DOCUMENT RESUME

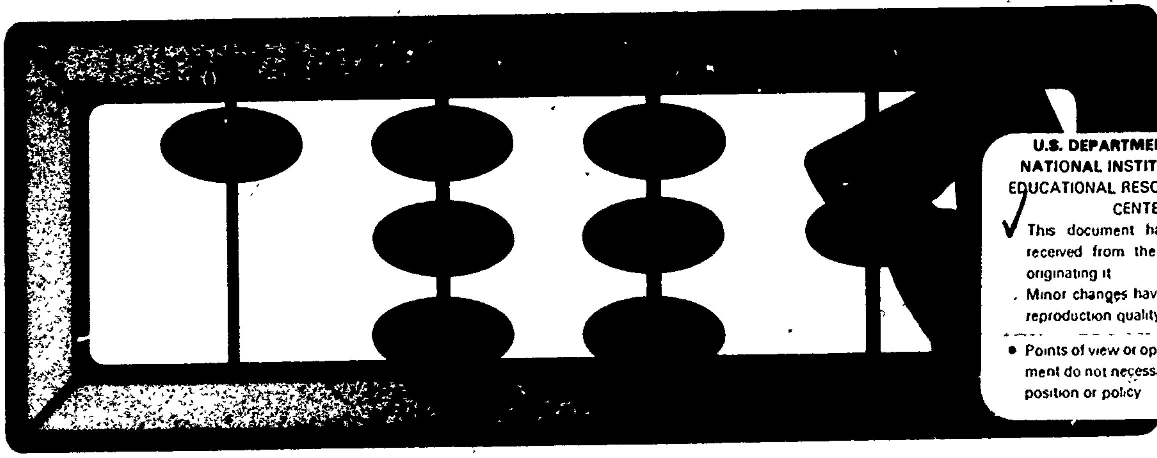
ED 228 049 SE 040 925

TITLE The Third National Mathematics Assessment: Results, Trends and Issues. [1981-82 Assessment] Advance Copy.
INSTITUTION Education Commission of the States, Denver, Colo. National Assessment of Educational Progress.
SPONS AGENCY National Inst. of Education (ED), Washington, DC.
REPORT NO ISBN-0-89398-149-4; NAEP-13-MA-01
PUB DATE Apr 83
GRANT NIE-G-80-0003
NOTE 77p.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC04 Plus Postage.
DESCRIPTORS Computation; *Educational Assessment; Educational Research; Educational Technology; Educational Trends; *Elementary School Mathematics; Elementary Secondary Education; *Mathematics Achievement; Mathematics Curriculum; *Mathematics Instruction; Minority Groups; Problem Solving; *Secondary School Mathematics; Sex Differences; Testing
IDENTIFIERS Mathematics Education Research; *National Assessment of Educational Progress; *Third Mathematics Assessment (1982)

ABSTRACT

This report describes and interprets findings from the 1981-82 national mathematics assessment, the third such assessment conducted by the National Assessment of Educational Progress. The results show a leveling off of the performance of American 17-year-olds, who had shown a decline between 1978 and 1982; 9-year-olds' performance has changed little from assessment to assessment. These findings are described and interpreted by a panel of mathematics educators. Besides discussing the overall results, the authors examine findings in the following categories: knowledge, skills and concepts; problem solving, applications and attitudes toward mathematics; computers and technology; minorities and mathematics; and sex differences in achievement. Findings are presented for different kinds of test items--those assessing knowledge, skills, understanding, and application--and for different subpopulations: Blacks; Hispanics; students in heavily minority schools, students in different achievement categories, males, females and students in advantaged-urban, disadvantaged-urban and rural schools. The report contains an executive summary and, in the appendixes, some discussion of other test results and other information about mathematics education, mathematics course taking, and steps being taken across the country to improve science and mathematics education. (Author)



The Third National Mathematics Assessment: RESULTS, TRENDS and ISSUES

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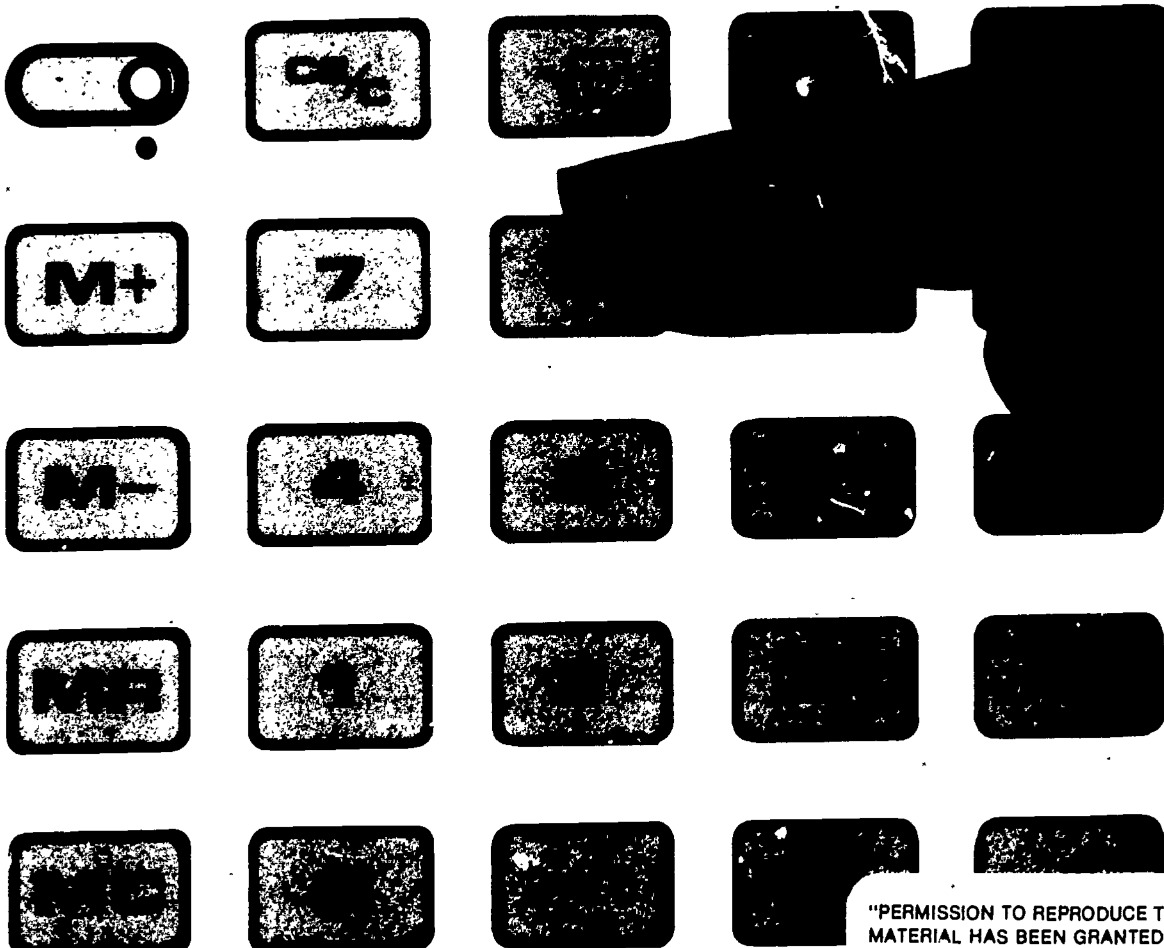

National Assessment of Educational Progress

The Third National Mathematics Assessment: Results, Trends and Issues

Report No. 13-MA-01

by the
National Assessment of Educational Progress

Education Commission of the States
Suite 700, 1860 Lincoln Street
Denver, Colorado 80295

April 1983

The National Assessment of Educational Progress is funded by the National Institute of Education, under a grant to the Education Commission of the States. It is the policy of the Education Commission of the States to take affirmative action to prevent discrimination in its policies, programs and employment practices.

Library of Congress
Catalog Card Number: 72-183665

Although a few early National Assessment reports have individual catalog card numbers, all recent reports have been assigned the above series number.

ISBN 0-89398-149-4

The National Assessment of Educational Progress is an education research project mandated by Congress to collect and report data, over time, on the performance of young Americans in various learning areas. National Assessment makes available information on assessment procedures and materials to state and local education agencies and others.

The work upon which this publication is based was performed pursuant to Grant No. NIE-G-80-0003 of the National Institute of Education. It does not, however, necessarily reflect the views of those agencies.

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Foreword

When the U.S. Office of Education was chartered in 1867, one charge to its commissioners was to determine the nation's progress in education. The National Assessment of Educational Progress (NAEP) was initiated a century later to address, in a systematic way, that charge.

Since 1969, NAEP has gathered information about levels of educational achievement across the country and reported its findings to the nation. It has surveyed the attainments of 9-, 13- and 17-year-olds and sometimes adults in art, career and occupational development, citizenship, literature, mathematics, music, reading, science, social studies and writing. All areas except career and occupational development have been periodically reassessed to detect any important changes. To date, National Assessment has interviewed and tested more than 1,000,000 young Americans.

Learning-area assessments evolve from a consensus process. Each assessment is the product of several years of work by a great many educators, scholars and lay persons from all over the nation. Initially, these people design objectives for each subject area, proposing general goals they feel Americans should be achieving in the course of their education. After careful reviews, these objectives are given to exercise (item) writers, whose task it is to create instruments that appropriately measure performance on the objectives.

When the exercises have passed extensive reviews by subject-matter specialists, measurement experts and lay persons, they are administered to probability samples. The people who compose these samples are chosen in such a way that the results of their assessment can be generalized to an entire national population. That is, on the basis of the performance of about 2,000 9-year-olds on a given exercise, we can make generalizations about the probable performance of all 9-year-olds in the nation.

After assessment data have been collected, scored and analyzed, the National Assessment publishes reports and disseminates the results as widely as possible. Some of the exercises used in each assessment are published and made available to anyone interested in studying or using them. The rest are kept secure so they can be used in future assessments.

Acknowledgments

Many organizations and individuals have made substantial contributions to the three mathematics assessments. Not the least of those to be gratefully acknowledged are the administrators, teachers and students who cooperated so generously during the collection of the data.

Special acknowledgment must go to the many mathematics educators and specialists who provided their expertise in the development, review and selection of the assessment objectives and exercises. Development of the mathematics assessment was coordinated by Jane Armstrong.

Administration of the mathematics assessment was conducted by the Research Triangle Institute, Raleigh, North Carolina. Scoring and processing were carried out by Westinghouse Information Services, Iowa City, Iowa, and the National Assessment staff of the Education Commission of the States.

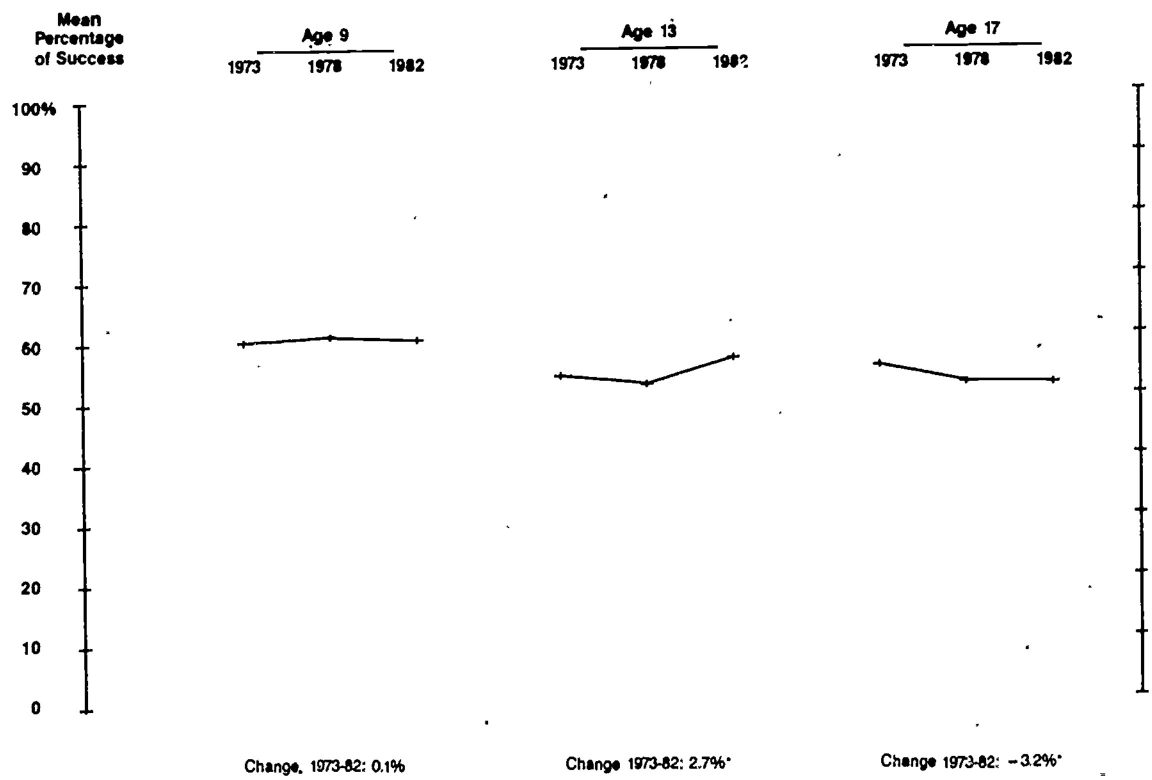
The actual preparation of this report was a collaborative effort of the National Assessment staff and members of the National Council of Teachers of Mathematics (NCTM) Task Force for Interpretation of National Assessments. Special thanks must go to the following National Assessment and Education Commission people: Valerie Thomas, for helping reduce and organize the data; Nancy Murphy and Jane Armstrong, for collecting information about the policy context of the assessment; Judy Bray and Judi Worker, for helping prepare the manuscript; Gwen Edwards, for data processing support; Pamela Thayer, for technical support; Marci Reser, for production of the report; Don Phillips, for statistical analysis and support throughout the writing of the report; and Rexford Brown, for writing and editorial direction. Special thanks, too, to the NCTM Task Force responsible for writing Chapters 1-6 and interpreting the results in the context of mathematics education: Thomas Carpenter, professor of curriculum and instruction, University of Wisconsin; Mary Lindquist, professor of mathematics, National College of Education; Westina Matthews, staff associate, Chicago Community Trust; and Edward Silver, associate professor of mathematics, San Diego State University.

Beverly Anderson
Director

Graphic Highlights and Executive Summary

Exhibit 1 displays performance trends based on exercises used in all three mathematics assessments. Nine-year-olds' overall performance was relatively stable over the nine years, neither declining nor improving significantly. Thirteen-year-olds' performance declined about 2 percentage points between the first two assessments and then improved more than 4 points between the second and third. Seventeen-year-olds' performance declined about 4 percentage points between the first and second assessments, and then stayed at about the same level between the second and third (Tables 1, 2 and 3).

Exhibit 1.
Changes in Mean Percentages of Success on 1973, 1978 and 1982 Mathematics Assessments, Ages 9, 13, 17.†



* Change is significant at the .05 level.

† Exact percentages appear in Table 1. Table 2 presents mean changes between 1973 and 1978 on a larger cluster of items, and Table 3 presents mean changes between 1978 and 1982 on a larger cluster of items. In both cases, the direction and magnitude of the changes parallel those displayed in this exhibit.

Table 1.
Percentages of Success on Common Mathematics Exercises, 1973, 1978, 1982

Age	Number of Items	Percentage of Success 1973	1978	1982	Change 1973 to 1982
9	23	39.8%	39.1%	39.9%	0.1%
13	43	53.7	52.2	56.4	2.7*
17	61	55.0	52.1	51.8	−3.2*

* Change is significant at the .05 level (see Introduction).

Table 2.
Percentages of Success on Common Mathematics Exercises, 1973 to 1978

Age	Number of Items	Percentage of Success 1973	1978	Change 1973 to 1978
9	55	38.1%	36.8%	−1.3%
13	77	52.6	50.6	−2.0*
17	102	51.7	48.1	−3.6*

* Change is significant at the .05 level.

Table 3.
Percentages of Success on Common Mathematics Exercises, 1978 to 1982

Age	Number of Items	Percentage of Success# 1978	1982	Change 1978 to 1982
9	233	55.4%	56.4%	−1.0%
13	383	56.7	60.5	3.8*
17	383	60.3	60.2	−0.1

* Change is significant at the .05 level.

Percentages in Tables 1, 2 and 3 differ because each is based upon a different set of exercises.

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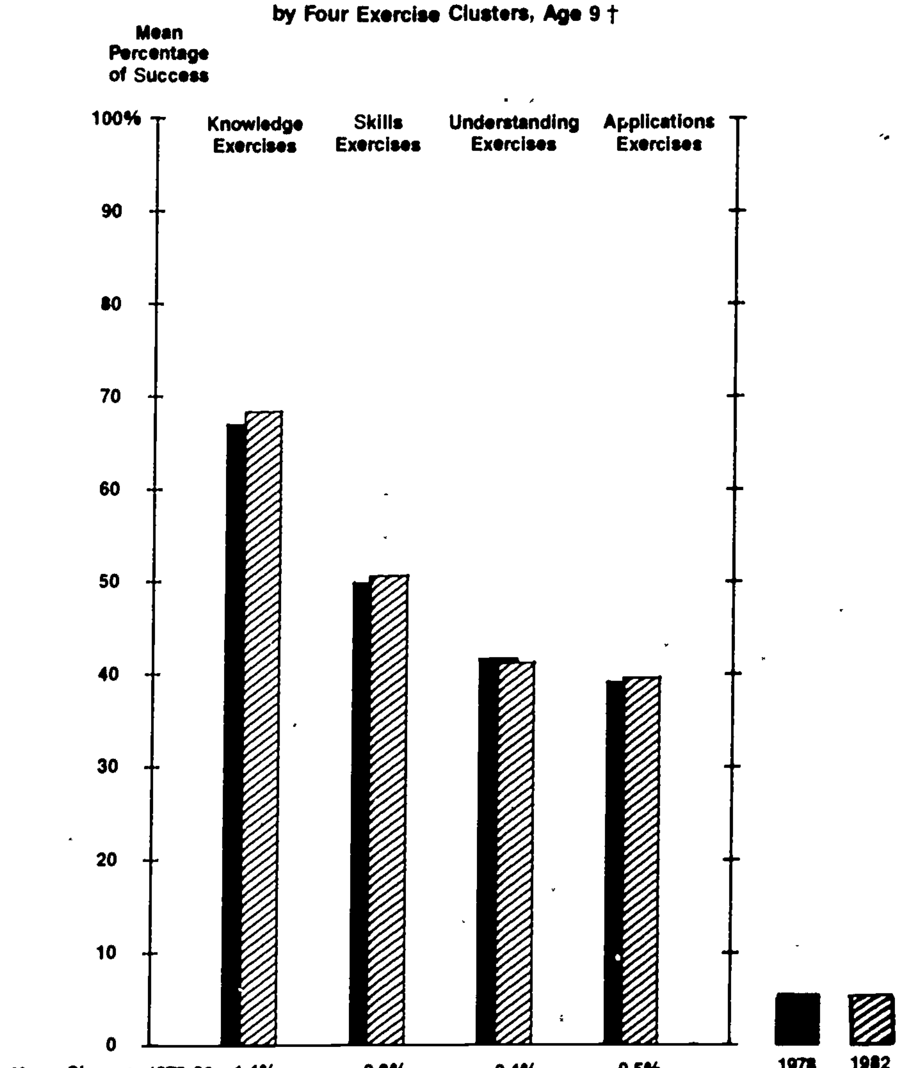
Exhibits 2-4 display changes in performance between 1978 and 1982 on four types of exercises: those assessing mathematical knowledge, skills and understanding, and those assessing ability to apply mathematics.

At ages 9 and 17, there were no significant average gains or losses on any of these exercise clusters.

Thirteen-year-olds improved significantly in all four areas. However, they improved most on the knowledge, skills and understanding exercises, and least on the applications exercises. Further study shows that their improvements in understanding came on exercises judged relatively easy by a panel of mathematics educators; performance levels on exercises calling for *deeper* understanding showed little or no improvement.

Exhibit 2.

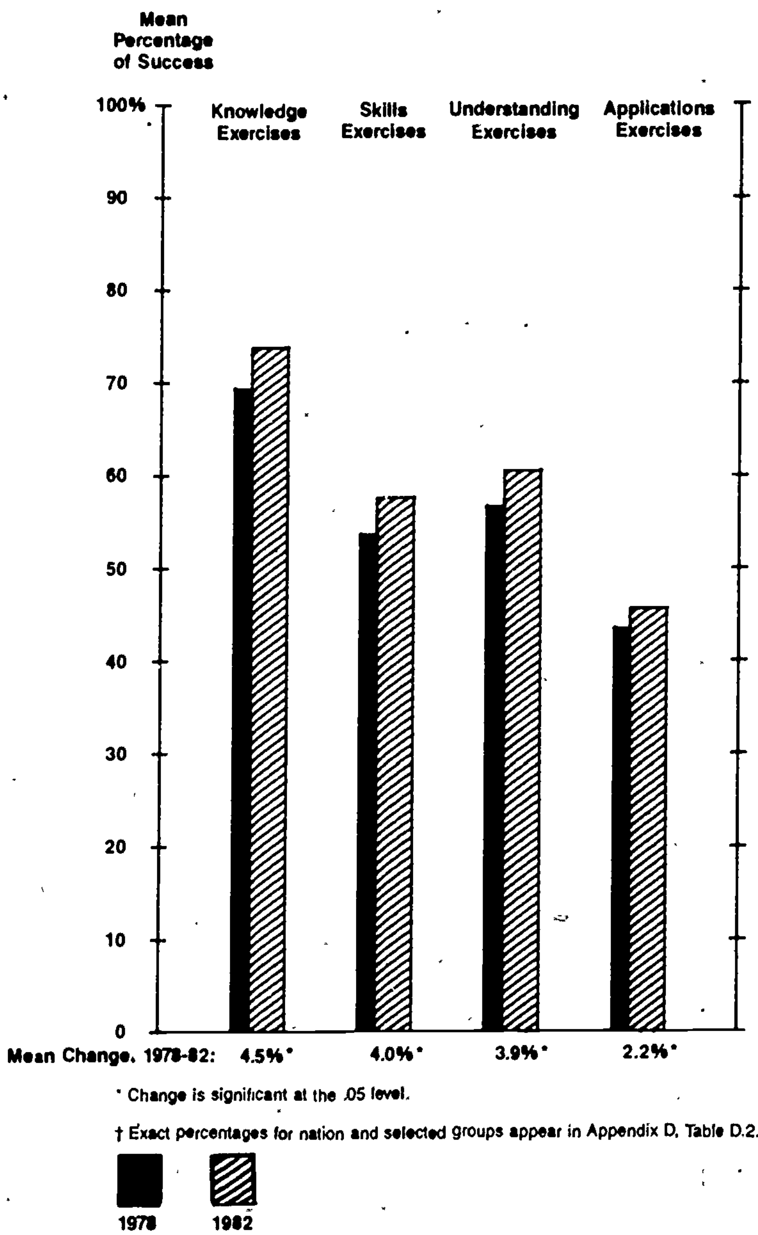
Mean Changes in Percentages of Success Between 1978 and 1982 Mathematics Assessments by Four Exercise Clusters, Age 9 †



† Exact percentages for nation and selected groups appear in Appendix D, Table D.1.

Exhibit 3.

Mean Changes in Percentages of Success Between 1978 and 1982 Mathematics Assessments by Four Exercise Clusters, Age 13 †



* Change is significant at the .05 level.

† Exact percentages for nation and selected groups appear in Appendix D, Table D.2.

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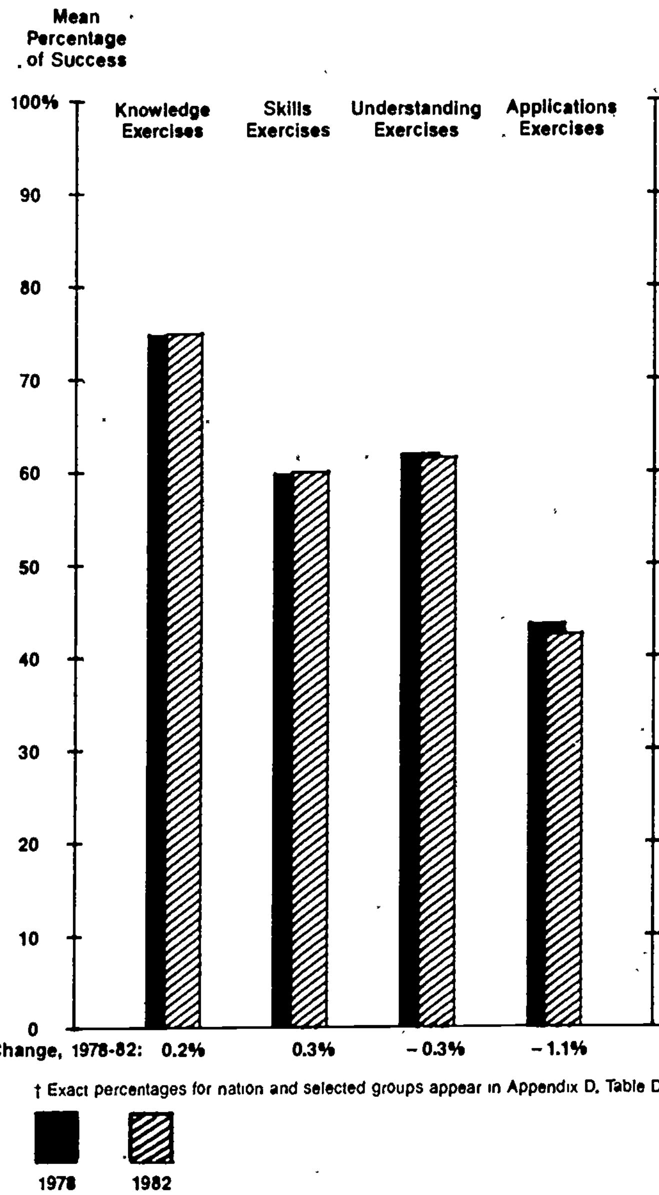
Exhibit 4.

Mean Changes in Percentages of Success Between 1978 and 1982 Mathematics Assessments by Four Exercise Clusters, Age 17 †



Mean Change, 1978-82: 0.2% 0.3% −0.3% −1.1%

† Exact percentages for nation and selected groups appear in Appendix D, Table D.3.

Exhibits 5-7 display changes in performance between 1978 and 1982 for White, Black and Hispanic students; for each achievement quartile (25%) of students (the lowest quartile = the lowest performing 25% of the students, the highest quartile = the highest performing 25%); for students in schools with large minority enrollments and schools that enroll mostly White students; and for students attending schools in rural, disadvantaged-urban and advantaged-urban communities.

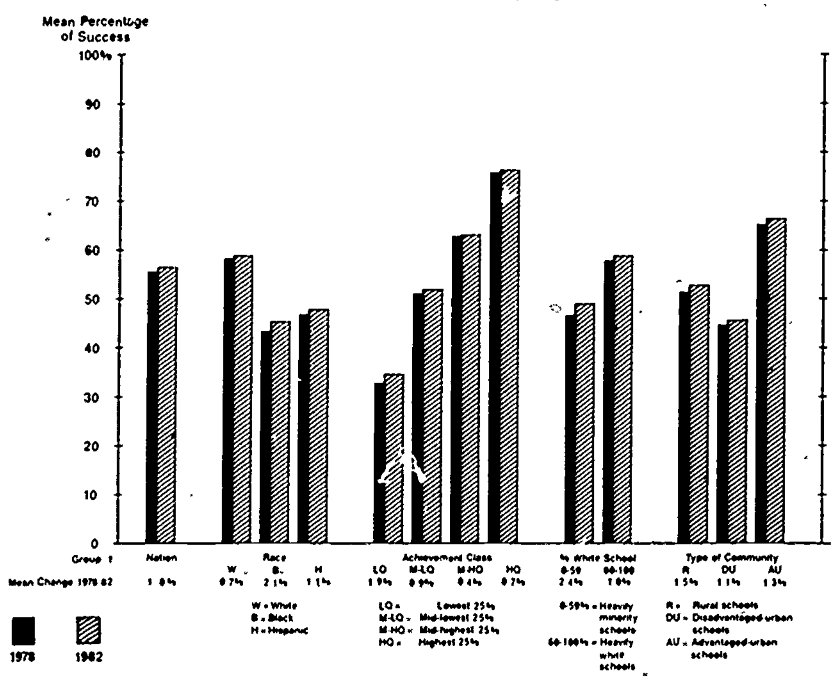
At age 9, none of these groups showed a significant change in average performance on all 233 exercises used to measure change in the assessment. However, Black students and students in the lowest performance quartile improved on exercises assessing mathematical knowledge (Appendix D, Tables D.1, D.4).

At age 13, all of these groups showed significant gains in average performance on 383 exercises. Students in largely minority schools showed an ov"rall gain that was twice as large as the national gain, while Black and Hispanic students and students attending both disadvantaged- and advantaged-urban schools all gained close to 6 percentage points, compared to a national gain of about 4 points. Students attending disadvantaged-urban schools registered larger-than-national gains on exercises assessing skills, understanding and application of mathematics (Appendix D, Tables D.2, D.4).

At age 17, the only group registering a significant gain in average performance on 383 exercises was the students attending schools with large minority enrollments. That group improved 5 percentage points, while the national population of 17-year-olds made no gain at all. Appendix D, Table D.3 reveals that the gains made by students in the minority schools were on exercises assessing knowledge, skills and applications.

In general, the mathematics educators who reviewed these results were heartened by the

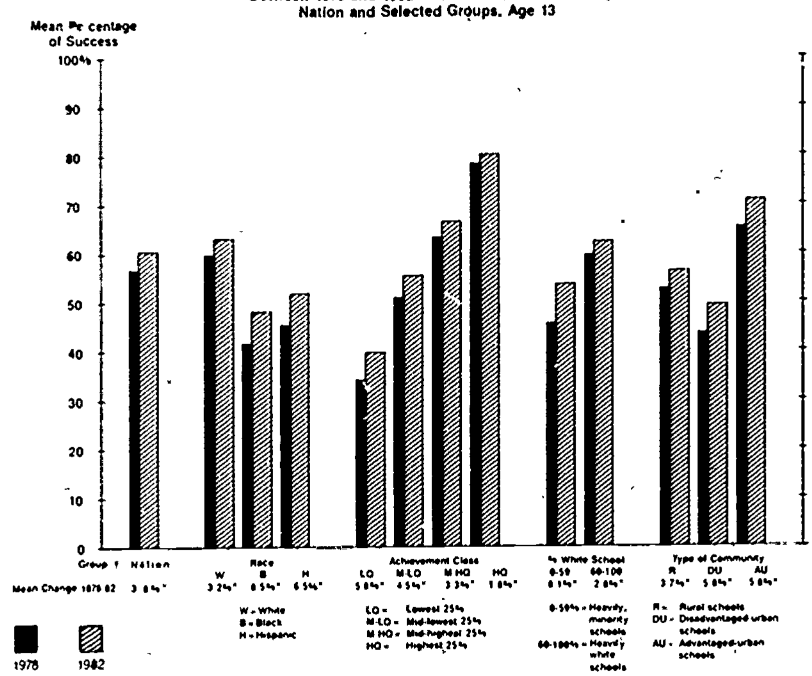
Exhibit 5.
Mean Changes in Percentages of Success Between 1978 and 1982 Mathematics Assessments, Nation and Selected Groups, Age 9



† See Appendix C for definitions of groups and Appendix D for exact percentages of success.

Exhibit 6.

Mean Changes in Percentages of Success Between 1978 and 1982 Mathematics Assessments, Nation and Selected Groups, Age 13

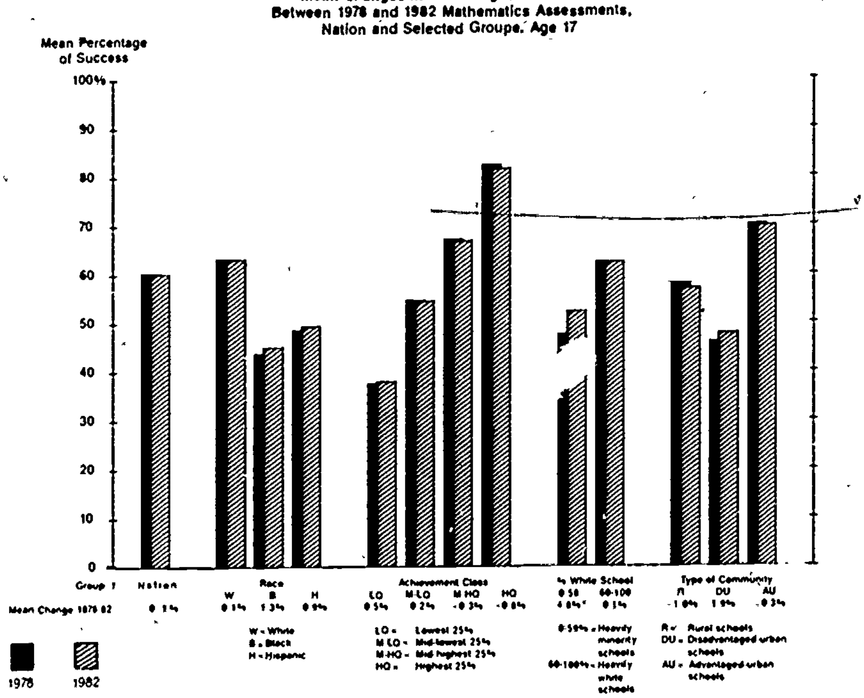


* Change is significant at the .05 level

† See Appendix C for definitions of groups and Appendix D for exact percentages of success

Exhibit 7.

Mean Changes in Percentages of Success Between 1978 and 1982 Mathematics Assessments, Nation and Selected Groups, Age 17



* Change is significant at the .05 level

† See Appendix C for definitions of groups and Appendix D for exact percentages of success

fact that 17-year-olds' performance has stopped declining and 13-year-olds' performance has improved dramatically since 1978. However, they point out in this report that improvements have been largely in the knowledge, skills and understanding exercises assessing things most easily taught and learned by rote. They express concern that performance on nonroutine problems and on problem solving in general continues to be unacceptably low.

Methodological Introduction

General Background

Mathematics performance of 9-, 13- and 17-year-olds in school has been nationally assessed three times: in the 1973 school year, in 1978 and in 1982. For the sake of convenience, each assessment will be referred to by the last half of the school year in which it took place—1973, 1978 and 1982. Each assessment contained a broad range of exercises (items) measuring performance in relation to sets of national objectives for mathematics education developed by many panels of educators, scholars and parents from across the country (NAEP, 1981). Although changes were made from assessment to assessment in order to broaden or shift coverage of particular topics, a small set of exercises has been kept constant in order to assess change from 1973 to 1982. Changes over the decade can also be appraised by examining changes in the broader item pools that linked the first to the second assessment and the second to the third. In this report, the emphasis is upon examining changes from the second to the third assessment, since those data are the most recent.

Content of the Mathematics Assessments

The 1982 mathematics assessment was designed to measure students' attitudes toward mathematics and their abilities in various content areas. The major content areas assessed were: numbers and numeration; variables and relationships; geometry (size, shape and position); measurement; probability and statistics; graphs and tables; and technology (including the use of calculators and computers). The development principles and content coverage of the mathematics assessments are more fully described in other NAEP publications (NAEP, 1980) In addition to their content coverage, the items spanned four cognitive process levels (Bloom, 1956).

The lowest cognitive process level—**knowledge**—primarily involves recall of facts and definitions, and knowledge of number order; place value; basic facts of addition, subtraction, multiplication and division; geometric figures; and measurement units.

Skills exercises involved the ability to use specific algorithms (procedures for adding, subtracting, etc.)—and manipulate mathematical symbols. Skills exercises assess computing with whole numbers, fractions, decimals, percents and integers; taking measurements; converting measurement units; reading graphs and tables; and manipulating geometric figures and algebraic expressions.

Understanding implies a higher level of cognitive process than simply recalling facts or using algorithms. Exercises assessing mathematical understanding require students to explain or illustrate different mathematical knowledges or skills, requiring a transformation of knowledge but not the application of that knowledge to solve a problem.

Application exercises require the use of mathematical knowledge, skills and/or understandings—typically in problem-solving activities.

This rough hierarchy is used to construct an assessment that spans all four cognitive processes, rather than only one or two. In principle, applications exercises are the most demanding because they require understanding and a mastery of requisite skills and knowledge. However, *within* each cognitive level there can also be both easy and difficult exercises. Some exercises assessing understanding, for example, require only an elementary understanding; others require a more comprehensive grasp of mathematical concepts. Consequently, one can discuss lower- or higher-order cognitive processes (knowledge and skills being lower, understanding and applications being higher), and one can also look at easier or more difficult tasks within any of the four clusters of items.

A final note on the contents of the assessment: most of the items used to assess 17-year-olds involve material typically learned in 8th, 9th and early 10th grade mathematics courses. Very few exercises depend upon material taught in Algebra 2 and trigonometry, and none require knowledge of calculus or other mathematics courses taught in grades 11 and 12. This is so because the National Assessment uses a probability sample of students, many of whom are no longer taking mathematics courses, and it would be both unfair and inefficient to ask them questions about subjects they have never studied.

Methodological Overview

To measure changes in achievement from one assessment to the next, testing conditions must be as nearly the same as possible in each assessment. National Assessment makes every effort to hold conditions constant from one assessment to the next by using a specially trained staff to administer each assessment. Items used to measure change are identical in format, wording and time allowed for response. Comparable samples of 9-, 13- and 17-year-old school students are drawn for each assessment year. Thirteen-year-olds are assessed in the fall of the school year, 9-year-olds in the winter and 17-year-olds in the spring.

Identical item sets—233 items at age 9 and 383 items at ages 13 and 17—were used to measure changes in achievement from the 1978 assessment. (In spite of identical numbers of items, the sets for ages 13 and 17 are *not* the same.) Some items—23 at age 9, 43 at age 13 and 61 at age 17—were used in all three mathematics assessments.

In the 1973 and 1978 assessments, approximately 2,100 to 2,500 students responded to each item. For the 1982 assessment, approximately 1,900 to 2,100 students responded to an item. The total number of individuals assessed in each assessment appears in Table 4.

Because of time restraints and respondent fatigue, each respondent answers only a small proportion of all the items assessed at an age. Each student takes only one booklet of exercises designed to be administered during a 45-minute class period.

Table 4.
Total Number of Students in Mathematics Assessments, 1973, 1978 and 1982

Age	1973	1978	1982
9	18,638	14,752	13,947
13	23,507	26,661	15,758
17*	25,865	26,757	16,319

* In 1973, 17-year-olds not enrolled in school were also sampled. In subsequent assessments, only enrolled students were assessed. The figures in the table are those for only the in-school portion of the 1973 age 17 sample.

Scoring

Item scoring must remain consistent across assessment years. About half of the exercises used to measure change from the second to the third assessment were in a multiple-choice format. These were scored by an optical scanning machine using the same keys for both years. The remaining half of the items were open-ended—i.e., students had to supply the correct answer. Responses to these were scored by trained scorers using the same guides in each assessment.

Measures of Achievement

The National Assessment's basic measure of achievement is the percentage of students responding acceptably to a given item. This percentage is a statistically adjusted estimate of the percentage of 9-, 13- or 17-year-olds who would respond acceptably to that item if every 9-, 13- or 17-year-old in the country were assessed.

In addition to providing results on individual items, National Assessment reports the average performance (mean) across groups of similar items for the learning area as a whole, or for particular objectives or subobjectives. This arithmetic average, the estimate of performance on a group of items, is called the mean percentage acceptable, or the mean performance level. The exercises used to calculate a mean percentage are usually located in several exercise booklets, and the same students do not take them all. Thus, the mean percentage should not be construed as an average test score.

To present a general picture of changes in achievement, NAEP reports describe the gains and losses on a group of exercises in terms of the differences in the average percentages of acceptable responses.

Unless the exercises summarized in the mean percentages of acceptable responses are identical, *the means of one age group should not be compared with the means of another* since their values reflect both the choice of exercises and the performance of the students. When only a few exercises are summarized by a mean, one should be especially cautious in interpreting results, since a small set of exercises might not adequately cover the wide range of potential behaviors included under a given objective or subobjective.

In addition to providing national results, National Assessment reports the achievement of various subpopulations of interest. Groups are defined by region of the country, sex, race/ethnicity, size of community lived in, type of community lived in, grade and level of parents' education. Definitions of the groups discussed in this report appear in Appendix C.

Estimating Variability in Achievement Measures

National Assessment uses a national probability sample at each age level to estimate the proportion of people who would complete an exercise in a certain way. The particular sample selected is one of a large number of all possible samples of the same size that could have been selected with the same sample design. Since an achievement measure computed from each of the possible samples would differ from one sample to another, the standard error of this statistic is used as a measure of the sampling variability among achievement measures from all possible samples.

The standard error provides an estimate of sampling reliability for NAEP achievement measures. It is composed of sampling error and other random error associated with the assessment of a specific item or set of items. Random error includes all possible nonsystematic error associated with administering specific exercises to specific students in specific situations. For open-ended items, random differences among scorers are also reflected in the standard errors.

Differences in performance between assessment years or between a reporting group and the nation are called "significant" and are highlighted with asterisks *only* if they are at least twice as large as their standard error. Differences this large would occur by chance in fewer than 5% of all possible replications of the sampling and data collection procedures for any particular reporting group or national estimate. Thus, they are called "significant at the .05 level," conforming with traditional statistical conventions.

Further and more detailed information about the mathematics assessments will appear in *Procedural Handbook, 1981-82: Mathematics Assessment*, available from the National Assessment of Educational Progress.

A Note About Interpretations and Value Judgments

Many things can cause apparent changes in student achievement levels besides such obvious factors as better (or worse) instruction, curriculum changes or harder work by students. Population shifts, for instance, can skew results; so can large-scale social changes having little or nothing to do with the schools. The National Assessment attempts to control or take into account any measurement or sampling phenomena that could cause apparent, but false, declines or improvements. But no one can control all possible variables; it is always conceivable that a particular change can be explained in many ways or cannot be explained at all. Interpreting the results—attempting to put them into a "real world" context and advance plausible explanations of their causes—will always be an art, not a science. All interpretive remarks in NAEP reports represent professional judgments that must stand the tests of reason and the reader's knowledge and experience. The conjectures of professionals are not always correct, but they *are* always ways of stimulating discussion that may ultimately lead to correct understanding of the results and appropriate action.

The interpretive remarks and value judgments in this report represent the best judgments of the National Council of Teachers of Mathematics Task Force on Interpreting National Assessments. The members of the task force—Thomas Carpenter, Mary Lindquist, Westina Matthews and Edward Silver—are distinguished mathematics educators who were asked to place the findings in the context of contemporary issues in mathematics education. Their views are their own and do not necessarily reflect the views of the National Assessment of Educational Progress, the Education Commission of the States, the National Council of Teachers of Mathematics or the National Institute of Education. Every effort has been made to present the NAEP findings objectively; interpretive judgments, however, are always challengeable, and readers are encouraged to form their own opinions about the meaning and implications of the findings.

Chapter 1

Synthesis of Findings and Discussion of Their Potential Implications

These are turbulent times for mathematics education in the United States. In general, there is renewed interest in mathematics as a school subject. There is evidence that college-bound students are voluntarily taking more mathematics courses in high school (CEEB, 1982), and many states are increasing the amount of mathematics coursework required for graduating from high school (Pipho, 1983). In some states (e.g., California), universities have recently instituted stricter entrance requirements with respect to the number of mathematics courses entering students must take in high school.

The reason most often advanced for this renewed attention is that mathematics is central to the physical, biological and information sciences, and these sciences are keys to America's economic revitalization. Whether employed as engineers, physicists, computer designers and programmers or information theorists, mathematically trained people will be making critical contributions to the country's growth over the next several decades.

In addition, it is clear that in our increasingly computerized society, all students—not just those planning careers in the sciences—need a stronger foundation in mathematics. Mathematical literacy, computer literacy, a general understanding of the language of mathematics—these are becoming prerequisites for informed citizenship in a high technology, information-oriented society. Those who do not understand the principles by which information is stored, analyzed and retrieved will have less power over their lives and less say in social decisions than those who have the requisite knowledge.

In spite of this need, however, recent surveys suggest that there is a serious shortage of qualified mathematics teachers. In 1981, 43 of 45 surveyed states indicated a shortage or a critical shortage of secondary mathematics teachers (Howe and Gerlovich, 1981). According to the National Science Teachers Association, 50% of the teachers newly employed by high schools to teach mathematics or science in 1981-82 were unqualified or were teaching with emergency credentials (NSTA, 1982).

Making mathematics courses available to more students, reexamining and modifying the curriculum in light of currently available technology and alleviating the shortage of trained mathematics teachers all require an increased national commitment to mathematics education. Yet these needs have arisen at a time of diminishing resources for education. Many states have reduced their educational expenditures over the past few years, and most others have been barely able to maintain current levels of funding.

Major Findings

It is against this backdrop (detailed more fully in Appendixes A and B) that the results of the third mathematics assessment must be interpreted. Table 1.1 presents a summary of the mean performance levels on items that were administered in all three assessments. Table 1.2 summarizes the mean performance levels on all items administered in the two most recent assessments. The most salient finding from both tables is that the decline noted between 1973 and 1978 has halted for 17-year-olds, and 13-year-olds have improved dramatically between 1978 and 1982.

One must certainly be encouraged by the impressive gains made by the 13-year-olds, yet a closer examination of their performance reveals a trend that is also characteristic of the 9- and 17-year-olds: much of the positive change that occurred in this assessment can be attributed to improved performance on rather routine items, such as computation and figure recognition. In general, students made much more modest gains, or no gains at all, on items assessing deep understanding or applications of mathematics.

Looking at these results across a wide range of tasks at all grade levels, it appears that American schools have been reasonably successful in teaching students to perform routine computational and measurement skills, and to answer questions assessing superficial knowledge about numbers and geometry. It is encouraging to note positive change on items assessing knowledge and skills not only in numerical computation, but also in geometry and measurement. On the other hand, it appears from the low percentages of success on some items that schools have thus far taught

Table 1.1.
Mean Performance Levels on Three Mathematics Assessments, Ages 9, 13, 17

Age	Number of Items	Mean Performance 1973	1978	1982	Change 1973-82
9	23	39.8%	39.1%	38.9%	−0.9%
13	43	53.7	52.[illegible]	56.4	2.7*
17	61	55.0	52.1	51.8	−3.2*

* Change is significant at the .05 level.

Table 1.2.
Mean Performance Levels on 1978 and 1982 Mathematics Assessments, Ages 9, 13, 17

Age	Number of Items	Mean Performance 1978#	1982	Change
9	233	55.4%	56.4%	1.0%
13	383	56.7	60.5	3.8*
17	383	60.4	60.2	−0.2

* Change is significant at the .05 level.
# The percentages for 1978 in Tabels 1.1 and 1.2 differ because each is based upon a different set of exercises.

only a small percentage of students how to analyze mathematical problems or apply mathematics to nonroutine situations.

Some other major findings, discussed more fully in the following chapters, appear below. Because the last two assessments were the most recent and because there are so many more common exercises included in the last two assessments than in all three assessments, the findings highlighted below apply only to changes between 1978 and 1982.

- Although the mean performance for Black and Hispanic students continued to be below the national mean, 13-year-old Black and Hispanic students made substantial gains in performance (about 7 percentage points) between the last assessment and this one. Moreover, the gains made by Black and Hispanic students were usually substantially larger than those made by their White counterparts. In general, the most significant gains were on exercises assessing the lower cognitive levels of knowledge and skills (see Appendix D).
- Students in schools with heavy minority enrollment tended to perform below the national level, but these students made significant performance gains since the second assessment. For example, 13-year-olds attending schools with 0-59% White enrollment (heavily minority schools) had a mean performance level that was 8 percentage points higher in 1982 than in 1978.
- The performance pattern of males and females on this assessment was very much like that reported in the last assessment. At ages 9 and 13, there was very little difference in performance; but at age 17, males tended to outperform females. There was very little difference between the sexes in their attitudes toward mathematics or in their taking of mathematics courses through Algebra 2, although males were somewhat more likely than females to have studied trigonometry, calculus or computer programming in high school.
- The results of the self-report inventory for 13- and 17-year-olds suggest that there has been a significant increase in computer usage in the schools. Between 1978 and 1982, the number of students reporting access to computers for learning mathematics increased from 12% to 23% (age 13) and from 24% to 49% (age 17). During that same period, the number of students who reported that they knew how to program a computer also rose dramatically—from 8% to 20% at age 13 and from 12% to 22% at age 17. In general, attitudes toward computers were positive and were more favorable than those reported in the last assessment.
- Although enrollment in computer science classes doubled in the four-year period between the assessments, there was no significant change in enrollment in traditional mathematics courses. (Table 1.3).

Table 1.3.
Percentages of 17-Year-Olds Who Have Completed at Least ½ Year of Specific Courses

Course	1978	1982
General or business mathematics	45.6%	50.0%
Pre-algebra	45.8	44.3
Algebra	72.1	70.9
Geometry	51.3	51.8
Algebra 2	36.9	38.4
Trigonometry	12.9	13.8
Pre-calculus/calculus	3.9	4.2
Computer science	5.0	9.7

- There was no significant change for 9-year-olds overall, but when the 9-year-olds' results are analyzed separately for 9-year-olds in third grade and in fourth grade, a significant 1.6 percentage point positive overall change appears for 9-year-olds in the fourth grade.
- Between 1978 and 1982, students' familiarity with the metric system of measurement increased dramatically. At all age levels,

students performed better on items testing knowledge of metric units. Nine-year-olds were up 2 percentage points, 13-year-olds were up 9 points and 17-year-olds were up 4 points. There was a corresponding decrease on items assessing knowledge of the English system: 9-year-olds were down 3 percentage points, 13-year-olds were down 1 point and 17-year-olds were down 5 points.

Explaining the Change

National Assessment is designed to document changes in performance and to provide some insights into the nature and scope of those changes. The Assessment is not designed to identify the underlying causes of change, however, and any attempt to do so must include reference to other data and a certain amount of speculation.

We would speculate that a major factor contributing to the significant gains of 13-year-olds and the leveling-off of the 17-year-olds' decline is an increased emphasis on academics in schools and society that began in the late 1970s. Recent studies of school effectiveness suggest that schools with high levels of achievement have high expectations of the students (Cohen, 1982; Edmonds, 1979); perhaps the nation as a whole has had higher expectations of its students since 1978, and these expectations have led to higher achievement.

Why did significant gains only occur at age 13? Probably because that is the age group most likely to be affected by the increased attention given to mathematics education during the last four years. The mathematics curriculum is most diverse in grades 4-8, spanning almost all of the topics covered in national assessments. The primary school curriculum is largely limited to whole number operations, upon which 9-year-olds already perform rather well. This is not to say that important learning does not occur before age 9 or that children's learning at this age cannot be influenced. It is simply to say that whatever has been happening to primary school children, it has not been reflected in dramatic changes on nationally administered assessments or achievement tests.

Since the assessment focuses on topics covered in grades 7-9, it is not particularly sensitive to changes in the high school curriculum. Many 17-year-olds will have studied the topics covered in the assessment three or four years before the assessment was administered, and previous to the national push for higher standards. We might hypothesize, though, that if 13-year-olds really have gained a more solid grounding in mathematics, 17-year-olds' performance may rise on the next assessment.

Implications and Suggestions

In general, the results of this assessment indicate that schools are doing a good job of teaching those mathematical topics that are relatively easy to teach, such as figure recognition, and fairly low-level cognitive tasks, such as routine computation. The positive changes found in this assessment are encouraging, but they represent only a first step toward improved mathematics education. Since there was very little change in topics that are relatively difficult to teach, such as nonroutine problem solving, the next step will not be as easy to take. We cannot hope to improve substantially higher-level cognitive skills and understanding simply by teaching students lower-level knowledge and skills; we cannot attain maximum competencies by teaching minimum competencies. Changes at the higher cognitive levels will occur only when higher-level cognitive activity becomes a curricular and instructional focus.

. . .for Local, State and Federal Education Policy Makers

Education policy needs to reflect the fundamental importance of mathematical understanding and problem solving. In particular, policy pertaining to the following areas needs to be carefully examined:

- **The testing program.** Many states and virtually all school districts have some form of standardized testing for students. These tests are a direct reflection of what is valued in schools. However, the very things that are difficult to teach are very often difficult or expensive to test. Educational leaders need to pressure test developers to include items that reflect the higher-level objectives of the curriculum. The age-by-age comparisons of findings also suggest that many mathematical skills develop over a long period of time, often after the years in which they are emphasized in the curriculum. Minimum competency tests that impose a rigid time frame for the learning of basic skills have tended to ignore this fact, thereby providing misleading data and perhaps distorting curricular priorities.

- **The shortage of qualified mathematics teachers.** Qualified mathematics teachers are leaving the field at an alarming rate, and very few young men or women are currently preparing to become mathematics teachers (see Appendix A for more on this). If schools are to improve students' understanding of mathematics and their ability to solve problems, then this must be accomplished by teachers who are well trained in mathematics and the teaching of mathematics. Although programs for retraining teachers from other disciplines may reduce the shortage of credentialed mathematics teachers, they represent only a short-term, "quick-fix" solution. Teachers who have studied only a few mathematics courses, and whose training is in other disciplines, will not have a sufficiently deep understanding of mathematics to communicate to their students. Our emphasis should be upon the recruitment and retention of *highly qualified*, not marginally competent, mathematics teachers.

- **The need for inservice training.** Experienced mathematics teachers at all levels need to be provided with inservice opportunities to improve their teaching skills and broaden their knowledge, especially in the areas of applications and problem solving. Furthermore, mathematics teachers need opportunities to become acquainted with available computer technologies and their potential for improving mathematics teaching. These opportunities for systematic inservice training can probably best be managed at the district, county or state levels.

- **Support for basic research on mathematical cognition.** Over the past decade, researchers have made considerable progress in understanding the links between childrens' mastery of computational skills and their understanding of underlying concepts, or the processes they use to solve mathematical problems. Such research on basic learning, understanding and problem-solving processes must continue if efforts to address many recommendations emerging from this assessment are to be successful.

- **Support for programs that assist disadvantaged students.** The results of this assessment join a swelling body of data that document major improvements in the achievement of Black, Hispanic and disadvantaged young people over the last decade. These improvements undoubtedly stem from many factors, but chief among them must be the federal commitment to improve minority students' educational opportunities reflected in the entitlement programs of the seventies. With this strong circumstantial evidence that concerted attention to the educational needs of minorities can, indeed, bring about positive changes, policy debate can shift from the question of *whether* such programs work to the matter of *how* we can continue the progress made thus far. The performance of minorities still remains far below the national level and much remains to be done.

. . .for Textbook Publishers and Curriculum Developers

The principal method of presenting mathematics to students is through written text materials. Thus, the objectives of a textbook or textbook series can have a profound effect on the kind of learning that results. The findings of this assessment suggest several areas that deserve serious attention from textbook publishers and curriculum developers.

- **Emphasis on higher-level cognitive objectives.** Text materials and curricula need to reflect an emphasis on understanding mathematical concepts and applying knowledge and skills to solve mathematical problems in both routine and nonroutine situations.

- **Assessment of the current curriculum.** The findings of this assessment suggest that the mathematics curriculum needs reexamination in light of currently available computational technology. The hand-held calculator is nearly ubiquitous, and students are much more likely to be using computers today than they were even a few years ago. If computations can be done by machine, students need a much better understanding of the relationship between problem situations and the operations necessary for finding solutions. Moreoever, a technologically literate population needs to be skilled in analyzing and managing quantitative data, estimating answers to calculations and judging the reasonableness of results.

  These new skills should not simply be appended to the existing curriculum. It is likely that some of the current curriculum could be revised or even eliminated. For example, the computational algorithms taught in school are designed to produce rapid, accurate answers in a paper-and-pencil environment. Yet these algorithms are often difficult to relate to associated mathematical concepts. Given current computational technologies, it is time to reconsider the utility of such algorithms and examine alternate, easier-to-understand procedures. Furthermore, the secondary school mathematics curriculum is designed to prepare students to study calculus, but some mathematicians and computer scientists question the importance of calculus both in computer science and in the increasingly computer-based university mathematics curricula.

  The results of this assessment also suggest that underlying concepts and skills in geometry and measurement have not been mastered by enough students. This suggests a need for a carefully developed curriculum sequence of concepts and skills in the areas of measurement and geometry just as we have for whole numbers, fractions and other areas of mathematics.

...for School Administrators

Recent research on effective schools indicates that strong leadership from a school's administrator is an essential ingredient in that school's educational excellence (Edmonds, 1979; Cohen 1982). Thus, if schools are to be successful in efforts to teach those aspects of mathematics that are difficult to teach, such as problem solving, then school administrators will need to provide leadership, especially in the following areas:

- **Maintaining high expectations.** Research generally suggests that expectations play an important role in student achievement. If students are expected to learn, they tend to learn; if they are expected to fail, they tend to fail. Administrators need to convey a school's commitment to achievement of higher-level mathematics objectives, and they must convey this belief that all students can learn.

- **Institutional support for teachers.** Incorporating problem solving and higher-level cognitive objectives into the curriculum is time consuming, especially in planning new classroom activities. Administrators can assist teachers in their efforts by providing institutional support in the form of smaller class size, reduced course load, released time or paraprofessional aides.

- **Inservice training.** School inservice programs should provide opportunities for teachers to learn about current research in mathematics education, programs that teach higher-level objectives and helpful supplemental curriculum materials.

- **Encouraging minorities.** Given the data on minority student course taking in Chapter 5, administrators should consider various counseling and incentive programs for encouraging greater minority participation in mathematics courses.

...for Classroom Teachers

The classroom teacher plays the most crucial role in education. The results of this assessment suggest that teachers should ask themselves the following questions:

- **Textbooks.** Do the textbooks used in your mathematics classes sufficiently stress the higher-level objectives, or do they dwell too heavily on routine knowledge at the expense of material that could deepen understanding? What supplemental materials are available to extend the text in the direction of higher-level objectives?

- **Problem-solving emphasis.** To what extent do students have opportunities to engage in real problem solving? Are students regularly challenged to apply mathematics to problem situations, or are they generally asked to memorize and repeat?

- **Teaching techniques.** Does class discussion focus on the variety of interpretations or representations that might be possible for a given problem, or do students see only a single solution for a problem? Are students asked to defend their reasoning, or justify an answer, or explain why a particular result is reasonable?

The answers to such questions can point the way toward an enhanced mathematics program.

Chapter 2

Knowledge, Skills and Concepts

This chapter presents the primary results of exercises assessing mathematical knowledge and skills. Sometimes the results of exercises assessing understanding clarify the results of the knowledge and skills exercises, and in such cases, the understanding findings are also discussed.

In general, there was a common pattern of results across the exercises discussed in this chapter: students improved most on easier knowledge and skills exercises, least on those that required a more complete grasp of mathematics or more sophisticated skills.

Computational Concepts and Skills

Since numbers are central to much of elementary mathematics, it is natural to ask how students are doing with number concepts and skills. This section focuses on whole numbers, fractions, decimals and percents.

Whole Numbers

Children begin their study of mathematics by learning about whole numbers and using them in computation. The amount of time and effort spent on whole numbers in school is evident in the fact that students did well on these exercises, and in general, their performance was the same as or higher than in the previous assessments. Looking at particular exercises, the results actually vary somewhat, depending upon whether one is looking at exercises assessing knowledge of the numbers themselves, the basic facts or computational skills.

The numbers themselves. Besides computation, 9-year-old students must learn about such concepts as counting, reading numbers and place-value. On average, almost two-thirds of the 9-year-olds showed proficiency in these skills when the task involved considering the number as a whole. Almost all knew that 67 is sixty-seven, or that 243 is two hundred forty-three. However, when the task called upon the child to consider place-value (e.g., 38 is 3 tens and 8 ones), fewer of the 9-year-olds (about three-fourths) were successful. An understanding of these more sophisticated ideas can greatly help in developing computational skills. Yet, the 9-year-olds' performance decreased slightly on this type of exercise, while it increased on non-place-value exercises and on place-value ideas that are easily learned by rote (Table 2.1, for example).

Basic facts. Quick recall of basic facts, such as $3 + 4$ or 5×8, is necessary for all other computation and for estimating answers. Thus, it is a skill area that should be highly developed, and the results suggest that it is.

Nine-year-olds' performance, as could be expected, was strongest on addition facts, slightly lower on subtraction facts and lowest on multiplication facts (Table 2.2).

Altogether, there was a slight improvement—2 percentage points—on the basic facts exercises between 1978 and 1982, largely due to the improvements in multiplication.

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Table 2.1.
Percentages of Success on Two Place-Value Exercises, Age 9

Exercise	Percentage Correct 1978	1982	Change
A. What does the 5 stand for in the number 3,517? (Answer: 5 hundreds)	80.1%	85.1%	5.0%*
**B. Which of the following represents "seven tens"? ☐7 ■70 ☐700 ☐710 ☐I don't know	79.9	73.0	−6.9*

* Significant at the .05 level.
** Similar to an unreleased exercise.

Table 2.2.
Average Percentages Correct on Exercises Assessing Basic Facts, Age 9

Type of Basic Fact	Number of Exercises	Average Percentage Correct 1978	1982	Change
Addition	12	87.8%	88.1%	0.3%
Subtraction	12	78.6	79.1	0.5
Multiplication	12	59.7	66.3	6.6*

* Change is significant at the .05 level.

A cursory look at performance on the multiplication facts shows 9-year-olds performing in the 70-85% range on easier facts and in the 50-60% range on harder facts. These results are not surprising, since it is in third and fourth grades that multiplication facts are mastered. A closer look reveals two positive trends. First, performance has increased about 10 percentage points on the multiplication facts; second, fourth graders performed, on the average, about 40 percentage points better than the third graders.

Both 13- and 17-year-olds are in the 90%-or-above range on addition, subtraction and multiplication facts. While performance on division facts is slightly lower, 13-year-olds increased about 10 percentage points from the last assessment. Seventeen-year-olds showed no change from the last assessment.

Computation. Nine-year-olds' whole number computation did not change appreciably between the last two assessments; however, they did improve slightly in subtraction.

When a subtraction exercise was presented horizontally, however—e.g., "Subtract 237 from 504"—only a third as many 9-year-olds could solve the problem correctly as could when the same problem was presented vertically (17% compared to 48%).

Table 2.3.
Percentages of 9-Year-Olds Who Perceived Mathematics Topics To Be Easy

Topic	Percentage Saying Topic Easy 1978	1982	Change
A. Learning about money	52.3%	62.8%	10.5%*
B. Doing additional problems	65.3	73.9	8.6*
C. Solving mathematics word problems	31.1	41.3	10.2*
D. Learning multiplication or times tables	38.1	52.3	14.2*
E. Learning how to measure things with a ruler	62.0	64.2	2.2*

* Change is significant at the .05 level.

The fact that 9-year-olds are performing better on a few types of tasks, but are declining or showing no improvement on others, leads to speculation that they are being exposed to a narrower range of tasks in the classroom. It is interesting to note that the 1982 9-year-olds in the third assessment thought math was easier than did the students in the second assessment (see Table 2.3) This was the only outstanding change in their attitudes.

By age 13, addition and subtraction of whole numbers has stabilized. While at age 9 the performance varies greatly according to the difficulty of the exercises, by age 13, 80% of the students could answer these exercises successfully.

Thirteen-year-olds' performance improved on paper-and-pencil multiplication and division exercises between 1978 and 1982. They performed at or above the 75% level on easier division exercises, but on a more difficult problem (three-digit quotients with a zero in the quotient), performance was below 60% (see Table 2.4., part C).

While the performance of 13-year-olds and 17-year-olds was about the same on paper-and-pencil division exercises, the 17-year-olds were more adept at doing division mentally. Exercises such as the ones in Table 2.5 illustrate the level of each age group.

Table 2.4.
Percentages of Success on Three Division Exercises, Age 13

Divide	Answer	Percentage Correct 1982
A. $3\overline{)304}$	101,R1	77.0%
B. $5\overline{)150}$	30	90.7
C. $12\overline{)2496}$	208	57.0

Table 2.5.
Percentages of Success on Mental Division Exercises, 1982

Exercise	Answer	Percentage Correct Age 13	Age 17
** A. 480/16	30	32.6%	58.4%
** B. 3500/35	100	38.8	63.2

** Similar to unreleased exercises. These exercises were not seen by the students; the problems were read to them and they were given 10 seconds for each response.

Fractions and Decimals

In recent years, there has been movement to present decimals earlier and to do less with fraction operations because of the availability of calculators. This movement might have been responsible for the fact that performance changed considerably in this area. While 13-year-olds increased on almost every other computational skill, they showed no significant improvement on fraction computation. On the other nand, they showed an 8 percentage point improvement on decimal computation.

Fractions. Nine-year-olds improved on the simple fraction exercises. On exercises such as writing the symbol for one-third, for instance, they improved from about 51% to 56%. However, on an exercise that asked which of several fractions is the same as ⅓, their performance dropped from 22% to 18%.

Thirteen-year-olds showed improvement on exercises that could build understanding about fractions. For example, on two exercises that required changing a mixed number to an improper fraction (such as in Table 2.6), they improved about 6 or 7 percentage points. However, one sees little evidence that students connected these skills to operations with fractions.

Table 2.6.
Percentages of Success Changing Mixed Numbers to Improper Fractions, Age 13

Exercise	Answer	Percentage Correct 1978	Percentage Correct 1982	Change
**A. 1 1/5 =	6/5	60.7%	67.3%	6.6%*
**B. 2 5/6 =	15/6	56.4	63.8	7.4*

* Change is significant at the .05 level.
** Similar to an unreleased exercise.

Table 2.7.
Percentages of Success on Exercises Assessing Operations With Fractions, Ages 13 and 17, 1978 and 1982

		Percentage Correct			
		Age 13		Age 17	
Exercise	Answer	1978	1982	1978	1982
**Subtraction					
A. 3 1/3 - 3 1/4	1/12	34.7%	36.1%	60.5%	60.4%
B. 6 1/5 - 3 1/2	2 7/10	18.0	17.9	38.8	39.1
**Multiplication					
A. 7/8 x 5/3	35/24	57.5	60.1	64.7	66.3
B. 7/14 x 4/5	28/70, 4/10	48.8	52.4	59.1	61.5
C. 8/11 x 3/16	24/176, 12/88	49.7	53.9	61.2	65.3

**Similar to unreleased exercises.

The performance on fraction computation is low and students seem to have done their computation with little understanding. The level of this performance is illustrated by the results in Table 2.7.

One trend in the change data prompts speculation that fractions are perhaps being taught more by rote than they used to be. If it is understood that 10 is 10/1, then the computation involved in exercises A and B (Table 2.8) is the same. Yet students made gains on the exercise that can done routinely (exercise A), while they lost ground in the exercise that requires more understanding.

There is one bright spot in the performance of 13-year-olds with fractions. More of them (a 6 percentage point gain) understood the concept of a fraction as a division—i.e., that 5 divided by 6 could be expressed by 5/6 (see exercise A in Table 2.9). Correspondingly, the same sort of gain appeared in the skill of using this relationship when changing a fraction to a decimal (see exercise B in Table 2.9). Although performance is low, it does point to consistent gains on related concepts and skills. In general, performance on exercises requiring conversion from fractions to decimals went up 7 percentage points at age 13 and remained stable at age 17.

Decimals. The performance on decimals was more positive than on fractions.

Even though decimals are being introduced earlier than they used to be, most 9-year-olds have not had much experience with them. Performance has not improved since the last assessment, and 9-year-olds can still be characterized as having little understanding of or facility with decimals.

Table 2.8.
Percentage Gains and Losses on Two Kinds of Multiplication Exercises, Ages 13 and 17

Exercises	Age 13 Average Change 1978-82	Age 17 Average Change 1978-82
A. Fraction times fraction	gain of 3.5%	gain of 2.7%
B. Whole number times fraction	loss of 4.4	loss of 5.7

Table 2.9.
Percentages of Students Demonstrating Knowledge of Decimal Fraction Equivalence, Ages 13 and 17

		Percentage of Success			
		1978		1982	
Exercise	Answer	Age 13	Age 17	Age 13	Age 17
A. Write "5 divided by 6" as a fraction.	5/6	47.7%	+	54.2%	+
B. What decimal is equal to 5/6?	.833	26.0	53.3%	33.6	58.0%

+ Not administered at age 17.

Table 2.10.
Changing Percentages of Success on Three Decimal Exercises, Ages 13 and 17

Write in decimal form:		1978	1982	Change
A. Six and three thousandths	Age 13	49.5%	65.3%	15.8%*
Answer: 6.003	Age 17	64.5	64.7	0.2
Forty-two ten thousandths	Age 13	25.7	37.0	11.3*
Answer: .00042	Age 17	37.8	32.8	-5.0*
Eight and six hundredths	Age 13	48.3	64.3	16.0*
Answer: 8.06	Age 17	61.9	63.4	1.5

* Change is significant at the .05 level.

On the other hand, 13-year-olds appear to be gaining in understanding and facility with decimals. The exercise in Table 2.10 shows the level of performance on translating words to symbols.

Note that on this exercise not only have 13-year-olds improved, they are at the level of 17-year-olds. However, on three exercises that require a deeper understanding of decimals, the 17-year-olds' performance was about 18 to 25 percentage points higher than the 13-year-olds.

On simple decimal computation, both 13- and 17-year-olds are performing at the 80 to 90% level. However, performance drops for both groups when the computation involves more understanding of decimals. Contrast the results of three division problems in Table 2.11.

Students' problems with basic decimal concepts are also illustrated by performance on an estimation exercise (Table 2.12). Although 57% of the 13-year-olds and 72% of the 17-year-olds *calculated* the answer to a similar exercise, significantly fewer made a reasonable *estimate* for the exercise in Table 2.12. The 13-year-olds' responses were almost at the level of guessing, and performance by 17-year-olds was not much better.

Percents. About seven-eighths of the 13-year-olds could identify an object that represented a

Table 2.11.
Percentages of Success on Three Decimal Division Exercises, 1982

Exercise	Answer	Percentage of Success Age 13	Age 17
** A. 8.4/4	2.1	85.9%	89.4%
** B. 8.4/.04	210	39.2	56.0
** B. 6.03/.3	20.1	50.1	63.9

** Similar to unreleased exercises.

Table 2.12.
Percentages of Success on Decimal Estimation Exercise, Ages 13 and 17, 1982

ESTIMATE the answer to 3.04 x 5.3

Response (multiple-choice)	Percentage Responding Age 13	Age 17
☐ 1.6	27.8%	21.1%
■ 16	20.7	36.6
☐ 160	17.8	17.1
☐ 1600	22.8	11.2
☐ I don't know	8.6	12.2

37

given percentage, five-sixths could change a decimal to a percent and three-fourths could relate percents to hundreds. Their performance on percent computation exercises was not high; however, there was a marked difference between 13-year-olds in grade 7 and in grade 8 (see Table 2.13).

On five other items dealing with percentages, the 17-year-olds' performance averaged about 16 percentage points higher than the 13-year-olds'. The 17-year-olds showed an overall improvement of about 3 points from the previous assessment.

Continued Development of Computational Skills

In examining the results of the numerical exercises, it is important to recognize that most computational skills are learned over an extended period of time. The results summarized in Table 2.14 suggest that most skills are mastered *after* the period of primary emphasis in the curriculum. For example, even though most mathematics programs expect students to learn subtraction facts by age 9, there is significant improvement in performance from age 9 to 13. Addition, subtraction and multiplication are skills that are used in a variety of contexts (e.g., in division problems), so students continue to have experiences with them in the curriculum long after they have been the focus of instruction.

The assessment results suggest that rigid programs that hold children back until they have demonstrated mastery of a given set of skills may, in fact, be depriving them of the very experiences that would lead to long-term mastery.

Table 2.13.
Percentages of Success on Three Percent Exercises, 13-Year-Olds in Grades 7 and 8, 1982

Exercise	Answer	Percentage Correct Grade 7	Percentage Correct Grade 8
A. What is 10% of 50?	5	24.4%	47.6%
B. What is 60% of 50?	30	15.4	31.5
C. What is 75% of 12?	9	14.3	27.7

Table 2.14.
Improvement in Performance by Age, 1982

Problem	Answer	Percentage Correct Age 9	Age 13	Age 17
Basic subtraction facts		77.2%	93.0%	94.8%
Three-digit subtraction		45.8	87.2	89.8
**5/8 - 3/8	1/4		83.6	89.6
4 1/3 - 4 1/4	1/12		36.1	60.4

**Similar to an unreleased exercise.

Noncomputational Skills

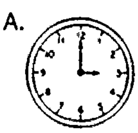
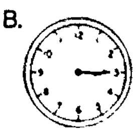
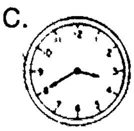
A Broader Basis for Basis Skills

The National Council of Teachers of Mathematics (NCTM, 1980) and the National Council of Supervisors of Mathematics (NCSM, 1978) make a compelling argument that basic skills in mathematics encompass more than computation. The mathematics assessment included exercises in a number of other basic skill areas including geometry, measurement, probability and statistics, and interpreting graphs and tables. Performance was generally

Table 2.15.
Percentages of Success on Exercises Requiring Identification of Geometric Figures, All Ages, 1978 to 1982

	Percentage Correct					
	Age 9		Age 13		Age 17	
Figure	1978	1982	1978	1982	1978	1982
Square	92.7%	92.9%	95.9%	95.7%		
Triangle	87.9	92.4				
Rectangle	83.4	90.1	92.6	93.7		
Octagon			64.3	72.2	85.7%	88.4%
Acute angle			36.2	46.0		
Parallel lines	56.9	51.1	89.5	91.4	94.6	96.2
Cube	85.1	97.0	95.8	98.2	98.7	98.6
Cylinder	39.6	52.2	78.5	85.0	92.5	92.0

Table 2.16.
Percentages of Success on Time-Telling Exercises, Age 9, 1978 and 1982

		Percentage Response	
		1978	1982
A. 	☐ 12:00	0.9%	0.8%
	☐ 12:03	3.3	3.5
	■ 3:00	93.4	94.3
	☐ 3:12	1.7	1.2
	☐ I don't know	0.3	0.1
B. 	☐ 3:00	2.6	1.6
	☐ 3:03	8.2	5.9
	■ 3:15	85.9	90.1
	☐ 3:20	1.5	1.7
	☐ I don't know	1.6	0.7
C. 	☐ 3:20	6.0	3.9
	■ 3:40	67.8	77.0
	☐ 4:08	8.2	4.3
	☐ 8:20	12.3	11.5
	☐ I don't know	5.0	3.1

high on exercises assessing knowledge and skills like:

- Recognizing common geometric shapes (Table 2.15)
- Using a ruler to make linear measurements
- Telling time (Table 2.16)
- Recognizing common units of measure
- Reading simple graphs and tables (Table 2.17).

In general, these skills are relatively straightforward and easily learned. They do not require any deep understanding of underlying mathematical concepts, and, in many cases, they can be learned in isolation without having to build upon related skills and concepts. Many of these skills are also used and reinforced outside of school.

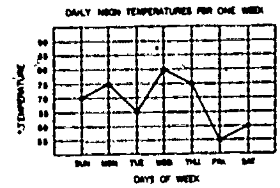
Performance was low on exercises assessing more complex concepts and skills that require some understanding of underlying mathematical principles. The results for three area exercises summarized in Table 2.18 illustrate this pattern. Area concepts and skills cannot be learned piecemeal. More advanced concepts and skills are based on underlying concepts and skills, which require time to be fully assimilated. The 17-year-olds' inability to calculate the area of a parallelogram probably does not simply reflect a failure to learn an isolated formula for calculating area; it may also reflect a failure to develop a cohesive concept of area throughout the curriculum.

At the most basic level, area is defined as the number of units, usually square units, required to cover a given region (see Table 2.18, part A). When students calculate area by multiplying various linear dimensions of figures, they should understand that such operations are a short-cut for finding the number of units in a unit covering.

Only about 25% of the 9-year-olds and about two-thirds of the 13-year-olds demonstrated an understanding of areas as the number of units covering a region (Table 2.18, part A). Over half of the 13-year-olds could not calculate the area of a rectangle from its dimensions (Table 2.18, part B).

Students' apparent failure to learn concepts and skills that go beyond recognition, recall, simple manipulations or general intuition is reflected in performance on a number of other measurement and geometry exercises. Although most students at all age levels could identify common geometric shapes, relatively few demonstrated knowledge of basic properties of these shapes. For example, the

Table 2.17.
Percentages of Success on Graph-Reading Exercise, Ages 9, 13 and 17



A. Which day was the warmest at noon?
Correct answer: Wednesday

	Percentage Correct	
	1978	**1982**
Age 9	65.2%	75.9%
Age 13	93.3	95.5
Age 17	97.6	98.3

B. Which two days had the same noon temperature?
Correct answer: Monday and Thursday

	Percentage Correct	
	1978	**1982**
Age 9	59.7%	68.9%
Age 13	92.4	93.5
Age 17	96.7	96.5

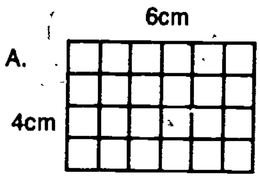
C. How many days was the noon temperature 70° or above?
Correct answer: 4

	Percentage Correct	
	1978	**1982**
Age 9	41.8%	41.6%
Age 13	79.7	80.7
Age 17	91.5	90.8

Table 2.18.

Percentages of Success on Three Area Exercises, Ages 9, 13 and 17

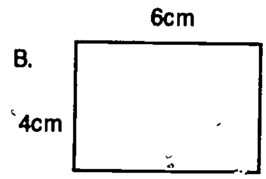
A.



What is the area of this rectangle?
Correct answer:
24 sq. cm

	Percentage Correct 1978	1982
Age 9	27.6%	24.7%
Age 13	70.4	64.0

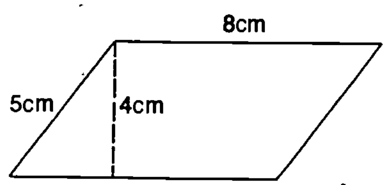
B.



What is the area of this rectangle?
Correct answer:
24 sq. cm

	Percentage Correct 1978	1982
Age 9	3.4%	8.4%
Age 13	51.4	48.4
Age 17	73.4	73.8

C.



The dotted line is an altitude of the parallelogram. What is the area of the parallelogram?

Correct answer: 32 sq. cm

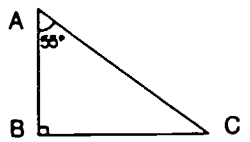
	Percentage Correct 1978	1982
Age 17	19.2%	19.2%

solution to the exercise in Table 2.19 is based on the knowledge that the sum of the measures of the angles of a triangle is 180°.

The Pythagorean theorem is another basic principle that most students apparently have not learned (see Table 2.20).

Metric measurement. If performance is affected by changes in emphasis in the mathematics curriculum, one area in which significant changes should be observed would be familiarity with the metric system. Between the second and third mathematics assessments, the number of 13-year-olds reporting that they often use the metric system in their mathematics classes more than doubled from 16% to 34%. The gain in metric use outside of school was more modest: 49% of the 13-year-olds reported that they *never* used metric measures outside of school, as opposed to 54% four years earlier. Significant gains in familiarity with metric units were reported at all three age levels (Table 2.21). The corresponding deemphasis in standard English units was reflected in a performance decline on

Table 2.19.
Percentage of Success on Angle Exercise, Ages 13 and 17

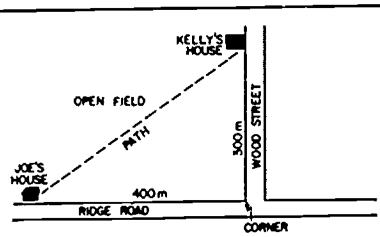


ABC is a right triangle. What is the measure of ∠ACB?

Correct answer: 35°

	Percentage Correct 1978	1982
Age 13	10.4%	9.6%
Age 17	41.3	43.6

Table 2.20.
Percentage of Success on Exercise Assessing Knowledge of Pythagorean Theorem, Ages 13 and 17



Joe's house on Ridge Road is 400 meters from the corner of Ridge Road and Wood Street. Kelly's house is on Wood Street and is 300 meters from the same corner. When Joe goes to Kelly's house, he walks through the open field. How many meters does he walk?

Correct answer: 500

	Percentage Correct 1978	1982
Age 13	18.9%	20.0%
Age 17	36.2	39.0

exercises assessing knowledge of English units. This pattern of performance on exercises involving metric and English units is consistent with the trend observed between the first and second assessments.

Table 2.21.
Average Change in Performance for Exercises Assessing Familiarity With Metric and English Units, All Ages

	Age 9 Change	Age 13 Change	Age 17 Change
Metric units	2.2%*	8.6%*	4.3%*
English units	-2.6	-1.3	-4.6*

* Change is significant at the .05 level.

Change Within Different Content Areas

Overall change within each content area of the assessment is summarized in Table 2.22.

At age 9, performance went up in two content areas, geometry and graphs and tables. The greatest gains in geometry were in recognizing common geometric shapes, where the average increase was about 5 percentage points. In most other clusters of exercises within geometry, there was no clear pattern of gains.

A similar pattern of change accounts for the increase in performance on the graph and table exercises. Performance increased almost uniformly on all exercises that required 9-year-olds to read data from a graph or table; there was virtually no change on exercises that required students to interpret information or draw conclusions about it. Thus, for both geometry and graphs and tables, the greatest gains in performance came on exercises assessing the skills that were already mastered by the highest proportion of students.

At age 13, performance improved over every content area. However, as at age 9, the most consistent and largest increases came on exercises assessing recall or relatively low-level skills. For example, 13-year-olds' performance on exercises assessing recognition of common geometric shapes was up an average of 4

Table 2.22.
Change Within Content Areas, All Ages, 1978 to 1982

Content Area	Age 9	Age 13	Age 17
Number and numeration	0.7%	4.0%*	0.0%
Variables and relations	-1.3	3.2*	-0.5
Geometry	2.0*	4.7*	0.0
Measurement	0.8	2.6*	-1.3
Probability and statistics	0.2	3.7*	1.2
Graphs and tables	3.0*	3.2*	0.4

* Change is significant at the .05 level.

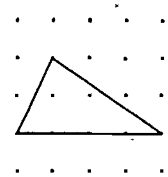
percentage points, and virtually every exercise showed an increase. Thirteen-year-olds' performance was also consistently up on geometry exercises that involved informal geometry concepts or could be solved intuitively without much formal knowledge of geometric principles (e.g., Table 2.23). Exercises requiring specific knowledge of geometry showed more mixed results. Those requiring knowledge of specific geometry theorems showed about as many declines as increases, and the increases were relatively modest (e.g., Tables 2.19 and 2.20).

At age 17, there was no clear pattern of change in any content area. Much of the assessment focused on basic skills taught in junior high school, but there was a reasonable sample of topics from high school algebra and some exercises include material covered in a high school geometry course. These topics also showed no significant change.

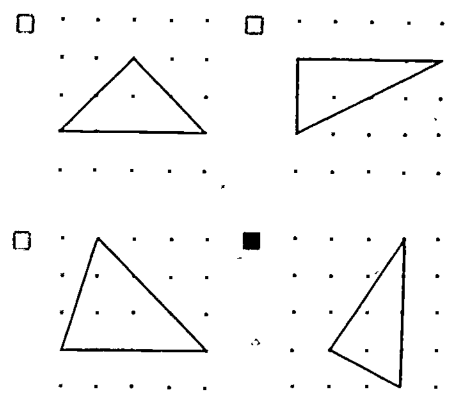
Summary Comments

Mathematics is a hierarchically ordered discipline. Knowledge of basic number facts is required to perform computations, and the ability to apply algorithms may depend upon the ability to use other algorithms that are embedded in the process. For example, the common division algorithms use both subtraction and multiplication. The mathematics

Table 2.23.
Percentages of Success on Informal Geometry Exercise, Ages 9, 13 and 17



Suppose you cut out the above triangle. On top of which triangle shown below would it fit exactly? Fill the box beside the triangle you choose.



☐ I don't know.

	Percentage Correct 1978	1982
Age 9	54.2%	58.0%
Age 13	73.1	82.2
Age 17	83.6	82.2

curriculum is designed to teach this hierarchy, and the assessment results indicate that schools are making most progress doing so with respect to number skills. At the earlier ages, performance is high on number fact recall items and simple addition and subtraction problems. Whole number operations are learned by most students by age 13, and 13- and 17-year-olds' results indicate an increasing mastery of fraction and decimal skills.

However, the hierarchy of concepts and skills that underlie geometry and measurement does *not* appear to have been learned very well. Furthermore, the greatest gains in these areas are on topics that can be learned as isolated skills. It is much more difficult to teach area and volume concepts than it is to teach figure recognition. Improvement in performance will be much more difficult to achieve for skills and concepts that are based on understanding fundamental mathematical ideas than it has been for recall and simple skill problems. We *are* making progress. But we need to begin to attack the more fundamental problems, not just the ones that are easiest to teach.

Chapter 3

Problem Solving, Applications and Attitudes Toward Mathematics as a Discipline

Developing students' ability to apply mathematical knowledge and skills in solving problems is widely recognized as one of the major goals of mathematics instruction. In 1978, the National Council of Supervisors of Mathematics identified problem solving and applications as 2 of the 10 *basic skills* in the mathematics curriculum. More recently, the National Council of Teachers of Mathematics (1980) selected problem solving as the area of primary concern for the 1980s.

The third mathematics assessment is the first one conducted since those calls for increased attention to problem solving were made. In general, the results from this assessment indicate that more effort and time is needed in order to affect substantial changes in the problem-solving ability of school age children. With one exception, there was very little change in problem-solving performance between 1978 and 1982. The one exception is that 13-year-olds showed significant growth in solving routine problems—i.e., word problems of the type usually found in textbooks and practiced in school. This chapter highlights those findings as well as other findings about skill in solving routine problems, skill in solving nonroutine problems and attitudes toward both problem solving and mathematics in general.

Routine Problems

Students at all ages are fairly successful in solving routine, one-step verbal problems such as those often found in their textbooks. The overall performance of the 9-year-olds was about the same as on the second assessment. In particular, most 9-year-olds appear able to solve simple addition and subtraction verbal problems involving whole numbers or money. They do far less well on multiplication and division word problems, but most 9-year-olds have only just begun to master these operations.

The average performance of 17-year-olds on routine problems also did not change much from the last assessment. Performance averaged about 60% on problems involving whole numbers, but it was lower on problems involving fractions, decimals, percents and integers (about 40%). Table 3.1 illustrates performance on a fraction verbal problem and a percent verbal problem. There were some performance gains on problems involving percents, but there were complementary losses for 17-year-olds on problems involving fractions.

As was true in many other areas of the assessment, 13-year-olds' performance on routine problems improved significantly (about 2%), with general improvement on routine problems involving whole numbers, fractions and decimals. Performance on problems involving percents was quite low, but 13-year-olds are just beginning to learn the concepts and skills associated with percent.

Students at all three age levels experienced more difficulty with multistep verbal problems than with one-step problems. Thirteen-year-olds' performance in solving multistep verbal problems improved 2 percentage points, although the performance level on many items was still below that which would generally be

Table 3.1.
Percentages of Success on Two Word Problems, Ages 13 and 17, 1978 and 1982

A. A store is offering a discount of 15 percent on fishing rods What is the amount a customer will save on a rod regularly priced at $25.00?
Correct Answer: $3.75

	Percentage Correct	
	1978	1982
Age 13	10.1%	14.0%
Age 17	40.1	44.0

B. Pam has 4 3/4 cups of flour. If she uses 2 1/2 cups to make a cake, how much flour will she have left?
Correct Answer: 2 1/4

	Percentage Correct	
	1978	1982
Age 13	52.6%	54.7%
Age 17	77.4	74.1

considered educationally acceptable (see Table 3.2 for an example of a multistep problem).

In general, 17-year-olds performed about 16 percentage points higher than 13-year-olds on routine problems administered to both groups. The gap was somewhat larger for problems involving variables or percent, and somewhat smaller for problems that required only a single operation of addition, subtraction or multiplication of whole numbers.

One new type of item that was included in this assessment involved pictorial presentation of a problem, with a minimum of written text. These items were estimation items, on which students were instructed not to use paper and pencil. In general, 17- and 13-year-olds performed about as well on these items as they did on related routine problems presented in standard written form. Table 3.2 presents one of the estimation items and a related standard written problem that called for similar computation. There was very little difference in performance on these two problems.

Nonroutine Problems

Some of the tasks in this assessment required students to apply their mathematical knowledge and skills in problems that were different from those usually found in their textbooks. These problems are referred to hereafter as nonroutine problems.

As was true in previous assessments, there was a marked discrepancy between performance on routine problems and problems that required some analysis and nonstandard application of knowledge or skill. Table 3.3 presents performance data for two nonroutine problems given in this assessment.

One important aspect of problem solving is the identification of relevant information in a given problem. Performance on item A indicates that 9-year-olds do not have a clear understanding of this crucial aspect of problem solving. Although performance on this item improved between 1978 and 1982, only about one student in three was able to correctly and completely identify the missing information in this relatively simple task. On a related exercise asking students to identify extraneous information in a problem statement, only about one student in four was able to do so.

The poor performance of 17-year-olds on item B suggests that their understanding of variables is incomplete. Although this item is nonroutine for most high school students, a rudimentary

Table 3.2.
Percentages of Success on an Estimation Problem and a Computation Problem, Ages 13 and 17, 1978 and 1982

A. Suppose you want to bake some cakes for a party. Two cake recipes require the following amounts of flour:

Pineapple Swirl Cake
2 1/3 cups flour

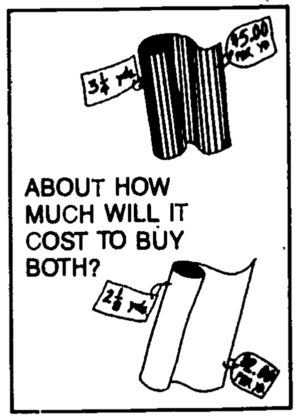
Chocolate Velvet Cake
2 1/2 cups flour

How much flour will be needed to make three Pineapple Swirl Cakes and two Chocolate Velvet Cakes?

Correct Answer: 12

	Percentage Correct 1978	1982
Age 13	34.2%	36.2%
Age 17	67.1	60.0

B.



Correct Answer: $20.50

	Percentage Correct 1982
Age 13	31.3%
Age 17	53.9

Table 3.3.
Percentages of Success on Two Nonroutine Problems, Ages 9 and 17, 1978 and 1982

A. Jason bought 3 boxes of pencils. What else do you need to know to find out how many pencils he bought?

Correct Answer: How many pencils were in each box

	Percentage correct 1978	1982
Age 9	28.4%	35.0%

B. Which of the following numbers could be written in the form 4m + 3 where m is a counting number?

☐25 ☐28 ■31 ☐80

	Percentage Correct 1978	1982
Age 17	32.4%	27.9%

knowledge of elementary algebra, together with a trial-and-error testing of the answer choices, should have resulted in considerably higher performance than the level of chance (i.e., random guessing).

Even the 13-year-olds, who made significant gains on routine problem solving, showed no change in their performance on nonroutine problems. Data from several of the problems suggest that students do not carefully analyze the problems they are asked to solve. The errors made on several of the problems suggest that students generally try to use all the numbers given in a problem statement in their calculation, without regard for the relationship of either the given numbers or the resulting answer to the problem situation.

Although this assessment was not designed to test hypotheses about problem solving, the findings provide partial answers to two interesting and important questions about the relationship of mathematical problem solving to computational skill and to understanding.

What Is the Relationship Between Computational Skill and Problem Solving?

Logic suggests that growth in a computational skill is a necessary condition for growth in the ability to apply that skill to a problem. However, it is also clear that computational skill alone is not sufficient to guarantee successful problem solving. Consider the following problem:

> George had 3/4 of a pie. He ate 3/5 of that. How much pie did he eat?

Only 17% of the 13-year-olds and 29% of the 17-year-olds were able to solve this problem correctly. On the other hand, 60% of the 13-year-olds and 66% of the 17-year-olds were able to solve a straightforward computation problem similar to $7/8 \times 3/2$.

These results suggest that students can mechanically compute the product of two fractions, but they may have little understanding of the relationship between fraction multiplication and physical situations that embody that operation. An indication of the robustness of this phenomenon may be seen by examining Table 3.4. The results indicate that even 17-year-olds with substantial mathematical experience may not understand the relationship between fraction multiplication and the situation presented in this problem.

What Is the Role of Understanding in Mathematical Problem Solving?

Data from this assessment suggest that students may not understand the problems they solve. Most of the routine verbal problems can be solved by mechanically applying a computational algorithm. In such problems, there is no need to understand the problem situation or why the particular computation is appropriate or whether the answer is reasonable. However, when students are given nonroutine problems in which those and other considerations are important, they do less well.

One of the first steps in understanding a problem is to identify the unknown. On several exercises, students gave answers suggesting that they had routinely performed the correct calculation without analyzing the problem sufficiently to determine the unknown. For example, 13-year-olds were given the following problem:

> An army bus holds 36 soldiers. If 1,128 soldiers are being bussed to their training site, how many buses are needed?

About 70% of the students performed the correct calculation, but about 29% gave the

Table 3.4.
Problem-Solving Skill on Two Exercises by Amount of Coursework, Age 17, 1982

A. George had 3/4 of a pie. He ate 3/5 of that. How much pie did he eat?

B. 7/8 X 3/2

Students' Level of Coursework	Percentage Correct A.	B.
Algebra 2	48.9%	84.4%
Algebra 2	31.6	73.9
Geometry	26.5	65.0
Algebra 1	23.7	60.9
Algebra 1	22.7	54.0

exact quotient (including the remainder) and another 18% ignored the remainder. These answers reveal a failure to understand the problem situation and the nature of the unknown. Those who gave the exact quotient response ignored the need for a whole number of buses and those who ignored the remainder failed to provide transportation for all the soldiers.

These results, together with the findings on problems concerning missing or extraneous data, suggest that more attention needs to be given to increasing students' understanding of mathematical problems. Students must be given opportunities to reflect on the problem situation and the relationships among the physical situations, the data, the unknown, the computation and the answer.

Attitudes Related to Problem Solving and Mathematics as a Discipline

This assessment included a number of items designed to assess students' attitudes toward problem solving and mathematics, and their perceptions of various aspects of learning mathematics. Table 3.5 contains a summary of students' attitudes toward problem solving.

In general, response frequencies are quite similar for 13- and 17-year-olds and indicate a generally favorable attitude toward various aspects of problem solving or (to some degree) a knowledge of what the desired responses are. There are some interesting aspects of the students' response pattern. For example,

Table 3.5.
Teenagers' Beliefs and Attitudes Toward Problem Solving, 1978 and 1982

		Percentage Responding					
		Disagree		Undecided		Agree	
Statement	Age	1978	1982	1978	1982	1978	1982
I feel good when I solve a mathematics problem by myself.	13	3.2%	5.1%	6.7%	7.4%	89.7%	87.0%
	17	2.5	3.6	7.0	6.1	90.2	90.1
There is always a rule to follow in solving mathematics problems.	13	5.9	4.6	5.0	4.7	88.3	90.4
	17	8.1	5.4	3.7	5.1	87.5	89.2
Knowing how to solve a problem is as important as getting a solution.	13	4.3	3.3	8.0	7.4	87.3	89.2
	17	2.7	3.1	4.8	3.9	91.9	92.7
Knowing why an answer is correct is as important as getting the correct answer.	13	4.4	4.6	7.3	6.9	87.9	87.8
	17	2.9	3.3	3.9	4.6	92.8	92.0
Trial and error can often be used to solve a mathematics problem.	13	13.3	13.9	30.8	33.1	55.6	52.3
	17	10.1	9.8	19.5	18.3	70.1	71.6
Exploring number patterns plays almost no part in mathematics.	13	63.6	66.2	21.3	22.3	13.7	10.2
	17	68.5	68.2	22.1	22.4	8.1	8.8

students felt very strongly that mathematics always gives a rule to follow to solve problems. Yet, they feel just as strongly that knowing how to solve a problem is as important as getting a solution and that knowing why an answer is correct is as important as getting the correct answer. This latter belief in the importance of understanding is encouraging.

Table 3.6 summarizes students' attitudes toward mathematics as a discipline. It is apparent that students do not have an accurate picture of mathematics as an intellectual activity, perhaps because of the way they have experienced mathematics in the classroom. For example, almost half of the teenaged students agree that learning mathematics is mostly memorizing, and only about half of them disagree that mathematics is made up of unrelated topics or that new discoveries are seldom made in mathematics. These attitudes are not surprising since, in fact, most of their mathematics learning has involved memorizing and has not made them aware of new discoveries or interrelationships among the mathematics topics they have studied (Fey, 1979).

It is interesting to note that, despite the fact that almost half of the students view mathematics as mostly memorizing, three-fourths of them agree that mathematics helps a person to think logically and more than three-fifths of them agree that justifying the statements one makes is an extremely important part of mathematics. These latter attitudes may reflect the beliefs of their teachers or a more general social view, rather than attitudes emerging from their own experience with school mathematics.

Table 3.6.
Students' Beliefs and Attitudes Related to Mathematics as a Discipline, Ages 13 and 17, 1978 and 1982

		Percentage Responding					
		Disagree		Undecided		Agree	
Statement	Age	1978	1982	1978	1982	1978	1982
Learning mathematics is mostly memorizing.	13 17	32.7% 39.7	35.9% 41.7	18.7% 14.5	17.0% 11.7	47.7% 44.5	46.5% 46.2
Doing mathematics requires lots of practice in following rules.	13 17	11.6 7.9	13.3 9.1	11.2 11.5	14.2 11.6	76.8 80.2	72.4 79.1
Justifying the mathematical statements a person makes is an extremely important part of mathematics.	13 17	4.2 4.9	6.7 4.7	31.2 27.6	30.7 25.9	64.5 67.4	62.2 69.1
Mathematicians work with symbols rather than ideas.	13 17	24.2 34.5	28.5 34.5	43.4 37.3	42.4 35.1	32.1 27.8	28.8 30.2
Mathematics is made up of unrelated topics.	13 17	49.4 58.5	52.3 61.7	32.2 29.2	32.4 26.9	18.0 11.7	14.7 11.1
Mathematics helps a person to think logically.	13 17	6.4 7.9	5.6 6.7	19.8 15.3	19.2 14.1	73.1 76.4	74.2 78.3
New discoveries are seldom made in mathematics.	13 17	41.4 52.4	43.7 51.0	22.0 28.5	24.7 25.0	35.4 18.7	30.0 22.9

Chapter 4

Computers and Technology

In the last few years, many elementary and junior high schools have started programs that expose students to computers. This exposure may take the form of a short one- or two-week course or an enrichment activity. In high school, students' exposure to computers is usually limited to a semester or year-long course. Although some mathematics or science teachers use computers in their courses, this is not yet common practice. Thus, only high school students who elect to take a full semester course are likely to have any contact with computers. In contrast, it appears that many 13-year-olds are getting at least limited hands-on experience with computers.

The data summarized in Table 4.1 confirm that there has been a significant increase in computer usage in schools.

Some caution is necessary in interpreting these results as some students may have interpreted the term "computer" to include calculators and/or video games. With this caution in mind, some observations can be made. The number of students having access to computers for

Table 4.1.
Availability and Use of Computers, Ages 13 and 17, 1978 and 1982

	Percentage Answering Yes			
	Age 13		Age 17	
Question	1978	1982	1978	1982
Do you have access to a computer terminal in your school for learning mathematics?	12.2%	22.7%	24.3%	49.3%
Do you know how to program a computer?	8.2	19.9	11.8	21.5
Have you studied mathematics through computer instruction?	14.4	23.5	12.2	18.9
Have you ever used a computer to solve a mathematical problem?	55.9	65.6	45.9	51.1
Have you ever written a computer program to solve a mathematics problem?	28.4	39.6	18.1	24.2
Have you ever used a computer to play a game?	39.8	80.4	49.8	80.1
Have you ever written a computer program to play a game?	21.1	41.7	13.4	26.3

learning mathematics doubled in the four years between 1978 and 1982. Almost a fourth of the 13-year-olds and half of the 17-year-olds now say they have access to a computer in school. During this same period, the number of 17-year-old students who reported that they completed a course in computer science also doubled (Table 1.3), as did the number who said they know how to program a computer (Table 4.1). Although almost twice as many 17-year-olds as 13-year-olds reported that they have access to a computer in school, 13-year-olds reported about the same level of computer use as 17-year-olds.

Thirteen- and 17-year-olds exhibited generally positive attitudes towards computers. About three-fourths of the students at both ages thought that computers were useful for teaching mathematics and make mathematics more interesting. Some 80% also believe that a knowledge of computers would help a person get a better job, and 50% to 60% believe that computers would probably create as many jobs as they eliminate. Not only were responses very positive in these areas, they also were generally up about 10 percentage points from the last assessment.

The results summarized in Table 4.2 indicate that a substantial number of students continue to hold a variety of misconceptions about what computers can do and how they work. Although most students recognized that computers store instructions and information, many did not realize that computers require special languages or that they are suited for doing repetitive, monotonous tasks. One-third of the 13-year-olds and one-fifth of the 17-year-olds believed that computers have a mind of their own.

Mathematics for Tomorrow

The technological innovations of the last decade are transforming the ways in which people use mathematics. The mathematics needed today is not the mathematics that was needed a century ago, but for the most part, that is the mathematics still being taught in schools.

Certainly, we cannot abandon the teaching of computational skills, but we need to seriously rethink what level of computational proficiency is necessary and what kinds of skills should be emphasized. Some shifts in emphasis can be observed in the assessment results. Between the second and third assessments, there was a general increase in performance on exercises involving computation with decimal fractions and a slight decrease in performance in computation with common fractions. But this simply represents a slight change in emphasis

Table 4.2.
Knowledge About Computers, Ages 13 and 17, 1978 and 1982

	Percentage Who Agree or Strongly Agree			
	Age 13		Age 17	
	1978	1982	1978	1982
Computers store instructions and information.	84.7%	91.3%	91.5%	94.1%
Computers require special languages for people to communicate with them.	51.5	55.8	60.5	64.5
Computers are suited for doing repetitive, monotonous tasks.	37.5	38.7	63.1	54.5
Computers have a mind of their own.	29.2	33.8	17.1	20.9

on computational skills that have traditionally been included in the mathematics curriculum. More radical changes are called for.

Time devoted to developing skills in calculating with long columns of numbers is time that cannot be spent developing skills and understandings that may be more critical in today's world. We must decide which has higher priority.

For example, some assessment results indicate that, in spite of the extensive instruction provided on whole number division, fewer than 60% of the 13-year-olds are reasonably proficient in division. This at least raises some serious questions as to whether the time spent drilling on division is a productive use of time and effort that might otherwise be devoted to other topics. It certainly is clear that our current approach to teaching division is not effective for many students.

The division algorithm, like most of the other algorithms that we teach in school, is designed to produce rapid, accurate calculation procedures. Given rapid, accurate calculators and computers, it does not seem that this should still be a high priority. Certainly, computation is important; but what is needed are algorithms that students will remember and will be able to generalize to new situations. Students are more likely to remember and be able to generalize and apply algorithms if they understand how they work. Thus, it would seem appropriate to begin to shift to computational algorithms that can be easily understood even if they are less efficient.

The results summarized in Table 4.3 illustrate why it is necessary to rethink our approach to computation. Most students recognized that th problem requires division, but the majority of them did not know what to do with their answer once they had divided. This is the central issue. There are machines that can do the calculations, but it is still necessary to know what questions to ask and how to use the results. The assessment results clearly show that these are the abilities that are most lacking.

As calculators and computers assume greater prominence in our lives, estimation skills become ever more critical. At all ages, the performance involving an estimation of a computation was considerably lower than corresponding computation exercises. For

Table 4.3.
Percentages of Success on Word Problem With and Without Calculator, Ages 13 and 17, 1982

Problem: An army bus holds 36 soldiers. If 1,128 soldiers are being bussed to their training site, how many buses are needed?

Response	Without Calculator†† Age 13	Using Calculator Age 13	Using Calculator Age 17
32*	23.9%	7.1%	17.7%
31.33, 31 1/3, etc.	28.9	16.2	23.8
31	17.5	25.3	37.7
Wrong operation	7.1	20.2	4.0

* Correct response.
†† Not administered to age 17 without a calculator.

example, 13-year-olds' performance on choosing an estimate for a multiplication of whole numbers was 20-30 percentage points lower than actual computation of the same problem (see Table 4.4).

Estimation requires a certain facility with mental arithmetic. In general, students were not very successful in performing calculations in their heads. For example, only about 20% of the 9-year-olds could do a problem such as 58 − 9 in their heads, while about 70% could subtract two-digit numbers using paper and pencil. Only about 55% of the 9-year-olds could mentally add two numbers like 53 and 30, in contrast to almost 90% who could add two-digit numbers involving no carrying. At age 13, the difference between mental and paper-and-pencil computation on a two-digit subtraction problem was 15 percentage points.

The situation improves at age 17, where more than 85% of the students can perform simple addition and subtraction problems mentally. Mental multiplication of two-digit numbers like 90 × 70, however, yielded lower results: about 55% were successful. Performance drops another 10 points for a mental computation such as 4 × 625.

These results further illustrate the importance of a firm understanding of numbers and operations. With such a background, a 9-year-old should be able to reason that 9 from any number can easily be found by taking away 10 and adding 1. Likewise, the older students would be able to use their ability with whole numbers to make reasonable estimates of computation with decimals and fractions.

Computers also have implications for the teaching of algebra. A number of studies have suggested that students are more successful in learning to manipulate algebraic equations than they are in learning to produce equations in a meaningful way (see, for example, Clement, 1982). Computers can solve equations, but students still need the ability to generate mathematical expressions using variables in order to program a computer.

Table 4.4.
Percentages of Success on Parallel Estimation and Computation Exercises, Age 13, 1982

Exercise Type	Percentage Correct Computation	Percentage Correct Estimation
Subtraction of whole numbers	81.6%	53.5%
Multiplication of whole numbers	74.6	50.3

Chapter 5

Minorities and Mathematics

An important objective of the National Assessment is to describe the performance of major groups within the national population. These groups are identified on the basis of region of the country, sex, race, size and type of community, racial composition of the school, parental education and other characteristics. Following is a discussion of the results for White, Black and Hispanic students. Other minority racial groups are not included because they were not sampled in sufficient numbers to provide reliable measures of performance. Approximately 79-81% (depending on age) of the sample were White students, 12-14% Black, 5% Hispano and 2% were identified as other minorities. These classifications were based on appearance and surname.

Although the focus of this section is on Black and Hispanic students, selected results based on racial composition of the school, achievement class and type of community have been included because they provide an additional perspective on minority results. Racial composition of the school is based on the percentage of Whites enrolled in the school, as reported by the principal. Achievement class is reported by quartile and is determined by the students' performance on assessment booklets. The type-of-community classification is based on the occupational profile of the area served by a school as well as by the size of the community in which the school is located (see Appendix C for more detail on all these groups). Although these variables represent an imperfect measure of students' background, patterns can still be identified that might assist in understanding any racial differences in performance.

Mathematics Achievement and Change

Consistent with the earlier assessments, Black and Hispanic students performed below the national level while their White counterparts performed above the national level. The results in Table 5.1 indicate that, at age 9, Black students were about 11 percentage points below the national level. At age 13, the difference was 12 percentage points; at age 17, it was 15 percentage points. A somewhat similar, but less pronounced, pattern was found for Hispanic students. At ages 9 and 13, Hispanic students were about 9 percentage points below the national level, and at age 17, the difference increased to 11 percentage points.

While the position of White, Black and Hispanic students did not change relative to national levels of performance, the rate of change for each group did. At each age, Black and Hispanic students made greater gains in performance (even if not statistically significant) than their White counterparts. At ages 9 and 17, Black students registered a slightly higher gain than Hispanic students.

This pattern continues when the data are examined by the four cognitive levels within which the mathematics exercises have been categorized. As shown in Appendix Tables D.1-D.4, Black and Hispanic students appear to have made substantial gains on the lowest cognitive level, knowledge. These gains are generally greater than their White counterparts'. There was little change in the performance in the higher cognitive levels of understanding and application for 9- and 17-year-old Black and Hispanic students. At age 13, however, Black and Hispanic students made substantial gains in both understanding and application.

Results by Student Background Variables

In an earlier NAEP report (Holmes et al, 1982), it was found that certain measures of student background variables were related to changes in performance for Black and White students. Consistent with that study, the results of the third assessment indicate that achievement class, racial composition of the school and type of community are related to mathematics achievement. Table 5.2 presents the change in average performance for students according to these variables. These data are particularly interesting since, at each age assessed, about 40-60% of the Black and Hispanic students were found in the lowest quartile of

Table 5.1.
Mean Performance Changes for White, Black and Hispanic Students, 1978 to 1982, Ages 9, 13 and 17

	Age 9 Average Performance		Change Perf.	Age 13 Average Performance		Change Perf.	Age 17 Average Performance		Change Perf.
	1978	1982		1978	1982		1978	1982	
Nation	55.4%	56.4%	1.0%	56.6%	60.5%	3.9%*	60.4%	60.2%	-0.2%
White	58.1	58.8	0.7	59.9	63.1	3.2*	63.2	63.1	-0.2
Black	43.1	45.2	2.1	41.7	48.2	6.5*	43.7	45.0	1.3
Hispanic	46.6	47.7	1.1	45.4	51.9	6.5*	48.5	49.4	0.9

* Change is significant at the .05 level.

Table 5.2.
Mean Performance Changes for Achievement Classes, Percent-White Schools and Types of Community, Ages 9, 13 and 17

	Age 9 Average Performance		Change Perf.	Age 13 Average Performance		Change Perf.	Age 17 Average Performance		Change Perf.
	1978	1982		1978	1982		1978	1982	
Achievement class									
Lowest quartile	32.6%	34.4%	1.8%*	34.2%	39.8%	5.6%*	37.4%	37.9%	0.5%
Highest quartile	75.6	76.2	0.6	78.2	80.1	1.9*	82.3	81.7	-0.6
% White school									
0-59% white	46.4	48.8	2.4	45.5	53.6	8.1*	47.5	52.3	4.8*
60-100% white	57.6	58.6	1.0	59.6	62.4	2.8*	62.4	62.4	0.1
Type of community									
Disadvantaged urban	44.4	45.5	1.1	43.5	49.3	5.8*	45.8	47.7	1.9
Advantaged urban	65.0	66.3	1.3	65.1	70.7	5.6*	70.0	69.7	-0.3

* Change is significant at the .05 level.

achievement class, about two-thirds were found in the 0-59% White schools and from 15-30% in the disadvantaged-urban community schools. The White students were more evenly distributed among the achievement classes, but about 10% were found in the 0-59% White schools and 2 or 3% were in the disadvantaged-urban (Appendix E).

At ages 9 and 13, students in the lowest quartile made significant gains in their mathematics performance. Teenaged students in the 0-59% White schools (heavily minority) showed substantial gains in performance, whereas those in the 60-100% White schools did only at age 13. Of particular significance is the 8 percentage point gain for 13-year-olds in heavily minority schools—a gain higher than either the national average or any racial group.

Course Background

As indicated by the achievement data, the difference in performance between Black and White students increased with age. By age 17, Black students performed 18 percentage points below their White counterparts. One explanation of this difference is the underrepresentation of Black students in the more advanced mathematics courses. Table 5.3 shows that, while there is a slight increase in the enrollment of Black students in Algebra 1 courses and above, the differences between Black and White students are still substantial. It also seems that performance is directly related to the amount of mathematics studied. As the data in Table 5.4 indicate, for each additional amount of course work taken, there is a substantial increase in the level of performance for both Black and White students.

Performance of Hispanic students of each level of course taking are not reported because when the Hispanic population is broken down into these subgroups, the number of individuals in each cell is too small to make stable estimates.

Table 5.3.
Changes in Percentages of Black and White Students Taking Mathematics Courses, Age 17

Course	Percentage of 17-Year-Olds Who Have Taken at Least 1/2 Year	
	White	Black
General or business mathematics		
1978	44.7%	51.3%
1982	49.4	54.8
Prealgebra		
1978	45.6	46.5
1982	43.4	47.4
Algebra		
1978	75.4	54.5
1982	73.9	56.9
Geometry		
1978	54.9	31.2
1982	55.1	34.1
Algebra 2		
1978	39.1	24.4
1982	40.9	27.7
Trigonometry		
1978	13.8	6.8
1982	14.9	8.2
Precalulus/ calculus		
1978	4.0	2.8
1982	4.4	2.8
Computer		
1978	4.9	5.2
1982	9.6	11.3

Implications of the Findings

It is widely recognized that certain minority groups have consistently scored below national norms in mathematics, and while the results of this assessment do not contradict this phenomenon, there is evidence to suggest that

Table 5.4.
Performance Levels of Black and White 17-Year-Olds by Highest Course Taken and Exercise Type, 1982

Level	All 17-Year-Olds	Algebra 1	Algebra 1	Geometry	Algebra 2	Algebra 2
Knowledge						
White	77.3%	65.7%	72.1%	78.5%	83.6%	88.9%
Black	62.6	56.6	61.4	66.6	71.2	75.3
Skills						
White	63.0	46.9	57.5	61.8	72.0	79.2
Black	44.2	36.6	43.4	47.7	55.0	59.1
Understanding						
White	64.7	47.7	57.5	66.4	73.6	81.1
Black	44.8	35.6	43.9	49.6	55.5	61.4
Applications						
White	45.5	30.4	38.2	44.7	53.1	63.8
Black	26.0	20.9	24.9	27.7	31.2	38.8

the gap is narrowing, especially at age 13. The implications of these findings are two-fold.

First, more Black and Hispanic students are learning mathematics. Although the gains appear to be largely accounted for by the increase in the lower cognitive levels, i.e., knowledge and skills, these gains should not be dismissed as insignificant. For it has been just this deficiency in the basic areas that has contributed so significantly to the reported mathematics illiteracy of minorities. Further, the gains found for the disadvantaged-urban students, for predominantly minority schools and for the lowest quartile students provide additional support that schools are making advantageous use of additional resources allocated to these special student populations.

Second, the improvement found for Black and Hispanic students is important but not sufficient. Not only must educators continue to stress the basic skills, they must now begin to build upon these strengths and focus on the teaching of the higher cognitive areas of understanding and application. Moreover, minorities must continue to be encouraged to enroll in advanced mathematics classes. Although there was modest improvement in the participation of Black students in mathematics courses, they still lag behind White students. Over half of the 17-year-old Black students had taken at least one-half year of Algebra 1, compared with about 70% of their White counterparts.

The concentrated energies and efforts of educators appear to have made a difference for minorities, and this difference is not only positive, it is encouraging. Educators should be able to view these changes as a reaffirmation of the importance of a sound mathematics education and the potential for improvement in the learning of all students.

Chapter 6

Sex Differences in Achievement

At each age level, differences between the average performance of males and females remained relatively stable between 1978 and 1982. The data in Table 6.1 indicate that, at ages 9 and 13, the overall performance of males and females is not significantly different. However, at age 17, males scored higher by about 3 percentage points.

When the percentages for the sets of items representing the four cognitive levels were examined, no clear pattern of differences in achievement was apparent at ages 9 and 13 (Table 6.2). At age 17, the average performance of males exceeded that of females on every cognitive level. Nine-year-old females did show an appreciable improvement on knowledge exercises, performing 2 percentage points higher than males. At age 13, both males and females improved at the same rate and are not significantly different from each other in performance at any cognitive level. At age 17, no statistically significant changes were found at any cognitive level.

Course Taking

Consistent with the previous assessment, there was very little difference between females and males in the mathematics courses taken in the early years of the high school mathematics

Table 6.1.
Changes in Mean Performance for Males and Females, 1978 and 1982, Ages 9, 13 and 17

	Age 9 Average Performance			Age 13 Average Performance			Age 17 Average Performance		
	1978	1982	Change	1978	1982	Change	1978	1982	Change
Male	55.3%	55.8%	0.5%	56.4%	60.4%	4.0%*	62.0%	61.6%	-0.4%
Female	55.5	56.9	1.4*	56.9	60.6	3.7*	58.8	58.9	0.1

* Change is significant at the .05 level.

sequence. Table 6.3 shows the percentages of 17-year-old males and females who reported that they had enrolled for at least half a year in specific mathematics courses. The only appreciable change was that both males and females increased their participation in computer courses.

Even when course background was held constant (Table 6.4), achievement differences still existed. For each course background category, male achievement exceeded that of females. Not surprisingly, for additional course work, there was an increase in the level of performance for both males and females.

Conclusions

As was found in the 1977-78 NAEP mathematics assessment, few sex differences in achievement exist for 9- or 13-year-olds. The only notable exception is that females at age 9 tended to outperform males on exercises measuring knowledge, and this difference increased slightly in 1982.

Table 6.2.
Changes in Mean Performance for Males and Females, by Exercise Type, 1978 and 1982, Ages 9, 13 and 17

	Average Performance and Change								
	Age 9			Age 13			Age 17		
	1978	1982	Change	1978	1982	Change	1978	1982	Change
Knowledge									
Male	66.4%	67.4%	1.0%	69.4%	73.8%	4.4%*	75.9%	75.9%	0.0%
Female	67.4	69.3	1.9*	69.3	73.8	4.5*	73.5	73.9	0.4
Skills									
Male	49.7	50.2	0.5	52.8	57.0	4.2*	60.9	61.1	0.2
Female	49.9	51.1	1.2	54.4	58.2	3.8*	58.5	58.9	0.4
Understanding									
Male	42.3	41.0	-1.3	56.6	60.8	4.2*	64.1	63.1	-1.0
Female	41.0	41.4	0.4	56.5	60.2	3.7*	59.8	60.0	0.2
Applications									
Male	39.6	40.0	0.4	44.0	46.1	2.2*	45.9	44.6	-1.3
Female	38.6	39.2	0.6	42.8	45.1	2.3*	41.3	40.2	-1.1

* Change is significant at the .05 level.

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Table 6.3.
Changes in Percentages of Males and Females Taking Mathematics Courses, Age 17

Course	Percentage of 17-Year-Olds Who Have Taken at Least 1/2 Year	
	Males	Female
General or business mathematics		
1978	44.3%	46.8%
1982	47.3	52.6
Prealgebra		
1978	46.4	45.3
1982	44.3	44.5
Algebra 1		
1978	70.7	73.6
1982	69.4	72.2
Geometry		
1978	52.1	50.5
1982	51.8	51.8
Algebra 2		
1978	37.8	36.1
1982	38.9	38.0
Trigonometry		
1978	14.7	11.1
1982	15.0	12.7
Precalulus/ calculus		
1978	4.7	3.1
1982	4.7	3.6
Computer		
1978	5.9	4.1
1982	11.1	8.6

Thirteen-year-old males and females made similar gains in achievement between assessments. Although few changes in achievement were found for 17-year-olds between the two assessments, significant sex differences favoring males on high level mathematics did persist. In 1978, males did better on exercises measuring skills, understanding and applications—and these differences continue. Even when the amount of course work is taken into account so that males and females with similar mathematics backgrounds are compared, males perform somewhat better and continue to have an edge on mathematical tasks requiring understanding and applications. Additionally, 17-year-old males still tend to take more advanced mathematics courses than females.

These recent results suggest there still may be a continuing inequity in mathematics education. Only slightly more females are taking mathematics courses in 1982 than in 1978. Even adjusting for course background, females are not performing as well as males on more difficult mathematics exercises.

Reasons for these differences and possible intervention procedures have been the object of numerous research studies during the past few years. Hopefully, the findings of these studies and new programs for increasing mathematics participation and achievement will help provide an equitable education in mathematics for females and males.

Table 6.4.

Performance Levels of Male and Female 17-Year-Olds by Highest Course Taken and Exercise Type, 1982

Cognitive Level	Algebra 1	Algebra 1	Geometry	Algebra 2	Algebra 2
Knowledge					
Males	64.5%	71.8%*	79.1%*	83.4%	87.7%
Females	62.8	69.1*	75.4*	81.1	87.6
Skills					
Males	45.1	56.3	62.8*	71.2*	77.7
Females	43.8	53.8	58.2*	68.8*	76.8
Understanding					
Males	46.0	57.4*	67.5*	73.2*	80.0
Females	44.0	52.6*	62.2*	69.6*	78.7
Applications					
Males	29.6*	38.0*	45.8*	53.6*	62.9*
Females	27.0*	33.7*	40.1*	48.0*	59.8*

* Male-female difference is significant at the .05 level.

Appendix A

Other Findings About Mathematics Achievement, Course Taking and Instruction

How do the NAEP findings compare with findings from other tests? What do we know about trends in course taking, the teacher supply or computer use in the school? Following are some facts about these subjects that help put the NAEP findings into perspective.

Other Major Test Results

The NAEP findings generally square with other indicators of mathematics achievement in America.

- The Scholastic Aptitude Test (SAT) in mathematics, administered to a nonrandom sample of college-bound seniors, declined from a mean of 492 (out of 800 possible points) in 1967 to between 466 and 470, where it has remained for the last 5 years. In 1982, it was 467 (CEEB, 1982, p. 5).
- Between 1967 and 1975, the proportion of students scoring above 650 on the SAT mathematics test declined 23% (*Chronicle*, 1983, p. 7).
- The American College Testing Program (ACT), also involving a nonrandom sample of college-bound seniors, reports trends similar to the SAT and NAEP. In 1969-70, the ACT mathematics mean was 20.0 (out of 36); in 1972-73, it was 19.1; in 1977-78, it was 17.5; and in 1981-82, it was 17.2. The mean has not changed significantly since the mid-seventies.

  At the same time, the average mathematics grades for high school seniors who take the ACT have been rising. According to the ACT, the average mathematics grade of ACT test takers in 1969-70 was 2.37 (out of a possible 4.0). In 1972-73, it was 2.53; in 1977-78, it was 2.76; and in 1981-82, it was 2.77 (ACT, 1982).
- Between 1968 and the present, mean scores on Advanced Placement examinations in mathematics have increased consistently (Jones, 1981).
- Scores on levels 1 and 2 of SAT mathematics achievement tests (different from the general mathematics SAT) both increased in 1982. These positive changes may indicate that the serious mathematics students are even better today than they used to be (CEEB, 1982).

Course Taking

- Children in grades 1-6 receive about four hours per week of mathematics instruction (Weiss, 1978, p. 51).
- All young people in grades 7-9 take general mathematics and about two-thirds take 9th grade Algebra (Weiss, 1978, p. 53).

- By the end of high school, about half the college-bound students and one-third of all students have taken three years of mathematics; about two-thirds of all students have taken two years of high school mathematics; and about one-fourth of the students have taken trigonometry (NCES, 1982).
- Half of all high school graduates take no mathematics beyond 10th grade (Hurd, 1982).
- Between 1960-77, the proportion of high school students enrolled in mathematics courses declined (NSB, 1982).
- Enrollment in remedial mathematics courses in colleges and universities rose 72% between 1975 and 1980 (NSB, 1982).
- In public two-year postsecondary institutions, 42% of the mathematics courses offered are remedial (NSB, 1982).
- The 1982 graduating class of students who took the SAT indicated that they were taking more mathematics classes than other recent classes have taken. Fifty-five percent of the males and 45% of the females indicated that they had taken four years of mathematics; 14% and 9%, respectively, indicated that they had taken five or more years of mathematics (CEEB, 1982).
- The most dramatic growth in 1982 seniors' intended college study (according to the College Board) was in computer science. Since 1975, interest in that subject has quintupled. Business and commerce remains the most popular area of intended study; with 18.7% of the high school seniors expressing an interest in majoring in that area. Health and medicine follow (14.2%), then engineering (12.6%), computer science/systems analysis (7.7%), the social sciences (7.2%) and education (5%). (CEEB, 1982.)

Teacher Shortages

- In 1981, 43 of 45 reporting states indicated a shortage or a critical shortage of secondary mathematics teachers (Howe and Gerlovich, 1981).
- Experienced science and mathematics teachers left classroom teaching for nonteaching jobs at a rate of 4% per year in the years 1980 and 1981. In addition, 25% of those currently teaching have stated that they expect to leave teaching in the near future (NSB, 1982).
- The Association for School, College and University Staffing indicates that 22% of all high school teaching posts in mathematics are vacant at the present time.
- From 1971 to 1980, student teachers in science and mathematics decreased in number—threefold in science and fourfold in mathematics—and only half of these have been entering the teaching profession (NSB, 1982).
- The National Science Teachers Association reports that 26% of all secondary mathematics positions are filled by teachers who are not certified, or are only temporarily certified, to teach mathematics. This pool appears to be expanding, for among the newly employed secondary mathematics teachers, 52% were uncertified to teach science or mathematics (National Science Teachers Association, 1981).

Appendix B

Recent Federal and State Initiatives To Improve Mathematics Education

Many steps are being taken around the country to remedy the current situation in mathematics and science through a number of avenues.

Federal or National Activities

Over 50 bills relating to precollege education in mathematics, science and technology were introduced in the 97th Congress. At least as many will reappear in the 98th Congress.

All the federal approaches fall into seven general concepts:

1. **Special commissions.** All manner of special executive and legislative commissions have been proposed, with powers ranging from advisory only to power over the allocation of funds for improving mathematics and science education.
2. **Low-interest loans and loan forgiveness for students who become mathematics or science teachers.** These loans would be within or outside of available current student loan programs.
3. **Assistance to state and local agencies.** State help includes funds for program evaluation, technical assistance, courseware evaluation and teacher training assistance. Local help includes staff development, teacher training assistance and curriculum revision.
4. **Assistance to postsecondary institutions and schools of education.** This includes money for improving the teacher education curriculum, retraining teachers and developing new resources.
5. **Tax credits.** Tax credits are proposed for gifts to schools, employing teachers part-time and getting former mathematics teachers *back* into the schools.
6. **Research and development.** Various grants and incentives have been proposed to encourage research and development in instructional improvement.
7. **Demonstration projects and seed money for "lighthouse" efforts, summer institutes and course improvement.**

It is important to stress the fact that these are all *proposals;* few of them have yet to be translated into actual funded activities. Nonetheless, they demonstrate the *breadth* of approaches being considered, and they constitute considerable promise that something is definitely going to be done.

Other national efforts to improve mathematics education are worthy of note. In December 1982, the Council of Chief State School Officers issued a policy statement entitled, "The Need for a New 'National Defense Education Act.' " The paper asserts that "there is a federal role in support of improved instruction in mathematics and science," and it outlines elements of that role, including:

- Creating incentives for persons to become mathematics teachers

- Providing funds for inservice training, updated equipment and the creation of support groups (e.g., business/education partnerships)
- Encouraging new mathematics curricular sequences that match the stages of children's intellectual development
- Fostering cooperation, rather than competition, between the private and military sectors as they both seek highly skilled personnel
- Supporting programs that insure that the needs of women and minorities are being adequately met as all of the above actions materialize.

The National Council of Teachers of Mathematics (NCTM) has also made a statement about the federal role in mathematics education. Among its key recommendations are the following:

- "**Using technology.** The increasing availability of computing technology requires a reexamination of the mathematics curriculum and an adaptation of instructional methodologies. Using calculators and computers in imaginative ways to explore, discover and develop mathematical concepts will require new materials that take full advantage of the potential of the new media. There is simply not an adequate pool of talent to enable local districts to produce high-quality courseware adapted to the new technologies. Prototype materials must be developed and tested to provide a resource from which local schools may draw in their attempts to incorporate technological advances. Fundamental research is needed to explore the potential of such technology to improve learning and instruction. Local initiatives will be more efficient and effective if reliable resources are available."
- "**Developing materials that emphasize problem solving.** Most current materials are inadequate to support or implement a problem-solving approach. Although there is general agreement within the profession and among the concerned public that the mathematics curriculum should be reorganized to focus on problem-solving skills, there is no clear agreement on exactly what that would mean or how it should be done. Many local districts are attempting such a reorganization. These efforts are largely uncoordinated. They represent a wasteful duplication of effort. A variety of examples of problem-solving materials must be developed and disseminated as models from which local districts may draw in revising their local curriculum. The scope of what constitutes "good problems" or teachable problem-solving strategies at each school level must be verified through research.
- **Increasing mathematics study.** When students discontinue the study of mathematics early in their high school program, many options in college or vocational training programs or in job opportunities are foreclosed. More mathematics should be required, but it is not enough simply to increase the number of years of study. The program of courses must be modified to accommodate a larger variety of student needs. This challenge to develop a more diverse set of course offerings, more fully differentiated according to students' needs, will present a major problem for curriculum developers. Few local school districts have adequate resources to develop such alternatives carefully. Prototype programs must be made available to such districts. Exemplary programs that exist or that may be developed in response to this challenge must be identified and disseminated. The NCTM, through its membership, its affilliated groups and its publications program, would provide a network for such identification and dissemination. Appropriate federal roles for addressing the problems of mathematics education in the 1980s can be identified under the following three categories: materials, research and teachers. . . .We are concerned here with ways in which federal effort may provide a catalyst or facilitate the collective efforts of the states, local districts and individual teachers. For all these

categories, federal attention can help create and maintain a public awareness of the issues and problems of mathematics education."

- "**Materials.** There are many recommended actions. . .that require the development of instructional materials. Innovative materials are needed to facilitate the use of such technology as computers, calculators and video discs. Alternative materials are needed to provide three years of secondary school mathematics for every student. Examples of such alternatives could be consumer-oriented, statistics-related or computer-based approaches to mathematics. Materials that fully incorporate genuine problem solving and applications are not now available.

  "The federal role in assisting with the develompent of instructional materials should have two general thrusts: first, the identification and writing of materials for which the private sector cannot take the risk or for which the individual schools do not have the resources; and second, the dissemination of innovative materials. Each of these two areas has broad potential impact for a relatively small investment of federal resources, especially when coordinated with the efforts of local schools, the private sector and professional organizations."

- "**Research.** High priority should be given to research on how children learn mathematics; the nature of problem solving and how problem solving can be developed in children; problems of learning in mathematics; learning mathematics in the context of various technologies (e.g., computers, calculators and video discs); how to teach mathematics to enhance problem solving, applications and the use of technologies; and other topics relevant to mathematics education. . . .NCTM has a unique role in the reporting, interpretation and dissemination of research through its journals (the *Journal for Research in Mathematics Education*, the *Mathematics Teacher*, the *Arithmetic Teacher*), its other publications and its professional meetings and conferences. It is critical that, in the 1980s, research on the teaching and learning of mathematics be supported to assist with providing the best possible mathematics education.

  "There is very little local or state support of research on the teaching and learning of mathematics. This fact, coupled with the central importance of mathematics learning in our society, argues for a federal role in supporting research on the teaching and learning of mathematics."

- "**Teachers.** There is a mathematics teacher shortage. Its resolution is a decade-long problem. Further, the necessary changes in mathematics education in the 1980s call for more mathematics teachers (even with projected declining school enrollments) and more extensive education of teachers. Local education agencies cannot solve the teacher shortage by their own action except to assign teachers not fully certified in mathematics to mathematics classes.

  "We believe there is a federal role to assist local school districts and colleges of teacher education to address the issues of mathematics teacher education. . . ." (NCTM, 1981).

State Activities

Many governors mentioned the mathematics situation or referred to the need to improve education for a high technology economy in their 1983 state-of-the-state addresses. In addition, many state legislatures are dealing with bills aimed at improving mathematics education, attacking the teacher shortage or addressing various technology issues. Specifically,

- At least 15 states are considering legislation addressing teacher shortages either through differential pay of mathematics teachers, loan forgiveness programs or a variety of incentive programs.
- At least seven states are considering raising graduation requirements in mathematics.

- Five states are considering legislation giving tax breaks to businesses that donate computer equipment to the schools.
- Seven states are debating various means of encouraging or regulating computer use in the schools or making computer literacy a curricular requirement.

A recent ECS survey of state initiatives to improve economic growth through an enhanced education system found considerable statewide activity that could improve mathematics education in the years ahead (ECS, 1982). Although each state reported a unique set of initiatives for each problem area, most of the responses fit into one of three categories:

- **Task forces** to study the issues, define the problems, needs and opportunities, and recommend new policies and programs
- **Programs to enhance quality and quantity** of curriculum, facilities, students and teachers
- **Programs to encourage broader involvement in education** by citizens, business and industry

A brief overview of the types of activities in each of these areas is presented below. In some cases, only a few states are actively pursuing particular paths; nevertheless, their activities are worth noting if only to present the wide range of approaches being pursued across the country. Details are available from ECS upon request.

Task Forces

The task forces reported in the survey dealt with nearly every conceivable education issue facing the states. Task force agendas included consideration of:

- Programs to achieve excellence in the high schools, including reevaluating the nature and role of the high school, determining appropriate goals and curricula for various categories of students and revising high school graduation requirements
- The structure of the teaching profession, and nature of teacher training programs and the future supply and demand of teachers, especially in science and mathematics
- Computer literacy goals and curricula for students in grades K-12 and for teachers and administrators
- A central statewide advisory service for computer software
- Mechanisms to involve business, education and labor in educational planning and policy, decision making and evaluation
- Strategies to expand a state's economy and employment rate through improvements in education and training
- Manpower projections and vocational and technical education to meet the needs of the state's industries

Enhancing Program Quality and Quantity

Every state has implemented programs to improve educational quality. To increase the effectiveness of educational systems, states are:

- Developing new or revised curricula, especially in science, mathematics and computer literacy in grades K-12. Most states have developed curriculum guides, statements of minimal competencies or curricular goals that students should meet upon graduation. Many states are moving beyond minimal competencies and are requiring "standards of excellence" and strengthening existing programs
- Emphasizing a shift in the curriculum to teach concepts, applications, problem solving and critical thinking
- Providing technical assistance (using on-site workshops or regional centers) in such areas as computer literacy, clarifying course goals,

curriculum design, student and program evaluation and the use of the results of research on effective education practices

- Introducing college level courses in high school (such as Calculus)
- Introducing computer assisted instruction (CAI) or national information systems such as Project BEST to increase student learning and achievement
- Purchasing new equipment, including computers and software
- Reducing the number of "at-risk" students by providing additional training and job placement services
- Establishing local pilot programs to develop strategies that can be effective on a statewide basis

Enhancing Student Quality and Quantity

States have attempted to increase the number of students taking science and mathematics and to improve their skills, abilities and capabilities by:

- Increasing high school graduation requirements so that students are required to take three to four years of high school mathematics, two to three years of science, four years of English and one or more years of a foreign language
- Increasing the entry requirements in science and mathematics in state supported colleges and universities
- Lengthening the school year
- Implementing student testing for assessment and minimal competency purposes. Some testing is aimed at minimal competencies for graduation; other state assessment programs cover a wide variety of learning areas and are used to locate student or program weaknesses
- Creating special schools or specific centers to promote learning by gifted students in areas such as science, mathematics and computer literacy
- Initiating child development programs for preschool children, home-based programs for parents of preschool children and workshops for parents on early childhood development

Enhancing Teacher Quality and Quantity

Issues regarding teacher quality and shortages are being addressed by:

- Revising teacher certification requirements. Depending on the state, teachers may be required to pass an entry exam prior to being enrolled in an undergraduate teacher education program, pass a written examination before being certified as a teacher, pass practice teaching standards, take additional courses in the subject area, successfully teach for two years after graduation and obtain a certain amount of additional training every five years to keep a teaching certificate current.
- Providing tuition and scholarship programs. In some instances, states have set aside funds to assist teachers in obtaining training in areas where there are shortages of teachers, such as in science and mathematics.
- Making student loans available to prospective teachers. In many cases, the loans are forgiven if the teachers remain in the state to teach for several years.
- Providing 12-month contracts in critical areas (science, mathematics, vocational education). The summer months are used for curriculum development, retraining, course preparation and special group instruction.
- Providing internship to alleviate the teacher shortage in science and mathematics. In

several states, science and mathematics teachers work for private industry as "interns" during the summer, helping them to increase their salary and learn new skills. On the other side, many large corporations are allowing their qualified professionals to teach science or mathematics classes several hours per week in local schools.

- Providing inservice programs so that teachers' skills can be updated and improved. Some states are initiating summer institutes at universities where teachers can enroll to update their skills. Many states offer traveling workshops to school districts where technical assistance is provided, especially in the teaching of computer literacy.
- Increasing teachers' salaries, either across the board or in areas of teacher shortages.
- Working with high school guidance counselors to help recruit good students into the field of teaching.
- Assessing the present and future teacher supply and demand for future planning.

Broader Involvement of Citizens, Business and Industry

A number of initiatives have been undertaken to encourage participation by broader segments of the community in the educational process. Initiatives reported are:

- Statewide and local task forces involving business, industry and labor leaders as well as parents and concerned citizens in all areas of educational planning, decision making, implementation and evaluation
- Advisory councils for vocational education programs
- Efforts to obtain input on educational priorities from a broad cross-section of interested parties
- The matching of state funds with private sector donations to secure faculty, equipment and up-to-date programs at state institutions
- Statutory changes to promote cooperative research and development efforts between colleges and universities and industry
- Training of school personnel by industry technicians to use state-of-the-art equipment
- Adopt-a-school and partnership programs with business and industry; citizen volunteer programs in schools
- Team teaching using teachers and industry employees
- Customized job training to meet specific needs of industry within a state
- Workshops to involve parents in helping their children learn to read

Clearly, a good deal is happening and the movement to upgrade the quality of education is broad-based. All that remains is for readers of this report to select the activity they would most like to play a role in and commit themselves to its success.

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Appendix C

Definitions of Reporting Groups

Sex

Results are reported for males and females.

Race/Ethnicity

Results are presented for Black, White and Hispanic students.

Type of Community

Communities in this category are defined by an occupational profile of the area served by a school, as well as by the size of the community in which the school is located. About one-third of the students fall into the categories listed below. Results for the remaining two-thirds are not included in this report, since their performance is similar to that of the nation.

Advantaged-urban communities. Students in this group attend schools in or around cities having populations greater than 200,000 and a high proportion of residents in professional or managerial positions.

Disadvantaged-urban communities. Students in this group attend schools in or around cities having populations greater than 200,000 and a relatively high proportion of residents on welfare or not regularly employed.

Rural communities. Students in this group attend schools in areas with populations under 10,000 and many residents who are farmers or farm workers.

Grade in School

Results are reported for 9-year-olds in grade 3 or 4; 13-year-olds in grade 7 or 8; and 17-year-olds in grade 10, 11 or 12.

Percent-White Student Enrollment

Results are presented for students in schools with relatively heavy minority enrollments (0-59% White) and in schools with heavy White enrollments (60-100% White).

Achievement Class

Results are presented for students in four quartiles of achievement, based on their performance on a booklet of NAEP exercises. Particular attention is paid to students in the lowest quartile (the bottom 25%) and the top quartile (the top 25%).

Other student background variables such as region of the country, size of community or parental education are not discussed in this report.

Appendix D

Back-Up Tables for Exhibits 2 Through 7

Table D.1.
National Percentages of Success on Exercises Assessing Mathematical Knowledge, Skills, Understanding and Applications, Nation and Selected Groups, Age 9

	Knowledge 1978	Knowledge 1982	Change 1978-82	Skills 1978	Skills 1982	Change 1978-82	Under-standing 1978	Under-standing 1982	Change 1978-82	Appli-cation 1978	Appli-cation 1982	Change 1978-82
Nation	66.9	68.3	1.4	49.8	50.6	0.8	41.6	41.2	-0.4	39.1	39.6	0.5
White	69.6	70.8	1.2	52.5	53.1	0.6	44.2	43.4	-0.8	41.8	42.4	0.6
Black	54.3	57.8	3.5*	37.1	38.7	1.6	30.5	31.4	0.9	27.6	27.0	-0.6
Hispanic	58.7	58.7	0.0	41.3	43.8	2.5	32.6	32.4	-0.2	29.9	30.5	0.6
Low quartile	42.5	44.5	2.0*	27.8	30.1	2.3	21.6	22.1	0.5	17.5	18.5	1.0
Mid-low quartile	64.1	65.7	1.6	44.9	45.6	0.7	34.4	33.5	-0.9	31.0	31.5	0.5
Mid-high quartile	75.4	76.6	1.2	56.4	56.3	-0.1	46.8	45.0	-1.8	44.7	45.4	0.7
High quartile	85.6	86.4	0.8	69.9	70.6	0.7	63.7	64.1	0.4	63.1	63.1	0.0
0-59% white	58.1	60.7	2.6	40.6	43.2	2.6	33.0	34.7	1.7	30.4	31.4	1.0
60-100% white	69.0	70.7	1.7	52.0	52.9	0.9	43.8	43.2	-0.6	41.2	42.1	0.9
Rural	62.8	65.1	2.3	45.5	46.5	1.0	37.1	35.6	-1.5	34.1	37.3	3.2
Disadvantaged-urban	54.5	56.6	2.1	38.7	40.1	1.4	33.0	31.7	-1.3	30.8	29.6	-1.2
Advantaged-urban	76.0	77.9	1.9	59.3	60.1	0.8	52.3	53.0	0.7	49.1	50.0	0.9

*Change is significant at the .05 level.

Table D.2.
Percentages of Success on Exercises Assessing Mathematical Knowledge, Skills, Understanding and Applications, Nation and Selected Groups, Age 13

	Knowledge 1978	Knowledge 1982	Change 1978-82	Skills 1978	Skills 1982	Change 1978-82	Under-standing 1978	Under-standing 1982	Change 1978-82	Appli-cation 1978	Appli-cation 1982	Change 1978-82
Nation	69.3	73.8	4.5*	53.6	57.6	4.0*	56.6	60.5	3.9*	43.4	45.6	2.2*
White	72.2	76.1	3.9*	57.0	60.4	3.4*	60.0	63.6	3.6*	46.3	47.9	1.6*
Black	55.8	63.8	8.0*	37.3	44.0	6.7*	40.5	46.4	5.9*	30.4	34.8	4.4*
Hispanic	59.0	65.3	6.3*	42.0	49.2	7.2*	43.8	49.7	5.9*	32.8	38.8	6.0*
Low quartile	48.6	56.2	7.6*	29.4	34.6	5.2*	32.6	37.7	5.1*	23.9	27.8	3.9*
Mid-low quartile	65.8	71.4	5.6*	47.1	51.7	4.6*	49.9	54.6	4.7*	37.3	39.6	2.3*
Mid-high quartile	75.5	79.3	3.8*	60.5	64.3	3.8*	64.2	67.4	3.2*	48.6	50.1	1.5
High quartile	87.4	88.4	1.0	77.4	79.7	2.3*	79.5	82.3	2.8*	64.0	65.0	1.0
0-59% white	58.8	67.7	8.9*	41.6	50.1	8.5*	45.0	52.7	7.7*	33.9	40.0	6.1*
60-100% white	72.1	75.5	3.4*	56.7	59.6	2.9*	59.5	62.6	3.1*	45.9	47.2	1.3
Rural	65.6	70.6	5.0*	48.9	53.0	4.1*	52.6	54.5	1.9	40.6	42.9	2.3
Disadvantaged-urban	57.0	63.9	6.9*	39.3	45.6	6.3*	42.6	47.7	5.1*	32.6	35.8	3.2
Advantaged-urban	77.1	81.1	4.0*	62.7	69.6	6.9*	65.7	72.2	6.5*	51.1	54.7	3.6

*Change is significant at the .05 level.

Table D.3.
Percentages of Success on Exercises Assessing Mathematical Knowledge, Skills, Understanding and Applications, Nation and Selected Groups, Age 17

	Knowledge 1978	Knowledge 1982	Change 1978-82	Skills 1978	Skills 1982	Change 1978-82	Understanding 1978	Understanding 1982	Change 1978-82	Application 1978	Application 1982	Change 1978-82
Nation	74.7	74.9	0.2	59.7	60.0	0.3	61.8	61.5	-0.3	43.5	42.4	-1.1
White	77.3	77.3	0.0	62.7	63.0	0.3	64.8	64.7	-0.1	46.5	45.5	-1.0
Black	59.6	62.6	3.0	42.4	44.2	1.8	45.0	44.8	-0.2	26.2	26.0	-0.2
Hispanic	64.1	66.1	2.0	47.9	48.4	0.5	48.9	49.7	0.8	31.0	31.4	0.4
Low quartile	54.1	55.7	1.6	35.4	36.2	0.8	37.0	36.7	-0.3	21.9	21.5	-0.4
Mid-low quartile	72.0	71.7	-0.3	53.7	54.1	0.4	55.4	55.5	0.1	35.3	34.0	-1.3
Mid-high quartile	81.5	81.3	-0.2	66.9	67.2	0.3	69.7	69.3	-0.4	48.2	46.7	-1.5
High quartile	91.2	90.9	-0.3	82.8	82.3	-0.5	85.3	84.5	-0.8	68.6	67.2	-1.4
0-59% white	63.0	68.4	5.4*	46.1	51.9	5.8*	49.7	52.4	2.7	30.3	33.8	3.5*
60-100% white	76.6	76.8	0.2	61.8	62.2	0.4	63.8	64.1	0.3	45.6	44.8	-0.8
Rural	73.6	72.2	-1.4	56.8	57.2	0.4	59.1	57.1	-2.0	41.2	38.5	-2.7
Disadvantaged-urban	60.9	64.7	3.8*	44.6	46.4	1.8	48.1	47.8	-0.3	28.5	30.1	1.6
Advantaged-urban	82.0	82.5	0.5	69.8	69.7	-0.1	72.6	72.3	-0.3	54.2	52.8	-1.4

*Change is significant at the .05 level.

Table D.4.
Mean Percentages of Success on All Mathematics Exercises, Selected Groups, Ages 9, 13, 17

	Age 9 1978	Age 9 1982	Age 13 1978	Age 13 1982	Age 17 1978	Age 17 1982
Nation	55.4	56.4	56.6	60.5	60.4	60.3
White	58.1	58.8	59.9	63.1	63.2	63.1
Black	43.1	45.2	41.7	48.2	43.7	45.0
Hispanic	46.6	47.7	45.4	51.9	48.5	49.4
Low quartile	32.6	34.5	34.2	39.8	37.4	37.9
Mid-low quartile	50.9	51.8	51.0	55.5	54.6	54.4
Mid-high quartile	62.6	63.0	63.2	66.5	67.1	66.8
High quartile	75.5	76.2	78.2	80.1	82.3	81.7
0-59% white	46.4	48.8	45.5	53.6	47.5	52.3
60-100% white	57.6	58.6	59.6	62.4	62.3	62.4
Rural	51.2	52.7	52.6	56.3	58.0	57.0
Disadvantaged-urban	44.4	45.5	43.5	49.3	45.8	47.7
Advantaged-urban	65.0	66.3	65.1	70.7	70.0	69.7

Appendix E

Percentages of Racial/Ethnic Groups in Various Categories by Age, 1982

	Black			Hispanic			White		
	Age 9	Age 13	Age 17	Age 9	Age 13	Age 17	Age 9	Age 13	Age 17
Rural	9.4%	9.7%	6.4%	1.7%	10.7%	7.0%	11.6%	8.8%	8.2%
Disadvantaged-urban	26.6	25.5	29.9	21.9	22.1	16.5	2.0	3.0	3.2
Advantaged-urban	3.0	2.3	3.8	8.5	6.8	2.9	10.9	10.0	10.8
Low quartile	48.0	54.3	59.1	42.0	42.7	48.9	19.9	18.8	18.4
Mid-low quartile	28.4	26.1	25.2	25.7	28.1	27.7	24.4	24.6	24.9
Mid-high quartile	16.9	14.7	11.5	21.3	19.0	16.3	26.9	27.4	27.5
High quartile	6.7	4.9	4.2	11.0	10.3	7.2	28.8	29.2	29.2
0-59 white	72.5	56.2	64.5	67.9	69.0	65.5	10.2	10.2	11.7
60-100 white	27.5	43.8	35.5	32.1	31.0	34.5	89.8	89.8	88.3

Glossary

Algorithm—A mechanical, step-by-step procedure that will yield an answer. Usually, but not always, algorithms are computation procedures.

English units—Measurement units from the English system of measurement, e.g., feet, yards, pounds, etc.

Integers—Whole numbers and their negatives. . . −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5 . . . .

Place value—Our numeration system is a base-ten place value system. A digit is a numeral that takes its value from its position. For example, in 521, five represents 5 hundred.

Quotient—The answer to division. In $6/2 = 3$, the number 3 is the quotient.

Regrouping—Regrouping refers to the operations once called borrowing and carrying in arithmetic computation.

Unit covering—The area of a surface can be approximated by covering the surface with squares of one unit per side. For example, a rectangle 2 feet wide and 3 feet long could be covered by 6 squares one foot on a side. These squares each would be a unit covering of 1 square foot.

Whole numbers—The counting numbers: 1, 2, 3, 4 . . . and 0.

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