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ABSTRACT

A questionnaire with 70 closed and 10 open questions was administered to 230 students enrolled in grades 9 through 12, the majority of whom were enrolled in Math 10 geometry courses. Aspects of the questionnaire dealt with: (1) attributions of success or failure; (2) students' comparative perceptions of mathematics, English, and social studies; (3) the nature of mathematics as a discipline; and (4) mathematics attitude. The results paint a disturbing picture of students' perceptions of mathematics as a whole. The data (which are tied closely to a series of empirical studies) suggest the resolution of contradictory patterns of data in other attitude surveys, where students simultaneously claim that mathematics is mostly memorizing but also that mathematics is a creative and useful discipline in which they learn to think. (Author/JN)

STUDENTS' BELIEFS ABOUT MATHEMATICS AND THEIR EFFECTS ON MATHEMATICAL PERFORMANCE: A QUESTIONNAIRE ANALYSIS

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Running head: Belief Questionnaire

Abstract

A questionnaire with seventy closed and ten open questions was administered to 230 students enrolled in grades 9 through 12, the majority of whom were enrolled in Math 10 geometry courses. Aspects of the questionnaire dealt with: attributions of success or failure; student's comparative perceptions of mathematics, English and social studies; the nature of mathematics as a discipline; and mathematics attitude. The results paint a disturbing picture of students' perceptions of mathematics a. a whole. The data--which are tied closely to a series of empirical studies--suggest the resolution of contradictory patterns of data in other attitude surveys, where students simultaneously claim that mathematics is mostly memorizing but that mathematics is a creative and useful discipline in which they learn to think.

Students' Beliefs A'out Mathematics and Their Effects
on Mathematical Performance:
A Questionnaire Analysis

The Context for this Study

This paper describes one of four components of a research project exploring student's understandings of formal mathematical procedures and the effects of those understandings on the students' mathematical performance. The major focus of these studies is on students' understanding of the role of formal argumentation ("proof") in geometry. The three other components of the project, which establish the context for this study, are the following:

1. The analysis of videotapes of problem solving sessions, and subsequent clinical interviews. In these sessions students were asked to work both proof problems and straightedge-and-compass construction problems, to discuss the relationships between them, and to explain why they had approached the problems that they had worked in the ways that they did.

2. The systematic observations of high school geometry classrooms. For the entirety of a school year one 10th grade geometry class (the "target class") was observed extensively, and videotaped periodically. The entire units on locus and constructions were videotaped and analyzed. Other classes dealing with the same and other mathematical subject matter were visited periodically, to determine the typicality of instruction in the target class.

3. The construction of detailed computational models of students'

hypothesis generation in straightedge-and-compass construction problems. As described in (Schoenfeld, 1983), these models provide detailed characterizations of students' geometrical empiricism.

As described below, these fine-grained studies yielded some consistent and disturbing evidence both regarding students' perceptions of the utility of mathematical argumentation and regarding the students' use (or lack of use) of mathematical deduction in problem solving situations. All of these studies, however, dealt with relatively small numbers of students; all used data gathering and analysis methods that have been considered subjective in nature. For those reasons the questionnaire described in this report was developed. The questionnaire permitted the gathering of objective data from a larger sample of students. This allowed for a comparison of the typicality of the target class's responses with the responses from other classes, and also allowed the researchers to situate the data obtained from this larger sample (n = 230) with regard to attitude data obtained from other, much larger studies such as the NAEP Secondary School Study (Carpenter, Lindquist, Matthews & Silver, 1983) and the Second International Mathematics Study (McNight, Travers & Dossey, 1985).

The Phenomena Explored in this Study

The issues at hand may be best introduced by a piece of anecdotal evidence. In early 1983 the author gave a seminar on mathematical cognition to an audience of about fifteen very talented undergraduate cognitive science majors, who had each put together their own interdisciplinary majors, heavily based in mathematics and computer science.

At the beginning of the seminar the students were asked to solve problems 1 and 2 below (see Figure 1), working as a group. They produced correct proofs in less than three minutes.

Insert Figure 1 here

The proofs were written on the blackboard in the seminar room, and were left there. The students were then asked to solve problem 3 (see Figure 2).

Insert Figure 2 here

Students came to the board and made the following conjectures, in order:

a. The center of the desired circle lies at the midpoint of PQ, where Q is the point on the bottom line that lies the same distance from V as P (see Figure 3a).

b. The center of the desired circle lies at the midpoint of the segment of the arc drawn through P that lies between the two given lines (see Figure 3b).

c. The center of the desired circle lies at the midpoint of the segment of the perpendicular drawn through P that lies between the two given lines (see Figure 3c).

d. The center of the desired circle lies at the intersection of the perpendicular drawn through P and the bisector of the vertex angle V (see Figure 3d).

Insert Figure 3 about here

Asked which of these conjectures might be correct, these students argued for more than ten minutes on purely empirical grounds. (The second student argued, for example, that the center of the circle in Figure 3a was not far enough to the left; thus that it was necessary instead to use the arc he had suggested in Figure 3b.) The issue was left unresolved by their discussions -- despite the fact that the proof that resolved the issue was still on the board. (Indeed, the proof ruled out the first three conjectures a priori.)

This problem session, although described anecdotally and although somewhat more dramatic than most of the problem solving sessions one usually sees, typifies a large body of rigorously gathered data. In studies conducted with both high school and college students, (Schoenfeld, 1983; Chapter 5 of Schoenfeld, in press) more than 90% of the subjects asked to solve Problem 3 (which was given first in most of the problem sessions) did so by trial-and-error, testing their hypotheses by carrying out the constructions and accepting or rejecting them on purely empirical grounds. (A small number of incorrect solutions were accepted because they looked good and a small number of correct solutions were rejected because they looked bad.) The vast majority of the students examined were able to solve Problems 1 and 2 in short order. Moreover, subsequent interviews with the students indicated in most cases that these students were fully aware of the ramifications of their proofs; they understood, for example that the results of problems 1 and 2 applied to all similarly shaped figures. The students simply failed to use that knowledge.

The working hypothesis generated by these studies is that, despite the time devoted to both proofs and constructions in geometry, students see little or no connection between them. To state the hypothesis more provocatively, it may be the case that the students (consciously or otherwise) believe proof to be irrelevant to discovery -- and, therefore, ignore the results of problems 1 and 2 when asked to work problem 3. The classroom observation studies conducted in parallel with the interviews (see Chapter 10 of Schoenfeld, in press) tended to substantiate that hypothesis. They also suggested the origins of some additional beliefs that, it is hypothesized, students hold about the nature of mathematics. Two further examples of classroom behavior, and the hypothesized student beliefs related to them, are as follows.

First, typical examinations in the classes that were observed contained as many as 25 "problems" to be worked in 54 minutes. Homework assignments usually contained a similar number of problems, allowing students to get the impression that problems in mathematics can all be solved very rapidly if you have learned the material. (This belief, if held, would lead students to stop working on problems they had failed to solve within a few minutes.) Second, the students received mixed messages in their classes regarding the best ways to learn mathematics. On the one hand, it was consistently stressed that it is important to understand the mathematics being studied. Yet the classroom message was often different, giving students the impression that memorizing is more important than understanding. Before an examination, for example, students were told the following. "You'll have to know all your constructions cold so you won't have to spend a lot of time thinking about them. This is where practice at home comes in. . . . Mainly with constructions it is all going home and practicing." The questionnaire was designed to gather objective data regarding students' views of these and other aspects of mathematics.

Some of the Relevant Literature

The issues of primary concern in this research lie at the intersection of what have traditionally been called the cognitive and affective domains. [It may be appropriate at this point to recall Piaget's (1954, p.14) commentary on the relationship between the two: "1) Il n'y a pas de mecanisme cognitif sans elements affectifs. . . . 2) Il n'y a pas non plus d'etat affectif pur, sans element cognitif."] There is an extensive body of literature within the cognitive domain that deals with the acquisition of geometric knowledge, in particular with regard to the understanding of proof. In recent years the bulk of work in mathematics education on the topic has been based in, or shaped by, the pioneering work of Pierre van Hiele (1957) and Dina von Hiele-Geldof (1957). A recent translation of much of the van Hieles' work (Fuys, Geddes & Tischler, 1984) part the Brooklyn College geometry project, now makes that work much more accessible to English-speaking audiences. Work on projects based in Chicago, Brooklyn, and Oregon (Hoffer, 1983) has served to flesh out the structure of what have come to be known as "van Hiele Levels" in geometry. Unfortunately the categorization found in this empirical work does not address the paradoxical problem that prompted this research: that students who demonstrate clear Level 3 understandings of mathematical argumentation will, in the context of geometric construction problems, behave in a way that demonstrably contradicts those understandings.

Although couched in different language, work in information processing psychology has addressed issues similar to those raised in the van Hieles' work, specifically those of the transitions between levels. Loosely speaking, the van Hieles' notion of moving from one level to the next is that the ideas or concepts that the student struggles to understand at level N, once understood, become comfortably accessible parts of the knowledge base at level (N+1). In Greeno's (1983) terms, these ideas or concepts have become "conceptual entities;" a series of papers by Greeno and colleagues (Greeno, Magone and Chaiklin, 1979; Greeno and Simon, 1984) explores the processes by which this takes place. So too does a large body of work on "knowledge compilation" by Anderson (1980, 1981, 1982) and colleagues. Anderson's work includes the development of a computer-based tutor for writing geometry proofs. Like the work in mathematics education, however, the information processing work on geometry has not addressed the use of proof-related knowledge in contexts that call for invention or discovery.

To the author's knowledge there are no studies within the domain of mathematics learning that straddle the cognitive/affective boundaries as precisely discussed here. (See, however, the discussion of mathematics attitude below.) There are some relevant studies on unusual "reasoning practices" in other domains. The misconception literature in physics (Clement, 1983; Lochhead, 1983; diSessa, 1983; McCloskey, Caramazza, and Green, 1983; McCloskey, 1983) documents ways in which people's interpretations of real-world phenomena are at variance with their formal knowledge regarding those phenomena. Similarly, work in decision theory (Kahneman, Slovic and Tversky, 1982) indicates that people with training in probability and statistics will in certain situations make predictions that clearly violate the laws of probability. In one experiment, for example, students read a brief passage describing an intelligent, socially concerned woman. They are then asked which of the following two options was more probable:

A) The person described is a bank teller.

B) The person described is a bank teller who is active in the feminist movement.

Despite the rule that $A \supset B = P(A) \geq P(B)$, 50% of a sample of statistically sophisticated psychology graduate students chose B as being more probable. To sum up these results briefly, people frequently make judgments that contradict their formal knowledge -- and those people can live comfortably with the contradictions.

The literature on affect in mathematics is extensive. Areas in which affect has a clear inhibitory effect on performance are mathematics anxiety (Suinn, Edie, Nicoletti, & Spinelli, 1972; Tobias, 1978; Buxton, 1981) and fear of sucess (Tresemer, 1976; Leder, 1980); these two domains cover only a small fraction of the area known as mathematics attitude. Major literature reviews on that topic may be found in papers by Aiken (1970, 1976) and Kulm (1980). As discussed below, The National Assessment (Carpenter, Lindquist, Matthews, & Silver, 1983) provided data on mathematics attitude directly relevant to this study. Other areas in the affective realm that may bear on the issue discussed here are those of motivation (Atkinson and Raynor, 1974; Ball, 1977) perceived personal control (Weiner, 1974; Stipek and Weisz, 1981; Lefcourt, 1982) and individual differences (Fennema and Behr, 1980). To be honest, however, the relationship between some of these affective variables and some of the "reasoning practices" explored in this paper is unclear at best.

The Population

The survey was administered, on a volunteer basis, to 230 students in three high schools in upstate New York: an inner city "magnet" school (two math 10 classes), a suburban school with a high proportion of high SES, college-bound students (one advanced math 9, four math 10 including the target class, one math eleven, two math 12), and a private suburban school with a mixed population of students (one math 10, one math 11). The teachers of the various classes administered the questionnaires to their students. On the questionnaires some of the questions were in "positive" and some in "negative" form. For ease in data presentation the positive forms are used below. Data from a subset of the questions are presented.

Results and Commentary

Attributions of success or failure.

Mean scores for these ten questions are reported in Table 1. These questions produced the strongest and least ambiguous responses on the questionnaire. In brief, students consider mathematics to be an objective discipline that can be mastered; they claim that it is work and not luck that accounts for good grades and that teachers' attitudes towards them are not a factor in grading. If the students do badly, they believe it to be their fault.

Insert Table 1 here

A comparison of Mathematics, English and Social Studies.

The purpose of this section of the questionnaire was to compare students' perceptions of these three desciplines. Summary data are given in Table 2.

Insert Table 2 here

This category produced some surprises. One might have predicted the pattern of responses to the first question, for the notion of "mathematical ability" is a commonly accepted part of our folk culture. Likewise, one might have expected to find the results for the second question: The general sentiment appears to be that providing training in a discipline is more appropriate for the sciences than for the humanities. It is somewhat surprising, however, to find the pattern of responses to the "it's right or it's wrong" statements in the third question: Students agreed this was the case in English; they were neutral in social studies; they disagreed in mathematics. Even more surprising is the very strong "agree" rating (1.45, the second most extreme rating on the questionnaire) with the statement that "good mathematics teachers show students lots of different ways to look at the same question." The reason for the author's surprise is that very little of such teacher behavior was observed in the classroom studies conducted in parallel with this study. The vast majority of the students in the survey had not (at least during our observations that year) experienced that kind of teacher behavior. Their response suggests either a strong acceptance of part of the mythology about teaching, or some strong degree of wishful thinking.

Questions about the nature of mathematics (mathematics attitude) and mathematics teaching.

These questions revealed some interesting contradictions, typified by the following two responses. With 1 representing strong agreement and 4 representing strong disagreement, students gave "you have to memorize the way to do constructions" a mean score of 1.87. However, they also gave "a construction is easy to figure out even if I've forgotten exactly how to do it " a mean score of 2.15. This pattern was replicated in the answers to other problems. On the one hand, there was a tendency to regard mathematics learning largely as a matter of memorization: see Table 3. On the other hand, the students expressed significant support for the idea that mathematics is interesting and challenging, allowing a great deal of room for discovery: See Table 4.

Insert Tables 3 and 4 here

These data reflect the contradictory patterns of mathematical attitude data reported in the NAEP Secondary Study and the Second International Mathematics Study. In discussing the NAEP data, Carpenter et al (1983) made the following comments:

> Some aspects of the patterns of student's responses are interesting. For example, students felt very strongly that mathematics always gives a rule to follow to solve problems. Yet, they felt just as strongly that knowing how to solve a problem is as important as getting the solution and that knowing why an answer is correct is as important as getting the correct answer. . . .

> Despite the fact that almost half the students view mathematics as mostly memorizing, three fourths of them thought that mathematics helps a person to think logically, and more than three-fifths thought that justifying the statements one makes is an extremely important part of mathematics. These latter attitudes may reflect the beliefs of their teachers or a more general social view rather than emerge from their own experience with school mathematics. (Carpenter, Lindquist, Matthews, & Silver, pp. 656-657)

The classroom studies (Chapter 10 of Schoenfeld, in press)support these hypotheses and document their origins in instruction. As noted above, students receive contradictory messages in their mathematics classes. On the one hand they are told to memorize, and they find memorization an essential survival skill. On the other hand, there is a continual classroom rhetoric dealing with the importance of understanding and the value of the thinking skills that one learns in mathematics. As noted above, students can live comfortably with such contradictions -- not noticing them, in fact. For example, a significant proportion of the students in the empirical studies ignored or contradicted the results of proofs they had just completed when working construction problems. Yet most students disagreed with the statement that "constructions have little to do with other things in geometry like proofs and theorems" (mean score 2.88).

Open-ended questions.

The questionnaire included ten open-ended questions dealing with various aspects of the nature of mathematics. Tables 5, 6 and 7 present randomly selected responses (the 4th response from a randomly typed list of responses, from each of the 12 classes polled) to three of the issues most germane to the current discussion. These responses speak for themselves, and in support of the hypotheses advanced above.

Insert Tables 5, 6, 7 here

Secondary Analysis: Factor analyses and correlations with demographic data.

The first sixty items on the questionnaire dealt with the following categories: attribution of success of failure, mathematics attitude, the comparison of mathematics with English and social studies, teacher's classroom behavior, and motivation. Factor analysis on the responses to these sixty questions revealed two primary factors.

The first factor will be referred to as "mathematics is closed," a shorthand for the f owing summary assertion: "mathematics is a rigid and closed discipline, inaccessible to discovery by students and best learned by memorizing." The five questions comprising this factor are given in Table 8. The second factor will be referred to as "mathematics is useful," a shorthand for "mathematics is useful, enjoyable, and helps me to understand things." The six questions comprising this factor are given in Table 9. The correlations between the factors and their component questions are given in Table 10.

Insert Tables 8, 9, 10 here

Correlations were then examined between average scores on the factors (for a justification of this procedure see Wackwitz & Horn, 1971) and various demographic data. Those correlations are given in Table 11.

Insert Table 11 here

These data offer few surprises. The students who stay with mathematics (a self-selected group) tend to find it less rigid, although no more enjoyable. Loosely speaking, those who got better grades both in and out of mathematics, thought of themselves as good mathematics students, and those who worked hard at the subject, thought mathematics to be less rigid and more enjoyable. Two curiosities occurred in the correlations to the last three questions. First: The students' perceived importance of doing well in mathematics did not correlate significantly with their perceptions of the rigidity of the discipline, although it did correlate positively with their enjoyment of it. Second: The more important the students' mothers felt it for the students to do well in mathematics, the more rigid the students found the discipline to be. I have no explanation for this.

Discussion and Conclusions

Perhaps the most important result of this study is that it documents the typicality of the target class that was used in the empirical studies, and by doing so provides a clear resolution of the contradictory patterns in the mathematics data reported in the NAEP secondary study. As noted above (see chapter 10 of Schoenfeld, in press, for details), students in the target class received contradictory messages about the nature of mathematics. On the one hand they were instructed to memorize (in fact, told they would not have time to think on tests) and they did so. Based on their own empirical experience, students believe that mathematics is a discipline mostly to be memorized. On the other hand, the students were continually subjected to a classroom rhetoric that stressed the importance of understanding and the utility of mathematics. They bought the message, at the rhetorical level. (The empirical studies indicate that this rhetoric is ignored when the students engage in mathematical tasks in which they might exploit the results of formal mathematics, but ignore them instead.) Observations of other classes indicated the typicality of instruction in the target class, and the questionnaires indicate the typicality of their responses. The pattern of responses in the target class (n = 21) was very much like the pattern in the questionnaire sample (n = 230), which reflected the pattern of mathematics attitude data on the NAEP secondary study (n = 45,000). This permits the unfortunate conclusion that most of our students experience mathematics as a discipline to be memorized, an experience couched in a rhetorical presentation that stresses the importance of understanding and the utility of mathematics but that provides students with little experience of either. If this is the case, the fact that the students learn to repeat the rhetoric is hardly consoling -- especially since their mathematical behavior stands in stark contrast to their avowed profession of the importance and utility of mathematics.

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Table 1

Attribution Data

When I get a good grade in math...		
		mean score*,#
1.	It's because I work hard	1.52
2.	It's because the teacher likes me	3.44
3.	It's just a matter of luck	3.04
4.	It's because I'm always good at math	2.50
5.	I never know how it happens	3.49
When I get a bad grade in math...		
6.	It's because I don't study hard enough	1.75
7.	It's because the teacher doesn't like me	3.68
8.	It's just bad luck	3.21
9.	It's because I'm just not good at math	3.04
10.	It's because of careless mistakes	1.75

*Scoring: 1 = very true; 2 = sort of true;
3 = not very true; 4 = not at all true

#n = 230.

Table 2

Students' Perceptions of Three Disciplines*,#

	Math	English	Social Studies
Some people are good at ____ and some just aren't.	1.66[1,$]	1.83	2.03
Good ____ teachers show the exact ways to answer the questions you'll be tested on.	2.28[2,$]	2.70	2.49
In ____ it's either right or it's wrong.	2.94[3,$]	1.93	2.42
Good ____ teachers show students lots of different ways to look at the same question.	1.45[4,$]	1.67	1.85

*Scoring: 1 = very true; 2 = sort of true;
3 = not very true; 4 = not at all true

#Mean scores (n = 230) tested for differences using a repeated measures analysis of variance.

[1] ($F(230) = 30.16$, $p < .0001$)

[2] ($F(230) = 63.40$, $p < .0001$)

[3] ($F(230) = 18.61$, $p < .0001$)

[4] ($F(230) = 24.41$, $p < .0001$)

$Post hoc planned comparisons tests indicate that the mathematics mean scores differ significantly from the others.

Table 3

Memorization Questions*

	Mean Score#
The math that I learn in school is mostly facts and procedures that have to be memorized.	1.75
When the teacher asks a question in math class the students who understand only need a few seconds to answer correctly.	2.07
The best way to do well in math is to memorize all the formulas.	2.13
You have to memorize the way to do constructions.	1.87

*Scoring: 1 = very true; 2 = sort of true;
3 = not very true; 4 = not at all true

#n = 230.

Table 4

Understanding and Creativity in Mathematics*

	Mean Score#
The math that I learn in school is thought provoking.	2.01
In mathematics you can be creative and discover things by yourself.	2.02
When I do a geometry proof I get a better understanding of mathematical thinking.	1.99
When I do a geometry proof I can discover things about geometry I haven't been taught.	2.21
The reason I try to learn mathematics is to help me think more clearly in general.	2.33

*Scoring: 1 = very true; 2 = sort of true;
3 = not very true; 4 = not at all true

#n = 230.

Table 5

Randomly selected responses to the question "How important is memorizing in learning mathematics? If anything else is important, please explain how."

A. Memorizing is very important. Also, derivations can help you if you know how to derive a formula.

B. It is important to memorize in learning, but understanding the concept is more important.

C. It's very important, many problems cannot be solved unless you've memorized a formula.

D. Not very important, understanding should be stressed instead of having to memorize.

E. In geometry, memorizing is the key that will get you through the course.

F. I think memorizing is very important.

G. Memorizing is important.

H. You must know certain rules which are a part of all mathematics. Without knowing these rules, you cannot successfully solve a problem.

I. It is very important because there are many vital formulas you must know.

J. It is very important, especially in geometry.

K. Memorizing is very important, and in geometry, especially for the final exam, because I am required to write proofs from memory.

L. Memorization of equations and formulas are essential in mathematics. If you memorize those then you plug in your variable and solve what you're looking for.

Table 6

Randomly selected responses to the question "In what way, if any, is the mathematics you've studied useful? The arithmetic, the algebra, the geometry?"

A. For doing shopping, for doing proportionment measurements, and comparative pricing.

B. It is helpful in chemistry and physics.

C. All of the math courses taken in high school are useful for certain professions.

D. Geometry helps me think more.

E. Math is useful by getting us into good colleges, and having better reasoning.

F. I really don't find geometry useful at all. Algebra can help you with science sometimes.

G. Math has helped me to think more logically.

H. If you were to become an engineer or technician you need the basic rules to follow for measuring things and estimating things.

I. It helps you with many everyday tasks.

J. Math requires you to really think about what you're doing. I think this affects your life and how you think about things.

K. I use arithmetic all the time when I need to figure my money situation. Algebra and geometry are not useful but they prepare me for the rest of high school and college.

L. The arithmetic is always useful. Algebra helps you in business and geometry gets your deductive skills up.

Table 7

Randomly selected responses to the question "If you understand the material how long should it take to solve a typical homework problem? What is a reasonable amount of time to work on a problem before you know it's impossible?

A. 2-5 minutes. 15-20 minutes is reasonable before giving up.

B. Up to 2 or 3 minutes. I would work on a problem for about 10 minutes before deciding it's impossible.

C. Less than 5 minutes. The most time is 10 minutes.

D. A typical homework problem would take about 45 seconds. About 10 minutes for the impossible problem.

E. 1 minute. 5 minutes.

F. It should take a few minutes or less for a typical problem. About 2 or 3 minutes is reasonable before you know it's impossible.

G. 1-2 minutes. 5 minutes.

H. If I know it really well I can rip right through it in 1 minute or less. If I have difficulty a couple of minutes.

I. If you understand it, it should take between 1-2 minutes depending on the problem. 3 minutes tops.

J. 3 minutes. 10 minutes.

K. It would probably take from 30 seconds to 2 minutes. I usually give up after 3 or 4 minutes if I can't do it.

L. It should only take a few minutes if you understand it. No more than 10-15 minutes should be spent on a problem.

Table 8

Factor 1: "Mathematics is a closed discipline inaccessible to discovery by students and best learned by memorizing."*

Q33.	Everything important about mathematics is already known by mathematicians.
Q35.	Math problem can be done correctly in only one way.
Q37.	To solve math problems you have to be taught the right procedure, or you can't do anything.
Q38.	The best way to do well in math is to memorize all the formulas.
Q39.	When you get the wrong answer to a math problem it's absolutely wrong -- there's no room for argument.

*Scoring: 1 = very true; 2 = sort of true;
3 = not very true; 4 = not at all true

Table 9

Factor 2: "Mathematics is useful and enjoyable, and helps me to understand things." *

Q44.	When I do a geometry proof I get a better understanding of mathematical thinking.
Q47A.	NOT: When I do a geometry proof I'm doing school math that has nothing to do with the real world.
Q48.	When I do a geometry proof I feel like I'm doing something useful.
Q49.	Geometry constructions are fun to do.
Q53.	The reason I try to learn mathematics is to help me think more clearly in general.
Q56.	The reason I try to learn mathematics is that it's interesting.

*Scoring: 1 = very true; 2 = sort of true;
3 = not very true; 4 = not at all true

Table 10

Correlations of Factors 1 and 2 with each other and with each other's components

	Factor 1	Factor 2
Factor 1	1.000	-.293
Factor 2	-.293	1.000
Q33	.62	-.20
Q35	.73	-.15
Q37	.67	-.20
Q38	.62	-.24
Q39	.61	-.13
Q44	-.17	.66
Q47A	-.37	.62
Q48	-.21	.76
Q49	-.00	.58
Q53	-.14	.57
Q55	-.36	.74

Table 11, Part 1

Correlations of Factors 1 and 2 with demographic data

	Factor 1	P<	Factor 2	P<
Q61	.28	.0001	-.00	.999
Q62	-.06	.33	-.02	.77
Q63	.14	.025	-.20	.002
Q64	.22	.0009	-.37	.0001
Q65	-.32	.0001	.27	.0001
Q66	-.17	.01	.23	.0005
Q67	.06	.31	.14	.033
Q68	-.07	.25	.35	.0001
Q69	.17	.009	.12	.06
Q70	.01	.87	.13	.047

Table 11, Part 2

Details of Demographic Items 61 through 70

Q61: I am in grade:

9/10/11/12

Q62: I am:

1 = Female, 2 = Male

Q63: My overall grade average on the last report card was about:

F/D/C/B/A

Q64: This marking period I expect the following grade in math:

F/D/C/B/A

Q65: Compared to other students in Math I'm about:

top 10%/above average/average/below average/bottom 10%

Q66: Compared to how hard other students work at math I'm:

top 10%/above average/average/below average/bottom 10%

Q67: During the year I've done my homework:

always/most/half/occasionally/rarely/never

Q68: How important do you think it is to do well in math?

very/sort of/not very/not at all

Q69: How important does your mother you think it is?

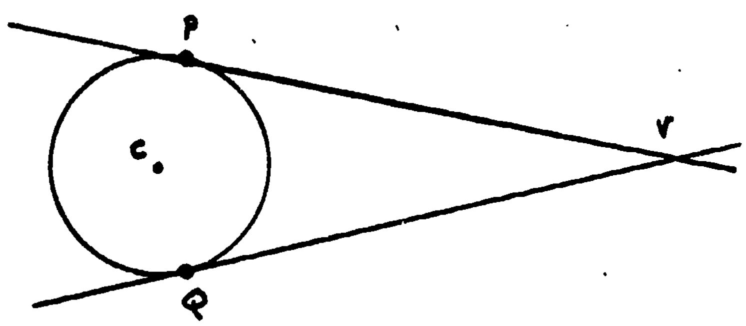
very/sort of/not very/not at all

Q70: How important does your father you think it is?

very/sort of/not very/not at all

Figure 1

In the drawing below, the circle with center C is tangent to the top and bottom lines at the points P and Q respectively.



Problem 1. Prove that $\overline{PV} = \overline{QV}$.

Problem 2. Prove that the line segment CV bisects angle PVQ.

Figure 2

Problem 3. Using straightedge and compass only, you would like to construct the circle that is tangent to both of the lines in the drawing below, and that has the given point P as the circle's point of tangency to the top line. How would you do the construction?

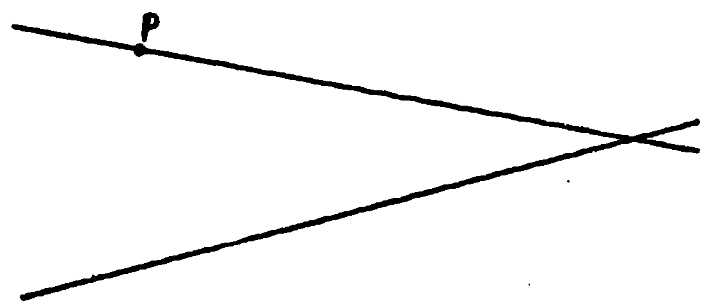
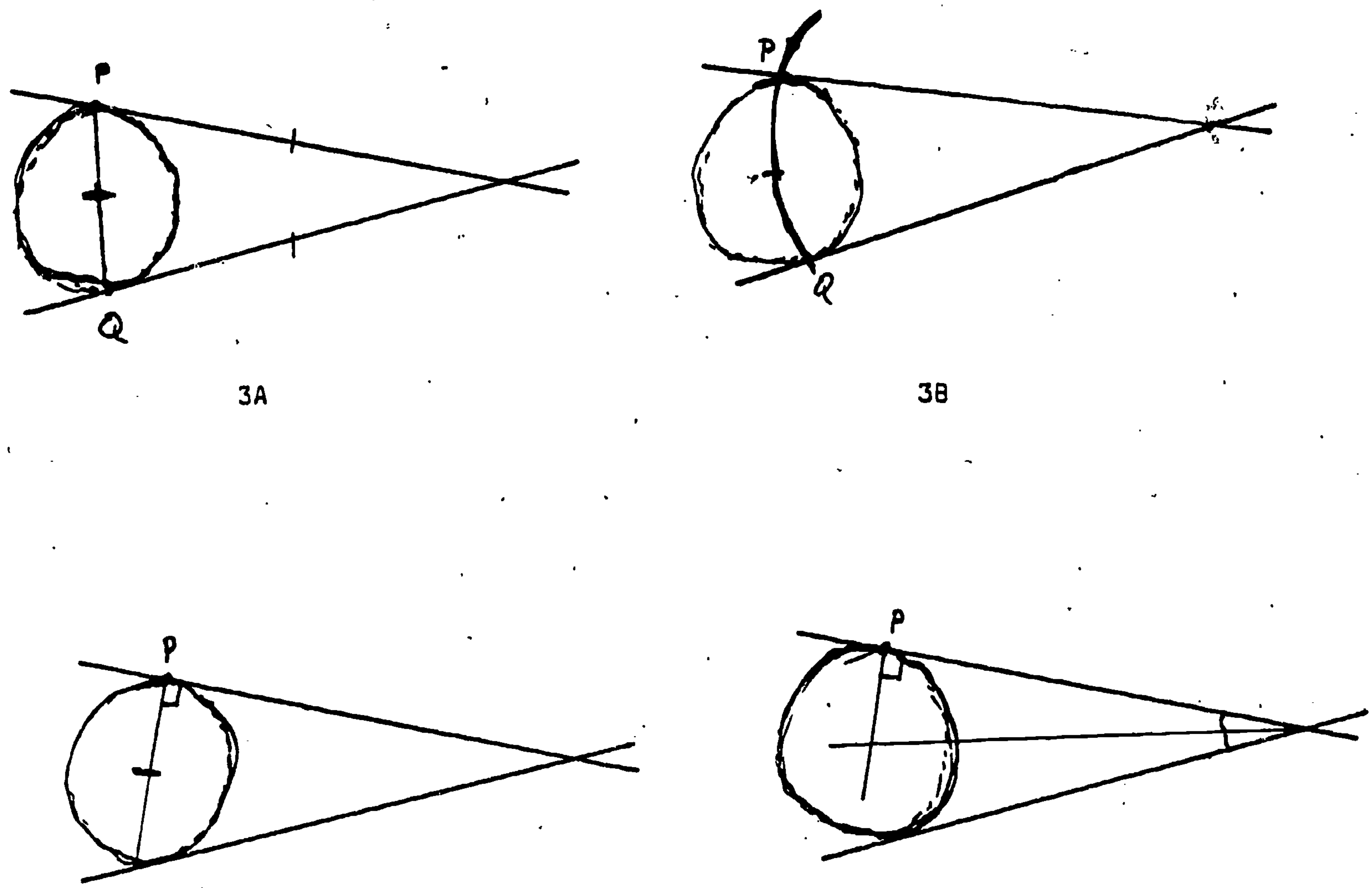


Figure 3



3A 3B

3C 3D