that some forms of prior knowledge are facilitative while others are not. It appears that prior instruction that provided our students with factual or conceptual knowledge supported their attempts to engage in semantic analyses during (and after) our instructional sequence. However, prior instruction that encouraged the routinization of syntactic rules seemed to interfere with, and prevented the adoption of, semantic analyses on the affected tasks. This result adds empirical support to the claim that it is preferable to develop meanings before practicing syntactic routines (cf. Carpenter, in press; Resnick & Omanson, in press). Such claims, of course, are limited by the set of tasks and the same from which the results were collected.

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Table 1

Percent of Students Responding Correctly

Item*	Grade 5		Grade 6	
	Fall	Spring	Fall	Spring
Computation				
4.6 + 2.3	67	92	76	81
5.1 + 4.6	8	44	39	50
.64 + 2	11	32	46	54
4 + .3	18	38	46	56
6 + .32	12	40	48	56
5.3 + 2.42	10	54	50	64
.2 + 5	14	38	46	54
25 + .42	16	30	46	52
.78 - .35	63	86	91	85
4.7 - .24	4	36	14	44

*Similar item administered in the Spring

49 fifth graders in the Fall and 50 in the Spring testing

46 sixth graders in the Fall and 52 in the Spring testing

Table 1 continued

Item*	Grade 5		Grade 6	
	Fall	Spring	Fall	Spring
6 - .3	0	30	17	40
1.47 - .2	6	44	48	56
7 - .15	8	32	22	25
.60 - .36	57	70	87	83
.86 - .3	8	44	44	64
9.6 - 4	4	36	37	50
6 x .4	14	64	37	81
2 x 3.12	49	78	70	88
8 x .06	55	74	61	88
.34 x 2.1	2	44	13	60
.4 x .2	9	48	11	56
.05 x .4	6	42	15	64
.8 x 3	10	56	30	85
2.1 ÷ 3	35	78	50	73
.24 ÷ .03	16	20	4	64
.56 ÷ 7	6	44	11	56

Table 1 continued

Item*	Grade 5		Grade 6	
	Fall	Spring	Fall	Spring
.028 ÷ .4	6	14	11	40
48 ÷ .8	0	8	2	19
3 ÷ .6	0	10	2	19
.8 ÷ .02	0	10	4	29
1 ÷ 12.5	0	10	0	12
Ordering (Circle the Larger/Largest or Both if Equal)				
.02 .020	12	56	28	58
.42 .5	20	52	24	60
1/4 1.4	37	46	37	64
0.7 .07	51	46	59	66
.003 .030	76	72	74	69
3.5 3/5	31	42	39	64
.8 .008	26	52	56	65

Table 1 continued

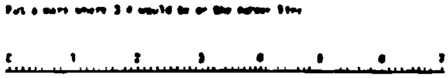
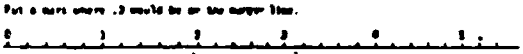
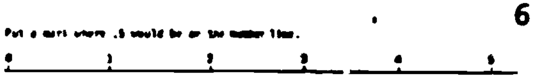
Item*	Grade 5		Grade 6	
	Fall	Spring	Fall	Spring
2.0 2	33	72	54	69
0.6 .6	49	76	70	88
.3, .09, .385, .1814	2	36	13	29
.899, 1.1, 1.062, 1.0095	12	40	46	56
Circle all numbers smaller than .06: .4, .065, .0096, .064	4	24	15	31
Representing a Decimal: Number Line				
	49	72	63	75
	2	6	6	15
	6	22	17	36

Table 1 continued

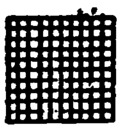
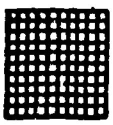
Item*	Grade 5		Grade 6	
	Fall	Spring	Fall	Spring
	Representing a Decimal: Area			
Shade .7 	59	86	72	88
Shade .08 	36	58	48	69
Shade .4 	26	58	24	42

Table 1 continued

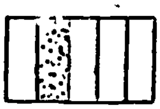
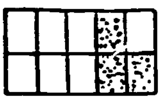
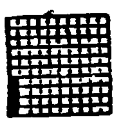
Item*	Grade 5		Grade 6	
	Fall	Spring	Fall	Spring
Writes Decimal for a Representation				
	26	34	24	25
	25	52	30	64
	12	42	10	50

Table 1 continued

Item*	Grade 5		Grade 6	
	Fall	Spring	Fall	Spring
Writes Decimal for Fraction or Fraction for Decimal				
.8	2	46	13	72
.07	3	48	11	67
1.6	0	22	6	54
4/10	9	58	22	75
3/100	4	40	17	65
43/10	4	28	[illegible]	44
83/100	4	56	35	56

Table 1 continued

Item*	Grade 5		Grade 6	
	Fall	Spring	Fall	Spring
	Place Value			
Writes number 10 times as big as 437	35	56	56	69
Writes number 10 times as big as 36.58	6	20	6	31
Writes number 10 times as big as .2	2	24	9	31
Writes number one-tenth as big as 829	2	16	4	14
Writes number one-tenth as big as 4	6	26	13	22
Writes number one-tenth as big as .3	8	18	9	17
Circles number nearest to .16				
.02, 20, 2, .2, .01, 1	12	36	24	33

Table 2

Number of Students Who Performed Correctly and Incorrectly Before (B) and After (A) Instruction

	Had Previous or Concurrent Instruction in Decimals[a]				Had No Previous Instruction in Decimals[b]			
Task	B-cor/ A-inc	B-cor/ A-cor	B-inc/ A-inc	B-inc/ A-cor	B-cor/ A-inc	B-cor/ A-cor	B-inc/ A-inc	B-inc/ A-cor
				Direct Measures				
Represent (B).32 (A) 1.04 with blocks	0	3	1	11	0	0	4	10
Compute:								
(B)1.3 + .25 (A)2.3 + .62	0	7	0	8	0	1	0	13
(B,A)5 + .3	0	8	2	5	0	3	3	8

Table 2 continued

	Had Previous or Concurrent Instruction in Decimals[a]				Had No Previous Instruction in Decimals[b]			
Task	B-cor/ A-inc	B-cor/ A-cor	B-inc/ A-inc	B-inc/ A-cor	B-cor/ A-inc	B-cor/ A-cor	B-inc/ A-inc	B-inc/ A-cor
				Transfer Measures				
Select larger (B) .5, .42 (A) .8, .34	0	4	6	5	1	3	4	6
Write fraction for (B).7, (A).5	0	2	6	7	0	0	9	5
Write decimal for (B) 6/100 (A) 8/100	2	1	6	6	0	0	11	3

[a] $\underline{n}$ = 15 [b] $\underline{n}$ = 14

Table 3

Number of Students Who Used Place Value Meaning to Perform Tasks Before (B) and After (A) Instruction

Task	Had Previous or Concurrent Instruction in Decimals[a]				Had No Previous Instruction in Decimals[b]			
	B-yes/ A-no	B-yes/ A-yes	B-no/ A-no	B-no/ A-yes	B-yes/ A-no	B-yes/ A-yes	B-no/ A-no	B-no/ A-yes
Direct Measures								
Represent written number with blocks	0	4	1	10	0	0	5	9
Compute	0	1	7	7	0	0	3	11

Table 3 continued

Task	Had Previous or Concurrent Instruction in Decimals[a]				Had No Previous Instruction in Decimals[b]			
	B-yes/ A-no	B-yes/ A-yes	B-no/ A-no	B-no/ A-yes	B-yes/ A-no	B-yes/ A-yes	B-no/ A-no	B-no/ A-yes
				Transfer Measures				
Order two decimal numbers	0	1	4	10	0	1	4	9
Translate between decimals and fractions	0	1	4	10	0	0	8	6
Represent 1.503 and 1.0623 with			A-no	A-yes			A-no	A-yes
blocks			2	13			7	7

[a] $\underline{n} = 15$ [b] $\underline{n} = 14$

Table 4

Descriptions of Solution Processes Given by Two Students

Task	Before Instruction	After Instruction
	Amanda - Cognitive Change Over Instruction	
Select Blocks (pre) .32 (post) 1.04	1 tenth's block, 2 hundreth's blocks: This (tenth) is 30 and 2 more.	1 whole block, 4 hundredth's blocks: This is a whole and these are four hundreths. (Why are they called hundredths?) Because there are a hundred of them in a whole.
Compute (pre) 1.3 + .25 (post) 2.3 + .62	1/3: Because this (the 3) is one-third bigger than this (the 1 in 1.3)	2.3 .62 2.92 You first put down the 2 because there's nothing to add to it and then you add the 3 and the 6 and you get 9 and you bring down the other 2. (Why don't you add the 6 to the 2 (of 2.3)?) Because this (6) is a tenth and that (2) is a whole.

Table 4 continued

Task	Before Instruction	After Instruction
(pre and post) 5 + .3	2: The 5 has no point and it's 2 bigger than 3.	5.3: Here's the 5 and you put the 3 and you get 5 here and 3 here.
Select larger (pre) .5, .42 (post) .8, .34	.42: It looks bigger.	.8: This is 8 tenths and this is 3 tenths so this is higher.
Write fraction for decimal (pre) .7 (post) .5	1/3: I don't know.	5/10: Five is in the tenth's place so that is five-tenths.
Write decimal for fraction (pre) 6/100 (post) 8/100	60: The two zeros can be by themselves.	.08: Eight hundredths is after tenths so I put it after the zero.

Table 4 continued

Task	Before Instruction	After Instruction
	Arthur - No Cognitive Change Over Instruction	
Select blocks (pre) .32 (post) 1.04	3 tenth's blocks and 2 hundredth's blocks: The big block would be before the decimal point and the little ones (thousandth's blocks) don't look right but maybe it could be 3 of these (tenth's blocks) and 2 of these (thousandth's) or 3 of those (hundredth's) and 2 of those (thousandth's). It's one of those, I know for sure.	1 whole block, 4 hundredth's blocks: One big one and some of these (hundredth's blocks) on top. These are ones (whole block) and we haven't used those yet (thousandth's block). (How much are these (hundredth's blocks) worth?) Tenths, no these (tenth's blocks) are tenths. I haven't done this for awhile. I forget.

Table 4 continued

Task	Before Instruction	After Instruction
Compute (pre) 1.3 + .25 (post) 2.3 + .62	1.30 / .25 / 1.55 You always match up the decimal points. The teacher said to always match them up. You have 1 decimal point 3 and the 2 and the 5 so I put the 0 in and added them up.	2.30 / .62 / 2.92 That's easy. I remember from last year that you do it that way. (Why do you line up the decimal points?) Because the teacher told me that last year and I got it right. I get good grades so it must be right.
(pre and post) 5 + .3	5.0 / 0.3 / 5.3 The teacher said 70 times you put the decimal point after the number. I think the teacher said you're not supposed to put 0 in but I do it anyway to feel more comfortable.	5.0 / 0.3 / 5.3 There really is a decimal here but they didn't make it and then I just added them up. Zero plus 3 is 3 and 5 plus 0 .s 5.

Table 4 continued

Task	Before Instruction	After Instruction
Select larger (pre) .5, .42 (post) .8, .34	.42: Four and a two is bigger than 5.	.8: I would say the lesser amount is greater in this case because of the decimals. Because if you lower it any more, it will become 1. I think in decimals, 8, the lower amount, is bigger than the higher number. (Is that always true?) Yes, unless the numbers are tied up.
Writes fraction for decimal (pre) .7 (post) .5	7: I'm not sure about this.	5: Don't say I did that wrong. It's been a long time and I forget really.

Table 4 continued

Task	Before Instruction	After Instruction
Write decimal for fraction (pre) 6/100 (post) 8/100	6.100: I remember that from the test. Cause before you showed 1.2 and that was 1/2 (referring to his first response on the inter-view when asked to represent a number with blocks) so I just reversed it. For 1,2, I would just put the 1, then a line, and then put the 2 on the bottom.	8.100: I forgot how to do this. Why I did that, I do not know. (Do you have any ideas?) I would say the higher number goes before the decimal.

Table 5

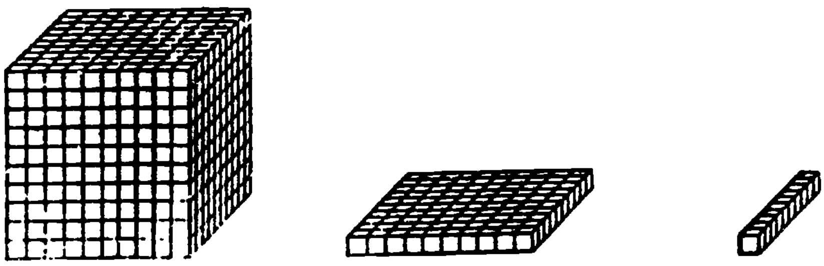
Number of Students Classified by Meaning Use and Correctness After Instruction[a]

Task	Did Not Use Place Value Meaning		Used Place Value Meaning	
	Incorrect	Correct	Incorrect	Correct
	Direct Measures			
Compute				
2.3 + .62	0	12	0	17
5 + .3	4	11	0	14
Select larger				
.8, .34	11	4	3	11
Write fraction for .5	11	4	3	11
Write decimal for 8/100	17	0	2	10

[a] n = 29

Figure Caption

Figure 1 Values and symbols assigned to base-ten blocks.



one	tenth	hundreth
1	.1	.01

APPENDIX A

GROUP INSTRUCTION

Activity 1

Materials: Base 10 blocks (ones, tenths, and hundredths)

The students are introduced to the base 10 blocks in this activity. They use them to represent a verbally stated decimal number (through tenths) and to verbally state the decimal number corresponding to a given block representation, including representations requiring regrouping.

1. Give the students the blocks and have them look for relationships among them (e.g., how big the different size blocks are compared to each other, how many of one size block it takes to make the next size). Discuss with them what they have discovered.

   Tell the students we are going to use the blocks to represent (show) numbers. The large block is the one's block so if we wanted to show 2 using the blocks, we would put out two of the large blocks.

   Ask the students which block would show tenths if the large block is the one's block. Lead the students to the answer, if they are having difficulty, by asking them what does a tenth mean (one part of something that has been divided into 10 equal parts), which of the blocks seems to fit that description, etc.

   Show a representation (e.g., 1 and 4 tenths) and have the students describe verbally how much is shown. Do additional examples (e.g., 5 tenths, 2 and 3 tenths).

   Ask the students to represent a specified amount (e.g., 6 tenths). Do additional examples (e.g., 1 and 3 tenths).

   Have the students work in small groups in which they either show a representation and the other children tell how much is shown or state an amount and the other children represent it.

2. Ask the children to construct 1 and 14 tenths with their blocks. Ask the children if there is another way to show the same amount. Lead the children to the response of 2 and 4 tenths (by showing 10 tenths is the same as one whole) and 24 tenths. Do other examples (e.g., 2 and 12 tenths, 1 and 6 tenths).

   Ask the children to show 17 tenths. Ask how many different ways they could show the same amount. Do the same for 1 and 4 tenths.

   Have the children work together in small groups doing examples in which they show a specified amount in as many ways as possible or, when shown a specified amount, state how much is shown in as many ways as possible.

Activity 2

Materials: Base 10 blocks (ones, tenths, and hundredths)

In this activity, the symbol for decimals is introduced. The students construct block representations of given decimal numbers through tenths and write the decimal corresponding to a given block representation, including representations requiring regrouping.

1. Tell the students you are going to write a number using a decimal point. Show the students 2 and 3 tenths with the blocks. Have them tell you how much is shown. Ask them to guess how you would write this number using a decimal point. Then show the students how it is written stressing that the decimal point separates the wholes from the fractional part of a whole.

   Give the students examples with blocks and ask them to write the corresponding nuber (e.g., 1 and 4 tenths, 7 tenths). Have the children read their answers. Do not use "point" when reading the answers but rather 1 and 7 tenths, for example.

   Give the students decimal numbers in written form (e.g., 1.6, 0.7, 2.3) and have them read the answers and then represent each of them with blocks.

   Have the students work in small groups in which they either write a decimal and the others represent it or a child shows a representation and the others write the corresponding number. Ask the children to read their numbers aloud when they write them or represent them with blocks.

2. Show the children 13 tenths with blocks. Have the children construct this with their blocks. Ask the children to tell you how much is shown. Lead the children to the response "one and three tenths" and then ask them to write the number using a decimal point. Do other similar examples (17 tenths, 4 tenths, 1 and 16 tenths).

3. Show the children a number (e.g., 1.2) and ask them to represent this number in as many written ways as possible (1 and 2 tenths, 12 tenths). Do other similar examples. Ask the children to suggest examples.

Activity 3

Materials: Base 10 blocks (ones, tenths, hundredths, and thousandths)

The students are introduced to the hundredths place in this activity. The students state the decimal number corresponding to a given representation or, when given a stated decimal number involving tenths and hundredths, constructs the appropriate representation.

1. Review the previous work with tenths including reading a decimal number, representing a decimal with blocks, and writing a decimal.

   Ask the students which of the blocks is a tenth of the tenth's piece. Ask what part of the one's block is this new piece. Lead the students to see that the piece is one-hundredth of the one's block. Ask the students what they think this new piece should be called.

   Show the students a block representation (1 and 2 tenths, 3 hundredths) and ask them to state verbally how much is shown. (Use the phrasing "one and two tenths, three hundredths.") Do other examples (e.g., 3 tenths, 2 hundreths; 1 and 6 hundredths).

   Ask the students to represent a verbally stated quantity with blocks. (e.g., 1 and 2 tenths, and 4 hundredths; 2 and 8 hundredths).

   Have the children work in small groups in which one child shows a given representation with blocks and the others tell how much is shown or a student states a quantity verbally and the other represent it with blocks.

2. Show the students 2 tenths with blocks. Ask them how much is shown. Have them show this with their blocks. Ask them to show the same amount using the hundredth's pieces. Ask the children to show 35 hundredths in as many ways as possible. Have them describe verbally each block representation. Talk about the different ways you can say the same amount (e.g., 3 tenths, 5 hundredths; 35 hundredths; .2). Do similar examples in which either an amount is stated verbally and the students represent with blocks it or they are shown a representation with blocks and they try to show the same amount using other combinations of the blocks, verbally describing each combination.

Activity 4

Materials: Base 10 blocks (ones, tenths, hundredths, and thousandths)

The students write decimal numbers through hundredths in this activity.

1. Review previous work by showing the students a block representation involving ones, tenths, and hundredths and asking them to state verbally how much is represented. Also ask the students to represent with blocks a verbally stated quantity (e.g., 2 and 4 tenths; 7 hundredths; and 3 hundredths).

2. Show the students 2 and 3 tenths, 7 hundredths with blocks. Ask them to tell you how much is shown. Ask them to write a decimal number showing this amount. If the students introduce two decimal points (one in the usual position, and one separating tenths from hundredths), retain the second point but use a big dot for the true decimal point. Have the students read their numbers and talk about the two ways the number could be read (2 and 3 tenths, 7 hundredths; 2 and 37 hundredths). Show the students additional block representations (e.g., 1 and 5 tenths, 3 hundredths; 3 and 7 hundredths; 8 tenths, 5 hundredths; 9 hundredths) and ask them to write the corresponding decimal number and to read their numbers. Let the children continue to use two decimal points at this time with the true decimal point being larger than the other one.

   Give the students a written decimal number to represent with blocks. Have the students read their answers.

3. Show the students examples in which regrouping is necessary. For example, show the students 1 and 2 tenths, and 18 hundredths with blocks. Have the students construct this representation with their own blocks. Ask the students to write the corresponding number. Note how the students are recording their numbers and if they are using intermediate steps such as 1.218 or 1.2.18 and then writing 1.38 or 1.3.8.

   Lead students who do not regroup (students whose numbers are either 1.218 or 1.2.18) by asking them if there is another way they could write the same number. Lead the students to regroup the blocks and write the associated decimal.

   Do other similar examples such as 3 and 14 hundredths; 16 tenths, 3 hundredths; 2 and 12 tenths, 15 hundredths). Have the students read their final answers.

Activity 5

Materials: Meter sticks and masking tape

The students divide meter sticks into tenths and hundredths. The students use the meter sticks to determine and represent lengths to the nearest tenth or hundredth of a meter.

1. Give the students meter sticks with masking tape covering both sides of the sticks. The tape should cover the sticks to the extent that the markings on the stick are not visible through the tape. Write "one meter" on the masking tape covering the side with English units (if both sides of the stick are not in metric units).

2. Have the students show how long 3 meters would be using their sticks.

3. Ask the students to show a length of 1.6 meters (1.6 is written on a card). Lead the students to realize they must divide the meter stick into 10 equal parts. Have the students make marks on the meter sticks corresponding to the 10 divisions. Then have the students show the length of 1.6 meters. Next have the students measure something in the room correct to the nearest "tenth". Have the students find other measurements, such as 0.7 meters, and measure other objects in the room correct to the nearest "tenth" of a meter.

4. Ask the students to show a length of 1.36 meters. Again, lead the students to realize they must divide their tenths into 10 equal parts to show hundredths. Have them do this for two of the tenths (including between .3 and .4) and then have them show 1.36 meters. Have the students measure something in the room correct to the nearest "hundredth" in which the students will put in the marks necessary to obtain hundredths. Ask the students to show a length of .84 meters. Have the students measure something correct to the nearest "tenth" and to the nearest "hundredth".

5. Ask the students what they would need to do to show a length of 1.364 (written on a card). As before, lead the students to realize they must divide each "hundredth" into "thousandths". Then ask what they would need to do to show 1.3648. Lead the students to see that successive divisions of each unit into 10 equal parts yield the next unit.

6. Now have the students remove the masking tape from the side of the meter stick where they made their marks. Do this in such a way the students can compare their markings on the masking tape to the tenths and hundredths on the meter sticks. Discuss the meanings of the numbers on the meter

sticks. For example, ask the students what they think the 80 on the meter stick refers to (80 hundredths) and how do we write this as part of a meter (.8 or .80).

7. Have the students show a length of .7 and .54 meters. Then have the students work in pairs to find the lengths of various objects. Have the students find the length of something (the room, for example) correct to the nearest meter, nearest tenth of a meter, the nearest hundredth of a meter. Have the students work in pairs to find their heights correct to the nearest tenth and hundredth of a meter.

Activity 6

Materials: Base 10 blocks (ones, tenths, and hundredths)

The students use the blocks to solve addition and subtractions sentences in which regrouping is not necessary.

1. Write the numbers 1.3 and 2.4. Have the students read the numbers. Ask the students to make a representation of each of the numbers with their blocks. Then ask them to write the number which tells how much there is altogether. When the students have finished, show them the sentence 1.3 + 2.4 and ask if this is what they did. Talk about the meaning of the sentence (adding the quantities 1.3 and 2.4 together). Have the students discuss what they did to find out how much there was altogether (added tenths to tenths and ones to ones). Do another similar example, this time showing the sentence (e.g., 2.3 + 1.2) and telling the students to first represent each number with blocks, combine them to determine how much there is altogether. Have the students read their answers. Stress how like parts were combined.

   Do an example involving hundredths (1.36 + 1.23) in which the students first represent each number with blocks, combine them, and then write their answer. Have the children discuss what they did to find the answer (combined liked things).

   Do other examples (e.g., 2.3 + 1.46, 4 + .7, 1.03 + 1.6) in which tenths, hundredths, or the ones place may be missing but no regrouping is necessary. Continue to stress how like things are combined to determine how much there is in the sum of the two quantities.

   Students who are still using two decimal points may continue to do so in this activity.

2. Show the students the number 2.7, have them read the number, and then represent it with their blocks. Ask the students to write the number which tells how much you would have left if you took away 1.2, showing the students this number. After the students have completed their work, show the students the sentence 2.7 - 1.2 ask them if this is what they did or alternately, ask the students to write a sentence which tells what they did. As in the case of addition, have the students discuss what they did to find the answer, stressing that you take tenths away from tenths and whole blocks from whole blocks. Do another similar example (e.g., 3.4 - .2).

   Give the students an example containing hundredths. Show the students the sentence 2.58 - 1.32. Have the students solve the sentence using the blocks. Tell students that are having difficulty to represent the first number (2.58) and then take away the second number. When the students are finished, have them read their answers. Then talk about what they did to find the answer, encouraging them to state they took hundredths away from hundredths, tenths away from tenths, etc.

Do other similar examples in which regrouping is not necessary but include examples in which some of the places may be omitted, for example 2.79 - 1.4, 1.46 - .03.

3. Give the students examples of both addition and subtraction sentences in which regrouping is not necessary. Have the students read the sentences and their solutions. Continue to emphasize that like parts are being combined, that is, hundredths with hundredths, tenths with tenths, and ones with ones.

4. Show the students the written number .23. Ask if you could write this number as the sum of two quantities, that is, .23 = .2 + __, or .23 = .1 + .13. Repeat the questions for the number 4.6.

Activity 7

Materials: Base 10 blocks (ones, tenths, and hundredths)

The students solve addition problems in which regrouping is necessary.

1. Review previous work by writing the sentence 2.3 + 1.41. Have students read the sentence, represent each of the numbers with blocks, and then write their solution. Have the students read their solution. Encourage the children to discuss what they did to find the answer (combine like quantities).

2. Show the students a sentence in which regrouping is necessary (1.3 + 1.8). Have the students represent each of the quantities with blocks and write the number that tells how much there is altogether. Have the students read their answers. Ask the students who have written 2.11 if there is another way to write their answer. Students who are using two decimal points may have written 2.11 and read "two and 11 tenths." Ask the student if there is another way they could write their answer. Assist the students to see that 10 tenths is the same as one whole.

   Give the students additional examples in which regrouping is necessary, both in the tenths and the hundredths places, for example: .7 + 2.8, 1.79 + .18, 1.66 + 1.48, 1.05 + .05, 1.67 + .33, 3.4 + .6, 2.3 + .78, 2 + .4, 5 + .63.

Activity 8

Materials: Base 10 blocks (ones, tenths, and hundredths)

The students solve subtraction problems in which regrouping is necessary.

1. Show the students the sentence 1.2 - .8. Have the students read the sentence. Have them find the answer using their blocks by first representing 1.2 and then taking away .8. When the students see they cannot take away 8 tenths from 2 tenths, ask the students what they could do with their blocks in order to make the subtraction. Lead the students to see that if they exchanged the one for 10 tenths, then they would be able to perform the subtraction. Have the students read their solution. Discuss with them what they did.

   Do other similar examples in which regrouping is necessary: 2.43 - 1.16, 1.44 - .56, 2 - .4, 2 - .82, 1.08 - .42, 3.2 - .47.

2. Provide additional examples of addition and subtraction, some involving regrouping and some not. Have the students work all of the examples with their blocks, read and write their solutions. Continue to emphasize that like things are combined.

   At this point, students who are still using double decimal points should be encouraged to write their final answer using a single decimal point. The double points could be used to assist in regrouping, however.

Activity 9

Materials: Base 10 blocks (ones, tenths, and hundredths)

The students write horizontal sentences in vertical format and use the blocks to validate their solutions.

1. Give the students the sentence 2.3 + 1.5. Have the students write what they think the answer is and then validate their "guess" using the blocks. Ask them how they got their original answer. Discuss how we add tenths to tenths and ones to ones.

   Show the students the sentence 1.42 + .5. Have them try to write their answer without using the blocks and then validate their answer with the blocks. Discuss how they got the answer. Ask how they knew to combine to 4 and the 5 to get 9 tenths (both were tenths).

   Refer back to the first sentence (2.3 + 1.5) and ask the students to write the sentence vertically, that is, one number over the other in such a way as to show which numbers you add together. Do the same for the sentence 1.42 + .5, stressing that you write the numbers to show which ones you combine.

   Give the students the sentence 1.6 + .34 and have them write the problem vertically, find their answer, and then validate their answer using the blocks. Again stress that you place one number under the other to show which digits you combine (e.g., you combine the 6 and 3 since they are both tenths).

   Show the students the sentence 4 + .3, have them write the problem vertically, then validate their solution using the blocks.

2. Give the students the sentence 2.8 - 1.3, have them write the problem vertically, and then validate their answer using the blocks. Discuss why you put the 3 under the 8 and the 1 under the 2 (because you are subtracting like things, tenths from tenths and ones from ones).

   Show the students the sentence 2.5 - .4, have them write it vertically, and then validate their solution. Continue to stress that you put like kinds of quantities in the same columns, that is, you combine tenths with tenths and hundredths with hundredths.

   Show the students the sentence 2.46 - 1.3, have them write the problem vertically, and then validate their solutions with the blocks. Do other examples such as 1.57 - .02, 5 - .3, etc., in which they first write it vertically, solve it, and then validate their solutions with the blocks.

APPENDIX B

GROUP TEST

ADD

4.6 + 2.3 = ________

5.1 + .46 = ________

.64 + 2 = ________

4 + .3 = ________

ADD

6 + .32 = ________

5.3 + 2.42 = ________

.2 + 5 = ________

25 + .42 = ________

SUBTRACT

.78 - .35 = ________

4.7 - .24 = ________

6 - .3 = ________

1.47 - .2 = ________

SUBTRACT

7 - .15 = ________

.60 - .36 = ________

.86 - .3 = ________

9.6 - 4 = ________

MULTIPLY

6 x .4 = ________

2 x 3.12 = ________

8 x .06 = ________

.34 x 2.1 = ________

MULTIPLY

.4 x .2 = ________

.05 x .4 = ________

.8 x 3 = ________

DIVIDE

2.1 ÷ 3 = ________

.24 ÷ .03 = ________

.56 ÷ 7 = ________

.028 ÷ .4 = ________

DIVIDE

$48 \div .8 =$ ________

$3 \div .6 =$ ________

$.8 \div .02 =$ ________

$1 \div 12.5 =$ ________

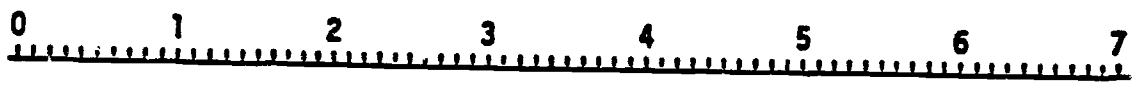
Circle the larger number in each pair of numbers. If the two numbers are equal, circle both numbers.

.02	.020
.42	.5
$\frac{1}{4}$	1.4
0.7	.07
,003	.030

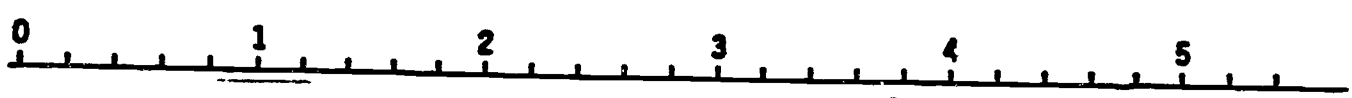
Circle the larger number in each pair of numbers. If the two numbers are equal, circle both numbers.

3.5	$\frac{3}{5}$
.8	.008
2.0	2
0.6	.6

Put a mark where 3.4 would be on the number line.



Put a mark where .3 would be on the number line.



Put a mark where .5 would be on the number line.

Circle the largest number.

.3 .09 .385 .1814

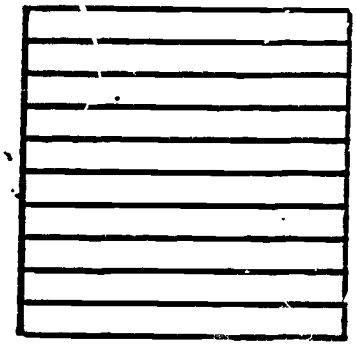
Circle the largest number.

.899 1.1 1.062 1.0095

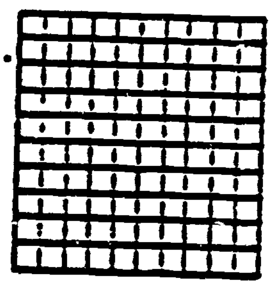
Circle all the numbers <u>smaller</u> than .06

.4 .065 .0096 .0666

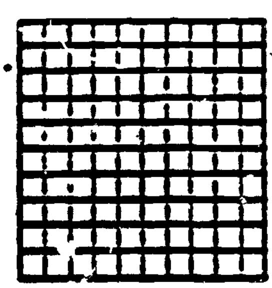
Shade .7 of the figure.



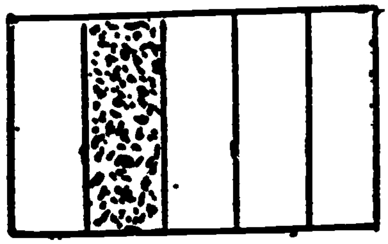
Shade .08 of the figure.



Shade .4 of the figure.



Write the <u>decimal</u> that tells what part of the figure is shaded.



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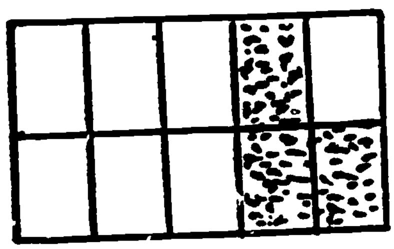
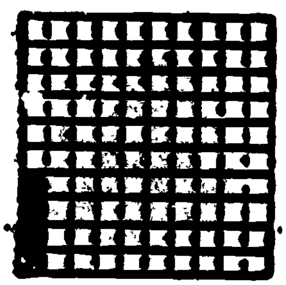
Write the <u>decimal</u> that tells what part of the figure is shaded.



____

Write the <u>decimal</u> that tells what part of the figure is shaded.



____

Write a __fraction__ that is equal to .8 _______

Write a __fraction__ that is equal to .07 _______

Write a __fraction__ that is equal to 1.6 _______

Write a decimal that is equal to $\frac{4}{10}$ ________

Write a decimal that is equal to $\frac{3}{100}$ ________

Write a decimal that is equal to $\frac{43}{10}$ ________

Write a decimal that is equal to $\frac{83}{100}$ ________

Write a number that is ten times as big as 437 ________

Write a number that is ten times as big as 36.58 ________

Write a number that is ten times as big as .2 ________

Write a number that is one-tenth as big as 829 ________

Write a number that is one-tenth as big as 4 ________

Write a number that is one-tenth as big as .3 ________

Circle the number nearest to .16

.02

20

2

.2

.01

.1

APPENDIX C

INTERVIEW

Interview 1

1. The base-ten blocks are on the table including the large cube, the flat, the long and the small cube.
   a. Show me this number (1.6 written on a card) using the blocks. You may use any of the blocks that you wish.
   b. Why did you pick those blocks?
   c. How much do you think each of your blocks is worth?
   d. If this block (large cube) is worth one, how much would this one (flat) be worth? Why?

2. Same materials as in Question 1.
   a. Show me this number (.32 written on a card) using the blocks. you may use any of the blocks that you wish.
   b. Why did you pick those blocks?

3. Student is given a sheet with three problems (if sixth grade) or two problems (if fifth or fourth grade) on it.

   I want you to try to do all of these problems. You can write as much as you want on this page.

   a. 1.3 + .25

      Tell me how you did the problem.

      Is there any other way we could do the problem?

      Would you get the same answer?

   b. 5 + .3

      Tell me how you did the problem.

c. 4.7 - .25

Tell me how you did the problem.

4.

a. Which number is the larger (.5 and .42)? Why?

b. Which number is the smaller (.06 and .065)? Why?

5. Student is given a number line divided into fifths.

Put a mark where this number (.7) would be on the number line. Why did you pick that place?

6.

a. Write a fraction to go with this number (.7).

b. How did you know what to write?

7.

a. Write a decimal to go with this fraction (6/100).

b. How did you know what to write?

8. If someone made a mistake on this problem (2.3 + .76), what do you think it would be?

Interview 2

1. The base-ten blocks are on the table including the large cube, the flat, the long, and the small cube.
   a. Show me this number (1.04) using the blocks. You may use any of the blocks that you wish.
   b. Why did you pick those blocks?
   c. Why is that block called a _____?

2. Same materials as in Question 1.
   a. Show me this number (2.503) using the blocks. You may use any of the blocks that you wish.
   b. Why did you pick those blocks?
   c. What is the smallest block called? Why?

3. Same materials as in Question 1.
   a. Show me this number (1.0623) using the blocks. You may use any of the blocks that you wish.
   b. (If realized not enough blocks) What would the next block look like?

      How could we make the next block?

      What would you call the next block?

4. Student is given a sheet with four problems on it.

   a. 2.3 + .62

      Tell me how you did the problem.

      (If student says to line up the decimal points) Why would you do this?

   b. 5 + .3

      Tell me how you did the problem.

   c. 6.8 - .15

      Tell me how you did the problem.

      If someone were to do this problem wrong, how would they do it?

      What would you say to help the person do the problem correctly?

   d. 7 - .4

      Tell me how you did the problem.

5. The student is given a sheet with pairs of numbers and asked to circle the larger of each pair. On a subset of these, the students is asked to justify their answer.

   a. Why did you circle this number (.8 and .34)?

   b. Why did you circle this number (.05 and .056)?

   c. Why did you circle this number (3.0 and 3)?

   d. Why did you circle this number (.4 and .40)?

6. The student is given a sheet with fractions written on it and asked to write the decimal number that went with the fraction, that is, to write the decimal number that has the same meaning or value.

a. 7/10

How did you know what to write?

b. 8/100

How did you know what to write?

c. 27/10

How did you know what to write?

7. Student is given a sheet with decimals written on it and asked to write the fraction that went with the decimal.

a. .5

How did you know what to write?

b. .03

How did you know what to write?

c. 4.3

How did you know what to write?

8. Student is given a number line divided into fifths.

a. Put a mark where this number (.3) would be on the number line. Why did you pick that place?