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ABSTRACT

Presented here are two models that can be used to solve distribution problems, such as assigning teachers to students, teachers to courses, or special students to schools. The models, the assignment model and the transportation model, are termed evaluation models under a definition of evaluation that delineates its function as that of serving decision-making. The paper offers step-by-step procedures for the use of both models. The models are applicable to assignment problems where there is a variable to be optimized, such as teacher satisfaction. In the example used to demonstrate the assignment mode, students are assigned to teachers in a way that matches them up with the students they request as much as possible. The transportation model is like the assignment model but with added constraints. The example of the transportation model given assigns teachers to sections and courses when a certain number of sections must be taught and a certain number of class periods are available. The author concludes that the advantages of these models are that they give a better solution than can be obtained by inspection and they take teachers' wishes into account regarding assignment. Computer use of the models is mentioned. (Author/JM)

paper and report series

No. 68 THE USE OF ASSIGNMENT AND TRANSPORTATION MODELS IN EVALUATION

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PREFACE

The Research on Evaluation Program is a Northwest Regional Educational Laboratory project of research, development, testing, and training designed to create new evaluation methodologies for use in education. This document is one of a series of papers and reports produced by program staff, visiting scholars, adjunct scholars, and project collaborators--all members of a cooperative network of colleagues working on the development of new methodologies.

What is the nature of the assignment and transportation models from operations research? How might these models be used in evaluation? Darrel Caulley considers these questions in this paper. For each of the two models, Dr. Caulley gives algorithms on how the models can be solved. Using examples, he gives step-by-step procedures for finding optimal solutions to certain evaluation problems.

Nick L. Smith, Editor
Paper and Report Series

CONTENTS

THE USE OF ASSIGNMENT AND TRANSPORTATION MODELS IN EVALUATION

Transportation and assignment models can both be used to solve distribution problems. In education, distribution problems arise in such cases as assigning teachers to students or teachers to courses to teach. The problem is to assign the teachers in such a way that some criteria such as teacher satisfaction is optimized. If teachers give a numerical rating to the desirability of teaching certain students or of teaching certain courses, the assignment which caters optimally to the teachers' wishes is sought. Other examples of distribution problems are assigning bilingual students to schools, limited audiovisual material to classrooms, principals to schools, students to occupational education experiences, reading specialists to schools, and microcomputers to classrooms. In each example, for the models to be applicable, there must be some variable that is to be optimized.

Stufflebeam, et al. (1971) define evaluation as "the process of delineating, obtaining, and providing useful information for judging decision alternatives" (p. 10). Thus, the function of evaluation is to serve decision making. The transportation and assignment models serve decision making, so they could be termed evaluation models.

This paper is intended to give step-by-step procedures for solving a class of problems which has to do with the assignment of something. These problems could be solved by an appropriate computer program. However, the algorithms given for the solution of the problems can readily be solved by hand without the problem of gaining access to a computer. This paper first looks at the assignment model and then the transportation model. While the assignment model is a special case of the transportation model, they are solved by entirely different algorithms. Two examples will be solved for each model.

Assignment Model

As the name of the model implies, something is assigned to something. This could be the assignment of tutors to students or the assignment of teachers to courses. Another example would be where we have buses at various locations and we want to minimize the miles travelled to pick up pupils at various sites. Suppose we take the example of four tutors assigned to four students. For the solution to the problem the number of tutors assigned must equal the number of places to be filled. We will show later how this requirement may be circumvented. The students have been interviewed by the tutors who have then assigned ratings to each student, in which 1 is high desire to teach and 7 is low desire to teach. We then wish to assign the four tutors to the four students so as to minimize the sum of these ratings so that we obtain the most desirable assignment of tutors to students. Suppose the ratings given are as in the following table.

Students

		1	2	3	4
	A	4	7	2	2
Tutors	B	3	7	6	3
	C	6	7	6	4
	D	1	4	7	2

Figure 1 (adapted from Eck, 1976, p. 261) represents the algorithm for solving an assignment problem. The solution for the above example is as follows:

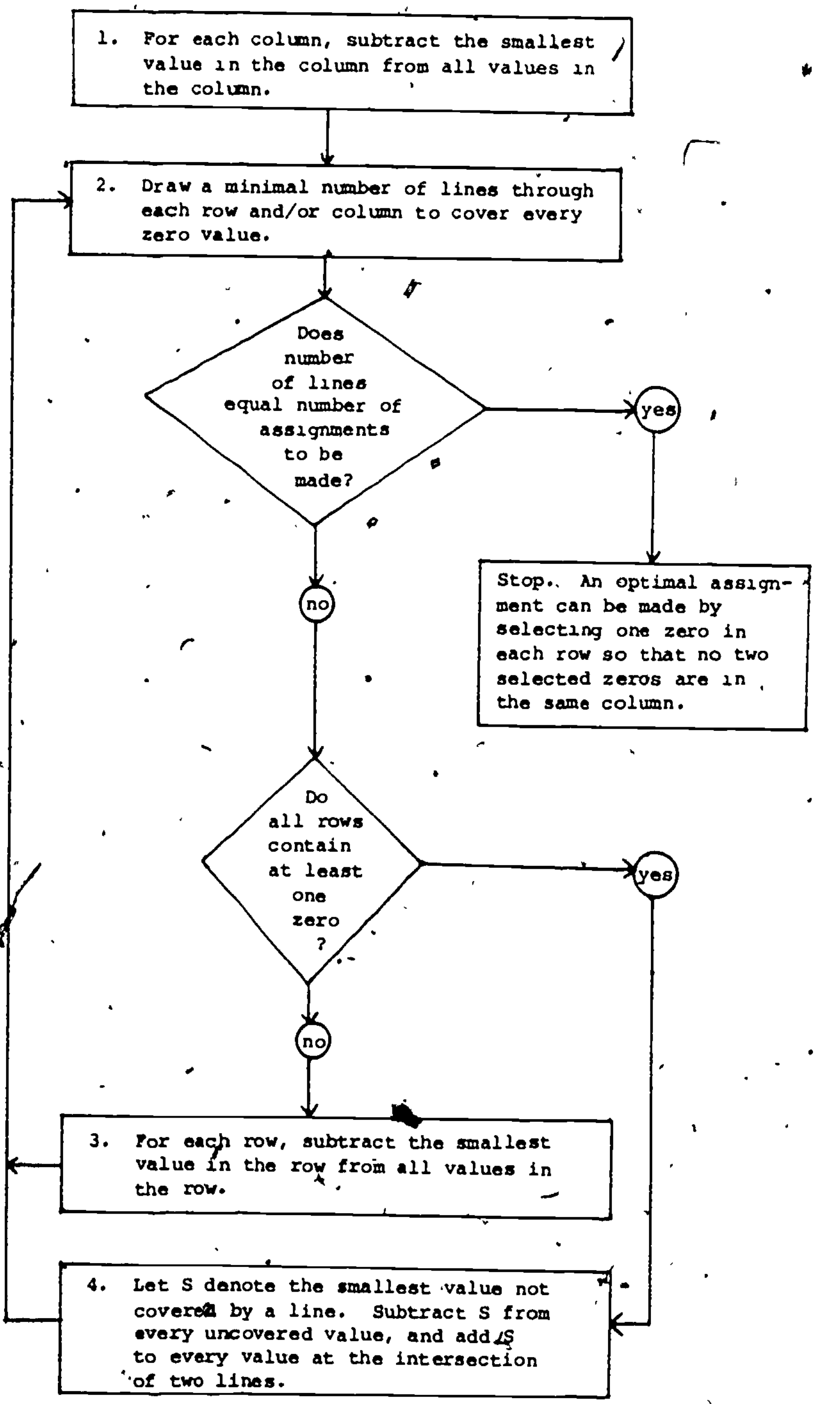
Step 1

	1	2	3	4
A	3	3	0	0
B	2	3	4	1
C	5	3	4	2
D	0	0	5	0

Step 2

	1	2	3	4
A	~~3~~	~~3~~	~~0~~	~~0~~
B	2	3	4	1
C	5	3	4	2
D	~~0~~	~~0~~	~~5~~	~~0~~

Figure 1. The Assignment Algorithm



The number of lines (2) does not equal the number of assignments to be made. Rows B and C do not contain zero values. Accordingly, to to Step 3.

Step 3

	1	2	3	4
A	3	2	0	0
B	1	2	3	0
C	3	1	2	0
D	0	0	0	0

Return to Step 2.

Step 2 (repeated)

	1	2	3	4
A	3	2	0	0
B	1	2	3	0
C	3	1	2	0
D	0	0	5	0

The number of lines does not yet equal the number of assignments to be made, but all rows now contain at least one zero. Go to Step 4.

Step 4, S=1

	1	2	3	4
A	3	2	0	1
B	0	1	2	0
C	2	0	2	0
D	0	0	5	1

Return to Step 2.

Step 2 (repeated)

	1	2	3	4
A	3	2	0	1
B	0	1	2	0
C	2	0	2	0
D	0	0	5	1

The minimal number of lines now equals the number of assignments to be made, so stop. An optimal assignment can be made by selecting one zero in each row so that no two selected zeros are in the same column. In this case there are two optimal assignments.

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1. Tutor A is assigned Student 3
   Tutor B is assigned Student 1
   Tutor C is assigned Student 4
   Tutor D is assigned Student 2

2. Tutor A is assigned Student 3
   Tutor B is assigned Student 4
   Tutor C is assigned Student 2
   Tutor D is assigned Student 1

The minimal sum of ratings for solution one is 2+3+4+4 = 13, and for solution two is 2+3+7+1 = 13. The second solution is not an obvious one since rating 7, the lowest rating, is chosen.

In this problem we aimed to minimize the assignment values, but we could just as easily maximize in such a problem. For example, the ratings could be given in the opposite direction so that 1 is low and 7 is high. Only Steps 1 and 3 would be changed. Step 1 would read: For each column, subtract each cell from the greatest value in the column. Step 3 would read: For each row, subtreact each cell from the greatest value in the row.

Let us consider another example. Suppose there are four teachers to be assigned to three courses. Clearly the end result will be that one of the four teachers will not be assigned a course. The four teachers have given a rating to each course according to their desire to teach the course so that 1 is high and 7 is low.

Courses

		1	2	3	4
Teachers	A	7	4	1	0
	B	2	4	6	0
	C	1	3	L	0
	D	3	4	2	0

In order to find a solution, the number teachers assigned must equal the courses. So, course 4 is a dummy course with all zeros in the cells. Also teacher C cannot teach course 3, so L is placed in the cell. L stands for a large number which is so large that it would never be chosen. The steps in the solution are as follows.

Step 1. According to Step 1 of the algorithm, 1 is subtracted from each value in column 1, 3 is subtracted from each value in column 2, etc. The result is

Step 1

	1	2	3	4
A	6	1	0	0
B	1	1	5	0
C	0	0	L	0
D	2	1	1	0

Step 2

	1	2	3	4
A	6	1	6	0
B	1	1	5	0
C	0	0	L	0
D	2	1	1	0

Step 4. We are to assign four teachers to four courses, so that four assignments are to be made. Three lines are needed to cover all zeros. Because the number of lines is not equal to the number of assignments, we next ask, "Do all rows contain at least one zero?" The answer is yes, so we proceed to Step 4. One is the smallest value not covered by a line. So subtract one from each value that is not covered by a line, and add one to each value that is at the intersection of two lines in the above array. The result is

	1	2	3	4
A	6	1	6	1
B	0	0	4	0
C	0	0	L	1
D	1	0	0	0

Step 2 (repeated). We now return to Step 2 and redraw a minimal number of lines to cover all zeros on the previous array.

	1	2	3	4
A	6	1	6	1
B	0	0	4	0
C	0	0	L	1
D	1	0	0	0

The number of lines does not equal the number of assignments to be made. The next question is do all the rows contain at least one zero. The answer is no, so proceed to Step 3, and for

each row subtract the smallest value in the row from all values in the row.

Step 3

	1	2	3	4
A	5	0	5	0
B	0	0	4	0
C	0	0	L	1
D	1	0	0	0

Step 2 (repeated)

	1	2	3	4
A	~~5~~	~~0~~	~~5~~	~~0~~
B	~~0~~	~~0~~	~~4~~	~~0~~
C	~~0~~	~~0~~	~~L~~	~~1~~
D	~~1~~	~~0~~	~~0~~	~~0~~

The number of lines now equals the number of assignments to be made so that the process can be stopped. There are three optimal solutions.

Teacher A to dummy 4
Teacher B to course 1
Teacher C to course 2
Teacher D to course 3
Sum of ratings =
0+2+3+2 = 7

Teacher A to dummy 4
Teacher B to course 2
Teacher C to course 1
Techer D to course 3
Sum of ratings =
0+4+1+2 = 7

Teacher A to course 2
Teacher B to dummy 4
Techer C to course 1
Teacher D to course 3
Sum of ratings =
4+0+1+2 = 7

This completes the discussion of the assignment model. Next will be a discussion of the transportation model.

The Transportation Model

Although the transportation model is used to solve transport problems, the example given here is one from education.

	Course 1	Course 2	Course 3	No. of Class Periods Available
Teachers 1	7	3	2	3
Teachers 2	4	1	6	4
No. of Sections Required to be Taught	2	2	3	(7)

Figure 2. The Transportation Model for Assigning Teachers to Courses

Suppose there are three courses to be taught and two teachers to do the teaching. The teachers have available various numbers of class periods. The number of sections for each course that are required to be taught are also given in the above figure. It is an assumption of this model that the total number of units available (total number of class periods) equals the total required (total number of sections). For each course the teachers have given a preference rating according to whether they would like to teach a course. For the seven-point rating, 1 is high and 7 is low. The ratings are given in the top right-hand corners of the cells of Figure 2. The problem is to assign teachers to sections and courses so as to minimize the sum of the ratings.

The transportation model is like the assignment model but with two added constraints. In the above example, the constraints are the number of class periods available and the number of sections required to be taught.

The transportation algorithm is initiated by finding a first, not necessarily optimal, solution (Page, Note 1).

(a) Find the lowest cell in Figure 2. If there is a tie, make an arbitrary choice of the lowest cells. In this case, the lowest preference is 1, which is found for teacher 2 and course 2.

(b) Compare the available units in that row (Row 2) with the required units in that column (Column 2).

(i) If available units are less than required units, assign the available units to the chosen cell and delete the row (by crossing out the outer cells). Now adjust the column demand.

(ii) If the required units are less than the available units, we assign the required units to the cell in question, delete the column and adjust the available units in the row. (See Figure 3)

	1	2	3	
1	7	X 3	2	3
2	4	2 1	6	~~4~~ 2
	2	~~2~~ 0	3	

Figure 3. Selection of the First Assignment

(iii) If the available units are equal to the required units for a cell and this is the last cell chosen, then STOP, as a solution has been reached. If they are equal and this is not the last cell chosen, then the solution is degenerate. If the solution is degenerate, assign the required units to the cell in question, delete the row and column and adjust the required and available units in the column and row respectively.

(iv) Now return to step (a), considering only the remaining rows and column in the Figure 2.

Figure 4 shows the second assignment which is for cell x_{13}. (i.e. for row one and column three). Note that for this cell, the number of available units equals the number of units required and thus the solution is degenerate.

	1	2	3	
1	X 7	X 3	3 2	~~3~~ 0
2	4	2 1	X 6	~~4~~ 2
	2	~~2~~ 0	~~3~~ 0	

Figure 4. <u>Selection of the Second Assignment</u>

Figure 5 shows the third assignment which is for cell x_{21} which is the last cell.

	1	2	3	
1	X 7	X 3	3 2	~~3~~ 0
2	2 4	2 1	X 6	~~4~~ ~~2~~ 0
	~~2~~ 0	~~2~~ 0	~~3~~ 0	

Figure 5. <u>Selection of the Third Assignment</u>

The solution is that Teacher 1 teaches course 3 and Teacher 2 teaches courses 1 and 2. We still have to test whether this is the optimal solution.

If there is m teachers and n courses, it can be shown that there is m+n-1 cells in the solution. A degenerate solution will have fewer than m+n-1 cells having non-zero values. In the above example the solution is degenerate m+n-1 = 2+3-1 = 4 and only three cells have positive values. (The deleted cells are considered to have a value of zero.)

To avoid degenerate basic solutions, one or more cells can be increased in value from zero to a slightly positive amount <u>p</u>. It is understood that <u>p</u> is greater than 0 and <u>p</u> approaches zero in value. If a cell is assigned a value of <u>p</u> to avoid degeneracy, the value will be treated like any other strictly positive-valued basic cell. Cells that have values of <u>p</u> will be interpreted to have values so close to zero that no teacher will be assigned to a course (Eck, 1976).

Having found a first feasible solution, it is now necessary to determine whether or not there exists a better (lower rating) solution. To evaluate other possible solutions, attention is directed to cells of the table where the x_{ij}'s in the initial solution have values of zero. Since the solution is degenerate, choose one of the x_{ij}'s equal to zero to have a value of p. Suppose we make an arbitrary choice of x_{12} to have the value p. (Figure 6)

		Courses 1	2	3	Available
Teachers	1	7 $x_{11}=0$	3 $x_{12}=p$	2 $x_{13}=3$	3 0
	2	4 $x_{21}=2$	1 $x_{22}=2$	6 $x_{23}=0$	4 2 0
Required		2	2	3	

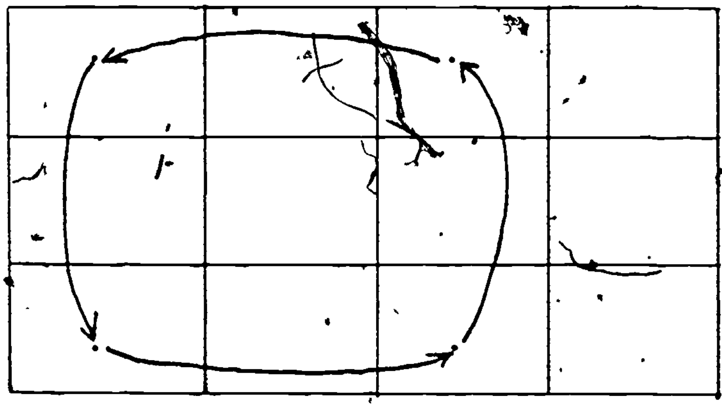
Figure 6. Selection of a Cell to Have a p Value

Suppose that one unit is added to cell x_{11} so x_{11} increases in value from 0 to 1. To accommodate this change and still satisfy the rim requirements, x_{12} can be reduced by one unit, x_{22} can be increased by one unit, and x_{21} can be reduced by one unit. The quantities shown in the row labeled "Required" and in the column labeled "Available" are called rim requirements. The net change in the total rating due to the modifications will be 7-3+2-4 = +2, which indicates an increase in total rating. Hence this solution is inferior to the solution first found. Another reason why it is inferior is that x_{12} cannot be reduced by one unit without it becoming negative, which has no meaning. Notice that the search for a new improved solution was started by looking at a cell where x_{ij} had a value of zero, and by making modifications to all of the x_{ij}'s along a loop. Eck (1976) gives a formal definition of a loop.

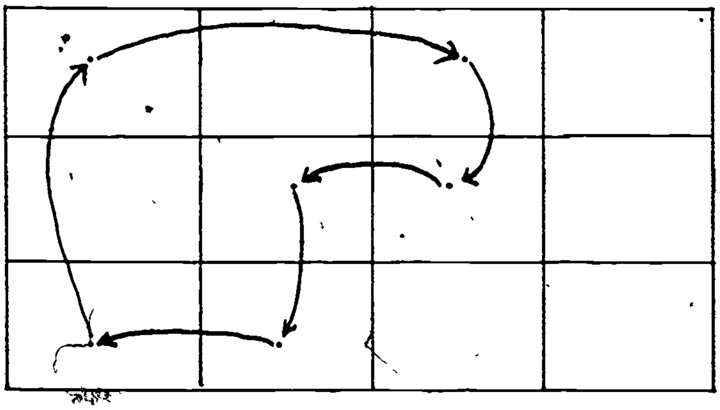
> A loop in a transportation rating matrix is a sequence of four or more variables, where the first variable in the sequence follows the last variable in the sequence when
>
> (a) no more than two consecutive variables in the sequence belong to the same row or column, and
>
> (b) any two consecutive variables in the sequence belong to either the same row or same column. (p. 250)

Examples of valid and invalid loops are as follows.

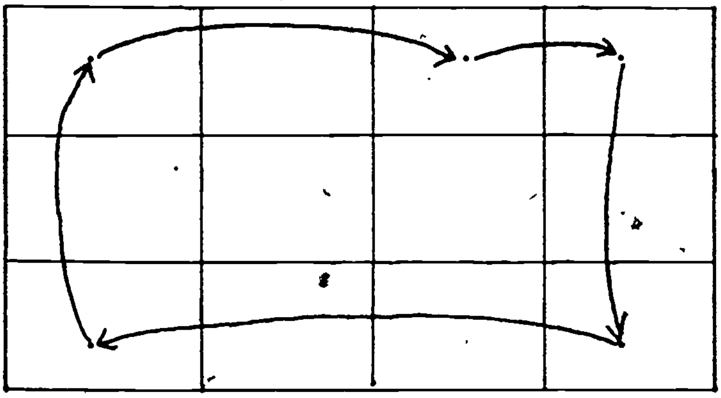
Valid loop:



Valid loop:



Invalid loop:



To find a better solution, pick a cell with value zero to enter the solution. We next obtain a loop that includes the entering cell and other cells (having values other than zero). If it is impossible to form a loop in which all cells other than the entering cell have strictly positive values, some cells may

be increased to p. Once a suitable loop has been obtained, the entering cell is increased in value to the level where some other cell in the loop must be decreased to zero to satisfy rim requirements.

Another example will be given to show how the loop works. Figure 7 shows the solution to a problem.

		Courses 1	Courses 2	Courses 3	Available
Teachers	1	(2) $x_{11}=0$	(3) $x_{12}=0$	(7) $x_{13}=4$	4
	2	(2) $x_{21}=1$	(1) $x_{22}=5$	(4) $x_{23}=1$	7
	3	(6) $x_{31}=0$	(5) $x_{32}=0$	(3) $x_{33}=3$	3
Required		1	5	8	

Figure 7. A Solution to a Rating Matrix Problem

Consider the cell x_{12}. Suppose that one unit is added to x_{12}. To accommodate this change and still satisfy the rim requirements, x_{13} can be reduced by one unit, x_{23} can be increased by one unit and x_{22} can be reduced by one unit. The net change in total ratings due to the modifications will be

$$3-7+4-1 = -1$$

which indicates a desired decrease in total cost.

If instead of increasing x_{12} from zero to a value of 1, the value of x_{12} is increased as much as possible (while x_{13}, x_{23}, and x_{22} are suitably modified), total cost will be further reduced. It can be noticed that x_{13} will become negative if value x_{12} is increased to a value in excess of 4. It follows that x_{12} cannot have a value greater than 4. If x_{12} is increased to a value of 4, then x_{13} must be reduced by 4 (to $x_{13} = 0$), x_{23} must be increased by 4 (to $x_{23} = 5$) and x_{22} must be decreased by 4 (to $x_{22} = 1$). Figure 8 shows the new improved solution.

	Courses 1	2	3	Available
Teachers 1	2 $x_{11}=0$	3 $x_{12}=4$	7 $x_{13}=0$	4
2	2 $x_{21}=1$	1 $x_{22}=1$	4 $x_{23}=5$	7
3	6 $x_{31}=0$	5 $x_{32}=0$	3 $x_{33}=3$	3
Required	1	5	8	

Figure 8. <u>New Improved Solution</u>

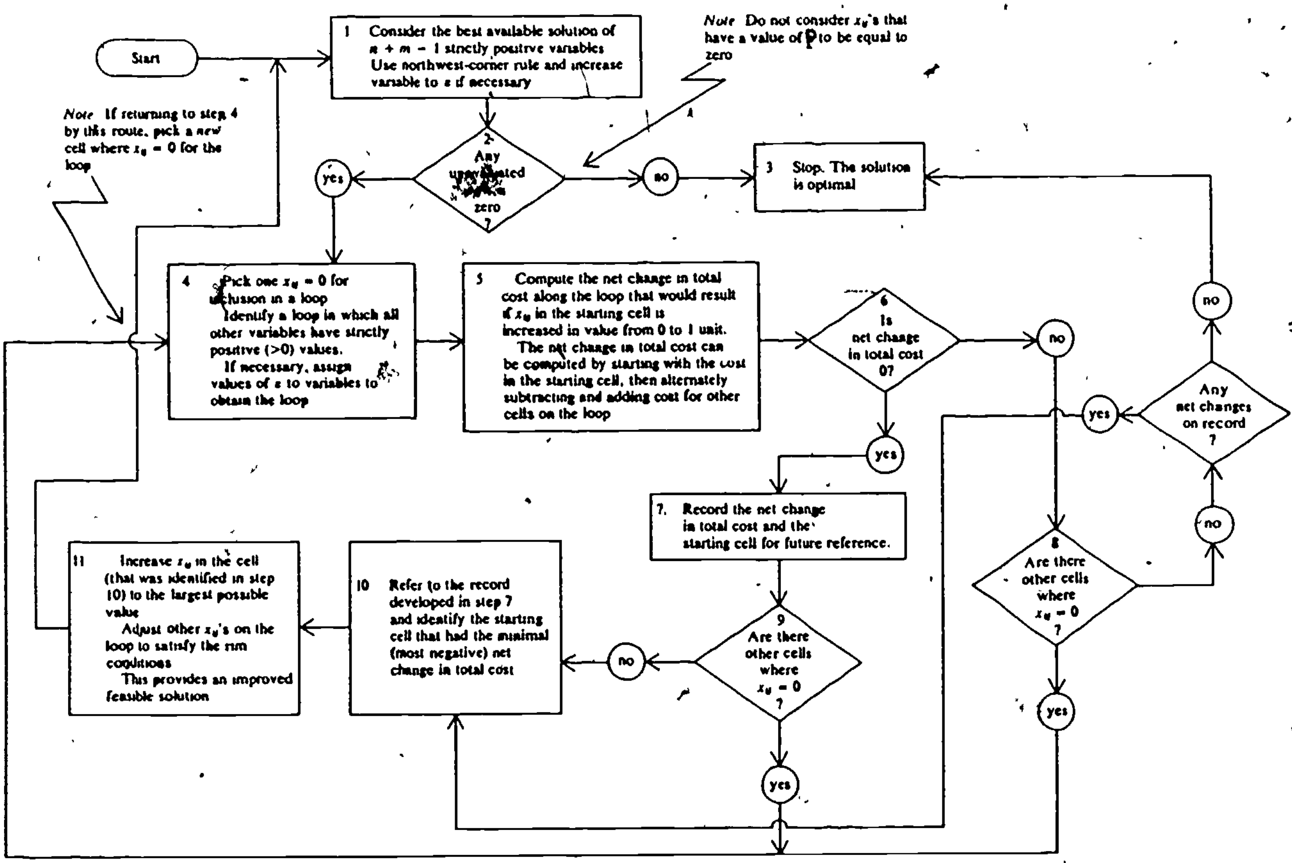
Figure 10 shows an algorithm for finding a solution for a rating matrix. To illustrate the algorithm two examples will be used. The first example is shown in Figure 9.

	Courses 1	2	
Teachers 1	1	2	1
2	3	7	5
	4	2	

Figure 9. <u>An Example Rating Matrix</u>

The solution is as follows.

Figure 10. A Transportation Algorithm
(Adapted from Eck, 1976, p. 252)



Step 1. Figure 11 shows the initial solution found.

Courses

Teachers	1	2	
1	1 $x_{11}=1$	2 $x_{12}=0$	1
2	3 $x_{21}=3$	7 $x_{22}=2$	5
	4	2	

Figure 11. <u>An Initial Solution</u>

<u>Step 2</u>. x_{12} has a value of zero. Go to Step 4

<u>Step 4</u>. An appropriate loop is x_{12} to x_{11} to x_{21} to x_{22}. Go to Step 5.

<u>Step 5</u>. Net change = 2-1+3-7 = -3. Go to Step 6.

<u>Step 6</u>. Net change is less than zero. Go to Step 7.

<u>Step 7</u>. Record: Start of loop = x_{12} with net change -3. Go to Step 9.

<u>Step 9</u>. There is no other cells with x_{ij} equal to 0. Go to Step 10.

<u>Step 10</u>. The starting cell on record is x_{12}. Go to Step 11.

<u>Step 11</u>. The largest possible value that x_{12} can be adjusted to is 1. Thus

$$x_{12} = 0+1 = 1$$
$$x_{11} = 1-1 = 0$$
$$x_{21} = 3+1 = 4$$
$$x_{22} = 2-1 = 1$$

Figure 12 shows the improved solution.

Courses

Teachers	1	2	
1	1 $x_{11}=0$	2 $x_{12}=1$	1
2	3 $x_{21}=4$	7 $x_{22}=1$	5
	4	2	

Figure 12. <u>An Improved Solution</u>

Step 1. The above improved solution is now the best solution under consideration. Go to Step 2.

Step 2. x_{11} has a value of zero. Go to Step 4.

Step 4. An appropriate loop is x_{11} to x_{12} to x_{22} to x_{21}. Go to Step 5.

Step 5. The net change in rating is 1-2+7-3 = +3. Go to Step 6.

Step 6. The net change is greater than zero. Go to Step 8.

Step 8. There are no other cells where x_{ij} = 0. To to Step 3

Step 3. Stop. Figure 12 shows the optimal solution.

The example in Figure 2 is now used to illustrate the algorithm.

Step 1. Figure 6 shows the best available solution. Go to Step 2.

Step 2. There are unevaluated x_{ij}'s. Go to Step 4.

Step 4. Pick x_{11} (an arbitrary choice). A suitable loop is x_{11} to x_{21} to x_{22} to x_{12}. Go to Step 5.

Step 5. The net change in rating is 7-4+1-3 = +1. Go to Step 6.

Step 6. The net change is greater than zero. Go to Step 8.

Step 8. Cell x_{23} equals zero. Go to Step 4.

Step 4. A suitable loop for x_{23} is x_{23} to x_{13} to x_{12} to x_{22}. Go to Step 5.

Step 5. The net change in rating = +6-2+3-1 = +6. Go to Step 6.

Step 6. The net change is greater than 0. Go to Step 8.

Step 8. There are no more cells with x_{ij} equal to zero. Go to Step 3.

Step 3. Stop. The initial solution is the optimal solution. Interpret x_{12} to have a value that is essentially zero.

References on the solution of assignment and transportation problems that can be used are Chapter 6 of Hillier and Liberman (1967), Chapter 8 of Trueman (1977), and Chapter 8 of Eck (1976). The best of these references is Eck (1976).

The transportation model can be used whenever something is assigned to something under two marginal restrictions. For example, suppose that students are to be assigned to occupational experiences taking into account the preferential rating of students. The marginal restrictions are that the students are

restricted according to the number of class periods per week they have available, and according to the number of class periods per week the experiences are available for. Figure 13 illustrates the example. The numbers in the cells are the preferential ratings of students.

Students	Occupational Experiences Available 1	2	3	No. of Class Periods Available
1	2	4	3	5
2	1	7	5	3
No. of Periods Experiences Are Available For	3	1	4	

Figure 13. <u>An Example of a Transportation Problem</u>

The disadvantage of the transportation and assignment models is that they are a little tricky to learn. The advantages is that they give a better solution than can be obtained by inspection, and they take the teachers' wishes into account regarding assignment to students or courses. These models assist decision makers and thus are a part of the storehouse of evaluation methods. The examples given in this paper are small and can be solved by hand. However, in practice the problems to be solved are larger and a computer can be used to reduce the amount of tedium involved in solving the larger problems.

At least two computer programs are available which will carry out the assignment and the transportation models. One computer program is the DSZ1IP algorithm of the MPOS program. MPOS runs only on the CDC 6000/CYBER series of computers. This program is available from Northwestern University, Vogelback Computing Centre, 2129 Sheridan Road, Evanston, Illinois 60201. Another program available is UKILT. UKILT runs only on UNIVAC 1100 series computers (contact your local SPERRY UNIVAC office to find out if there is a UNIVAC 1100 series computer with UKILT available.

Reference Notes

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