DOCUMENT RESUME

ED 251 419 SP 025 425

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TITLE	Instructional Risk Reduction: An Alternative to Instructional Remediation.
INSTITUTION	Southwest Regional Laboratory for Educational Research and Development, Los Alamitos, Calif.
SPONS AGENCY	National Inst. of Education (ED), Washington, DC.
REPORT NO	SWRL-TR-81
PUB DATE	1 Sep 83
CONTRACT	NEC-00-3-0064
NOTE	42p.
PUB TYPE	Reports - Descriptive (141) -- Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Academic Ability; Educational Strategies; Elementary Education; *High Risk Students; *Instructional Development; Learning Activities; *Learning Problems; *Learning Strategies; Process Education; *Skill Development; Teaching Methods; Textbooks

ABSTRACT

There is a risk that some students will progressively slip further behind the pace of instruction across grade levels, particularly in the second through fourth grades. A combination of factors are related to instructional risk: (1) structure of skill development across more than one grade level; (2) organization of textbook lessons that provide instruction and practice within a particular grade level; and (3) the way teachers move students through a year's worth of lessons--especially how they make adjustments in what to cover for students who may not complete all topics within the textbook by the end of the school year. The first step in reducing instructional risk is identifying pivotal skills which have strong instructional linkages across grade levels. Risks to further learning are increased when there is little practice on skill units which are strongly linked to next year's instruction, when practice is located near the end of textbooks, or when practice is badly fragmented. Once pivotal skills are identified, students must be instructed in these skills at the appropriate time, and intensive practice on these skills should be provided. Sample specifications are presented for risk reduction resources, and types of exercises that may be used in sequence by the classroom teacher are provided. (JD)

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SWRL EDUCATIONAL RESEARCH AND DEVELOPMENT

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TR 81 September 1, 1983

SWRL

SWRL EDUCATIONAL RESEARCH AND DEVELOPMENT

TECHNICAL REPORT 81

September 1, 1983

INSTRUCTIONAL RISK REDUCTION: AN ALTERNATIVE TO INSTRUCTIONAL REMEDIATION

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ABSTRACT

This paper looks at the risk that students will progressively slip further behind the pace of instruction across grade levels and how this risk can be reduced. The logic is illustrated by a design for Instructional Risk Reduction Resources that concentrate direct practice on a very small number of pivotal skills at grades 2, 3, and 4.

INSTRUCTIONAL RISK REDUCTION: AN ALTERNATIVE TO INSTRUCTIONAL REMEDIATION

Remediation is thought to be an unavoidable instructional necessity for many students and consequently an unavoidable instructional burden for their teachers. As students move from one grade to another, an increasing number "fall behind." These students either spend a lot of time on school lessons they can't do, or they receive remediation on work that is below grade level. The remediation works for some students, but despite the best efforts of students and teachers, it doesn't work overall. At each succeeding grade, the number of students requiring remediation increases.

Currently popular "reforms" such as longer school days and years, homework, teacher incentives, and such are sensible, but taken together are too weak to remediate remediation. This paper offers an alternative to instructional remediation termed instructional risk reduction.

The paper is in three parts. The first part explains what constitutes instructional risk. The second part describes how to go about reducing risk. The third part presents sample specifications for risk reduction resources. The paper throughout uses elementary school mathematics instruction for reference and illustration purposes. However, risk reduction is not restricted to mathematics. SWRL has inquiry in progress related to risk reduction in reading and essential language. In principle, risk reduction is applicable to schooling in general. In actuality, the risk to schooling success in all areas of subject matter can be reduced by a small set of risk reduction resources. Possibly

more than mathematics, reading, and essential language will eventually be entailed, but not a whole lot more are likely to be necessary.

ETIOLOGY of INSTRUCTIONAL RISKS

While remediation relies on analytic diagnosis and prescription, with the individual student as the analytic unit, instructional risk reduction relies on a different logic. It considers skill complexes across grades and skill units within skill complexes. This logic holds that instructional risk is due to a combination of factors related to:

(1) the way skill development is structured across more than one grade level,

(2) the way lessons that provide instruction and practice within a particular grade level are organized in textbooks that teachers use,

(3) the way teachers move students through a year's worth or lessons--especially how they make adjustments in what to cover for students who may not complete all topics within the textbook by the end of the school year.

We will discuss each of these determinants in some detail.

Structure of Skill Development Across Grade Levels

It is not possible to teach everything in a single year. For pedagogical reasons, a lot of instruction consists of several large and complicated skill complexes that are decomposed into a lot of smaller units and then taught one by one at succeeding grade levels. Sometimes, the decomposition is a fairly arbitrary one and the resulting units of skill development have weak linkages from one grade level to another. The practical significance of such weak linkages within a skill complex

is that, when students have missed the earlier instruction, it isn't much of a trick to teach both skill units at about the same time. A good example of a weak linkage in mathematics involves the teaching of denominations of money in the primary grades. Often students are taught to recognize the value of pennies, nickels, and dimes along with simple combinations of these coins in grade 1. Quarters usually come along in grade 2. Since these two units both involve "measures" of money, instruction in grade 2 can, and usually does, go back over identification of pennies, nickels, and dimes with little added cost in time.

Other skill complexes involve strong linkages rather than weak ones between skill units that are distributed over several grade levels. The most important characteristic of a strong linkage is the embedding of one skill unit within another across two grade levels. When a skill unit, ostensibly covered at the previous grade level, is embedded in instruction at the current grade level, instructional risk is inherently increased. Since one unit of skill development is tightly embedded in another, there is less likelihood that a limited amount of instruction can develop both at the same time. Students who have not had adequate earlier learning opportunity run a high instructional risk.

Computation algorithms provide some of the best examples of strong linkages in mathematics in the elementary grades. For example, addition of whole numbers with renaming, which is normally instructed in grades 2 and 3, is embedded in multiplication of 2-digit or 3-digit numbers by a 1-digit number, which is normally instructed in grade 3, or more likely,

in grade 4. Some students might be able to learn addition with regrouping early in the year and then use it in the same year to learn how to multiply 2-digit and 3-digit numbers, but most can't handle this kind of quick turn around. The latter, at the time they begin to learn the multiplication algorithm, need to have thoroughly learned the basic addition algorithm. If they encounter difficulty in learning the multiplication algorithm, weak proficiency in addition shouldn't be the cause.

The key to identifying risk within cross-grade-level structures is to look for strong linkages between units of skill complexes at different grade levels. The best place to see how these linkages work is in school textbooks. Finding the embedding of one skill unit within another across adjacent grade levels is not difficult, but determining which grade levels constitute "fail safe" points where all or very nearly all students should have received concentrated instruction and practice on key links can be tricky. The reason is the repetition of a given skill unit across two or three grades. This is done mainly to accommodate students who may not have reached the instruction on that skill unit before the end of the previous school year or who did receive a certain amount of instruction but now need a review. While enabling remediation, this pedagogical accommodation creates instructional risk by bringing with it a certain fuzziness over when different skill units with strong linkages across grade levels need to be taught to all or nearly all students.

Organization of Instruction and Practice Within Grade Levels

A certain amount of risk in elementary school instruction resides in the organization of subject-matter within textbooks for a particular

grade level. Quite simply, skill units at the end of the textbook are most likely to be omitted. More often than not in mathematics texts, these lessons involve skill units that have strong linkages to units at the next grade level. It's ironic that matters placed near the end of the textbook are usually intended for "accelerated" students, who could conceivably wait until next year and, in effect, work on larger chunks of the skill complex. "Accelerated" students in the upper elementary grades thus typically have two or three opportunities in the mainstream of instruction to learn skill units, while "average and "slow" students must move backward in the mainstream in order to get multiple opportunities for learning.

Another source of risk is the small amount of practice provided on some skill units that have strong linkages to next year's instruction. Textbooks deal with a limited framework of lesson space. Usually, 160 lessons or so are provided in the primary to intermediate grades, with perhaps as many as 175 lessons provided in the upper grades. At the same time, the number of skill units increases substantially in the intermediate to upper grades, thus reducing the amount of practice provided for any given unit. Paradoxically, the amount of instruction and practice per skill unit is decreasing at the same time that these skill units have still more skill units from earlier grade levels embedded in them. For example, skill units in grades 5 or 6 that involve addition and subtraction of mixed numbers with renaming seldom have more than three or four lessons, in contrast to addition facts in grades 1 and 2 which may have ten or twenty. Yet they have many skill units from grades 2, 3, and 4 embedded in them.

Fragmented practice also presents instructional risks, again, in part, because of a relatively limited amount of lesson space in textbooks. When instruction on a skill unit is first presented, practice is most likely to occur in short segments of homogeneous items. For example, addition of whole numbers in the early grades is most likely to be practiced along with other addition items. Mixed practice, where students are learning to make subtle but critical discriminations between different nuances of the same skill unit, is often too skimpy.

Teacher Adjustments in How Much Students will Cover

This third factor operates in conjunction with the other two sources of risk described above. There is a tendency to expect less of students who are unlikely to cover all of the major topics in a syllabus or textbook. Usually, this takes the form of deferring some skill units that have strong linkages across grade levels until "next year," and it is often done with less than complete understanding of major skill units that are already in the mainstream of next year's instruction. As a consequence, many students obtain most of their learning opportunities from the first half of school textbooks at each grade level. In other words, some students never get to the second half of the textbook where most new skill units are introduced. They get to them next year when these units are retaught at an earlier point in the textbook.

LOGIC OF INSTRUCTIONAL RISK REDUCTION

All of the risk in elementary school instruction cannot be eliminated, but much of it can be broken into chunks whose potential

impact on student progress can be reduced. Instruction at all grade levels is already full as far as the demands on both teachers and students are concerned. In other words, there isn't a lot of latitude to add or reapportion. There is always more that could be fixed than there is time to fix it. Therefore, the effort to reduce risk must be sparing in the amount of time taken from existing instruction, if it is to be productive and workable.

Identification of Pivotal Skills

The logic for reducing instructional risk requires designation of a small number of skill units. We will term these units pivotal skills. The adjective "pivotal" is used to avoid confusion with "minimum," "essential," "key," "priority," and other such selective designations. Pivotal skills differ from other skills not because they are intrinsically more important or valuable than other skills. Identifying pivotal skills simply relies on the understanding that skill units with strong structural linkages across grade levels are most susceptible to instructional risk.

Each instance where a skill unit at one grade level is embedded in units taught at the next grade level has a potential for risk, but the imminence of risk is often mitigated by relatively large amounts of practice that occur in textbooks prior to the last quarter in the sequence of lessons intended to constitute a school year's worth of work. It is the imminence of risk that is central to the designation of a pivotal skill and imminence of risk is precipitated by factors identified earlier:

*Amount of practice provided in grade level textbooks
*Location of practice in the sequence of textbook lessons
*Concentration with which practice is provided

Where the amount of practice on skill units with strong linkages to next year's instruction is low, where practice is located near the end of textbooks, and/or where practice that is provided is badly fragmented, then there is no. only great potential for risk to future learning opportunities, but the imminence of risk is greatly increased by the likelihood that teachers will defer instruction and practice until next year, and/or that whatever instruction and practice is provided will be inadequate.

The application of risk reduction logic to elementary school mathematics yields at least seven pivotal skills. Other skill units might also qualify, but these are good places to start:

(1) Addition and subtraction of 2-digit numbers with renaming, at grade 2

(2) Solving simple word problems that involve addition and subtraction, especially what's known as "comparison" subtraction, at grade 2

(3) Multiplication and division facts involving multiplication and division by 6, 7, 8, and 9 (commonly thought of as hard facts) at grade 3

(4) Solving simple word problems that involve multiplication and division, especially what's known as "partitive" division, at grade 3

(5) Recognizing common fractions and mixed numbers for illustrations that involve parts of regions, parts of sets, and points on the number line, at grade 3

(6) Multiplication of numbers up to 4 digits and division of numbers up to 3 digits by a 1-digit number, at grade 4

(7) Identifying and generating equivalent common fractions or mixed numbers, and rewriting two fractions so they have common denominators, at grade 4

All of these pivotal skills occur at grades 2-4. Skill units in grade 1 with linkages to grade 2 produce relatively less risk because they tend to receive a lot of instruction and practice fairly early in the school year. For example, "easy" addition and subtraction facts have a strong linkage to addition and subtraction with renaming, to solving simple word problems, and to learning simple representations for "easy" multiplication facts at grade 2, but this skill unit also gets more instruction and practice in grade 1 than almost any other. By grade 5, students have either successfully avoided risks in grades 2, 3, and 4, or they are in formal remedial work for much of the year. Risk reduction resources may find a use in remedial instruction in grades 5 and 6, but instructional risk reduction inherently entails earlier attention.

The reason these seven skill units have been designated as pivotal varies a little from skill to skill, but it's mainly because they are embedded in a lot of instruction at the next grade level and because instruction and practice are likely to come late in the school year, if provided at all. This is especially true of addition and subtraction with renaming at grade 2, "hard" multiplication facts at grade 3, and the skills involving fractions at grades 3 and 4. Multiplication and division of larger numbers at grade 4 produces a little different risk. Quite a lot of practice is provided on this skill unit at the end of grade 3. The instruction and practice are basically repeated at

grade 4, at an earlier point in the sequence of lessons, and repeated again at grade 5--but to a lesser extent. The skill unit is designated as pivotal at grade 4 because it has embedded in it the recall of multiplication and division facts that are initially taught at grade 3. Were the skill unit to be designated as pivotal at grade 3, this would mean that students would have to learn multiplication and division facts and then be expected to use them in learning the multiplication and division algorithms--all at the same grade level. This kind of "doubling up" is not pedagogically sensible.

The rationale is a little different for the two pivotal skills involving word problems. Here, fragmented practice is more a factor than with other designated pivotal skills. We will describe the phenomenon for problems involving multiplication and division, but the risk works out the same way for word problems involving addition and subtraction. By the time all regular textbook lessons involving multiplication and division have been completed on word problems at grade 3, students have had some opportunity for two different kinds of practice using two or three different kinds of word problems. The first type of practice provides examples of "real-world" concepts of multiplication or division and usually begins right after work on computation facts is first introduced. Sometimes this kind of practice involves two or three practice items that occur at the end of a lesson on computation facts; less often, it involves an entire lesson consisting of as many as ten to fifteen word problems. Either way, all of the word problems in a practice exercise at

this point in instruction always require a single operation, multiplication or division, the same operation involved in intensive practice that students usually get on various computation facts. The second type of exercise involves mixed practice. Here, not all items require the same operation, but the practice is still related closely to the learning of computation facts. If, for example, students are learning multiplication facts, some word problems require the use of addition or subtraction. Textbooks at grade 4 mainly provide more of the same kind of practice that was introduced in grade 3, but the syntactical formats for word problems get a little more sophisticated--and a little easier to misinterpret.

One reason multiplication and division word problems are pivotal at grade 3 is because the opportunity students have for practice is relatively brief compared to all of the work they are doing on computation facts. Their opportunity for practice is also quite fragmented, especially when it comes to distinguishing among problems that require addition, multiplication, and different types of division. Altogether, it is unusual for students to have more than five to ten items of practice on partitive division* in all of grade 3. Some textbooks may only get around to one or two items of this type in a mixed practice exercise. Within any single exercise, there is unlikely to be more than one or two items of any one type, especially partitive division.

*In partitive division, students are given a total number that is divided into a certain number of subsets and they are asked to find the number in each subset. This contrasts with measurement division, which is easier, where students are given a total number that is divided into subsets if a certain size and then asked to determine how many subsets there will be.

Although most textbooks at grade 3 take some special precautions to get partitive division introduced, even if it's done late in the year and practice is fragmented; at grade 4, they don't. Ensuring that students in grade 3 get additional practice on distinguishing among different types of word problems before they move on to grade 4 is something that some teachers do now, by expanding on instruction from their textbook, but most teachers don't.

Adjusting the Within-Year Sequence of Skill Units

Once pivotal skills have been identified, the next step in the logic for risk reduction is to make certain that students get instruction on a pivotal skill at about the right location at a given grade. This step is important for two reasons. First, students need to get all of the basic instruction on the pivotal skill provided by their textbook before the school year runs out, and second, they need to get that instruction early enough so that regular textbook lessons can be augmented by additional direct practice. Given the sequence of most elementary mathematics textbooks, this means moving instruction a little closer to the front of the book. For the pivotal skills designated here, moving the instruction ahead by two or three months should not have any negative effect on the instruction of other skills at a given grade. Mathematics textbooks are designed so that teachers can either use them from "front to back," if they want to, or to rearrange chapters in a more convenient sequence. Some skill units need to be taught before others are begun, but, as long as this order is preserved, things can be moved around--which many teachers do. The adjustments to within-year sequences that are entailed by risk reduction are all well within this pedagogical discretion.

Concentrating Existing Instruction and Practice

The third step in risk reduction logic is to ensure that students get enough practice on pivotal skills once basic instruction using textbook materials is completed. This usually means intensive practice within a one or two week period. But not just any practice will do--it must be systematically varied so that the range of conditions within which students can perform a pivotal skill can be expanded and strengthened. If this kind of practice seems a bit steep, it should be kept in mind that only two or three pivotal skills are involved at any given grade.

Concentrating instruction also means providing very brief segments of followup practice distributed at regular intervals over several weeks so that the newly learned pivotal skill is reviewed briefly and strengthened over time. This can be done with no more than two or three practice items to be completed during the first five minutes or so of a regular classroom period one or two times a week for several weeks.

SAMPLE SPECIFICATIONS FOR RISK REDUCTION RESOURCES

The logic of instructional risk reduction provide that students at a particular grade level will complete all of the work on a pivotal skill provided in their regular textbook. Instructional Risk Reduction Resources (IR^3) are then introduced to round out the instruction on the pivotal skill.

The IR^3 materials specified here include two kinds of practice within each kit focused on a given skill. First, there are ten to twelve

practice exercises, simply called "Exercises," that are relatively long. These Exercises are to be used in sequence by the classroom teacher as soon as regular textbook lessons that introduce the pivotal skill are completed. The other kind of practice involves a series of abbreviated Mini Exercises that consist of no more than two or three items intended to keep the newly-learned pivotal skill active and in shape. Mini Exercises are to be used twice a week after the longer Exercises are completed.

The sets of longer Exercises have been developed as five or, in one or two cases, six pairs of parallel exercises. For example, Exercises 1 and 2 cover the same types of practice items and so do Exercises 3 and 4, 5 and 6, and so on. Each pair of Exercises gets progressively more complicated in the kinds of practice that are provided, although none exceeds, to any significant degree, the complexity of more difficult problems found in regular textbook materials. The difference here is that more opportunity for practice is provided so that students get sufficient repetition in making important distinctions between different kinds of practice problems. The second Exercise in each pair is almost identical to the first, so students get two solid opportunities to practice the critical steps in skill development represented by each pair.

In all kits but one, the first Exercise in each pair requires students to write out solutions to problems as constructed responses, much like they respond to exercises in their textbook. The second Exercise involves multiple choice problems where students select

the correct answer from three or four alternatives. This permits students to get two slightly different kinds of practice, and it provides students with some experience in avoiding wrong answers that seem very attractive. In the early stages of learning, students often go for choices that entail "quick fixes" that have worked well for them in the past, especially on multiple choice items where some answer choices involve straightforward and easy-to-see operations on numbers that are part of a problem or task, even though the operations are incorrect.

Use of the IR^3 kits requires some advanced planning by the classroom teacher at the beginning of the year so that regular instruction on the pivotal skill can be completed before the end of the third quarter in school, or sometime prior to the end of March. Otherwise, there will not be enough time for the kind of extended, albeit brief, followup, using Mini Exercises, before school is out. With some textbooks, teachers will only need to keep up with the recommended sequence of lessons and chapters to complete regular work on the pivotal skill prior to the end of March. With other texts, teachers will need to do a little rearranging of the "front to back" sequence.

Exercises within the kit should not require a great deal of formal teaching because the kit materials are to be used after pertinent instruction from a regular textbook is completed. Although practice within the kit has a very careful and precise structure to it, it is compatible with whatever method was used to develop the key skill in regular instruction. Practice in the kit begins where regular instruction leaves off, so all practice problems are ones that students will

have seen before, but not in the sequence or concentration that they are presented in the Exercise.

Teachers can use a certain amount of flexibility in assigning the longer Exercises. Minimally, they should set aside a week of class time for doing in class the initial Exercise in each of the five pairs. The second Exercise in each pair can be assigned as homework. It is possible, although not recommended, that some teachers may want to have their students work through both exercises in each pair in a single extended class period. Other teachers may want to set aside two weeks to work through the Exercises in a kit.

Enough materials are provided within a kit for a typical classroom of 25 to 30 students. The exercises are reproduced on low-cost paper, so that the entire kit is disposable after classroom use. In other words, students can do all of their work directly on materials provided in the kit. As materials are used, they should either be sent home or discarded. No special record keeping is required or recommended other than routine checking of student work. That extra demand on teacher time is explicitly discouraged. The important thing is to use the longer practice exercises all at once and then move on to other topics with abbreviated followup practice every two or three teaching days using one of the Mini Exercises during the first five minutes or so of a class period.

The specifications for each of the illustrated kits are shown on the following pages.

ADDITION AND SUBTRACTION WITH RENAMING

1. The focus of this kit is addition and subtraction of 2-digit numbers with renaming ("carrying and borrowing"). All of the exercises involve different aspects of two-column addition and subtraction, but the renaming involves mostly addition and subtraction facts with sums and subtrahends no larger than 12 (e.g. $\begin{array}{r}23\\+18\\\hline\end{array}$ and $\begin{array}{r}42\\-15\\\hline\end{array}$)

2. 12 Exercises written as pairs of parallel exercises.

3. 10 Mini Exercises.

4. The first two Exercises have 20 items. All other Exercises have ten items.

5. All Mini Exercises have three items.

6. Exercises X and Y are matching exercises. Exercises 1, 3, 5, 7, and 9 require a constructed answer.

7. Exercises 2, 4, 6, 8, and 10 require students to select the appropriate answer from three choices.

Preliminary Exercises X and Y

These exercises require students to discriminate between addition and subtraction problems that require renaming and ones that don't. No computation is necessary. For each item, students identify a matching item from a set of solved pattern problems shown at the top of the exercise.

Exercises 1 and 2

Items involve addition and subtraction of 2-digit numbers. All items require renaming of place values--with the following restrictions:

* Addition or subtraction in "ones" place involves basic facts to 12.
* Addition or subtraction in "tens" place involves easy computation such as "doubles" (e.g. 4 + 4 or 6 - 3), or addition or subtraction of 1.
* Addition problems have 2-digit answers (sums less than 100).
* Pattern problems showing how to rename place values are provided at the top of the exercise.

Exercises 3 and 4

Items involve mixed addition and subtraction. Some items require renaming; some don't. Other restrictions on items are the same as in Exercises 1 and 2.

Exercises 5 and 6

Items involve mixed addition and subtraction with harder basic facts. All items require regrouping--with the following restrictions:

* Addition and subtraction in "one's" place involves basic facts beyond 12 (e.g. 8 + 7 or 17 - 8).
* Addition and subtraction in "ten's" place still involves relatively easy computation facts such as "doubles" or addition and subtraction of 1.
* Addition problems may now have 3-digit answers (sums greater than 100).
* "Harder" basic facts are provided in a look up table.
* Pattern problems showing how to rename are still provided.

Exercises 7 and 8

Items involve mixed addition and subtraction. Most items require renaming, but some don't. All item represent the following restrictions:

* Addition and subtraction in "one's" place involve basic facts to 12 (e.g. 8 + 3 and 12 - 4).
* Addition and subtraction in "ten's" place involves easy computation such as "doubles" (e.g. 4 + 4) and addition or subtraction of 1.
* Addition problems have 2-digit answers (sums less than 100).
* No pattern problems of tables of basic facts are provided at the top of the exercise.

Exercises 9 and 10

Items involve mixed addition and subtraction. Most items require renaming. No pattern problems or tables of basic facts are provided as references. There are no restrictions on the difficulty of basic facts in the "one's" or "ten's" column and no restrictions on the size of the answer.

ADDITION AND SUBTRACTION WORD PROBLEMS: GRADE 2

General Specifications

1. The focus is on simple word problems involving addition and subtraction facts, with special emphasis on what is known as "comparison" subtraction problems: "John has 8 marbles. Susan has 13 marbles. How many more marbles does Susan have?"

2. All items use easy addition and subtraction facts--A + B = C and C - A = B, where A and B are whole numbers less than 10 and C is no larger than 12--or addition and subtraction of 2-digit numbers where no renaming ("borrowing" or "carrying") is required.

3. Vocabulary for these items is focused on grade 1, but some special terms may occasionally extend beyond grade 1.

4. Ten Exercises will be written as pairs of parallel exercises.

5. Ten Mini Exercises will also be written.

6. All Exercises have ten items.

7. All Mini Exercises have three items.

8. Exercises 1, 3, 5, 7, and 9 require a constructed answer.

9. Exercises 2, 4, 6, 8, and 10 require students to select the appropriate answer from three choices.

10. Each Exercise involves mixed addition and subtraction items. In early Exercises, all items in each Exercise involve one of the addition or subtraction facts which are given as reference facts at the top of the page.

Exercises 1 and 2 Items for this exercise can draw from the following types of problems:

JOHN HAS 5 MARBLES.
HE GETS 3 MARBLES.
HOW MANY IN ALL?

JOHN HAS 8 MARBLES.
HE GIVES AWAY 3 MARBLES.
HOW MANY ARE LEFT?

JOHN HAS 5 CUPS.
HE ALSO HAS 8 SPOONS.
DOES HE HAVE MORE CUPS OR SPOONS?
HOW MANY MORE?

JOHN SEES 5 DUCKS.
3 DUCKS GO AWAY.
HOW MANY ARE LEFT?

JOHN SEES 5 WHITE DUCKS.
HE SEES 8 BROWN DUCKS.
HOW MANY MORE BROWN DUCKS DOES HE SEE?

JOHN HAS 6 WHITE DUCKS.
HE HAS 4 BROWN DUCKS.
HOW MANY DUCKS DOES HE HAVE IN ALL?

In this exercise, all of the action words have the following forms:

HAS/GETS
HAS/GIVES AWAY
HAS/HAS
SEES/SEES
SEES/GOES AWAY

The relevant addition and subtraction facts will be shown as reference facts at the top of the exercise.

Exercises 3 and 4

Items in this exercise can draw from the following item types:

JOHN HAS 5 MARBLES.
JULIE HAS 3 MARBLES
HOW MANY DO THEY HAVE IN ALL?

JOHN HAS 5 MARBLES.
JULIE TAKES 3 OF THEM.
HOW MANY DOES JOHN HAVE LEFT?

JOHN HAS 5 MARBLES.
JULIE HAS 3 MARBLES.
WHO HAS MORE MARBLES?
HOW MANY MORE?

JOHN HAS 5 MARBLES.
HE FINDS 2 MARBLES.
HOW MANY MARBLES DOES HE HAVE NOW?

JOHN HAS 5 MARBLES.
HE LOSES 2 OF THEM.
HOW MANY MARBLES DOES HE HAVE NOW?

The formats for these items are quite similar to formats for Exercises 1 and 2, except that two people are involved in three out of the five item types and the action words HAS/TAKES, HAS/FINDS, and HAS/LOSES have been added.

Relevant addition and subtraction facts will be shown as reference facts at the top of the exercise.

Exercises 5 and 6

The items in these exercises can draw from the following item types:

JOHN HAS 5 MARBLES.
HE WANTS 8 MARBLES.
HOW MANY MORE DOES HE NEED?

JOHN WANTS 8 MARBLES.
HE ALREADY HAS 3 MARBLES.
HOW MANY MORE DOES HE NEED?

JOHN HAS 5 MARBLES.
HE GETS 3 MORE MARBLES.
HOW MANY DOES HE HAVE NOW?

JOHN NEEDS 8 MARBLES.
HE ALREADY HAS 3 MARBLES.
HOW MANY MORE MUST HE FIND?

JOHN HAS 5 MARBLES.
JOYCE HAS 3 MORE MARBLES THAN JOHN.
HOW MANY DOES JOYCE HAVE?

The main distinctions between these item types and the ones specified for Exercises 1-4 are:

(1) The action words HAS/WANTS, WANTS/HAS, NEEDS/HAS, and FINDS/LOSES have been added.
(2) The time sequence has been changed in some item types to include the "already has" phrase in the second sentence of the item's statement.

The four relevant addition and subtraction facts are shown as reference facts at the top of the exercise.

Exercises 7 and 8

Items in these exercises can draw from the following item types representing applications of addition and subtraction to situations involving length and distance.

JOHN IS 43 INCHES TALL.
JULIE IS 45 INCHES TALL.
HOW MUCH TALLER IS JULIE?

GEORGE IS 45 INCHES TALL.
JACK IS 43 INCHES TALL.
HOW MUCH SHORTER IS JACK?

JOYCE WALKS 3 BLOCKS TO SCHOOL.
SHE WALKS 5 BLOCKS TO JAN'S HOUSE.
HOW MUCH FARTHER IS IT TO JAN'S HOUSE?

JOYCE TAKES 15 MINUTES TO GO TO JAN'S HOUSE.
THEN IT TAKES HER 10 MORE MINUTES TO GO ON TO SCHOOL.
HOW LONG DOES IT TAKE JOYCE TO GET TO SCHOOL?

Key words for the first two item types also include LONG/LONGER.

No addition and subtraction facts will be included as reference facts at the top of the exercise.

Exercises 9 and 10

Items in these exercises will draw from the following formats, all involving key words related to money and purchases.

JOHN SPENDS 3 DOLLARS FOR A BOOK.
HE SPENDS 2 DOLLARS FOR A BALLOON.
HOW MUCH DOES HE SPEND IN ALL?

JOHN HAS 3 DOLLARS.
HE SPENDS 2 DOLLARS.
HOW MUCH MONEY DOES HE HAVE LEFT?

JOHN BUYS A BOOK FOR 3 DOLLARS.
HE PAYS 5 DOLLARS.
HOW MUCH MONEY WILL HE GET BACK?

A BOOK COSTS 3 DOLLARS.
A BALL COSTS 2 DOLLARS.
HOW MUCH MORE DOES THE BOOK COST?

JOHN SPENDS 5 DOLLARS.
JOYCE SPENDS 3 DOLLARS MORE THAN JOHN.
HOW MUCH DOES JOYCE SPEND?

No addition and subtraction facts will be included as reference facts at the top of the exercise.

MULTIPLICATION AND DIVISION FACTS: GRADE 3

General Specifications

1. The primary focus (90% of the items) is on "hard" multiplication and division facts. This includes multiplication of numbers up to 9 by 5, 6, 7, 8, and 9, and division by 5, 6, 7, 8, and 9. The secondary focus (10% of the items) is on "easy" multiplication and division facts, which includes multiplication and division of numbers up to 9 by 2, 3, 4, 1, and 0.

2. 10 Exercises written as pairs of parallel exercises

3. 10 Mini Exercises

4. All Exercises have 15 to 20 items.

5. All Mini Exercises have three items.

6. Exercises 1, 3, 5, 7, and 9 require a constructed answer.

7. Exercises 2, 4, 6, 8, and 10 require students to select the appropriate answer from three or four choices.

8. The items on each Exercise have a random mix of horizontal (equation) and vertical (working) form.

9. The following items or item forms should get heavier emphasis:

6 X 6	7 X 6	8 X 6	9 X 6
6 X 7	7 X 7	8 X 7	9 X 7
6 X 8	7 X 8	8 X 8	9 X 8
6 X 9	7 X 9	8 X 9	9 X 9

6	7	8	9
X6	X6	X6	X6
6	7	8	9
X7	X7	X7	X7
6	7	8	9
X8	X8	X8	X8
6	7	8	9
X9	X9	X9	X9

DIVISION BY 6, 7, 8, AND 9; ALL QUOTIENTS

10. Some Exercises should include items that mix multiplication and division in about the following proportions:

*"Easy" multiplication and division facts--20%
*"Hard" multiplication facts--40%
*"Hard" division facts--40%

Some Exercises will include only division facts that have been cued by related multiplication facts (shown as reference problems at the top of the page).

Exercises 1 and 2 Multiplication by 6 and 7.

Part I. (4 items) Items involve "easy" facts: both numbers are less than 6. Items are in random order with horizontal (equation) format.

Part II. (8 items) Items involve "medium/hard" facts:

in A X B = ____

or B
XA
—

formats, A can be 6 or 7, while B is 5 or less). Items are in random order with mixed horizontal and vertical formats.

Part III. (16 items) Items involve "hard" facts:

in A X B = ____

or B
XA

formats, A is always 6 or 7 while B can be 6, 7, 8, or 9). Items are in random order with each legitimate combination of numbers used twice: once in horizontal form and once in vertical.

Exercises 3 and 4 Division by 6 and 7 (cued).

Part I. (8 items) These items involve eight "hard" multiplication facts in horizontal form: A X B = ____, where A is 6 or 7 and B is 6, 7, 8, or 9.

Part II. (8 items) These items involve eight "medium/hard" division facts in mixed horizontal or working form. This means that the divisor is 6 or 7, while the dividend is any number up to 35 (i.e. a number that yields quotients up to 5 when they are divided by 6 or 7).

Part III. (16 items) These items involve eight "hard" division facts (each done in two different formats). This means that the divisor is 6 or 7, while the dividend is any number greater than 35 that yields quotients of 6, 7, 8, or 9, when divided by 6 or 7. Horizontal and working formats for these items are mixed at random.

Exercises 5 and 6

Multiplication and division by 6 and 7.

26 items that include about equal numbers of:

* "hard" division facts
* "hard" multiplication facts
* "medium/hard" multiplication facts
* "medium/hard" division facts

All items are ordered at random according to multiplication or division fact and the type of format (horizontal or working format) in which it's presented.

Exercises 7 and 8

Multiplication by 8 and 9.

Specifications for Exercises and Mini Exercises are parallel to specifications given for Exercises 1 and 2 except dividends can go to 45.

Exercises 9 and 10

Division by 8 and 9 (cued).

Specifications for Exercises and Mini Exercises are parallel to specifications given for Exercises 3 and 4.

Exercises 11 and 12 Multiplication and division by 8 and 9.

Specifications for Exercises and Mini Exercises are essentially parallel to specifications given for Exercises 5 and 6. The only difference is that Exercises 11 and 12 will mix in a small number (three to five) of items involving multiplication and division by numbers less than 8.

MULTIPLICATION AND DIVISION WORD PROBLEMS: GRADE 3

General Specifications

1. 10 Exercises written as pairs of parallel exercises.
2. 10 Mini Exercises.
3. All Exercises have ten items.
4. All Mini Exercises have three items.
5. Exercises 1, 3, 5, 7, and 9 require a constructed answer.
6. Exercises 2, 4, 6, 8, and 10 require students to select the appropriate answer from three or four choices.

Exercises 1 and 2

Exercises 1 and 2 focus on mixed practice with multiplication and division, no addition and subtraction. Word problems have limited syntax like the following:

> There are 40 race cars on 8 tracks.
> Each track has the same number of cars.
> How many cars are on each track.

This item format involves partitive division, which is the type of word problem we are most concerned about. Other word problems for measurement division and for multiplication will parallel this structure as closely as possible without being awkward or confusing. The only variation in syntax in this practice exercise will be within the structure of individual sentences more or less as follows:

> 40 race cars are on on 8 tracks.
> Each track has the same number of cars.
> How many cars are on each track.

The content of items in this exercise can involve several topics which lend themselves to decomposition of a large set into several smaller ones of equal size, but it does not yet include applications involving money, such as "total cost" and "cost for one," or measurement applications involving length and distance. Both of these very important and often used applications will be the

principal focus of practice exercises that come a little later. For now the content deals with things that fit well into sentences using verbs such as "are" or "has."

All practice items in these two exercises have their numerical information in the same order as the order for computation. The "action" in all items is present tense.

Exercises 3 and 4

Exercises 3 and 4 move to on mixed practice that also includes addition and subtraction. There are only three items that require addition or subtraction, but this is enough to provide some contrast for students who are learning to identify differences between multiplication and division word problems and problems that require other operations.

Exercises 3 and 4 feature a longer syntax than exercises 2 and 3. Instead of having all items in the basic "There are..." format, we now use more regular "story" formats involving names of other people:

> Mr. Robert has 12 apples.
> He gives them to 3 children so that each child gets the same number of apples.
> How many apples will each child get?

With longer syntax, there is more variety in action verbs and in the structure of individual sentences, although all items still have the same basic three-sentence format.

All items in this exercise still have their action in present tense, and they have numerical information in the same sequence as the order for computation. No items use applications, such as money, lengths, or time, that involve measurement.

Exercises 5 and 6

Exercises 5 and 6 focus on money topics, and they use situations that involve "total cost," and "cost for one." Applications involving money and "cost" are enormously important in elementary school mathematics, because they show up repeatedly in the learning of concepts and skills at later grade levels, and because they use many nouns and verbs that do not show up in more general "There are..." and "Marie has..." problems.

Like Exercises 3 and 4, these exercises use a longer syntax, but, this time, the order of numerical information in word problems may vary from the order in which the numbers will enter into computation.

Exercises 7 and 8

Exercises 7 and 8 have the same structure as Exercises 5 and 6, but the content of all word problems involve measurement of length and time. Together with "money" and "cost" items, "length" and "time" items represent most of the measurement applications that are used throughout grades 3 through 6 in development of new concepts and skills.

Exercises 9 and 10

Exercises 9 and 10 mix the content of practice items represented in Exercises 3 through 8. Except for this change in content, there are no changes in item structure.

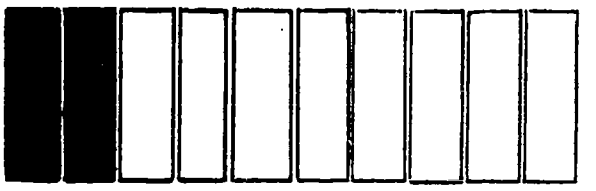
RECOGNITION OF FRACTIONS FOR REGIONS, SETS AND THE NUMBER LINE: GRADE 3

General Specifications

1. Ten Exercises written as pairs of parallel exercises
2. Ten Mini Exercises
3. All Exercises have 20 to 25 items.
4. All Mini Exercises have three items.
5. Exercises 1, 3, 5, 7, and 9 require constructed answer.
6. Exercises 2, 4, 6, 8, and 10 require students to select the appropriate answer from three or four choices.

Exercises 1 and 2

Students write common fractions (1/2, 5/6, etc.) for illustrations involving regions and sets. All illustrations show fractions for numbers from 0 (0/4, 0/6, etc.) to 1 (4/4, 6/6, etc.). Two example items are provided at the top of the page. Illustrations show fractions with denominators up to 16. The distinction between regions and sets in the illustrations for most items should be quite minimal. For example, the following illustration could connote either region or set.



The main cue to the student would be:

(1) whether or not the shaded parts come one after the other, and
(2) whether the set up uses the term how much, as in "How much is shaded?" or what part, as in "What part is shaded?"

In this exercise, items should vary as to whether their set up is how much or what part.

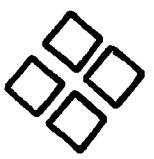
Altogether there should be 20 to 25 items.

The first ten items should be written in twos or threes where those items have the same numerator or denominator. At first, items with the same

denominator should have illustrations about the same size. Later, connected items having the same denominator should have illustrations with a different size.

For the first few items, shaded parts should be together. Later items should have them separated.

Almost all of the set items should have parts that fit together like a unit



some should be close together; others should be farther apart--but 10% to 20% can have a freer arrangement, especially items that involve sets of real objects.

Exercises 3 and 4

Exercise 3 (approximately 25 items): Students write common fractions for regions, sets, and points on the number line (from 0 to 1, although the number line can go a little past 1, and up to 2 on later items).

Most items should involve the number line (approximately 60%), with region and set items mixed in to provide continuity from Exercises 1 and 2.

Use about the same restrictions given for Exercises 1 and 2.

Exercise 4 (approximately 25 items): Students are to select the appropriate fraction (four choices) for illustrations that are about the same as ones used in Exercise 3 - with one exception. Some illustrations should show more units in the denominator than the appropriate answer. For example, the illustration might show "halves" and "eighths" while the only appropriate answer choice is in "halves."

In developing wrong answer choices, be sure that one of them is a ratio of one kind of object to the remaining objects (rather than one kind of object to all objects--the appropriate answer).

Exercises 5 and 6

(25 items): Students write common fractions or mixed numbers for regions, sets, and points on the number line.

Use restrictions from Exercises 3 and 4. Be very careful in using sets to illustrate a mixed number. Be sure the illustrated whole number unit looks like a unit. (In this case, sets of real objects work best if they're in cartons.)

Restrictions should be comparable to Exercises 3 and 4.

Exercises 7 and 8

(20 to 30 items) Students write two fractions or mixed numbers for the same region, set, or point on the number line.

Restrictions are comparable to those for Exercises 5 and 6.

The first 10 to 12 items should involve only regions or points on the number line. Then you can start to mix in sets.

The first two items should be regions that represent fractions that are less than 1.

Exercises 9 and 10

Exercise 9 (20 to 25 items): Students write a fraction or mixed number, a decimal, or both for a region, set, or point on the number line.

Two example items are provided at the top of the page.

All illustrations should show tenths.

Exercise 10 (20 to 25 items): Students select appropriate fraction, mixed number, or decimal (from four choices) for an illustration. Except for items where a decimal is to be the appropriate choice, illustrations may show any denominator up to 16.

EQUIVALENT FRACTIONS: GRADE 4

General Specifications

1. Ten Exercises written as pairs of parallel exercises.
2. Ten Mini Exercises.
3. All Exercises have 20 to 25 items.
4. All Mini Exerices have three items.

Exercises 1 and 2 Tell which fractions are equal to a given fraction.

Format: A fraction is given, followed by four six answer choices. The fraction is a common fraction from 0 (0/3, 0/8, etc.) to 1 (3/3, 8/8, etc.) and so are the answer choices.

Exercise is divided into two parts.

Part 1 (10 items, 3 answer choices): Either the given fraction or each of the answer choices has 1 as the numerator (1/2, 1/3, 1/4 . . .).

Part 2 (15 items, 6 answer choices): Given fraction and answer choices have numerators that are different from 1.

Restrictions: (1) Denominators can be 2, 3, 4, 5, 6, 8, 10, 12, or 16.
(2) Items in Part 2 are written in three's so that all three given fractions have the same denominator.

Exercises 3 and 4 Tell which fractions are equal to a given fraction.

Format: Same as Lessons 1 and 2 except that "fractions" can now be common fractions, improper common fractions (numerator is larger than the denominator), or mixed numbers (3-3/5, 5-1/8, etc.)

Exercise is in two parts.

Part 1 (10 to 15 items): All items involve twelfths in either the "prompt" or the choices.

Part 2 (15 items): All items involve denominators that are 16 or less.

Exercises 5 and 6 Generate fractions that are equal to a given fraction.

Format: Part 1 (10 items): For each given fraction, students are to write two equal fractions. having either higher or lower terms.

Part 2 (10 items): Tell which of four fractions (or mixed numbers) is in lowest terms.

Part 3 (10 items): For each given fraction, students are told to provide an equal fraction in higher, lower, or "lowest" terms.

Exercises 7 and 8 Rewrite fractions with new given denominators.

Format: Students are given sets of two to five common fractions, improper fractions, or mixed numbers and told to rewrite them so that each fraction has the same, given denominator.

E.G., rewrite each of these fractions so the denominator is 12.

1/2, 3/4, 5-1/6, 4/6

Restrictions: All denominators are 16 or less.

Exercises 9 and 10 Rewrite pairs of fractions so they have the same (but not given) denominator.

"Rewrite each pair of fractions so their denominators are alike."

1. 3/4 and 1/8
2. 5/12 and 5-1/2

Exercise should be divided into two parts.

Part 1: One of the two denominators is a common denominator.

Part 2: For most pairs of fractions neither denominator is a common denominator.

Restrictions: All denominators are 16 or less.

MULTIPLICATION and DIVISION OF 2-, 3-, and 4-DIGIT NUMBERS BY A 1-DIGIT NUMBER: GRADE 4

General Specifications

1. 10 Exercises written as pairs of parallel exercises.
2. 10 Mini Exercises.
3. All Exercises have 20 to 25 items.
4. All Mini Exercises have three items.
5. Exercises 1, 3, 5, 7, and 9 require a constructed answer.
6. Exercises 2, 4, 6, 8 and 10 require students to select an appropriate answer from three to four answer choices.

Exercises 1 and 2 Students multiply and divide 2-digit numbers by 1-digit numbers. Multiplication involves regrouping (carrying) in 90% of the items. Division items have a 2-digit quotient with remainder. There are two pattern problems at the beginning that look about like this:

```
 1          22 r1
 35       4)89
 x3         8
105         9
            8
            1
```

All items avoid hard facts.

		6 x 7,	6 x 8,	6 x 9
	7 x 6,	7 x 7,	7 x 8,	7 x 9
	8 x 6,	8 x 7,	8 x 8,	8 x 9
9 x 5,	9 x 6,	9 x 7,	9 x 8,	9 x 9

Items are written in twos and threes, more or less like this:

```
1)      2)      3)
  26      76      77
  x2      x2      x2

4)      5)      6)      7)
  3)64    3)74    3)70    3)61
```

Altogether, there should be about 25 to 30 items that mix multiplication and division in this way.

All of the items in Exercises 1 and 2 should have a vertical (or working) format.

Exercises 3 and 4

Students multiply or divide 2- and 3-digit numbers by a 1-digit number. Multiplication items have no middle 0's. For example:

```
 305
 x5
```

Division items have no 0's in 3-digit dividends and most of them have a 2-digit quotient, with or without remainder. About 3/4 of all division items should have remainders. Two pattern problems should come first, more or less like this:

```
 166          84 r1
 x2        3)253
            24
             13
             12
              1
```

Multiplication items with 3-digit multiplicands may require 1 or 2 regroupings (carries).

Items are written in groups of three or four, with multiplication and division alternating from group to group.

```
1)      2)      3)      4)
  137     136     163     166
  x2      x2      x2      x2

5)      6)      7)      8)
  2)153   2)253   2)155   2)250
```

Altogether there should be about 25 to 30 items that mix multiplication and division as shown.

All items should be in vertical (working) form.

Use of hard multiplication and division "facts" should be avoided. (See specifications for Exercises 1 and 2.)

Exercises 5 and 6

Students multiply or divide 2- and 3-digit numbers by a 1-digit number. Multiplication items may have a 0 in the multiplicand. For example:

$$\begin{array}{r} 503 \\ \underline{\times 4} \end{array} \qquad \begin{array}{r} 530 \\ \underline{\times 4} \end{array}$$

Division items may have 2- or 3-digit quotients, with or without remainders.

These exercises should have no pattern problems.

Almost all multiplication items should have 3-digit multiplicands and may require 1 or 2 regroupings (carries).

Items should be written in groups about like this:

$$\begin{array}{ll} 1) & \begin{array}{r} 137 \\ \underline{\times 2} \end{array} \\ 2) & \begin{array}{r} 107 \\ \underline{\times 2} \end{array} \\ 3) & \begin{array}{r} 120 \\ \underline{\times 2} \end{array} \\ 4) & \begin{array}{r} 177 \\ \underline{\times 2} \end{array} \\ 5) & 3\overline{)50} \\ 6) & 3\overline{)505} \\ 7) & 3\overline{)500} \\ 8) & 3\overline{)200} \end{array}$$

There should be about 25 to 30 items that mix multiplication and division in about this way.

All items should be in vertical (working) form.

Use of hard multiplication and division "facts" should be avoided. (See specifications for Exercises 1 and 2.)

Exercises 7 and 8

Students multiply 2-, 3-, and 4-digit numbers or divide 2- and 3-digit numbers by a 1-digit number. Multiplication items may have 0's in the multiplicand and may require up to three regroupings. Division items may have a 2- or 3-digit quotient and may have 0's in the quotient. Most division problems will have remainders, but two or three may not.

These exercises should have two pattern problems at the beginning that look about like this:

```
 1 1                 104 r2
 2022              3)314
   x7                3
14154                 1
                      0
                      14
                      12
                       2
```

Altogether, there should be 25 to 30 items that alternate between small sets of multiplication and division items that look about like this:

```
1)        2)        3)
  22        202       2022
  x7          x7        x7

4)        5)        6)        7)
  3)31      3)301     3)321     3)121
```

All items should be in vertical (working) form.

Use of hard multiplication and division "facts" should be avoided. (See specifications for Exercises 1 and 2.)

Exercises 9 and 10

Students multiply and divide by a 1-digit number. Multiplication items may have multiplicands up to 4-digits. Division items may have dividends up to 3-digits. All but one or two multiplication items should involve regrouping.

Items should alternate between multiplication and division at random, more or less, and they should also mix in both vertical and horizontal formats.

Altogether there should be about 15 to 20 items.

Use of hard multiplication and division "facts" should be avoided. (See specifications for Exercises 1 and 2.)