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ABSTRACT

This study examined the aspirations of the Developing Mathematical Processes (DMP) program and sought to ascertain the extent to which it has been implemented in observed classrooms. DMP was intended to reshape conceptions of mathematical knowledge and school work and to create a pedagogy in which children would be active in creating and testing mathematical knowledge through an inquiry approach requiring exploration, investigation, choice, and judgment. Interviews in connection with a field study in 1978-80, when teachers taught each revised DMP topic for the first time, were replicated and extended with those teachers continuing in 1981-82 to teach the revised topics at the same grade levels. These interviews were complemented by a series of classroom observations. Among other findings, it was apparent that teachers tended to treat the mathematical content of DMP as a fixed body of knowledge which they were to transmit to students. As a result, the mathematics which was taught often differed markedly from what the DMP developers had intended. Moreover, teachers preferred to interact directly with students through group process and so reduced discussion and collaboration among students. Only rarely were the content and methods of DMP modified to meet the needs of students more effectively, or to better implement the mathematical goals of DMP. (AUthor/MNS)

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MATHEMATICAL KNOWLEDGE AND SCHOOL WORK

A CASE STUDY OF THE TEACHING OF DEVELOPING MATHEMATICAL PROCESSES (DMP)

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Abstract

This study asked what meaning has been given to knowing and doing mathematics in those classrooms which comprised the classroom observational study conducted by the Wisconsin Center for Education Research during 1978-1981. This study examined the aspirations of the program Developing Mathematical Processes (DMP) for the reform of teaching and learning of mathematics in elementary schools. In seeking to ascertain the extent to which DMP has been implemented in those classrooms, and the degree to which its authors' aspirations had been achieved, this study interpreted the work of teachers, the work of students, and what constitutes appropriate mathematical knowledge for children to learn.

DMP was intended to reshape conceptions of mathematical knowledge and school work. It sought to create a pedagogy in which children would be active in the creating and testing of mathematical knowledge. Its developers saw mathematical inquiry as requiring exploration, investigation, choice, and judgment. They believed that children could be assisted by their teachers to approach mathematical inquiry in this spirit.

A field study was employed in the two schools where revised topics of DMP had been taught. Data were gathered from ten teachers using classroom observations and interviews. This study also had access to interviews which had been recorded by members of the Mathematics Work Group of the WCER with teachers in the years 1978-1980 preceding this study. These earlier interviews were conducted when each of the revised DMP topics had been taught for the first time. They were intended to ascertain the degree of importance which teachers had given to the revised topics, and also to elicit from teachers specific features of their own teaching.

In the present study, these original interviews were replicated and extended with those teachers who were continuing in 1981-1982 to teach the revised topics of DMP at the same grade levels as before. In this replicated series of interviews, the researchers also sought to elicit from teachers the extent to which they had changed their approach to the content and methods of DMP, and the beliefs, purposes and values which were implicit in these changes. These interviews were complemented by a series of classroom observations in order to determine the extent to which the beliefs, purposes and values which had been articulated by teachers were embodied in the curriculum as taught.

The developers of DMP did not reckon with pervasive features of the culture of schools which would need to be challenged if DMP was to have the effect which they intended. By adopting both the instructional programming features of Individually Guided Education (IGE) and a center-out approach to the implementation of DMP in schools, the craft features of mathematical inquiry were often curtailed so that mathematical knowledge became represented as a set of crystallized logical entities. These aspects of development and implementation diverted attention away from the social and intellectual processes by which mathematical knowledge is created and tested.

The notion of school work as a social and ethical construct was indispensible for showing that children in school learn not only the subject matter of mathematics, but through their work, they are also taught the appropriate forms in which to cast their knowledge. In the predominant pattern of teaching the revised topics of DMP, teachers accentuated their supervisory and managerial role over children's learning. They preferred to interact directly with children through group processes and so reduced discussion and collaboration among children themselves. Whenever this approach to instruction prevailed, children had limited opportunities to engage in creating and testing mathematical knowledge for themselves.

Teachers said that they had modified the activities and methods of DMP in order to meet the perceived needs of students. However, the predominant pattern was one of technical change. Teachers tended to treat the mathematical content of DMP as a fixed body of knowledge which they were to transmit to students. Moreover, their concerns for orderly classroom management and control shaped the processes by which mathematical knowledge was transmitted to students. As a result, the mathematics which was taught often differed markedly from what the developers of DMP had intended.

Only rarely were the content and methods of DMP modified directly to meet the needs of students more effectively, or better to implement the mathematical goals of DMP. In the few instances of constructive change, teachers displayed a sense of ownership and control over what they were teaching. Likewise, children were also helped to bring personal meaning to what they had learned. These features seemed to be absent from those classrooms where a management approach to instruction has prevailed.

A management approach to instruction and its corresponding pattern of technical change over time are related both to the IGE instructional programming model and to a-center-out model of curriculum implementation. In its turn, these models of curriculum development and implementation are related to a wider perspective where teachers and children are regarded essentially as consumers of predefined curriculum knowledge.

MATHEMATICAL KNOWLEDGE AND SCHOOL WORK

INTRODUCTION

In a tentative conclusion to his study, Tasks and Social Relationships in Classrooms, Bossert (1979), having examined the effect of different patterns of task organization on patterns of social relationships in the classroom, suggests that how tasks are organized in classrooms must be pertinent to the moral or normative role of the school in the socialization of children. Absent, however, from Bossert's (1979) study was any attempt to explore the links between conceptions of work and knowledge and different patterns of "task organization". While it may seem commonplace for him to refer to classrooms as "places where teachers and pupils work" (p. 7), he fails to recognize that conceptions of work are related to a network of moral and social considerations which his study leaves unstated. As Popkewitz (1982) argues, notions of children's competency are inextricably related to a normative view of society in terms of which competency is located.

While it is obvious that all learning is rooted in a social process (Berger & Luckmann, 1976), there is considerable disputation about the moral, psychological and social conditions under which knowledge is developed. For example, a management perspective of instruction depicts children needing to be managed in their introduction to a body of knowledge extrinsically conceived; they are depicted as learners or consumers of knowledge; and as needing to be tested in order to ascertain whether they can reproduce or apply what they have received. The fact that items of knowledge have been cast in predefined terms, the fact that children have to forego their own preferences and choices in demonstrating what they have learned; the fact that they required to enter into competition with others, or are isolated from others as they learn, cannot be set aside as mere incidental features to the processes of teaching and learning. Those features define teaching and learning as social events by incorporating assumptions as to how children are to relate to each other and their teacher, and how the content of instruction is defined.

One is struck in reflecting upon Bossert's (1979) study, how the social and ethical dimensions of school work and knowledge are filtered out by focusing exclusively on considerations of task organization and classroom management.

It is an irony of Bossert's study of the sociology of classroom organization that he fails to consider the interplay between the social context, which defines the work of teachers and students, and the conceptions of knowledge which are embedded in that context.

THE TEACHING OF DEVELOPING MATHEMATICAL PROCESSES (DMP)

In this current study, teachers have brought assumptions about their own work and about appropriate work of students to their teaching of Developing Mathematical Processes (DMP) (Romberg, Harvey, Moser, & Montgomery, 1974, 75, 76). With few exceptions, their approach to instruction has reflected a management perspective. The beliefs, purposes and values which underpin this perspective were an impediment to the constructive implementation of DMP, and have served as a barrier behind which assumptions about school work and mathematical knowledge can remain unquestioned and unchanged.

Through interviews with teachers and observations of their lessons, a predominant pattern of implementation of DMP was depicted by:

- the imposition of a pattern of whole-group instruction within which teachers have accentuated their supervisory and managerial role over children's learning, and within which teachers have preferred to interact directly with children through group processes and to reduce contact among children themselves;
- a strong tendency for teachers to treat the mathematical content of DMP as subordinate to their concerns for orderly classroom management and control;
- where these two features have led teachers to change the instructional activities of DMP, these changes have, in general, been confined to changes in the procedures by which the content of DMP has been presented, with no direct attempt to modify the nature of the content itself;
- modification of instructional activities in order to make children's responses more uniform and to ensure that children were given more direction, more reinforcement, and more constant assessment;
- elimination of activities which were believed to be "too challenging" or "too demanding", and the transformation of small-group or individual activities into whole-group instruction;

- introduction of fixed rules and procedures in teaching children to solve story problems, and persistence with a single method of analysis and solution even when children's difficulties with that method had made its continued use questionable;
- a disinclination to have children work collaboratively in order to overcome reading difficulties encountered by individual children; and an appeal to reading difficulties as a rationale for whole-group presentations;
- a tendency on the part of teachers to decide in advance what would be in students' best interests, and to preempt choices which might have been made by them.

Within this pattern of teaching, there was an almost exclusive focus upon the teacher as the one who presents instruction, and the one who develops concepts and skills for those being taught. Indeed, in their interviews, teachers consistently argued that this direct style of instruction was what their children really needed. Teachers preferred to adopt a whole-group approach to instruction because they hoped to monitor more closely what children were doing. Within this pattern of instruction, teachers were able to provide the kind of immediate supervision which they saw pupils as needing. When children had completed assigned seatwork, for example, they usually presented their teacher with work for correction during the lesson, or their folders containing completed work were collected at the end of the lesson. In many cases, children needed to make corrections to their mistakes before they were allowed to move on to the next activity. These salient features of classroom activity during the teaching of DMP reinforced teachers' perceptions of their role as managers of children and as transmitters of a fixed body of knowledge.

MATHEMATICAL KNOWLEDGE

Knowing and doing mathematics need to be related to the creation and testing of mathematical knowledge within a public domain. There, mathematical inquiry can be seen as an intellectual craft which is practiced and developed in a community whose function is to legitimate standards of acceptable work and what constitutes appropriate questions and standards of proof. This picture of mathematical inquiry is often ignored by those who present mathematics and a collection of logical entities, themselves beyond dispute, and mathematical inquiry as an isolated activity of applying these logical artifacts and generating fresh ones in accordance with fixed rules.

The dynamic and social character of mathematical inquiry is well illustrated in three of the six goals proposed for mathematics instruction by Buck (1965). These goals are intended to be more or less independent of specific courses of mathematics instruction, and are intended to "help to supply an answer to the person who asks: 'Aside from its technological importance, what are the educational values of mathematics?'":

> Goal 2: To convey the fact that mathematics is built upon intuitive understandings and agreed conventions, and that these are not eternally fixed.
>
> Goal 3: To demonstrate that mathematics is a human activity and that its history is marked by inventions, discoveries, guesses, both good and bad, and that the frontier of its growth is covered by interesting unanswered questions.
>
> Goal 4: To contrast "argument by authority" and "argument by evidence and proof"; to explain the difference between "not proved" and "disproved", and between a constructive proof and nonconstructive proof. (Buck, 1965, pp. 949-952)

These goals, as Romberg and Harvey (1969) note,

> reflect a belief that the primary value of mathematics is in its relationship to reality, that mathematics is an abstract but humanly created image of reality. From this perspective, it is our belief that these goals can only be attained through the human activity of creating mathematics. (p. 3)

IMPLICATIONS FOR DMP

How might this vision of mathematical inquiry as an intellectual craft be reflected in classrooms as Romberg and Harvey (1969) hoped it would? Children would need to sense that they were participating in the creation and testing of mathematical knowledge. They would need also, to some extent, to become their own authorities in dealing with mathematical ideas. They could need to understand that abstracting, inventing, proving, and applying mathematics are activities which take place in a context of mutually agreed and developing standards as to what constitutes acceptable abstractions, proofs, inventions, and applications of mathematics.

How was this vision of mathematical inquiry as an intellectual craft reflected in DMP? There were two separate and contrasting strands in how the authors of DMP conceived of mathematical inquiry. On the one hand, DMP was developed as the mathematics program for Individually Guided Education (IGE) (Romberg, 1977). The IGE model of instructional programming emphasizes specifying objectives, grouping students in terms of need, and assessemnt. From this model there was a tendency to portray mathematical knowledge as a set of crystallized logical forms to which children were introduced by their teacher. On the other hand, the DMP authors recognized that children's mathematical investigations needed to advance beyond these predefined patterns of analysis to a point where children could try out their own strategies for solving number sentences and story problems (Romberg, Harvey, Moser & Montgomery, 1975, p. 50).

In reflecting upon the implementation of DMP, it is, however, all too easy to point to shortcomings in the way in which the course materials were presented to teachers, and so to argue that DMP would have been implemented differently than it was if only the authors had been more explicit and consistent in what they wanted to achieve. To focus on these issues is to miss the mark entirely. Although DMP was intended to transform the teaching and learning of mathematics in the elementary school, its authors did not adequately identify the traditions of schooling which DMP was to challenge, nor did they recognize that the IGE instructional management procedures could dictate how mathematical inquiry was conceived. The conceptions of work and knowledge which most teachers brought to the implementation of DMP were embedded in a management perspective of instruction where their focus was on the efficient transmission of a fixed body of subject matter to the children who comprised the classroom group.

IMPLICATIONS FOR MATHEMATICAL INSTRUCTION

This predominant pattern of instruction which was reinforced by the IGE management procedures led teachers to focus their attention on the management of the classroom group. Their attention was diverted from the processes used by individual students, and directed instead to ascertaining whether the outcomes of the work of individual students conformed to those patterns which had been prescribed for the whole group. Most teachers acted as though their students comprised a relatively homogeneous ability group. Especially in groups where children were depicted as comprising a "low" or "slow" group, they were taught as though they were deficient in mathematical knowledge and had nothing of their own to contribute to the creation and testing of mathematical knowledge. When these children were experiencing difficulty in grasping a new concept or skill, their difficulties were interpreted as a call for more intensive practice, or for the presentation of more simplified examples, rather than as a sign that an alternative approach to the content of DMP might be warranted. In all, the content of DMP was treated as a collection of crystallized forms, and presented as a series of tasks to which children needed to be introduced. If pupils were seen as comprising a "bright" group, they were expected to exercise more responsibility in completing their own work, but they were not expected to exercise choice or judgement beyond what was required by the text.

Thus, the implementation of DMP was assimilated into an existing network of beliefs, purposes, and values embedded in a management perspective of instruction. This was so even though this study was conducted in schools which were not IGE schools. However, the IGE instructional programming model clearly supports the pervasive management perspectives held by teachers in these schools. Adapted by technical changes to conform to this predominant pattern of instruction, DMP was in fact assimilated into existing patterns of mathematical instruction. Thus, DMP seemed to lose its reformative influence on the teaching of mathematics, and was adopted as a collection of ameliorative changes to existing practices.

These ameliorative changes which accompanied the implementation of DMP were noted by the researchers on the many occasions when teachers followed recommended activities, especially when children used blocks, counters, or other manipulative materials to represent the mathematical transformations of combining, separating, joining and comparing objects. Likewise, counters and other visual devices were used as recommended in order, for example, to present addition and subtraction when regrouping between tens and units was required. In these instances, it was hoped that children would be helped to develop an understanding of the mathematical concepts embodied in these concrete situations.

However, the predominant management perspective tended to re-assert itself when children were being introduced to concepts and skills which teachers saw as more demanding, or when teachers anticipated that their group would experience difficulty with a given activity, even though the same activity might not be thought difficult for a "brighter" group. That same perspective was also evident in those classes where teachers preferred to demonstrate the uses of manipulative materials for the whole class rather than have children attempt to use manipulatives themselves or in small groups. These observations were most frequent in classes which teachers described as "slow" or "low".

Whenever a management approach to instruction emerged in the teaching of DMP, its presence usually indicated that those elements of a given activity which were intended to provide a constructivist framework for children's learning had been abandoned or substantially modified. As a result, the content of the activity was treated as prescribed material to be mastered. Often, too, the concept or skill to be taught was presented as a task separated from the mathematical context which gave it meaning. This was more clearly demonstrated in the tendency to treat the Part-Part-Whole analysis as a logical entity in its own right and thus divorced from the writing of a number sentence in order to solve a story problem. Under these circumstances it was difficult to describe children as creating mathematical knowledge.

Likewise, a management approach to instruction, which was usually accompanied by patterns of teacher-directed and whole-group instruction, narrowed opportunities for children to test mathematical knowledge. Their opportunities were usually limited to the validation of answers to addition and subtraction problems. Frequently children needed to present their answers to the teacher for checking. That process extended not only to ascertaining whether children had the correct answer, but to checking whether they had used the prescribed method prescribed for them.

In order to exemplify more specifically the creation and testing of mathematical knowledge within the teaching of DMP, one may refer to Romberg (1983) where four related activities are presented which are special to mathematical inquiry: abstracting, inventing, proving and applying. A management approach to the teaching of DMP limited the opportunities for children to engage in these four activities; and, furthermore, that it imposed a more restrictive and limited definition on those activities.

ABSTRACTING

The fundamental processes of DMP--describing, classifying, comparing, ordering, joining, separating, and grouping--are all instances of mathematical abstraction. Many of the activities of DMP required children to use manipulative materials to represent and validate mathematical transformations in which these processes have been embodied. Teachers reported that, in general, they were comfortable with those activities which incorporated the use of manipulatives. Likewise, they said that they had implemented many measurement activities, including introductions to geometry and fractions, much as the DMP booklets had recommended. Although there were occasions when teachers did tend to limit the scope and variety of measurement activities, it could be argued that, even where a management approach to instruction had intruded into the teaching of DMP, abstracting remained a strong feature of the implementation of DMP.

For example, in the teaching of Part-Part-Whole, children were able to see that the abstract arithmetical operations of addition and subtraction applied with remarkable generality to a wide range of story problems. However, it should be noted that children were often taught to rely exclusively on the Part-Part-Whole analysis in order to abstract a mathematical transformation from the semantic structure of a story problem. This almost total reliance upon one method of analysis limited children's experience in using a variety of approaches for penetrating the semantic structure of story problems. Wtih the exception of one teacher, children were not encouraged to explore different approaches in analyzing the structure of story problems, where abstracting becomes intertwined with inventing, providing, and applying.

It is possible to view abstracting from a psychological perspective as a process in which an individual engages. However, mathematical inquiry cannot be identified with a psychological process of abstracting. It is indeed a necessary condition of mathematical inquiry that one engages in abstracting. What makes that inquiry mathematical are the kinds of abstractions--concepts and skills--and the norms and standards by which abstraction is regulated. Likewise, one must avoid treating inventing, proving, and applying as psychological processes if one wishes to use these activities to identify mathematical inquiry. For that purpose, these activities need to be seen as public mathematical performances. As mathematical performances these activities need to be seen in a context of agreed rules as to what constitutes a successful attempt at inventing, proving, and applying. In its relation to these three performances, a pervasive management approach to instruction not only presents a restricted range of experiences, as in the case of abstracting, but it tends to redefine what constitutes inventing, proving, and applying in radically different ways.

INVENTING

Romberg (1983) defines inventing as creating a law or a relationship. In a similar spirit, the authors of DMP hoped

> That exposing children to a wide variety of problems will lead to a willingness to tackle new problems, confidence in their ability to handle new problems, and the ability to apply problem solving techniques. (Romberg, Harvey, Moser & Montgomery, 1975, p. 50)

Romberg (1983) claims that inventing may be seen as "discovering relationships which lead to abstractions, theorems, models, and so forth, known to the mathematical community but not to the student". This sense of "inventing" appears especially relevant to school mathematics. However, the emphasis placed by most teachers on standardized and fixed procedures had an immediate impact on children's experience of inventing. This was most clearly illustrated in the teaching of Part-Part-Whole. When that one method of analyzing and solving story problems was presented as the only method which children could use, there was little inventiveness to be seen in pupils application of a predetermined pattern of analysis in order to establish a mathematical relationship between "parts" and "whole".

The authors of DMP had hoped that children, after becoming confident in a part-part-whole analysis, would indeed be helped to invent alternative approaches to the solution. However, when children were not introduced to any other method, their task was to become competent users of that single pattern of analysis. Thus, a management approach to instruction has imposed such limitations on what children learned, on how they were to learn, and how their learning was evaluated, that "inventing" in any strong sense was inhibited.

Moreover, the activity of inventing cannot b divorced from having one's invention or discovery recognized as such. But in the predominant pattern of instruction, the individual child was often separated from discussion with other students and submerged in a group process, or required to carry out tasks specifically assigned to that student by the teacher.

Because students had so few opportunities for mutual collaboration, variations in methods of problem solving were unlikely to be fostered. Indeed, such variations as might have led to inventions were likely to be seen as deviations from a pattern of response which the class had been taught to follow. It was not surprising, therefore, that teachers reported that children did not use alternative strategies in solving problems; or when teachers reported that their children did use alternative strategies in solving word problems, they added that the children were usually wrong.

PROVING

Many instances were observed of children proving relationships between numbers, but all too often these instances were confined to validating an answer to a number problem. Validation or checking was often performed using counters or blocks to represent a mathematical transformation. In Grade 3, children were encouraged to validate answers to subtraction problems, for example, by adding the answer and the number which had been subtracted (the subtrahend). But these instances of proving were only a tiny element of what the DMP authors aspired to when they urged that "Children need to be left alone at times with objects or pictures or pencil and paper to try their own methods" (Romberg, Harvey, Moser & Montgomery, 1975, p. 50).

Leaving children on their own, or guiding them to attempt their own methods do not constitute proving. But these activities are a seedbed upon which experiences in proving can be developed. Yet these activities tended to be avoided by teachers, especially by those who saw their group as "low" or "slow". Only one teacher showed clear signs of developing a sense of proving when she asked her children to estimate whether the solution to a story problem was likely to be greater or less than the number given; to record their prediction; and then to prove whether their prediction was correct by using whatever method worked best for them. Here, a context of agreed rules established which allowed pupils to ascertain whether their estimate was correct. Unlike the single predetermined pattern of analysis as used in other classrooms, this context of agreed rules did allow children to explore their own methods and to recognize when their methods were correct.

In this way, children were able to develop a sense that they were their own authorities in dealing with mathematical ideas. Opportunities such as this, to develop a sense of personal responsibility and control over mathematical knowledge, were not observed in other classes.

APPLYING

The notion of applying is so pervasive throughout DMP that one might think that this feature of mathematical inquiry was least affected by a management approach to instruction. Did not children apply mathematical techniques to the solution of story problems? Did they not make use of measurement activities in order to represent and validate mathematical transformations? Did they not apply mathematics to their investigations of space and shape?

Yet if one asks who did the applying and under what circumstances were these applications made, one can see how the predominant pattern of instruction might redefine the very idea of applying mathematics. One was forced to ask whether pupils were applying mathematics when they watched their teacher demonstrate applications for the whole group. One also had to ask what kind of applications were being made, when the range and variety of measurements was reduced in order to keep instruction more efficient and orderly, or when imprecision in children's measurements was deliberately precluded for the same reasons. The effect of this latter kind of technical change was not merely to make instruction more orderly and efficient, it also altered the nature of the applications which children performed. For example, by providing objects whose length could be measured exactly so that children were not confronted by imprecision in their measurements, two fundamental elements of the process of measurement had been set aside. These were the possibility of using a systematic procedure of measuring to the nearest unit, and of establishing that if more exact measurement is desired, smaller units need to be used (cf. Romberg, Harvey, Moser & Montgomery, 1975, p. 34).

Mathematical applications are made in a physical world where there is variety, ambiguity and imprecision. For that reason, one's mathematical models always embody some simplifications of the physical data, but those very simplifications are warranted in order that one's model can be understood and can work. But to simplify one's data in advance in order to make teaching more efficient and orderly is to simplify data for the wrong reason and to confuse children about the nature of the physical data. If they are to apply mathematics, children need to know that the data are not always precise, and that one has to come up with acceptable procedures for handling imprecision; for example, by agreeing to measure to the nearest unit, or to accept as equally correct the two units which are on either side of the measurement.

SUMMARY

This paper has presented mathematical inquiry as an intellectual craft which takes place in a context of rules, some agreed upon and some still evolving, which are maintained and developed by a community of mathematicians. While it is true that a classroom group does not constitute a community of mathematicians, it is our contention that the same features of intellectual craftmanship and debate which characterize the mathematical community, should be exemplified in the teaching and learning of mathematics in schools.

This vision of mathematics was endorsed by the authors of DMP, and it was realized in some of the classrooms where DMP was implemented. However, whenever a management perspective of instruction predominated, mathematical knowledge became extrinsic to students and mathematical inquiry became crystallized: opportunities for children to participate in the development of agreed rules for mathematical inquiry were severely curtailed; their experiences of abstracting were often limited by the priorities and procedures for efficient classroom management. Severe limitations also applied to the opportunities available to children for inventing, proving and applying mathematical relationships. But, more importantly, children's experiences of these mathematical activities were redefined by a management approach to instruction.

To reinterpret Bossert (1979), the very patterns of task organization within the classroom have profoundly changed what it means to know and do mathematics.

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