DOCUMENT RESUME

ED 259 949 SE 045 930


AUTHOR Shalin, Valerie L.; Bee, Nancy V.


TITLE Structural Differences Between Two-Step Word Problems.


INSTITUTION Pittsburgh Univ., Pa. Learning Research and Development Center.

SPONS AGENCY Office of Naval Research, Washington, D.C.

PUB DATE Apr 85

CONTRACT ONR-N00014-82-K-0613

NOTE 26p.; Paper presented at the Annual Meeting of the American Educational Research Association (69th, Chicago, IL, March 31-April 4, 1985).

PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.

DESCRIPTORS Elementary Education; *Elementary School Mathematics; Mathematics Education; *Problem Solving

IDENTIFIERS *Mathematics Education Research; *Word Problems (Mathematics)



ABSTRACT

A logical analysis of word problems provides an hypothesis about the kinds of knowledge that may be necessary for solving multi-step word problems. This hypothesis is that knowledge of the overall pattern of relations in a problem may function in problem-solving apart from knowledge of the particular entities and the local relations in which they participate. This overall pattern or structure determines equivalence classes of problems, problems whose solutions differ in the specific mathematical operations utilized but which are of similar difficulty. To illustrate this analysis, a notation system for one-step word problems is introduced and a description of two-step word problems according to this notation is provided. Experimental results (using third-, fourth-, and fifth-grade students as subjects) are then presented that verify the psychological validity of the analysis, namely, that for all grade levels the problem structure (the overall pattern of relations between the quantities in the problem) has an effect on problem-solving success. Speculation on the representation of problem-solving knowledge that could account for these results is included. (Author/JN)

# Abstract

## Structural Differences Between Two-Step Word Problems

Valerie L. Shalin and Nancy V. Bee

Learning Research and Development Center

University of Pittsburgh

1985 Meeting of the American Educational Research Association

A logical analysis of word problems provides an hypothesis about the kinds of knowledge that may be necessary for solving multi-step word problems: Knowledge of the overall pattern of relations in a problem may function in problem solving apart from knowledge of the particular entities and the local relations in which they participate. This overall pattern or structure determines equivalence classes of problems--problems whose solutions differ in the specific mathematical operations utilized but are of similar difficulty.

To illustrate this analysis we introduce a notation system for one-step word problems and then describe two-step word problems accordingly. Experimental results are then presented that verify the psychological validity of the analysis.

# Structural Differences Between Two-Step Word Problems[1]

Valerie L. Shalin and Nancy V. Bee
Learning Research and Development Center
University of Pittsburgh

1985 Meeting of the American Educational Research Association

Existing analyses of problem solving knowledge involve either the rational decomposition of a task in terms of the procedures required (Gagne, 1963) or the identification of components of a problem solving model that are sufficient for solving the task (Greeno, 1976). We present a logical analysis of word problems that provides additional hypotheses about the kinds of knowledge that may be necessary for solving multi-step word problems: Knowledge about global problem structure may affect problem solving success. Our analysis of two-step arithmetic word problems is based on possible logical combinations of one-step word problems. We build on previous analyses of one-step word problems, but emphasize the importance of the overall pattern of relations between quantities in multi-step word problems. The general importance of knowledge about patterns of relations between quantities has been identified previously in the context of the geometry problems first studied by Wertheimer (1945/1959) and later by Greeno and Anderson (Greeno, 1982). We suggest that knowledge of global relations in a problem may function in problems that are not particularly spatial in nature, and that these global patterns of relations constitute knowledge apart from knowledge of the particular entities involved and the local relations in which they participate. This overall pattern or *structure* determines equivalence classes of problems--problems whose solutions differ in the specific mathematical operations utilized but are of similar difficulty.

In the following sections we first introduce a notation system for one-step word problems. Then we describe two-step word problems according to this notation. Next we present experimental results that verify the psychological validity of the analysis. In the final section we speculate on the representation of problem solving knowledge that could account for these results.

*One-step word problems*

The analysis begins with the identification of some basic types of quantities and relationships used in one-step word problems, many of which have been extensively analyzed in the cognitive literature (Riley, Greeno & Heller, 1983; Schwartz, 1976). These particular relationships are meant to be illustrative, rather than exhaustive, and many more could be identified. We selected four different relationships for description here and defined corresponding notation for them: simple additive combination, a simple multiplicative relation involving a single intensive quantity, multiplication involving three intensives, and multiplicative comparison.

We propose the following notation for part-whole set relations in additive combination problems, illustrated in Figure 1. Each quantity is represented in a box with a rectangular top. Each quantity box has a name (in the upper portion of the box) and a value (in the lower portion of the box). The three quantities in one-step additive combination word problems are represented

[1]This research was sponsored by the Office of Naval Research, under Contract No. N00014-82-K-0613. We gratefully acknowledge the guidance of James Greeno, and helpful interactions with Mark Detweiler, Matt Lewis, Ernest Rees and Charles Perfetti.

in a triad: the whole quantity is at the top and the two parts are at the bottom.

Figure 1 shows two different word problems and their notations according to the same part-whole structure. The problems differ in terms of the role in the structure of the quantity with the unknown value. A problem whose solution is obtained by adding is represented by a triad in which the values of the two parts are known and the value of the whole is determined by moving up the triad and adding the known values. A subtraction problem is represented as one in which the values of a whole and one of the parts are known and the answer is obtained by moving down the triad, subtracting the known part value from the known whole value.

One relationship involving multiplication uses intensive, non-additive quantities that relate two extensive quantities (see Figure 2). The multiplicative triad illustrated here involves extensives associated by groups or collections (e.g., wagons and trains) and an intensive "mapping function" quantitatively relating the two (e.g., wagons in each train). As above, extensive quantities are written in boxes with rectangular tops. Intensive quantities are written in boxes with semicircular tops. Triads for three different word problems, according to the selection of known and unknown quantities, are shown in Figure 2. Note that there are two types of division problems, one for determining the value of "wagons per train" given the values of "wagons" and "trains", and another for determining the value of "trains" given the values of "wagons" and "wagons per train".

A second multiplicative relation refers to two grouping relations, involving, for example, horses and wagons (grouped as horses per wagon), and trains (which group the wagons, as in wagons per train). This introduces a new quantity, horses per train. The relation shown in Figure 3 involves three such intensive grouping quantities. As with the multiplicative relation just described, this structure generates one multiplication problem and two different division problems, depending upon the role of quantities with known and unknown values.

A third multiplicative relation involves a comparison between extensives. In this triad a box with a hexagonal top is used for the comparison factor "times more white eggs than brown eggs", as shown in Figure 4. One of the extensive quantities, "white eggs", is at the top, and the other extensive quantity, "brown eggs", is at the bottom. As with the other multiplicative relations described above, there are two types of division problems, depending upon the role of the unknown.

*Two-step word problems*

Two-step word problems can be modeled by combining two of these triads based on local relations. Each of the different ways of combining these triads constitutes a different global problem structure. For example, the following problem is derived from a hierarchical arrangement of triads (shown in Figure 5) which we call a hierarchical structure:

> A neighborhood is planning its annual cat show. There are 72 siamese, angora and persian cats altogether, and 26 of these are persian cats. If 24 of the cats are angora cats, how many siamese cats are there?

There are two other ways to combine binary triads (shared-whole in Figure 6 and shared-part in Figure 7) and these are exemplified by the following problems:

> A sporting goods shopkeeper is checking her inventory list. She has 13 soccer and tennis balls altogether, and 9 golf balls. If 14 of the balls are soccer and golf balls, how many tennis balls

does she have?

Robert is fixing snacks for his men's club meeting. He has 208 onion and rye crackers altogether, and 116 of these are rye crackers. If Robert has 324 onion, rye and wheat crackers a... ..ther, how many rye and wheat crackers are there?

These different structural patterns also pertain to triads based on other local relations, such as the shared-part structure based on one multiplicative and one additive relation, shown in Figure 8. The several problems shown in this figure illustrate the idea of *equivalence classes of problems* derived from a common problem structure. All of these problems differ in terms of the role played by the quantities with known and unknown values, and the specific mathematical operations that are used to solve them, but they are all derived from the same structure.

The experiment reported below tests the hypothesis that problem structure--or the overall pattern in which quantities are organized--is a significant factor in problem solving success and hence the problem solving process. It explicitly tests the hypothesis that equivalence classes of problems may be defined on the basis of problem structure, although different operators and even quantity types may be involved. Elementary school subjects attempted to solve the 34 problems involving additive and multiplicative relations in Table 1 within a limited period of time, with percent correct and written problem solution as the dependent measures.

*Method*

*Subjects*. 82 Falk Elementary School third, fourth and fifth grade students participated in the study. Because Falk permits students to study advanced math if they are able, the ultimate grade assignment was based on the textbook that a subject was using at the time of testing. Textbook levels ranged from grades three to seven.

*Design*. Word problems were generated from 11 problem structures, summarized in Table 2. These structures were obtained by crossing triads of operator type (two additive, one additive and one multiplicative, or two multiplicative) with problem structure (hierarchical, shared-whole quantity or shared-part). In all cases the additive triad concerned simple combination. In most cases the multiplicative triad concerned simple multiplication. The exception was the use of a multiplicative comparison in the "shared-part" multiplicative problems. To ensure that this cell did not differ simply because of the use of this relation, other problems were designed in which the shared-part was an intensive quantity. However, these problems were also exceptional because they contained an assumption that the intensive was appropriate for both types of quantities, e.g., that there are the same number of roses in red bunches of roses as in white bunches of roses.

*Materials*. The thirty-four word problems that represent eleven problem structures with their variations are presented in Table 1. Each cover story consisted of a setting line that had no quantitative information in it. For example: "A sporting goods shopkeeper is checking her inventory list" or "A neighborhood is planning its annual cat show". The next line of text gave the two quantities that enter into an appropriate first step, in language that was as terse as possible to control for biases within problems due to text wording. The final line of text introduced the third quantity and the question. Because of the terse language and great difficulty in selecting and ensuring that adjectives were unambiguously non-overlapping, the word "altogether" was used systematically to clarify the situation as much as possible.

Most of the word problems studied here can be solved using one of two competing conceptualizations. In general, and illustrated in Figure 9, hierarchical problems can be solved by alternative hierarchies. Shared-whole quantity problems can be solved by corresponding shared-part structures. Special shared-part problems can only be represented by other shared-part structures: those in which the two outer parts overlap conceptually, although they are not illustrated as such in the network. To ensure that a subject must attempt an additive shared-part problem, special shared-part problems must be utilized. Word order was used to bias a simpler or preferred structure. For example, word order on the convertible shared-whole problems was biased toward a shared-whole conceptualization by arranging the first two quantities for this structure in the first quantitative sentence.

To unconfound structural pattern with number facts, number set was crossed with problem structure, inasmuch as possible. Three number sets were available for additive and multiplicative word problems and four were available for mixed problems to be crossed with hierarchical, shared-whole quantity and shared-part structures. Because the different patterns required different relationships to be calculated, each structure could not accomodate exactly the same numbers as another structure. Rather a number set is best regarded as a number family, which was applied as appropriate to structures, requiring some adjustments to fit a structure. In cases where the use of a number set was incompatible, suitable numbers were utilized at random. There were therefore three different versions of the problem set, each utilizing a different assignment of number family to problem structure.

There were ten different random orders of the 34 problems to prevent order biases in solving the problems and to ensure that each structure had an equally likely opportunity to be attempted.

Each of the 30 different problem sets (10 orders x 3 number sets) were presented in booklets, with one problem at the top of an otherwise blank page.

*Procedure*. Subjects were tested in the classroom during a 40 minute session. They were instructed to do as many problems as possible and to show the relevant work. This included showing work for only part of a problem if that was all they could do, or skipping a problem that they could not do.

### *Results*

*Preliminary Analysis*. Subjects were grouped by textbook level and the problems were scored as attempted, skipped or not attempted, with the latter categorization applying for all problems following the last problem with written work. Attempted problems were scored as correct, partially correct or incorrect. Correct problems had completely correct answers, or a complete sequence of mathematical expressions that may have included a mathematical error. Partially correct problems had at least one correct expression along a possibly correct solution path. Partial credit was not assigned to individual numbers not supported by evidence of a mathematical expression. In all other cases a problem was scored as incorrect.

Because subjects did not necessarily attempt the same problems, the analysis of variance was performed on proportion scores consisting of the number of correct answers over the number of attempts. Thus, for each text level there was a set of 34 proportion scores, one for each problem in the design. These scores were transformed according to the following formula: arcsin (sqrt(x)). The transformed scores were submitted to a two-way, three x three analysis of variance of problem structure by operator set.

In the following subsection we first describe general performance statistics across all problems by text level, including the number of problems attempted and the probability of success given an attempt. Then, in the five following subsections we treat each grade separately and consider the significance of structure, operator set and structure x operator set effects. In general, the structure effect was always significant with the effects of operator set and the interaction varying by text level.

*General performance statistics.* The average number of problems attempted per child and the probability of success given an attempt changed across text levels. Third graders and fourth graders attempted about the same number of problems (13.33 and 13.96 respectively) but fourth graders showed a higher probability of success (.09 and .20 respectively). Fifth graders attempted 14.81 problems each with a .40 probability of success given an attempt. Sixth graders and seventh graders performed similarly, having attempted an average of 21.1 and 21.2 problems each with a .76 and .73 probability of success respectively. This similarity in performance persists in the more detailed analysis presented below.

*Third grade text level performance.* The summary proportion scores of the 12 third graders are presented in Table 3. The scores presented in each cell are collapsed across the several members of the equivalence class in that cell. Table 2 may be consulted for the number of problems in each cell. The scores presented as row and column totals are derived from the total number of successes over the total number of attempts in that section.

The individual proportion scores for each problem were transformed and submitted to an analysis of variance. The effect of structure was significant, $F(2, 25) = 4.06$, $p < .05$. The effect of operator set was not significant, $F(2, 25) = 1.36$, but the interaction between structure and operator set was significant at this text level, $F(4, 25) = 4.72$, $p < .01$.

*Fourth grade text level performance.* The summary proportion scores of the 25 fourth graders are presented in Table 4. As previously described, the individual proportion scores were transformed and submitted to an analysis of variance. The effect of structure was significant, $F(2, 25) = 4.6$, $p < .025$. The effect of operator was not significant, $F(2, 25) = .61$, but the interaction between structure and operator set was significant, $F(4, 25) = 5.22$, $p < .01$.

Thus, although fourth graders performed somewhat better than the third graders overall, their performance patterns are similar, with both showing a significant effect of structure and a significant interaction between operator set and structure, with no main effect of operator.

*Fifth grade text level performance.* The summary proportion scores for the 27 fifth graders are presented in Table 5. The proportion scores for each problem were transformed and submitted to an analysis of variance. In this text level both main effects and the interaction were significant, $F(2, 25) = 4.85$, $p < .025$ (for structure), $F(2, 25) = 4.97$, $p < .025$ (operator set) and $F(4, 25) = 3.90$, $p < .025$ (structure x operator set). This pattern of performance is unique to this text level.

*Sixth grade text level performance.* The summary proportion scores of the 10 sixth graders are presented in Table 6. As above, the proportion scores for each problem were transformed and submitted to an analysis of variance. Only the effect of structure is significant, $F(2, 25) = 4.30$, $p < .025$ and $F(2, 25) = 1.7$ (operator set), $F(4, 25) = 1.71$ (structure x operator set).

*Seventh grade text level performance.* The summary proportion scores of the 7 seventh graders are presented in Table 7. The analysis of variance of the transformed problem scores indicates a pattern of performance identical to that found at the sixth grade level; only the effect of structure is significant, $F(2, 25) = 7.54$, $p < .01$ and $F(2, 25) = 1.89$ (operator set) and $F(4, 25) = 2.02$ (structure x operator set).

*Summary and additional performance features common to all text levels.* Thus, in all text levels there was a significant main effect of problem structure. In text levels six and seven this was the only significant effect. In text levels three and four there was also a significant interaction. In text level five there was also both a significant interaction and a main effect for operator set.

By examining Tables 4 - 7 it appears that the main effect of structure is due to [illegible] advantage of the hierarchical problems. Not shown in the tables, but apparent in the [illegible], are differences in success probabilities between the two different mixed operat[illegible]archies. It appears that one cell of purely additive problems (cell 1,3--the additive shared-part problems) is consistently harder than one cell of purely multiplicative problems (cell 3,1--the multiplicative hierarchical problems). It also appears that the source of the interaction effects is due at least in part to the advantage that ratio problems (cell 3,3) have in comparison to the other problems with a shared-part structure.

## Discussion

In this experiment we demonstrated the psychological validity of the structural analysis for two-step word problems. That is, for all grade levels problem structure--the overall pattern of relations between the quantities in the problem--has an effect on problem solving success. Of course, these results are not sufficient for clarifying the *role* of problem structure in the problem solving process. However, there is good reason to believe that the problem structure effect has something to do with knowledge. According to our preliminary analysis of textbook problems, the easier hierarchical structures are in fact practiced more than the harder structures, except for the ratio structures, which also yield high success rates. In addition, the cell of mixed operator hierarchies consists of two different hierarchies; the more practiced hierarchy yields greater problem solving success. Should this effect be entirely due to practice, it is still essential that a theory of problem solving in this domain accounts for differences in problem solving success on problems that differ in the sequence in which the steps are executed; this is, on close inspection, one way of describing the difference between problems derived from different structures.

Further analysis of the existing data may clarify the role of problem structure in problem solving. Each individual structure might be thought of as a schema, in which the types of quantities involved and the local relations between them are represented with the additional information about the way in which the five quantities are arranged, according to the three possibilities illuminated by our structural analysis. This schema might be thought to guide operator selection in the manner of procedural attachment (Bobrow & Winograd, 1977). Further, particular problems within a structure may be thought to yield differential success depending upon the association between the schema and the required operator. Such an explanation would be supported by patterns of performance within individual subjects such that correct responses are clustered within--rather than spread between--structures.

We are intrigued by the changes in performance across grade levels. Of course, since we cannot explain the general effect, our *post hoc* speculations about grade changes are even more tentative than usual. Nevertheless, we do wish to point out that performance at the two highest text levels is quite different than performance at other levels since they are both more successful and show only a main effect of structure with no interaction. The schemas that operate here may not represent individual structure-local relation combinations but rather patterns that are independent of local relations. Should this speculation prove correct, we emphasize the fact that the subjects in these groups are not true sixth and seventh graders, but rather advanced fourth and fifth graders classified by textbook level. Differences in performance may therefore be due to either general increases in expertise or the special talents of these subjects.

### *Summary*

We have described our analysis of problem structure based on the possible logical combinations of one-step word problems to generate two-step word problems. We provided evidence that the different problem structures define equivalence classes of problems and affect problem solving difficulty. The other possible effects (operator set and structure x operator set) were differentially significant across grade levels. We suggested that further analysis may support the idea that each problem structure consitutes an individual schema that guides operator selection and hence problem solving success.

### *References*

Bobrow, D. G., & Winograd, T. (1977). An overview of KRL, a knowledge representation language. *Cognitive Science*, 1, 3-46.

Gagne R. M. (1983). Learning hierarchies. *Educational Psychologist*, 6, 1-9.

Greeno, J. G. (1976) Cognitive objectives of instruction: Theory of knowledge for solving problems and answering questions. In D. Klahr (Ed.), *Cognition and instruction*. Hillsdale, N. J.: Lawrence Erlbaum Associates.

Greeno, J. G (1982) Forms of understanding in mathematical problem solving. In S. Paris, G. M. Olson & H. W. Stevenson (Eds.), *Learning and motivation in the classroom*. Hillsdale, N. J.: Lawrence Erlbaum Associates.

Riley, M. S., Greeno, J. G. & Heller, J. I. (1983) Development of children's problem-solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking*. New York: Academic Press.

Schwartz, J. (1976) Semantic aspects of quantity. M. I. T. Division for Study and Research in Education.

Wertheimer, M. (1959) Productive thinking (enlarged edition). New York: Harper (first edition published 1945).

Table 1

Word Problems Used in the Study

***Problems With Two Additive Relations***

*Hierarchical Problems*

A neighborhood is planning its annual cat show. There are 22 siamese, angora and persian cats altogether, and 8 of these are persian cats. If 9 of the cats are angora cats, how many siamese cats are there?

Jenny is buying candies to fill her candy jar. She buys 25 lime candies and 35 cherry candies. If Jenny buys 32 grape candies, how many lime, cherry and grape candies will she have altogether?

*Shared-whole Problems*

A sporting goods shopkeeper is checking her inventory list. She has 114 soccer and tennis balls altogether, and 62 golf balls. If 117 of the balls are soccer and golf balls, how many tennis balls does she have?

Jim is preparing breakfast orders for his diner customers. He has orders for 18 scrambled and fried eggs altogether, and for 5 boiled eggs. If 11 of the orders are for fried eggs, how many orders are there for scrambled and boiled eggs altogether?

*Shared-part Problems*

Susan is counting the stickers in her collection. She has 205 sniffy, puffy and fuzzy stickers altogether, and 122 of these are sniffy and puffy stickers. If 162 of the stickers are fuzzy and sniffy stickers, how many sniffy stickers does she have?

Robert is fixing snacks for his men's club meeting. He has 206 onion and rye crackers altogether, and 116 of these are rye crackers. If Robert has 324 onion, rye and wheat crackers altogether, how many rye and wheat crackers are there?

Maggie picked up her family's clothing from the clearners. She brought home 16 white and blue shirts altogether, and 9 of these are white shirts. If 14 of the shirts are white and pink shirts, how many white, blue and pink shirts did she bring home altogether?

Greg is counting the records in his collection. He has 72 country, rock, and soul records altogether, and 46 of these are country and rock records. If 28 of the records are country records, how many country and soul records does he have altogether?

***Problems with One Additive and One Multiplicative Relation***

*Hierarchical Problems* TYPE 1

Mr. Brown is planting fruit trees in his orchards. He plants 30 rows of peach trees and 25 rows of apple trees. If there are 16 trees in each row of trees, how many peach and apple trees does Mr. Brown plant?

Tom is restocking the shelves in his grocery store. He has 840 diet and regular cokes altogether, and there are 8 cokes in each carton of cokes. If this makes 65 cartons of regular coke, how many cartons of diet coke does Tom have?

The Ace Building Company is drawing plans to build a sports field. They plan to install 22 rows with red seats and 18 rows with blue seats. If there are 400 red and blue seats altogether, how many seats will there be in each row of seats?

Peggy is planting flower seeds in her garden. She has 17 packages of petunia and marigold seeds altogether, and 8 of these packages are petunia seeds. If there are 990 marigold seeds altogether, how many seeds are in each package?

*Hierarchical Problems* TYPE 2

The Reading Railroad Company is trying to decide if it has enough seats on its trains. There are 4 cars of first class seats on its trains, and 14 seats in each car of seats. If there are 168 first and second class seats altogether, how many second class seats are there?

Grandma Clausen is making pickles this spring. There are 100 pickles in each barrel of pickles and 3 barrels of dill pickles. If there are 200 sweet pickles, how many dill and sweet pickles is she making altogether?

The cook for Camp Pocono is putting together a fruit salad using canned fruit. He uses 240 purple and yellow plums altogether, and 180 of these are yellow plums. If the purple plums came from 3 cans, how many plums are in each can of plums?

June delivers morning and evening newspapers. There are 140 morning and evening newspapers altogether and 60 of these are evening newspapers. If there are 20 newspapers in each bundle of newspapers, how many bundles of morning newspapers does she deliver?

*Shared-whole problems*

Mr. Candyman is grouping his jelly beans in jars. He has 240 mint jelly beans and 300 spice jelly beans. If there are 60 jelly beans in each jar of jelly beans, how many jars of spice and mint jelly beans does he have altogether?

The Girl Scouts are selling cookies for their annual fund raiser. There are 24 cookies in each box of cookies and 34 boxes of chocolate and vanilla cookies. If there are 360 vanilla cookies in these boxes, how many chocolate cookies are there?

A fruit market owner stores his grapefruits in bins. He has 125 pink grapefruits and 175 white grapefruits. If he has 5 bins of pink and white grapefruits, how many grapefruits are in each bin of grapefruits?

*Shared-part problems*

Peggy is planting flower seeds in her garden. She has 17 packages of petunia and marigold seeds altogether, and 8 of these packages are petunia seeds. If there are 990 marigold seeds altogether, how many seeds are in each package?

Paul works in a donut shop. He made 72 glazed donuts and put 12 donuts in each box of donuts. If he made 14 boxes of glazed and powdered donuts altogether, how many boxes of powdered donuts did he make?

Mrs. Bell baked homemade buns for a family gathering. She baked 48 sesame buns and put 6 buns in each packages of buns. If Mrs. Bell made 4 packages of plain buns, how many packages of sesame and plain buns are there altogether?

Sally is wrapping dishes in paper towels. She has 9 rolls of yellow and green paper towels altogether, and 6 of these are rolls of green paper towels. If there are 180 paper towels in each roll, how many yellow paper towels does Sally have?

## *Problems with Two Multiplicative Relations.*

### *Hierarchical problems*

Jackie is looking at the photos in her photo albums. She has 300 photos altogether and 4 photos can be fit on each page of photos. If she has 15 pages in each album of photos, how many albums of photos does she have?

The children's baseball committee is organizing the players into teams and leagues. There are 90 children altogether, and 9 children on each team. If there are 2 leagues, how many teams are there in each league?

A western club is planning to cross the United States in covered wagons. There are 7 wagons in each train, and 3 trains. If there are 4 horses for each wagon, how many horses will there be?

A fruit dealer is packaging berries for sale. There are 3 cases of berries and 8 cartons of berries in each case. If there are 480 berries, how many berries are there in each carton?

### *Shared-whole problems*

The members of the model train club made houses out of popsicle sticks for their next display. They made 35 houses, using 8 popsicle sticks for each house. If there were 7 members altogether, how many popsicle sticks did each member have in the beginning?

Dr. Wizard has discovered a group of monsters living in a dark cave in South America. He has counted 7 monsters and there are 8 fingers on each monster. If he has counted 14 monster hands, how many fingers are there on each monster hand?

The Pittsburgh recreation department adds fish to its ponds every year. There are 3 parks altogether, and 200 fish in each park. If there are 100 fish in each pond, how many ponds are there?

Sam is making meat sandwiches for children going on a school picnic. He is making 60 sandwiches using 2 slices of meat on each sandwich. If he planned on using 4 slices of meat for each child, how many children is he expecting?

### *Shared-part problems* TYPE 1 - PROBLEMS USING INTENSIVES

Ginny is selling bunches of roses for her garden club. She has 18 bunches of red roses and 6 bunches of white roses. If there are 72 red roses, how many white roses does Ginny have?

An egg farm is selling brown and white eggs by the carton. They have 96 white eggs and 48 brown eggs. If there are 4 cartons of brown eggs, how many cartons of white eggs are there?

### *Shared-part problems* TYPE 2 - RATIO PROBLEMS

Andy is mixing punch for a party. He mixes 6 cans of orange juice and 2 cans of pineapple juice. If Andy uses 8 cans of pineapple juice for a bigger batch, how many cans of orange juice will he use?

Deborah feeds the birds in a pet shop. She makes special bird seed with 10 bags of sunflower seeds and 2 bags of flax seeds. If Deborah makes the special bird seed using 40 bags of sunflower seeds, how many bags of flax seeds will she use?

Table 2

A Summary of the Word Problems Derived from Problem Structures

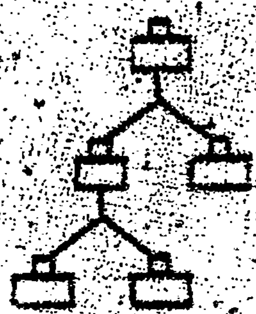
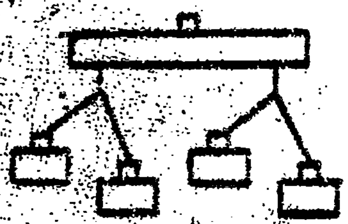
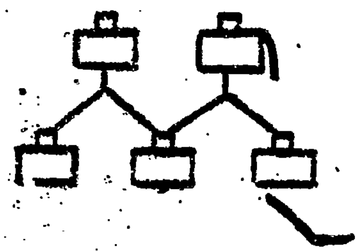
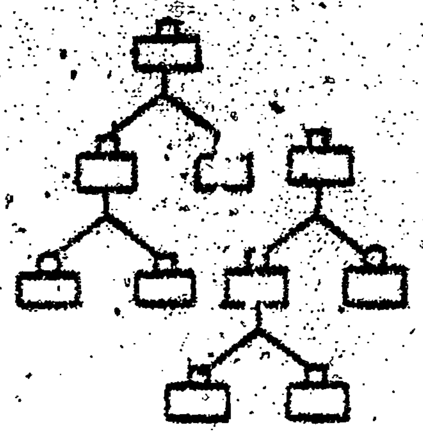
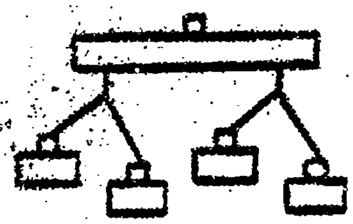
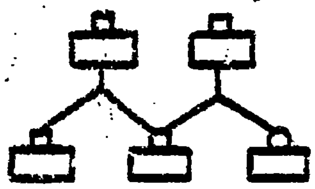
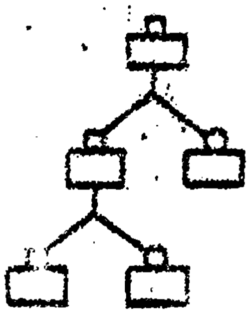
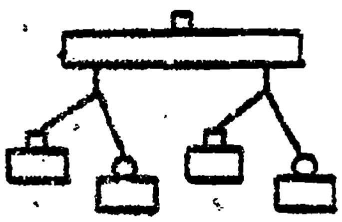
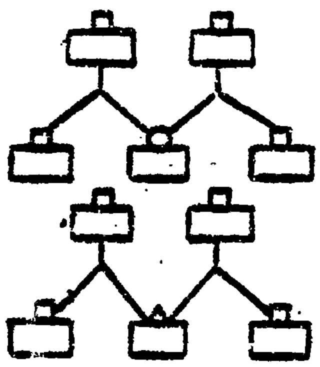
| | Structure Type | | |
|---|---|---|---|
| Operator Set | Hierarchy | Shared-whole | Shared-part |
| +,+ |  2 Problems |  2 Problems |  4 Problems |
| +,x |  7 Problems |  3 Problems |  4 Problems |
| x,x |  4 Problems |  4 Problems |  4 Problems |

Table 3

Third Grade Text Level

Structure Type

| Operator Set | Hierarchy | Shared-whole | Shared-part | Row Total |
|---|---|---|---|---|
| +,+ | .44 | 0 | 0 | .11 |
| +,x | .03 | 0 | .08 | .03 |
| x,x | .22 | 0 | .10 | .10 |
| Column Total | .15 | 0 | .05 | .08 |

Table 4

Fourth Grade Text Level

Structure Type

| Operator Set | Hierarchy | Shared-whole | Shared-part | Row Total |
|---|---|---|---|---|
| +,+ | .75 | .12 | .05 | .29 |
| +,x | .21 | .23 | .19 | .19 |
| x,x | .17 | .14 | .13 | .15 |
| Column Total | .30 | .16 | .10 | .20 |

## Table 5

### Fifth Grade Text Level

Structure Type

| Operator Set | Hierarchy | Shared-whole | Shared-part | Row Total |
|---|---|---|---|---|
| +,+ | .79 | .56 | .32 | .49 |
| +,x | .44 | .45 | .46 | .44 |
| x,x | .43 | .14 | .35 | .28 |
| Column Total | .56 | .34 | .37 | .40 |

## Table 6

### Sixth Grade Text Level

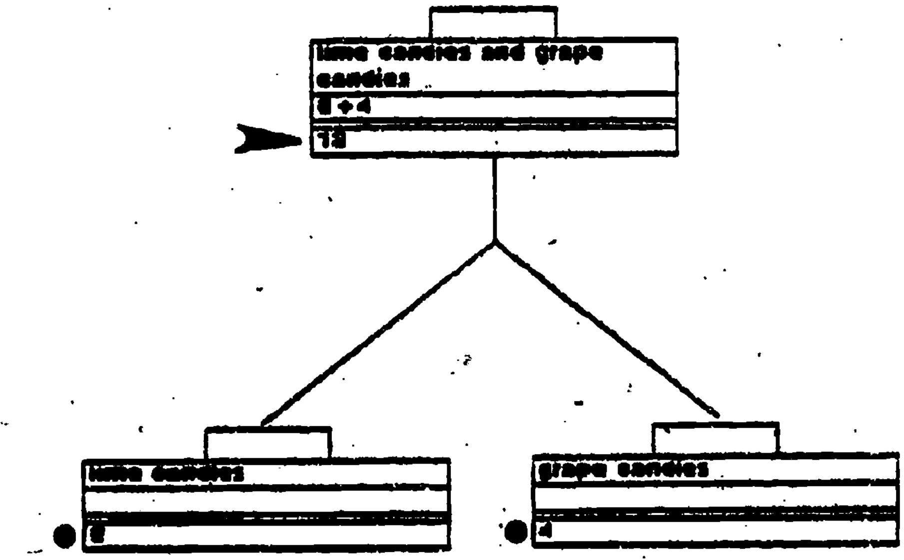
Structure Type

| Operator Set | Hierarchy | Shared-whole | Shared-part | Row Total |
|---|---|---|---|---|
| +,+ | .92 | .88 | .46 | .70 |
| +,x | .88 | .77 | .73 | .82 |
| x,x | .83 | .60 | .78 | .74 |
| Column Total | .87 | .72 | .66 | .76 |

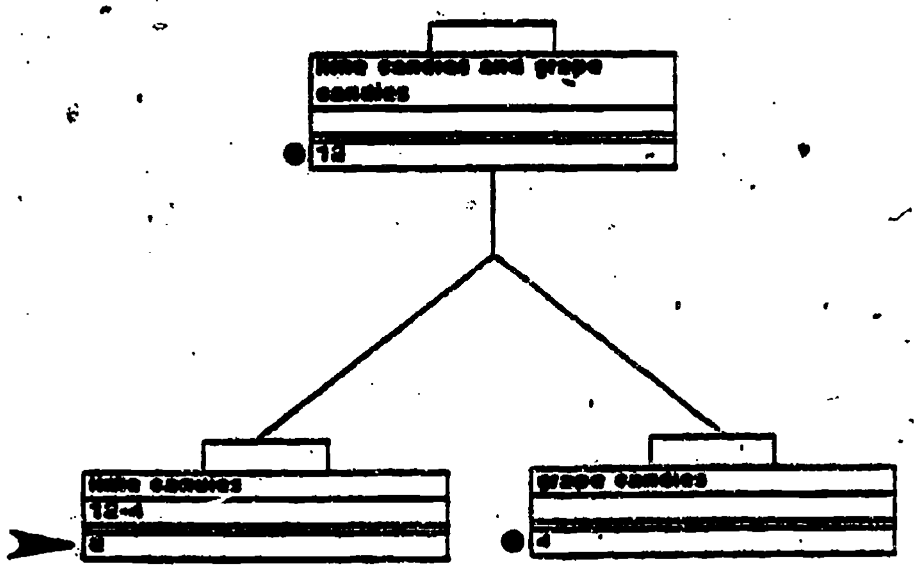
Table 7

Seventh Grade Text Level

| | Structure Type | | | |
|---|---|---|---|---|
| | Hierarchy | Shared-whole | Shared-part | |
| Operator Set | | | | Row Total |
| +,+ | .89 | .67 | .44 | .62 |
| +,x | .82 | .86 | .47 | .75 |
| x,x | 1.00 | .59 | .82 | .80 |
| Column Total | .88 | .70 | .58 | .73 |

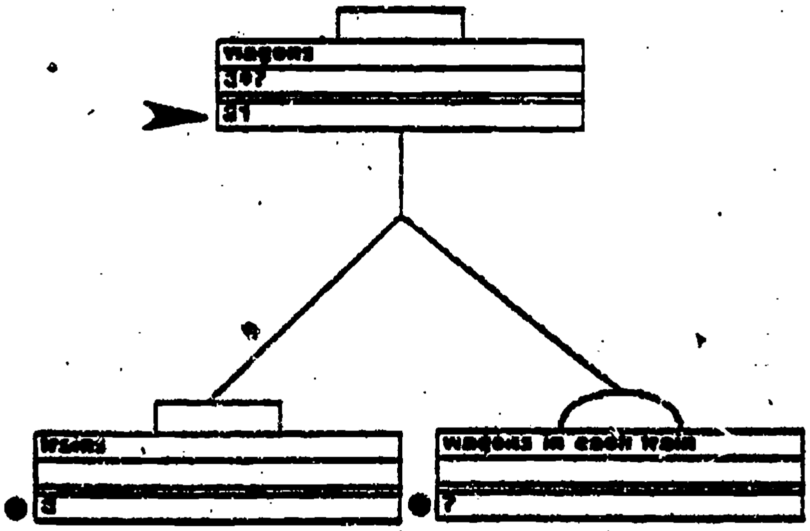



Jenny is buying candy. There are 8 lime candies and 4 grape candies. How many lime and grape candies are there altogether?

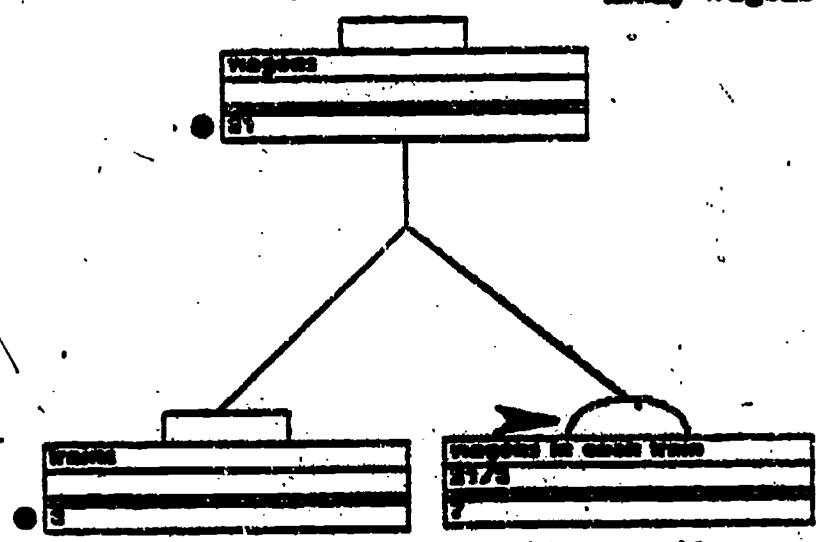



Jenny is buying candy. There are 12 lime and grape candies altogether, and 4 of these are grape candies. How many lime candies are there?

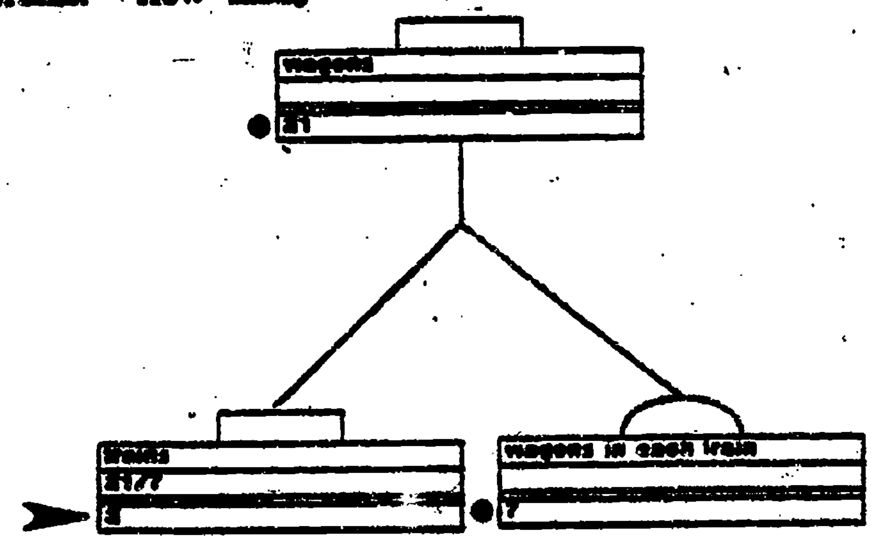
Figure 1. A network notation for two different additive combination problems.




**A western club is planning a trip in covered wagons. There are 7 wagons in each train and 3 trains. How many wagons are there?**

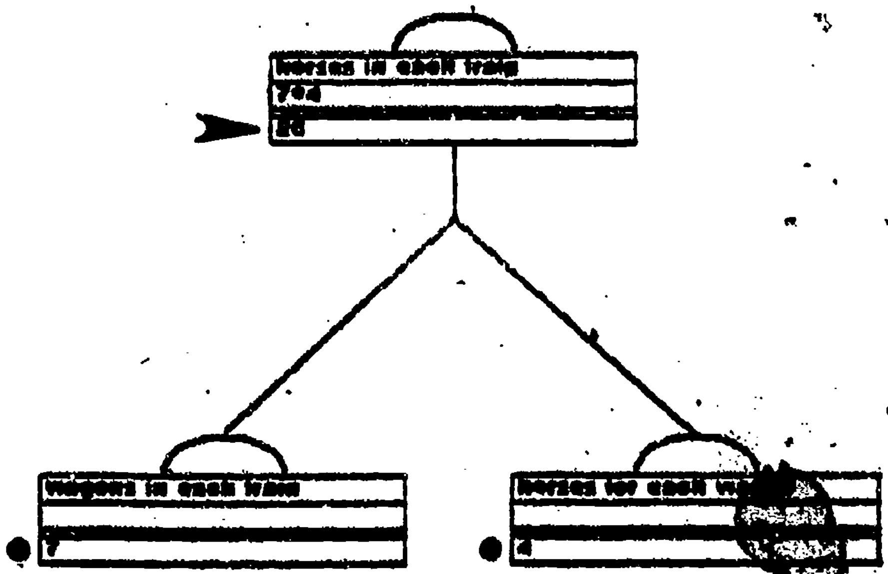



**A western club is planning a trip in covered wagons. There are 21 wagons and 3 trains. How many wagons are there in each train?**

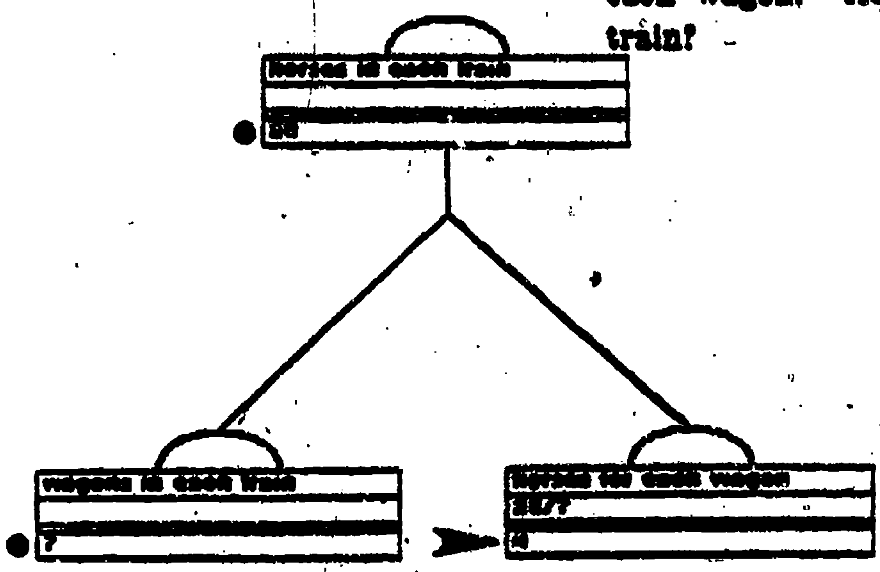



**A western club is planning a trip in covered wagons. There are 21 wagons and 7 wagons in each train. How many trains are there?**

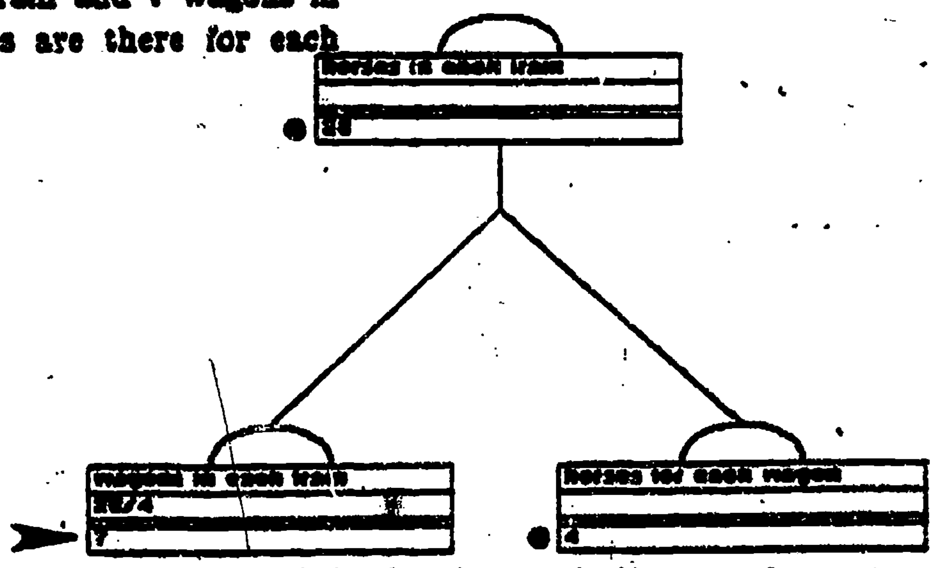
**Figure 2. Network notation for three different simple multiplication problems.**



A western club is planning a trip in covered wagons. There are 7 wagons in each train and 4 horses for each wagon. How many horses are there in each train?

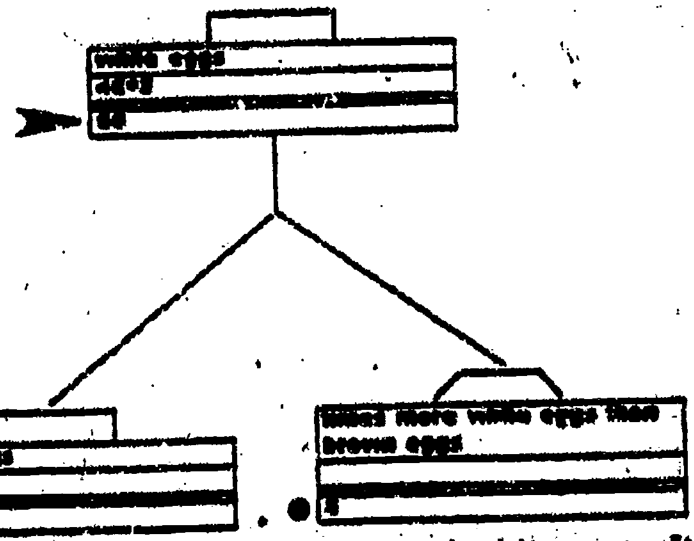


A western club is planning a trip in covered wagons. There are 28 horses in each train and 7 wagons in each train. How many horses are there for each wagon?

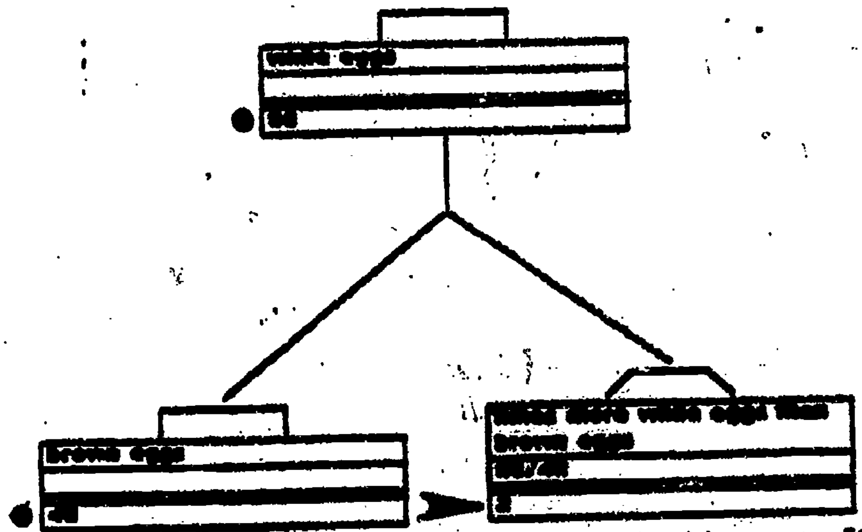


A western club is planning a trip in covered wagons. There are 28 horses in each train and 4 horses for each wagon. How many wagons are there in each train?

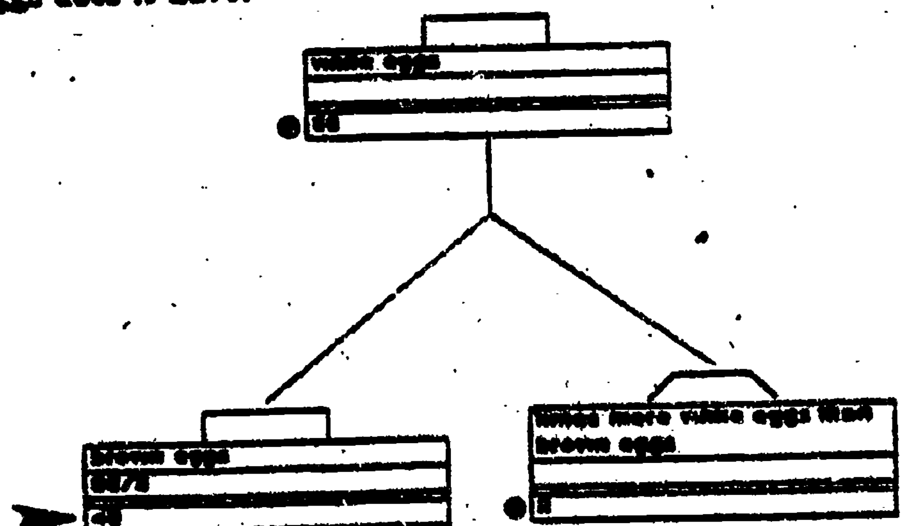
Figure 3. Network notation for three different multiplication problems with three intensive quantities.



An egg farm is selling brown eggs and white eggs. It has 48 brown eggs and 2 times more white eggs than brown eggs. How many white eggs does it have?



An egg farm is selling brown eggs and white eggs. It has 96 white eggs and 2 times more white eggs than brown eggs. How many brown eggs does it have?



An egg farm is selling brown eggs and white eggs. It has 96 white eggs and 48 brown eggs. How many times more white eggs that brown eggs does it have?

Figure 4. Network notation for three multiplicative comparison problems.

A neighborhood is planning its annual cat show. There are 72 siamese, angora and persian cats altogether, and 26 of these are persian cats. If 24 of the cats are angora cats, how many siamese cats are there?

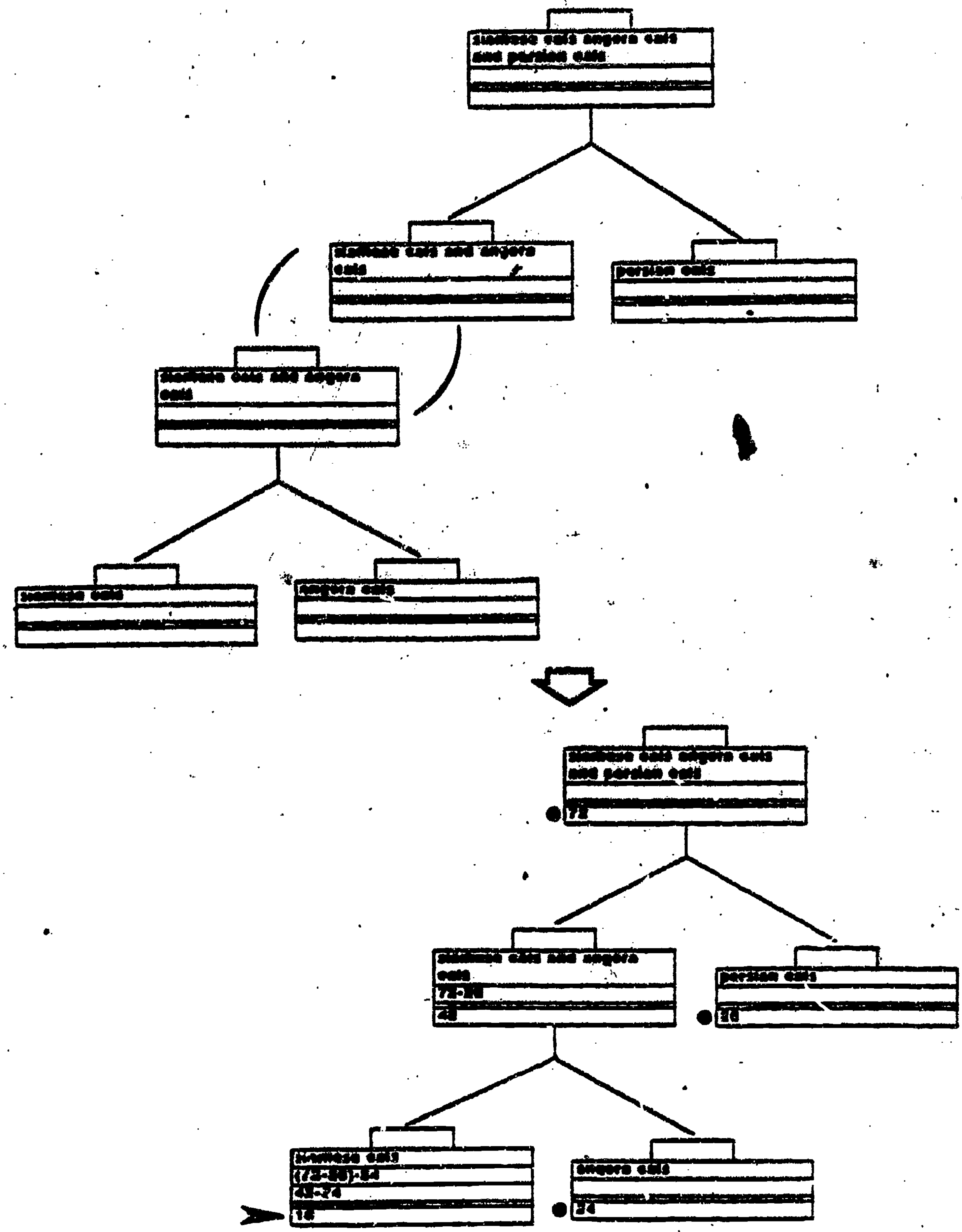



Figure 5. An additive hierarchy formed by stacking two additive triads.

A sporting goods shopkeeper is checking her inventory list. She has 13 soccer and tennis balls altogether, and 9 golf balls. If 14 of the balls are soccer and golf balls, how many tennis balls does she have?

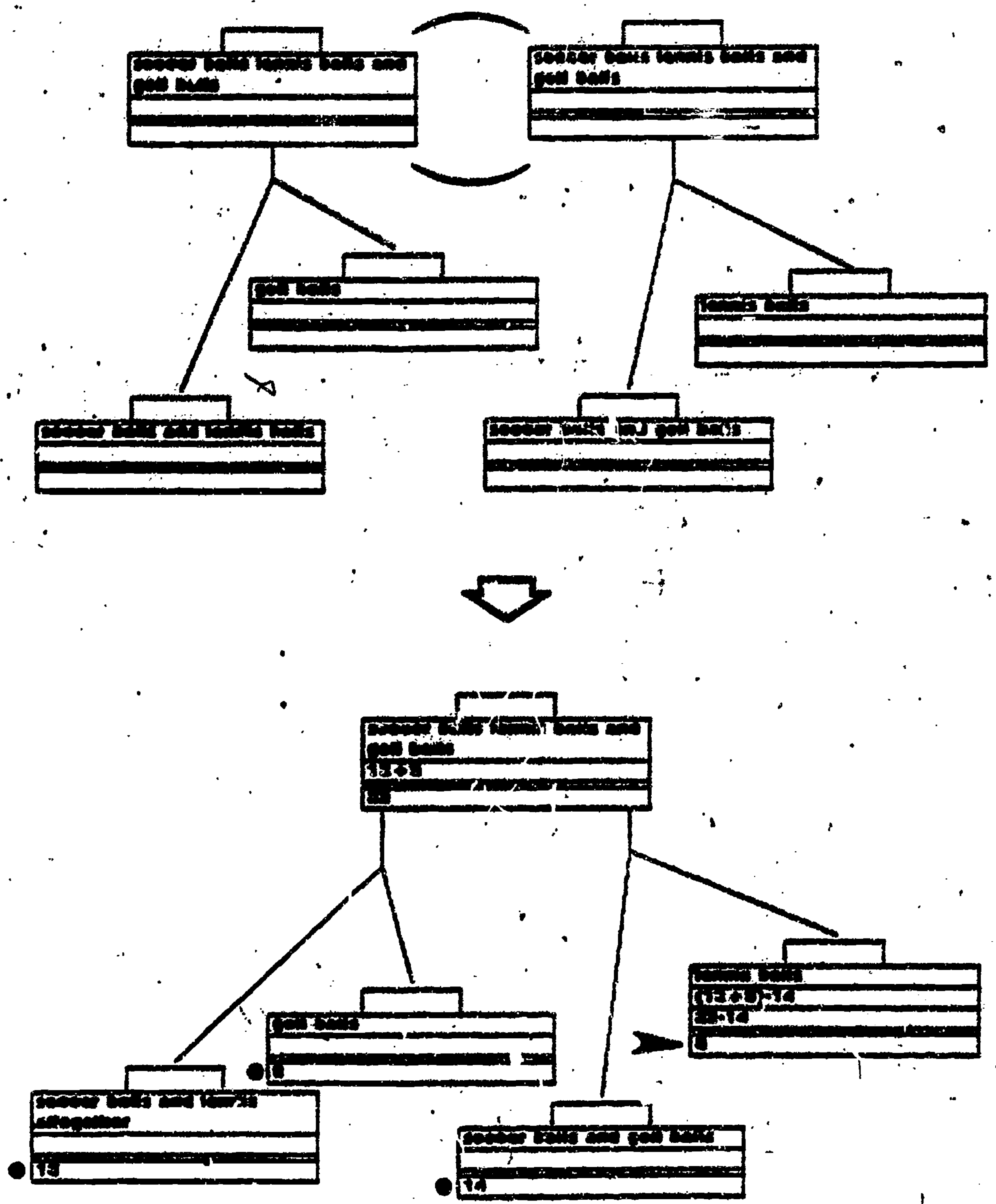



Figure 6. An additive shared-whole structure formed by two additive triads that share the same whole quantity.

Robert is fixing snacks for his men's club meeting. He has 206 onion and rye crackers altogether, and 116 of these are rye crackers. If Robert has 324 onion, rye and wheat crackers altogether, how many rye and wheat crackers are there?

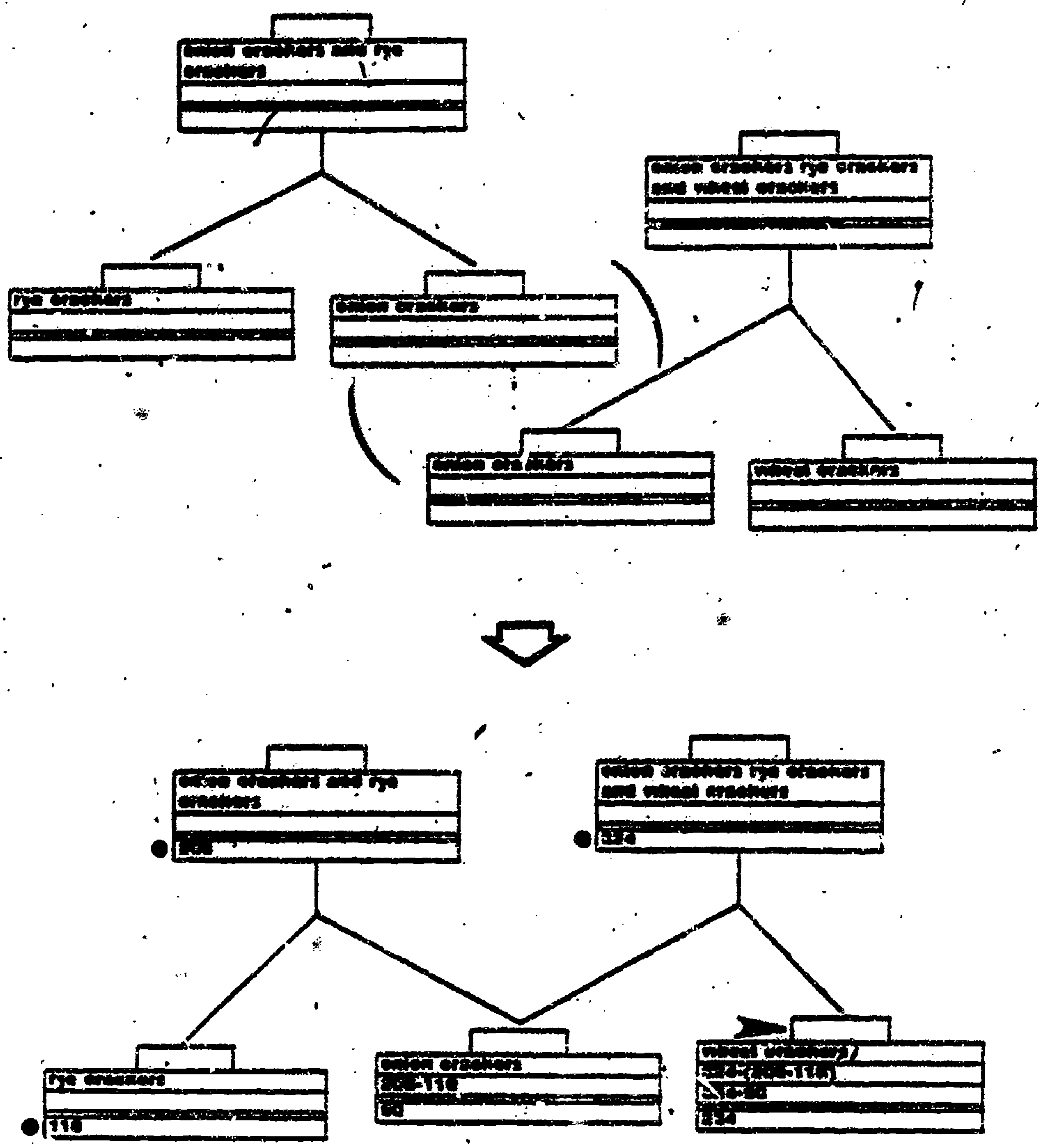
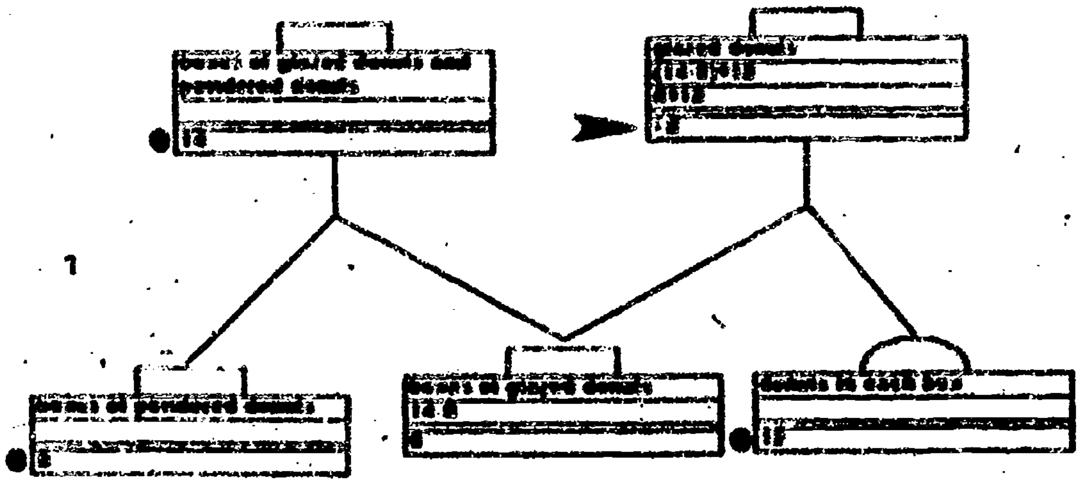
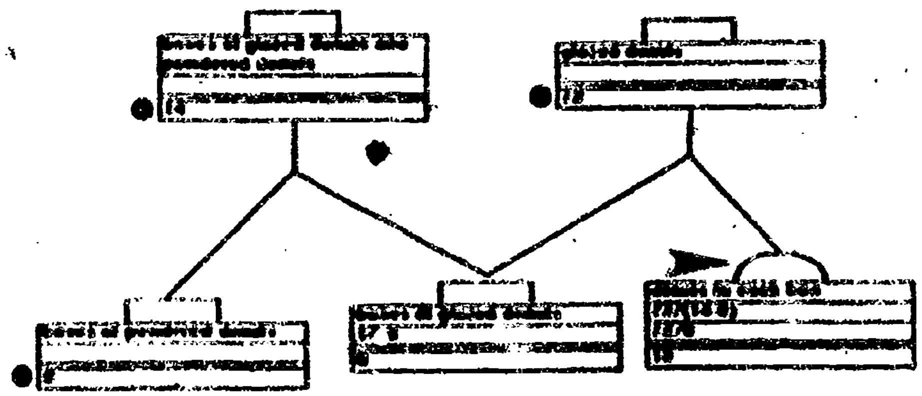


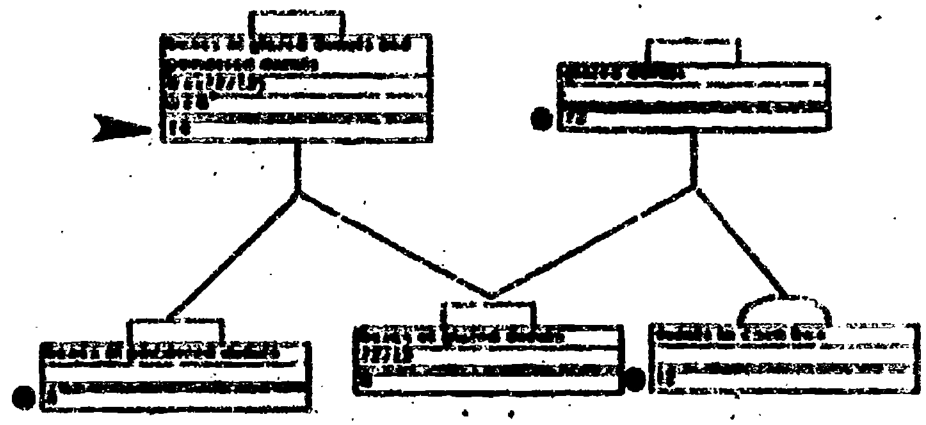
Figure 7. An additive shared-part structure formed by two additive triads that share the same part.



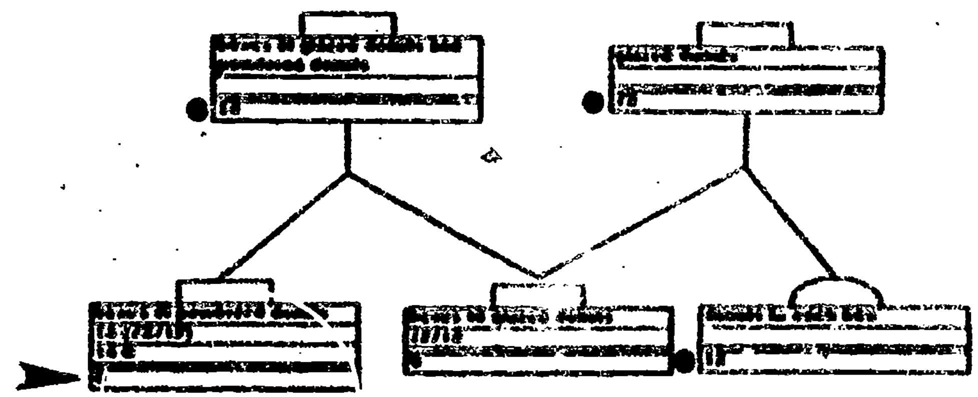
Paul works in a donut shop. He made 14 boxes of glazed and powdered donuts and 8 of these boxes were powdered donuts. If he put 12 donuts in each box, how many glazed donuts did he make?



Paul works in a donut shop. He made 14 boxes of glazed and powdered donuts and 8 of these boxes were powdered donuts. If he made 72 glazed donuts, how many donuts are there in each box?



Paul works in a donut shop. He made 72 glazed donuts and put 12 donuts in each box of donuts. If he made 8 boxes of powdered donuts, how many boxes of powdered and glazed donuts did he make altogether?



Paul works in a donut shop. He made 72 glazed donuts and put 12 donuts in each box of donuts. If he made 14 boxes of glazed and powdered donuts altogether, how many boxes of powdered donuts did he make?

Figure 8. The equivalence class of mixed relation shared-part problems.

Jackie is looking at the photos in her photo albums. She has 300 photos altogether and 4 photos can be fit on each page of photos. If she has 15 pages in each album of photos, how many albums of photos does she have?

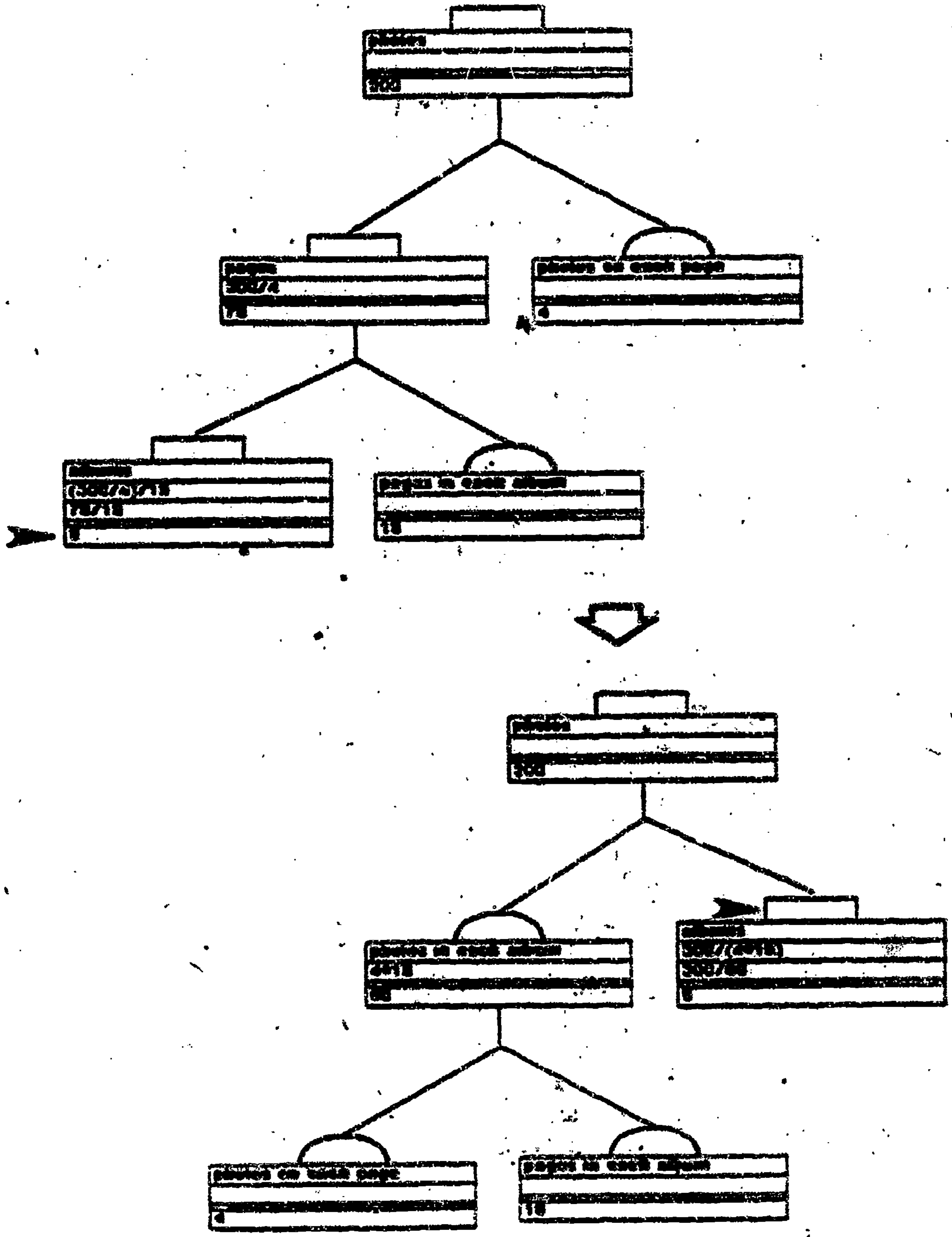


Figure 9. Converting a multiplicative hierarchical problem.