

DOCUMENT RESUME

ED 223 965

CG 016 374

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TITLE An Approach to the Modeling of Long-Range Stability in Marital Dyads: The Use of Computer Simulation Methodology. Preliminary Draft.
SPONS AGENCY National Inst. of Mental Health (DHHS), Bethesda, Md.
PUB DATE 13 Oct 81
GRANT NIMH-T32-MH-14594
NOTE 20p.; Paper presented at the Annual Meeting of the National Council on Family Relations (Milwaukee, WI, October 13-17, 1981). Figures 2-10 are not filmed due to small print size.
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Adults; Behavior Change; *Behavior Patterns; *Computer Oriented Programs; Interaction Process Analysis; *Marriage; *Mathematical Models; Models; *Prediction; *Simulation; Spouses
IDENTIFIERS *System Dynamics

ABSTRACT

Researchers are now attempting to improve techniques for studying stable behavior patterns in family systems over long periods of time, e.g., the marital dyad across several levels of function and dysfunction covering time periods of months to years. One approach to improving this research design is based on the assumptions that longitudinal studies will be improved if the expected time course of the behavior patterns can be forecast in advance, and that computer simulation is one of the best technical approaches available for problems in long-range forecasting. System Dynamics, a computer simulation approach that is designed to deal with behavior patterns of complex systems over extended time periods (even when those patterns involve relatively severe nonlinearities), was used to develop a model for dyadic interaction. The forecasting of behavior patterns in a marital dyad began from a mathematical model of the dyad, which contained two "individuals," each represented by a mathematical model of personality function and a linking equation describing how each "individual" was influenced by behavior changes in the other "individual." Data analyses of the simulation model supported the hypothesis that, assuming the validity of the Freudian model of individual personality function and the accuracy of its translation into a mathematical model, equations exist that link two "individuals" in such a way as to produce equilibrium for both "individuals," even when neither "individual" model will tend to equilibrium, given the same starting conditions, without being linked in a dyad. Further investigation of this approach is needed. (HLM)

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ED223965

Preliminary Draft

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AN APPROACH TO THE MODELING OF
LONG-RANGE STABILITY IN MARITAL DYADS:
THE USE OF COMPUTER SIMULATION METHODOLOGY

C 5-116

Prepared for

Theory Construction and Research Methodology
NCFR Meeting

October 13, 1981
Milwaukee, Wisconsin

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The authors would like to acknowledge the partial support
of this research by NIMH Grant No. T32 MH14594,
and the assistance of Dr. Cees Wegman.

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INTRODUCTION

This paper describes a project aimed at improving research techniques for studying stable behavior patterns in family systems over long periods of time. The early work is restricted to the study of the marital dyad across several levels of function and dysfunction, covering time periods of months to years.

The approach to improving research design is based on two assumptions:

- Longitudinal studies will be improved if the expected time course of the behavior patterns ("baseline" patterns and rates) can be forecast in advance; and,
- Computer simulation is one of the best technical approaches available for problems in long-range forecasting.

Modeling baseline behavior patterns and rates against which to compare observed changes presents two problems. First, the baseline itself may change over time, a problem generally discussed in terms of maturation effects or historical effects as threats to internal validity of experimental design (Cook & Campbell, 1979). Second, baseline patterns of dyadic behavior may best be compared to the baseline patterns of each individual had that person not married, since this comparison seems to be implicit in many studies of marital stability and satisfaction (Roloff, 1981).

The computer simulation approach used in this study is System Dynamics (Forrester, 1968). This is a methodology designed specifically to deal with behavior patterns of complex systems over extended time periods, even when those behavior patterns involve relatively severe nonlinearities. This method has the advantage of building on previous work (Wegman, 1977). The interlocking of numerous positive and negative feedback loops results in a system not easily described in words and having behavior patterns which can not be forecast by simple linear extrapolation from present conditions. Although numerous investigators have commented that the study of human behavior is hindered by just such considerations (Strauss, Bartko, & Carpenter, 1981), little quantitative methodology designed to overcome these problems has yet emerged. It is these methodological deficits that the present project addresses.

A PROPOSED MODEL FOR DYADIC INTERACTION

The forecasting of behavior patterns in a marital dyad starts from a mathematical model of that dyad. The model contains two "individuals," each represented by a mathematical model of personality function, and a linking equation which describes how each individual is influenced by the behavior changes in the other individual.

The behavior of the dyadic model is forecast over long periods of time. A range of different initial conditions and a variety of different linking equations are examined.

The source of the model for each of the individuals is Freud's Counterwill Theory translated into a mathematical model by Wegman (1977). The behavior of this model over time was described under three different starting conditions (Figures 2 and 7).

The sources of the various linking equations include laboratory research in social psychology (Reiss, 1980; Gottman, 1979), clinical work in family therapy (Bowen, 1966; Wertheim, 1973), survey research in family sociology (Olson et al., 1979), and clinical behavior therapy (Weiss, Hops, & Patterson, 1973, in Gottman, 1979).* Although the ideas for the linking equations come from the sources cited, the responsibility for translating them into mathematical terms, and from equations into the conventions used in the System Dynamics method, rests with the investigators in the present project.

* Note: When an abstract of this project was proposed for acceptance in the present workshop, it was hoped that some of the linking equations could come from Social Exchange Theory (Roloff, 1981) and Interpersonal Perception Theory (Malone, 1975). However, both of these approaches require complex matrix calculations in order to represent the interaction processes, and proved too difficult to complete in the time available prior to the workshop.

Building upon Wegman's individual models, which were described in isolation from external inputs, the present dyadic model describes the ongoing effects of the interpersonal interaction upon two internal emotional "economies." Thus, the present dyadic model assumes that the details of internal psychological conditions, or personality system "states" involving the emotions and cognitions of each individual, are extremely important in understanding the behavior of each member of the dyad (Guthrie, 1938). Important aspects of interpersonal relationships modeled by other investigators are not dealt with in this model, but may be added in later stages of this project. Among these issues are:

- a) The social context of the dyadic relationship, i.e., the influence of a third person or of larger groups;
- b) the possibility of simultaneous exchange of material resources along with emotional resources, i.e., material or socioeconomic environment, and, simultaneously, mutual validation of self-concept (which is modeled in the present investigation);
- c) classes or types of behaviors, rather than just the rates of behaviors (which are modeled in the present investigation);
- d) cognitive processes using complex decision rules and strategies from game theory and other long-range or anticipatory behavior on the part of individuals; and
- e. developmental processes such as interpersonal attraction or commitment, maturation or aging of either or both individuals, or changes in the relationship over time (changes in the linking equation) caused by perturbations imposed from external sources.

Therefore, the primary hypotheses of this investigation, which are derived from the assumed existence of a mathematical model of dyadic interaction that will forecast baseline behavior patterns and rates of each member of the dyad, are:

- 1) Assuming the validity of the Freudian model of individual personality function and the accuracy of its translation into a mathematical model by Wegman, there should exist equations which will link two individuals in such a way as to produce equilibrium for both individuals, even when neither individual model would tend to equilibrium given the same starting conditions without being linked in a dyad.*

- 2) Furthermore, several different equations could produce this result, since different types of dyadic relationships might have selective advantages (higher probability of tending toward equilibrium for both individuals) in different social contexts.

- 3) Last, such equations should represent the mathematical translations of already existing theories of interpersonal relationships in marital dyads.

* The use of the concept of equilibrium here rests on the work of several previous investigators. First, equilibrium is defined to mean states in which most or all of the level variables in the system being modeled evolve toward relatively constant values, which they then hold over long periods of time, usually in a form involving gentle oscillations around the mean value. This corresponds to the concept of family homeostasis in the way that Jackson (1957) adapted it from physiology and general system theory to family dynamics. In family homeostasis, each individual personality system is at equilibrium, and each person is therefore at the level of comfort which he or she is seeking, at all times, by means of all behaviors. An additional concept is added here, that of "reciprocal complementary neurotic interaction" (Mittleman, 1956) in which it is assumed that certain dyadic relationships might be expected to be stable, and might even be favored in their formation and long-range stability, because individuals who might have great difficulty achieving personal homeostasis as individuals might be able to form just the proper marital match so that both could achieve that equilibrium.

STRUCTURE OF THE SIMULATION MODEL

Individual Models

The model of dyadic interaction consists of mathematical representations of two individual personalities linked by one of several versions of a single interaction equation. The details of the basic individual personality model are described by Wegman (1977). Each individual model is, briefly, a dynamic system consisting primarily of five level variables. In mathematical terms, this corresponds to a set of five first-order difference equations plus all necessary definitions and auxiliary equations. In addition, each level variable has one or more input rates and one or more output rates. These rates determine the value of that level variable at any given point in time. Together, the level and rate variables, along with the auxiliary variables required to determine the rates, plus the initial values of the level equations, require a total of thirty six simultaneous equations. At this point it is easy to see why an attempt to understand the properties of this system is only feasible by means of computer simulation approaches, that is, numerical approximation methods. Analytical solutions to this set of equations are unlikely. Even approximate solutions via numerical methods are difficult, time consuming, and essentially impractical without computer methods.

A flow diagram which graphically displays the relationships among the variables in the individual personality model is included as Figure 1. The set of equations for this diagram is listed in its computer program format as equations Q1 through Q9.2 in Appendix 1. The computer language used here is called DYNAMO II, version 4.04, based on AED (Algol Extended for Design) with a small amount of assembly language (Pugh, 1976). Each individual model takes approximately forty program steps. Two individuals with the linking equations and other necessary instructions require approximately 120 program steps. A sample of this program appears as Appendix 2. When the program was revised from the individual personality model to the model of the dyad, a notation change was made. One individual was designated as Q, and the other individual was designated as Z. Thus each variable in the dyad program has a Q or a Z at the end of its original name to designate whether it specifies the value for individual Q or individual Z.

It is at equation 2.2 that the difference occurs in structure between the individual model and the dyadic model. This can be seen in flow diagram form in Figure 1 (the area enclosed in the central rectangle), or in Appendix 1 at equation Q2.2. In the individual model, level variable 2, or equation 2, is termed antithetic ideas (AI.K). This level represents the cognitive process of doubt, cognitive inhibition, or negative expectations about the outcome of an action, such as, "I intend to do such and such, but I shall probably fail." The value of this level variable depends on two inflow rates, and one outflow rate.

$$(2) \quad AI.K = AI.J+(DT)(INCR.JK+AIPE.JK-SUP.JK)$$

One of the inflow rates, equation 2.2, is termed antithetic ideas from perceived effectiveness (AIPE.KL). Equation 2.2 in the individual version is reproduced below.

$$(2.2) \quad AIPE.KL = (1/PT.K)(OBH.K-BH.K)(W)$$

Antithetic ideas derived from the individual's own perception of his or her effectiveness means that, for a given task, the closer the individual's behavior rate to his or her own estimation of the ideal behavior rate, the less doubt or negative thinking will be added to the level of negative thinking already present. Thus, increases in perceived effectiveness (decreasing gap) will lower the rate of gain.*

* NOTE: The term "gain" describes a rate process, but not as precisely as is generally used in mathematical systems theory. In this and following discussions the term is generally used to express the relative change in output from a given rate-determining equation that one obtains from a change in input. The higher (Cont.)

But, increases in perceived ineffectiveness (increasing gap between perceived effectiveness and expected effectiveness) will increase the gain.

Linking Equations--A Sample

In the present project, Equation 2.2 was rewritten to provide two components to the rate of increase or decrease in the antithetic idea level depending upon perceived effectiveness. The first component is the individual's perception of his or her own effectiveness, as in the individual model. The second component is the individual's perception of his or her own effectiveness determined from the behavior of the partner. This form of the equation expresses the concept that the first individual increases his or her self-esteem in part by reducing self-doubt through evaluating the effectiveness of a combination of his or her own behavior and the behavior of his or her partner.

The first individual may evaluate the other's behavior in a number of different ways and may give different relative weights to his or her own and the other's behaviors. To represent these possibilities, there are a number of variations of the dyadic version of equation 2.2. But the two-component principle remains the same throughout. One variation of this equation appears (dyadic form) appears below.

$$(2.2-1-A) \quad \text{AIPEQ.KL} = (1/\text{PTQ.K})(\text{OBHQ.K} - \text{BHQ.K})(\text{WQS}) + \\ (1/\text{PTQ.K})(\text{OBHQ.K} - \text{BHZ.K})(\text{WQO})$$

In this equation, AIPEQ.KL is the rate of increase in antithetic ideas of person Q derived from the perception of own and other effectiveness in one specific time period; PTQ.K is the perception time of person Q at the beginning of this time period; OBHQ.K is individual Q's expected behavior rate of himself for this task at the beginning of this time interval; BHQ.K is the actual behavior rate of Q as observed by Q at the beginning of this time period; WQS is the weight that Q attaches to own effectiveness relative to the weight attached to other effectiveness; BHZ.K is the behavior rate of individual Z as observed by individual Q at the beginning of this time period; and WQO is the weight given by individual Q to the perceived effectiveness of individual Z. The units of measurement of each of the variables in the dyadic form of the equation are the same as those of the original individual model (Wegman, 1977).

In this version of the linking equation (2.2-1-A), the two components of perceived effectiveness are linked by a positive sign. This means that the smaller the discrepancy between the behavior that Q expects from self and the behavior that Q actually observes from self, the fewer antithetic ideas will be added to Q's existing level of antithetic ideas in that time period. (Recall that these are behavior rates for any given task, but the task is not specified here.) In other words, all other things being equal, Q will add to his or her own level of antithetic ideas by the discrepancy in the first component of the equation. No matter what Q does, he is

NOTE (cont.) the ratio, the greater the gain. Gain is of importance because, in systems involving multiple feedback loops, it is usually found that increasing the gain in any feedback loop "destabilizes" not only the level variable in that loop, but many other level variables as well.

Here, the term "stabilize" is also used in a looser sense than in mathematical systems theory. It means that a level variable undergoes little or no change over time, or experiences a gradually decreasing rate of change. If the level variable is oscillating, it is said to become more stable if the oscillations are lower in amplitude and lower in frequency than previously.

The term "stabilize" is also used to indicate that large changes in input or output to a level variable do not cause the level variable to undergo sudden and dramatic changes in value, or to become impossible to calculate at all. When rate variables are referred to as being "stable," this means that they do not demonstrate wide or frequent oscillation around their equilibrium values. The system is said to be stabilized when most or all level variables are stable.

bound to add to his AI level, although he can minimize the addition. The bigger the discrepancy, the bigger will be the rate of increase or the gain in this feedback loop. But Q is also "responsible for" Z; that is, Q will also increase own antithetic ideas because of Z's ineffectiveness. This additional gain depends on the discrepancy between the expected behavior and observed behavior of Z, as perceived by Q.

It is also assumed that Q expects the same behavior rate from Z as he or she would expect from himself; hence the same variable (OBHQ.K) is used for the index level of expected behavior. Unless Z's behavior rate exceeds expected rates, Q can only experience increased gain in the feedback loop because of the positive sign between the two components of the equation. Small discrepancies between OBHQ.K and BHZ.K mean a slight increase in gain, and large discrepancies mean a big increase in gain.

Generally, reducing the gain in a feedback loop stabilizes the level variables. Therefore, Q loses some stabilization in the level of his or her own self-doubt in proportion to decreasing effectiveness in either own or other behavior, or both. This describes a collaborative but taxing situation. Yet this is seen in interpersonal relationships. Both Q and Z must be relatively effective in dealing with the task at hand or the problems posed by the environment for Q to experience stabilization. But there is one bright spot. If Z's behavior rate exceeds the expectations that Q has for this rate, then Q should experience significant stabilization in this feedback loop, and hence should do better, all other things being equal. (The same argument holds for Z to experience stabilization, or its opposite).

This equation is a quantitative expression of the verbal description given by Reiss (1980) of the environment-sensitive family type, which he defined by laboratory studies of family problem solving behavior.

Of course, all other things are not always equal, most obviously the relative weights (WQS and WQO) of the two components in this equation. The greater the ratio of WQO to WQS, the more important is the ineffectiveness of the partner to the self-concept of individual Q. "Experimenting" with different ratios of weights, and different absolute values of these weights, constitutes part of the simulation procedure discussed below. But whatever the relative weights, or their absolute values, this equation amounts to a quantitative expression of the social psychology hypothesis that the self-esteem of one individual depends in some part upon self-validation done through interpreting the behavior of significant others as well as through evaluating one's own effectiveness. These relative weights may correspond to such concepts from personality theory as internal versus external locus of control (Rotter, 1966) and introversion versus extraversion (Eysenck, 1976).

Reiss (1980) also described three other well-defined family types with respect to two dimensions of interpersonal communication and problem solving. Taking a second of these types, the achievement-sensitive family type, the present investigators would express this state of internal reality for the family in equation form as follows:

$$(2.2-1-B) \quad \text{AIPEQ.KL} = (1/\text{PTQ.K})(\text{OBHQ.K} - \text{BHQ.K})(\text{WQS}) - \\ (1/\text{PTQ.K})(\text{OBHQ.K} - \text{BHZ.K})(\text{WQO})$$

This equation, identical to 2.2-1-A except for the negative sign joining the two segments of the right side, represents a competitive rather than a collaborative interaction. Depending on the relative weights in the two components of the equations, a decrease in Z's effectiveness benefits Q by reducing the gain in this feedback loop for Q. Q cannot benefit much from Z's increased effectiveness. In fact, depending on the relative weights, Q can only benefit greatly if Z's behavior falls well below Q's expectations for Q's own level of effectiveness, and, hence, Z's expected level of effectiveness.

In these arguments and those which follow, all of the explanations of Equation 2.2 for Q apply to the corresponding equation for individual Z, with only changes in notation required. This mechanism establishes the reciprocal interdependency of each

individual upon the other. Q estimates the behavior rate of Z in order to determine own effectiveness. Having obtained this, Q can then "calculate" the values of all his or her level variables for the next time period. The output of Q's "calculations," in particular the value of Q's behavior rate, serves as the input into Z's "calculations." Z's calculations then result in a behavior rate for Z, which is an output value for Z but is an input into Q's subsequent calculations, and so on for each successive time period. Thus, the system of equations models an interdependency by which each individual adjusts his or her own internal states and behavior rates according to feedback obtained from the other individual (Powers, 1973).

In principle, dyadic interaction could be investigated by linking any other level variable, through one of its input or output flow rates, or by linking several level variables at once. But in this part of the project, only the level variable of antithetic ideas (AI) and one of its input rate variables, (AIPE), were investigated.

Linking Equations--Other Types

Gottman (1979) hypothesized that couples experiencing distress in their marital relationship would show more negative behavior towards one another than couples not experiencing distress. Thus, converting this idea to the conventions used above, each partner would evaluate the other partner as communicating nonapproval or nonvalidation to the self by means of his or her behavior (verbal and nonverbal). Gottman validated this hypothesis about marital interaction by laboratory methods involving observation and coding of verbal and nonverbal behavior between married couples reporting varying degrees of distress or satisfaction with their marital relationship. These studies covered time periods on the order of minutes to hours. In the present investigation, this rule for interpersonal exchange, extended over much longer time periods, would be operationalized by the following equation:

$$(2.2-2-A) \quad \text{AIPEQ.KL} = (1/\text{PTQ.K})(\text{OBHQ.K}) - \text{BHQ.K})(\text{WQS}) + \\ (1/\text{PTQ.K})(\text{CONVZ.K})(\text{WQO})$$

Here the expression CONVZ.K follows the convention developed by Wegman (1977) and represents the rate of behaviors of individual Z which are counterproductive to the successful completion of the task at hand. In Freudian theory these behaviors represent the eventual conversion of antithetic ideas, or negative thoughts, into behaviors which are not effective in dealing with the task at hand. Thus, in the dyadic case, it is assumed that Q would perceive this type of behavior by Z as a nonvalidating message to self. If a relatively great rate of these behaviors is produced by Z, and if these behaviors are responded to strongly and/or quickly by Q, then this exchange would constitute a mechanism by which one member of a distressed couple could, in part, communicate controlling or influencing messages to the other. The messages are sent by way of feedback taking the form of aversive reinforcement following the other's behavior.

Using Freudian terminology, this equation is an attempt to operationalize the verbal model by which certain behaviors of the dyadic partner are interpreted as disapproval of the first individual and act through the superego mechanism of the first individual, according to the quantitative rules of that individual's superego "calculations", to induce guilt, negative thoughts or doubts in that individual, and thereby exert social control over this first individual.

In this equation (2.2-2-A), Z's counterproductive behavior would be expected to add to the level of antithetic ideas of Q, or to increase the gain in this feedback loop and increase the probability of destabilization of Q.

A situation opposite to this one is described by the following equation for interaction:

$$(2.2-3-A) \quad \text{AIPEQ.KL} = (1/\text{PTQ.K})(\text{OBHQ.K}) - \text{BHQ.K})(\text{WQS}) + \\ (1/\text{PTQ.K})(\text{INEXZ.K})(\text{WQO})$$

In this situation, Q responds only to those behaviors of Z which Q perceives as intended by Z to deal with the task at hand, and as being successful in so doing. INEXZ.K represents this behavior rate on the part of Z, as perceived by Q, at a given time. As mentioned above, Gottman (1979) noted that Weiss et al. (1973) demonstrated that a number of couples who were trained to communicate an increased ratio of positive reinforcement over negative reinforcement to their spouse experienced an increase in marital satisfaction. It was also reported that the rates of positive and negative behaviors appeared to be independent, further substantiating the possibility of two separate equations as put forth above (2.2-2-A and 2.2-3-A). The ratio of the weights of the two components of each equation thus determines the extent to which individual satisfaction depends upon satisfaction in the marital relationship.

In the three equations discussed so far, each has been stated in the "collaborative" form. That is, each equation was stated in the form in which Q "takes some responsibility" for Z's behavior in the sense that Q requires Z to reduce counterproductive behaviors, or to increase effective behaviors, in order to accomplish a decrease in the gain in this loop for Q, and thus increase Q's self esteem by virtue of decreasing Q's level of antithetic ideas. Obviously, both equations can be stated in the opposite form, that is, in a "competitive" form, by changing the signs between the two components of each of the two different rate equations.* Mixed forms are also possible.

The present research group also formulated a group of linking equations based on verbal descriptions coming more from clinical family therapists than from laboratory studies of communication or other behavioral exchanges between marital dyads. These have to do with the concept of reactivity (Bowen, 1966). The present investigators interpret this concept to mean that Q responds more to Z's rate of reacting than to the actual behaviors that Q observes Z carrying out.

In the "calculations" required by all the equations previously, person Q must first discriminate either Z's rate of effective behaviors or Z's rate of ineffective behaviors or both, then calculate a net difference, and then react to this difference at the rate that Q would normally react to such a discrepancy in his or her own behavior. But in the "reactive" group of equations, Q will react to these same calculations at the rate that Z would have reacted, as estimated by Q. Since the model so far is entirely deterministic, it is assumed that perfect measurement of the variables in self and partner by each of the individuals are made here. It is also assumed that no other sources of random input into the calculations occur. Given these assumptions, "reactive" equations can be seen as one way of operationalizing the social psychology hypothesis that people may adjust their behavior rate, or activity level, to the activity level of those around them in order to maintain some optimum level of functioning (Simon, 1957). There is also some general support for this hypothesis in more recent laboratory work (Reiss, 1980) wherein two of the family types described by Reiss seem to be responding more to the other people in the family than to the solutions required by the task at hand. These were the interpersonal

* Note: The following convention is used throughout: The "collaborative" form of the equation is designated as the "A" form, and has a positive sign between the two components: The "competitive" form, which usually has a negative sign between major components, is called the "B" form. The two "mixed" forms designate situations in which both individual members of the dyad use the exact same equation except for a difference in sign between the two major components. The "C" form indicates the situation in which Q has a positive sign and Z has a negative sign, while the "D" form indicates the condition in which Q has a negative sign and Z has a positive sign. If Q and Z start with one level variable unequal (e.g., NESQ=750, NESZ=850), the letters A through D are used. If Q and Z start with all level variables equal (e.g., NESQ=NESZ=750), the letters E through H are used instead.

distance sensitive family type and the consensus sensitive family type. The equation is as follows:

$$(2.2-4-A) \quad \text{AIPEQ.KL} = (1/\text{PTQ.K})(\text{OBHQ.K}-\text{BHQ.K})(\text{WQS}) + \\ (1/\text{PTZ.K})(\text{OBHQ.K}-\text{BHZ.K})(\text{WQO})$$

Again, the equation is stated in collaborative form, but can be cast in a competitive form or mixed forms by a sign change.

Lastly, an even more complex or idiosyncratic set of equations was developed. It seems possible that person Q might further try to anticipate person Z not only by anticipating Z's perception time, or reaction rate, but also by anticipating Z's expectation of Q's behavior. Thus Q computes the other behavior discrepancy by means of the difference between Q's estimate of Z's expectation of Q, and Q's measurement of Q's own behavior rate. These equations are stated below, in the collaborative form:

$$(2.2-5-A) \quad \text{AIPEQ.KL} = (1/\text{PTQ.K})(\text{OBHQ.K}-\text{BHQ.K})(\text{WQS}) + \\ (1/\text{PTZ.K})(\text{OBHZ.K}-\text{BHQ.K})(\text{WQO})$$

Throughout the discussion of all equations above, the term "mixed form" (the "C" and "D" forms) refers only to dyads in which both individuals use the same equation, but one with a positive and one with a negative sign between the major components. The term "mixed type" does not yet refer to the condition in which partners may use both different signs and different equations, which will be studied in future investigations.

SIMULATION PROCEDURE

Simulation "experiments" consist of performing the calculations, and plotting or tabulating the results so as to describe the behavior of the dyadic system at all points in time, for as long as the experimenters choose to observe. Initial conditions are varied in each separate experiment to test different hypotheses. At some later time, stochastic variations and other types of perturbations can be added to experiment in order to make the behavior of the dyadic model approximate real behavioral systems more closely. In either the deterministic or stochastic case, system behaviors resulting from a wide variety of different initial conditions are classified into a small number of output types for purposes of further study.

For the present study, all four level variables except nervous energy supply (NESQ and NESZ) are initialized at time $t = 0$ in the same way as in the original Wegman article (1977). These procedures are not repeated here. However, the observation period is extended from 100 time units to 320 time units, in order better to observe any late appearing phenomenon in dyadic interaction that might not have been evident in individual behavior alone.

The differing initial conditions for the different simulation runs fall into three categories:

1. Initial value of the two level variables NESQ and NESZ.

Two different initial conditions are simulated. In the first, Q has a value NESQ=750 while Z has the value NESZ=850. In the second condition, NESQ=NESZ=750.

Higher level values, such as NESQ=NESZ=850, or NESQ=NESZ=1000, or combinations of these, were not investigated. The interest is in whether or not individuals with initial NES values so low as to predispose them to slowly deteriorating system states, or actual "breakdowns", can reach a system state such that one or both need not experience such "undesirable" courses. Preferably one or both individuals would reach equilibrium. These initial conditions are equivalent to the question of whether two individuals at the same low initial NES level can stabilize one another, or whether stabilization requires that at least one of the individuals be at a slightly higher level in order for there to be "resources" in the emotional sense) to exchange

between both partners in order to achieve stabilization.

2. Type and form of the linking equations.

This constitutes testing four variations of each of the types of linking equations. The choice of equation thus simulates the general type of the relationship and the specific rules for exchange. Twenty runs would thus be needed to investigate all possible combinations and permutations of equations which could characterize any given marital dyad.

3. Values of the weighting constants.

For any given initial values of NESQ and NESZ, and for any given equation, two more issues must be fixed in order for a simulation experiment to be run. These are the absolute values of the four weighting constants, and their ratios. Sixteen possible permutations for any given equation are possible, as shown at the bottom of Appendix 1 in the multiple run statements for each dyadic program.

With sixteen weighting permutations per equation form, and four equation forms per equation type, the examination of merely five different linking equations for each of the two initial value variations of NESQ and NESZ requires the examination of 740 simulation runs. This is not actually a difficult problem in computing, and is a common size and type of approach is simulation methods applied to social science problems (Hammel et al., 1979).

What is most necessary is a scoring system for condensing and ordering the results of the 740 runs. At this stage of the present investigation, the major hypothesis is only that there exist linking equations which produce equilibrium for both individuals. Accordingly, those runs which produce such a condition are given a score value of 1. Those runs which produce equilibrium for only one individual are given a score value of 2. Those runs which produce equilibrium for neither individual are given a score value of 3. Finally, those runs which produce equilibrium, or nearly so, for both individuals, but which hold this equilibrium only for 100 to 250 time units are given a score value of 4.

RESULTS OF THE SIMULATION RUNS

The results of the simulation runs are presented in Table 1 below.

TABLE 1 ABOUT HERE

Table 1 shows that the first hypothesis, the existence of linking equations leading to equilibrium for both members of the dyad, is confirmed. Thirty-one of the 740 simulation runs demonstrated such properties.

Table 1 also shows that the second hypothesis, the possibility of finding equilibrium for both partners under a wide range of initial conditions, and using a variety of different linking equations, was confirmed. There were at least a few instances of both partners achieving equilibrium for each of the five different types of linking equations under about half of the different kinds of starting conditions.

The third hypothesis was also confirmed, as can be seen in Figures 3-11. Three different types of equilibrium were seen for each individual. These were labeled "overfunction," "chronic dysfunction," "short-lived." The first two terms come from concepts proposed by Bowen (1966). All possible combinations of the six types of individual equilibria were seen. For example, the "overfunction-dysfunction" "reciprocity" of Bowen (1966) is one such combination.

Bowen further postulated that there were only two other types of basic marital dyad. A second type tended to achieve a stable "overfunctioning" equilibrium for both partners by focusing interest, energy, blame, etc., onto a child. The results above include instances of both members of the dyad achieving an overfunctional equilibrium.

A third marital type described by Bowen as being stable was "chronic conflict." This was operationalized in the results above by the finding of combinations in which both partners achieved dysfunctional equilibria, as if "drained" by chronic conflict, but not changing in response to that conflict.

TABLE 1: Results for the Five Different Types of AIPE equations

FORM	A	B	C	D	E	F	G	H	* Row Total

(2.2-1-A,H) "Rational" Type: *									
AIPEQ. KL=(1/PTQ.K)(OBHQ.K-BHQ.K)(WQS)+(1/PTQ.K)(OBH.K-BHZ.K)(WQO)									
-----*									
SCORE-1	0	0	1	0	2	2	1	1	* 7
-----*									
SCORE-2	4	5	3	4	2	0	3	3	* 24
-----*									
SCORE-3	12	9	11	12	8	12	9	9	* 82
-----*									
SCORE-4	0	2	1	0	4	2	3	3	* 15

(2.2-2-A,H) "Antagonistic" Type: *									
AIPEQ. KL=(1/PTQ.K)(OBHQ.K-BHQ.K)(WQS)+(1/PTQ.K)(CONVZ.KL)(WQO)									
-----*									
SCORE-1	0	1	1	1	0	1	2	2	* 8
-----*									
SCORE-2	4	1	1	4	4	0	1	1	* 16
-----*									
SCORE-3	12	14	14	11	8	11	10	10	* 90
-----*									
SCORE-4	0	0	0	0	4	4	3	3	* 14

(2.2-3-A,H) "Supportive" Type: *									
AIPEQ. KL=(1/PTQ.K)(OBHQ.K-BHQ.K)(WQS)+(1/PTQ.K)(INEXZ.KL)(WQO)									
-----*									
SCORE-1	0	0	0	0	1	1	1	1	* 4
-----*									
SCORE-2	4	4	4	4	4	4	4	4	* 32
-----*									
SCORE-3	8	8	8	8	9	9	9	9	* 68
-----*									
SCORE-4	4	4	4	4	2	2	2	2	* 24

(2.2-4-A,H) "Reactive" Type I: *									
AIPEQ. KL=(1/PTQ.K)(OBHQ.K-BHQ.K)(WQS)+(1/PTZ.K)(OBHQ.K-BHZ.K)(WQO)									
-----*									
SCORE-1	1	0	1	1	2	2	1	1	* 9
-----*									
SCORE-2	3	5	3	4	2	0	1	1	* 19
-----*									
SCORE-3	8	9	11	8	8	10	11	11	* 76
-----*									
SCORE-4	4	2	1	3	4	4	3	3	* 24

(2.2-5-A,H) "Reactive" Type II: *									
AIPEQ. KL=(1/PTQ.K)(OBHQ.K-BHQ.K)(WQS)+(1/PTZ.K)(OBHZ.K-BHQ.K)(WQO)									
-----*									
SCORE-1	0	1	0	0	0	2	0	0	* 3
-----*									
SCORE-2	2	7	1	6	0	8	3	4	* 31
-----*									
SCORE-3	14	6	9	8	9	4	11	10	* 71
-----*									
SCORE-4	0	2	6	2	7	2	2	2	* 23

These clinical descriptions are offered under many different names by many different observers, and are well-summarized by Olson (1979). For example, the overfunction-dysfunction reciprocity of Bowen appears to be a clinical description of the same phenomena that were labeled as marital skew by Lidz.

Not all possible combinations of the three equilibrium types were observed for each linking equation, and under all different initial conditions. But each type of combination was observed at least once, substantiating the third hypothesis that, at least some existing verbal descriptions of dyadic interaction would be operationalized successfully.

The "short-lived" equilibrium may also operationalize sociological descriptions of family systems in stability rather than clinical descriptions. Very short-lived equilibria, on the order of 40 - 100 time units, may represent phenomena having to do with marriages which do not evolve into stable family units. The relationships appear to be developing, that is, following the usual time course as seen by outside observers, and then suddenly deteriorate. When the same phenomenon is seen after very long periods, such as 150 to 250 time units, it might represent divorce, seen in later stages of the family life cycle, such as the "empty nest" syndrome. Since the combination of short-lived equilibria occur under only a few different types of initial conditions for each equation, these conditions might serve to design improved sampling strategies for empirical studies of the types of family systems which are prone to these disintegration scenarios.

Lastly, some interesting combinations were seen in those runs achieving a score of two. For example, it was seen that an individual Z with an initial NES=850 could "support" an individual Q with an NES=750 in either a chronic dysfunctional or chronic overfunctional mode. This would correspond to very different types of successful "caretaker" relationships. These relationships were achieved with only mild exacerbations of the pattern normally seen in an individual with a starting condition of NES=850. In several rare examples, both of these "caretaker" scenarios were seen to undergo late deterioration after long periods of stability, approximately 200 to 250 time units.

In summary, it seems reasonable to conclude that this approach toward improving experimental design is worth further investigation.

Based on these findings, several areas will be explored as part of the next step in this project. These include:

- 1) Examination of dyads in which each individual relates to the other using a different linking equation;
- 2) Extension of dyadic equations to triads;
- 3) Introduction of stochastic variability in order to see how much variance is necessary for the model to produce mode switches "by accident";
- 4) Introduction of perturbations, again to see what will produce "accidental" mode switches, under what conditions the model can "recover" from such stresses, and under what conditions the model will deteriorate;
- 5) Literature search to begin to assess what existing measurement instruments might serve to obtain data with which to validate the theoretical model;
- 6) Extension of this model using matrix calculations from game theory or learning theory so as to include classes of behavior as well as rates of behavior.
- 7) "Aging" of parameter values to search for "developmental stages" in the dyadic relationship.

Figures 2 - 10 are available from:

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APPENDIX 2: DYNAMO Program for Counterwill Dyad

```

* COUNTERWILL DYAD
*
*
(Q1)      L  INTQ.K=INTQ.J+(DT)(IPINTQ.JK-INEXQ.JK)
(Q1.1)    R  INEXQ.KL=(1/ATEX)(DISCQ.K)(DECQ.K)(POWQ.K)
(Q1.1.1)  A  DISCQ.K=INTQ.K-AIQ.K
(Q1.1.2)  A  DECQ.K=CLIP(1,0,DISCQ.K,0)
(Q1.1.3)  A  POWQ.K=CLIP(1,0,NESQ.K,EXCRIT)
(Q2)      L  AIQ.K=AIQ.J+(DT)(IPAIQ.JK+INCRQ.JK+AIPEQ.JK-SUPQ.JK)
(Q2.1)    R  INCRQ.KL=(1/DTAIQ.K)(AIQ.K)
(Q2.1.1)  A  DTAIQ.K=DTAIN-(F)*(AFFQ.K)+DCQ.K
(Q2.1.2)  A  DEQ.K=T*EXP((-ED)*(MAI-AIQ.K))
*
NOTE: Equations Q2.2 & Z2.2 link the Q AND Z monads.
*
(Q2.2)    R  AIPEQ.KL=(1/PTQ.K)(OBHQ.K-BHQ.K)(WQS) +/- {Depends on form}
(Q2.2-1-A,H) X (1/PTQ.K)(OBHQ.K-BHZ.K)(WQO) {"Rational" Type}
(Q2.2-2-A,H) X (1/PTQ.K)(CONVZ.KL)(WQO) {"Antagonistic" Type}
(Q2.2-3-A,H) X (1/PTQ.K)(INEXZ.KL)(WQO) {"Supportive" Type}
(Q2.2-4-A,H) X (1/PTZ.K)(OBHQ.K-BHZ.K)(WQO) {"Reactive" Type I}
(Q2.2-5-A,H) X (1/PTZ.K)(OBHZ.K-BHQ.K)(WQO) {"Reactive" Type II}
*
(Q2.2.1)  A  PTQ.K=NPTQ+DCQ.K
(Q2.3)    R  SUPQ.KL=SUPAQ.K
(Q2.3.1)  A  SUPAQ.K=(1/ATSUPQ.K)(AIQ.K)
(Q2.3.2)  A  ATSUPQ.K=CLIP(NATSUPQ.K,1,NESQ.K,EXCRIT)
(Q2.3.3)  A  NATSUPQ.K=MSD-(DR)*(SCQ.K)
(Q3)      L  AFFQ.K=AFFQ.J+(DT)(ARQ.JK)
(Q3.1)    R  ARQ.KL=(1/DT)(IAFFQ.K-AFFQ.K)
(Q3.1.1)  A  IAFFQ.K=NAF*AMNESQ.K*AMAIQ.K
(Q3.1.2)  A  AMAIQ.K=TABHL(TAMAIQ,AIQ.K,0,100,10)
(Q3.1.3)  T  TAMAIQ=1.00/1.12/1.18/1.22/1.24/1.26/1.27/1.27/1.27/1.27/1.28
(Q3.1.4)  A  AMNESQ.K=TABHL(TAMNSQ,NESQ.K,0,1000,100)
(Q3.1.5)  T  TAMNSQ=0/1.25/1.20/1.15/1.10/1.05/1.00/.95/.90/.85/.80
(Q4)      L  UAIQ.K=UAIQ.J+(DT)(SUPQ.JK-CONVQ.JK)
(Q4.1)    R  CONVQ.KL=(1/CONVT)(UAIQ.K)(CQ.K)
(Q4.1.1)  A  CQ.K=CLIP(0,1,NESQ.K,EXCRIT)
(Q5)      L  NESQ.K=NESQ.J+(DT)(RESTQ.JK-EXHQ.JK)
(Q5.1)    R  RESTQ.KL=(1/TREST)(NNESQ-NESQ.K)
(Q5.2)    R  EXHQ.KL=SUPAQ.K
(Q5.3)    A  SCQ.K=NESQ.K*SAC
(Q6)      A  BHQ.K=INEXQ.JK-CONVQ.JK
(Q7)      A  OBHQ.K=(1/ATEX)(INTQ.K)
(Q8)      R  IPINTQ.KL=(1/DT)(ININTQ.K-INTQ.K)(HQ.K)
(Q8.1)    A  HQ.K=SWITCH(0,1,ININTQ.K)
(Q8.2)    A  ININTQ.K=PULSE(100,0,20)
(Q9)      R  IPAIQ.KL=(1/DT)(IDAIQ.K-AIQ.K)(HQ.K)
(Q9.1)    A  IDAIQ.K=RQ.K*ININTQ.K
(Q9.2)    A  RQ.K=RMAX-(RD)(SCQ.K)
* END OF Q-EQUATIONS
    
```

APPENDIX 2: (Continued--Page 2)

(Z1) L INJZ.K=INTZ.J+(DT)(IPINTZ.JK-INEXZ.JK)

*

NOTE: Equations Z2.1--Z9.2 replicate Q2.1--Q9.2 with Q & Z interchanged.

*

(Z9.2) A RZ.K=RMAX-(RD)(SCZ.K)

* END OF Z-MONAD

C ATEX=5

C EXCRIT=500

C DTAIN=19

C F=16

C T=1

C ED=1

C MAI=300

C MSD=33

C DR=.03

C NAF=.625

C CONV=5

C TREST=20

C SAC=1

C RMAX=1.3

C RD=.001

* INITIAL VALUES OF Q

N INTQ=0

N AIQ=0

N AFFQ=.541

N UAIQ=0

C WQS=.1

C WQO=.3

C NPTQ=3

C MNESQ=750

N NESQ=MNESQ

* INITIAL VALUES OF Z

N INTZ=0

N AIZ=0

N AFFZ=.541

N UAIZ=0

C WZS=.1

C WZO=.3

C NPTZ=3

C MNESZ=850

N NESZ=MNESZ

PLOT NESQ=N(0,1000)/AIQ=A,INTQ=I(0,400)/UAIQ=U(0,2000)/

X AFFQ=F(-1,1)/BHQ=B(-200,200)

PLOT NESZ=N(0,1000)/AIZ=A,INTZ=I(0,400)/UAIZ=U(0,2000)/

X AFFZ=F(-1,1)/BHZ=B(-200,200)

PRINT 1)NESQ/2)INTQ/3)AIQ/4)AFFQ/5)UAIQ/6)BHQ/7)INEXQ/8)INCRQ/

X 9)AIPEQ/10)SUPQ/11)ARQ/12)CONVQ/13)RESTQ

PRINT 1)NESZ/2)INTZ/3)AIZ/4)AFFZ/5)UAIZ/6)BHZ/7)INEXZ/8)INCRZ/

X 9)AIPEZ/10)SUPZ/11)ARZ/12)CONVZ/13)RESTZ

PRINT 1)EXHQ/2)IPINTQ/3)IPAIQ/4)DISCQ/5)DTAIQ/6)OBHQ/7)IDAIQ/8)ININTQ

PRINT 1)EXHZ/2)IPINTZ/3)IPAIZ/4)DISCZ/5)DTAIZ/6)OBHZ/7)IDAIZ/8)ININTZ

SPEC DT=.5/LENGTH=320/PRTPER=0/PLTPER=2

APPENDIX 2: (Continued--Page 3)

NOTE: 16 Runs with varying weights in Equations Q2.2 and Z2.2

*

RUN 1-TYPE(FORM):WQS=.1/WQO=.3/WZS=.1/WZO=.3
RUN 2-TYPE(FORM):WQS=.3/WQO=.1/WZS=.3/WZO=.1
RUN 3-TYPE(FORM):WQS=.1/WQO=.3/WZS=.3/WZO=.1
RUN 4-TYPE(FORM):WQS=.3/WQO=.1/WZS=.1/WZO=.3
RUN 5-TYPE(FORM):WQS=1.0/WQO=3.0/WZS=.1/WZO=.3
RUN 6-TYPE(FORM):WQS=3.0/WQO=1.0/WZS=.3/WZO=.1
RUN 7-TYPE(FORM):WQS=1.0/WQO=3.0/WZS=.3/WZO=.1
RUN 8-TYPE(FORM):WQS=3.0/WQO=1.0/WZS=.1/WZO=.3
RUN 9-TYPE(FORM):WQS=.1/WQO=.3/WZS=1.0/WZO=3.0
RUN 10-TYPE(FORM):WQS=.3/WQO=.1/WZS=3.0/WZO=1.0
RUN 11-TYPE(FORM):WQS=.1/WQO=.3/WZS=3.0/WZO=1.0
RUN 12-TYPE(FORM):WQS=.3/WQO=.1/WZS=1.0/WZO=3.0
RUN 13-TYPE(FORM):WQS=1.0/WQO=3.0/WZS=1.0/WZO=3.0
RUN 14-TYPE(FORM):WQS=3.0/WQO=1.0/WZS=3.0/WZO=1.0
RUN 15-TYPE(FORM):WQS=1.0/WQO=3.0/WZS=3.0/WZO=1.0
RUN 16-TYPE(FORM):WQS=3.0/WQO=1.0/WZS=1.0/WZO=3.0

Mathematical Models for Family Systems

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