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ABSTRACT

A 3-year study was conducted to determine the logical reasoning processes necessary to construct line graphs. Responses obtained from middle and high school science and mathematics students were classified into one of nine categories. These categories ranged from "no attempt to make a graph" to "complete graph with a statement of a relationship between the variables." The categories in between represented increasingly more successful attempts at ordering data in one and both variables to correct scaling of the data on the axes. Middle school subjects exhibited behaviors mainly in the first four categories, 9th- and 10th-grade subjects overlapped with middle school subjects and the 11th- and 12th-grade subjects in the middle categories, and the 11th- and 12th-grade subjects exhibited behaviors mainly in the last five categories. These response categories also showed a close fit with Piagetian concrete operational structures for single and double seriation and formal operational structures for proportional reasoning and correlational reasoning. Teachers can use this information: (1) to determine what logical reasoning students will bring to a graphing situation; (2) to understand the reasons why students make certain mistakes when they make line graphs; and (3) to make interventions that will help students make their graphs correctly. (Author/JN)

The Logical Reasoning Necessary to Make Line Graphs

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Abstract

The Logical Reasoning Necessary to Make Line Graphs

A three year study was conducted to determine the logical reasoning processes necessary to construct line graphs. Three types of line graphs were used: a straight line with a positive slope, a straight line with a negative slope, and an exponentially increasing curve. This area was chosen for investigation because of the importance in science of displaying information graphically. Previous research by other investigators was concerned with the accuracy of reading various types of graphs and with measuring graphing skills. This research determined the underlying logical reasoning processes needed to make line graphs.

The three research instruments used consisted of a set of instructions, data to make a graph, and two unlined pieces of paper. The subjects used were middle school and high school science and mathematics students who were allowed to use a ruler if necessary and allowed to use as much time as needed to complete the task.

The responses were classified into one of nine categories. The categories ranged from no attempt to make a graph to complete graph with a statement of a relationship between the variables. The categories in between represented increasingly more successful attempts at ordering data in one and both variables to

correct scaling of the data on the axes. The middle categories also represented increasingly more successful attempts at establishing a one-to-one correspondence between the variables to pattern recognition leading to recognition of a relationship between the variables. Middle school subjects exhibited behaviors mainly in the first four categories, ninth and tenth grade subjects overlapped with the middle school subject and the eleventh and twelfth grade subjects in the middle categories, and the eleventh and twelfth grade subjects exhibited behaviors mainly in the last five categories.

The categories appeared to be valid with the three types of line graphs and the subject response patterns were the same on all the instruments. These response categories also showed a close fit with Piagetian concrete operational structures for single and double seriation and formal operational structures for proportional reasoning and correlational reasoning.

The implications of this research for science teaching are these. Teachers can now be aware of what logical reasoning students will bring to a graphing situation. Teachers will also be able to understand the reasons why students make certain mistakes when they make line graphs. Teachers will be able to make interventions that will not only help students make their graphs correctly but possibly will help students develop the logical reasoning to make their graphs correctly on future occasions.

The Logical Reasoning Necessary to Make Line Graphs

I. Statement of Problem

Over the past three years research has been conducted to make clear the types of reasoning processes students in middle and high school use when they attempt to make a graph. The first year of the study was used to determine what kind of reasoning patterns students use when graphing a line with a positive slope. The second year was used to determine if patterns found in the first year of the study reappeared when students graphed a line with a negative slope. The third year of the study was used to determine what effect graphing an exponential curve had on exhibited reasoning patterns.

II. Previous Research

Graphing is a tool used in science to display data and aid in the analysis of relationships between variables. It would be helpful to science teachers to understand the logical reasoning processses students use when making graphs so that teachers would know what the basis for faulty graph construction and misinterpretation. Previous research was concerned chiefly with ability to read various types of graphs and with measuring graphing skills. Peterson and Schramm (1954) determined accuracy of reading various types of graphs. Culbertson and Powers (1959) studied comprehension difficulties with graphs. Weintraub (1967) determined that children interpret graphs with increasing difficulty in the order: pictorial, circular, two-dimensional, and line graphs. McKenzie and Padilla (1982) developed a test to measure graphing skills in science. The research reported in this paper determined the logical reasoning patterns necessary to make line graphs.

III. Research Procedure

To measure student reasoning concerning graphing different research instruments were developed each year: Research Instrument 1 for a line with a positive slope; Research Instrument 2 for a line with a negative slope; and Research Instrument 3 for an exponential curve. On the research instrument the student was told to make a graph with the numbers on the sheets provided and to write down why they are doing what they do to make the graph. They are also asked to identify a pattern in the graphed points and state a relationship, if there is one. In each case the student was give a copy of the research instrument and two unlined sheets of paper. The student was told they could use a ruler and had as much time as they needed to complete the task. They were also told to do the task the way they understood what the research instrument said. No other instructions or help were given.

The research instruments were given to math and science classes in the middle and high school. With the exception of the physics and chemistry class, all of the classes were representative of the school population as a whole.

IV. Results

The responses were classified into one of the nine categories. Category 1 is no attempt to make a graph. In category 2 data is ordered in one or both columns, but no one-to-one correspondence is made between variables. In category 3 a one-to-one correspondence is established between the variables and the data is plotted on the graph in the order it appears on the research instrument. In category 4 one or both of the columns is ordered, but there is no scaling of either axis on the graph, instead the order numbers are equally spaced. In category 5 one axis is ordered and the other axis is scaled. In category 5.5 data in both columns is rearranged from high to low and a new one-to-one correspondence is established (used for lines with a negative slope) or one axis has one or more scaling breaks (used for an exponential curve). In category 6 axes on graph are correctly scaled, but student does not see a relationship between the variables. In category 7 the student states the correct relationship between the variables. Category 8 is characteristic of an exponential curve in that the student recognizes that not only does as one variable increases so does the other but also realizes that this is not a straight line relationship. Figures 1 through 10 depict student made graphs that are characteristic of these nine response categories.

The collected data showed obvious differences between middle school and high school responses. Most middle school students gave responses in the first four categories. Most high school students gave responses the last four or five (for the exponential curve) categories with a little overlap in categories one through four for grades nine and ten. It is significant that almost no middle school students responded in categories five and five-point-five.

The response categories for the research instruments show reasoning that can be characterized in Piagetian terms. Category 1 corresponds to properational reasoning. In categories 2 through 5 two parallel strands of concrete operational reasoning develop. In one strand seriation, first single then double, develops and in the other stand one-to-one correspondence leads into pattern recognition. These two parallel strands lead into formal operational reasoning in categories 5.5 through 8. The seriation strand becomes the proportional reasoning needed for scaling data on axes and the pattern recognition becomes the early correlational reasoning needed to recognize the relationship between variables.

This study supports the following conclusions. First, the response categories appear to be valid with the three types of graphs the data on the research instruments represent with the addition of category 5.5 and 8 for lines with negative slopes and exponential curves. Second, the student response patterns for grades six through twelve are similar for all three research instruments.

The above conclusions need to be viewed with caution. The eleventh and twelfth grade samples are not representative of the classes from which they are taken.

V. Implications

In teaching graphing and having students use it as a tool for classwork, teachers need to be aware of the reasoning processes students bring to these situations. Teachers can become familiar with typical mistakes and flaws in logic and seek appropriate means to help students understand their errors in graph construction. The logical progression from simple to complex reasoning involved in graphing suggests approaches to teaching graphing through the middle school and high school years. These approaches need to be concerned with using data that can be graphed using the scaling and ordering reasoning processes that students are developing.

References

Culbertson, H. and Powers, R. A study of graph comprehension difficulties. A-V Communication Review, 1959, 7, 211-213.

McKenzie, D. and Padilla, M. Test of graphing in science (TOGS). An unpublished test developed at the University of Georgia, Athens, 1982.

Peterson, L. and Schramm, W. How accurately are different kinds of graphs read? A-V Communication Review, 1954, 2, 178-179.

Weintraub, S. Reading graphs, charts and diagrams. Reading Teacher, 1967, 20, 345-349.

Table 1

Research Instrument 1

Make a graph with the following numbers. On one of the sheets provided make your graph. On the other sheet write down what you are doing and thinking as you do it.

Temperature (^{o}C)	Length (cm.)
225	1020
312	1270
186	937
351	1416
290	1237
301	1206
194	976
200	956
212	998
337	1305
325	1340
346	1377
283	1176
254	1092
236	1043
268	1119
275	1147
247	1067

What kind of pattern do the points on the graph make? If there is a relationship, what does the graph show? State it in a sentence.

Table 2

Research Instrument 2

Make a graph with the following numbers. On one of the sheets provided make your graph. On the other sheet write down why you are doing what you do to make the graph.

Temperature (^{o}C)	Length (m)
312	102
301	105
325	99
247	123
212	134
283	104
268	114
346	97
186	141
236	120
254	117
200	130
225	125
290	106
194	137
351	93
337	95
275	111

What kind of pattern do the points on the graph make? If there is a relationship, what does the graph show? State it in a sentence.

Table 3

Research Instrument 3

Make a graph with the following numbers. On one of the sheets provided make your graph. On the other sheet write down why you are doing what you do to make the graph.

Distance (m.)	Time (sec.)
3230	184
637	81
91	32
999	99
2563	158
253	49
3613	189
361	61
1443	118
2886	166
42	19
810	93
2248	155
9	12
1209	112
4004	202
1950	137
1695	131

What kind of pattern do the points on the graph make? If there is a relationship, what does the graph show? State it in a sentence.

Table 4

Response Categories From the Research Instruments

Category	Criteria
1	No attempt made at graphing. A story may be made up about the data.
2	Data is ordered in one or both columns, but no attempt is made to establish a one-to-one correspondence between the two variables to be graphed.
3	A one-to-one correspondence is established between pairs of data variables. Data is plotted on the graph in the order it is on the research instrument.
4	Same as category 3, except one or both of the data columns is now ordered. There is no scaling on either axis of the graph. Instead, the numbers appear equally spaced in either ascending or descending order.
5	Same as category 4, except one axis is scaled and the other axis ordered.
5.5	Arranges in order both variables from low to high and establishes a new one-to-one correspondence. Both axes are usually scaled. (This is characteristic of graphing a line with a negative slope.) One axis has one or more scaling breaks. (This is characteristic of graphing exponential curves.)
6	Same as category 5.5, except the axes on the graph are now scaled correctly. The student says there is no relationship between the variables because of the irregularities in the pattern of the points on the graph.
7	Same as category 6, except that the relationship is now accurately stated. The irregularities do not destroy the overall pattern made by the variables.
8	For exponential curves, recognition that as one axis of the graph increases the other does too but in a nonlinear fashion.

Table 5

Summary of Data

Research Instrument 1

Middle School

Grade	7		8	
	M	F	M	F
Category				
1	0	0	2	1
2	3	1	6	5
3	5	10	4	12
4	3	4	4	4
5	0	0	0	0
6	1	1	1	0

N = 69

High School

Grade	9		10		11		12	
	M	F	M	F	M	F	M	F
Category								
1&2	3	2	0	0	0	0	1	0
3&4	3	3	0	1	1	0	0	0
5	3	0	0	0	0	0	0	0
6	10	5	1	0	10	0	2	2
7	0	1	2	1	11	5	10	6

N = 83

Table 6

Summary of Data

Research Instrument 2

Middle School

Grade	6		7		8	
Category	M	F	M	F	M	F
1	3	1	1	3	0	0
2	3	3	2	2	1	1
3	1	0	2	2	2	0
4	3	4	0	7	4	5
5	0	0	0	0	3	0
5.5	0	0	0	0	2	0
6	6	1	8	5	3	3
7	0	0	2	2	0	2

N = 84

High School

Grade	9		10		11		12	
Category	M	F	M	F	M	F	M	F
1&2	2	1	3	4	3	2	0	0
3&4	4	4	3	3	4	6	0	2
5	3	2	2	3	1	3	0	1
5.5	5	1	2	1	0	0	2	0
6	12	16	10	12	4	16	3	2
7	3	1	4	5	8	8	10	4

N = 195

Table 7

Summary of Data

Research Instrument 3

Middle School

Grade	6		7		8	
Category	M	F	M	F	M	F
1	12	12	8	4	4	4
2	0	1	2	0	0	3
3	2	3	0	2	1	5
4	4	3	4	7	1	13
5	0	1	0	2	0	1
5.5	0	0	2	5	1	1
6	1	0	10	3	11	4
7	0	0	0	0	3	0

N = 139

High School

Grade	9		10		11		12	
Category	M	F	M	F	M	F	M	F
1&2	1	0	2	0	0	0	0	0
3&4	5	6	2	6	1	1	0	0
5	2	3	3	1	0	1	0	0
5.5	0	3	0	5	1	1	1	0
6	7	6	10	5	8	8	6	3
7	7	7	7	8	5	8	0	7
8	2	1	3	0	0	1	6	6

N = 164

Table 8

Proposed Overlap of Piagetian Operations

for Graphing

Stage	Operation (left arrow)	Operation (right arrow)	Category
			1
Concrete Operational Reasoning	Seriation Single (Ordering) ↓	One-to One Correspondence ↓	2
			3
	Seriation Double ↓	↓	4
	↓	Pattern Recognition	5
Formal Operational Reasoning	Scaling (Proportional Reasoning) ↓	↓	5.5
			6
		(Early Correlational Reasoning) ↓	7
	↓	↓	8

Figure 1

Illustration of Response Category 1

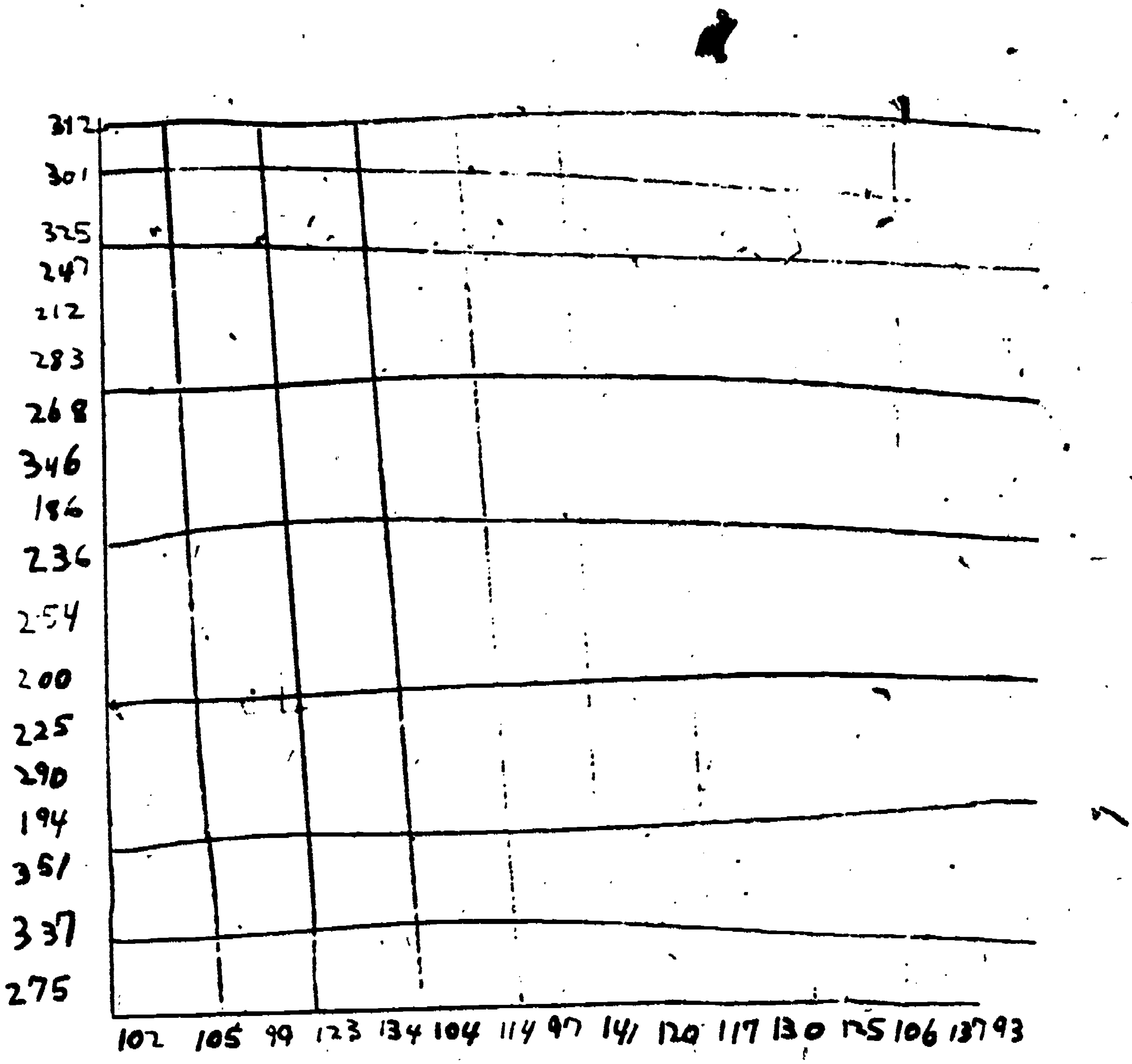

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Figure 2

Illustration of Response Category 2

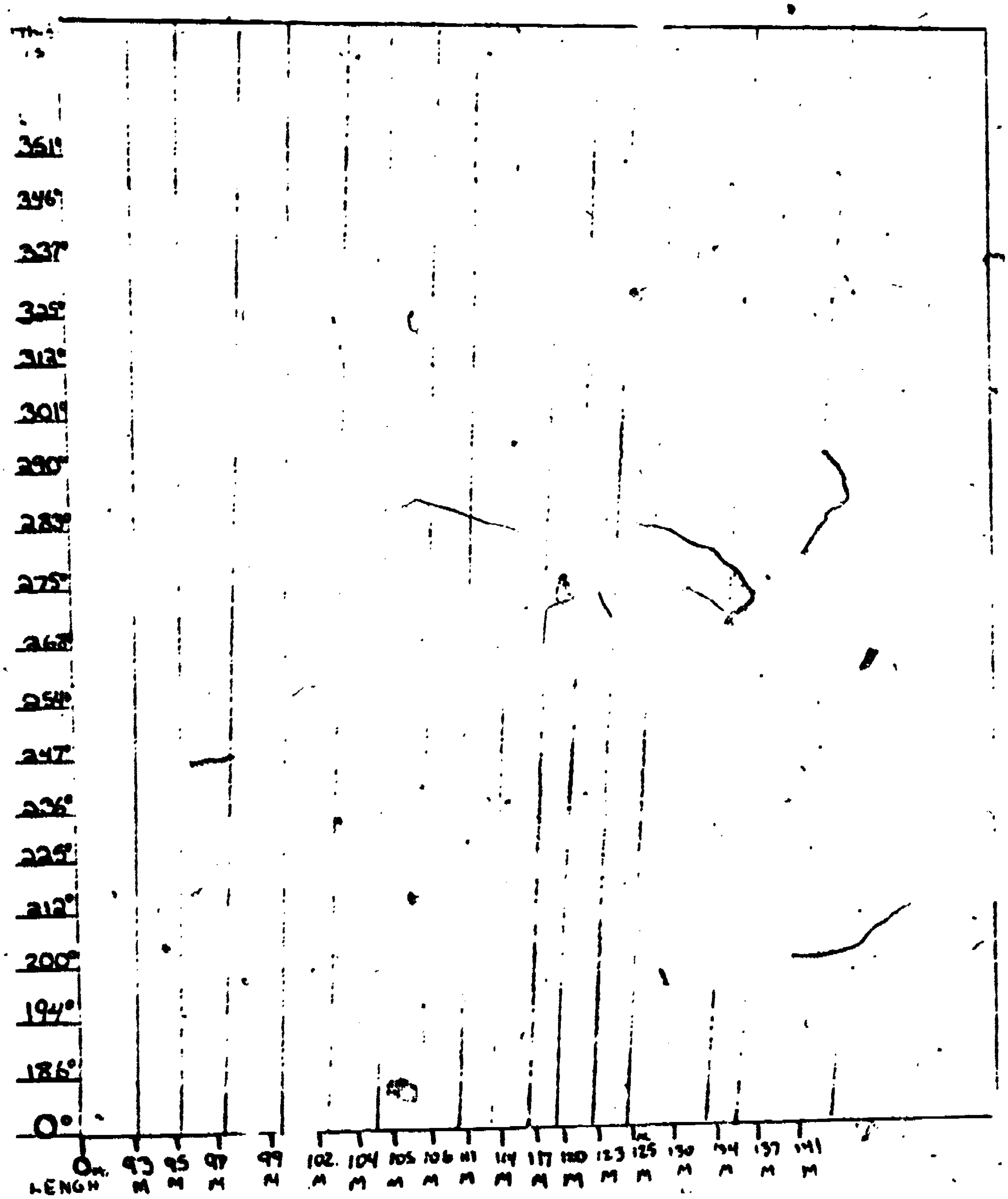

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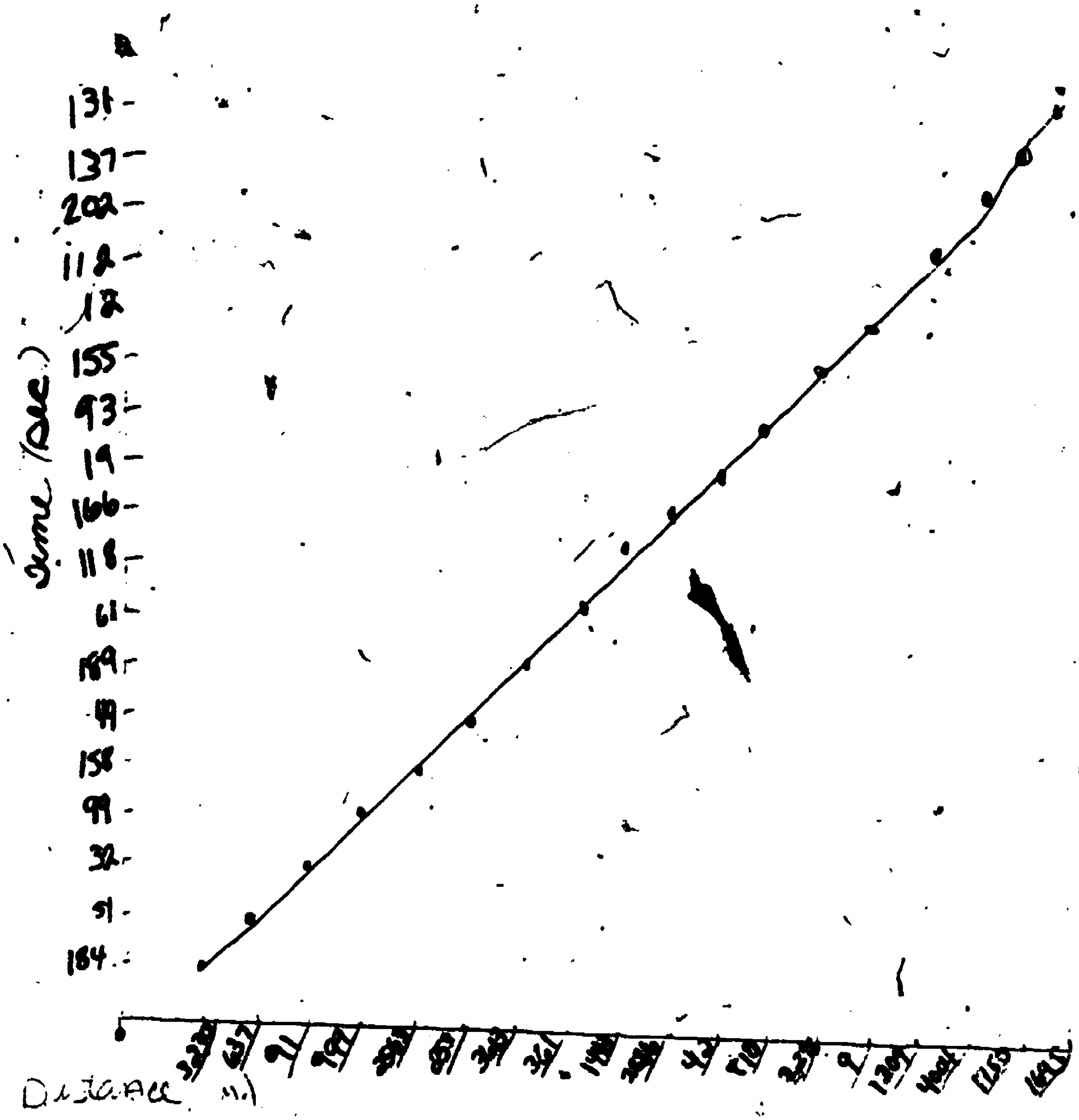
Figure 3

Illustration of Response Category 3



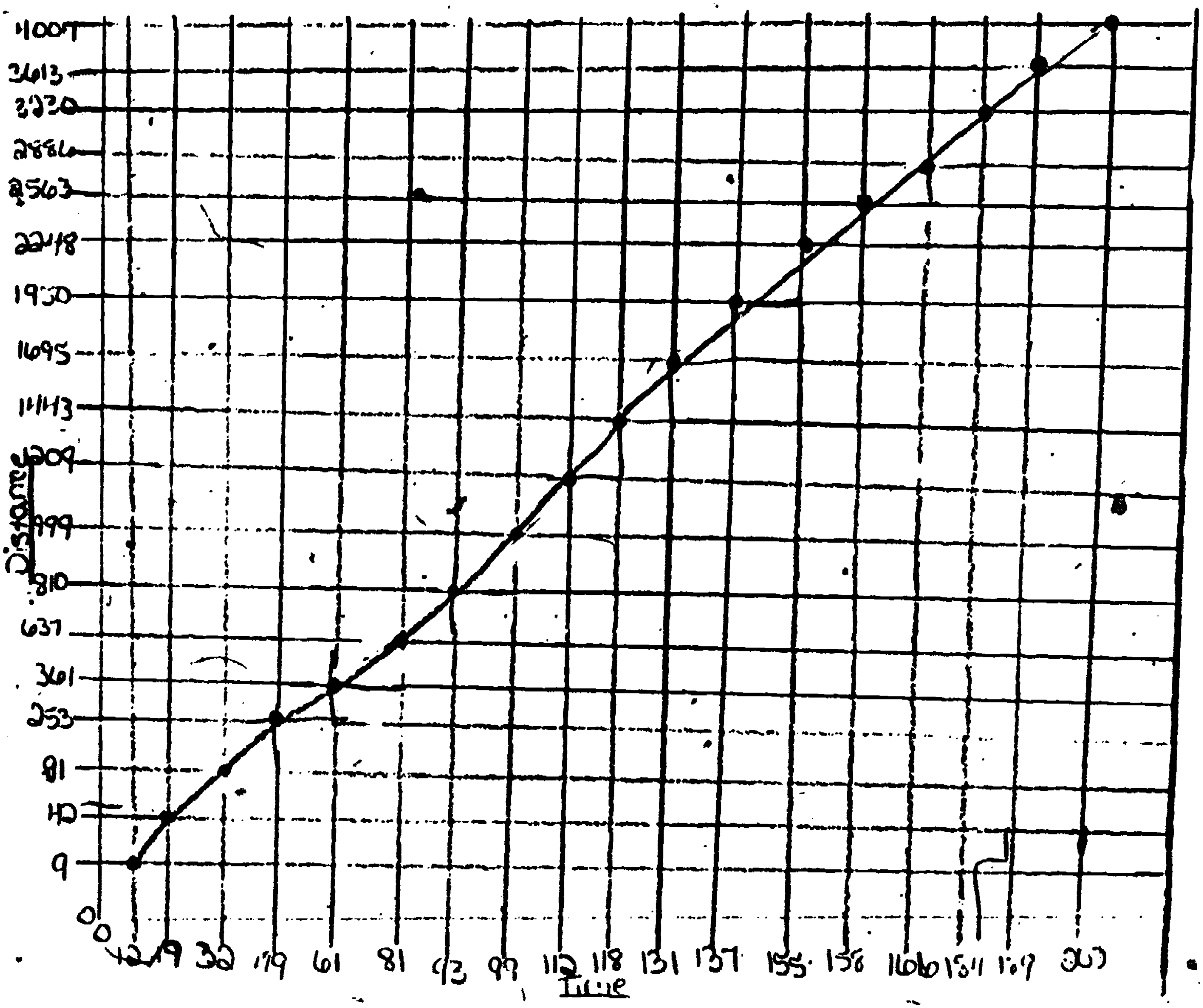


Figure 4

Illustration of Response Category 4

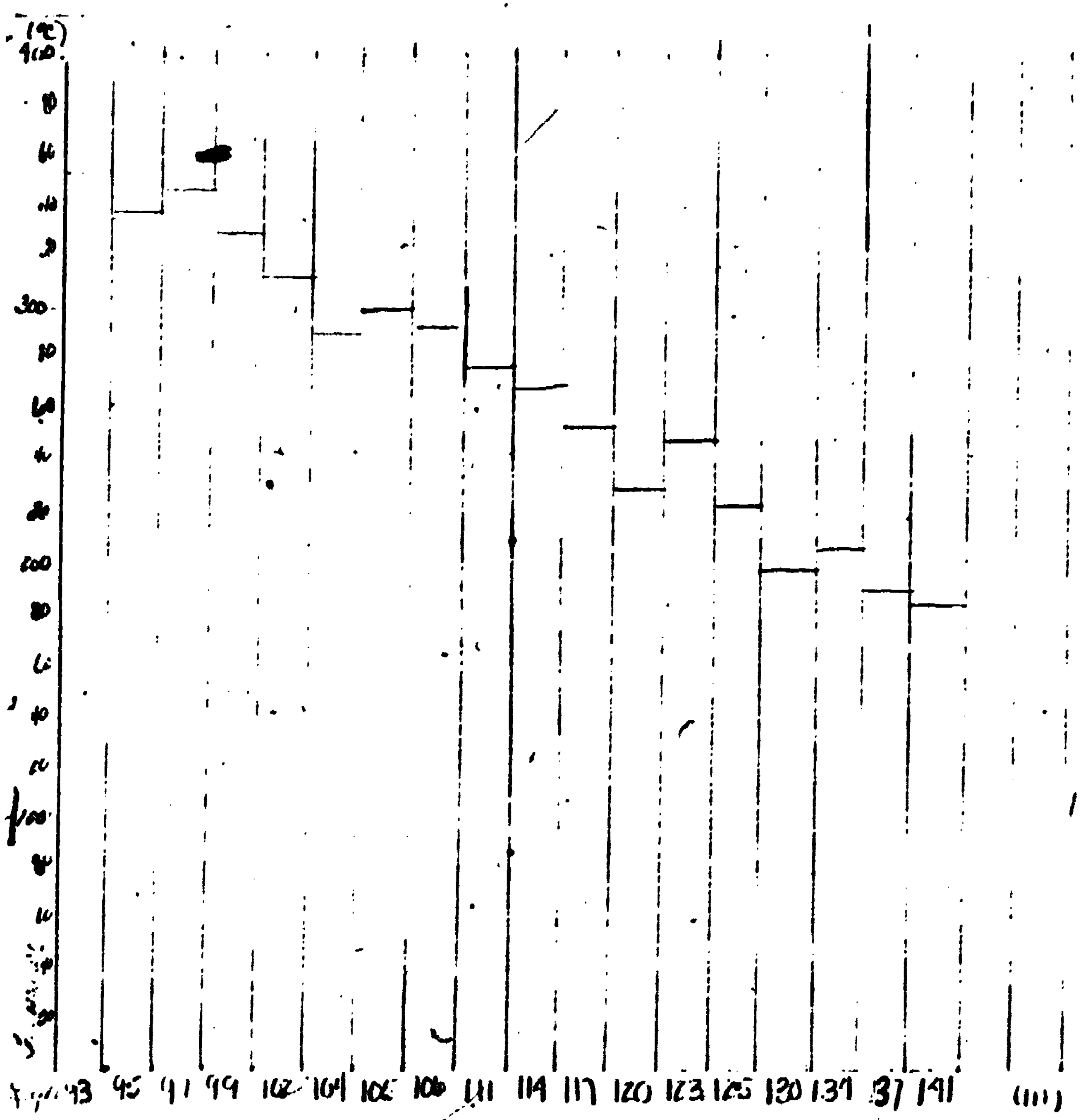


Figure 5

Illustration of Response Category 5

Figure 5

Illustration of Response Category 5.5

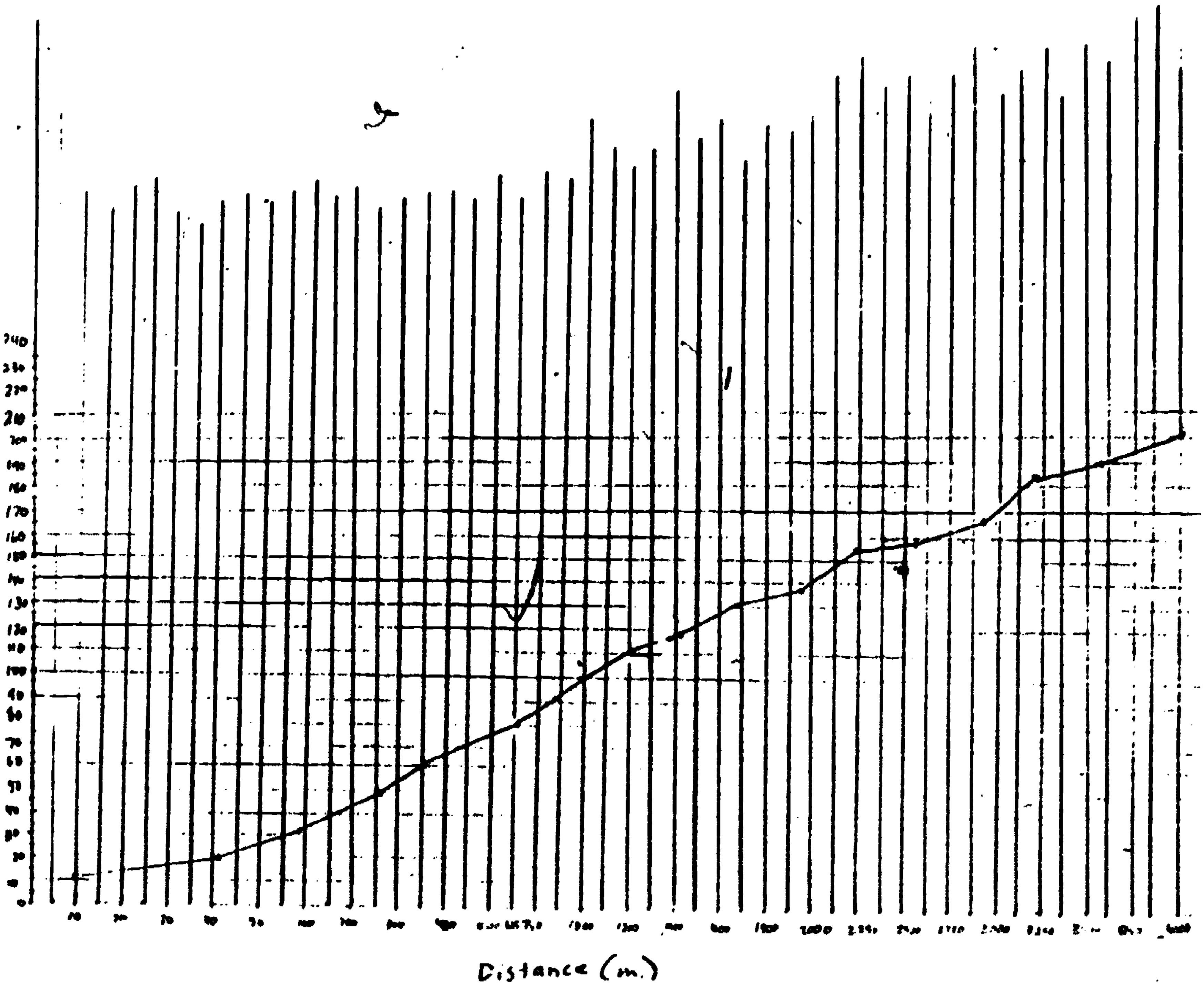


Figure 7

Illustration of Response Category 5.5

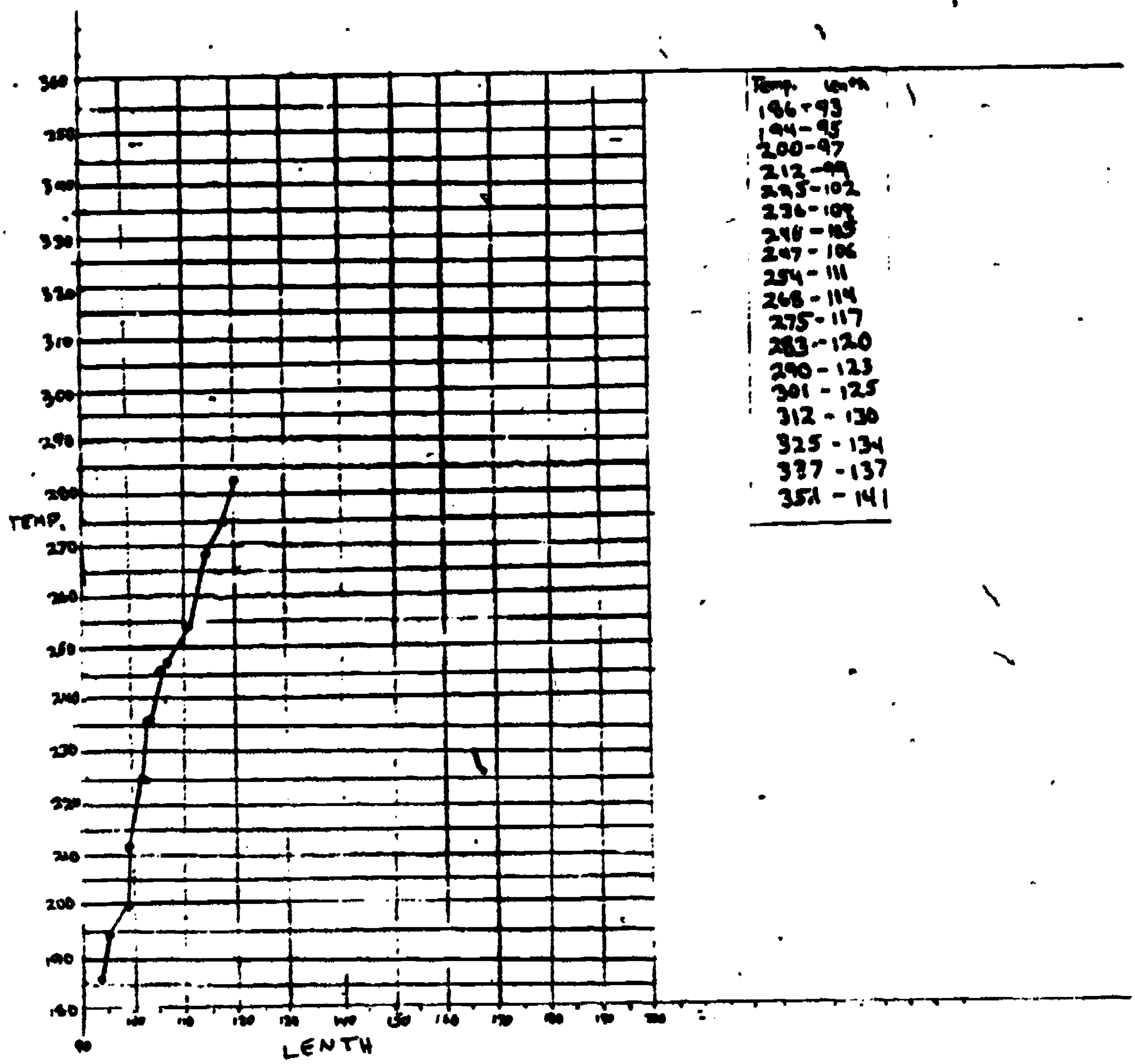
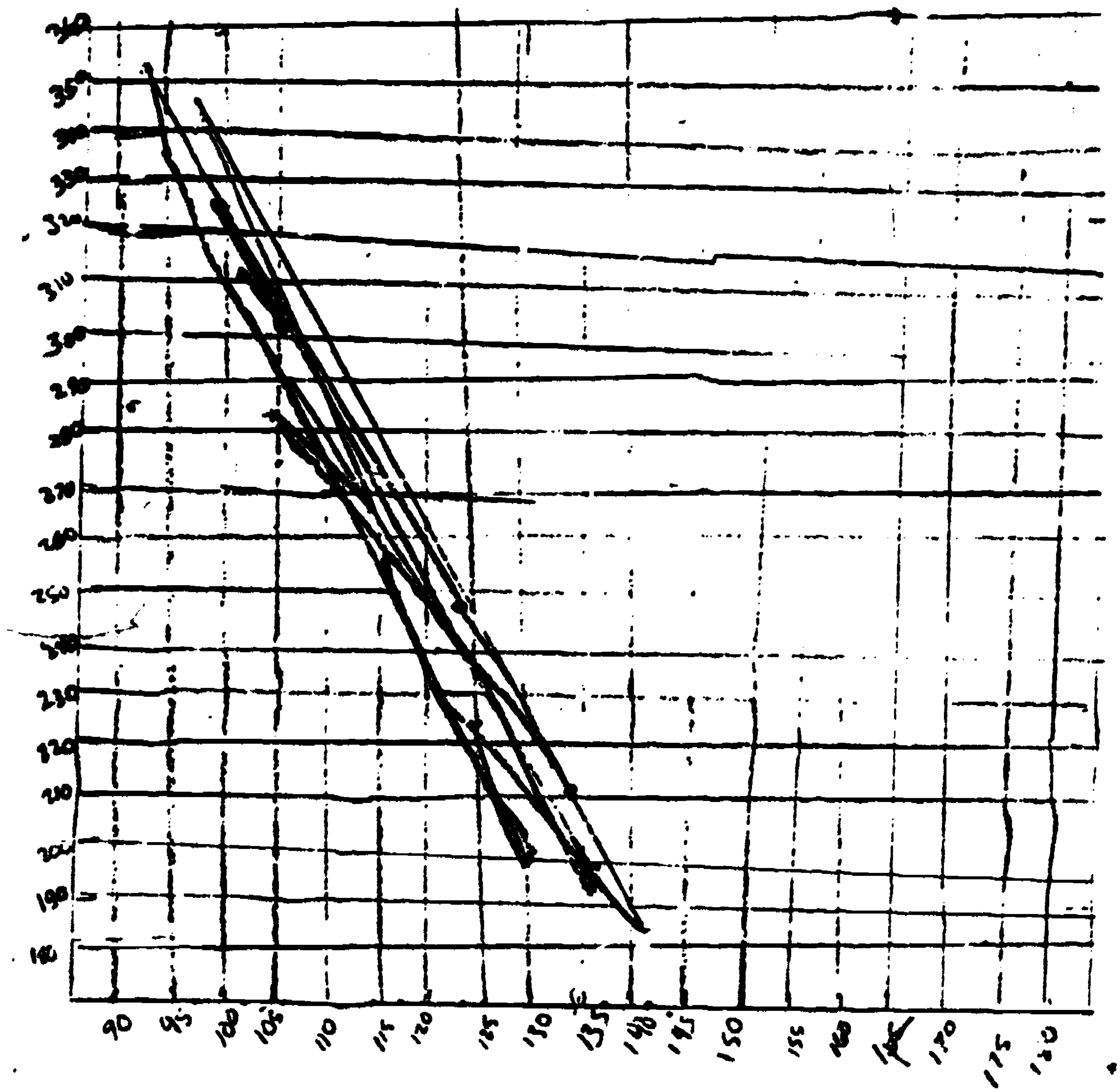


Figure 8

Illustration of Response Category 6



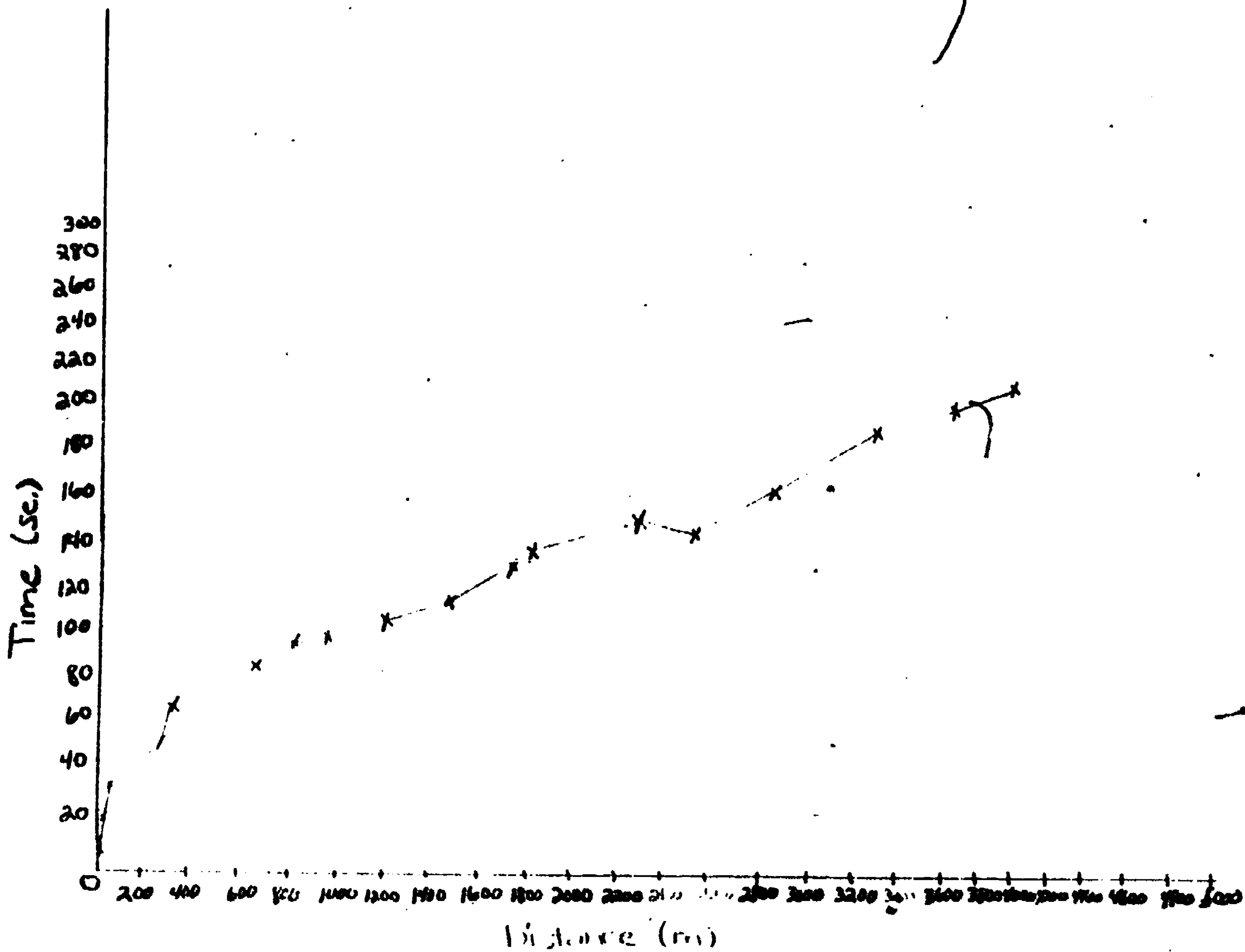


Figure 9

Illustration of Response Category 7

Figure 10

Illustration of Response Category 8