This issue contains abstracts and critical comments on 11 articles, plus an editorial comment entitled "A Teacher Shortage?". Studies included concern solution processes on addition and subtraction examples; numerical rule induction; individual differences in performance on manipulative, pictorial, and symbolic tasks; the effect of a calculator curriculum; proportionality rules in judgments of area; van Hiele levels of geometric thought; interaction patterns and attitudes; word problems with ratios; incremental development in algebra; instruction with two types of calculators; and errors in translation from sentence to equation. Research studies reported in "Resources in Education" from July to September 1983 are listed. (MNS)
INVESTIGATIONS IN MATHEMATICS EDUCATION

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The Ohio State University
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A Teacher Shortage?

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One of the significant issues being discussed in mathematics circles is the teacher shortage, "a matter of national survival." A great deal has been written about the topic, both in mathematics professional journals and in the popular press. There seems to be general agreement in the profession that there is a critical shortage. There is somewhat less general agreement that the solution is more money for mathematics teachers. The National Council of Teachers of Mathematics is quoted as supporting higher pay for (mathematics) teachers in school systems with the greatest shortage (1, p.153). The idea of "give them more money" brings to mind the H.L. Menken quote, "For every difficult problem there is a simple solution, and it's wrong." The entire issue of the mathematics teacher shortage needs to be examined objectively. What follows is a partial list of some of the questions that call for careful data-based research. Much of the present oft-quoted research is based on "hearsay evidence". When the statement that "43 of 45 states responding" report a "shortage of mathematics teachers" (1, p.146) is considered, there are many unanswered questions. Surely the state did not respond; rather, some person from that state responded. Who was that person, how reliable were his or her data, was the response based on data or opinion, and what constitutes a "shortage"? Some of the evidence seems contradictory. For instance, the mathematics teacher shortage in North Carolina has been dramatically detailed in the MATHEMATICS TEACHER and elsewhere (1, p.149) and yet in the study "Teacher Supply/Demand in the United States 1980," a survey done with placement offices of higher education, it was reported that there was a "balanced supply" of mathematics teachers in North Carolina.

It seems apparent that several of the major questions surrounding the teacher shortage issue are not only amenable to research, they
require research if the problem is to be approached logically. The profession desperately needs hard data, by state, that will provide a national look at the problem. (It may be that Houston's problems are not everybody's problems). The questions that should be researched include but are not limited to:

1. Is there a shortage of high school mathematics teachers?
2. Is there a shortage of qualified high school mathematics teachers?
3. Why are fewer people entering mathematics teaching?
4. Why are more people leaving mathematics teaching?
5. Are "retrained" teachers less effective than other mathematics teachers?
6. What are the potential effects of differential pay or other differential treatment of mathematics teachers?

IS THERE A SHORTAGE OF HIGH SCHOOL MATHEMATICS TEACHERS?

We know that there are some schools having difficulty getting mathematics teachers. Some of the school districts seem to be dealing effectively with the problem on a local level. Houston Independent School District, a district often cited in the teacher shortage literature for using an incentive pay plan and retraining program, recently reported: "...Houston was able dramatically to decrease the number of mathematics vacancies. For example, there were twenty-one vacancies in October 1981 and only four vacancies in October 1982." (2, p.644) Regrettably, the total number of mathematics teachers in Houston is not reported, so it is impossible to determine the percent drop in vacancies and obtain a better understanding of the real magnitude of the problem.

It is also known that there are some schools not having difficulty getting mathematics teachers.

It may be the case that the shortage of mathematics teachers is at least somewhat geographic and probably related to specific assignments. That is, within a given state there are probably a lot of people
who want to teach "upper-level" mathematics to college-bound
students in affluent suburban districts. There are probably fewer
people who want to teach ninth-grade general mathematics in inner-city
or poor rural situations. Answers to questions concerning how many
students actually receive mathematics instruction from teachers with-
out mathematics certification, what mathematics subjects are usually
taught by non-certified individuals, and in what geographic areas is
the shortage most severe should be sought.

IS THERE A SHORTAGE OF QUALIFIED HIGH SCHOOL MATHEMATICS TEACHERS?

This is, of course, an entirely different question than the first.
Certainly most people in the mathematics education field would answer
with a resounding yes. It may be the case that this is the question
that professionals are answering when the first question is asked.
Before attempting to answer this question is would be important for a
researcher to realize that there is probably not universal agreement
about what constitutes a "qualified high school mathematics teacher".
There is certainly disagreement about teacher education programs;
what constitutes an appropriate balance between content, methods, and
field experience is just one debated topic. How many college hours of
mathematics are necessary for a person to become "qualified"? The
important thing to remember when attempting to answer this question
with research is that certified does not equate with qualified. Does
this mean that a non-certified person might be qualified?

WHY ARE FEWER PEOPLE ENTERING MATHEMATICS TEACHING?

There does seem to be hard data indicating that many fewer people
are being trained as mathematics teachers than was the case several
years ago. In trying to determine the causes for and the meaning of
this fact it would be important to compare the drop in mathematics
with the change, if any, in the other subject areas. It is often
suggested that one reason people are not entering the field of
mathematics teaching is that in our technological society people with mathematics skills have many more options, often more lucrative options, than teaching mathematics. It appears to be a logical assumption. However, if data indicated a substantial drop in the number of people entering other teaching fields such as music or English, perhaps the assumption should be examined more closely. How important is the subject area in an individual decision to become a teacher? Did most teachers of mathematics decide to become teachers and then select mathematics, or did these people decide to choose a career somehow connected with mathematics and then choose teaching? Knowing this might help in an understanding of why enrollments are dropping and why some people stay in teaching and some people leave teaching.

WHY ARE MORE PEOPLE LEAVING MATHEMATICS TEACHING?

ARE more people leaving? The attrition rates quoted are high (1, p.148). How do they compare with the rates ten years ago or twenty years ago? It should probably be acknowledged that there are more opportunities for people with mathematics training than there were 25 years ago. When an individual teacher leaves teaching for business or industry, are they going for more money, a more interesting job, or a job with less stress and frustration and more prestige? The answer may be all of the above. Finding the answer may not be as easy as it appears. Certainly teachers should be paid more, and this may well be a contributing factor for many leaving the field. Yet, teachers have historically been underpaid, and the fact is well-known. When considering the other alternative, it is important to remember that while teaching is fraught with stress and frustration it is also fulfilling and exciting and just plain fun when the students are learning. The question of why good teachers leave the field needs careful study.
ARE "RETRAINED" TEACHERS LESS EFFECTIVE THAN OTHER MATHEMATICS TEACHERS?

Retraining, either of teachers from other fields or people with mathematical backgrounds who are not teachers, is an alternative often suggested for helping to alleviate the teacher shortage. The idea appears to have merit. Yet as this retraining is done in more and more places it would also appear that careful monitoring of the effectiveness of these individuals should take place. If these individuals are as effective as teachers who have completed an entire program in mathematics teacher training, perhaps the teacher training programs should be looked at. If they are not as effective, the problem of how to get these retrained people out of mathematics classes may be a real one. Perhaps they will be differentially effective; that is, maybe as effective as a mathematics-trained teacher in general mathematics but less effective as a trigonometry teacher. These things should be studied.

WHAT ARE THE POTENTIAL EFFECTS OF DIFFERENTIAL PAY OR OTHER DIFFERENTIAL TREATMENT OF MATHEMATICS TEACHERS?

Differential pay is a controversial topic hotly debated at professional meetings. It may be, however, that it is not hotly debated in high school faculty lounges. It appears that it is difficult for a mathematics teacher in the field to explain to his or her colleagues in English or social studies, that because of a shortage of mathematics teachers, mathematics teachers should be paid more than teachers in other fields. People suggesting differential pay need to consider whether a "bad" mathematics teacher should be paid more than a "good" English teacher. The effects of overall staff morale should be considered. Perhaps the question that needs the most careful examination is whether the differential pay being suggested in many places will attract large numbers to teaching. The pay differential between entering mathematics teachers and mathematics-trained people entering industry is substantial, ten to fifteen thousand dollars in some areas. The differential pay programs being suggested are seldom
more than a few thousand dollars and often less than that. If money is the primary incentive, then most places are just not offering enough to make a real difference.

There may well be a shortage of mathematics teachers, and a huge shortage of "qualified" mathematics teachers -- but hard data, numbers, are needed. What percent of students in the secondary schools in the United States receive mathematics instruction from teachers who are uncertified, or unqualified? Is the shortage geographic or related to assignment? What numbers substantiate or refute conjectures about this?

If mathematics education professionals believe that the teacher shortage is an important problem, "a matter of national survival", then careful, systematic, objective, data-based study of the issue, its causes, and its cures must be undertaken.

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De Corte, Erik and Verschaffel, Lieven. CHILDREN'S SOLUTION PROCESSES IN ELEMENTARY ARITHMETIC PROBLEMS: ANALYSIS AND IMPROVEMENT. Journal of Educational Psychology 73: 765-779; December 1981.

Abstract and comments prepared for I.M.E. by ROBERT D. BECHTEL, Purdue University - Calumet.

The researchers "tried to analyze and improve children's solution processes in elementary numerical addition and subtraction problems. Using error analysis and individual interviews, the problem-solving actions of first and second graders were analyzed. Important shortcomings of children's knowledge and solution strategies were discovered. It seemed that these shortcomings could be overcome by instruction; therefore, a teaching experiment was undertaken wherein instruction was given for two weeks to an experimental class, while in a control group, the usual arithmetic program was taught. Experimental instruction related mainly to three topics: an exact notion of the equality sign (=), the part-whole relation, and the action of verifying the outcome of an arithmetic operation."

The researchers defined three characteristics of the children's tasks:

(1) The operation: addition \(a + b = x\) versus subtraction \(a - b = x\).
(2) The complexity: simple \(a + b = x\) or complex \(a + b + c = x\).
(3) Direct or indirect: direct \(a + b = x\) or indirect \(a + x = b\).

They also analyzed the problem-solving dimension into a thinking phase, a technical or execution phase, and a verification phase.

In a preliminary study (Experiment 1: The Ascertaining Study), paper-and-pencil arithmetic tests as well as individual tests were administered to children. During the individual tests the children
were asked to think aloud while solving the problems. Findings from this assessment aided the researchers in designing the experiment and in developing the materials for classroom instruction. Quoting the researchers in a summary of the preliminary experiment, "it is our view that the traditional methods of arithmetic instruction are primarily responsible for children's errors. Through these traditional methods, children have acquired a set of specific isolated solution methods that are almost not transferable to new and unfamiliar tasks and situations. To give children an appropriate orientation basis for approaching such new problems, it is necessary, first, to impart to them general and powerful concepts and principles such as the equality concept and the part-whole relation schema and, second, to teach them to apply these concepts and principles during a thinking phase that involves problem analysis and precedes the technical phase of performing an arithmetic operation and also during a verification phase that follows the performance of the operation."

The design for the teaching experiment (Experiment 2) was a pretest-posttest for an experimental and control group. The 20-item pre-post test had 10 simple exercises, 8 complex exercises, and 2 word problems. There were 27 students in the experimental (second grade) class and 25 in the control (second grade) class. A small number of students from each group were also given an individual 10-item test to observe the children's problem-solving actions. A retention test was administered to the experimental class.

The stated results show that the experimental teaching program applied in a second-grade class has led to a strong decrease in children's thinking errors on elementary addition and subtraction problems. Data analysis shows that quantitative improvement in achievement is due to a qualitative development in children's problem-solving actions.
Abstractor's Comments

One sentence would really suffice to comment on this article:
The researchers have discovered that concept development precedes using symbolism to identify the concept.

The arithmetic exercises (called problems in the article) are highly symbolic, and cast in an algebraic style. A child must understand the meaning of the notation in order to work the exercise. Otherwise the child will see "numbers" and "+" and "-" operations, and will invent and work his or her own exercises. The researchers designed an instructional program which taught the concepts, which gave meaning to the test items. They used objects and graphic representations of objects!

Mathematics is not symbol manipulation. i.e. symbols have meaning. They are alive!
Holzman, Thomas G.; Pellegrino, James W.; and Glaser, Robert. COGNITIVE DIMENSIONS OF NUMERICAL RULE INDUCTION. Journal of Educational Psychology 74: 360-373; June 1982.

Abstract and comments prepared for I.M.E. by JOHN W. GREGORY, University of Florida.

1. Purpose
The investigation sought to develop a better understanding of academic intelligence as measured by standardized aptitude and intelligence tests, by examining process and content-knowledge requirements of number analogy items.

2. Rationale
From a basis that a) numerical induction problems (including number analogies) have been identified as a prominent class of problems on general aptitude and intelligence tests, and b) a research review which revealed that relatively little is known about performance on number analogies specifically, the investigators posited a model for solution strategies and possible major task variables. Since standardized tests "frequently confound the manipulation of several task dimensions and exhibit only a few instances of any item feature," this study sought to isolate the factors responsible for item difficulty and for age and skill differences on number analogy performance.

3. Research Design and Procedures
The sample reflected two levels: 18 college undergraduates (referred to as "adults") and 36 fourth- and fifth-grade students from two private schools. The children had been selected by taking the 18 highest IQ students available from one school (Stanford-Binet mean of 144) and the lowest IQ (but not learning disabled) students available from the other school (Otis-Lennon mean of 96). These groups were referred to as the "high-IQ" and "average-IQ" groups, respectively.

All subjects were administered all grade levels of the Analysis of
Learning Potential (ALP) Number Relations subtest, a specially designed 32-item arithmetic test (at the same time period as the ALP), and twice received the individually administered Wechsler Adult Intelligence Scale (WAIS) backward digit span subtest, with these administrations separated by at least several days, as pretests to the administration of a specially designed number analogies test. The pretests served to validate the experimental test as being a model of standardized tests of aptitude and intelligence, to provide a basis for assessment of the relationship between numerical content knowledge and number analogies performance, and to obtain a measure of the relative capacities of subjects' working memories, respectively.

The 70-item test of number analogies contained items of the type A:B :: C:D :: E:_, differing with regard to operation type(s), number of operations, ambiguity, and magnitude. Operation types were addition, subtraction, multiplication, and division for the children, and the additional skill of exponentiation for the adults. Forty-four items were common to both tests. Number of operations included zero (the identity relationship), one (e.g., for the analogy 2:5 :: 6:9 :: 4:_, the operation is "+3"), or two operations, which included rules involving two distinct operations (e.g., multiply by 2, then add 1), rules involving multiplication by a fraction (seen by the investigators as multiplying and then dividing), and rules involving cubes or cube roots.

Ambiguous items were those in which the first pair of numbers was related by multiplication or division, but the rule (being perceived from the second pair) was addition or subtraction (e.g., 11:22 :: 7:18 :: 28:__). All analogies with two operations were also classified as ambiguous since the first pair could be inferred as a single arithmetic relationship.

Magnitude is the term used to refer to the domain from which rule addends and rule factors were selected. For addition/subtraction items, low-magnitude domain used the domain of whole numbers less than 6; high-magnitude domain was numbers greater than 10 and less than 17. For multiplication/division items, domain was 2 to 5 for low magnitude and 6 to 9 for high magnitude.
The number analogies test was individually administered by computer-controlled video display and terminal. Subjects responded by typing numbers on the keyboard. Directions for the test were read silently by the subjects as the experimenter read them aloud. Ten practice problems were administered to ensure that subjects understood the task. Each item was initiated by an exhibit of the first pair only. The second and third pairs were successively added to the display as the subject pressed the space bar. If 23 seconds elapsed without a response from the subject, the succeeding pair was exhibited automatically. The subject had 25 seconds to type an answer once the third pair appeared on the screen. If the subject failed to respond within the time frame, the item was simply removed from the screen and the next analogy was begun. Computer data files were established for latencies as well as actual responses for each item.

4. Findings

Mean proportions of correct responses on the total 60-item test were .47, .58, and .80 for average IQ, high IQ, and adult groups, respectively. Mean latencies by item ranged from 3.84 to 45.35 seconds for average IQ, 3.62 to 33.82 seconds for high IQ, and 2.88 to 39.22 seconds for adults. Pearson correlations between proportion correct and mean latencies per item were consistently .82 or higher for the three groups.

Analyses of proportions correct were of two types: multiple regression to identify significant predictors of performance, and a repeated measures ANOVA to ascertain age and IQ differences.

Results of the multiple regression analysis for the 60-item test indicated that the number of operations served as the major predictor for all three groups. This analysis showed an increasing influence of ambiguity on performance: nonsignificant for average IQ, adding slightly to variance accounted for by high IQ, and of about equal influence with number of operations for adults.
Using only the 32 one-operation items, multiple regression identified magnitude to be the strongest predictor for the children's scores. Average IQ performance was also influenced significantly by both multiplication (versus addition) and division (versus addition). The coefficients indicated that for the average IQ group, multiplication items were easier than addition, whereas division was harder. The only significant operation predictor for adult performance was cube/cube root items.

The ANOVA and post hoc tests led to the following results:

1. Overall performance on the 44 common items differed on the basis of both age and IQ.

2. Adults outperformed children on both one- and two-operation items.

3. Working memory (WAIS digit span) was significantly related to number of operations, becoming significantly higher with greater numbers of operations (after partialling out items with fewer numbers of operations.)

4. Ambiguous items were harder than unambiguous items involving low-magnitude addition/subtraction, having no significant interaction with either age nor IQ.

5. Adults outperformed children on items of greater magnitude, although all groups were detrimentally influenced by this variable (higher-magnitude was a main effect).

6. Relative to operation types:
   a. High IQ outperformed low IQ on decrementing operations (subtraction and division),
   b. Addition performance was lower than multiplication for all groups (no significant interaction),
   c. Interactions occurred in contrasting subtraction and division items performance: average IQ subjects found subtraction easier, and both high IQ subjects and adults scored better on the division items.
5. Interpretations

The researchers felt that "the most pervasive influence on the accuracy of subjects' answers was the working memory load imposed by the item." This comment reflected the influence of number of operations on performance, and the significant relationship it had with digit span scores.

Although the ANOVA did not find a significant interaction between age and ambiguity of items, on the basis of the regression analyses the researchers suggest that "...adults were more susceptible to misleading first pair relationships than the children." The researchers were surprised that college adults failed to change their initial hypotheses regarding the solution rule after seeing the second pair of numbers, suggesting that the lack of systematic verification may be indicative of a more general neglect of cross-checking procedures in a variety of problem-solving situations.

The influence of both magnitude and operation types on performance suggests that performance on standard intelligence tests may be as dependent upon mathematical ability as it is on reasoning ability.

Abstractor's Comments

This investigation once again reaffirms the dangers inherent in interpreting intelligence test performance and IQs in general as being measures of abstract reasoning. Research which will lead to theoretical models explaining complex problem-solving processes, such as desired by the researchers of this investigation, should be furthered.

There are some questions raised by this investigation that may suggest alternate hypotheses for the findings as well as suggest alternate methods for future studies:

1. Would analysis of error-types, as opposed to proportion correct, give us greater insight into solution processes for number analogies?
The researchers used traditional design and statistical techniques that answer the question, "What kinds of problems can you do?" More diagnostic data were available, but not used, which could possibly lead to answering, "What did you do on the ones you missed?" Analysis of errors on the ambiguous items, for example, might have led to less simplistic interpretations of why these items were missed.

2. Would limiting the domains for the numbers serving as stimuli lead to less effect due to mathematical skill?

Controls were established for the addends and factors used in the rules to be induced. However, it is apparent from the few examples cited that these domain limitations were not applied for the numbers presented in the pairs. An analysis of the items might indicate that, for example, whereas the addition/subtraction items involved two-digit number pairs (non-basic fact skill), the multiplication/division items involved numbers found only in the basic facts. This alone could account for the finding that addition item performance was lower than multiplication item performance, since magnitude was found to be a strong predictor of performance.

3. Would similar results be achieved if the test administrations were less demanding?

All the pretesting, fatigue, test-wiseness, and Hawthorne effects may very well have accounted for some of the performance.

Also the use of computers to administer the number analogies test could be another source of performance variance. Not only would familiarity with computers affect facility of responses, but eliminated is the opportunity for alternate solution strategies to account for lower working memories (e.g., making notations of first-pair rule possibilities and subsequently eliminating options upon inspection of the second pair). Returning to uncompleted items or previewing the total test (two sound test-taking strategies) are not permitted with computer-administered tests. Similarly, anxiety about completing an item before it disappears from the screen could cause less attention to cross-checking the second pair with initial rule induction.
4. Might results be more generalizable in future studies if greater care is taken in sample selection?

Differences in class size, curriculum content, curriculum emphasis, and other features found to be related to student performance exist not only between private public and private schools (the latter being the source of the children sampled in the study), but also between two different private schools. The IQ levels used in the study are also questionable, since one group had Stanford-Binet scores (individually administered, dependent upon aural perception, and requiring little reading ability), while the other group had Otis-Lennon scores (group administered, highly dependent upon reading ability). Little is reported about the adult population other than the frequency of each sex.

5. What is the real influence of ambiguity upon number analogy performance?

Ambiguity entered as a predictor only after the number of operations in one analysis, was not a significant predictor in comparison with magnitude and operation type, and did not significantly interact with age or IQ level in the ANOVA, although was found to be a main effect on the 44 common items. Since cube/cube root items were classified also as ambiguous items for adults, and this operation type was the only significant one in predicting adult variance, are mathematics skills the truer source of difficulty for ambiguous items?

On the other hand, is it the influence of ambiguity that led to perceived influence of all other item types? From the report it appears as though most of the items should have been classified as ambiguous. Single-operation multiplication and division rule items were not classified as such, but should have been. Any multiplication/division item can be initially perceived as addition/subtraction. Thus the only items on the test that are not ambiguous would be the identity rule items, and one-operation addition/subtraction items in which the numbers in the first number pair are relatively prime. Any conclusion about the role of ambiguity upon the basis of the test used would be tenuous. New methods relative to discerning the role of ambiguity should be pursued.

Abstract and comments prepared for I.M.E. by BOYD D. HOLTAN, West Virginia University, Morgantown.

1. Purpose

Researchers have investigated student performance on mathematical tasks in manipulative, pictorial, and symbolic modes. This study investigated the effects of the individual difference variables of field dependence/independence and spatial visualization ability on the performance of college students on retention tests in (a) the pictorial, (b) the symbolic, and (c) mixed symbolic/pictorial modes. The study also investigated the extent to which the four variables (field dependence/independence, spatial visualization ability, symbolic mode retention test performance, and pictorial mode retention test performance) account for the variability in the retention test performance on tasks requiring an interplay between the symbolic and pictorial modes.

2. Rationale

Previous research suggests that there should be a rather continuous interplay between object manipulations, pictorial representations, and their symbolic recordings during the learning process. Students seem to proceed entirely through a problem solution in one mode or the other, and then, if requested, solve the problem in the other mode. If we consider a mixed mode task that requires an interplay between the pictorial and the symbolic modes for problem solution, the more field independent student is expected to disassociate the common aspects of the two representations from the mixed mode format more readily than the more field independent. It is expected that the more field independent student accomplishes the interplay between the pictorial and symbolic modes more easily than the less field independent student does.
Research Design and Procedures

Subjects
Two intact classes of 96 preservice elementary school teachers enrolled in a university course in methods of teaching elementary school mathematics were the subjects for the investigation.

Treatment
Paper-and-pencil tests were administered to all subjects to measure field dependence/independence using the Gottschaldt Hidden Figures Test (HFT) and spatial visualization abilities using the Purdue Spatial Visualization Test (SPV). A one-week instructional treatment on whole number addition algorithms used the counting stick manipulative aid and emphasized the relationship between the solution of addition problems using the manipulative and a symbolic algorithmic solution. Immediately following instruction, a learning test was administered. Three weeks later, the same test was administered to measure retention. Because the variance in the learning scores was very small, only retention test scores were later considered for analysis.

The retention test was a three-part, paper-and-pencil, multiple-choice, group-administered test. Only the pictorial mode was used in Part 1 and the symbolic mode in Part 2, and in Part 3 the presentation alternated randomly between pictorial and symbolic modes. Students were asked to choose the appropriate symbolic statement or pictorial presentation to suggest a step-by-step solution of each of the six addition algorithm problems.

Using stepwise regression analyses and repeated measures of analysis of variance, the investigators looked at the question of whether or not performance on HFT and SPV tests predicts performance on the pictorial, symbolic, and mixed modes retention test. They were also interested in whether the performance on both the symbolic and the pictorial modes, along with the HFT and SPV scores, would predict performance on mixed mode retention test.
4. Findings

With HFT and SPV scores as predictors of performance on the pictorial mode retention test, the symbolic mode retention test, and the mixed mode retention test, results of the stepwise regression analyses indicated that the HFT variable entered first on all three stepwise regression equations.

A stepwise regression analysis was conducted with the mixed mode retention test scores as the dependent measure and with the pictorial mode retention test scores, the symbolic mode retention test scores, the HFT scores, and the SPV scores as the independent measures. Performance on the symbolic mode retention test entered first in the stepwise regression equation and thus was the best predictor for success on the mixed mode retention test. Symbolic mode retention test scores accounted for 68.63% of the variability in the mixed mode retention test.

To investigate the effect of the individual difference variables of field dependence/independence and spatial visualization on subjects' performance on the three retention test modes, subjects scoring in the upper and lower thirds on the HFT and SPV were identified. The crossing of High (field independent) -Low (field dependent) HFT with the High-Low SPV scores resulted in four classes of subjects. The total number in these four classes of students was 61. The experimental design was 2x2x3 factorial with repeated measures of pictorial, symbolic, and mixed mode retention test scores as the third factor. These data were analyzed by a repeated measures analysis of variance. The results of the repeated measures ANOVA indicate that the main effect due to the retention test mode was significant. The order of the marginal means according to retention test mode, from highest to lowest, was in the order symbolic mode, mixed mode, and pictorial mode. All pairs were found to be significantly different when the differences between all possible pairings of these three means were tested.
A significant main effect due to spatial visualization was also found. This indicates that there are differences among the means of the scores on the three retention test modes when we compare the low level of spatial visualization and the high level of spatial visualization. The significant interaction effect between retention test mode and spatial visualization indicates that the effect of spatial visualization on retention test performance is not uniform across the retention test modes. Mean scores for the high spatial visualizers was higher on each of the three retention test modes. This difference was found to be significant for the pictorial mode but not significant for the symbolic and mixed modes.

There was no significant main effect due to field dependence/independence for the different modes of retention test performance.

5. Interpretations

The field dependence/independence scores correlated consistently higher than the spatial visualization scores with each of the three retention test modes -- pictorial, symbolic, and mixed.

When mixed mode retention scores of the total sample were regressed on HFT and SPV scores in conjunction with pictorial and symbolic mode retention scores, the regression coefficients of HFT and SPV were not significantly different than zero. Symbolic mode retention scores accounted for 68.6% of the variance of the mixed mode retention scores. The experimenters suggest that since the subjects were college students whose prior experience with addition algorithms may have been mainly in the symbolic mode, the results might have been different with younger subjects. They also suggest that the large proportion of female subjects (87%) may account for the higher performance on the symbolic mode.

Abstractor's Comments

The investigators have done extensive statistical analyses on the scores of a specially constructed retention test. Based on their analyses, it appears that there was not support for the assumption that the more field independent student accomplishes the interplay between
the pictorial and symbolic modes more easily than the less field independent student. I wonder whether this is really the case or are the tests not as precise as we need to investigate such relationships. The investigators have attempted to do a careful study and suggest that further studies be encouraged. I agree that such studies should be encouraged.

Abstract and comments prepared for I.M.E. by JOSEPH J. SHIELDS, Missouri Southern State College.

1. Purpose
The purpose of this study was to determine the effect of a calculator-based curriculum on achievement in computation and problem solving for sixth-grade boys and girls and to determine if participation in such a curriculum had a negative effect on posttest computation scores for students who did not use calculators during the posttest.

2. Rationale
Previous studies have suggested that the use of calculators in the classroom may improve computational skills and reasoning/problem-solving abilities. The authors wished to determine the impact of a particular calculator-based instructional program on children.

3. Research Design and Procedures
The subjects in this study were 16 boys and 17 girls in a randomly selected sixth-grade class. The California Test of Basic Skills (Tests 6, 7, and 8) was used as a pretest of the students' achievement in computation and problem solving. "During the next several weeks" the 33 students received instruction in the use of hand-held calculators and in problem solving. Each instructional period involved solving problems by mental estimation, hand computation, and calculator manipulations.

A posttest (Tests 6, 7, and 8 of the California Test of Basic Skills) was administered at the completion of the instructional program. Prior to the posttest, students were randomly assigned to
one of two groups, one of which used calculators during the posttest while the other group did not. The size of the two groups was not indicated.

4. Findings

Four hypotheses were tested by comparing pretest and posttest scores for the three subtests, (computation, concepts, applications) of the California Test of Basic Skills. Analysis of the data was made using Fisher's "t" for testing differences between non-independent means. Based on the results of this analysis, the investigators concluded:

1) A specially adapted calculator-based curriculum had an effect on subjects' achievement in computation.

2) This calculator-based curriculum had an effect on students' achievement in problem solving.

3) There is no significant difference between performance of males and females on either the pretest or the posttest.

4) A calculator-based curriculum did not adversely effect the achievement level of subjects who did not use calculators during the posttest.

5. Interpretations

The findings of this study support the use of hand-held calculators in instruction of sixth-grade children. Calculators can have a positive effect on students' achievement in problem solving and computation. Students using calculators spent more time analyzing and interpreting the information given in a problem.

Abstractor's Comments

There is little in this article which is of value to researchers or classroom teachers. Neither the design of the study nor the description of the treatment merits a positive endorsement.
The authors fail to include enough information to give the reader a feel for what occurred in the classroom during the instructional portion of the study. How many weeks did the study last; how many times per week and for what period of time were students involved in the calculator-based curriculum? What types of problems were solved by hand computation, mental estimation, and calculator manipulation? Since the purpose of the study was to investigate the effect of a calculator-based curriculum on students, this reader expected at least a superficial description of the curriculum.

The design and analysis of this uncontrolled study does not support the conclusions of the authors. The gains made by the students in this study could be attributed to a number of factors. Indeed, the improvement in achievement scores should be a consequence, at least in part, of the non-calculator component of the instructional program. The authors have pretested, carried out some unspecified classroom experience, posttested and concluded that the experience was beneficial because posttest scores were significantly different from pretest scores. This is not research and I question whether the article, in its present form, should have been published at all.

Abstract and comments prepared for I.M.E. by MARY M. LINDQUIST, National College of Education.

1. Purpose
This study examined how children made judgments about the areas of different regions and whether children who used a multiplicative strategy could use proportionality thinking to identify a missing dimension of a rectangle of a given area.

2. Rationale
Other studies of children's ability to use proportional thinking had not established that children had the prerequisite quantitative concepts. This series of experiments was designed to ascertain this before presenting tasks that required the use of proportions.

3. Research Design and Procedures
The first two of the three experiments of this study were concerned with finding the age at which children use the height times width rule in judging areas. The third experiment examined the use of proportions.

The first experiment used 16 rectangles made from combinations of heights (3, 6, 9, and 12 cm) and widths (3, 6, 9, and 12 cm) and 16 irregular figures of corresponding areas. Each 7-year-old was presented each figure twice and asked to make a judgment on a response scale. The response scale consisted of 12 circles increasing in diameter from 0.8 to 3.2 cm. Squares with sides 1.5 cm and 15 cm and corresponding equal-area irregular figures defined the small and large end of the response scale. Subjects were instructed to use circles between the small and large ones so that "the more black space (the regions were cut from black paper), the larger the circle you point to." The circle (numbered 1-12) pointed out by the subject was the response. Sixteen 7-year-olds and 16 adults were the
subjects.

The second experiment limited the stimuli to 12 rectangles and used
32 children from age 8 through 11.

In the third experiment, children were presented with a standard
rectangle and a horizontal line representing the width of a second rectangle.
They chose a length to indicate how tall the second rectangle would have to
to be in order for both rectangles to have the same area. Nine standard
rectangles (widths: 3, 6, and 9 cm; heights: 6, 9, and 15 cm) and three
horizontal widths (6, 9 and 18 cm) were presented for a total of 27 combina-
tions. The stimuli for the heights were 14 vertical bars arranged in
height from a 0.5 to 26.5 cm. The subjects were the same 32 children as
in Experiment 2.

Analysis of variance was used to analyze the results as well as an
examination of goodness of fit to the theoretical outcome.

4. Findings

The main result of Experiment 1 was that 7-year-olds used linear extent
as the basis of judging areas of rectangles and irregular figures. In the
second experiment only three subjects used linear extent and the others used the
height times width rule. There appeared to be no significant difference due
to age or sex, but a significant difference due to the type of rectangle.
The third experiment showed that all ages (8-11) used the proportionality
rule.

5. Interpretations

The experimenter states that two aspects of the data are of special
interest. "First, the transition of the extent to the height times width
rule. Second, the use of the proportionality rule by the 8- and 9-year-
olds. Both the timing and the substance of these developments are at
variance with Piaget's theory of cognitive development."

The experimenter ruled out that the results could be explained by any
iterative procedure or perceptual scanning strategy and suggested that it is
informational content and processing load rather than cognitive development
that limit the logical thinking of the child.

Abstractor's Comments

As the experimenter noted, the results of the first two experiments are predicated on whether the subjects understood the response scale. Although he was confident that they did, I had difficulty since there were only 9 distinct areas and 10 response circles. The theoretical curves were not linear as stated by the experimenter if 9 areas were used. These discrepancies made it difficult to accept fully some of the arguments proposed.

The change from a large set of stimuli to a much smaller set may be a partial explanation of the 8-year-olds' "success". Also, more information is needed before I would be truly convinced that a proportionality rule was used.

It is the type of study that I would want replicated before drawing strong conclusions about cognitive development versus information processing. It should serve as a caution to curriculum developers not to introduce area too early without the background needed.

Abstract and comments prepared for I.M.E. by EDWIN McCLINTOCK, Florida International University.

1. Purpose
The purpose of the research was to study the van Hiele levels of geometric thought. Two hypotheses were tested:

"H1: For each geometric concept, a student at level N will answer all questions at a level below N to criterion but will not meet the criterion on questions above level N.

"H2: A student will meet criterion at the same level on all geometric concepts tested."

2. Rationale
The implicit rationale for the study was that of needing direct confirmation in this country of the validity of van Hiele's theses. These are said to have been validated by the Soviet Academy of Pedagogical Sciences. Further, the practical concern of whether the levels tested form a hierarchy and whether there is "levels consensus" for different concepts is important to learning and teaching geometry. If the former but not the latter is the case, there is evidence that a level-wise sequential development of each separate concept is necessary.

3. Research Design and Procedures
For each of the five van Hiele levels and for each of seven geometric concepts (squares, right triangles, isosceles triangles, circles, parallel lines, similarity, and congruence), questions were designed and validated.

Basic Level
Geometric figures are perceived visually but properties of figures are not perceived.
Level I
Properties of geometric figures are perceived but relations between figures and properties are not perceived.

Level II
Relationships between figures and between their properties, as well as meaningfulness of definition, are perceived.

Level III
Meaningfulness of proof and of concepts of deduction are perceived.

Level IV
Perception is functional for laws of formalistic logic and abstract aspects of deductive geometric structures.

The subjects (19 elementary education undergraduate majors; 18 females and 1 male), thirteen of whom had completed a course in geometry, were interviewed using the questions. The two separate, approximately one-hour interviews were audiotaped for analysis. Altogether, 128 questions were asked of each subject. The questions were categorized by van Hiele level, with different success criteria attached to each level.

4. Findings
Geometric concept strands and van Hiele levels were used as variables for a level-by-concept matrix analysis of data. Criterion scores of 0 or 1 were determined for each subject (and validated by independent ratings), for each concept at each hypothesized level (a total of 665 scores). Mayberry's Table I summarizes these data. The geometric concepts are shown across the top, with the ordered quintuple of 0's and 1's representing the respective levels.
Table 1
Response Patterns, Errors, and Consensus for Geometry Concepts by Each Student

<table>
<thead>
<tr>
<th>Student no.</th>
<th>Squares</th>
<th>Right triangles</th>
<th>Isosceles triangles</th>
<th>Circles</th>
<th>Parallel lines</th>
<th>Similarity</th>
<th>Congruence</th>
<th>Consensus (%)</th>
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*One error.
Two errors.

The test of hypothesis one (H1) was through Guttman's scalogram analysis. The five scores for achievement of level of performance (van Hiele levels) were tested for hierarchial arrangement. This was done by applying the scalogram analysis to determine if the items measuring achievement of the levels formed a scale. Since the items did form a scale, the inference of hierarchial arrangement was made.

A test of ordinal consensus was used to investigate hypothesis two (H2). Using a measure described by Leik (1966), calculations of consensus among geometric concept attainment were done for each subject. These results are reported in Table 1.

H1 was supported, with scalogram coefficients ranging from .91 to 1.00. A value of .90 had been taken as the standard for scalability. H2 was not supported. As Table 1 indicates, consensus percentages ranged from .33 to .91. Only two subjects had consensus percentages of greater than .90. The level of significance had been set as a consensus of .90 for 90 percent of subjects.

Exploratory analyses were also performed. Observations about performance of subjects with respect to van Hiele levels were made, for each of the five levels. Level III observations, for example, indicate...
that making deductions was very difficult for the subjects. Since 13 of the 19 subjects had completed a course in geometry (the purpose of such a course, usually, is learning to make such deductions), the hope would be for van Hiele Level III performance. Mayberry notes that of the 91 responses of students of geometry, only 31 percent were Level III performances.

5. **Interpretations**

The researcher indicates that the study provides support for van Hiele's statements:

a. Students' functioning at a level is dependent upon intuitive performance at all lower levels.

b. Instructional language pitched at a level is not understood by subjects functioning at a lower level.

An immediate interpretation of the correspondence between these findings and van Hiele's statements is, according to Mayberry, that subjects who have not been brought to Level II intuitive performance prior to the formal geometry course may not benefit from the course.

**Abstractor's Comments**

The study by Mayberry provides a particularly valuable approach to studying geometric levels of thought of the preservice elementary teacher subjects. The valid determination of requisite levels of thought to allow benefit to be derived from a formal geometry course is much needed.

As a theoretical framework that shows strong parallels to Piagetian development, van Hiele's work can be used as an application model for development of thought. Mayberry has carefully operationalized the levels of the van Hieles. As the researcher points out, the development of representative questions for the levels of thought is a painstaking task, which has been done well here.

Not mentioned in the report is whether the geometry courses pursued by the thirteen students was, in fact, a formal geometry. Nationwide, informal geometry courses are frequently taught and are based on
textbooks that provide exercise in, rather than assume, Level II thought. Had some of these students' experiences been limited to this type of course, an explanation for their level of thought could be interpreted in different ways.

As the researcher points out, a limitation of the study was possibly the subjective determination of criterion level. With a range of success criterion varying from 50 percent to 80 percent, one wonders about the rationale for setting the levels and what effects would have resulted from a fixed criterion rate of, say, 75 percent.

Finally, the fruitful model provided here could be applied to a population of subjects who are not so apt to perform at low to low-middle thought levels. Mayberry has suggested and developed a productive direction for practical research with respect to geometric thought. This form of research needs the attention of those who have interest in bettering the quality of geometric teaching and learning.

References

Wirszup, I. Breakthroughs in the psychology of learning and teaching geometry. In J. L. Martin & D. A. Bradbard (Eds.), Space and geometry: Papers from a research workshop. Columbus, Ohio: ERIC Center for Science, Mathematics, and Environmental Education, 1976.
Purpose

Two questions were addressed: "(1) Does the sex of the student or the teacher's expectation for the student influence the nature of student-teacher interactions? (2) Do variations in teacher-student interaction patterns affect student attitudes?" (p.322).

Rationale

Evidence from earlier studies (e.g., those of Dweck and associates, and Heller and Parsons) has shown sex differences in students' expectations of success in mathematics. It has also been shown that, although teachers do not typically have lower expectations for girls, their patterns of classroom interaction do vary with the sex of the student. However, the existence of a link between teacher-student interaction patterns and student expectations of success or student self-concepts of ability has received little attention.

Research Design and Procedures

Seventeen mathematics classes in grades 5, 6, 7, and 9 were involved in the study. Five trained observers coded interactions between teachers and students during 10 class sessions per class. In addition, measures of past performance in mathematics; information about student expectancies, self-concepts of ability and concepts of task difficulty; and information about teachers' expectancies of the students were collected on 275 students in grades 7 and 9. Performance measures were obtained by standardizing (within the population) recent mathematics grades and available scores on the Michigan Educational Assessment Program or the California Achievement Test. Questionnaires
completed by students and teachers provided the information about self-concepts and expectations.

Two analyses were carried out. In Analysis I, analyses of variance were run pairing the interactional variables identified by the observers with the attitudinal variables from the student questionnaires and students' past performance scores. Stepwise regression analyses and partial correlations were also carried out to assess the magnitude of various teacher-student interaction patterns.

Analysis II investigated the possibility that the classroom climate might adversely affect girls' expectancies. In five of the classrooms, there was a significant sex difference in student expectations; these were compared with the five classrooms with the least sex difference in expectancies. ANOVAs (sex X classroom type X teacher expectancy) were run on the interactional variables, on student attitudinal variables, and on past performance scales. Additional analyses were carried out, using Tukey's HSD, with the sample further partitioned into high and low teacher-expectancy subsets.

4. Findings

Thirty-seven interactional variables were coded by the observers. These included behaviors such as teacher- and student-initiated interactions and teacher responses such as praise or criticism. In addition, there were three attitudinal variables from the student questionnaires and one from the teacher questionnaires.

Analysis I showed sex differences in attitude, with the females in the sample having lower future expectancies in mathematics, and perceiving it as more difficult than the males. The females also received less criticism and asked more questions. With respect to teacher expectancy, children for whom the teachers had high expectancy generally had higher self-concepts and perceived mathematics as easier, although high-expectancy females had slightly lower self-concepts than the males ($F = 3.19, p = .075$ [p. 329]) and
Teachers' assessments were related strongly to the children's past performance. High expectancy children generally "receive less total praise but have a higher proportion of their questions praised," but "high-expectancy females have the smallest proportion of their interactions praised" and low-expectancy males receive the most criticism" (p. 329). It was also found that "For both girls and boys high levels of work criticism in public response opportunities ... were related to high self-concepts of ability, low estimates of difficulty, and high future expectancies" (p. 329). The authors point out, however, that the actual incidence of both praise and criticism was low. Sex differences were also found in the number of questions asked by students: for boys, this was positively related to their perception of difficulty whereas for girls no such relationship was found.

In four of the eight seventh-grade classes and one of the six ninth-grade classes, there was a significant sex difference in the students' expectancies about future mathematics courses and mathematics related careers. These were paired with the five classrooms with the least sex difference (two seventh- and three ninth-grade classes) for Analysis II. The sex differences in the two types of classes were found to be a function of the girls' expectancies; no differences were found in the boys' expectancies in the two types of classroom. Classroom dynamics, however, did differ in the two types of classes. In the high-difference classrooms, teachers "used more criticism and praise, were more likely to rely on public response opportunities ..., and made more use of student volunteers for answers" (p. 335). These teachers also made less use of private interactions with individual students. Boys also interacted more in the high-difference classes; in the low-difference classes, girls "interacted more and received more praise" (p. 335). Within these types of classrooms, interactions with students for whom the teachers had high expectancy were compared with interactions with students for whom the teacher had low expectancy. In the low-difference classes, there was little difference
in the way boys and girls were treated; however, the high-expectancy girls initiated significantly more interactions, and had significantly more total interactions than either the high- or low-expectancy boys. In the high-difference classes, high-expectancy boys received significantly more praise and had significantly more response opportunities than high-expectancy girls; the low-expectancy girls received significantly more response praise than the high-expectancy girls. The low-expectancy girls also initiated significantly more interactions than the high-expectancy girls, and asked significantly more procedural questions than any other subset.

5. **Interpretation**

The authors carefully point out that they only investigated relationships, and are not making causal predictions. They also point out instances in which their findings differ from the results of other studies.

**Abstractor's Comments**

A number of interesting points are raised by the findings. As indicated, the authors consider that the incidence of both praise and criticism was low in the classes observed, yet the students with high personal expectancy received less praise than others. On the other hand, in classes with high sex difference in student expectancy, the teachers used more criticism and praise than teachers in the low-difference classes. The authors suggest that it may be the judicious use, rather than the frequency of use, that relates criticisms and praise to student expectancy and performance. They also point out that in the classes in which there were high sex differences in student expectancy, praise was positively related to teacher expectancy for boys, negatively related for girls.

Do variations in teacher-student interaction patterns affect student attitudes? This was one of the major questions addressed in the study, and the authors are reluctant to assign causality to their
findings. Nevertheless, there appears to be evidence from the study to make this an attractive hypothesis to test. The differential use of praise in the high-difference classrooms needs also to be addressed. Students in the high-difference classes were predominantly seventh-graders, in contrast to those in the low-difference classes. Is seventh grade a more critical time? To what extent does past classroom experience have an effect? These and other questions come easily to mind.

As mentioned in the abstract, 37 classroom interactional variables were coded, and the analyses were carried out on various collections of these. Readers who are interested in the topic are urged to consult the article so as to get a complete picture. In view of today's interest in encouraging girls to take more mathematics, the topic is timely.

Abstract and comments prepared for I.M.E. by LELAND F. WEBB, California State College, Bakersfield.

1. **Purpose**

   The purpose of this study was to analyze children's conceptual understanding in solving multi-step mathematical word problems with a ratio and to determine the specific sources of difficulty.

2. **Rationale**

   Mathematical word problems than cannot be solved with a single step are a real source of difficulty for children in the elementary grades. Most previous research studies analyzing children's difficulties in solving multi-step word problems have used very broad categories that are "frequently not informative (e.g., 'standardized word problem test'). In order to understand the child's behavior, it is important to have a clear description of the types of problems that are being solved, and it is important to consider the meaning of the concepts and relationships involved, i.e., the semantic aspect of the problem.

   With multi-step problems, in addition to choosing the operation, the child has to "organize the order of applying the operations and identify the pair of numbers to which each operation can be applied", creating an additional difficulty. Thus, there are two possible sources of difficulty: (1) difficulties in understanding the concepts and relationships; or (2) difficulties in planning and organizing the solution method.

3. **Research Design and Procedures**

   The types of problems used in the study were:

   - There are a x's for every b y's.
   - There are c y's.
   - How many x's are there?

   For example:

   - In the store they give 6 lollipops for every 2 chocolate bars you buy. John bought 10 chocolate bars. How many lollipops did they give John?
Understanding the ratio concept (6:2) as well as the equivalencies (6/2 = x/10) are important for problems of this type. Once understood, one of two methods is usually employed: (1) simplify the ratio and multiply (6/2 = 3/1; 3/1 x 10); or (2) find how many groups of 2 chocolate bars there are in 10 chocolate bars (10/2 = 5), and then multiply by 6. If one of the ratios is not evenly divisible, then this might affect the choice of methods to utilize.

 Twelve children were selected from each of grades 5, 6, and 7 in public schools in Puerto Rico. Two fifth, sixth, and seventh grades were selected in equal numbers from two public schools, while the seventh graders were similarly selected from an intermediate school.

 There were six word problems that were presented individually to each child, on cards, one at a time. Each child's session was audio-taped and was one-half hour in duration. Three of the word problems were the same for each child (two fillers and a one-step problem), while three two-step problems varied in context and numbers utilized. Various combinations were used such that in two of the problems one of the two possible ratios was an integer, and in the third problem both ratios resulted in an integer. Each child read the problem silently and solved it. Following this, the experimenter read the problem aloud and asked the child to repeat it from memory. This was done to "identify the children's representations of problems."

 After solving all problems, the child was asked to identify the correct solution from a set of drawings representing the situations described. Some choices were correct; others incorrect. The purpose of this was to obtain information on the child's representation of the problem.

 Three scoring procedures were used:
 a. "Solutions. A solution was considered correct if it was set up correctly, even though there might be computational errors.
 b. Repetition. A repetition of a problem was considered correct if it contained all the information needed to solve the problem.
 c. Recognition. A selection of one or more drawings was considered correct if it included only the correct representation."
4. **Findings**

For the one-step problems the results were as follows:

a. **Solution.** 29 of the 35 children (81%) solved the problem correctly.

b. **Repetition.** In 33 of 35 cases (94%), the children were consistent in their solution and their ability to repeat the problem (i.e., solution and repetition both correct or solution and repetition both incorrect).

c. **Recognition.** In 30 of 36 cases the solution and selection of drawings were both correct or both incorrect.

The two-step problems were much more difficult for the children but the results did not differ across the context or numbers used:

a. **Solution.** The number correct classified by context was 5/36, 8/36, and 8/36. The number correct classified by use of numbers was 6/36, 6/36, and 9/36.

b. **Repetition.** There was not as great an association between solution and repetition as with the one-step problem. In 69 of 105 total cases scored (66%), the solution and repetition were both correct or both incorrect. Distortion of the data resulted in more errors than omission errors.

c. **Recognition.** In 94 of 108 (87%) of the cases, the solution and selection of drawings were both correct or both incorrect.

5. **Interpretations**

Most of the children had difficulty with the two-step problems, but the sources of difficulty varied. There appear to be three levels of understanding:

a. "Children understand ratios that are equivalent to an integer. They can solve one-step but not two-step problems.

b. Children understand ratios that are not equivalent to an integer, but given a ratio they have difficulty finding an equivalent one.

c. Children can find equivalent ratios."

Abstractor's Comments

This appears to be a very thorough study regarding children's
solutions to two-step word problems involving a ratio. It should be noted that it was completed with support from the National Institute of Education. It is obvious from the amount of work done that the author spent a great deal of time in the conceptualization process. The study is quite complicated in its design, but it would be relatively easy to replicate. The controls established (use of audiotapes, a pilot study to determine the type of drawings to use for the model recognition, etc.) indicate that a great deal of time was spent thinking through the entire process.

There are a number of suggestions that might be mentioned:

1. It appears as though the language used throughout the study was Spanish, although there is only one reference to it: "The experimenter displayed the drawings on cards in front of the child together with the word problem they represented and said (in Spanish): ........(p. 105)". If, in fact, the study was conducted in Spanish, it should be stated.

2. The sample size (n = 36) is quite small. Although each student was audiotaped, it would have been better to have obtained a larger sample.

3. No reference is made as to whether or not the sample was random. Why were only two schools utilized for the fifth and sixth graders? Why were only two schools utilized for the seventh graders? Were they the only schools available? Some comments would have been appropriate here.

4. Mention is made of the sessions with the children being audiotaped, but there was no description of how the audiotapes were used. I assume that they were used to analyze the children's comments?

5. Mention is also made of the types of difficulties (page 103): (1) difficulties due to understanding the concepts and relationships; or (2) planning and organizing the method of solution; but there seems to be no reference to these types later in the article. Why were they included initially and not utilized later?

In general, this is a well-designed study, and although it would appear obvious that two-step problems are more difficult than one-step problems, it is good to determine where differences exist and exactly what difficulties exist when solving two-step word problems.

Replication with a larger population would be desirable.

Abstract and comments prepared for I.M.E. by JANE O. SWAFFORD, Northern Michigan University, Marquette.

1. **Purpose**
   To compare the achievements of students taught from Saxon's textbook to that of students taught from standard textbooks.

2. **Rationale**
   Algebra can be mastered if presented in increments with constant review that allows time for absorption and practice between the introduction and elaboration of a concept.

3. **Research Design and Procedures**
   Ninth-grade algebra students in 20 Oklahoma schools were divided into control (n = 841) and experimental groups (n = 541) each taught by the same teacher within their school. The experimental group was taught from Saxon's then unpublished textbook, *Algebra I: An Incremental Approach*. The control group was taught from whatever algebra textbook was normally used. The California Achievement Test was given for the purpose of dividing the subjects into four ability levels for the achievement analysis. Over a 3½ month period, 16 tests developed by the author were administered to both groups. Each test consisted of from 3 to 15 items derived from problems submitted by the participating teachers. A test was only administered after both textbooks had covered the topic. Two of the tests (on scientific notation and integral exponents) were administered to Algebra II students as an additional control group.

4. **Findings**
   On all 16 tests at each ability level, the experimental group outscored the control group. The experimental group also scored higher
than the Algebra II group on the two tests administered to them. In general, gains were greatest for lower ability students.

No tests of significance were reported for the differences observed. The results were reported in terms of "percent gain" of the experimental over the control. For example, if the experimental group's mean was 5.75 out of 10 as compared to the control group's 1.89 mean, then this difference of 38.4% was reported as a "gain" of 204%.

5. Interpretations

An incremental approach to the teaching of mathematics with continuous review is superior to the approach of the current textbooks. This approach will improve declining test scores, mathematics attitudes, and math avoidance.

Abstractor's Comments

Of the three footnotes to this article, one advertised Saxon's book for $12.80 and another offered a copy of the tests used in the study. I sent for both.

The textbook is a rather ordinary looking, single color, college remedial text organized into 126 lessons instead of chapters. The lessons are not grouped logically, but spiral through a particular topic almost at random through the course of the book. Each lesson contains 30 to 35 exercises of which less than 6 deal with the topic of the lesson. The rest drill topics taught in previous lessons (hence, the constant review). The emphasis of the text is on complicated symbol manipulations.

The set of 16 tests used in the study carries the same emphasis on complex computational skills. Together they contain 106 items of which the following are typical:

\[
\text{Simplify: } -2[(-2-2)-2(-2^2-1)]
\]
\[
\text{Simplify: } \frac{2}{x} \cdot \frac{2}{y} \cdot \frac{25}{y} \cdot \frac{2}{x} \cdot \frac{10}{y} \cdot \frac{2}{x}
\]
Solve: \[2(4-x) = -2(1-2x) + 6 - [4]\]

Evaluate: \(a[-c-(b)]\) if \(a = -1, \, b = -2, \, c = -2\)

Looking at the data, it would appear that Saxon's textbook is more effective than the standard textbook. However, these results may be misleading. The sample used may or may not have been appropriate. We are told nothing about either the control or experimental groups other than that they came from ninth-grade algebra classes in 20 Oklahoma schools. Whether these students are at all typical of the ninth-grade population in general is unknown. Although the California Achievement Test was given in the beginning, no attempt was made to use it to describe the sample relative to national norms. Further, we do not know how the control and experimental groups compared. No attempt was made to equate them on the basis of CAT scores. Neither was assignment to treatment random. There would appear to be some bias, since 55% more subjects were assigned to the control than to the experimental group. In the absence of evidence to the contrary, the achievement "gains" observed may only be the result of initial differences between the groups.

In addition to the question of sample bias and representativeness, there is also the question of teacher bias. On the surface, the design seems to control for teacher effects by using the same teacher for both the control and experimental groups. However, our experience (Swafford and Kepner, 1980) indicates that regular classes taught by an experimental teacher are contaminated by the very fact that the teacher is involved in the experiment. This is especially true when the teacher volunteers to be involved rather than is selected at random. Such a teacher is often already dissatisfied with the standard treatment and eager for any experimental treatment to work. One way to control for these teacher effects is to have two comparable teachers from the same school assigned at random to teach a control or an experimental class. Without adequate controls for teacher effects, the achievement
gains observed in the present study may only be the result of a teacher bias in favor of the Saxon treatment, rather than the result of that treatment itself.

Finally, the test instrument used to compare the two treatments was obviously biased in favor of the experimental text. The test was derived by Saxon from items submitted by the teachers, not from items selected from both textbooks. While the topics covered by the tests may be covered by standard textbooks, they are not covered at the same level of computational complexity as in the tests. Hence, the achievement gains reported may not reflect a more effective approach for teaching certain skills, but merely the fact that those skills were taught to the experimental group and not taught to the control group.

The failure to conduct any statistical test is troublesome, but not as troublesome as the rather unorthodox reporting of the mean raw score comparison as "percent gains." No doubt most of the differences would be significant. The author chose to dramatize these differences rather than judge performance against a standard, a practice he himself advocates (p. 484). For example, the experimental group on the average only answers 54.6% of the 106 items tested. While this is better than the control, it does not constitute mastery as claimed for an incremental approach. Other conclusions drawn by the author are even more suspect. There are no data to substantiate the impressions given that the experimental group had a more positive attitude toward mathematics, were more successful in subsequent mathematics courses, had higher SAT scores, or went on to take more non-required mathematics than the control group. In fact, given how little is known about the sample, it is impossible to draw any conclusion about the general ninth-grade algebra population, based on the results here. Furthermore, the sweeping generalizations made by Saxon about the teaching of mathematics in general are surely not warranted by the data.

However, for all of its design weaknesses, to me the study's most serious flaw is in its underlying assumption about the goals of algebra instruction. Back when it was believed that mathematics
trained the mind, proficiency in performing complex algebraic manipulation was an end in itself regardless of its subsequent usefulness. Saxon's incremental approach with constant review may well be a more effective way to teach manipulation skills. However, in today's increasing complex society, instructional time cannot be devoted to teaching students skills they will never use or need if we expect to equip them to cope with tomorrow. To do better what no longer needs to be done is not a breakthrough but a retreat.

Reference


Abstract and comments prepared for I.M.E. by CLYDE A. WILES, Indiana University Northwest.

1. Purpose

To investigate the effects of conventional hand-held (ch) calculators and programmed feedback (pf) calculators on the mathematics achievement of third- and fourth-grade compensatory students.

2. Rationale

An earlier study conducted by the authors with normally achieving third graders found that the use of the ch calculators provided greater achievement than either traditional instruction or the pf calculator, though there was some evidence that supplementary daily use of the pf calculator was more effective than traditional instruction alone in advancing computational skill.

It was postulated that these subjects simply knew too much at the onset for the pf calculator to have helped very much; hence, benefits to be derived from the immediate feedback of the pf calculator could be expected only with lower achieving groups. Though not cited in this report, aspects of S-R psychology are implicitly appealed to in support of this expectation.

3. Research Design and Procedures

Students from 8 third-grade and 7 fourth-grade Title I compensatory mathematics classes were assigned as intact classes to one of three instruction groups. Five classes were randomly assigned to a ch calculator group, 5 to a pf calculator group, and 5 to a regular curriculum group. The treatment period ran from October 1979 to the end of February 1980, with retention testing done in May. Calculators were not used in testing or made available after the posttest.
All students studied the regular compensatory curriculum for 45 minutes per day. The calculator groups devoted 8 to 10 minutes of that time to calculator activities. The pf group was restricted to the programmed functions of the pf calculator, while the ch group checked homework, drilled on basic facts, and completed selected calculator activities.

Teachers were oriented to the study and students were pretested in September. SRA tests of concepts, computations, and total mathematics achievement were used for the posttest and retention test. This produced six criterion variables. Each criterion measure was analyzed using a two-way analysis of covariance with treatment group (ch vs. pf vs. control) and grade (3rd vs. 4th) as main effects, and with subject's sex and pretest mathematical achievement scores (more than one?) as covariates. The adjusted means and standard deviations for the criterion variables, along with summaries of the ANCOVAs, were presented in this report. All comparisons were made at the .05 level of confidence.

4. Findings
Initial analysis of the adjusted means showed that

...the main effects for treatment were non-significant; however, the treatments by grade interaction was significant in every case. (p. 14)

Duncan's tests were then used to investigate grade-level differences with the following results for the posttest data:

...significant differences among treatment groups were not obtained for third-grade students; and significant differences generally favoring the group using the conventional hand-held calculator over the programmed feedback calculator were obtained for fourth-grade students. (p. 16)

To summarize the retention test data: the conventional hand-held calculators resulted in generally superior performance by fourth-grade students, whereas the programmed feedback calculators resulted in generally superior retention by third-grade students. (p. 16)
5. **Interpretations**

   i) "calculators appear to provide feedback which facilitates the acquisition and retention of mathematical achievement over what is achieved with traditional instruction alone" (p. 16).

   ii) "the feedback calculator seems to have its greatest impact for the least advanced students" (p. 16).

   iii) "the basic design of the calculators used in this study rather than their feedback function may account for the superiority of the hand-held calculator group over the feedback calculator among the underachieving fourth-grade students" (p. 17).

   And finally,

   iv) "...the results of this study suggest that calculators have a place in the instructional program for primary students within a remedial or compensatory setting, although the type of calculator used may depend on the ability of the students" (p. 17).

**Abstractor's Comments**

The feedback characteristics of calculators are appealed to as the factors that account for the significant differences in this study. I find this unconvincing. The pf calculator provides the most obvious feedback characteristic and it provides feedback for something that more closely resembles computation than concepts. Yet it is the difference for concepts on the retention test that is appealed to support the feedback idea. The adding of the concepts and computation scores to produce a total measure is redundant at best, and does not provide any additional evidence to support the interpretation. The failure of the obvious feedback machine to provide sensible, consistent differences, together with overall design weaknesses, render the feedback explanation speculative.

The existence of interaction effects justified looking for effects within grades. And, within the fourth-grade data there is clear evidence to support the use of calculators to promote either
concepts or computational skill as measured by the SRA tests. There is no evidence that children's learning was hindered by the introduction of calculators in any case.

The desire to place the study within the framework of a learning theory is commendable. However, if the study is taken as a test of reinforcement theory, it fails to support it. The best interpretation I can make of the data is that the introduction of conventional calculators within the compensatory program of the fourth-grade students provided the motivation and opportunity to learn both concepts and skills better than for students without this aid. The use of the machines in the third-grade compensatory curriculum failed to make any real difference in the variables measured.

Other questions that occurred during the close reading of the report follow.

1. What was the pretest? What, exactly, were the achievement scores measuring? Why was sex considered as a covariate? What other variables, if any, were considered, but not entered into the data analysis?

2. How did the covariates affect the means? Since grade level as a main effect was significant for the adjusted means, what were unadjusted means like? Perhaps the interaction was in the covariates rather than the criterion variables.

3. The data collection phase of the study called for 5 classes of third or fourth grades to be assigned to each group. The analysis then uses grade level as a main effect. If grade level was thought to be a main effect possibility at the outset, why weren't even numbers of grades assigned to each treatment? Why weren't grade differences on the unadjusted means considered as a part of the primary design?
1. **Purpose**

Approximately one out of two college students makes a reversal error in problems of the students and professors type like: Translate the sentence "There are n times as many blobs as there are gribs" into an equation relating B, the number of blobs, to G, the number of gribs. The purpose of the investigation was to explore sources for these reversal errors and to examine whether a short teaching intervention would influence students' performance.

2. **Rationale**

Reversal errors are errors leading to the answer $G = nB$ (or an equivalent answer) to the above problem. Clement et al. have shown that reversal errors may follow from two inappropriate thought patterns: A word order match (words are transliterated into a formula in the order in which they appear in the sentence to be translated) or a set match (B is taken to stand for "blobs" instead of "the number of blobs" and similarly for G; moreover, in a set match, the equal sign does not compare two equal quantities). Clement et al. have also observed that many students tend to shift between $B = nG$ and $G = nB$ before deciding on one of the two versions for an answer.

In order to get more detailed information about the observed shifts the author examined

(i) whether students understand English sentences of the students and professors type;

(ii) whether students understand equations of the type $x = 4y$;

(iii) to what extent students making a reversal error need to be prompted to correct this error; and

(iv) why these students failed to prompt themselves in the first place.
3. **Research Design and Procedures**

A sequence of six studies was designed. The subjects were all female college students enrolled in a science for teachers course. They were judged by the author to be "weak in mathematics." Twenty-five sections of the course participated in various stages of the investigation. The sample sizes in the six studies varied from 43 to 185, with partial overlaps.

The studies used questionnaires (studies 1, 2, 5 and 6), interviews (study 4), or both (study 3). Study 1 examined whether the students were able to understand an English sentence of the students and professors type. They were given a questionnaire with three questions about one sentence. Study 2 consisted of a similar questionnaire concerning the understanding of the equation \( x = 4y \). The central study, study 3, consisted of one written translation question followed by an interview with each of the 155 students in the study. In the interview the students were prompted to check their original answer by comparison questions ("Can you tell from the sentence which number is greater, B or G?"; "does your equation agree with this?") and computation questions ("Find number pairs satisfying the sentence/the equation"). The interview appears to have had many goals, which included determining

- on which thoughts the students had based her original answer;
- how certain she was about her answer;
- to what extent prompting was needed to make the student change her answer if it was wrong; and
- whether errors were of the word order match or of the set match type.

Study 4 aimed at finding out why some students failed to check their answers. The students were asked to think aloud while answering a translation question and to convince someone else that their answers were correct. In studies 5 and 6 comparison and/or computing questions were inserted into questionnaires in order to see to what extent student performance could be improved by a short, limited intervention.

4. **Findings**

Studies 1 and 2 showed that (almost) all students understood both the English sentence and the equation. In study 3, however, 52% gave a
wrong answer, 90% of these making a reversal error. In the follow-up interview it turned out that 60% of the students having given the wrong answer corrected themselves after minimal prompting (i.e., spontaneously or after being asked one comparison question). Moreover, about one-third of the students having written the correct answer stated having first thought of the reversed equation. Set match errors were found to be more stable than word order match errors. In study 4, about two-thirds of the students giving a correct answer could justify their answers by comparing the numbers involved, whereas only very few of the students giving a wrong answer tried to do so. No reason was found why these students failed to check their answers. In studies 5 and 6 it was found that the insertion of comparison and/or computation questions between the translation questions significantly improved the number of correct answers.

5. Interpretation

The investigator concluded that the principal sources of reversal errors are not deep-seated misconceptions but "haste, failure to think of checking the equation, failure to base the equation on the meaning of the sentence and use of nonalgebraic symbols." He reasoned that the students' performance should become consistently good, if they can learn to habitually ask themselves checking questions.

Abstractor's Comments

The results of this investigation allow a certain optimism: Although a large proportion of our college population make reversal errors on simple translation questions, these errors often are neither based on deep-seated misconceptions nor very stable; they could therefore be eradicated quite easily. Unfortunately, this optimism might be exaggerated for the following reasons:

(i) Not only the wrong answers are unstable but so are the correct ones. No attempt has been made by the investigator to examine whether the students' answers are consistently correct or consistently wrong.
Prompting for checking does cause correction on the spot, but is quite unlikely to eliminate the more serious misconceptions connected with a set match error. No attempt has been made to identify which students improve their performance in studies 5 and 6 and which don't. The information given on this question in study 3 is imprecise (How many students who made a word order match error corrected themselves after minimal prompting and how many who made a set match error?). Moreover, study 3 offers less convincing evidence for self-correction than studies 5 or 6, because students were pushed by the interviewer until they eventually corrected themselves.

The teaching idea proposed by the author, namely: teach them to check their answers, is certainly relevant. Checking is clearly not limited to this type of translation problem nor to translation problems in general. But how do you teach them to check? How many of your students check their solutions to a linear or a quadratic equation or to an indefinite integration? Mine don't - although I urge them frequently.

Several terms have been used in the paper without being accurately defined: What does it mean that the students are "weak in mathematics"? What is the difference between tacit cueing (study 5) and explicit cueing (study 6)? And, more importantly: How have the reasons for the errors been determined from the interview in study 3? What is meant by "teaching intervention"? At one place a 10-minute teaching sequence is referred to, whereas in connection with the improvement in studies 5 and 6 only a couple of questions asked in the questionnaire are mentioned.

In summary, this investigation points to the important fact that some, or even most, reversal errors might be due to superficiality rather than serious misconceptions; however, many questions remain unanswered about the identification of superficial versus more serious errors, and about the stability of the effect of a teaching intervention urging systematic checking of answers.
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