DOCUMENT RESUME

ED 241 304 SE 043 997

AUTHOR Hiebert, James; Carpenter, Thomas P.
TITLE Information Processing Capacity, Logical Reasoning Ability, and the Development of Measurement Concepts. Working Paper No. 299.
INSTITUTION Wisconsin Center for Education Research, Madison.
SPONS AGENCY National Inst. of Education (ED), Washington, DC.
PUB DATE Oct 80
GRANT OB-NIE-G-80-0117
NOTE 64p.; Produced by the Project on Studies in Mathematics.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Cognitive Development; *Cognitive Processes; *Concept Formation; Conservation (Concept); Educational Research; Elementary Education; *Elementary School Mathematics; Grade 1; Information Processing; *Mathematical Concepts; *Mathematics Instruction; *Measurement; Primary Education
IDENTIFIERS *Mathematics Education Research; Piagetian Tasks

ABSTRACT

This study investigated: (1) the relationship between the development of information processing capacity and certain Piagetian logical reasoning abilities; and (2) how the development of these cognitive abilities related to acquisition of certain measurement concepts. Forty first-grade children were individually administered tests of conservation of length and number, transitivity of length, information processing capacity, and basic length measurement concepts. The Piagetian measures of logical reasoning were positively correlated with information processing capacity, but the measures of information processing capacity failed to account for much of the variability of performance on the logical reasoning tasks. Some children at the highest levels of processing capacity failed the logical reasoning tasks and some at the lowest level passed them, suggesting that the logical reasoning tasks are not simply measures of information processing capacity. Furthermore, information processing measures accounted for 25% of the variance in performance on the linear measurement tasks, and length conservation accounted for an additional 23%. Although these two measures accounted for almost half of the variance, it is not clear that they represent prerequisites for learning basic length measurement concepts. Some children at low levels on both measures successfully completed the measurement tasks. (Author/MNS)

Working Paper No. 299

INFORMATION PROCESSING CAPACITY, LOGICAL REASONING ABILITY, AND THE DEVELOPMENT OF MEASUREMENT CONCEPTS

by

James Hiebert and Thomas P. Carpenter

Report from the Project on
Studies in Mathematics

Thomas A. Romberg and Thomas P. Carpenter
Faculty Associates

Wisconsin Research and Development Center
for Individualized Schooling
University of Wisconsin
Madison, Wisconsin

October 1980

Published by the Wisconsin Research and Development Center for Individualized Schooling. The project presented or reported herein was performed pursuant to a grant from the National Institute of Education, Department of Health, Education, and Welfare. However, the opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education, and no official endorsement by the National Institute of Education should be inferred.

Center Grant No. OB-NIE-G-80-0117

MISSION STATEMENT

The mission of the Wisconsin Research and Development Center is to improve the quality of education by addressing the full range of issues and problems related to individualized schooling. Teaching, learning, and the problems of individualization are given concurrent attention in the Center's efforts to discover processes and develop strategies and materials for use in the schools. The Center pursues its mission by

- conducting and synthesizing research to clarify the processes of school-age children's learning and development
- conducting and synthesizing research to clarify effective approaches to teaching students basic skills and concepts
- developing and demonstrating improved instructional strategies, processes, and materials for students, teachers, and school administrators
- providing assistance to educators which helps transfer the outcomes of research and development to improved practice in local schools and teacher education institutions

The Wisconsin Research and Development Center is supported with funds from the National Institute of Education and the University of Wisconsin.

WISCONSIN RESEARCH AND DEVELOPMENT
CENTER FOR INDIVIDUALIZED SCHOOLING

Table of Contents

List of Tables

Abstract

This study investigated the relation between the development of information processing capacity and the development of certain basic logical reasoning abilities characterized by Piaget and how the development of these sets of cognitive abilities related to children's acquisition of certain measurement concepts. A group of 40 first-grade children were individually administered tests of conservation of length, conservation of number, transitivity of length, three measures of information processing capacity, and a test of basic length measurement concepts.

The basic Piagetian measures of logical reasoning were positively correlated with information processing capacity, but the measures of information processing capacity failed to account for much of the variability of performance on the logical reasoning tasks. Some children at the highest levels of processing capacity failed the logical reasoning tasks and some at the lowest level passed them. This suggests that the logical reasoning tasks are not simply measures of information processing capacity. Furthermore, using step down regression techniques in which the information processing measures were entered first, conservation of length accounted for a significant portion of the variance on the mathematical concepts test not accounted for by information processing capacity. Information processing measures accounted for 25 percent of the variance in children's performance on the linear measurement tasks and length conservation accounted for an additional 23 percent. Although these two measures accounted for almost half of the variance, it is not clear that they represent prerequisites for learning basic length measurement concepts. Some children at low levels on both measures successfully completed the measurement tasks.

Information Processing Capacity, Logical Reasoning Ability, and the Development of Measurement Concepts

James Hiebert
University of Kentucky

Thomas P. Carpenter
University of Wisconsin

Purpose

The study of cognitive development has identified two fundamental dimensions along which development proceeds. The first is logical reasoning ability of the kind described in Piaget's work (cf. Piaget, 1952; Piaget, Inhelder & Szeminska, 1960). This includes the ability to conserve quantity under spatial transformations and to make transitive inferences. These two abilities in particular have received much attention in mathematics education research aimed at tracing the development of basic mathematical concepts (Carpenter, 1980).

The second major dimension of cognitive development can be described in terms of an increasing ability to deal with several alternatives simultaneously (Bruner, 1966; Carpenter, 1975; Inhelder, 1972). This has also been referred to as an increase in working memory or information processing capacity (Case, 1979; Pascual-Leone, 1970). Young children have an especially limited capacity to coordinate and integrate "chunks" of information, a limitation which may be critical in mathematics instruction contexts (Carpenter, 1979, 1980).

Logical reasoning ability and information processing capacity represent fundamental dimensions of cognitive development which may have important implications for how children learn mathematics. However, at present, little is known about the role they play in the acquisition of mathematical concepts. Furthermore, even though these dimensions are theoretically related (Inhelder, 1972; Pascual-Leone, 1970), little information is available which verifies this relationship. The purpose of this study is to provide this information by focusing on certain logical reasoning abilities, several measures of information processing capacity, and certain basic measurement concepts. There are three specific objectives of the study: (a) to examine the relationship between the two developmental dimensions and children's knowledge of linear measurement concepts; (b) to investigate the relationship between the developmental dimensions themselves; and, (c)to establish the degree of association between different measures of information processing capacity.

Background and Rationale

The development of logical reasoning ability has been studied most completely within the framework of Piaget's theory. Because many of these abilities, such as conservation and transitivity, are closely tied to mathematical concepts they have received much attention in the research literature. From a logical perspective, abilities like conservation and transitivity are required to solve a variety of arithmetic and measurement problems. For example, the notion of cardinal number is based on matching sets, and the matching relation assumes conservation. Therefore conservation should be required for any meaningful concept of number. Conservation and transitivity also play a critical role in measurement operations. It is difficult to see how lengths can be meaningfully compared or measured if the child believes that simply moving a length, or altering its path, will change its size, i.e., if the child fails to conserve. In addition, all indirect comparisons, as well as unit iteration, require transitive inferences between equalities or order relations.

The limitation of this analysis is that it is based only on logical considerations, and children's logic is different than adults' logic. If children are not asked specific conservation questions, these questions do not occur to them; and children ignore the fact that their judgements depend upon certain prerequisite knowledge that they lack. There is ev. .ence that children who are pre-operational in Piagetian terms can successfully apply a variety of number, measurement, and geometric concepts and skills (Carpenter, 1980).

Some research efforts have been directed toward establishing the relationship between basic Piagetian constructs and children's learning of mathematics. One approach has been to correlate performance on a test of Piagetian tasks with some measure of mathematics achievement (cf. Cathcart, 1971; Dimitrovsky & Almy, 1975; Kaufman & Kaufman, 1972; Steffe, 1970). These studies have found high positive correlations, even when IQ is held constant (Steffe, 1970). Furthermore performance on Piagetian batteries administered in kindergarten appears to be an excellent predictor of mathematics achievement as much as three years later (Bearison, 1975; Dimitrovsky & Almy, 1975). These high positive correlations do not imply, however, that the logical skills of conservation and transitivity are required for learning mathematical concepts. The high correlations may simply indicate that the Piagetian tasks are good measures of a general cognitive ability, such as the ability to process information. Both the Piagetian tasks and the mathematical tasks may require an ability to attend to, and process, several pieces of information at the same time. Rather than serving as genuine prerequisites for mathematics learning, these logical reasoning skills may simply indicate the level of processing capacity available to the child.

A general integration or information processing capacity has, in fact, been proposed as the fundamental characteristic of cognitive development (Case, 1979; Pascual-Leone, 1970). From this perspective, children's improved performance with age on both the Piagetian tasks and mathematical tasks presumably reflects a developmental increase in this processing capacity. Unfortunately, little evidence is available which might substantiate or refute this conjecture.

The relationship between information processing and mathematics learning has been scarcely researched. This is in spite of the fact that information processing capacity may be the critical limitation in children's mathematics learning (Carpenter, 1979; Case, 1975, 1978). Young children are still quite limited in their ability to deal with all of the information demands of complex tasks. Yet instructional tasks require children to receive, encode, and integrate a substantial amount of information. Children's performance in mathematics instruction situations may be limited by their restricted capacity to deal with all of the incoming information.

Research using specially designed instructional tasks has shown that children's learning is constrained by their information processing capacity (Case, 1972. 1974a, 1974b; Parkinson, 1975). After instruction, children were found to succeed only on those tasks which made processing demands within the range of their processing capacity. Although the limits of learning laboratory tasks have been predicted with impressive accuracy, little is known about the role of information processing capacity in performing school mathematics tasks. A major objective of this study was to investigate the relationship between information processing capacity and mathematical performance. The intent was to determine whether processing capacity accounts for a significant amount of variation in performance on mathematical tasks.

An equally important question was whether information processing capacity accounts for all explained variation on the mathematical tasks or whether logical reasoning ability accounts for an additional portion of variance. The Piagetian tasks may simply represent good measures of processing capacity, and may have little explanatory power beyond that captured by more conventional measures of processing capacity. To the extent that performance on Piagetian tasks accounts for variation in performance on mathematical tasks in addition to that explained by information processing capacity, the logical reasoning abilities identified by Piaget might be considered to be related to mathematics performance independently of processing capacity. In other words, Piagetian tasks would be measuring an ability important for mastering mathematical tasks which could not be completely explained in information processing terms.

In addition to examining the relative contributions of these two cognitive variables in explaining mathematical performance, the present study directly investigated their relationship. Although the development of logical abilities can be described in information processing terms (Carpenter, 1975; Inhelder, 1972; Klahr & Wallace, 1976; Pascual-Leone, 1970) little empirical data is available on the relationship between the two.

Several researchers have considered the relationship between information processing capacity and logical reasoning ability by analyzing Piagetian tasks in terms of their information processing demands. These include conservation of substance (Hamilton & Moss, 1974), conservation of quantity (Klahr & Wallace, 1976), transitivity (Klahr & Wallace, 1976), length seriation (Baylor & Lemoyne, 1975), weight seriation (Baylor & Gascon, 1974; Baylor & Lemoyne, 1975), class inclusion (Klahr & Wallace, 1976; Pascual-Leone & Smith, 1969), bending rods (Case, 1974b), and combinatorial reasoning (Scardamalia, 1977). All of these studies carried out a logical analysis of the task either for descriptive purposes or to identify critical parameters which could be manipulated empirically. However, none of the above studies included data on the relationship between a subject's information processing capacity and performance on the conventional Piagetian task in question.

Two recent studies contain slightly more information on this relationship. Hamilton and Launay (1976) administered both a conservation of substance task and an information processing task to the same group of subjects. They found, as hypothesized, significant differences between groups of subjects classified as good-, poor-, and non-conservers on the information processing task. However the conservation task was a nonstandard pictorial task and no correlational data are presented. A study by Lawson (1976) is apparently the only one which reports correlations between conservation ability and information processing capacity. He administered four conservation tasks (number, substance, liquid, and weight) and one information processing task (backward digit span) to 82 children ages 4.4 - 6.5 years. Pearson r correlations between M-space and conservation performance ranged from .17 for weight to .50 for number. With age partialed out the correlations ranged from .11 to .43. Although most correlations were significant, at least at the .05 level, they are not as high as one might expect if conservation ability is strictly a function of information processing capacity.

One objective of the current study was to investigate the relationship between logical reasoning ability and information processing capacity and to determine if some minimum capacity is required to conserve or reason transitively. The question of interest was whether Piagetian tasks tap a unique form of logical reasoning or whether they simply represent alternate measures of processing capacity.

A final objective of this study was to examine the interrelationship between various measures of information processing capacity. Although the development of this capacity is considered to be an important characteristic of cognitive growth, it has proven to be a difficult construct to operationalize. Information processing capacity has been conceptualized in different ways, but it is commonly recognized that working memory is a critical part of this capacity (Case, 1978). Information processing capacity is a type of short-term memory plus a transformation process. Information is not only stored but is also acted upon in some way. The problem is that in order to measure a child's capacity, one must know how much information must be processed to complete the given task. Since different subjects approach tasks in different ways, the demands of the task may change from subject to subject.

One way to begin working on this problem is to empirically determine the relationship between suggested measures of working memory. These measures have been specially designed so that children will approach them in similar ways, i.e., they will use similar strategies to solve them. Therefore, the processing demands of the tasks should be similar for all children. This means that they should be reliable measures of capacity, and as such should correlate highly.

Given this background information, the objectives of the study can be explicitly stated as follows:

1a. What percentage of variation in mathematical performance is accounted for by information processing capacity?

1b. What percentage of additional variation in mathematical performance is accounted for by logical reasoning ability?

2. Is there a significant relationship between information processing capacity and logical reasoning ability?

3. Is there a significant relationship between various measures of information processing capacity?

Procedures

Subjects

The sample consisted of 40 first-grade children drawn from two elementary schools located in rural communities in central Wisconsin. This age group was chosen since some variance on both types of developmental tasks could be expected with the children of this age. The Piagetian tasks would not be particularly interesting with older children, and younger children may have a difficult time with some of the information processing tasks.

Tasks

Three groups of tasks were included in this study: mathematical tasks, Piagetian tasks measuring logical reasoning ability, and measures of information processing capacity.

Mathematical tasks. The mathematical tasks selected for this study were three linear measurement tasks. The tasks involved several fundamental ideas of linear measurement and required more than the application of simple measuring skills or techniques. They required logical-mathematical knowledge, in the

Piagetian sense (Piaget, 1964), in that they involved some of the fundamental, logical notions of measurement. Recent research suggests that basic cognitive abilities may be more closely related to learning these kinds of mathematical concepts than to acquiring mathematical skills (Steffe, Spikes & Hirstein, Note 1). In order to maximize the possibility of uncovering significant relationships between the two cognitive dimensions and mathematical performance, tasks were chosen which embodied basic concepts of linear measurement and which were presumed to require logical-mathematical knowledge.

The first task involved the use of an intermediate, continuous representation to compare and order lengths. Children were asked to drive their toy car the same distance as a second car driven by the experimenter on an adjacent road. The roads were placed to prevent correct visual solutions and the children were given a blank strip to "help them measure."

The second measurement task involved the use of a discrete representation to construct a straight path equal in length to a given polygonal path. This task involved the concept of length additivity and the notion of length as the linear distance between endpoints. Children were given a collection of Cuisenaire rods and were asked to make a straight road on which there was "just as far to walk" as the crooked road.

The third measurement task required children to iterate units and focused on the idea of assigning numbers to lengths and using these numerical measures to compare and order lengths. Children were given a single Cuisenaire rod and were asked to determine which of two strips was longer. The strips were placed in a "T" to prevent correct visual comparisons.

A complete description of the tasks, the protocols used to administer them, and the scoring criteria are given in Appendix A.

Logical reasoning tasks. Logical reasoning abilities were assessed using Piagetian tasks of number conservation (two forms), length conservation (two forms), and length transitivity (two forms). From a logical perspective, length conservation and length transitivity are prerequisites for carrying out meaningful measurement operations. Consequently, performance on these tasks was expected to relate highly to performance on the measurement tasks. However, as stated previously, the anticipated correlations may result from the fact that these Piagetian tasks measure a general performance ability rather than representing genuine prerequisites. Number conservation tasks were included in the test battery in order to check this possibility. Since logical number skills are largely unrelated to the measurement tasks used in this study, number conservation should correlate lower than length conservation and length transitivity, provided that the latter two tasks measure prerequisite abilities. On the other hand, if the tasks are simply general ability measures, they should all correlate equally well with mathematical task performance.

In the number conservation tasks, children were asked to lay out a row of cubes which had just as many as the experimenter's row. In Form 1 of the task the cubes in one of the rows were spread apart and the child was asked about the equality of the two sets; in Form 2 the cubes in one of the rows were collapsed into two compact sets and the child was asked a similar question.

Form 1 of the length conservation task involved bending the longer of two wires so that it was transformed from initially appearing longer to ultimately appearing shorter than the other wire. After each of two transformations the child was asked about the relative lengths of the two wires. Form 2 of length conservation required the child to reaffirm the equality of two sticks after one of them was moved "ahead" of the other.

Both forms of the length transitivity task required the child to infer the relative lengths of sticks A and C after direct comparison had shown A to be longer than intermediate stick B, and B longer than C. Form 2 of the task contained a Müeller-Lyer illusion and Form 1 did not.

Complete descriptions of the tasks, protocols, and scoring procedures are present in Appendix A.

Information processing tasks. Three tasks were used to assess information processing capacity. The first was a backward digit span task modified from Lawson (1976). Digit span has traditionally been used to measure processing capacity because of the face validity of its requirement that the subject remember and operate on several units of information simultaneously. In the task used here, children were read a series of digits and were asked to repeat them backwards. Ten trials of length 2-, 3-, and 4-digits were included in the task. Testing stopped if the subject missed three consecutive trials, and subjects were scored by placing them into one of five categories based on the number of correct responses.

The second information processing task was a number sequencing task introduced by Case (1972). From a logical perspective, this task has requirements similar to the backward digit span task. Children are briefly shown a series of numbers presented successively and, except for the last, in ascending order of magnitude, e.g., 6, 9, 11, 7. The object is to place the final number in its correct position within the ordinal series. In this study, children were presented with five trials of 1-, 2-, and 3-digit series. Scoring procedures placed each child into one of four categories based on the largest series mastered.

The third information processing task was a symbol substitution task taken from Hamilton and Launay (1976). As in the previous two tasks, this task presumably requires the subject to receive, temporarily maintain, and transform or operate on several bits of information in order to respond appropriately. Children are asked to demonstrate this processing behavior by substituting a sequence of signs or symbols in chain-like fashion before selecting an appropriate response. The processing demand of the task is systematically manipulated by varying the number of substitutions required and the number of stimuli presented simultaneously. Children in this study were asked to make a series of two and three substitutions and were scored into one of three categories based on the largest number of substitutions mastered.

Complete descriptions of these tasks, protocols for administering them, and detailed scoring procedures are presented in Appendix A.

Design

The intent of this study was to explore the relationship between several variables. As such it was a descriptive study and the design was a simple one suitable for correlational procedures. All subjects received all of the tasks in an individual interview situation. The tasks were administered in two brief sessions several days apart. The tasks were partitioned into two sets by placing one form of each logical reasoning (Piagetian) task in each set and randomly assigning the measurement tasks and the information processing measures to one of the two sets. Half of the sample was randomly selected to receive Set 1 on the first day, the other half received Set 2 on the first day. Within each set the order of the tasks was randomized and then the tasks were administered in the same order to each subject.

Results

Order Effects

Effects of the presentation orders, Set 1 - Set 2 and Set 2 - Set 1 were assessed by comparing mean scores of the two groups on each variable. *T*-tests showed no significant differences ($\alpha = .01$) between the two groups on any of the variables. The two order groups were merged for the remaining analyses.

Information Processing Measures

Before relating the logical reasoning and information processing tasks, or determining the amount of variation in linear measurement performance accounted for by these cognitive abilities, it was necessary to select the "best" measure of information processing capacity. Cronbach's alpha was computed to assess the internal consistency of the three information processing tasks. The overall alpha was .53, but this rose to .60 with the symbol substitution task deleted. This task also showed a substantially lower corrected item-total correlation (.24) than the backward digit span task (.40) or the number sequencing task (.54). Consequently, most of the remaining analyses used backward digit span and number sequencing performance as measures of processing capacity.

Information Processing Capacity and Logical Reasoning Ability

For purposes of examining relationships between information processing capacity and logical reasoning ability, scores on the two forms of the Piagetian tasks were summed to create single scores for number conservation, length conservation, and length transitivity. Pearson correlation coefficients between these measures and the information processing tasks are shown in Table 1.

The symbol substitution task once again showed its unique character by the strikingly low correlations between performance on this task and performance on the logical reasoning tasks. An explanation for the lack of relationship between this task and the others is the ceiling effect generated by the high level of performance on this task. Thirty-six of the 40 children achieved a maximum score of 2 resulting in a mean score of 1.85 for this item.

Correlations between the other two information processing tasks and the logical reasoning tasks ranged from .19 between backward digit span and length transitivity to .54 between number sequencing and number conservation. Although most correlations were significant near the .01 level, performance on one task usually accounted for less than 20% of the variance in performance on the other.

In order to check whether some minimum level of information processing capacity was required to conserve or reason transitively, contingency tables were constructed for each logical reasoning task using backward digit span and number sequencing as measures of information processing capacity. Table 2 presents these results using backward digit span, and Table 3 presents the results with the number sequencing task.

Regardless of which measure of information processing capacity was used, some children who evidenced a low capacity were successful on the logical reasoning task. Furthermore, not all children who demonstrated a high capacity were completely successful on the logical reasoning tasks. In other words, the off-diagonal cells in these tables were not always empty.

Table 1

Correlations Between Performance On Logical Reasoning Tasks

and Information Processing Tasks

	Backward digit span	Number sequencing	Symbol substitution	Length conservation	Length transitivity	Number conservation
Backward digit span	1,000	.45 P= .00	.05 P= .38	.35 P= .01	.19 P= .12	.34 P= .02
Number sequencing		1,000	.32 P= .02	.38 P= .01	.40 P= .01	.54 P= .00
Symbol substitution			1,000	.04 P= .41	-.06 P= .36	.25 P= .06
Length conservation				1,000	.23 P= .08	.57 P= .00
Length transitivity					1,000	.20 P= .11

Table 2

Contingency Tables: Backward Digit Span

and Logical Reasoning Abilities

Developmental level	Backward digit span[a] 1	2	3	4
Number conservation				
0	4	3	2	0
1	0	1	1	0
2	4	1	0	0
3	0	0	0	0
4	3	10	8	3
Length conservation				
0	0	0	0	0
1	5	3	2	0
2	5	5	4	1
3	0	0	1	0
4	1	7	4	2
Length transitivity				
0	5	2	1	1
1	1	1	2	1
2	3	3	3	0
3	1	2	0	0
4	1	7	5	1

[a]Although five scoring categories were established, no subject scored in the lowest category.

Table 3

Contingency Tables: Number Sequencing

and Logical Reasoning Abilities

Developmental level	Number sequencing 1	2	3	4
Number conservation				
0	6	1	0	2
1	1	0	1	0
2	2	1	1	1
3	0	0	0	0
4	1	5	4	14
Length conservation				
0	0	0	0	0
1	5	2	1	2
2	3	3	3	6
3	1	0	0	0
4	1	2	2	9
Length transitivity				
0	4	4	0	1
1	1	1	1	2
2	2	1	2	4
3	1	0	0	2
4	2	1	3	8

Cognitive Abilities and Mathematical Performance

A single score for mathematical performance was created by summing the three measurement task scores. Correlations between the cognitive task scores and this composite mathematical score are presented in Table 4. Of the cognitive variables, length conservation was related most strongly to mathematical performance (r = .64) while length transitivity showed the weakest relationship (r = .23).

A regression analysis was run on mathematical performance using a hierarchical stepwise procedure in which the information processing measures were entered as the first set of predictors, followed by the set of logical reasoning tasks. This procedure permitted an assessment of the maximum amount of variation explainable by information processing variables, and the additional amount of variation, if any, explained by logical reasoning factors. It also provided information on the best single predictor within each set. Table 5 summarizes the results from this regression analysis.

Together, the information processing measures accounted for 25% of the variation in children's performance on the linear measurement tasks. Almost all of this portion was explained by the number sequencing task alone. An additional 23% of the variance was accounted for by the logical reasoning tasks, all of it by length conservation. The standardized regression coefficients support the interpretation that the number sequencing and length conservation tasks were the best predictors of measurement performance. Once the variation accounted for by these factors was removed, the other three tasks added little to the regression equation.

Although logical reasoning ability, in particular length conservation,

Table 4

Correlations Between Cognitive Abilities

and Mathematical Performance

Cognitive abilities task	Correlation with measurement score	*p*
Backward digit span	.32	.02
Number sequencing	.49	.00
Length conservation	.64	.00
Length transitivity	.23	.08
Number conservation	.47	.00

Table 5

Multiple Regression: Measurement

Performance on Cognitive Abilities

Variable	Multiple R	R^2	R^2 change	Standardized regression coefficient
Number sequencing	.49	.24	.24	.29
Backward digit span	.50	.25	.01	.01
Length conservation	.69	.48	.23	.53
Length transitivity	.69	.48	.00	-.02
Number conservation	.69	.48	.00	.01

did account for a substantial portion of variation in measurement performance not explained by the information processing measures, it is not yet clear whether they should be regarded as prerequisite abilities. Tables 6 and 7 are contingency tables which show measurement task performance at each level of length conservation and length transitivity, respectively. Number conservation was not included in this analysis since this ability does not logically appear to be a potential prerequisite for linear measurement.

Table 6 shows that most of the subjects who mastered the measurement tasks had achieved conservation. However there are a number of counter-examples which suggest that the forms of length conservation assessed here were not required to measure successfully. The data in Table 7 suggest that length transitivity is also not a prerequisite for learning many basic measurement concepts.

Discussion

The major goal of this study was to investigate the relation between the acquisition of certain basic measurement concepts and the development of more general cognitive abilities that might be prerequisites. From an instructional point of view, the question of whether the learning of certain mathematical concepts depends upon the development of basic cognitive abilities is important. There are potentially different instructional implications if certain concepts are closely linked to underlying cognitive abilities whose development is difficult to accelerate than if this is not the case.

Unfortunately, there does not appear to be a simple answer to this question. Although certain cognitive abilities were positively correlated

Table 6

Contingency Tables: Length Conservation

and Measurement Task Performance

	Length conservation				
Level	0	1	2	3	4
Measurement task 1					
0	0	8	12	1	4
1	0	2	2	0	6
2	0	0	1	0	4
Measurement task 2					
0	0	7	11	0	5
1	0	3	4	1	8
2	0	0	0	0	1
Measurement task 3					
0	0	8	5	0	0
1	0	2	7	1	7
2	0	0	3	0	7

Table 7

Contingency Tables: Length Transitivity

and Measurement Task Performance

	Length transitivity				
Level	0	1	2	3	4
Measurement task 1					
0	7	4	5	2	7
1	2	1	2	1	4
2	0	0	2	0	3
Measurement task 2					
0	6	2	6	2	7
1	3	3	3	1	6
2	0	0	0	0	1
Measurement task 3					
0	2	3	4	1	3
1	5	1	3	2	6
2	2	1	2	0	5

with students' knowledge of basic measurement concepts, some students at all levels of cognitive ability were able to successfully perform the measurement tasks. Several factors may account for these findings. One possibility is that minimum levels of the basic logical abilities and information processing capacity are not required to successfully learn the measurement concepts tested. From this point of view, one might argue that the positive correlations could result from the fact that the measures of cognitive ability and measurement concepts are all correlated with general intelligence.

On the other hand minimum levels of certain of the cognitive abilities may in fact be required to successfully learn certain mathematical concepts but there also may be a certain amount of measurement error in determining children's level of cognitive ability or knowledge of measurement concepts. A slightly different perspective on this explanation is that certain minimum levels of logical skills or information processing capacity are required to successfully perform any tasks but other factors intervene which affect performance. In other words it is not possible to get a pure measure of information processing capacity or logical abilities because task variables affect the way that a particular cognitive ability is applied to a given task. For example, less familiar tasks may require a greater amount of information processing capacity for the executive routines that control the solution process. Consequently, there is less processing capacity left to operate on the data in the problem to be solved. Thus, although processing capacity may limit performance in this case, a simple measure of processing capacity is not possible, because the processing capacity available to different problems may vary.

Thus, these cognitive abilities may represent valid theoretical constructs and may be directly related to children's ability to learn specific mathematical concepts. Whether these constructs have an important educational application is another question altogether. Although conservation of measures of cognitive abilities accounted for almost half of the variance on the measurement tasks, students at all levels of ability were able to solve each of the measurement tasks. Therefore, to say that a student who scored below a certain level on these measures of cognitive ability would be unable to or even unlikely to learn certain mathematical concepts would be inappropriate.

Thus, although certain of the abilities investigated in this study may help explain children's difficulty in learning certain mathematical concepts in general terms, it appears unlikely that a valid readiness test based on these constructs can be developed to effectively identify children who would not be able to learn a particular mathematics concept.

Reference Note

1. Steffe, L.P., Spikes, W.C., & Hirstein, J.J. Summary of quantitative comparisons and class inclusion as readiness variables for learning first grade arithmetical content. Athens, GA: The Georgia Center for the Study of Learning and Teaching Mathematics, 1976.

References

Baylor, G.W., & Gascon, J. An information processing theory of aspects of the development of weight seriation in children. Cognitive Psychology, 1974, 6, 1-40.

Baylor, G., & Lemoyne, G. Experiments in seriation with children: Towards an information processing explanation of the horizontal decalage. Canadian Journal of Behavioral Science, 1975, 7, 4-29.

Bearison, D.J. Induced versus spontaneous attainment of concrete operations and their relationship to school achievement. Journal of Educational Psychology, 1975, 67, 576-580.

Bruner, J.S. Toward a theory of instruction. New York: Norton, 1966.

Carpenter, T.P. The performance of first- and second-grade children on liquid conservation and measurement problems employing equivalence and order relations. In L.P. Steffe (Ed.), Research on mathematical thinking of young children. Reston, VA: National Council of Teachers of Mathematics, 1975.

Carpenter, T.P. Research on children's thinking and the design of mathematics instruction. In R.A. Lesh, D. Mierkiewicz, & M.G. Kantowski (Eds.), Applied mathematical problem solving. Columbia, OH: ERIC/SMEAC, 1979.

Carpenter, T.P. Research in cognitive development. In R.J. Shumway (Ed.), Research in mathematics education. Reston, VA: National Council of Teachers of Mathematics, 1980.

Case, R. Learning and development: A neo-Piagetian interpretation. Human Development, 1972, 15, 339-358.

Case, R. Mental strategies, mental capacity, and instruction: A neo-Piagetian investigation. Journal of Experimental Child Psychology, 1974, 18, 382-397. (a)

Case, R. Structures and strictures: Some functional limitations on the course of cognitive growth. Cognitive Psychology, 1974, 6, 544-573. (b)

Case, R. Gearing the demands of instruction to the developmental capacities of the learner. Review of Educational Research, 1975, 45, 59-87.

Case, R. Implications of developmental psychology for the design of effective instruction. In A.M. Lesgold, J.W. Pellegrino, S.D. Fokkema, and R. Glaser (Eds.), Cognitive psychology and instruction. New York: Plenum, 1978.

Case, R. The underlying mechanism of cognitive development. In J. Biggs and J. Kirby (Eds.), Instructional processes and individual differences in learning. New York: Academic Press, 1979.

Cathcart, G.W. The relationship between primary students' rationalization of conservation and their mathematical achievement. Child Development, 1971, 42, 755-765.

Dimitrovsky, L., & Almy, M. Early conservation as a predictor of arithmetic achievement. The Journal of Psychology, 1975, 91, 65-70.

Hamilton, V., & Launay, G. The role of information-processing capacity in the conserving operation. British Journal of Psychology, 1976, 67, 191-201.

Hamilton, V., & Moss, M. A method of scaling conservation-of-quantity problems by information content. Child Development, 1974, 45, 737-745.

Inhelder, B. Information processing tendencies - empirical studies. In S. Farnham-Diggory (Ed.), Information processing in children. New York: Academic Press, 1972.

Kaufman, A.S., & Kaufman, W.L. Tests built from Piaget and Gesell's tasks as predictors of first grade achievement. _Child Development_, 1972, _43_, 521-535.

Klahr, D., & Wallace, J.G. _Cognitive development: An information processing view._ Hillsdale, New Jersey: Lawrence Erlbaum Associates, 1976.

Lawson, A.E. M-space: Is it a constraint on conservation reasoning ability? _Journal of Experimental Child Psychology_, 1976, _22_, 40-49.

Parkinson, G.M. _The limits of learning: A quantitative investigation of intelligence_. Unpublished doctoral dissertation, York University, 1975.

Pascual-Leone, J. A mathematical model for the transition rule in Piaget's developmental stages. _Acta Psychologica_, 1970, _32_, 301-345.

Pascual-Leone, J., & Smith, J. The encoding and discoding of symbols by children: A new experimental paradigm and a neo-Piagetian model. _Journal of Experimental Child Psychology_, 1969, _8_, 328-355.

Piaget, J. _The child's conception of number_. London: Routledge & Kegan Paul, 1952.

Piaget, J. Development and learning. _Journal of Research in Science Teaching_, 1964, _2_, 176-186.

Piaget, J., Inhelder, B., & Szeminska, A. _The child's conception of geometry_. New York: Basic Books, 1960.

Scardamalia, M. Information processing capacity and the problem of horizontal decalage: A demonstration using combinatorial reasoning tasks. _Child Development_, 1977, _48_, 28-37.

Steffe, L.P. Differential performance of first-grade children when solving arithmetic addition problems. _Journal for Research in Mathematics Education,_ 1970, _1_, 144-161.

APPENDIX A

Task Descriptions and Scoring Criteria

Task Descriptions and Scoring Criteria

Set 1

Conservation of Length (Form 1)

Description. E lays out two straight wires of equal length with endpoints aligned and asks S to affirm their equal length. E then moves one of the wires slightly forward and asks S if one of the wires is longer or if they are the same length.

E returns the wires to their original positions, and after S reaffirms their equality E moves the other wire forward. S is again asked if one of the wires is longer or if they are the same length.

Protocol. E lays out two straight wires of equal length so the endpoints coincide.

LET'S PRETEND THAT THESE TWO WIRES ARE ROADS. IS THERE JUST AS FAR TO WALK ON THIS ROAD AS THIS ROAD, OR IS IT FARTHER ON ONE OF THE ROADS?

E moves one of the wires slightly forward.

NOW IS THERE AS FAR TO WALK ON THIS ROAD AS THIS ROAD, OR IS ONE OF THE ROADS FARTHER?

(If the response is unclear or if the child does not seem to understand the question, rephrase it as follows.)

IF TWO ANTS ARE WALKING, ONE ON THIS ROAD STARTING HERE, AND ONE ON THIS ROAD STARTING HERE, WOULD THEY BOTH WALK JUST AS FAR, OR WOULD ONE OF THEM WALK FARTHER?

WOULD ONE OF THEM BE MORE TIRED THAN THE OTHER?

WOULD ONE OF THEM HAVE TO TAKE MORE STEPS THAN THE OTHER?

E returns the road to its original position so the endpoints coincide.

NOW IS THERE JUST AS FAR TO WALK ON BOTH THE ROADS OR IS IT FARTHER ON ONE OF THEM?

E moves the <u>other</u> stick slightly forward.

NOW IS THER JUST AS FAR TO WALK ON BOTH THE ROADS OR IS IT FARTHER ON ONE OF THEM?

(Repeat clarification questions given above if necessary.)

<u>Scoring</u>. Two responses are required. A score of 0, 1, or 2 is assigned corresponding to the number of correct responses.

Conservation of Number (Form 1)

Description. E lays out a row of 8 markers and asks S to lay out a row of equal number (using different colored markers). E then spreads out one of the rows and asks S whether one of the rows has more or if they have the same number.

```
o o o o o o o o         o o o o o o o o
                 ⇒
o o o o o o o o     o    o    o    o    o   o   o   o
```

Protocol. E lays out a row of 8 white blocks.

PUT OUT AS MANY OF YOUR RED BLOCKS AS I'VE PUT WHITE ONES.

MAKE SURE THAT YOUR ROW HAS THE SAME NUMBER AS MINE.

E arranges blocks, if necessary, into two same length rows.

ARE THERE THE SAME NUMBER OF RED BLOCKS AS WHITE ONES?

If unsuccessful, terminate task.

WATCH NOW, I'M GOING TO MOVE THE WHITE ONES.

E spreads out white ones to form longer row.

ARE THERE THE SAME NUMBER OF WHITE BLOCKS AS RED BLOCKS, OR DOES ONE COLOR HAVE MORE?

WHY DO YOU THINK SO?

Scoring. A score of 0, 1, or 2 is assigned according to the following criteria:

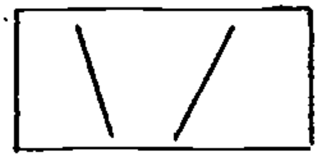
0: incorrect response

1: mixed response (e.g., changed response when giving explanation)

2: correct response

Transitivity of Length (Form 1)

Description. Two sticks of different colors, 35 cm. and 36 cm. are glued at slight angles on a cardboard backing.



E places a third stick of 35.5 cm. next to one of the glued sticks and asks S to identify the longer one. This is repeated with the second glued stick.

E then sets the third stick aside and asks S to decide whether the glued sticks are the same length or whether one of them is longer S is asked to give a reason for his/her response.

Protocol.

LET'S PLAY A LITTLE GAME WITH THESE STICKS.

E matches the intermediate measuring stick (B) with the longer of the two stationary sticks (A).

ARE THESE TWO STICKS (A & B) THE SAME LENGTH OR IS ONE OF THEM LONGER THAN THE OTHER? WHICH ONE?

E then matches the measuring stick (B) with the shorter of the two stationary sticks (C).

ARE THESE TWO STICKS (B & C) THE SAME LENGTH OR IS ONE OF THEM LONGER THAN THE OTHER? WHICH ONE?

SO THIS ONE (A) IS LONGER THAN THIS (B), AND THIS ONE (B) IS LONGER THAN THIS(C), E then removes the measuring stick and focuses attention to the table.

ARE THESE TWO STICKS (A & C) THE SAME LENGTH OR IS ONE OF THEM LONGER THAN THE OTHER? WHICH ONE?

WHY DO YOU THINK SO?

Scoring. A score of 0, 1, or 2 is given according to the following criteria:

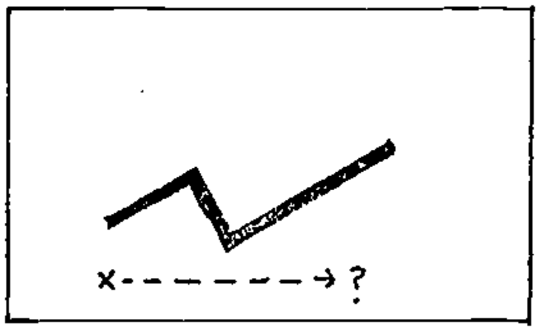
0: incorrect response

1: correct response with incorrect explanation (e.g., visual comparison)

2: correct response with correct explanation (i.e., an explanation indicating an application of the transitive rule: $A > B$ and $B > C \Rightarrow A > C$).

Measurement Task 2

Description. E asks S to pretend that the crooked strip glued to the cardboard backing is a curvy road or sidewalk. S is given an assortment of Cuisenaire rods and is asked to construct a straight road on which there is just as far to walk,



Protocol. E lays out an assortment of Cuisenaire rods along with the paper strip "road."

LET'S PRETEND THIS IS A ROAD, A CURVY ROAD. COULD YOU MAKE A STRAIGHT ROAD WITH THESE PIECES WHICH IS JUST AS LONG AS THE CURVY ROAD? MAKE YOUR ROAD SO THERE IS JUST AS FAR TO WALK ON YOUR STRAIGHT ONE AS THE CURVY ONE.

If S uses perceptual judgment, E lays out the first segment with the rods.

IF WE LAY THESE PIECES LIKE THIS, WE KNOW THAT THE FIRST PART OF THE CURVY ROAD IS JUST THIS FAR. COULD YOU FINISH BUILDING THIS STRAIGHT ROAD SO IT IS JUST AS LONG AS THE CURVY ROAD?

Scoring. A score of 0, 1, or 2 is assigned according to the following criteria:

0: used perceptual strategy, even after demonstration

1: achieved a partially correct solution, or measured correctly after demonstration.

2: measured correctly and achieved a correct solution before demonstration.

Number sequencing*

Description

Pretraining. Each subject (S) is seatd in front of the apparatus in Figure 1.

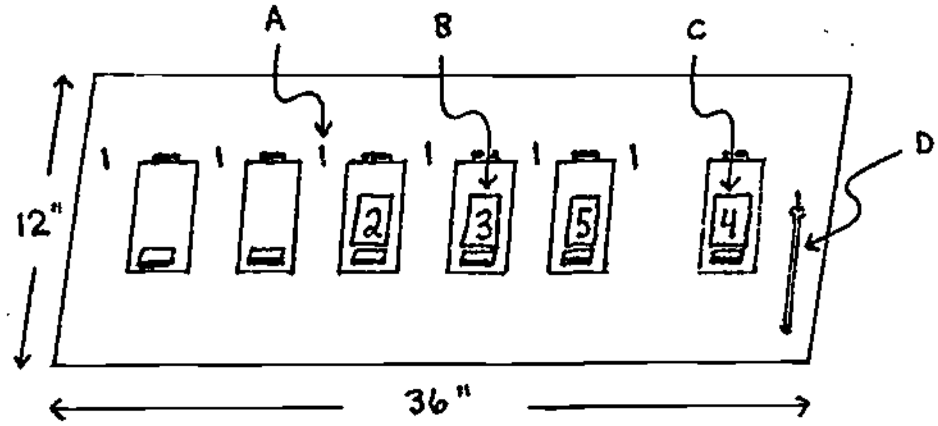


Figure 1. Experimental apparatus used for pretraining. A - hooks on which marker may be placed; B - cards on which ascending series is stenciled; C - card on which the final numeral is stenciled; D - marker used to represent final numeral.

The experimenter (E) asks S to read the mmbers aloud (2, 3, 5, 4) and to point to the place where the final number (4) belongs in the series. A marker is provided to represent the final number and S is asked to place the marker on the appropriate hook. Several trials are presented in which more than one number is omitted from the original series (e.g., 3, 6, 8, 5). The task remains the same, to indicate with

*taken from Case, R. Validation of a neo-Piagetian mental capacity construct. Journal of Experimental Child Psychology, 1972, 14, 287-302.

the marker the position in the original series where the final number belongs. Testing is begun after four trials have been passed in a row under conditions where the final number has to be ordered with respect to a single original number, rather than with respect to a series (e.g., 8, 5). Subjects who do not reach criterion are excluded from the study.

Testing. Each S is again asked to indicate the correct position of the final number with respect to the original series. This time, however, S sees only one number at a time since the cards on which the numbers are stenciled are placed within the apparatus instead of on top of it (see Figure 2).

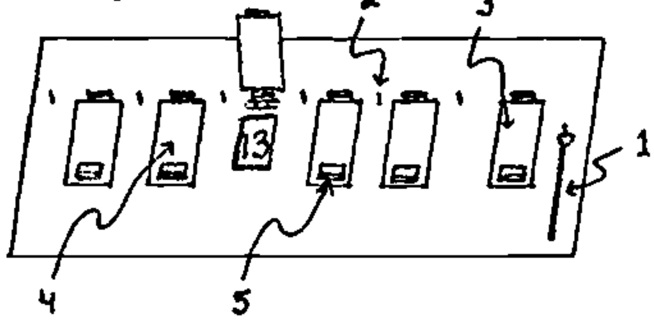


Figure 2. Experimental apparatus used for testing. 1 - marker; 2 - hooks on which it may be placed; 3 - door concealing final numeral; 4 - doors concealing ascending series; 5 - door handles.

E opens the doors from left to right for about 1 1/2 sec. each. As he exposes each number, E (who is sitting behind the apparatus) asks S to read it out to him. As soon as one door is closed, the next one is opened. The token cannot be placed until the final number has been read aloud, and the final door has been closed.

After one practice trial, five trials in each set are presented with the total number of numbers in each set (including the one to be placed) equal to two, three, and four. Testing is terminated if S misses three or more trials in any given set.

Protocol. S is seated in front of the apparatus which displays the series 2, 3, 5, - 4.

READ THESE NUMBERS ALOUD.

POINT TO THE SPACE WHERE THE 4 BELONGS. IF YOU COULD MOVE THE 4, WHERE WOULD YOU PUT IT SO ALL THE NUMBERS WOULD BE IN ORDER? PRETEND THIS MARKER IS THE 4. PUT THE MARKER IN THE RIGHT PLACE.

S is presented with the following 3 series and asked the same question. Feedback is provided to help S give the correct response.

3, 6, 8, - 5

4, 7, - 2

2, 8, 11, - 15

S is now presented with several one-digit series until four trials are passed in a row.

8 - 5

3 - 6

12 - 9

2 - 7

16 - 18

10 - 5

7 - 4

8 - 12

NOW WE ARE GOING TO PLAY THE SAME GAME ONLY THIS TIME YOU WILL NOT BE ABLE TO SEE ALL THE NUMBERS AT ONCE. I WILL SHOW THEM TO YOU ONE AT A TIME AND WHEN YOU HAVE SEEN ALL THE NUMBERS YOU CAN PUT THE MARKER IN THE RIGHT POSITION.

WHEN I OPEN THE DOORS, READ THE NUMBERS ALOUD.

E opens the door from left to right. The doors are opened for about 1 1/2 sec. each.

A practice trial is given with the series 6 - 9. Feedback is provided if needed.

The task series are then presented as shown on the response sheet.

Response Sheet. Mark + for correct, 0 for incorrect.

Practice Trials

8 - 5 ______

3 - 6 ______

12 - 9 ______

2 - 7 ______

16 - 18______

10 - 5 ______

7 - 4 ______

8 - 12______

Test Trials

5 - 7 ____	5, 9 - 12 ______	3, 8, 12 - 6 ______
13 - 9 ____	12, 19-15 ______	9, 11, 17-7 ______
8 - 13____	5, 8 - 3 ______	4, 11, 15-17______
15 - 19____	12, 16 - 4______	3, 9, 18 - 11______
10 - 4_____	10, 18 - 8______	5, 7, 11 - 9______

Scoring Terminate the task after three errors in a set of five trials. Assign a score of 0, 1, 2, or 3 based on the number of sets in which the subject obtained three or more correct responses.

Set 2

Conservation of Length (Form 2)

Description. E lays out two straight wires of unequal length so that one pair of endpoints coincide. After S has affirmed that one of the wires is longer E bends the longer one so that both pairs of end-points are aligned. E then bends the longer wire again so that the furthest endpoint of the shorter wire protrudes beyond the longer wire. After each of the two transformations E asks the question "Which one is longer or are they the same?"

Protocol. E lays out the two straight wires so that one pair of endpoints coincide.

LET'S PRETEND THAT THESE TWO WIRES ARE ROADS. IS THERE JUST AS FAR TO WALK ON THIS ROAD AS THIS ROAD, OR IS IT FARTHER ON ONE OF THE ROADS?

E bends road A so the endpoints coincide.

NOW IS THERE AS FAR TO WALK ON THIS ROAD AS THIS ROAD, OR IS ONE OF THE ROADS FARTHER?

(If the response is unclear or if the child does not seem to understand the question, rephrase it as follows.)

IF TWO ANTS ARE WALKING, ONE ON THIS ROAD AND ONE ON THIS ROAD, WOULD THEY BOTH WALK JUST AS FAR, OR WOULD ONE OF THEM WALK FARTHER?

WOULD ONE OF THEM BE MORE TIRED THAN THE OTHER?

WOULD ONE OF THEM HAVE TO TAKE MORE STEPS THAN THE OTHER?

E straightens road A to original position.

NOW IS THERE AS FAR TO WALK ON BOTH THE ROADS OR IS IT FARTHER ON ONE OF THEM?

E bends A so that the endpoint of B extends beyond that of A.

NOW IS THERE AS FAR TO WALK ON THIS ROAD AS THIS ROAD, OR IS IT FARTHER ON ONE OF THE ROADS?

(Repeat clarification questions given above if necessary.)

Scoring. Two responses are required. A score of 0, 1, or 2 is assigned corresponding to the number of correct responses.

Symbol substitution*

Description. Materials consist of stimulus cards, rule cards, and response cards. Stimuli are capital letters K and M for the Practice Set, A-D for Set 1 and E-H for Set 2. Responses are the numbers 11-14 for the Practice Set, 1-4 for Set 1, and the colors yellow, red, green, and blue for Set 2. Rules are the substitution cues which specify the correct response for each stimulus. The following rules are arbitrarily defined:

Practice Set	Set 1	Set 2
K = $\odot$; $\odot$ = 12	A = $\odot$; $\odot$ = 1	E = $\odot$; $\odot$ = 5 ; 5 = (yellow)
L = $\triangle$; $\triangle$ = 11	B = $\square$; $\square$ = 2	F = $\square$; $\square$ = 6 ; 6 = (red)
M = $\square$; $\square$ = 13	C = $\triangle$; $\triangle$ = 3	G = $\triangle$; $\triangle$ = 7 ; 7 = (green)
N = $\diamond$; $\diamond$ = 14	D = $\diamond$; $\diamond$ = 4	H = $\diamond$; $\diamond$ = 8 ; 8 = (blue)

After a brief training using stimuli, rules, and responses in the Practice Set, the experimenter lays out all essential rule cards and response cards for Set 1. The stimulus cards are then presented one at a time to the subject and s/he is asked to point to the correct response. No time limit is imposed but subjects are asked to work as quickly as they can. Set 2 is presented only if the subject is successful with Set 1.

*modified from Hamilton, V., & Launay, G. The role of information-processing capacity in the conserving operation. British Journal of Psychology, 1976, 67, 191-201.

<u>Protocol</u>. E lays out the practice response cards and the relevant rule cards in front of S.

WE ARE GOING TO PLAY A DETECTIVE GAME WITH THESE CARDS. THESE CARDS (RULES) GIVE YOU THE SECRET CODES WHICH HELP YOU SOLVE THE PROBLEM, AND THESE ARE THE ANSWER CARDS. YOU USE THEM TO SHOW YOU HAVE SOLVED THE PROBLEM.

LET'S DO ONE FOR PRACTICE.

E lays out stimulus card K. (K = ⊙ , ⊙ = 12)

READ THE CLUES CAREFULLY AND SEE IF YOU CAN FIND WHICH NUMBER GOES WITH K.

E provides the help necessary so that S identifies the appropriate response, and explains any procedure which appears unclear to S.

LET'S PRACTICE ANOTHER ONE.

E lays out card M. (M = □ , □ = 13)

NOW WE ARE GOING TO PLAY THE GAME FOR REAL. SEE IF YOU CAN SOLVE EACH MYSTERY AS QUICKLY AS YOU CAN BUT BE SURE YOU ARE RIGHT ON EACH ONE.

E lays out the Set 1 cards as indicated on the response sheet. The cards are layed out in columns but with linked clues non-adjacent.

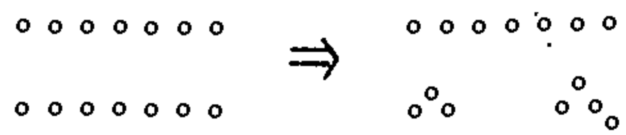
<u>Response Sheet</u>. Mark + for correct, 0 for incorrect.

A = ⊙ , ⊙ = 1 _____	E = ⊙ , ⊙ = 5, 5 = yellow _____
B = □ , □ = 2 _____	F = □ , □ = 6, 6 = red _____
C = △ , △ = 3 _____	G = △ , △ = 7, 7 = green _____
D = ◇ , ◇ = 4 _____	H = ◇ , ◇ = 8, 8 = blue _____

<u>Scoring</u>. Terminate the task after two errors in a set of four trials. Assign a score of 0, 1, or 2 based on the number of sets in which the subject obtained three or more correct responses.

<u>Conservation of Number</u> (Form 2)

<u>Description</u>. E lays out a row of 7 red markers and 7 white markers and asks S to affirm their equality. E then rearranges one of the rows into groups of 3 and 4 and asks S whether the two original sets are equal or whether one of them has more.



<u>Protocol</u>. E lays out a row of 7 white blocks and 7 red blocks

ARE THERE THE SAME NUMBER OF RED BLOCKS AS WHITE ONES?

If unsuccessful, terminate task.

WATCH NOW, I'M GOING TO MOVE THE WHITE ONES.

E groups the whites together into two fairly compact groups of 3 and 4 each.

ARE THERE THE SAME NUMBER OF WHITE BLOCKS AS RED BLOCKS, OR DOES ONE COLOR HAVE MORE?

WHY DO YOU THINK SO?

<u>Scoring</u>. A score of 0, 1, or 2 is assigned according to the following criteria:

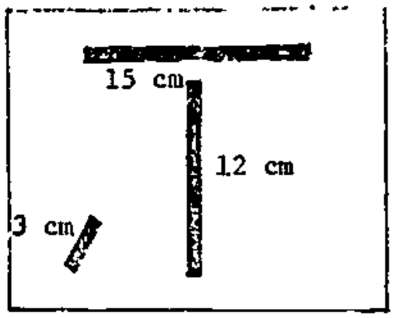
0: incorrect response

1: mixed response (e. g., changed response when giving explanation)

2: correct response

Measurement Task 3

Description. S is given a small Cuisenaire rod and is asked to find out which of the two strips is longer.



Protocol. E lays out cardboard with two perpendicular paper strips and a 3 cm. Cuisenaire rod.

COULD YOU FIND OUT WHICH ONE OF THESE STRIPS IS LONGER?

YOU CAN USE THIS LITTLE ROD TO HELP YOU FIND OUT.

If S uses perceptual judgment, . . .

CAN YOU USE THIS TO MAKE SURE YOU ARE RIGHT?

If S is unsuccessful, E lays out the rod several times along one of the strips.

PRETEND A MAN IS WALKING ALONG THIS ROAD AND THESE ARE THE STEPS HE IS TAKING. HOW MANY STEPS MUST HE TAKE TO WALK THE WHOLE WAY?

Scoring. A score of 0, 1, or 2 is assigned according to the following criteria:

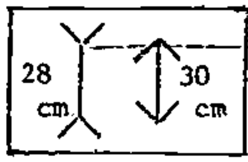
0: did not use unit iteration to compare the two lengths

1: iterated units but used inaccurate technique, or iterated units correctly after demonstration.

2: iterated correctly and achieved a correct solution before demonstration.

Transitivity of Length (Form 2)

Description. Two black-colored sticks, 28 cm. and 30 cm., are glued vertically about 25 cm. apart on a white cardboard backing. V-shaped figures of black cardboard are attached to the ends of the sticks to create a Mueller-Lyer illusion.



E places a blue stick of 29 cm. next to one of the glued sticks and asks S to identify the longer one (S is reminded that the lengths of the sticks do not include "the things on the ends"). This is repeated with the second glued stick.

E then sets the third stick aside and asks S to decide whether the glued sticks are the same length or whether one of them is longer.

Protocol.

LET'S PLAY A LITTLE GAME WITH THESE STICKS.

E matches the blue measuring stick with the longer of the two black ones.

ARE THESE TWO STICKS THE SAME LENGTH OR IS ONE OF THEM LONGER THAN THE OTHER? DON'T INCLUDE THE THINGS ON THE END OF THIS ONE: JUST LOOK AT THE STICKS. IS ONE LONGER? WHICH ONE?

E then matches the blue stick with the shorter black one.

ARE THESE TWO STICKS THE SAME LENGTH OR IS ONE OF THEM LONGER THAN THE OTHER? WHICH ONE?

SO THIS ONE IS LONGER THAN THIS, AND THIS ONE IS LONGER THAN THIS.

E removes the measuring stick and focuses attention to the table.

ARE THESE TWO STICKS THE SAME LENGTH OR IS ONE OF THEM LONGER?

JUST LOOK AT THE STICKS, NOT THE THINGS ON THE ENDS.

IS ONE OF THEM LONGER OR ARE THEY THE SAME?

WHY DO YOU THINK SO?

Scoring. A score of 0, 1, or 2 is given according to the following criteria:

0: incorrect response

1: correct response with incorrect or incomplete explanation

2: correct response with transitive-based explanation

Backward Digit Span

Description. The task requires subjects to repeat backward as many of 10 two-digit series, 10 three-digit series, and 10 four-digit series as possible. Testing is discontinued when the subject fails three consecutive series. Each series is presented verbally to the child at the rate of about one digit per second. The child is allowed as much time as needed to repeat the numbers in reverse order, but the series is read only once.

Protocol.

I WILL SAY SOME NUMBERS AND I WOULD LIKE YOU TO REPEAT THE SAME NUMBERS, ONLY YOU ARE TO SAY THEM BACKWARDS. LISTEN CAREFULLY TO THE NUMBERS I SAY. THEN SAY THE SAME NUMBERS ONLY REMEMBER TO SAY THEM BACKWARDS.

LET'S PRACTICE A FEW.

E presents the following 3 series and provides correct responses for those which S answers incorrectly.

4, 2

8, 0

1. 6. 2

THAT'S GOOD. NOW WE'LL TRY SOME MORE. LISTEN CAREFULLY AND REPEAT THE NUMBERS YOU HEAR ONLY REMEMBER TO SAY THEM BACKWARDS.

Response Sheet. Mark + for correct, 0 for incorrect.

7, 8 ______	7, 1, 3 ______	3, 4, 6, 9 ______
0, 7 ______	5, 8, 7 ______	1, 8, 4, 3 ______
4, 3 ______	8, 6, 2 ______	9, 5, 3, 2 ______
5, 1 ______	8, 1, 7 ______	9, 6, 7, 4 ______
6, 9 ______	0, 5, 3 ______	7, 3, 0, 5 ______
8, 2 ______	8, 4, 1 ______	3, 1, 2, 5 ______
5, 0 ______	2, 4, 3 ______	2, 3, 8, 1 ______
1, 4 ______	6, 2, 0 ______	6, 0, 2, 1 ______
9, 8 ______	1, 7, 6 ______	6, 5, 7, 9 ______
5, 6 ______	3, 8, 1 ______	8, 7, 4, 3 ______

Scoring. Terminate the task after 3 consecutive errors. Move to the next series after 5 consecutive correct responses and score a '1' for that column. Otherwise complete all 10 in the column. If 6 or more responses were correct, score a '1' for that column and move to the next column. If less than 6 responses were correct, record the number of correct responses, n, as a decimal, .n. Sum the scores for each series and assign a score to the total performance using the following criteria:

0 : 0 - 0.9

1 : 1.0 - 1.5

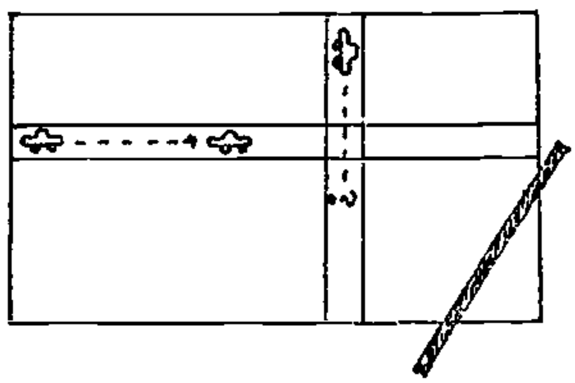
2 : 1.6 - 2.0

3 : 2.1 - 2.5

4 : 2.6 - 3.0

Measurement Task 1

Description. The experimenter (E) drives a toy car a predetermined distance along the east/west road and then asks the subject (S) to drive the other car the same distance on the north/south road. S is given an unmarked strip and E suggests using the strip to find out exactly how far to drive the car.



Protocol. E places "cars" in their "garages."

LET'S PRETEND THESE ARE REAL CARS AND THAT THEY ARE GOING FOR A DRIVE ON THESE ROADS. WATCH, THIS CAR IS GOING TO DRIVE THIS FAR.

E moves one of the cars a predetermined distance.

NOW, DO YOU THINK YOU CAN DRIVE THE OTHER CAR SO IT GOES JUST AS FAR? MAKE SURE YOUR CAR DRIVES JUST AS FAR ON YOUR ROAD AS MY CAR DROVE ON MINE.

E produces a blank strip.

THIS MAY HELP YOU TELL HOW FAR TO DRIVE YOUR CAR. USE IT TO MEASURE.

If S uses perceptual judgment, . . .

COULD YOU USE THIS STRIP TO MAKE SURE BOTH CARS WENT JUST AS FAR?

If S does not spontaneously use strip, E measures out distance of first car with the strip.

SEE, I CAN TELL THAT MY CAR WENT JUST THIS FAR. DO YOU THINK YOU COULD USE THE STRIP TO MAKE YOUR CAR GO JUST AS FAR AS MINE?

Scoring. A score of 0, 1, or 2 is assigned according to the following criteria:

0: used a perceptual strategy or used the strip in an non-measurement way, even after demonstration.

1: Measured with strip but did not achieve an accurate solution, or measured successfully only after demonstration.

2: Measured correctly with strip before demonstration, attended to both pairs of endpoints.

Center Planning and Policy Committee

Marshall S. Smith
Director

Dale D. Johnson
Area Chairperson
Studies in Language:
Reading and Communication

Marvin J. Fruth
Area Chairperson
Studies in Implementation
of Individualized Schooling

Penelope L. Peterson
Area Chairperson
Studies of Instructional Programming
for the Individual Student

James M. Lipham
Area Chairperson
Studies of Administration and
Organization for Instruction

Thomas A. Romberg
Area Chairperson
Studies in Mathematics and Evaluation
of Practices in Individualized Schooling

Associated Faculty

Vernon L. Allen
Professor
Psychology

B. Dean Bowles
Professor
Educational Administration

Thomas P. Carpenter
Professor
Curriculum and Instruction

W. Patrick Dickson
Assistant Professor
Child and Family Studies

Lloyd E. Frohreich
Professor
Educational Administration

Marvin J. Fruth
Professor
Educational Administration

Dale D. Johnson
Professor
Curriculum and Instruction

Herbert J. Klausmeier
V.A.C. Henmon Professor
Educational Psychology

Joel R. Levin
Professor
Educational Psychology

James M. Lipham
Professor
Educational Administration

Dominic W. Massaro
Professor
Psychology

Donald M. McIsaac
Professor
Educational Administration

Wayne Otto
Professor
Curriculum and Instruction

Penelope L. Peterson
Associate Professor
Educational Psychology

Thomas S. Popkewitz
Professor
Curriculum and Instruction

Gary G. Price
Assistant Professor
Curriculum and Instruction

W. Charles Read
Professor
English and Linguistics

Thomas A. Romberg
Professor
Curriculum and Instruction

Richard A. Rossmiller
Professor
Educational Administration

Peter A. Schreiber
Associate Professor
English and Linguistics

B. Robert Tabachnick
Professor
Curriculum and Instruction

Gary G. Wehlage
Professor
Curriculum and Instruction

Louise Cherry Wilkinson
Associate Professor
Educational Psychology

Steven R. Yussen
Professor
Educational Psychology