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ABSTRACT

Abstracts and comments are presented for 11 studies. There are two each on aspects of problem solving, mathematics achievement, and estimation. The remainder cover topics related to cognitive development, cognitive processes, evaluation, numeration, and student errors. Research as reported in RIE and CIJE between July and September 1982 is also noted. (MP)

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An Announcement . . .

TEACHING STATISTICS IN SCHOOLS THROUGHOUT THE WORLD

Ed. V. Barnett

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An editorial comment . . .

On Choosing Research Methods: The Pendulum Problem

Douglas B. McLeod
San Diego State University

In the not-too-distant past, researchers in mathematics education would typically conceptualize their research problems in the context of a traditional experimental study. They would probably gather data on two or more groups of students and analyze the data using traditional statistical methods.

Today a researcher in mathematics education is more likely to think about the solution to a research problem in quite different terms. Now the focus could be on whether to use clinical interviews, teaching experiments, case studies, or some other non-experimental approach. The popularity of these alternative methodologies (referred to here as "clinical methods") is a healthy sign, and I welcome it. Researchers need not and should not restrict themselves to the use of traditional experimental methods. But I am now becoming concerned that the pendulum may swing too far, and that researchers may avoid using traditional experimental methods even when they are appropriate.

We have seen this pendulum swing before. In the early part of this century, concerns over the weaknesses of introspective methods were followed by the dominance of behaviorist ideas and statistical techniques. But now researchers focus their concerns on the weaknesses of traditional statistical methods, and remember little but the strengths of clinical research.

The research literature, of course, includes a number of useful articles that discuss both the strengths and weaknesses of clinical methods. See, for example, the books and articles by Ginsburg, Simon, and others (e.g., Swanson, Schwartz, Ginsburg, and Kossan, 1981; Ericsson and Simon, 1980). This kind of careful analysis is very helpful for choosing the best research methods for a particular problem. But my concern is with the tendency that I find among some of our leaders to recommend clinical methods regardless of the requirements of the research problem.

The pressure to use clinical methods was particularly obvious to me back in the days when the federal government had some funds to support research in mathematics education. Reviewers of proposals frequently made suggestions that would increase the use of clinical methods. For example, some reviewers could be counted on to recommend that a study of early number concepts using quantitative methods with 100 students should be changed to a teaching experiment with six subjects, or to suggest that a proposal to analyze statistical data on teacher effectiveness should be recast into a small number of case studies. It was not clear to me that such changes would have improved these proposals, since quantitative methods seemed quite appropriate for the proposer's needs.

Sometimes the pressure to use clinical methods came from inside the government, as well as from outside. In a 1981 request for proposals (RFP), for example, a government agency asked researchers to submit proposals to investigate the learning of secondary school mathematics, especially algebra and geometry. These proposals were to focus on difficulties that minority students had with these courses. Researchers were instructed to search for patterns of differences in how majority and minority students learn mathematics.

From my point of view, this RFP had very worthy goals, and dealt with an important problem. But the next set of requirements of the RFP made me uncomfortable: Every proposal needed to use clinical methods. The project should use no more than twelve subjects. Data would be mostly qualitative in nature.

Such methodological restrictions struck me as both unusual and unfortunate. To require the use of clinical methods seemed inappropriate for at least some aspects of the problem that was being addressed, especially since the study was supposed to have patterns of individual differences as one of its major concerns. I doubt that one can develop a convincing pattern of individual differences with a limit of twelve subjects.

The RFP continued by noting that quantitative data could also be used for certain purposes, such as for judging inter-rater agreement and family size. When an RFP has to specify that such quantitative data are still allowable, then the pendulum has swung too far in favor of qualitative and clinical methods.

Although this example is taken from a government document, the ideas in the RFP reflect the views of a number of influential leaders in mathematics education. And given their influence, who can predict what this swing of the pendulum will bring us next? Can we expect, for example, that general courses on research methods will disappear from graduate schools, to be replaced by new courses which focus only on clinical methods? In the days when traditional experimental methods dominated our field, those methods were essentially all that was taught. That was certainly an unfortunate situation. But now that clinical methods are more fashionable, should we teach graduate students only clinical methods?

The answer, of course, is no. Students need to know that there are a variety of research methods from which they can choose, depending on the requirements of the research problem. As the NCTM volume on Research in Mathematics Education (Shumway, 1980) indicates, the first step of any research project should be to define the problem. Then one chooses the most appropriate set of research methods, keeping in mind that each method has its strengths and weaknesses. For another discussion of these and related ideas, see Lester and Kerr (1979).

So there is no reason to restrict oneself to a single research method, and certainly no justification for teaching only the one that happens to be most fashionable. Instead, researchers need to choose from a wide range of methods. The important point is to match the problem and the methods, making the decisions on the basis of the facts, not the fads, of the moment.

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Winter 1983

Beady, Charles H., Jr.; Slavin, Robert E.; and Fennessey, Gail M. ALTERNATIVE STUDENT EVALUATION STRUCTURES AND A FOCUSED SCHEDULE OF INSTRUCTION IN AN INNER CITY JUNIOR HIGH SCHOOL. Journal of Educational Psychology 73: 518-523; August 1982.

Abstract and comments prepared for I.M.E. by ROBERTA L. DEES, Purdue University Calumet, Hammond.

1. Purpose

The study

> examined the effects of a particular model of direct instruction, focused instruction (FI), and two alternative student evaluation structures, individual learning expectations (ILE) and relative standing (RS) on students' mathematics achievement and attitudes. (p. 518)

2. Rationale

In earlier work discussed at some length, one of the authors had found positive results with focused instruction (FI) and with the evaluation reward system referred to as "individual learning expectations" (ILE). Focused instruction is described as a highly structured schedule of teaching, followed by worksheet work, and then by assessment. Slavin (1980b) had found this teaching method to have "a strong effect on student achievement" (p. 519); the age level and content area are not mentioned.

Further, the FI method resembles other "direct instructional strategies that have been found to improve student achievement (Good and Grouws, 1979; Anderson, Evertson, and Brophy, 1979)" (p. 519). Slavin therefore used focused instruction in a later study (1980a) of the effects of reward procedures on language arts achievement. The reward consisted of recognition in a weekly class newsletter, based on students' improvement over their own previous scores. Since the control group did not receive any newsletter, "it is possible that the observed effects were due to provision of weekly recognition, not to the improvement score system per se" (p. 519). The present study was designed so that the separate effects of focused instruction and the reward-for-improvement system could be assessed.

3. Research Design and Procedures

Subjects were 307 students in 10 seventh-grade mathematics classes in an inner-city junior high school. The sample was approximately 48% black and 44% white; other categories were not included in the analyses. The study was conducted for 11 weeks. Two volunteer teachers each taught five classes which were randomly assigned to one of three treatments.

Two treatments consisted of focused instruction, but with different reward procedures; the third was a control. In focused instruction with individual learning expectations (FI-ILE), students followed a regular cycle of teaching, worksheets, quiz; the number of periods used for each was determined by the teachers. After the quiz, students received a score on the quiz and were also awarded points according to their improvement over their base scores. Certificates were distributed to those seven students in each class making the greatest number of points and/or having perfect papers. After every two quizzes, students' base scores were recalculated.

In focused instruction with relative standing (FI-RS), the same instruction method was used. After each quiz, certificates were given to the seven highest scoring students in each class.

The control group

> studied the same skills as those studied in the two focused instruction groups, but did not follow the same schedule of activities. Students were given percentage scores on whatever quizzes or tests they took and received only traditional grades, not certificates. (p. 520)

Students' records contained their scores on the Iowa Test of Basic Skills, given by the school system. The mathematics subscore (ITBS) was used as pretest. The posttest was

> a specially constructed test covering operations with whole numbers, fractions, and decimals, to which students in all 10 classes had been exposed. (p. 520)

Seven attitude scales were also administered before and after the study.

4. Findings

The authors report means and standard deviations for the ITBS and the posttest by treatment group and by race within each group. (There seems to be an error in the FI-ILE means, since the figures given are: Black 52.69; White, 51.95, and Total, 55.50.)

An analysis of covariance for treatment effects on achievement, with ITBS scores as covariate, showed a significant treatment effect ($p < .01$) in the three-way comparison. The similar two-way analysis comparing FI-ILE and FI-RS showed no significant difference in posttest means. However, the two-way comparison of FI-ILE + FI-RS versus the control group shows a significant effect ($p < .01$). Higher adjusted means (marginally significant) were found for black students than for white in the three-way analysis and in the FI-ILE versus FI-RS comparisons. However, treatment by race interactions did not reach significance levels. No significant differences on the attitude scales were reported.

5. Interpretations

The authors expected that students in the FI-ILE treatment group would exceed the other two groups in achievement and in favorable attitudes, and that both the FI groups would similarly exceed the control group. Both FI groups did exceed the control group on the achievement measure, but there was no significant difference between the two FI groups. This suggests that the focused instruction model, which included the awarding of certificates, had more impact on achievement than the basis on which the certificates were awarded.

The authors suggest possible reasons for this outcome, which contrasts with results of Slavin's earlier study (1980a): some irregularity in the ITBS tests, which were not monitored by the authors; the differences between the two population samples; and "perhaps the most likely reason" (p. 522), the difference in the frequency of feedback in the two studies. Since the FI-ILE groups did exceed the FI-RS groups (non-significantly), a longer period of study might have produced significant differences.

In instruction methods, however,

> the significant differences in favor of the structured, focused schedule of teaching, worksheet work, and quiz . . . provide further support for the utility of direct instructional strategies in which regular activities, frequent feedback, and emphasis on coverage play a major part. (p. 523)

The authors say that validation of this method in an inner city junior high school is important. It is further suggested that the reward-for-improvement system merits further study.

Abstractor's Comments

The first group of questions concerns information not available in the report:

What was the nature of the worksheets?

What did the individual quiz look like? In one place it is described as "a written, summative quiz rather than verbal responses or worksheet answers" (p. 520). What does this mean?

Of what did the "specially constructed" posttest consist? Was it similar to the six quizzes, or more like the ITBS?

What was the mathematical content being taught? Was it from a standard textbook series? Were concepts being taught, or only "operations with whole numbers, fractions, and decimals" (p. 520)?

If the control group "did not follow the same schedule of activities" (p. 520), then what did they do?

The next group of questions and comments indicate reservations I have about the study and/or the philosophy behind it:

Since the same teachers taught some control classes as well as some FI classes, how was contamination avoided? That is, what prevented the teachers using some of the materials/methods with the control classes? On the other hand, in the FI classes, material (worksheets and quizzes) was prepared by the researchers, making preparation for these classes quite simple for the teachers. We are given no information about the instruction in the control classes. Perhaps they received no instruction at all.

What do the authors think the treacher's expectations were of the FI classes versus the control classes?

Do the authors think the certificates really should make any difference? Consider this fact:

> While the ILE points and certificates were emphasized as indications of week-to-week improvements or decrements in performance, students still received individual grades based on their actual (unadjusted) performance relative to other students, not on their ILE scores. (p. 520)

Could the result, that there was no significant difference between FI-ILE and FI-RS, be due to the fact that students either

1) saw immediately that the certificates had no effect on what was written in the almighty grade book (and therefore it made no

difference on what basis they were awarded), or

2) learned and improved their scores because of intrinsic rewards (not needing M&M's)?

What other measures of achievement did the teachers us in their evaluations for final grades? What might be predicted about the retention of these students at a later date? Is this just another example of "teaching the test"?

How can we be sure that any of the groups understand the underlying concepts of our number system, of fractions, and of decimals?

Does the FI method transfer? That is, does it provide for learning <u>how</u> to learn mathematics in the absence of the worksheets?

Junior high school students certainly need instruction, structure, and feedback. However, in junior high schools throughout the country (inner city schools in particular), worksheets seem to abound already. I worry about this cut-and-dried system, this focused instruction. Persons desperately trying to raise their schools' achievement scores may grasp at this straw. If they do, when will their students ever get to see a film, take a field trip, have an argument, do an experiment, or solve a problem?

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Bednarz, Nadine and Janvier, Bernadette. THE UNDERSTANDING OF NUMERATION IN PRIMARY SCHOOL. Educational Studies in Mathematics 13: 33-57; February 1982.

Abstract and comments prepared by GLEN BLUME, The University of Iowa.

1. Purpose

The purpose of the study was to clarify the notion of numeration and what is meant by understanding numeration, to develop a framework for evaluating the understanding of numeration and for teaching numeration, and to identify children's difficulties with numeration and the strategies they use on numeration items.

2. Rationale

The authors characterize numeration as encompassing more than counting and identification of place value. Numeration is a process which consists of "moving from the number (associated with a given collection) to the representation of that number" (p. 34). This process requires skills that enable one to move among three states: the collection itself, the collection reorganized into groupings, and the representation of number associated with the collection. Some of these skills are: to make groupings, to 'unmake' groupings, to make groupings of groupings, to code (move from groupings to representation), to decode, and to discover the rule of grouping.

3. Research Design and Procedures

From March through June of 1979, first, third, and fourth graders (40, 75, and 45, respectively) from three schools (underprivileged, middle-class, and upper-class) were individually interviewed. First graders were given 14 tasks and third and fourth graders were given 12 tasks. Follow-up interviews in groups of two or three students used six tasks and were given to 42 third graders and 36 fourth graders.

The framework used to classify the tasks had three dimensions: degree of complexity, context, and "the representation related to a rule of grouping" (p. 36). Task complexity was easy, average, or difficult; an illustrative average complexity item had "three skills involved and a course of interplay between two states" (p. 38). Tasks were classified in one of two contexts: Use of conventional symbols or everyday situations such as those dealing with money defined the familiar context, and contexts such as candies, rolls of candies, and bags of rolls constituted an unfamiliar context. The relation between the representation and the rule of grouping in each task was apparent (visible or explicit), disguised, or conventional (e.g., using written symbols). Figure 1 summarizes the framework for describing the tasks and illustrates for each grade level where the majority of the tasks were concentrated.

The authors present a partial analysis of the results from six tasks from the third- and fourth-grade individual interviews in the following format: (1) a description of the task, (2) the aspect of understanding of numeration addressed by the task, (3) the task's classification according to Figure 1, and (4) the percentages of third and fourth graders who exhibited certain strategies and difficulties on the task.

4. Findings

Task 1. Using six tags containing the digits 0-5, students were asked to use all tags to make as large a number as possible and to compare their number to one formed by the interviewer.

This item addressed the importance the child placed on the position of a digit in a written number and was classified as familiar, easy, and conventional. Sixty-three percent (84%) of the third graders (fourth graders) were successful on this task. The main difficulties were in working with numbers greater than 10000 and with placement of the zero.

Task 2. By repeatedly tossing a die and using or omitting the digit produced, students were asked to construct a three digit number greater than 423.

This item addressed the role the child attributed to position when writing numbers and was classified as familiar, average, and conventional. "'Good understanding' of numeration" (p. 43) was exhibited by only 10% (80%) of the subjects in grade three (four); an example of this was recognition that a 5 in the hundreds position was sufficient. Although less than 20% of the students in each grade accepted all digits and wrote them (in order) to form their number, 40% (23%) of the third graders (fourth) attempted to construct a number in which each digit was greater than the corresponding digit in the given number.

Task 3. Given 20 tags containing entries such as '2 ones', '11 ones', '45 tens', or '5 hundreds', students were to use the tags to guess a number greater than 402 and less than 513 that had been selected by the interviewer. Students were told whether the interviewer's number was bigger or smaller than each successive guess. Children also were presented various subsets of the tags and again asked to perform the task and write the number.

This item attempted to demonstrate the meaning children attached to the words hundreds, tens, and ones and was classified as familiar, difficult, and conventional. Only 27% of the third graders and 44% of the fourth graders gave a meaning to ones, tens, and hundreds in terms of groupings. Over a third of the students worked exclusively with the digits, paying no attention to the words, and 30% (21%) of the students in grade three (four) related ones, tens, and hundreds to order in writing rather than to grouping.

Task 4. Students were shown a picture of four individual candies, three rolls of candies, and two bags of rolls, a second picture of those that were given away (1 bag, 7 rolls, and 8 candies), and were asked to draw a picture of what was left.

This item was classified as unfamiliar, average, and disguised, and addressed "what meaning (if any) children give to borrowing in subtraction" (p. 48) and whether children see groupings or groupings of groupings with this representation. Many students, [60% (32%) in grade three (four)], made no inquiry into the number of candies per roll or the number of rolls per bag, and only 30% (61%) of the students exhibited understanding (associated a number to the grouping or mentally 'unmade' the grouping).

Task 5. Students were asked to find the answer to 234 - 178 = .

This task was intended to address the meaning attributed to borrowing in conventional symbolism and to determine transfer from Task 4, and was classified as familiar, average, and conventional. Only 4% (10%) of the third graders (fourth) gave meaning to the borrowing in terms of groupings, although 11% (35%) of the third graders (fourth) verbally differentiated between the borrowing of tens and hundreds.

Task 6. After being shown a pack containing 10 candies, students were shown a picture of 5 rolls of candies, 6 candies, and 1 bag of rolls and asked whether someone buying 15 packs would have enough to give someone the candy shown in the picture. A second presentation used 15 packs with 9 in each pack.

This item determined whether students perceived groupings of groupings and was classified as unfamiliar, average, and disguised. At most 50% (78%) of the third graders (fourth) showed understanding of numeration on this item. Only 7% (10%) of the students made a direct comparison between the sets of objects, and 68% of the fourth graders used computations.

5. Interpretations

The authors concluded that children attribute meaning to hundreds, tens, and ones more in terms of order or position than in terms of groupings, and that they see numbers not as entities, but as "symbols placed side by side" (p. 54). The authors also concluded that the borrowing procedure is not transfered from conventional symbolism to other forms of representation and is rarely linked to the idea of exchanging. The main difficulities exhibited were those with zero, working with groupings of groupings or with two groupings simultaneously, and unmaking groupings and handling the exchange in borrowing. The authors found that fourth graders made more systematic use of computation, and their answers were more consistent than those of third graders.

The authors state, "One of the major outcomes of our analysis was the discovery of a striking similarity between the processes involved in numeration and in measure" (p. 55). They contend that their research reveals elements which characterize understanding of numeration and that their framework could be used for teaching and evaluation of numberation. The difficulties linked with different modes of representation are part of a larger question of representation in mathematics.

Abstractor's Comments

The authors are to be commended for their attempts to characterize aspects of numeration (individual skills that contribute to understanding numeration), for their description of children's procedures and errors on specific numeration tasks, and for their suggestions concerning teaching emphases for numeration. My comments will focus on aspects of these three areas.

The authors' theoretical framework characterizing numeration as a representation process is a worthwhile first step toward specifying in detail what an acceptable understanding of numeration should entail. From this framework tasks were classified according to complexity, familiarity of context, and type or representation, but it was not apparent that there was a direct link between these task categories and the three states and associated skills. For example, classification of task complexity as easy, average, or difficult reveals much less about the task than a classification which might specify precisely those skills or combinations of skills demanded by the task. What skill was lacking in a child who could solve an "average" task but not a "difficult" one? Also, I was not able easily to relate the objective of the first task, determining the importance a child places on the position of a digit in a written number, to any of the skills associated with the numeration process (making groupings, moving from groupings to symbols, etc.).

Some of the interview tasks were particularly interesting in their approach to assessing aspects of numeration, and the authors' descriptions of responses on each item seemed to "capture the essence" of their subjects' responses. However, no mention was made as to whether an a priori coding scheme was developed for recording students' responses. Also, the number of interviewers was not specified; if several were used, a measure of inter-rater agreement would have added credibility to the authors' classifications of student responses.

The authors report that children in grades three and four can use computation and conventional numerals, but have difficulty transferring their knowledge to situations using other forms of representation and difficulty attaching meaning to conventional symbols and computational procedures. Since results were reported for only a subset of the tasks used, I was tempted to speculate about the full range of results. Is it possible that first graders exhibit fewer such difficulties because they rely more heavily on counting and manipulating sets of objects than on symbolic procedures? Do older subjects perform poorly because they don't know how to manipulate the groupings or weren't taught to do so, or because they simply interpret the task as being one that can be solved using computation? The authors' promise of a report on the full range of tasks and grade levels was welcome; an interesting inclusion would be some description of individual subjects' performance across all tasks in addition to the group data on each task.

I agree with the authors' suggested teaching emphases -- instruction in the computational algorithms should be based on linking the symbolic procedures to grouping and exchanging, children should be given the opportunity to work with a variety of representational modes, and measurement can serve as the basis for developing representation processes. It was not clear to me, however, that the task framework used for the interviews necessarily could serve as a basis for teaching numeration and ensuring that students understood numeration. I viewed the tasks somewhat as isolated tasks which pointed out difficulties with some aspects of numeration rather than as a coherent framework for defining what constitutes understanding of numeration or varying degrees thereof. This was especially true of those instances in which the authors defined understanding by listing behaviors which described lack of understanding. As more data and additional refinements of the authors' framework for numeration become available, our knowledge about children's conceptualizations of numeration will be enhanced.

Concentration of Tasks by Grades

		CONTEXT					
		Familiar			Non-familiar		
		COMPLEXITY			COMPLEXITY		
		Easy	Average	Difficult	Easy	Average	Difficult
REPRESENTATION	Apparent	1st	1st	1st	1st	1st	1st
	Disguised	1st	3rd, 4th	3rd, 4th	1st	3rd, 4th	3rd, 4th
	Conventional	1st	3rd, 4th	3rd, 4th	1st	3rd, 4th	3rd, 4th

Figure 1.

Carry, L. Ray; and Others. PSYCHOLOGY OF EQUATION SOLVING: AN INFORMATION PROCESSING STUDY. Final Technical Report. August: University of Texas, Department of Curriculum and Instruction, 1979. ERIC: ED 186 243.

Abstract and comments prepared for I.M.E. by ROBERT B. ASHLOCK, RTS Graduate School of Education, Jackson, Mississippi.

1. Purpose and Rationale

The authors believe that what is learned in school mathematics differs sharply from the outcomes desired by educators who tend to organize instruction in terms of the logical network of mathematical conceptualization they themselves posses. Therefore, the authors have attempted to describe how college students actually go about solving algebraic equations. In so doing they attempted to evaluate and extend recent work in the psychology of skill.

2. Research Design and Procedures

Data were collected from two groups of university students. One group of 15 students expected to be good solvers of elementary algebra equations was selected from a population of engineering and mathematics education students. The other group, with many poor solvers, was an unselected sample of 19 volunteers from an introductory psychology course.

Through pilot testing an instrument was developed containing seven pairs of equations, each pair designed to probe for specific errors. The problems were given to subjects in the first of two sessions. For the second session, seven triples of equations similar to those used during the first session were prepared. Students in the unselected group were given equations related to those on which they had made errors at the first session. Also, four pairs of more complex equations were administered to skilled solvers. An instrument for screening the more proficient, fluent solvers on the basis of speed and accuracy was administered.

Students were video-taped as they individually attempted to solve the equations. For part of the equations they were asked to try to describe their problem-related thoughts while they worked. For other equations they were asked to explain the solution process as if they were assisting someone who needed help with homework. Written work and spoken comments were retained for analysis, then the comments were keyed to the written work with the help of the video recordings.

To organize the data, the authors used an artificial model of the solution process based on the work of Bundy (1975). When interpreting data they looked for three kinds of conclusions relevant to education. First they tried to identify the difficulties students had, and to guess the mechanism that produced those difficulties. Second, by comparing the work of successful and unsuccessful solvers they sought to identify ideas that might help make more solvers successful. And third, they tried to identify what must actually be learned by the student of equation solving.

3. <u>Findings</u>

The authors found very different thoughts and acts, even among successful students, than they believed should logically be expected. Few college students perceived algebra as generalized arithmetic. The authors found students mixing non-insightful and insightful ways of equation solving, and in some specific cases observed what understanding does and does not do.

The strategies used by solvers were recorded and the errors of non-solvers analyzed, primarily in terms of the Bundy strategy. It appeared that solvers need at least two strategies, one for linear and one for quadratic equations, with a means of selecting the appropriate one.

The Bundy model was used as a framework for tabulating errors in the study. Errors exhibited were tentatively tabulated in three categories: operator errors, applicability errors, and execution errors. Operator errors examined include various deletions, transpositions, recombinations, cross multiplications, splitting an equation with fractions, reciprocals, divisions by zero, splitting quadratic equations, square roots, extraneous roots, arithmetic error, and operator gaps.

A shift in prevalence of error types was observed as accuracy increased with execution errors relatively more frequent and operator errors less frequent. Some errors tended to occur consistently for given students. Many errors seemed to be consistent with the student's statement of what should be done, suggesting that his or her knowledge was faulty rather than the execution. Errors were systematic in form; few were random distortions of correct performance.

The Bundy model was found to be limited in that subjects do not simply solve; they also explore, evaluate, and check. Therefore, the comments and activities that surrounded solving itself were considered. Evidences of hierarchial organization were noted, as were some characteristics of erroneous operators. It was inferred that operators are not units of knowledge that either are known or unknown as suggested in the Bundy model. Rather, students monitored the progress of their solutions, making evaluations of the states they reached, often backtracking. Both local and global checking methods were observed. Some are not, beyond appeal to authority.

4. Interpretations

The authors noted that the three types of errors observed require different preventive or remedial measures. Operator errors seem to reflect incorrect or incomplete knowledge, and applicability errors usually involve mishandling of parentheses. However, execution errors did not cluster as did the others, possibly because they are a trade-off of speed and accuracy in relation to other tasks.

Furthermore, operators seem to have structure, and students may have only partial knowledge of the parts of that structure. For example, part of the knowledge of adding and multiplying seems to be that they accumulate things, while subtraction and division both take away things. Such partial knowledge can determine the form and occurrence of errors. It appears that strategic knowledge is not as simple as the Bundy model.

Students need to know that they will form misconceptions and make errors despite their best efforts, and that there are specific actions they can take to deal with these difficulties. For example, poor solvers will be helped if they can see checking as a way of evaluating their knowledge rather than their answers. Although one can do algebra without understanding it, meaning is important because of the specific role it can play in allowing the validity of operators to be tested.

Abstractor's Comments

Although this is a descriptive study with no interventions, the authors were correct in believing that such a line of investigation "should ultimately yield implications for learning and teaching algebra." In fact, there are implications for arithmetic as well as algebra. The study is a rich source of hypotheses for both clinical intervention research and experimental studies.

As you read the report, you cannot help but be struck by the need for an emphasis on meanings of operators and not just on manipulation of entities.

Reference

Bundy, A. Analysing Mathematical proofs. DAI Research Report No. 2, Department of Artificial Intelligence, University of Edinburgh, 1975.

Clements, M. M. CARELESS ERRORS MADE BY SIXTH-GRADE CHILDREN ON WRITTEN MATHEMATICAL TASKS. Journal for Research in Mathematics Education 13: 136-144; March 1982.

Abstract and comments prepared for I.M.E. by JOHN R. KOLB, North Carolina State University, Raleigh.

1. Purpose

Children's errors on a set of written mathematical tasks were investigated to determine the relationship of careless errors to total mathematical errors. Also, some associations are sought between personality and cognitive characteristcs of the students and the tendency to make careless errors.

2. Rationale

The author contends that "while there is a large and growing literature on process skills errors, and especially on errors due to the application of inappropriate or faulty arithmetic processes..., the literature on careless errors is relatively sparse" (p. 136). While some investigators have suggested that most, if not all, errors are systematic process errors, the work of other researchers suggests that careless errors and process errors occur with about the same frequency.

3. Research Design and Procedures

Twenty-six male and 24 female sixth graders attending an international school in Lae, Papua New Guinea, participated in the study. Kagan's cognitive style test, Matching Familiar Figures (MFF), was administered individually to each student in the sample. The errors and response time on the MFF were used to identify students as impulsive or reflective. One week later four paper-and-pencil tests were given to the students on two successive days in a group setting. The tests were:

a) Arithmetic Competency Test (ACT). A 25-item test involving computational exercises with whole numbers, fractions, and decimals.
b) Mathematics Language Test (MLT). A 17-item test assessing understanding of comparative terms like more or less in worded problems.
c) Mathematics Confidence Test (MCT). A 25-item test of word problems and calculations with an associated five-point Likert response

scale where students were required to report their degree of confidence in the answers they constructed for each test item.

d) Monash Assessment of Mathematical Performance Test (MAMP). A 36-item test of numeration, geometry, and logic that was administered to all students twice, once on each of the group-testing days.

In addition to the time to respond and errors on the MFF, and the scores on the four achievement tests, two more variables were defined. One variable was called total time and it consisted of the time to the nearest minute each student needed to complete the ACT, the MLT, and the MAMP (first administration only). The other derived variable was called misplaced confidence. Whenever students missed a question on the MCT, but indicated on the associated Likert scale that they were certain of their answers being correct, they received a misplaced confidence score of 2. A score of 1 was given for an incorrect answer for which students thought their answers were correct (but not certain). All other cases were scored a 0 on misplaced confidence. About one week following the group testing, the author interviewed the subjects individually using a technique the author identifies as the newman interview technique.

The author used the double administration of the MAMP tests as the source of data for careless errors. Errors were defined in this study as careless:

a) if on a given item a wrong answer appeared on one administration of the MAMP but a correct answer appeared on the other administration, and

b) if when presented the two original answers during the interview, the student obtained the correct answer without assistance from the interviewer and provided further probing did not indicate that the student was uncertain which answer was correct.

4. Findings

Errors. Considering both administrations of the MAMP, there were 903 incorrect responses out of a possible 3600.

- Double errors (incorrect response both times) totaled 638 of which 76% were stable double errors. (Same incorrect response both times.)
- Single errors totaled 265 of which 190 were judged to be careless using the definition applied in this study. Of the careless errors, 122 (64%)

occurred the first time and 68 (36%) occurred on the second administration of the MAMP.

Relation of Errors to Mathematical Performance.

- The number of careless errors and double errors correlated negatively with better performance on the ACT, MLT, and the MCT, but correlated positively with misplaced confidence and total time.
- The proportion of careless errors to total errors correlated positively with better performance on the ACT, MLT, and MCT and negatively with misplaced confidence and total time.

Relation of Errors to Impulsivity. Seventeen of the subjects were identified as impulsive and 12 were classified as reflective from their performance on the MFF test.

- Impulsive students tended to make a greater number of careless and double errors than reflective students, but not significantly more.
- The proportion of careless errors was nearly the same for both groups.
- The mean time taken by a student to select a picture in Kagan's MFF test correlated negatively (but not significantly different from zero) with total time in the first MAMP, ACT, and MLT.

5. Interpretations

Students who are mathematically weak and who are confident of their answers when they shouldn't be tend to make more careless errors and even more systematic errors. Students who take less time on mathematical tasks and who are generally confident about their answers make fewer process errors but only marginally fewer careless errors.

When the ratio of careless errors is considered, a different pattern emerges. Students who are mathematically competent, who are faster workers, who have confidence in their results, and who do not misplace their confidence tend to have a greater ratio of careless errors to total errors than do "slower, less confident, and mathematically weaker students" (p. 141).

Impulsivity-reflectivity as measured by the MFF does not indicate a dimension that is highly related to careless errors or proportion of careless errors. In fact, students who were identified as impulsive from the MFF were not impulsive on mathematical tasks. Similarly, students identified as reflective from the MFF were not reflective on mathematical tasks.

Abstractor's Comments

The author describes this study as "exploratory" involving "rather blunt statistical tools." With this I agree. The results are primarily in terms of simple correlations and need to be considered with great care. However, it is an interesting study that is well worth examining with the view toward further investigation.

The main results of this study are not inconsistent with what one would intuitively expect. The more errors, careless and otherwise, students make on one type of mathematical task, the more errors they tend to make on other types of mathematical tasks (while no correlations between the various mathematics achievement tests are reported, one suspects a large positive correlation among them.) Moreover, more able students mathematically who are confident of their answers make fewer careless errors and fewer systematic errors.

The use of the proportion of careless errors as a dependent variable can be misleading. It was found that students who were able on the mathematics achievement tests, who worked quickly, and who were confident of their answers tended to make a higher proportion of careless errors. This seems contradictory to the last statement in the previous paragraph, yet it is not contradictory, only an artifact of the use of the ratio of careless errors as a dependent measure. Clearly the more able students made fewer careless errors than their less able cohorts and also made fewer systematic errors in relation to their careless errors. Thus, the greater proportion of careless errors is primarily due to a smaller number of systematic errors and not due to a large number of careless errors.

Some of the relationships uncovered in the study may have been less predictable at the outset. One may have expected more careless errors from individuals classified as impulsive, yet this was not found. Individuals classified as impulsive did tend toward more double errors than reflective subjects, though not enough for statistical significance. In fact, the relationships uncovered between double errors and other variables seemed to be much stronger than for careless errors. For example, both mathematical confidence and total time were both significantly related to the number of double errors ($^{-}0.59$ and 0.50 respectively). Most of these strong relationships probably occur because of the basic finding that process errors in the form of double errors were three and one-half times more prevalent than careless errors.

The author raises the question, "What are careless errors?" Despite a careful operational definition, no attempt is made to discuss this question. From the review of the literature in this report, children's errors seem to fall into two categories: a) errors due to faulty processes or inappropriate applications of process skills and b) careless errors. Yet, in this study we find four empirical categories of errors: a) stable double errors, b) unstable double errors, c) single careless errors, and d) single non-careless errors. Are all errors other than simple careless errors to be considered systematic process errors? The author does not say, nor does he discuss two of the four types. While we are given an operational definition for careless errors we are not given a rationale for why this definition should be considered the test for whether an error is careless. From the operational definition of careless errors used in the study one infers that the author means that a careless error is characterized by a) sporatic occurrence as opposed to consistent appearance, b) ready identification as incorrect by the student and corrected without teacher assistance, and c) steadfast belief by the student in the correctness of the answer in the face of teacher probings. Yet must a careless error be sporatic in occurrence? Particularly, must it occur only once in two trials?

The Newman interview technique is referred to often in this report and plays an integral role in the identification of careless errors. No description is given of this technique and the only reference is to an article in an Australian publication. Some capsule description of this interviewing technique would have been helpful to the reader who has no access to the cited publication.

The author asserts that his interest in knowing the nature, extent, and causes of careless errors stems from a desire to find effective means to eradicate them. An interesting bit of data in the study suggests that perhaps a way has been found to eradicate careless errors without understanding their cause or nature. Clements reports that on the first administration of the MAMP, 122 careless errors occurred while on the second administration only 68 careless errors occurred. One might hypothesize that if the MAMP continued to be administered, the number of careless errors would continue to decrease, thus moving toward the eradication of careless errors. Meanwhile, the number of non-careless errors did not decrease nearly as much. From the first to the second administration, the number of errors decreased by about 80, of which 54 were careless errors and the remaining 26 were scattered among the other three categories. Clearly, errors classified as careless in this study were more affected by a repeat administration of the MAMP than were errors considered to be process errors.

If one considers the repeated administration of the MAMP as an example of repetitive practice or drill, we see that drill improves performance with respect to careless errors much more than it does for process errors. Thus, drill or repetitive practice might be hypothesized to reduce careless errors and increase proficiency. Also, we may expect that drill or repetitive practice would not serve to reduce errors in process skills.

Hiebert, James; Carpenter, Thomas P.; and Moser, James M. COGNITIVE DEVELOPMENT AND CHILDREN'S SOLUTIONS TO VERBAL ARITHMETIC PROBLEMS. Journal for Research in Mathematics Education 13: 83-98; March 1982.

Abstract and comments prepared for I.M.E. by DOUGLAS R. M. EDGE, University of British Columbia, Vancouver.

1. Purpose

The purpose of this study was to investigate the relationship between "general cognitive developmental abilities" (p. 83) as measured by an information processing capacity and three Piagetian tasks (number conservation, class inclusion, and transitive reasoning), and performance of first-grade children on verbal addition and subtraction problems.

2. Rationale

The authors argued that a logical analysis of simple arithmetic problems suggests that certain "basic cognitive abilities may be needed to interpret and solve them correctly" (p. 85). They examined each of the three Piagetian tasks and the information processing variable in terms of their relation to addition and subtraction, simple problem solving. They noted that the supporting empirical evidence for the logical analysis was either inconclusive, contradictory, or, with regard to information processing capacity and arithmetic problem solving, sparce. They concluded that it was not clear that any of these cognitive variables was in fact needed for successful performance or "whether the relationships are simply the result of a global common factor, such as general intelligence" (p. 85). The study was an attempt to clarify these ambiguities.

3. Research Design and Procedures

The 149 first-grade children interviewed in this study were drawn from three elementary schools in middle-class neighborhoods in Madison, Wisconsin. For their instructional program, the schools used a modified version of Developing Mathematical Processes (Romberg et al., 1974). As the students were tested in January, they had received several lessons on solving different arithmetic problems where modelling the situation with objects was a recommended solution strategy.

Each child received two forms of a standard protocol for number conservation, class inclusion, and length transitivity tasks. Although the children were asking to provide examinations for their answers, the responses were scored as either correct or incorrect, resulting in scores of 0, 1, or 2 for each of the Piagetian reasoning abilities. The backward digit span task used to measure information processing capacity consisted of 10 two-digit, 10 three-digit, and 10 four-digit trials. Performance was scored in such a way that for data analysis purposes four levels of backward digit span were established, resulting in scores of 0, 1, 2, or 3.

Six verbal addition and subtraction problems, "each drawn from categories representing different semantic structures" (p. 87), were selected for the study. The problem types used were: Join (addition), Separate (subtraction), Combine (subtraction), Combine (addition), Compare (subtraction), and Join (subtraction). Each child was presented with a total of 24 problems — each of the six problem types was presented under four conditions. The conditions resulted from crossing the variable number size with availability of physical objects to help solve the problem.

The data were obtained by interviewing each child individually over three fifteen-minute sessions. The first interview consisted of the 12 smaller number problems, the second the 12 larger number problems, and the third the cognitive ability tasks.

As each of the six problem types was treated separately, summing across the four problems of each type resulted in a score of 0 to 4 for each subject on each problem type. Hence, each subject, classified into a developmental level for each cognitive ability, had a score of 0-4 for each problem type.

Based on the children's explanations, the authors identified a variety of solution strategies. For addition: counting all, counting on from the first (smaller) number, counting on from the larger number, known fact, and derived fact. For subtraction: separating from, separating to, adding on, matching, counting down from, counting down to, counting up from given, known fact, and derived fact. It was suggested that the last three strategies in the list for addition and the last five for subtraction were more advanced than the first strategies.

4. Findings

Although no cognitive ability seemed to be an absolute prerequisite for solving the problems, the children who had developed a particular ability had higher mean scores than those who did not.

Resulting from a multivariate analysis of variance for each cognitive variable across all problem types to check between group differences in overall performance, only class inclusion and backward digit span "sorted children into developmental groups that differed significantly in their arithmetic performance when all six problem types were considered simultaneously" (p. 90).

Six multiple regressions, one for each problem type, were run to examine the possibility that some combination of the cognitive variables may best explain performance on the arithmetic tasks. The amount of variation in performance on the problems explained by the regression models has R^2s ranging from .07 for the Compare (subtraction) problems to .23 for the Combine (subtraction) problems, with the single best prediction for five of the six problem types being the information processing variable (backward digit span). Number conservation, the best predictor in the remaining case, was included in three other equations as the second variable.

The accuracy scores data were also analyzed to focus on the conditions under which the problems were solved (number size crossed with presence of physical objects). As in the original analysis no cognitive variable was prerequisite for solving at least some of the problems under any given condition. All cognitive variables (except transitivity) sorted children into significantly different groups when physical objects were available. Number conservation and backward digit span, however, were the only discriminating variables when the physical objects were not available. Regarding the number size condition, although all variables had significant discriminating power on the small number problems, only backward digit span did on the large number problems. The R^2s which resulted from the stepwise multiple regression analyses were significantly different from zero for each of the four sets and ranged from .11 (large number problems) to .24 (small number problems). Backward digit span was entered first in all four regression models.

From an examination of the solution strategies used by the children, both for addition and subtraction, no cognitive ability was prerequisite for acquiring any particular strategy. However, the low developmental children used the appropriate strategies less often than the more advanced groups. Finally, using the forward stepwise multiple regression procedure to establish the relative contribution of each cognitive variable in predicting the use of advanced strategies resulted in the following regression model, significant at the .01 level and accounting for 17% of the variation:

$$y = 3.37 + 3.59\ (\text{backward digit span}) + 1.98\ (\text{transitive reasoning})$$

5. Interpretations

Although the results indicated that children who had developed a particular cognitive ability performed better on all types of problems and under all conditions, two features of the results suggest it would be unwise to use the cognitive variables as readiness indicators in the classroom. First, the regression analyses showed that only 10% to 20% of the variance in performance could be accounted for on the arithmetic scores and in the use of advanced strategies. Second, no one of the cognitive abilities appeared to be prerequisite for solving any of the arithmetic problems.

One explanation offered for this lack of a strong relationship between the cognitive abilities and arithmetic performance is related to the validity of the tasks. Are the tasks a valid measure of the cognitive constructs in question? A second explanation suggested that perhaps the processes that the children use to solve arithmetic problems are not directly captured by the Piagetian tasks.

Finally, it was noted that as information processing capacity was most consistently related to arithmetic performance, and as examining new children's mathematical behavior from this perspective is a relatively new approach, further research in this area is recommended.

Abstractor's Comments

This study is noteworthy and exemplary for several reasons. It helps place recent and current research on early arithmetic concepts in some perspective. It suggests how seeming inconsistencies in the literature could be accounted for in that a particular construct may be statistically significant even though it may explain as little as 10% to 20% of the variance of the measure. Some children may be able to solve certain examples or problems simply due to the presence of some other factors or capability. Further, the study in its own right makes a relevant contribution to the growing body of knowledge in this area.

The study was very thoroughly done. The rationale was clear. The design was appropriate. The use of the statistical analyses along with the analysis of the solution strategies was effective and helpful.

The article was well written. All essential information was provided, thus facilitating replication. Extraneous information was avoided. Descriptions of protocols were complete. Decisions requiring justification were defended (for example, the use of backward digit span as a measure of information processing capacity).

Although a few questions could be asked, such as 'What effect did the several lessons on solving certain arithmetic problems prior to the time of testing have on the outcome?', I would suggest that these are of a minor nature and do not detract from the overall high quality of the study.

Ibarra, Cheryl Gibbons and Lindvall, C. Mauritz. FACTORS ASSOCIATED WITH THE ABILITY OF KINDERGARTEN CHILDREN TO SOLVE SIMPLE ARITHMETIC STORY PROBLEMS. Journal of Educational Research 75: 145-155; January/February 1982.

Abstract and comments prepared for I.M.E. by AARON D. BUCHANAN, Southwest Regional Laboratory, Los Alamitos.

1. Purpose

The main questions in the study were these:

Are more kindergarten students able to solve simple story problems in addition and subtraction if they are shown or provided with something they can manipulate?

Is the presence of some manipulative aid more important for some kinds of problems than others?

2. Rationale

A lot of previous research has shown in one way or another that students solve story problems by building some kind of model. For problems involving the standard operations of arithmetic, this usually means the use of some simple counting model involving sets. Presumably, students are aided in this process when their instruction includes such things as teacher demonstrations involving actual objects or pictures of objects grouped according to the important details given in the story problem. For very young students, especially those in kindergarten, the issue is an important one because their first experiences with number sentences and simple computation involve some kind of simple story used by the teacher as illustration. Some children may be ready to proceed with work on addition and subtraction combinations (e.g., addition facts to 5) and the number sentences used to represent them. Others may need a lot more preliminary help to comprehend what the words mean in the simple stories that teachers use to represent different addition and subtraction combinations.

3. Research Design and Procedures

One-hundred-thirteen kindergarten students were tested individually on 30 different kinds of simple story problems involving simple addition and subtraction combinations. Ostensibly, these students had not yet had any formal instruction in either of these arithmetic operations. Each story problem represented one of five different degrees of concreteness determined by the kind of aids students were provided for counting and manipulation of objects:

C1) reading of the story only (no aids available)
C2) reading of story plus manipulatives available to children
C3) reading of story and examiner shows the sets described
C4) reading of story and a three-panel sequence of pictures, depicting the actions, is shown to the child
C5) reading of story and examiner shows the sets and actions described

Each story problem also represented one of six different levels of difficulty, depending on the kind of number sentence that the story was supposed to represent. These included:

P1) a + b = ___ where sets described in the story were transformed ("I have ___ apples. You give me ___ more apples. How many apples do I have altogether?")

P2) a + b = ___ where sets described in the story were static ("You have ___ fish. I have ___ fish. How many fish do we have altogether?")

P3) a - b = ___ where sets represent a "take away" transformation ("I have ___ flowers. I give you ___ of my flowers. How many flowers do I have left?")

P4) a + ___ = c where sets described in the story were transformed ("You have ___ buttons. I give you some more buttons. Now you have ___ buttons. How many buttons did I give you?")

P5) a + ___ = c where sets described in the story were static ("I made ___ snowmen. You made some snowmen. Together we made ___ snowmen. How many snowmen did you make?")

P6) a - ___ = c ("You have ___ toy houses. I take some of them from you. Now you only have ___ toy houses left. How many toy houses did I take?")

Students were selected from five different schools representing five different communities. During the study, each student was asked to solve each of the 30 different problems. Results were based on the proportion of students answering each story problem correctly.

4. Findings

Overall, student performance on story problems went up dramatically as the mode for presentation went up in its degree of concreteness. On the average, problems presented in mode C1 were answered correctly by about 30% of the students; problems presented with some intermediate degree of concreteness (modes C2 - C4) were answered correctly by 40% to 45%; while problems presented in mode C5, where students were basically shown how to find the answer by physically combining or separating sets, were answered correctly by about 75%.

"Missing addend" problems (types P3 - P6) were generally harder for students to solve than regular problems (types P1 - P3), where students are converging on the results of the problem's action rather than trying to determine what happened at some intermediate point. There was a significant interaction between the mode of presentation and the type of problem that students were asked to solve. However, most of this interaction occurred with problem type P3, a basic problem in "take away" subtraction, that students were generally unable to solve unless it was demonstrated to them (mode C5).

5. Interpretation

The authors' interpretation was that most kindergarten students must be ready to begin to work with simple number sentences for addition and subtraction facts, because the students in their study could comprehend basic addition and subtraction story problems when they were given enough aid with concrete materials. On the other hand, kindergarten students are apparently a lot less ready to begin to work on number sentences with missing addends; many of them simply can't come up with a solution even when they're shown how to get it.

Abstractor's Comments

The results of this study shows that some kind of illustration or concrete aid is necessary for students to be able to make much sense out of their first experiences with addition and subtraction. This shouldn't be too surprising, but it does need to be said. Young children generally need to have things demonstrated for them in pretty complete detail, at least the first time or two, and I don't think many kindergarten teachers would seriously try to introduce addition and subtraction of whole numbers by simply reading a story problem. However, some of them might be inclined to simply show sets of objects or, even more likely, to use static pictures, and the results of this study clearly show that it won't work.

I am concerned about two things in the study, and I think many readers might be also. First, the report on the study doesn't say enough about the five different modes of presentation. The descriptions of C1 through C5 that I've included in this abstract were taken verbatim from the journal report. But experience with these kinds of verbal tasks suggests that you really need to have the problems stated exactly as they were presented to students in order to interpret results. Lots of times, types of problems that seem to be about the same are really quite different when you see exactly how the task was stated to students. A seemingly subtle change in a word or phrase often results in two problems or tasks that are completely different in terms of what young students know and know how to do. Second, I'm concerned by how frequently the authors refer to readiness in their interpretation of the results. I would be more cautious in generaliziang about what students are ready or not ready to learn based on their performance on a single problem. (Recall that each problem in the study represents what would be the beginning of a fairly large piece of instruction in the classroom.) For example, 35% of the students could not solve problem type P1, where one set is being added to another one, in mode C5, where they were essentially being shown how to do it step by step. Does this means that they aren't ready yet to begin instruction on basic addition? I suspect not. So far as we know, they've only had one chance. (Recall that students in this study were not supposed to have had previous formal instruction on addition and subtraction.) Maybe, if you demonstrated the process of combining one set with another two or three more times, most of the students who missed it the first time would do a lot better. And I suspect this is what most kindergarten teachers do.

Leder, Gilah C. MATHEMATICS ACHIEVEMENT AND FEAR OF SUCCESS. Journal for Research in Mathematics Education 13: 124-135; March 1982.

Abstract and comments prepared for I.M.E. by J. PHILIP SMITH, Southern Connecticut State College.

1. Purpose

The purpose of the study was to address four questions: (1) Are there sex differences in mathematics performance? (2) Are there differences in the proportions of boys and girls planning to continue with mathematics? (3) Is there a relationship between FS (fear of success) and expressed mathematics course-taking intention? (4) Is there a relationship between FS and performance in mathematics?

2. Rationale

A growing body of research is concerned with sex differences in performance and participation in mathematics. Although over the last decade a number of studies have focused on Horner's (1968) construct of the motive to avoid success, or fear of success, the present study grew out of an apparent contradiction in Horner's original work. Horner postulated that FS should be most evident in females who had successfully competed at tasks generally perceived as masculine ones -- e.g., mathematics. Yet, when Horner's high FS students were compared with her low FS ones, the great majority of the former were humanities majors, whereas more than half of the latter were majoring in the sciences and mathematics.

3. Research Design and Procedures

The sample consisted of 258 boys and 233 girls from 20 classes in 11 randomly selected coeducational schools of greater Melbourne, Australia. A total of 155 subjects were in Grade 7, 245 were in Grade 10, and 89 were in Grade 11. All of the Grade 10 and 11 students were taking faster-track, nonterminal mathematics classes.

Parallel forms of TRIM (Tests of Reasoning in Mathematics), constructed by the Australian Council for Educational Research, were administered to the students. Additional data regarding the eleventh graders were obtained 15 months later when they sat for a statewide mathematics achievement examination. An investigator-administered FS measure was given to all students: they were asked to write brief stories in response to three verbal cues describing a person of the same sex as the respondent. Scores on this measure can range from -6 to 24. Finally, all Grade 10 and 11 students were asked to complete a questionnaire inquiring about their future educational plans and about which subjects they intended to study the following year.

4. Findings

The mean TRIM scores of boys was significantly higher ($p < .05$) than that of girls at the Grade 7 and the Grade 10 levels. At the eleventh-grade level, no significant TRIM score differences existed.

In both Grades 10 and 11, proportionately more boys than girls intended to take two more mathematics courses. (In Grade 10 the respective percentages were 29 and 20; in Grade 11 they were 35 and 19.) At both grade levels, proportionately more girls than boys planned to continue in school and yet take no more mathematics. (In Grade 10 the respective percentages were 17 and 2; in Grade 11 they were 22 and 14.) When the Grade 10 and 11 students were divided into thirds on their TRIM scores and the high and low groups examined, the data show, in general, similar results. For the high TRIM group, however, the gap between boys and girls planning to take no more mathematics has narrowed to within a percentage point.

Does a relationship exist between FS and expressed mathematics course intention? Only data for the high TRIM groups are reported. Among students planning to take one or two more courses, girls had mean FS scores a point or two higher than boys. Among those few high TRIM students planning to continue without taking more mathematics, boys had a mean FS score of 6.25; girls, a mean FS score of 5.00. The highest mean FS score reported for boys was 7.50, achieved by four students in the "intending to leave school" category.

Correlations between FS and TRIM scores show increasing correlations with grade level for boys, with $r = 0.08$ at Grade 7, 0.18 at Grade 10, and 0.22 at Grade 11. (The last two r's are significant at the .05 level.) For girls, the analogous data are $r = .50$ at Grade 7, 0.20 at Grade 10, and

0.22 at Grade 11. (The first two r's are significant with $p < .001$ and $p < .05$, respectively.) Data for the high TRIM groups shows mean FS increasing by grade level for boys and decreasing by grade level for girls.

Finally, strong positive correlations existed for both boys and girls when eleventh-grade TRIM scores were compared with mathematics scores on the statewide examination held in the twelfth grade. There was little correlation between FS scores and scores on the statewide examination for boys. For girls, there was a high positive correlation between FS and examination scores for those taking the exams for the applied maths or the pure maths courses (the statistics were .88 and .64, respectively, both significant at the .05 level), but not for those girls who had elected the softer option of general maths.

5. Interpretations.

The author observes that the tendency of females to take fewer mathematics courses after the ninth or tenth grades is continuing, and is all the more striking given that, in this study, proportionately as many girls as boys planned to undertake postsecondary education. Leder suggests the existence of a more complex relationship between FS and grade level for girls than boys. It is hypothesized (after Hoffman, 1974) that for boys, the observed increase in FS with grade level may be a by-product of concerns growing out of an ever-increasing need to make career decisions. For girls, a similar increase in FS is not evident -- confounded, perhaps, by the tendency of high FS females to abandon higher-level mathematics. Leder suggests that for high TRIM groups in the sample FS is associated, for both boys and girls, with choosing a less traditional path -- higher mathematics for girls, and dropping out or taking no mathematics for boys.

In view of the high FS scores of those girls who subsequently took the twelfth-grade applied or pure maths courses, Leder concludes, "...it appears that for girls high FS is more likely to be associated with performing well in a traditional male field and not a type of post hoc rationalization for opting out of serious competition in that field" (p. 133). On the other hand, in view of the female FS data and the grade-by-grade drop in the FS scores of high TRIM girls, the author observes that "...it appears that girls who perform well in mathematics are more likely to be high in FS and yet that, for some, high FS tends to be incompatible with continued high performance in mathematics" (p. 133).

Abstractor's Comments

Although Leder's study breaks no new ground and contains few surprises, it does provide data from another country in an area of research currently of strong interest in the United States. The FS variable is puzzling in many respects, but it revolves around an important problem in mathematics education and probably deserves continued attention from researchers.

The article will not be particularly helpful to those unacquainted with the FS construct. Because of replication difficulties, Horner's construct has been questioned, and researchers do not agree, even in broad terms, on the meaning or existence of FS. The brief discussion of the FS instrument used in the present study will not help in understanding what FS is. Some indication of the FS scoring procedure would aid in evaluating the importance of the data, which generally show a one- or two-point difference in mean FS scores. If scores range from -6 to 24, how does a score of 6 compare with a score of 7? Is this an important (as opposed to significant) difference in "fear" levels?

One wonders whether the procedures of administering the various instruments might have affected the results of the study. Was TRIM administered just before the FS measure? Just after? No such procedural details are provided. Student selection criteria were also unclear. The schools involved were chosen randomly. Did the population consist of all students at the appropriate grade levels in those schools? Leder suggests that some schools were reluctant to let certain students -- particularly eleventh graders -- participate, but the possible effects of such actions is difficult to assess.

The data are nicely displayed and relatively easy to follow, although significance tests were not available in all cases and the type of correlations is not specified. Also, as the author observes, the n's are very low (4 or fewer) in certain of the categories, and a number of conclusions must be tempered by this fact.

Despite a number of such flaws, the study raises some interesting and potentially significant questions; certainly far more questions are raised than resolved. It appears that the FS construct is related to mathematics performance and participation, but just what is being measured under the FS label is unclear. Is FS some type of hybrid composed of ultimately identifiable "apples" and "oranges"? If we grant the legitimacy of FS, we should, based on Leder's conclusions, expect it to affect the performance of males in traditionally female domains. Does it?

Another important question revolves around the interaction of age and FS. Why did Horner (1968) find that a majority of those females low in anxiety were enrolled in science/mathematics areas? Do those students who do not leave non-traditional areas "come to terms" with FS over time? (There are, after all, two ways to overcome a fear of water: avoid it or learn how to swim.) If such is the case, we would expect consistently different results at the secondary and the post-secondary levels.

Finally, the intractable matter of individual differences comes to the fore. If both females who discontinue their mathematics studies and those who do continue are high in FS, then what, if anything, does FS tell us about performance and participation in mathematics? Surely other variables, at least, must come into play here.

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Schoen, Harold L.; Friesen, Charles D.; Jarrett, Joscelyn A.; and Urbatsch, Tonya D. INSTRUCTION IN ESTIMATING SOLUTIONS OF WHOLE NUMBER COMPUTATION. Journal for Research in Mathematics Education 12: 163-178; May 1981.

Abstract and comments prepared for I.M.E. by ROBERT E. REYS, University of Missouri-Columbia.

1. Purpose

Two related studies are reported. Their purpose was to examine "the effects of several methods and modes of teaching estimation in whole number computation" (p. 175).

2. Rationale

The need for teaching computational estimation skills is receiving widespread acceptance. Although several methods and activities for teaching estimation have been proposed, there is a very limited research base to support or repute their effectiveness.

3. Research Design and Procedures: Study 1

Study 1 involved two average ability fourth grades of 21 students each. Researchers developed detailed teacher directions and student worksheets for five 45-minute estimation lessons. The lessons focused on estimating products with one-, two-, and three-digit factors. One group received the estimation instruction while the other group did a unit on multiplication of two-digit numbers. After one week these groups interchanged treatments. Three equivalent forms of the following tests were used during different phases of the study for all students (see Figure 1):

Mental Computation (MC) -- consisted of 10 multiple-choice items involving products of one-, two-, and three-digit factors.

Estimation Test (ET) -- consisted of 10 multiple-choice estimation problems. The stem of each problem was read to students who were asked to choose the best answer.

Problem Solving (PS) -- consisted of 8 multiple-choice work problems requiring students to estimate answers involving addition or multiplication of two or more numbers.

Equivalent forms of this test were given to selected students:

Estimation Interview (EI) -- given to the best five estimators as determined by their ET score. Later it was given to these same students as well as the five students showing the greatest gain on the EP.

In addition, this test was given after the treatment:

Estimation Process Test (EP) -- designed to gain insight into the processes used after the treatment was completed. It contained 6 open-ended problems involving one-, two-, and three-digit factors.

Day 4	Day 7	Day 13
MC1	MC2	MC3
ET1	ET2	ET3
PS1	PS2	PS3
EI1	EI2	EI3
	EP1	EP2

Figure 1. Overview of Tests and Assessment Schedule

All instruction materials and testing instruments were field tested.

An ANOVA, followed by Fishers' Method of least squares where appropriate, was done on the MC, ET, and PS. The data from the EP were categorized and a chi square test was done to analyze process change. The data gained from EI were descriptively presented.

3. Research Design and Procedures: Study 2

Study 2 involved six equivalent groups (3 fifth- and 3 sixth-grade groups) totaling 100 students. A stratified sampling based on composite ITBS scores was used to assign students to treatments which were completed in five consecutive days.

The three treatment groups were defined as follows:

T1 — students completed two computer-assisted lessons on exact computation of sums involving two- and three-digit addends; two similar lessons with multiplication involving one-, two-, and three-digit factors; and a review lesson combining addition and multiplication problems. Each lesson controlled the time per question with the total lesson requiring 10-15 minutes each.

T2 -- students completed similar lessons as T1 except that the practice items involved estimation rather than exact computation.

T3 -- students completed five rounding and estimating lessons built upon the instructional treatment used in Study 1. Students used calculators and completed the researcher-designed worksheets.

The total instructional time for each treatment was about the same.

A total of eight short tests were used:

Test 1 -- contained 12 items and was similar to ET in Study 1, except that it included addition as well as multiplication. (Retention)

Test 2 -- contained 5 items and was similar to PS in Study 1. (Retention)

Test 3 -- contained 12 items and was similar to MC in Study 1, except it included addition as well as multiplication.

Test 4 -- contained 12 items and required the rounding of addends or factors to obtain a result.

Test 5 -- contained 5 items and required student to pick the number sentence closest to a solution of the given word problem.

Test 6 -- contained 12 exact computation items in an open-ended format. (Retention)

Test 7 -- contained 12 word problems with an open-ended response format.

Test 8 -- contained 6 items and was similar to EP in Study 1, except only products were included. (Retention)

All of these tests (total test time was 35 minutes) were given on the day immediately following completion of the treatment. Three weeks later the retention tests were given.

A MANOVA, followed by Fisher t tests were appropriate, was done on all tests except Test 8. The results from Test 8 were handled similarly to the procedure described earlier in Study 1 for the EP.

4. Findings

The results in both studies were consistent and suggested the following conclusions:

1. "All methods of teaching estimation are effective as measured by immediate estimation posttests.
2. This effect was still evident after 1 week (in Study 1) and 3 weeks (in Study 2).
3. There is no evidence that any of the treatments transferred to, or

interfered with, exact computation skills (speeded or unspeeded), or that computation drill affected estimation skills.

4. There is some evidence that meaningful estimation instruction, but not drill only, transfered to estimation in verbal problems. On the other hand, there is no evidence that estimation instruction transfered to problem-solving skill.
5. After estimation instruction most students did adopt the valid estimation strategies they had been taught" (p. 176).

5. <u>Interpretations</u>

This research demonstrates that whole number estimation can be taught in a manner which influences both strategies used and performance reached in a week. There was also significant evidence of retention of both estimation strategies and skills for at least 3 weeks.

There was no evidence that instruction in estimation either improved or interfered with traditional paper/pencil computational skills. In fact, students taught estimation could estimate better and compute as well as a group which spent the same amount of time on estimation alone. This suggests that some instruction on estimation could be substituted for computational drill without any adverse affect on computational skills.

Abstractor's Comments

The authors of these studies addressed a very difficult and timely topic. In so doing they had to do some pioneering work to construct specific instructional materials for the treatment, as well as different measures of computational estimation. In regard to the latter, it is to their credit that a number of different dependent measures were used in this research. These provide a balance and insightful look at treatment effects. In particular, the provisions for interviewing some students to examine their estimation processes deserves a special pat on the back. This was a difficult segment to develop and orchestrate, yet a very necessary consideration when assessing the instructional impact on students' thinking. The attempts to control time on several of the tests is also worth special commendation.. Such time limitations strengthen the validity of the data reported. The inclusion of two separate studies within a single article is somewhat unique. Although this article is a bit more difficult to digest (and abstract), this approach with tandem studies strengthens the research and should be encouraged in more studies. Overall this is an excellent research study which deserves careful reading by researchers as well as classroom teachers.

Having said this, I will now raise several questions regarding these studies:

1. What is computational estimation? There was no mention of any orientation for students and/or teachers. Perhaps it was included in each treatment, or maybe it was handled in the testing directions. My experience suggests that the "mental set of estimation" held by students varies greatly and may be far different from our own. Yet how students perceive estimation and their tolerance for error will greatly influence their solution approach and answers. This issue may have been addressed within the treatment, but it is not clear from the discussion that it was considered.
2. The scope of the study was narrow. The exclusive focus on adding and multiplying whole numbers of the size reported is questionable. It would seem not only appropriate but essential to have extended the strategies used to larger numbers. This would seem to fit naturally within the instructional treatments and would help students better appreciate the power of the estimation strategies being developed.
3. The duration of the treatment and indeed the entire study was short. In particular, the case for retention would be much more convincing if more than 3 weeks had elapsed between the end of the treatment and the retention test. In fact, Study 1 provided only a one-week interval.
4. The researchers relied very heavily on multiple-choice questions to assess estimation skills. Yet some research has reported very different estimation techniques are used in multiple choice items than in open-ended questions. The use of several different measures including individual interviews suggests that the researchers are sensitive to these differences, but it was never acknowledged or mentioned in the report.

Siegel, Alexander W.; Goldsmith, Lynn T.; and Madson, Camilla R. SKILL IN ESTIMATION PROBLEMS OF EXTENT AND NUMEROSITY. Journal for Research in Mathematics Education 13: 211-232; May 1982.

Abstract and comments prepared for I.M.E. by CHARLES E. LAMB, The University of Texas at Austin.

1. Purpose

The study was done in order to "(a) assess developmental differences in children's estimation skills by collecting normative data on their performances on a variety of problems, (b) assess the validity of a proposed model of estimation on the basis of the children's performance, and (c) suggest modification of the model, that is, specify problem dimensions that could guide further research."

2. Rationale

The background section of the article points to the fact that the skill of estimation is an important factor in mathematical measurement activities as well as useful in everyday life. In earlier works (Siegel and Zacharias, 1979), a competence model of the estimation process had been developed. The proposed model concentrated on two important processes in estimation: benchmark estimation and decomposition/recomposition. The proposed model was based on a rational task analysis (a la Gagné).
Note: Benchmark estimation involves the use of a known standard. For example, a piece of notebook paper is about the length of a foot ruler. Decomposition/recomposition involves the breaking up of the to-be-estimated object into workable pieces and then recombining the separate estimates.

The present study used "one dimensional" problems involving linear extent (length and height) and numerosity. There were six types of problems. They were:

(1) Benchmark extent;
(2) Fractional benchmark-extent;
(3) Regular decomposition-extent (RDC-E);
(4) Regular decomposition-numerosity (RDC-N);
(5) Irregular decomposition-extent (IDC-E);
(6) Irregular decomposition-numerosity (IDC-N).

3. Research Design and Procedures

Twenty children, 10 of each sex, from each of the grades 2-8, took part in the study. To provide data relative to mature performance on the problems, a group of ten adults was tested.

A broad sample of children was questioned to determine topics of high interest to them. Topics picked most often were animals, things to read, food, and sports. Interview tasks were designed around these topic areas.

For each of the content areas, six problems were designed, one for each of the problem types. Children were tested individually by one or two experimenters. Children were asked to "think outloud" as they worked. Each interview lasted approximately 40 minutes. Sessions were recorded and transcribed for analysis. Adults were tested in a modified paper-and-pencil format.

4. Findings

The results were reported in sections.

(a) Reasonableness and Accuracy

The criteria for reasonableness was set at plus or minus an order of magnitude of the actual value. Accuracy was defined as an estimate that was within 50% of the actual value. For benchmark and RDC problems, the percentage of unreasonable estimates (UE's) was very low. There were a lot more UE's given on the IDC problems. Children gave accurate estimation (AE's) to more benchmark problems than to RDC problems and more to RDC problems than to IDC problems. Children found fractional benchmarks harder than benchmark problems. For RDC problems, extent problems were easier than numerosity tasks. For IDC problems, extent and numerosity tasks were equally trying for the subjects. In general, adults performed as well or better than did children.

(b) Strategies and Performance

A taxonomy of 10 strategies was devised from a sampling of protocols. They were:

(1) Don't know;

(2) Guess;

(3) Eyeball;

(4) Range;

(5) Benchmark comparison;

(6) Benchmark;

(7) Fractional benchmark;

(8) Multiple benchmark;

(9) Pseudo-composition; and

(10) Decomposition/recomposition.

Children's strategies for solution were compared to the accuracy of their estimates.

In general, benchmark strategies were used the majority of the time on benchmark problems. Children made AE's more for benchmark than for fractional benchmark problems. IDC problems were harder for kids than the RDC problems. The DC strategy was used less on IDC than on RDC problems. Adults' performance on RDC and IDC patterns reflected a similar pattern.

5. Interpretation

In general, the results of the testing confirm the predictions of the proposed hierarchically-ordered model. Revisions in the model were desirable to achieve a better fit of appropriate strategy to accurate estimates. A new, revised model might also allow for the diversity of children's performance. The authors present a revised model.

Note: In the interest of space, the diagrams of the original and revised models are not presented.

Abstractor's Comments

(1) Model building and revision are important processes in the study of mathematics behavior.

(2) The report is extremely thorough and well-written.

(3) The authors have left the interested researcher with many ideas to build on: Their revised model is general enough to be used for time, volume, and other areas of measurement.

(4) It may be possible to combine their results with other measurement work to provide instructional suggestions in the area of measurement.

(5) Their distinction between approximation (19 x 21 is about 400) and estimation (there are about 80,000 people in the stadium) appears to be a useful one.

(6) A personal note: I have abstracted several articles for IME. This has certainly been one of the most pleasurable. This is due to my own personal interests in the measurement behavior of children and the thorough manner in which the authors conducted and reported their work.

Smith, Lehi T. and Haley, J. M. INSERVICE EDUCATION: TEACHER RESPONSE AND STUDENT ACHIEVEMENT. School Science and Mathematics 81: 189-194; March 1981.

Abstract and comments prepared for I.M.E. by ROBERT C. CLARK, Florida State University, Tallahasse.

1. Purpose

The authors' objective was to "evaluate a successful inservice program for teachers of elementary mathematics" (p. 190) on the effectiveness of certain program characteristics and student achievement.

2. Rationale

Inservice education has received increased attention as a result of reduced teacher turnover, increased need for technical knowledge, and the limitations of preservice education programs. Although there is a lack of agreement as to the characteristics of effective inservice, the authors identify four factors common to successful programs:

1. collaborative planning by university faculties and local school personnel,
2. teacher leadership,
3. relevance of the program to actual classroom activities, and
4. convenient location of the inservice classes. (p. 189)

3. Research Design and Procedures

During the 1977-78 school year, administrators from eight Phoenix-area elementary school districts and two university faculty members selected District Resource Leaders (DRLs) from the elementary school faculties. The DRLs and administrators surveyed teachers' inservice needs and formulated programs to meet these needs. In the summer of 1977 and 1978, the two university faculty members worked with the DRLs to prepare inservice programs to be conducted during the 1978-79 academic year.

Approximately 800 volunteer teachers participated in the program, with each district planning and scheduling its own classes. Although programs differed from one district to another, the following goals were common to most of the programs:

1. increasing teacher's understanding of district minimum competency standards,
2. sharing of teaching strategies between teachers,

3. introducing classroom teachers to interesting applications of mathematics, and
4. providing in-class opportunity to construct games and activities appropriate for giving computational drill (p. 190).

Evaluation instruments consisted of a questionnaire for teachers participating in the program and the Stanford Achievement Test: Math Battery, Intermediate Level I for fifth-grade students in the participating districts. All participating teachers completed the questionnaire, which covered attitudes on the quality of the inservice program, changes in attitudes toward mathematics, and changes in teaching style or method. The Stanford Achievement Test was administered to all fifth-grade students in the participating districts at the beginning and end of the school year. Fifth-grade students of teachers involved in the inservice program (n = 198) were considered the experimental group, while comparable classes of teachers not involved in the program (n = 219) made up the control group. Analysis of covariance was used to control for initial differences in mathematics achievement.

4. Findings

The completed evaluation forms of 127 teacher participants indicated changes in understanding the attitude toward mathematics. Seventy-five percent of the teachers completing the evaluation indicated changes in teaching methods, while eighty percent rated the inservice program as "Excellent" or "Good". A chi-square test showed no statistical significance in variance over grade levels, supporting homogeneity with respect to teacher grade level.

The analysis of covariance on the Stanford Achievement Test scores indicated statistical significance favoring the experimental group of fifth-grade students in the Math Computation and Math Application sections. No statistical difference was found in the Math Concepts section of the test.

5. Interpretations

The authors found that teacher response to the inservice program and student achievement supported the program design factors selected. The authors recommend the program as a successful model for the inservice training of elementary teachers and the efficient use of university faculty.

Abstractor's Comments

The paper reports on an evaluation study of an inservice program. The evaluation study paradigm is an appropriate way to study the effects of such an inservice program. With this study the authors have added evidence in support of the acceptability and efficiency of an inservice training design previously supported primarily by intuition.

There are several limitations to this study. First, decisions on program effectiveness were based on the opinions of a small group of participants. It is difficult to extend the results of the attitude survey to all elementary mathematics teachers when the study was limited to volunteer participants. The fact that less than sixteen percent of the program participants completed the questionnaire makes the findings less credible. Participants who had negative attitudes toward the program could have had a number of reasons for not completing the questionnaire.

Second, the handling of student achievement measures leaves a number of concerns. The analysis of covariance was used inappropriately as there were no experimental hypotheses, no prior selection of acceptable limits to type I and type II errors, and no choice of a practical effect size. Since these conditions were not met, no practical significance may be attached to the statistical significance shown. It should also be noted that analysis of covariance was used to control only initial differences in mathematics achievement of the two groups of students. This control does not extend to initial differences in teaching skills of the two groups of teachers. There is every reason to believe that the characteristics of teachers who would volunteer for such a program could account for the measured student differences had there been no inservice program.

Third, in using an experimental research technique to "sanctify" the differences in an evaluation study the authors lost sight of the most important aspects of the situation. Effective use of the evaluation paradigm demands a description of "... how the programs in 'experimental' and 'control' situations actually differ from one another" (Chapters and Jones, 1973). That teachers reported a change in their behavior is not sufficient. What evidence is there that new teaching skills were developed and that these skills were used in the classroom? The authors provide no evidence that the inservice program had any effect on classroom activities.

A fourth limitation of the study is that the goals of the program indicate a heavy emphasis on computational procedures. There can be no doubt that a significant portion of the student population needs to improve such skills, but the study makes no attempt to identify which students demonstrated these gains. Gains in scores of students with adequate computational skills are of questionable value. Further, the emphasis on computational skills may also account for positive teacher attitudes toward the program. The teacher attitude results may not be extended to inservice programs which emphasize a more comprehensive view of student learning.

Finally, it should not be assumed that the results of the study may be extended to inservice programs which use delivery systems that differ significantly from those described. The findings which support the success of the reported program do not demonstrate failings of other delivery systems.

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Abstract and comments prepared for I.M.E. by CAROL NOVILLIS LARSON, University of Arizona.

1. Purpose

The author states: "The purpose of this study was to investigate the mathematical problem solving attitudes of fourth grade students and teachers as related to mathematical problem solving performances of the students" (p. 217). Five questions were investigated related to this purpose.

2. Rationale

Low positive correlations have been reported between student and teacher attitudes toward mathematics and student achievement. Researchers have used single, global measures of attitude toward mathematics, but Aiken (1970) has suggested that attitudes may vary toward different aspects of mathematics, such as computation and problem solving.

3. Research Design and Procedures

Subjects: The subjects consisted of fifteen fourth-grade mathematics classes selected non-randomly from six Wisconsin schools. The classes were utilizing the mathematics program Developing Mathematical Processess (DMP).

Instruments: Three instruments were used in the study. The Mathematical Problem Solving Test consists of 22 three-part items, with each item testing comprehension, application, and problem solving. The Student Mathematical Problem Solving Attitude Scale is a 36-item scale with a modified Likert format. The Teacher Mathematical Problem Solving Attitude Scale is a 40-item scale with a modified Likert format. Thirty-one of the items focus on the teachers' attitudes toward problem solving and nine on their attitudes toward teaching various problem-solving skills and processes. Both attitude scales were developed using a procedure similar to the one used in developing the NLSMA attitude scales.

Procedures: The three instruments were administered twice with an intervening "treatment" period of 12 weeks. In describing the treatment, the author states:

> The treatment was not rigidly controlled ... (and) consisted of instruction.in the regular sequence of DMP topics for fourth grade, with the restriction that teachers select at least one topic from the problem solving strand of the DMP program. (p. 220).

Analysis: In an effort to answer questions of cause and effect between teacher attitudes and student attitudes, and between teacher attitudes and students' problem-solving performance, the researcher used a quasi-experimental design called cross-lagged panel correlation. This design uses time as a third variable.

4. Findings

The results of student and teacher responses to the three instruments are reported, but it is not stated whether the results are from the testing at Time 1, at Time 2, or the two pooled together. The fourth-grade students and teachers were judged to have favorable attitudes toward mathematical problem solving.

The mean scores on the three parts of the Mathematical Problem Solving Test were: Comprehension, 15.52; Application, 9.99; and Problem Solving, 3.27. The possible range of scores on each part of the test is 0 - 22.

The four cross-lagged panel correlations were significant. They are:

1. The three correlations between student performances on each part of the problem-solving test at Time 1 and teacher attitude at Time 2 were significantly more positive than the corresponding three correlations between teacher attitude at Time 1 and student performance on each of the three parts of the problem-solving test at Time 2. All correlations were negative: -0.25 to -0.72.
2. The correlation between teacher attitude at Time 1 and student attitude at Time 2 (0.13) is significantly more positive than the correlation between teacher attitude at Time 2 and student attitude at Time 1 (-0.37).

5. Interpretations

Students in an activity-oriented setting possess favorable attitudes toward mathematical problem solving. Even though this was not a random sample, the author claims that the generalizability of these findings is strengthened by the large number of students tested. All teachers in the study indicated positive attitudes toward mathematical problem solving, but since there were only 15 participating teachers, no generalization was made.

Fourth-grade students performed well on the comprehension and application parts of the Mathematical Problem Solving Test. "However, results suggest that students may not perform as well on multi-step problems whose solutions or methods of solution are not immediately obvious" (p.223).

The author calls for a replication of the study based on the results that student performance had a greater effect on teacher attitude than teacher attitude had on student performance.

Abstractor's Comments

An interesting aspect of this study is the use of cross-lagged panel correlations to show cause and effect relationships between the variables investigated. This research design, discussed by Campbell and Stanley (1963), appears to be used appropriately in this study. Campbell and Stanley claim that repeated testing is not a serious weakness in this design "unless an interaction or testing effect specific to but one of the variables were plausible" (p. 69). The author in the discussion section reiterates his findings from the cross-lagged panel correlations and claims it is valuable information. This abstractor would like to have seen these findings discussed in greater length, specifically those related to teacher attitudes and student performance. Also, the negative correlations between teacher attitude and student performance should be addressed since "low, positive correlations" (p. 217) are more normal.

It was worth reading this study to discover the three instruments described. Two technical reports cited in the paper appear to describe in depth the development of these instruments. Perhaps the most valuable part of this study was the instrument development. From the author's brief characterization, it appears that further study of these two technical reports would be worthwhile.

A major problem with this article is that it lacked critical information. In the procedure section, two testing periods of twelve weeks apart are described. Yet in the section on analyses and findings only one set of data is discussed. It is impossible to tell if the data described is from testing at Time 1, at Time 2, or the two pooled together. Not only should this have been clearly stated, but the means and standard deviations of the data from both testing periods should have been presented. These data would not only have provided a clear description of the sample at the two testing times, but would also have provided the reader with needed information for understanding the results of the various correlations presented.

In addition, the treatment does not seem to be a treatment at all. The regular curriculum was taught "with the restriction that teachers select at least one topic from the problem solving strand of the DMP program" (p. 220). Was the treatment supposed to interact in some way with the variables being studied? Given the lack of a control group, no common treatment in problem solving across classes, and the additional fact that no mention is ever made of the "treatment" in other than the procedure section, the answer would seem to be "no". Since there really wasn't any treatment except to continue teaching the regular program, it is hard to know why the pretest was given the fourth month of the year. It would be logical to expect that the greatest impact of one of the variables on another would be more likely to occur during the first 12 weeks of the school year rather than the second 12 weeks.

It is difficult to evaluate the study given the problems in the written report:

1. the results selected to be included are incomplete, and
2. there is an inadequate discussion of the findings, the purpose and impact of the treatment, and needed research in addition to replication.

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