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ABSTRACT

This study documented the spontaneously derived strategies that children use which may play a key role in the transition from counting to recall of number facts. For these strategies, the child uses a small set of known number facts to derive the solution to unknown number facts. How training in the use of derived strategies influences the solution strategies used was also studied, with special note of changes in solution strategies from counting to strategies based on relations among facts. A teaching experiment was conducted with one second-grade class (N=23) for eight weeks beginning in early September. Four interviews were conducted with each child, involving a pretest, a mid-instruction interview, a posttest, and a long-range effects test, in addition to short daily interviews and three group timed tests. The instructional unit and strategies taught are described, as are the interviews, observations, timed test, and student workbooks. Derived fact strategies accounted for about 20 percent of the children's responses on the pretest, and for one-half of the responses to addition combinations and over one-fourth of the answers to subtraction combinations following instruction. (MNS)

Derived Facts Strategies in Learning Addition and Subtraction

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Derived Facts Strategies in Learning Addition and Subtraction

BACKGROUND

Extensive research has been conducted in recent years on processes that children use to solve addition and subtraction problems (See reviews and models by Briars & Larkin, 1984; Carpenter & Moser, 1982; Carpenter, Moser, & Romberg, 1982; Nesher, Greeno, & Riley, 1982; Riley, Greeno, & Heller, 1983). This research has described the development of increasingly efficient counting strategies by children.

Although children's counting strategies have been clearly documented, very little is known about the strategies they use beyond counting and about the transition from counting to recall of number facts. This study concentrates on strategies that may play a key role in this transition. For these strategies, the child uses a small set of known number facts to find or derive the solution to unknown number facts. These strategies have been labeled "derived facts" (Carpenter & Moser, 1983). They have also been called "thinking strategies" (Cobb, 1983; Thornton, 1978; Rathmell, 1978, 1979) "indirect solutions" or "indirect memory" (Steffe, 1979; Houlihan & Ginsburg, 1981), and "heuristic strategies" (Carpenter, 1980).

There is evidence that young children use derived facts spontaneously (Beattie, 1979; Blume, 1981; Carpenter, 1980; Carpenter & Moser, 1982; Houlihan & Ginsburg, 1981; Lankford, 1974); howe[illegible], only a general description of most strategies is available and only a few of them have been described in detail. Although derived facts strategies are not used frequently, there is evidence that they may represent a transition strategy for a substantial number of children which precedes the learning of number facts at a recall level (Carpenter & Moser, in press). Furthermore, relating number facts to each other can lend an organizational framework to the set of facts and might provide a logical basis for storing addition and subtraction combinations in long-term memory (Carpenter, 1980; Carpenter & Moser, 1983). Even when a child recalls the facts, it is important that he or she be able to justify the answers and to reason why they are true (Brownell, 1928).

There has been little research to investigate how instruction affects the thinking processes children use in solving addition and subtraction problems, and how it affects the use of derived facts strategies in particular. Data from Japan suggest that most young Japanese children rely on derived facts rather than on counting (Hatano, 1980). Since the instruction in Japan encourages the use of regrouping strategies, this suggests that instruction may influence the solution strategies of children. In a study with American second grade children, Thornton (1978) found that teaching addition and subtraction derived facts strategies resulted in increased achievement, but little information was provided about the influence of instruction on the solution strategies the children used. Cobb (1983) encouraged six second graders to learn derived facts strategies by presenting related facts in successive order, but he did not explicitly teach or explain these strategies to the children. E.C. Rathmell (Personal Communication, February 1984) observed that children were able to learn derived facts strategies when taught how to use them.

PURPOSE OF THE STUDY

The purpose of this study was to document the spontaneous derived facts strategies that children use and to investigate how training in the use of derived facts strategies influences the solution strategies children use to solve addition and subtraction problems. Of special interest was whether children would change their solution strategies from reliance on counting to useing strategies based on relations among number facts.

METHOD

In order to obtain a detailed picture of children's solution processes and the influence of instruction on these processes, a clinical study in the model of a "teaching experiment" was done. This research paradigm is common in the Soviet Union and has provoked considerable interest in recent years among researchers in mathematics education (Cobb & Steffe, in preparation; Romberg, in preparation). The sample consisted of one second-grade class (N=23) in a middle-class neighborhood in Madison, Wisconsin. The small sample size made it possible to analyze in detail the changes that occurred in each child's solution strategies. The instructional unit was taught for eight weeks by the regular classroom teacher beginning in early September. The teacher was trained before and during the instruction. Four interviews were conducted with each child: a pretest, an interview in the middle of the instructional unit, a posttest, and a long range effects test. Short daily interviews were also conducted. Additionally, three group timed tests were administered concurrently with the first three interviews. All the lessons taught were observed and anecdotal data were collected.

Instructional Unit

The instructional unit presented the number facts in a sequence that was different than is common in current textbooks. Instead of presenting the combinations in a successive order (e.g., 5 + all addends, 6 + all addends, etc.), or as "families" of combinations that sum to a particular number, the facts were presented in relation to derived facts strategies. Number facts with similar structure were grouped together (Similar proposals for presenting the numbers facts were also suggested by Heddens, 1980; Rathmell, 1978; and Thornton, 1978).

Addition. All the doubles (e.g., 4 + 4, 5 + 5) were presented first. Then facts that can be related to doubles were presented: doubles ± 1 (e.g., 6 + 7, 7 + 8), and doubles ± 2 (e.g., 6 + 8, 6 + 4). Then combinations that can be easily related to the number 10 were presented (e.g., 8 + 5 = (8 + 2) + 3). The second half of the unit was devoted to teaching strategies for subtraction problems. The children demonstrated the different addition strategies by using manipulative materials; the strategies were then discussed orally without the manipulatives. A workbook by Thornton and Noxon (1977) was used throughout the instructional unit.

Doubles + 1 and Doubles - 1 are strategies that exploit the easily remembered doubles facts to solve addition problems in which the addends differ by 1. For example, the problem 6 + 7 can be solved as one more than 6 + 6 (doubles + 1) or one less than 7 + 7 (doubles - 1).

To demonstrate these strategies, the children modeled the problems with unifix cubes of two different colors.

Three strategies were taught for combinations of addends that differ by 2 (e.g., 6 + 8). The first two strategies, doubles + 2 and doubles - 2, are analogous to the doubles + 1 and doubles - 1. The third strategy, "sharing" (Thornton, 1978), uses the idea of compensation. To solve 6 + 8, for example, one cube was removed from 8 and was given to the 6 to create 7 + 7, the "double in between".

The going through 10 strategy relates number facts to the number 10. To model the strategy, an egg carton cut into 10 spaces and unifix cubes were used. To solve the problem 9 + 5, for example, 9 cubes of one color were put in the carton and five cubes of another color were placed outside. Then one cube from outside was placed in the empty space, creating the representation of 10 + 4 (see Table 1).

Subtraction. The main strategy in subtraction was based on the relationship between addition and subtraction. First, the "number family" idea was stressed. Then the "reverse doubles" and "reverse doubles + 1" problems were introduced to demonstrate the "think addition" strategy. For example, to solve 13 - 6, the children were encouraged to think of "what number should be added to 6 to get 13?" The addition "going through 10" strategy was the main strategy that was stressed to help solve subtraction problems (see also Table 1). The number of lessons spent on each derived fact strategy taught was approximately four.

Evaluation

Several measures were used to evaluate the children's solution strategies and the influence of the instruction on them, as well as the children's achievement.

Interviews. Four interviews were conducted with every child in the class: a pretest, an interview in the middle of the instructional unit, a posttest, and a test of long-range effect two months after the end of the instruction.

The interviews had two parts. The first part included five addition and subtraction word problems with different semantic structures and the second part included eight addition and five subtraction number combinations. The second interview consisted of only the second part but the other three interviews included both parts. The structures for the word problems were: Joining, Separating, Compare, Join Missing Addend, and Missing Minuend. Examples of these problems appear in Table 2. The sums and minuends of each problem ranged from 11 to 15.

The eight addition number combinations were chosen to represent the derived facts strategies that were taught. The following combinations were chosen: 7 + 6, 7 + 8, 7 + 5, 6 + 8, 6 + 9, 9 + 5, 5 + 8, 8 + 4.

Of the two triplets chosen for each strategy, one started with the bigger addend and the second with the smaller addend. For each triplet, half the children were given one addend first and half of the children were given the reverse combinations. For example, half of the students were asked the problem 6 + 7 and the other half were asked 7 + 6. The five subtraction number pairs were chosen to represent the equivalent addition strategies taught. The triplets that were chosen were: 15 - 8, 14 - 6, 13 - 8, 12 - 4, and 14 - 9. In order to control for any possible influence of the order in which the combinations were presented, each child was assigned the addition and subtraction number combinations in a random order. Each child was presented with the same number triplets in the same order every interview.

On the word problems, five different number triplets were used. The number pairs were chosen to represent the addition derived facts strategies. The number triplets that were chosen were: 6,7,13; 9,4,13; 7,4,11; 7,5,12; and 8,3,11. The five number triplets were permuted using a latin square design so that every number triplet appeared with each of the five word problems, creating five different number triplet-word problem groups.

The order of the word problems was also varied. The first problem for each child was a Joining problem in order to ensure an easy problem at the beginning of the interview. The order of the four remaining problems was randomly assigned for each child. In all three interviews in which word problems were presented, each child was given word problems with the same basic structure and numbers, but with different names and situations.

Each interview lasted 15-25 minutes. Responses were audiotaped and coded using the procedures and coding system developed by Carpenter and Moser (1982). The experimenter conducted all the interviews after an extensive training in the interview procedures and coding system.

Daily Interviews. Short interviews that included one or two number combinations were conducted daily, with each third of the class.

These interviews were conducted during the regular math time while the child was working at his or her desk.

Observations. All the lessons during the teaching experiment were observed and anecdotal descriptions were collected on the solution strategies the children used during class, the time that was necessary to teach new strategies, the errors the children made, and the exact content that was covered during the instruction.

Time Test. Three short group tests were administered concurrently with the main interviews. The timed test included 12 addition and 12 subtraction combinations that were recorded on an audiotape. Children were given two-seconds to write the answer to each problem. Half of the problems had sums and minuends under 10 and half between 10 and 20.

Students' Workbooks. The workbooks were corrected daily by the experimenter. Notes were taken on the level of correct answers, systematic errors that were apparent, and other hints from the workbooks that helped shed light on the students' understanding and misconceptions of the material.

RESULTS

Pretest Strategies

Derived facts strategies accounted for about 20% of the children's responses on the pretest (see Tables 3 and 4). The children used a variety of derived facts strategies, some of which had not been identified prior to the study. Contrary to previous predictions (Carpenter, 1980), only 58% of the spontaneous addition derived facts were based on doubles or 10 as an intermediate number, while 42% were based on other number facts known to the individual children (see Table 5 and 6).

All the addition derived facts strategies that were included in the instructional unit were used by some children in the pretest interview. In addition to these strategies, the other fact strategy was also used. This strategy involved relating the problem to another known fact that did not involve a double or 10. An example can be seen in a child's words solving the problem 7 + 6: "7 + 5 = 12, 7 + 6 must be 13."

In subtraction, 39% of the derived facts used in the pretest were based on addition number facts (see Table 6). A few of the subtraction strategies involved a combination of known facts and counting (see also Table 1). These strategies accounted for a large percent of derived facts in the prestest because of their consistent use by a small number of children. One such strategy is Subtract from 10 (with counting). An example of this strategy can be seen in a child's words solving the problem 12 - 4: "(I have) 10 (fingers) on the table. I took away 4 (from 10 fingers) and I have 2 in my head and 6 on the table, that makes 8." (see also Table 1) This example can be summarized by: 12 - 4 = (10 - 4) + 2.

Influence of Instruction

Interviews

Number Combinations. The use of derived facts strategies more than doubled during the instruction, accounting for half of the answers to addition number combinations following instruction (see Table 3). The increased use of derived facts strategies was much larger for addition problems than for subtraction problems (see Tables 3 and 4). The increase in the use of the derived facts was accompanied by a decrease in counting strategies (see Tables 3 and 4) (The terminology for the counting and other derived facts strategies in the tables is taken from Carpenter & Moser, 1982). The level of use of derived facts strategies remained high two months after the end of the instruction.

Paired t-tests were calculated to test for an increase in the use of derived facts strategies from the pretest to interview 2 and from interview 2 to interview 3. Separate tests were done for addition and subtraction number combinations. The change in the use of derived facts strategies from interview 1 to interview 2 and from interview 2 to interview 3 were calculated for each child and these differences were used to carry out the t-test. In calculating these differences, the proportion, P, of answers found using derived facts strategies was transformed to f(p) = arcsin (sqrt P), to make the proportions more closely resemble normally distributed data (Snedecor & Cochran, 1980, p. 290). The paired t-test for addition from the pretest to interview 2 showed a statistically significant increase at the $\alpha < 0.05$ level ($t = 3.32$, $df = 22$). No significant difference was found in the use of derived facts strategies in addition between interviews 2 and 3 ($t = .92$, $df = 22$).

The paired t-tests for the subtraction number combinations showed no significant difference between the pretest and interview 2 ($t = .63$, $df = 22$) or between interview 2 and interview 3 ($t = 1.52$, $df = 22$).

The number of children who used derived facts also increased substantially. Ninety one percent of the children used derived facts at least once, 75% of them used derived facts on more than 35% of the addition number combinations in at least one interview and 61% of the children used derived facts on more than 60% of these problems in at least one interview. The number of children who used derived facts strategies with subtraction number combinations, though smaller than in addition, was still high. Eighty-three percent of the children used derived facts at least once, and 61% of the children solved two or more of the five subtraction problems in at least one interview by using derived facts.

Following the instruction, the children started using more doubles related strategies and 10 as an intermediate point for addition problems. In subtraction, the proportion of the derived facts based on additive strategies increased to 61%. The order of presentation of addends (e.g., 6 + 7 vs. 7 + 6) was found to have no effect on the use of derived facts strategies.

Word Problems. Although the derived facts strategies were taught in the context of number combinations, the children transferred their use to word problems (Table 7). The percent use of derived facts with the Joining and Separating problems was very similar to the percent use with addition and subtraction number combinations, respectively.

The use of derived facts with the Missing Addend and the Missing Minuend problems is especially striking. The children used more derived facts with the Missing Addend problem on the pretest (35%) than with any other problem. The Missing Minuend problem, however, attracted no use of derived facts on the pretest.

Strategy Profiles. Examination of the children's individual patterns on addition number combinations revealed three clusters of strategy profiles. The first group included six children who frequently used derived facts strategies on the pretest. These children used a greater variety of derived facts strategies after the instruction than they did on the pretest and their use of derived facts strategies with subtraction problems increased substantially. Almost all the children in this group relied primarily on recall of facts in the later interviews. No other group of children consistently used recall of facts.

The second group included 12 children who moved from mainly using counting on the pretest addition problems to mainly using derived facts strategies in the later interviews. Eight of these children showed a large change in their strategies' use and the fraction of answers they found by use of derived facts increased by more than 0.6. Some of these children consistently used the most basic counting strategies (e.g., Counting All and Counting From Smaller in addition) before the instruction. Thus, high level skills in counting did not appear to be a prerequisite for learning derived facts strategies.

The third group included five children who used little or no derived facts strategies throughout the study. Four of the five children were very good counters and used the Counting On From Larger strategy in addition and other counting strategies in subtraction very quickly and efficiently. Two of these children got perfect scores on the addition part of the time test, even though they reported using counting almost exclusively in the interviews. The fifth child was from a learning disabled group, and could not use his fingers to count.

Observations and Daily Interviews

The classroom observations and the daily interviews revealed many specific results about the difficulties children encountered at different points of the program, the progress they made in relation to the instruction, prerequisite skills that were needed and insights many children showed. A few general results will be reported here.

There was a delay of two to four lessons between the time a new strategy was taught in class and the time the children started using it by choice. This was true despite the fact that most of the children could successfully use the new strategy in the first or second lesson when asked by the teacher or the researcher.

A few children treated some of the new derived facts strategies taught in class as rote procedural rules without understanding the relationships among the number facts. For example, for addends that differ by 1, like 6 + 7, four children used the rule: "double the first addend and add 1". They continued to use this rule for combinations like 7 + 6, where the first addend was larger for two lessons. However, other children were able to transfer the strategies they learned in class to new situations. For example, after learning the doubles + 1 strategy (e.g., 6 + 7 = (6 + 6) + 1), four children solved problems like 5 + 9 by applying the idea of doubles (e.g., 5 + 9 = (5 + 5) + 4).

Most children encountered difficulties in learning to use derived facts strategies for subtraction problems. In particular, understanding and applying the relationship between addition and subtraction problems were difficult. After four weeks of instruction only 20% of the children started using this approach consistently to solve subtraction problems; another 40% of the children never used it.

Time Tests

Paired t-tests on the arcsine transformation of the proportion of correct answers on the timed test showed a statistically significant increase in both addition and subtraction. The increase in the proportion of correct answers from Test 1 to Test 2 was significant at the $\alpha < .001$ level ($df = 19$) for both addition and subtraction problems with t values of 5.76 in addition and 7.57 in subtraction. The increase in the proportion of correct answers between Test 2 and Test 3 was significant at the $\alpha < .001$ level for subtraction ($t = 3.62$, $df = 19$), but not for addition ($t = .79$, $df = 19$).

While the children in this study solved more problems on the timed test at the end of the instruction, there is evidence that the gains may not reflect just an increase in the use of recall of facts. Some children were observed counting during the test. Additionally, some children who never used recall of facts during the interviews were successful on the time test.

Correlations between the percentage of derived facts answers given in the interviews and the rate of success on the corresponding Time Tests (after transforming the proportions to the arcsine square root scale) were low. Thus, it is not possible to conclude that using derived facts strategies is a quicker method of solution than counting.

DISCUSSION

This study supports the findings of recent research that young children are good problem solvers when they solve addition and subtraction problems. Even before the instruction, many of the children were very creative and innovative in their use of strategies and showed good understanding of many mathematical concepts. The study documented the children's spontaneous use of derived facts strategies.

The children in the study changed their solution strategies considerably during the period of instruction, from mainly using counting strategies to using derived facts. This large change was seen both in the percent of problems that were solved by derived facts strategies and the number of children who used derived facts.

The percent of addition problems solved by derived facts increased from 20 percent to 50 percent after four weeks of instruction and was still at this high level three months later. About 40 percent of the subtraction problems were solved by derived facts strategies after the beginning of the instruction. By contrast, only 0-15 percent of the answers to addition and subtraction problems were found by using derived facts in other studies. Thus, although counting plays an important role in children's cognitive development, many children can be taught to also use alternative noncounting strategies.

Furthermore, teaching derived facts does not have to wait until children reach high-level counting skills. Even children who have attained only low-level counting skills showed a large change and began to use derived facts strategies.

Cobb (1983) argued that children with low-level counting skills will learn derived facts only in a rote way. There is evidence in this study, however, that the learning of derived facts was meaningful even to the children with only low-level counting skills. They were able to decide which strategy best fit certain number combinations and were able to use the strategies to solve problems. The children used the derived facts strategies for a long time and also showed some ability to transfer the strategies they had been taught. This was especially evident in the use of "other facts" strategies, in which the children used number facts individually known to them to generate answers to facts that they did not know.

The issue of the prerequisites needed to learn derived facts strategies has not been completely resolved by this study, since there were only a few children who used low-level counting strategies. An investigation with more children at low-level counting skills, perhaps with first grade students, will be helpful. Cobb (1983) argued that children's general cognitive development might also be a prerequisite for learning derived facts. The children in this study were beyond this point.

It is not clear that children with high-level counting skills will readily adapt to derived facts strategies. In fact, developed counting skills might even interfere with the learning of derived facts strategies. Four of the five children who did not use or rarely used derived facts during the study were skilled counters. It might be that some children become so proficient in their counting that they do not see the need and are unwilling to invest the effort to learn new strategies that might be slower and less accurate when first used. It is possible that if the derived facts strategies are taught from an early age, as in the Japanese curriculum, the children will stop using counting altogether.

Subtraction

The increased use of derived facts strategies on subtraction problems was not as large as on addition problems. This might be attributed to the fact that less time was spent teaching the subtraction derived facts strategies in the study. However, it seems that the subtraction derived facts strategies are more difficult to learn and involve the understanding of more general concepts. The strategies that were taught in this study required the children to first convert the subtraction problem into an addition missing addend problem and then to use additive derived facts strategies to solve it. Further, solving a missing addend problem using derived facts is more difficult than solving a regular addition combination and involves a somewhat different process. Rather than using derived facts to find a sum, the child must find the appropriate addend. For example, to solve the problem 6 + __ = 13 by using doubles, the 6 is doubled and 1 is added to one of the 6s creating a 7.

Thus, solving subtraction problems using additive derived facts strategies involves understanding and applying more complex concepts and requires holding more steps in memory for execution. Although applying the additive derived facts for subtraction problems seems difficult, the use of direct subtractive derived facts was seen in the pilot study to be equally difficult (Steinberg, 1983).

Although the increase in the use of derived facts strategies in subtraction was not large, there was a large increase in the proportional use of additive derived facts in solving subtraction problems (from 37% of the derived facts in interview 1 to 65% in interview 3). This increase indicated that the instruction on the connection between addition and subtraction and on additive strategies was partially effective.

Relationship to Recall of Facts

The data from this study are not conclusive as to whether extensive use of derived facts strategies leads to recall of number facts. As with Thornton's (1978) data, the children in this study solved more number fact problems on the timed test after the instruction on derived facts. This kind of test is usually used to measure recall of facts. However, there is evidence that the children might have successfully used other strategies on the Time Test.

Other evidence hints that the use of derived facts might indeed lead to recall of facts. Almost all the children who used many derived facts strategies at the beginning of the study moved to primarily using recall of facts at the end of the study. This was the only group of children that consistently used recall or facts. This, of course, does not necessarily describe a causal relationship.

A few children who first found 9s combinations by relating them to 10s, later recalled the 9s combinations more than other facts and used their knowledge of 9s facts as a reference to find other facts. It could be that the use of derived facts strategies eventually becomes automatic and turns into a retrieval process from memory, in which the derived fact is used unconsciously.

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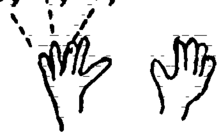
Table 1

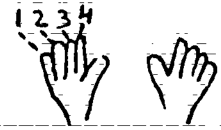
Derived Facts Strategies

Strategy	Example
Addition	
Doubles+1	6 + 7 = (6 + 6) + 1
Doubles-1	6 + 7 = (7 + 7) - 1
Doubles+2	7 + 5 = (5 + 5) + 2
Doubles-2	7 + 5 = (7 + 7) - 2
Sharing	7 + 5 = 6 + 6
Going Through 10	8 + 5 = (8 + 2) + 3
	5 + 9 = (9 + 1) + 4
Other Fact	7 + 5 = (7 + 4) + 1

Strategy	Example
Subtraction	
Additive	
Doubles +1	13 - 6 -> 6 + = 13 -> 6 + (6 + 1) = 13
Doubles-1	11 - 6 -> 6 + = 11 -> 6 + (6 - 1) = 11
Sharing	12 - 7 -> 6 + 6 = 12 so 7 + 5 = 12
Going Through 10	13 - 6 -> 6 + 4 = 10, 10 + 3 = 13 -> 4 + 3 = 7
Subtractive	
Down Other Fact	12 - 7 -> 12 - 8 = 4, 12 - 7 = 5
Subtract Through 10	12 - 7 -> 12 - 2 = 10 10 - 5 = 5
Subtract Through 10 (with counting)	15 - 8 -> 5; 6, 7, 8, 7 are left
Subtract From 10 (with counting)	12 - 4 = (10 - 4) + 2



1234



6 fingers are left, plus 2 in head is 8

Table 2

Word Problems Examples

1. Joining

Bob had 6 chocolate cupcakes.
His brother gave him 7 more.
How many chocolate cupcakes does Bob have altogether?

2. Separating

Mary had 11 flowers. She gave 7 of them to her sister.
How many flowers did Mary have left?

3. Join Missing Addend

Tim has 9 pet fish in his tank.
How many more fish does he have to put with them so there will be 13 fish?

4. Compare

Rachel won 11 prizes at the fair.
Her brother Ralph won 8 prizes at the fair.
How many more prizes did Rachel win than Ralph?

5. Missing Minuend

There were some birds sitting on a wire.
5 of the birds flew away. Then there were 7 birds left.
How many birds were there sitting on the wire before any flew away?

Table 3

Percent Use of Strategies in Addition

Number Combinations (8 problems, N = 23)

Strategies	Interview 1	Interview 2	Interview 3	Interview 4
Derived Facts	21	49	48	41
Counting All	10	4	7	3
Counting On From Smaller	4	1	4	2
Counting on From Larger	55	37	24	32
Number Fact	6	9	16	21
Memory Fail	2	0	0	0
No Response	1	0	0	0
Uncodable	1	0	0	0
% Correct	85	90	98	91

Table 4

Percent Use of Strategies in Subtraction

Number Combinations (5 problems, N = 23)

Strategies	Interview 1	Interview 2	Interview 3	Interview 4
Derived Facts	25	29	38	29
Separate From	0	4	2	0
Counting Down From	44	36	34	35
Counting Down To	0	3	2	2
Adding On	0	0	0	0
Counting Up From Given	9	8	4	10
Number Fact (Add)	10	10	13	16
Number Fact (Sub)	1	3	4	5
Memory Fail	1	3	0	0
No Response	3	1	2	1
Uncodable	2	0	0	0
Guess	3	3	0	1
% Correct	69	76	80	78

Table 5

Frequencies and Percent Use (in parentheses) of
Different Strategies Among the Derived Facts Strategies
Used on Addition Combinations

Strategies	Interview 1	Interview 2	Interview 3	Interview 4
Doubles+1	2 (5)	18 (20)	16 (19)	12 (7)
Doubles-1	7 (18)	4 (4)	3 (3)	3 (4)
Doubles+2	2 (5)	12 (13)	3 (3)	2 (3)
Doubles-2	0 (0)	2 (2)	1 (1)	1 (1)
Doubles+3	1 (3)	9 (10)	3 (3)	1 (1)
Doubles-3	0 (0)	1 (1)	0 (0)	1 (1)
Doubles+4	1 (3)	1 (1)	0 (0)	0 (0)
Sharing	0 (0)	9 (10)	4 (5)	3 (4)
Going Through 10	9 (24)	20 (22)	35 (41)	31 (43)
Other Fact	16 (42)	13 (15)	20 (24)	18 (25)
Total	38	89	85	72

Table 6

Frequencies and Percent Use (in parentheses) of
Different Strategies Among the Derived Facts Strategies
Used on Subtraction Combinations

Strategies	Interview 1	Interview 2	Interview 3	Interview 4
Additive				
Doubles+1	0 (0)	0 (0)	1 (2)	1 (3)
Doubles-1	0 (0)	0 (0)	3 (7)	1 (3)
Doubles+2	1 (3)	0 (0)	4 (9)	2 (6)
Sharing	3 (10)	0 (0)	4 (7)	3 (9)
Going Through 10	3 (10)	6 (19)	3 (7)	5 (16)
Other Fact	5 (16)	7 (22)	13 (29)	2 (6)
Subtractive				
Down Other Fact	3 (10)	0 (0)	2 (4)	0 (0)
Subtract Through 10 (with counting)	5 (16)	9 (28)	5 (11)	16 (50)
Subtract Through 10	1 (3)	2 (6)	7 (16)	0 (0)
Subtract From 10 (with counting)	10 (32)	8 (25)	1 (2)	0 (0)
Total	31	32	44	32

Table 7

Percent Use of Strategies on Word Problems

	Counting Int 1	Counting Int 3	Derived Facts Int 1	Derived Facts Int 3	Recall of Facts Int 1	Recall of Facts Int 3	Percent Correct Int 1	Percent Correct Int 3
Joining	69	35	17	52	9	13	83	100
Separating	56	34	22	35	9	17	70	87
Join Missing Addend	43	52	35	39	4	17	84	96
Compare	66	60	17	35	4	4	71	96
Missing Minuend	69	34	0	35	13	17	84	91