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ABSTRACT

The principal focus of this brief report is on findings and observations from relatively recent research on the learning of mathematics. The distinction between learning and the concepts of curriculum and instruction is first reviewed. Then Piaget's theory of intellectual development is discussed, with the conclusion that it does not provide a basis for determining readiness for mathematics instruction. The contribution of information processing theory is then noted. Four additional observations from contemporary research concern the following: the focus on individuals (versus groups), behavioral objectives, invention or construction of knowledge by young children, and metacognition. A list of references is included. (MNS)

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The Learning of Mathematics

Twenty years ago Macdonald (1965) pointed to the unfortunate fact (equally true years previously and years since) that "There is no consistently clear distinction in the use of much educational terminology One definition of *curriculum* may well turn out to be the same as the next definition of *instruction*, and this definition of *instruction* could quite likely be synonymous with another's definition of *teaching*" (p 2) Macdonald then characterized teaching " as the behavior of the teacher, learning as the change in learner behavior, [and] instruction as the pupil-teacher interaction situation" (p 6)

The distinction between teaching and learning, for instance, has been phrased in somewhat similar ways by others at later times. "By *teaching* I mean any activity on the part of one person intended to influence learning on the part of another" (Gage, 1978, p 14), and "Learning is a change in human disposition or capability, which persists over a period of time, and which is not simply ascribable to processes of growth" (Gagné, 1977, p 3)

In keeping with such distinctions, the principal focus of this present report is upon findings and observations associated with relatively recent research on the *learning* of mathematics and is not intended to embrace research on mathematics teaching and instruction

Closely related to these distinctions is another one stressed by Gage (1964) and by Bruner (1964) a *theory* of learning is descriptive rather than prescriptive, as is the case for a theory of teaching or a theory of instruction

Speaking of theories

Piaget and the Learning of Mathematics

Piaget's "theory" is a theory of *intellectual development* It is not a theory of learning or of teaching or of instruction or of curriculum—neither in general nor with respect to mathematics in particular But many investigations and much writing have been directed toward hypothesized implications of Piaget's theory for mathematics learning, teaching and instruction, for the nature and sequence of curricular content, and for classroom structure, organization and management! And concerns have been expressed regarding ways in which Piaget's theory has been applied to aspects of education in general (Sullivan, 1967) and to aspects of mathematics education in particular (Weaver, 1972)

Frequently it has been suggested that certain Piagetian tasks (conservation tasks, for instance) provide a good basis for determining students' readiness for learning one aspect or another of school mathematics But Hiebert and Carpenter (1982) have indicated that "The available research evidence suggests that . . . Piagetian tasks are not useful readiness measures [With respect to the question of whether] Piagetian tasks can be used to identify children who will be unable to benefit from instruction in mathematics, all the available evidence clearly suggests that the answer is no, many children who fail Piagetian tasks are able to learn mathematical concepts and skills" (pp 339-340)

Such evidence is not to be construed as refuting Piaget's theory of intellectual development, nor is it to be construed as reflecting unfavorably upon the nature of school mathematics. It simply indicates that a certain kind of "match" simply does not seem to exist And this is part of the reason why many persons now are turning away from, rather than looking toward, Piaget for help in dealing with issues associated with mathematics learning (and teaching and instruction)

A Different Direction

Kirby and Biggs (1980) indicated that "Cognition returned to psychology in the 1960s and flourished in the 1970s, permeating most areas of psychology Its metaphor, information processing, became dominant in that discipline" (p. xi)

More directly to the point, Groen and Kieran (1983) pointed out that "A few years ago, research on children's mathematics was dominated by Piaget To many in the field, the task was to extend Piaget's theory or reinterpret it . In contrast, the research described in this volume relegates Piaget to the background. Information-processing theory, broadly conceived, has replaced the Piagetian framework as a broad explanatory model The significance of information-processing theory in cognitive development has grown concomitantly with a retreat from the Piagetian framework" (pp. 351-352)

Within the contemporary context of "cognitive science" emphasis is placed upon "understanding" and "comprehension"—recalling Brownell (1935) and his exposition of the "meaning theory" 50 years ago! The learning of mathematics, and research associated with such learning, is more and more commonly being *described* or *explained* in relation to a *system* that includes provision for the intake of information, for its "chunking" within a working memory, and for interaction there with other information stored within and retrieved from a long-term memory

It is not at all surprising that an information-processing system often is associated closely with, and described in terms related to, a computer or microcomputer system

Some Additional Noteworthy Observations

1 Contemporary research associated with the learning of mathematics *is much more likely to focus upon individuals than upon groups* That is to say, learning is an individual phenomenon, and although much learning takes place within the context of groups such as classes, the focus of attention is not upon the group/class but upon individuals.

2 Contemporary research associated with the learning of mathematics *looks beyond observable performance on so-called "behavioral objectives" for its data base* Romberg and Carpenter (in press) have indicated that "internal cognitive processes are acknowledged Rational task analysis, which is based on a logical analysis by experts, has evolved to empirical task analysis, which focuses on what children actually do when they solve mathematics problems" (p 852) Some of this "doing" may be observable in a student's overt actions, but much of the doing may be terms of a student's observable but self-reportable thinking.

3 There is a growing body of evidence that *young children invent or construct much of their own mathematical knowledge*, and that they come to school with some well-developed, although understandably immature, ways of dealing with various mathematical situations We do not seem to take full advantage of children's conceptions in our programs of mathematics teaching and instruction

4 The term *metacognition* refers to a person's awareness of and sensitivity to her or his own thought processes, and includes the ability to monitor and control (to some degree) such processes. There is growing evidence that *learning associated with problem solving is facilitated or enhanced by student's increased awareness of metacognitive aspects of the problem-solving process*. Also, there is evidence that some of the differences between expert and novice problem solvers may be attributed to differences in metacognitive skills

In Conclusion

Romberg and Carpenter (in press) believe that "We currently know a great deal more about how children learn mathematics than we know about how to apply this knowledge to mathematics instruction. Research is clearly needed to explore how knowledge of children's learning of mathematics can be applied to the design of instruction" (p 859) Furthermore, "Although the emphasis in research on learning has changed dramatically in the last 15 years, the connection between theories of instruction and theories of learning remains an issue" (p 851)

In addition to references already cited, you may wish to peruse these as they relate to some of the things just identified: Brainerd (1982), Carpenter, Moser, and Romberg (1982), Case (1983), Davis (1984), Ginsburg (1983), Good, Grouws, and Ebmeier (1983), Lesh and Landau (1983), Lester and Garofalo (1982), Resnick and Ford (1981), Shumway (1980), Silver (1985), and Steffe, von Glaserfeld, Richards, and Cobb (1983) These citations include aspects of teaching and instruction along with aspects of learning

REFERENCES

Brainerd, C J (Ed) *Children's Logical and Mathematical Cognition Progress in Cognitive Development Research.* New York Springer-Verlag. 1982

Brownell, W A "Psychological Considerations in the Learning and the Teaching of Arithmetic " In W D Reeve (Ed), *The Teaching of Arithmetic* (Tenth Yearbook of the National Council of Teachers of Mathematics, pp 1-31) New York Teachers College, Columbia University, 1935

Bruner, J S "Some Theorems on Instruction Illustrated with Reference to Mathematics" In E R Hilgard (Ed), *Theories of Learning and Instruction* (Sixty-third Yearbook of the National Society for the Study of Education, Part I, pp. 306-335). Chicago National Society for the Study of Education, 1964

Case, R *Intellectual Development A Systematic Reinterpretation* New York Academic Press, 1983

Carpenter, T P , J M Moser, and T A Romberg, (Eds) *Addition and Subtraction A Cognitive Perspective* Hillsdale, NJ Lawrence Erlbaum Associates, 1982

Davis, R B *Learning Mathematics The Cognitive Science Approach to Mathematics Education* Norwood, NJ Ablex, 1984

Gage, N L "Theories of Teaching " In E R Hilgard (Ed) , *Theories of Learning and Instruction* (Sixty-third Yearbook of the National Society for the Study of Education, Part I, pp 268-285), 1964 Chicago National Society for the Study of Education

Gage, N L *The Scientific Basis of the Art of Teaching* New York Teachers College Press, 1978

Gagne, R M *The Conditions of Learning* (3rd ed) New York Holt, Rinehart and Winston, 1977

Ginsburg, H P (Ed) *The Development of Mathematical Thinking* New York Academic Press, 1983

Good, T L , D A Grouws, and H Ebmeier *Active Mathematics Teaching* New York Longman, 1983

Groen, G , and C Kieran "The Many Facets of Piaget " In H P Ginsburg (Ed) , *The Development of Mathematical Thinking* New York Academic Press, 1983

Hiebert, J , and T P Carpenter "Piagetian Tasks as Readiness Measures in Mathematics Instruction A Critical Review " *Educational Studies in Mathematics*, 13 329-345, 1982

Kirby, J R , and J B Biggs, (Eds) *Cognition, Development, and Instruction* New York Academic Press, 1980.

Lesh, R., and M Landau, (Eds.) *Acquisition of Mathematics Concepts and Processes* New York Academic Press, 1983

Lester, F K , Jr , and J Garofalo, (Eds) *Mathematical Problem Solving Issues in Research* Philadelphia Franklin Institute Press, 1982

Macdonald, J B "Educational Models for Instruction—Introduction " In J B Macdonald and R R Leeper (Eds), *Theories of Instruction* (pp 1-7) Washington, DC Association for Supervision and Curriculum Development, 1965.

Resnick, L B , and W W Ford *The Psychology of Mathematics for Instruction* Hillsdale, NJ Lawrence Erlbaum Associates, 1981

Romberg, T A , and T P Carpenter "Research on Teaching and Learning Mathematics Two Disciplines of Scientific Inquiry " In M C Wittrock (Ed) , *Third Handbook of Research on Teaching* (pp 850-873), in press

Shumway, R J (Ed) *Research in Mathematics Education* Reston, VA National Council of Teachers of Mathematics, 1980

Silver, E A (Ed) *Learning and Teaching Mathematical Problem Solving Multiple Research Perspectives.* Hillsdale, NJ Lawrence Erlbaum Associates, 1985.

Steffe, L. P , E von Glaserfeld, J Richards, and P Cobb *Children's Counting Types Philosophy, Theory, and Application.* New York Praeger, 1983

Sullivan, E V *Piaget and the School Curriculum—A Critical Appraisal* (Bulletin No 2) Toronto Ontario Institute for Studies in Education, 1967

Weaver, J F "Some Concerns About the Application of Piaget's Theory and Research to Mathematical Learning and Instruction " *Arithmetic Teacher*, 19 263-270, 1972.

This digest was prepared by J F Weaver, Professor Emeritus, University of Wisconsin-Madison It provides information on the current status of thinking about how mathematics is learned. While it refers to the relationship of learning with several specific curriculum and instruction concerns, its primary focus is on how theories of learning mathematics have changed direction and the implications of such changes for teachers and others who study how children learn mathematics

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