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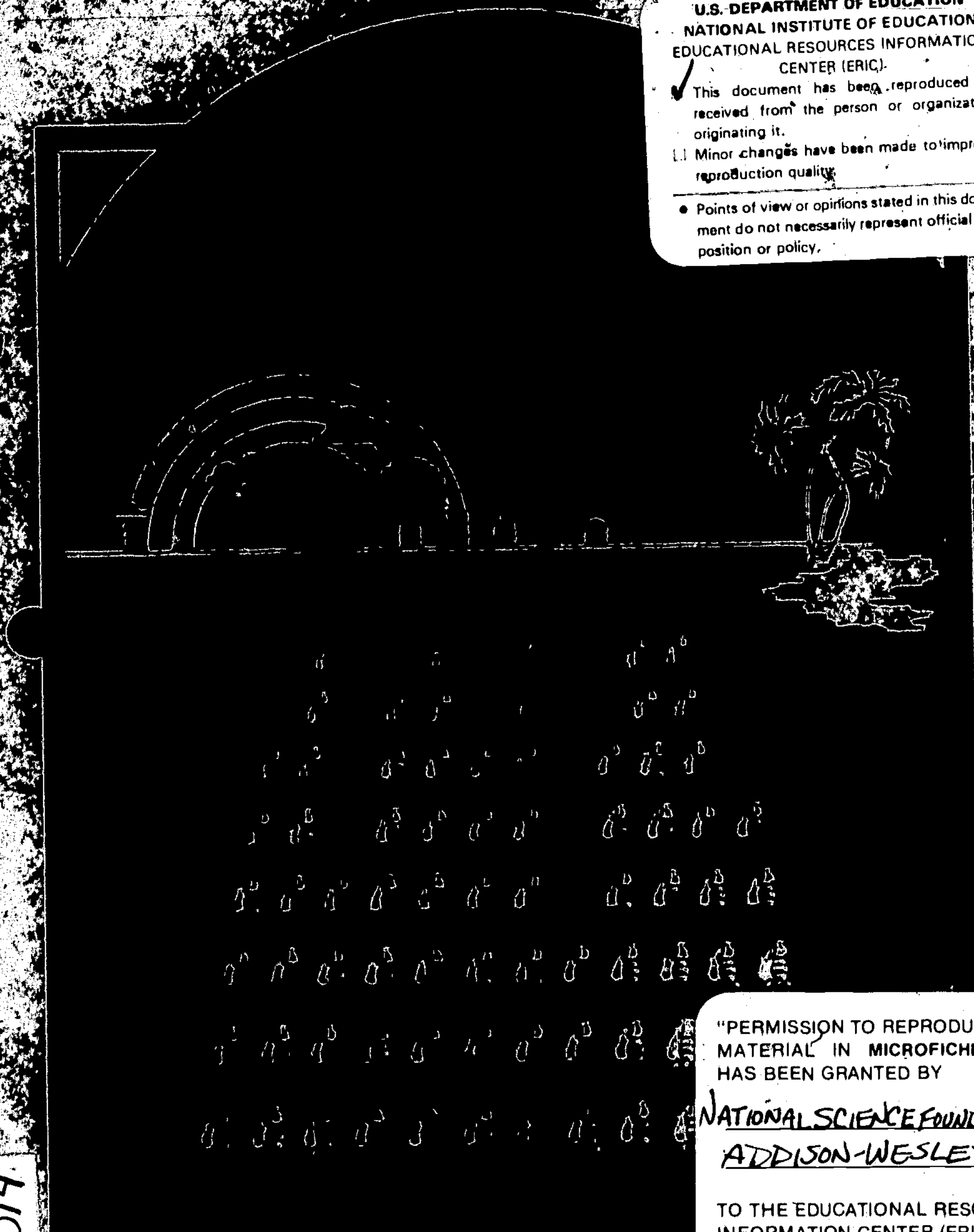
ABSTRACT

This unit is 1 of 12 developed for the university classroom portion of the Mathematics-Methods Program (MMP), created by the Indiana University Mathematics Education Development Center (MEDC) as an innovative program for the mathematics training of prospective elementary school teachers (PSTs). Each unit is written in an activity format that involves the PST in doing mathematics with an eye toward application of that mathematics in the elementary school. This document is one of four units that are devoted to mathematical topics for the elementary teacher. In addition to an introduction to the unit and an overview of graphs in the elementary school, the text has sections on picturing data, locations, relations, and functions and an appendix on graphing self-evaluation questions. (MP)

GRAPHS

The Picturing of Information



Mathematics-Methods Program
units written under the direction of

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GRAPHS: THE PICTURING OF INFORMATION

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PREFACE

The Mathematics-Methods Program (MMP) has been developed by the Indiana University Mathematics Education Development Center (MEDC) during the years 1971-75. The development of the MMP was funded by the UPSTEP program of the National Science Foundation, with the goal of producing an innovative program for the mathematics training of prospective elementary school teachers (PSTs).

The primary features of the MMP are:

- It combines the mathematics training and the methods training of PSTs.
- It promotes a hands-on, laboratory approach to teaching in which PSTs learn mathematics and methods by doing rather than by listening, taking notes or memorizing.
- It involves the PST in using techniques and materials that are appropriate for use with children.
- It focuses on the real-world mathematical concerns of children and the real-world mathematical and pedagogical concerns of PSTs.

The MMP, as developed at the MEDC, involves a university classroom component and a related public school teaching component. The university classroom component combines the mathematics content courses and methods courses normally taken by PSTs, while the public school teaching component provides the PST with a chance to gain experience with children and insight into their mathematical thinking.

A model has been developed for the implementation of the public school teaching component of the MMP. Materials have been developed for the university classroom portion of the MMP. These include 12 instructional units with the following titles:

Numeration

Addition and Subtraction

Multiplication and Division

Rational Numbers with Integers and Reals

Awareness Geometry

Transformational Geometry

Analysis of Shapes

Measurement

Number Theory

Probability and Statistics

Graphs: the Picturing of Information

Experiences in Problem Solving

These units are written in an activity format that involves the PST in doing mathematics with an eye toward the application of ~~that~~ mathematics in the elementary school. The units are almost entirely independent of one another, and any selection of them can be done, in any order. It is worth noting that the first four units listed pertain to the basic number work in the elementary school; the second four to the geometry of the elementary school; and the final four to mathematical topics for the elementary teacher.

For purposes of formative evaluation and dissemination, the MMP has been field-tested at over 40 colleges and universities. The field implementation formats have varied widely. They include the following:

- Use in mathematics department as the mathematics content program, or as a portion of that program;
- Use in the education school as the methods program, or as a portion of that program,
- Combined mathematics content and methods program taught in

either the mathematics department, or the education school, or jointly;

- Any of the above, with or without the public school teaching experience.

Common to most of the field implementations was a small-group format for the university classroom experience and an emphasis on the use of concrete materials. The various centers that have implemented all or part of the MMP have made a number of suggestions for change, many of which are reflected in the final form of the program. It is fair to say that there has been a general feeling of satisfaction with, and enthusiasm for, MMP from those who have been involved in field-testing.

A list of the field-test centers of the MMP is as follows:

ALVIN JUNIOR COLLEGE
Alvin, Texas

BLUE MOUNTAIN COMMUNITY COLLEGE
Pendleton, Oregon

BOISE STATE UNIVERSITY
Boise, Idaho

BRIDGEWATER COLLEGE
Bridgewater, Virginia

CALIFORNIA STATE UNIVERSITY, CHICO

CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

CLARKE COLLEGE
Dubuque, Iowa

UNIVERSITY OF COLORADO
Boulder, Colorado

UNIVERSITY OF COLORADO AT DENVER

CONCORDIA TEACHERS COLLEGE
River Forest, Illinois

GRAMBLING STATE UNIVERSITY
Grambling, Louisiana

ILLINOIS STATE UNIVERSITY
Normal, Illinois

INDIANA STATE UNIVERSITY EVANSVILLE

INDIANA STATE UNIVERSITY
Terre Haute, Indiana

INDIANA UNIVERSITY
Bloomington, Indiana

INDIANA UNIVERSITY NORTHWEST
Gary, Indiana

MACALESTER COLLEGE
St. Paul, Minnesota

UNIVERSITY OF MAINE AT FARMINGTON

UNIVERSITY OF MAINE AT PORTLAND-GORHAM

THE UNIVERSITY OF MANITOBA
Winnipeg, Manitoba, CANADA

MICHIGAN STATE UNIVERSITY
East Lansing, Michigan

UNIVERSITY OF NORTHERN IOWA
Cedar Falls, Iowa

NORTHERN MICHIGAN UNIVERSITY
Marquette, Michigan

NORTHWEST MISSOURI STATE UNIVERSITY
Maryville, Missouri

NORTHWESTERN UNIVERSITY
Evanston, Illinois

OAKLAND CITY COLLEGE
Oakland City, Indiana

UNIVERSITY OF OREGON
Eugene, Oregon

RHODE ISLAND COLLEGE
Providence, Rhode Island

SAINT XAVIER COLLEGE
Chicago, Illinois

SAN DIEGO STATE UNIVERSITY
San Diego, California

SAN FRANCISCO STATE UNIVERSITY
San Francisco, California

SHELBY STATE COMMUNITY COLLEGE
Memphis, Tennessee

UNIVERSITY OF SOUTHERN MISSISSIPPI
Hattiesburg, Mississippi

SYRACUSE UNIVERSITY
Syracuse, New York

TEXAS SOUTHERN UNIVERSITY
Houston, Texas

WALTERS STATE COMMUNITY COLLEGE
Morristown, Tennessee

WARTBURG COLLEGE
Waverly, Iowa

WESTERN MICHIGAN UNIVERSITY
Kalamazoo, Michigan

WHITTIER COLLEGE
Whittier, California

UNIVERSITY OF WISCONSIN--RIVER FALLS

UNIVERSITY OF WISCONSIN/STEVENS POINT

THE UNIVERSITY OF WYOMING
Laramie, Wyoming

CONTENTS

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INTRODUCTION TO THE GRAPHS UNIT

That "a picture is worth a thousand words" seems to be an accepted canon of human communication. This unit addresses itself to the communication of information by picturing it with graphs. Following the Overview, there are four sections of the unit, each focusing on the picturing of a different kind of information. Section I studies bar, line, and circle graphs and pictographs as means of picturing data. Section II uses rectangular and other less commonly recognized coordinate systems to picture locations. Section III considers the non-traditional material of picturing relations using directed graphs, networks, and Papygrams, and Section IV discusses the picturing of functions by means of graphs.

This approach to graphing as picturing information seems to be appropriate since it emphasizes the potential for graphing activities in the elementary school. Graphing activities, as you will see, can have a much wider scope than just the mathematics curriculum. Since other subjects such as science and social studies involve data, locations, relations, and even functions, graphs can have wide applicability and can provide a bridge between other curriculum areas and mathematics.

The unit begins with an overview of graphs in the elementary school.

OVERVIEW

FOCUS:

This overview activity is intended to broaden the reader's view of graphs and to provide the reader with some ideas concerning the possible role of graphs in the elementary curriculum.

MATERIALS:

(Optional) The Mathematics-Methods Program slide-tape overview entitled "Graphs in the Elementary School."

DIRECTIONS:

Do one of (1) or (3) and then discuss the questions in (2).

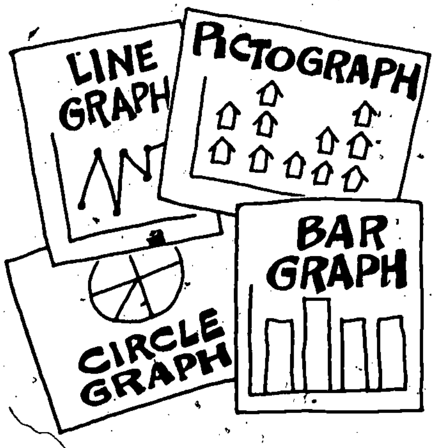
1. View the slide-tape overview entitled "Graphs in the Elementary School."
2. Discuss the following questions:
   a) As a result of the overview, what new ideas have you gained concerning graphs?
   b) In what ways could graphing provide a link between other school subjects and mathematics? (Be as specific as possible)
   c) In what ways do graphs enter into your "adult" life?
   d) What experiences did you have with graphs in your schooling,
      - elementary?
      - secondary?
      - college?
3. Read the short essay that follows. The essay is intended to broaden your view of what graphs are and to provide you with some perspective on the possible uses of graphs in the elementary school curriculum.

OVERVIEW OF
GRAPHS IN THE ELEMENTARY SCHOOL

Communicating information is an important human activity. We tend to rely most heavily on words, written, spoken, or sung, for communicating. We are also relying more and more heavily on numbers and equations to communicate, but their effectiveness in communicating seems to be limited for the nonspecialist.

Pictures also communicate information. An artist can convey a lot of information quickly and effectively with a few strokes of a pen or brush. In this unit we are concerned with graphs, which are certain kinds of formal pictures and which are used to convey certain kinds of information. As with most kinds of pictures, graphs have the advantage that they communicate through our highly developed visual senses, and they transcend many language barriers whether the barriers be due to nationality, to culture, or to educational background. Also, as with most kinds of pictures, making and interpreting graphs requires some training and experience.

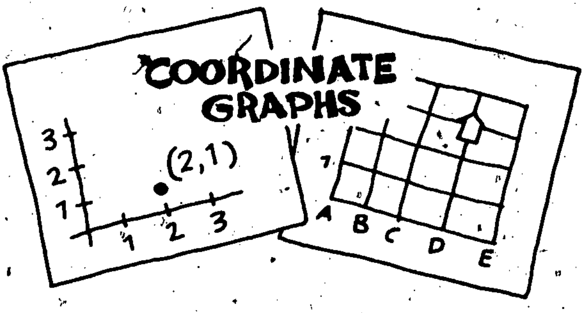
Many people have a somewhat narrower view of graphs than that presented in this unit. Also, many people fail to see the potential for graphing activities with children. This overview is intended to introduce you to the view of graphs that is presented in this unit and to provide you with a glimpse of some possible graphing activities for children.



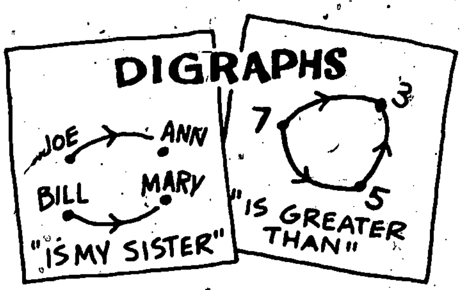
As you will see, this training and experience with graphs can be a very natural part of the elementary school curriculum. Different kinds of graphs can be used to portray at least four kinds of information:

1) Graphs such as the bar, line, circle and pictographs shown at the right can be used to <u>organize and display data</u> generated in classroom activities.

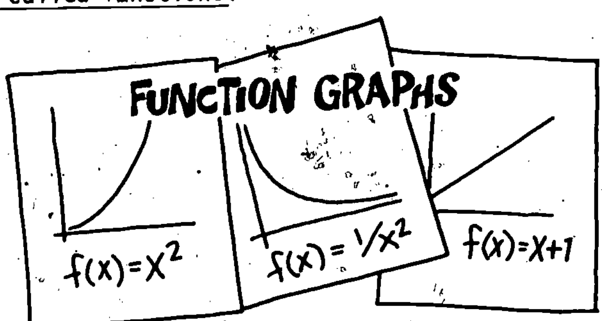
2) Coordinate graphs can locate points or objects.



3) Digraphs can picture relations such as "is my sister" or "is greater than."



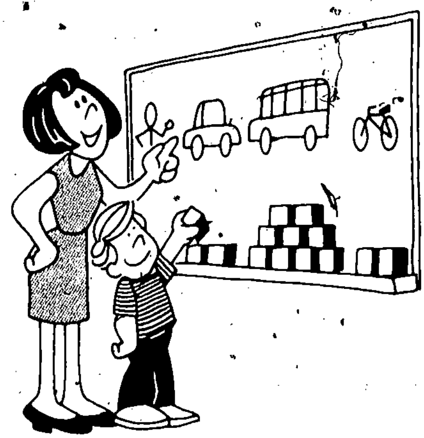
4) Graphs of functions can picture those certain special relations that are called functions.



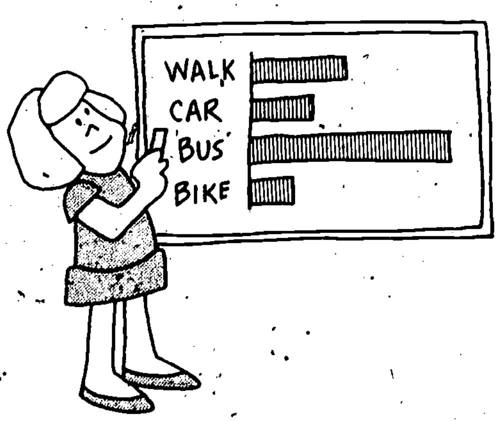
The remainder of this overview will attempt to go into more detail on each of these kinds of graphs and on its potential role in helping children to learn to picture information.

GRAPHS THAT PICTURE DATA

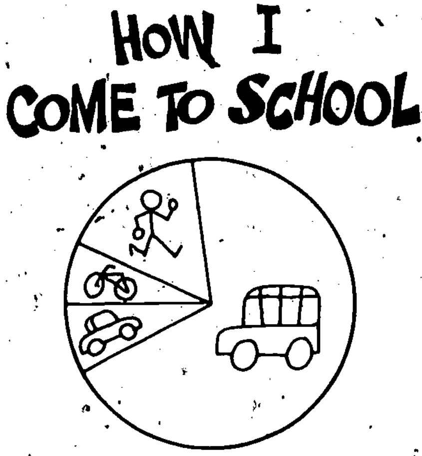
Every child gets to school one way or another. In a discussion of different modes of transportation, a class can collect data on the different ways in which the children come to school. Some walk, some ride a bike, and others are driven in a bus or a car. Each child can place a block on the chalkboard in front of the picture which shows how he or she comes to school. For example, in this illustration the class can see from the piles of blocks that most of the children come to school by bus.



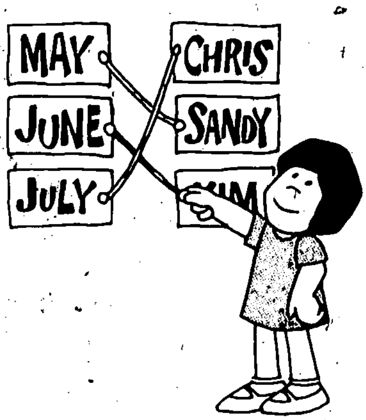
This activity could be extended to having the class display the data on a large bar graph. The teacher can also take advantage of this activity to enhance skills with counting and numbers.



In later grades, when the children have studied percentages and angle measurement and have further developed their drawing skills, one could use "How I Come to School" data to compile a circle graph.

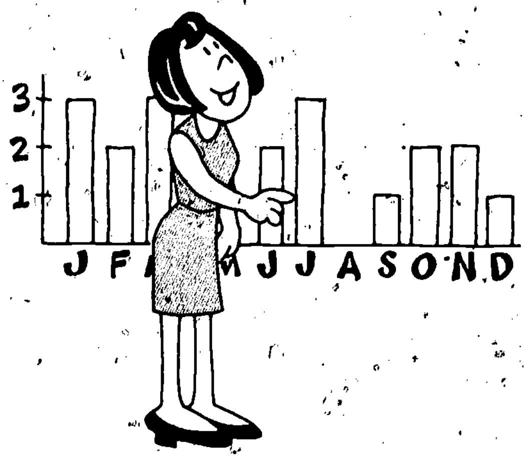


Another data-collecting activity could be generated by listing the birthdays of the children in a class. In this graphing experience, each child is asked to match his name with the month in which his birthday occurs by attaching a piece of yarn from his name to the proper month.



Then the class can have a discussion and answer several questions by looking at the yarn picture. Which month has the most birthdays?

How do we know that this month has the most birthdays? Which month has the least number of birthdays? Are there any months in which no one has a birthday? Does each child have a birthday? Do any children have two birthdays? Notice that there are many questions which children can answer easily by reading data pictured in this "graph." In the lower grades the teacher can construct a bar graph that will correspond to the student-collected data.

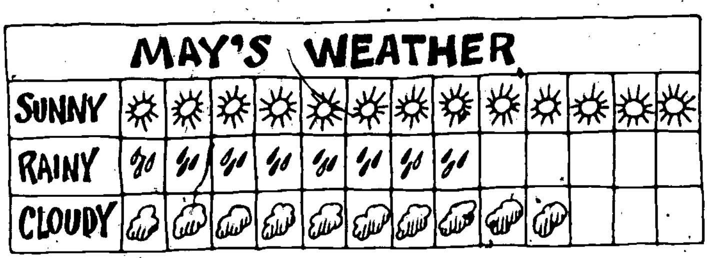


Questions can also be asked about this graph. Which month has the most birthdays? Which has the least? A question such as "Can we tell from this graph when Patty's birthday is?" can illustrate the strengths and shortcomings of particular kinds of graphs. In higher grades the students themselves can construct these bar graphs.

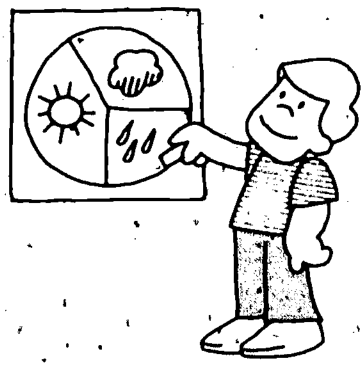
In many elementary classes, keeping a record of the daily weather report is a routine part of every day. This activity can be used in a graphing lesson. Students look at the sky and note the outdoor temperature. Then a student records the temperature and the visible weather picture on a class weather calendar.

At the end of each month, the weather calendar can be used for discussion and graphing purposes.

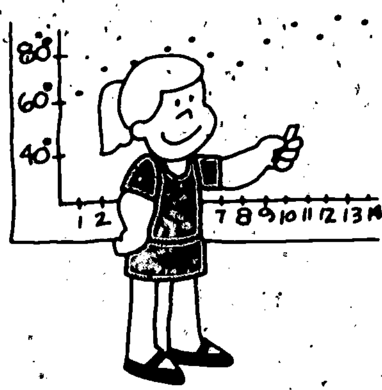
In the lower elementary grades, the weather data may be collected on a pictograph, providing an opportunity to ask the class such questions as: "What type of weather occurred most often in May?" and "Were there more sunny days than rainy or cloudy days?" and so on.



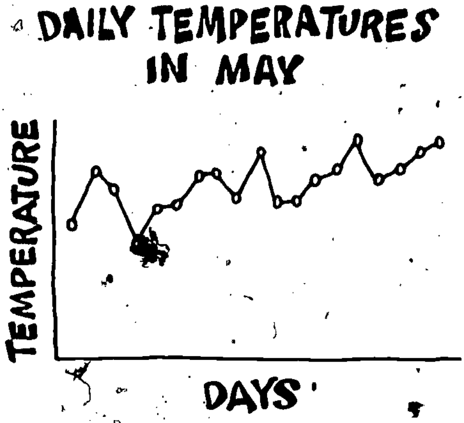
In later elementary grades the same data might be displayed by the student on a circle graph.



The temperature data on the calendar afford a good opportunity to have the upper elementary student construct a line graph. The student first transfers the data to a number grid, as shown below.



The points are then connected by straight line segments in order to make the detection of temperature trends and contrasts more evident.



The class could be asked such questions as: "What day of the month was warmest?" "What day was coldest?" "What was the temperature range for the month?" "What week had the greatest drop in temperature?" "Overall, did it get colder or warmer during the month?" And so on.

GRAPHS THAT PICTURE LOCATIONS

Children enjoy playing the "Where is the ?" game, which serves as a good introduction to working with coordinate systems. An object is placed on a number grid, and the children are asked to describe the location of the object in terms of the numbers labeling the lines. The teacher might ask, "Where is the tree?" and the student would locate the object.

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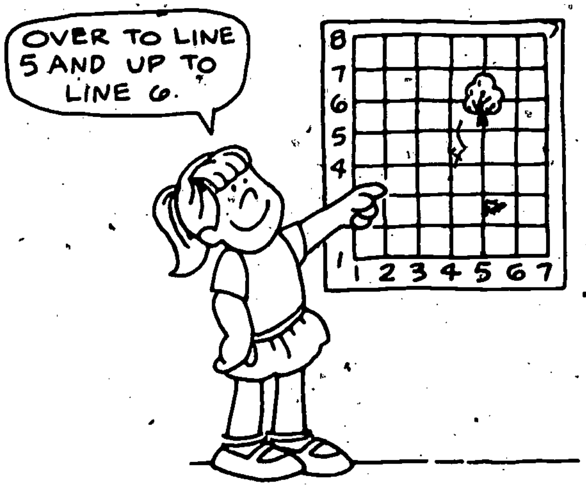
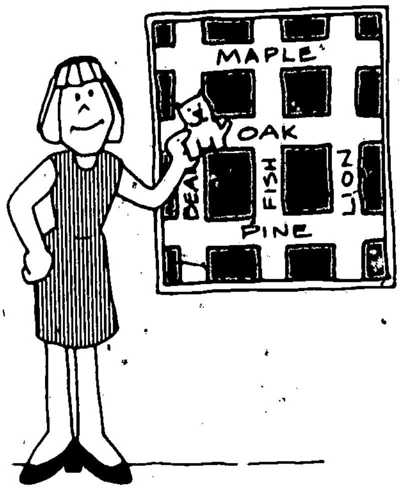


This game could be continued by calling a student up to the grid to place the object and ask his classmates to tell the location.

Another grid system that could be used for the "Where is the ?" game is a neighborhood street map. The teacher might place a small object at one of the street intersections, and then ask the class to describe the location of the object.

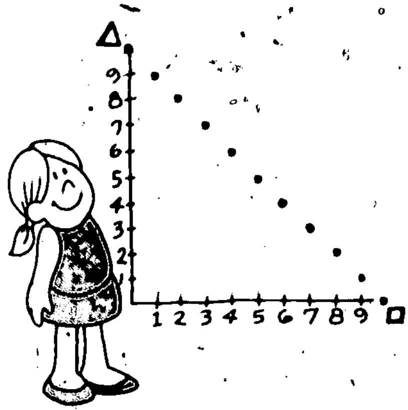


Skill in locating points can be used to picture information contained in equations and functions. For example, in the primary grades, children are often asked to find whole-number solutions to equations such as

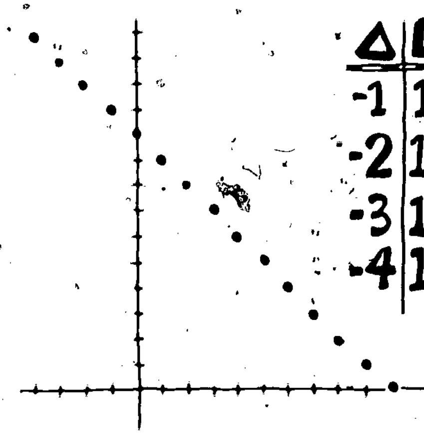
$$\triangle + \square = 10$$

A table can be made on the chalkboard showing several of the values of △ and □ that can be put into the equation to make it true. If the class has had some previous work with grids, the children can locate the points corresponding to the tabular data. When the graph is completed, the class can be asked if they see any pattern on the graph.

△	□
5	5
10	0
9	1
8	2



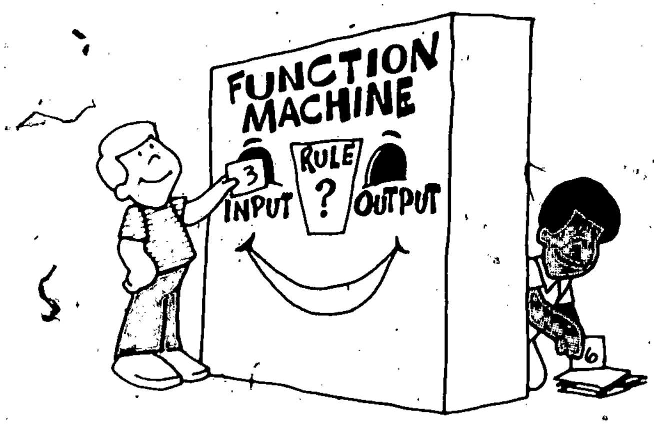
The primary-grade child will plot only whole-number values for △ and □. Later in the elementary school, graphing fractional values can be taught, as well as using negative integers and plotting points outside the first quadrant.



△	□
-1	11
-2	12
-3	13
-4	14

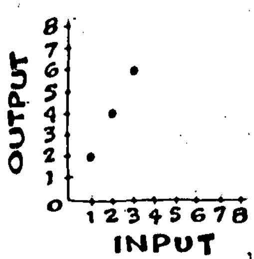
GRAPHS FOR PICTURING FUNCTIONS

Another use of the skill of locating points is graphing functions. For example, children enjoy graphing functions that are introduced through the "What's My Rule?" game. This game is played by placing a child inside a large box called a function machine. The child in the machine chooses a rule, e.g., "double." Then, as a basis for guessing the rule, the class gives the machine various input numbers and the child in the machine supplies the appropriate output numbers.



By recording the input-output pairs in a table and by plotting the pairs on a grid, the class can help make its guessing easier and more systematic. When the rule has finally been discovered, the game can be replayed, using a different rule.

INPUT	OUTPUT
1	2
2	4
3	6

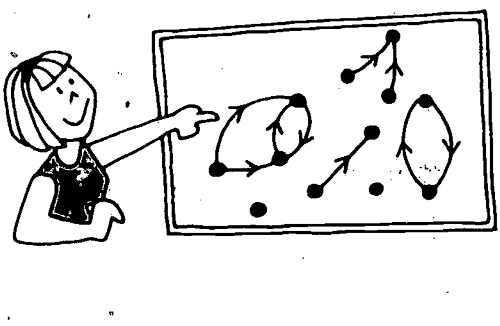


GRAPHS FOR PICTURING RELATIONS

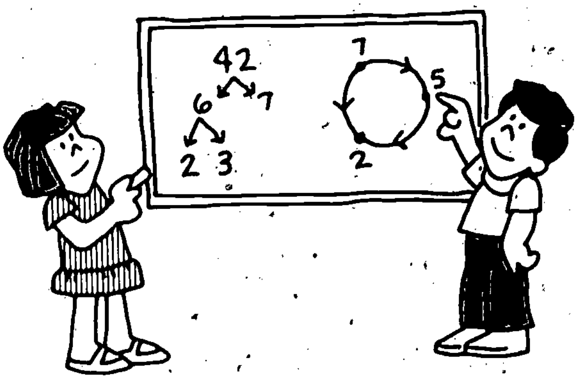
A somewhat less common kind of graph is the directed graph or digraph. This kind of graph can be used to picture relations and, unlike the others, does not depend on numerical information.

Children tend to be interested in various familial relationships. This interest can be capitalized on in working with digraphs. For example, the directed graph shown below is being used to symbolize the relation "is my sister."

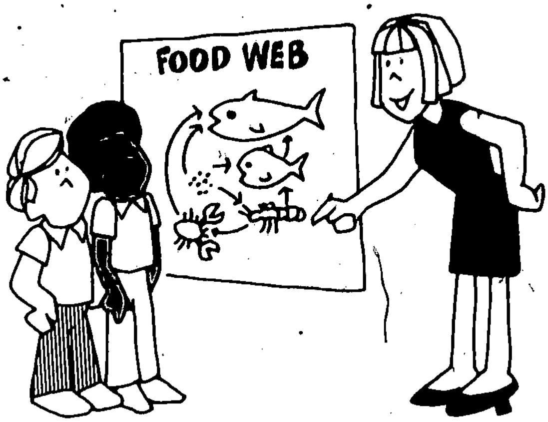
The teacher might show the class a directed graph representing a playground where everyone who is able to has pointed to his or her sister. She could then ask the members of the class to go to the graph and answer such questions as: "Where is a sister?" "Where is a brother?" "Does everyone shown have a sister on the playground?"



Directed graph activities can also be done with numerical relationships. The student on the right below is pointing to a directed graph illustrating the relation "is greater than." Digraphs can also be used in factoring composite numbers into their prime factors, as the student on the left has done.



Digraphs can also be used to picture ecological relationships between various animals, as in this illustration:



The use of digraphs in the elementary school is in its infancy, but it may become more widespread.

This has been a long list of examples. The intent of the list is to introduce you to different kinds of graphs, and to suggest to you that the making and interpreting of graphs is feasible and desirable in the elementary curriculum. The unit that follows is designed to broaden and deepen your skills with making and using graphs in order to help you help children use graphs to communicate information.

Section I

PICTURING DATA

This section explores the picturing of information that is in the form of data. Data arises in our lives in many different circumstances. Some of it you gather informally as in "I miss that turn half of the time." Some people such as pollsters, statisticians and even weathermen make a profession of gathering, organizing, analyzing, and drawing inferences from data. The use of graphs to help organize and present data is a common and important practice.

Activity 1 presents four types of data graphs (bar, line, circle and pictograph) and provides experience with reading and constructing each type. Also the point is made that the impression conveyed by a bar or line graph or pictograph can be altered by altering the horizontal and vertical scales. The data presented in Activity 1 is chosen to be much like that which one would encounter in work with children. In Activity 2 you collect data to substantiate a point of view and you present the data on graphs that are appropriate to that point of view.

There is an assignment connected with Activity 2 that must be begun prior to doing the activity in class. You should turn to direction 1 on pages 29-30 to get the assignment.

While there are questions concerning graphing activities for children throughout Section I, Activity 3 focuses on the use of data graphs with children.

If you want some additional work with data graphs, you can turn to the Graphing Self-Evaluation in the Appendix.

MAJOR QUESTIONS

1. Describe steps one might go through in collecting data to support a point of view and then in organizing and picturing that data. List some considerations one makes in picturing data. In particular, discuss the effect of the choice of types of graphs and scales.
2. Choose a level, primary (1-3) or upper elementary (4-6). List the objectives that you might have for a class at that level with respect to bar, line, circle, and pictographs. Outline a sample data-graphing activity that you might do with the children while studying some topic besides mathematics. Be sure to include some key questions that you would ask the children, in order to bring out the salient features of the graphs in question. Also be sure that the children have the prerequisite skills for the activity.

ACTIVITY 1

BAR, LINE, CIRCLE AND PICTOGRAPHS AND THE EFFECTS OF SCALING

FOCUS:

Bar graphs, line graphs, circle graphs, and pictographs can all be used to picture data. In this activity you will have experience reading and constructing each type of graph. You will also consider the characteristics of each type of graph and the effects of scaling on the message of a graph. Many of the graphs presented might be used in an elementary school classroom.

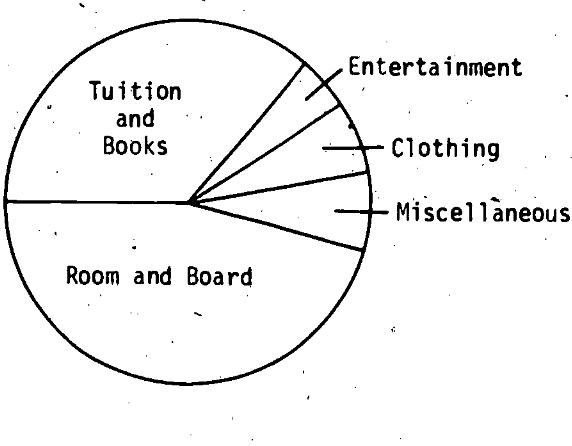
MATERIALS:

Graph paper, ruler, compass, and protractor.

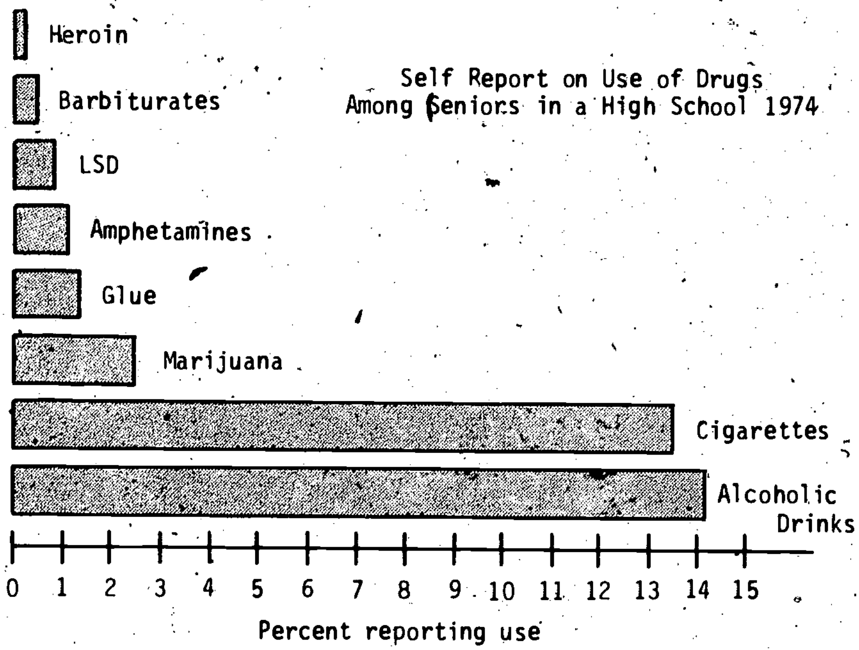
PART A: Bar, Line, Circle and Pictographs

DISCUSSION:

You have probably seen examples of each of the four kinds of graphs mentioned in the title. For example, circle graphs are often used to picture a totality of a quantity and to indicate how portions of that totality are allocated. Here is a circle graph indicating how one college student spent his budget.



Bar graphs and pictographs facilitate comparisons of quantities. This bar graph is intended to emphasize a comparison:



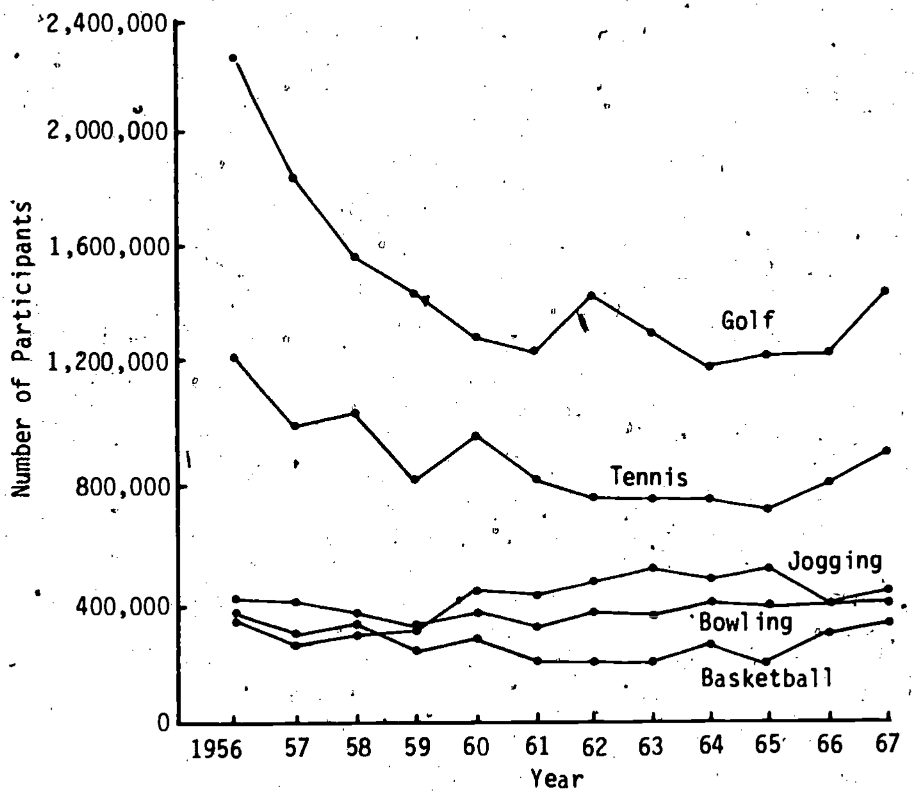
while this pictograph compares quantities and can be easily updated.

Money Accumulated For Classroom Projects
(Each $ represents two dollars)

Class Donation to United Fund	$ $ $ $ $
Class Gift to School	$ $ $
Class Picnic	$ $ $ $ $
New Volleyball	$ $

Line graphs can also be used for comparison and for expressing allocations of resources, but they seem to be particularly useful for communicating <u>trends</u>. Here is a line graph that compares trends.

Data of Sports Participants in a State, 1956-67

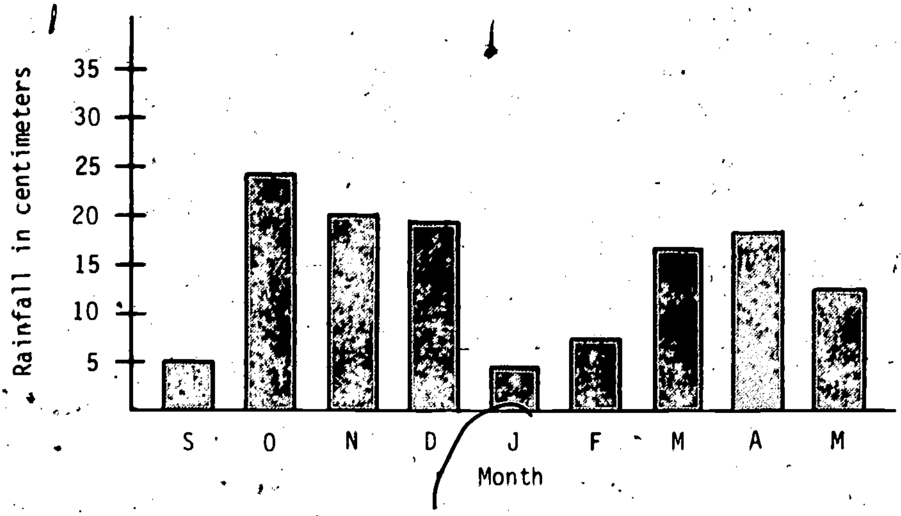


The questions in this activity should either help to introduce you to or remind you of these four types of graphs. If you feel the need for further experiences with the data graphs presented in this section, you can turn to the Graphing Self-Evaluation in the Appendix.

You should note that the data and graphs of this activity are chosen at an elementary school level. This is done to provide you with a resource of ideas to use in graphing activities with children.

DIRECTIONS:

1. The students in a fourth-grade class constructed the following rainfall graph during one school year.



Answer the following questions about the graph.

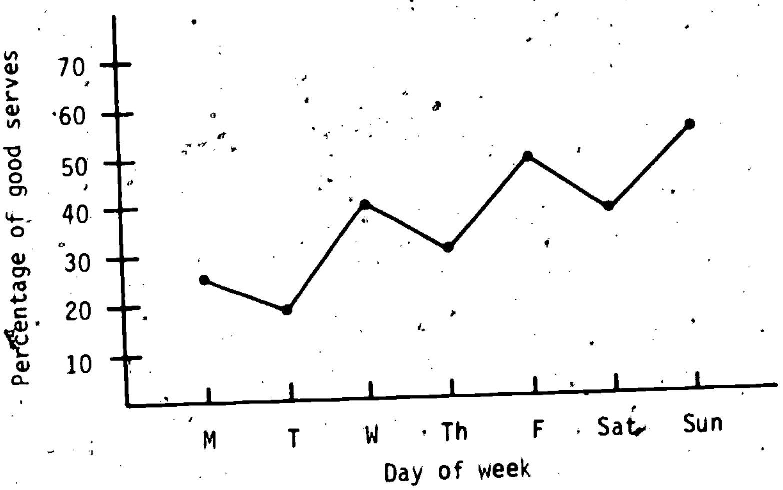
a) Which month had the most rain?

b) What were the approximate amounts of rainfall in September, December, and April?

c) What part of the school year would you call the rainy season?

d) About what percentage of the total rainfall fell in December?

2. Mary was concerned with her tendency to double-fault with her tennis serve. As part of a program to improve her serve, she decided to make a graph of her daily serving success. Here is her graph for one week.



Answer the following questions concerning this graph.

a) Is Mary improving?

b) Which was Mary's best serving day? Her worst?

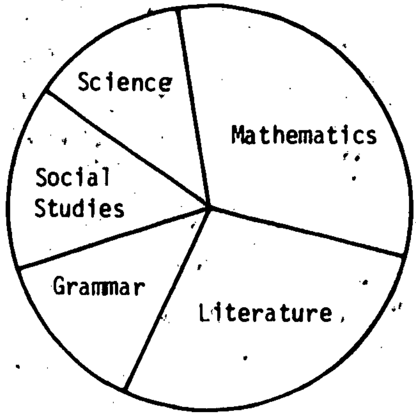
c) Approximately what percentage of her total serves of the week were good? (Analyze this question carefully.)

d) Fill in the following table.

Day	% of good serves
M	
T	
W	
Th	
F	
Sat	
Sun	

3. An eighth grade had classes in science, social studies, mathematics, and language arts (split by the teacher into literature and grammar). The class was upset about the length of its mathematics assignments, so they decided to make a presentation that

would convince their mathematics teacher that she was using an unfair share of their study time. Here is the graph that the class presented.

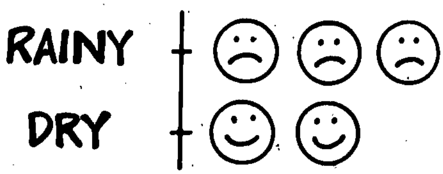


a) The alert mathematics teacher told the class that they were complaining to the wrong teacher. Which teacher do you suppose she had in mind?

b) About what percentage of study time did the class feel that they spent on each of their subjects?

c) What special skills are involved in reading a circle graph that are not required for the other three types of graphs?

4. A second-grade class wanted to keep track of the number of dry and rainy days during a week, so they used the following method of representing this data, using cut-out faces.



Answer the following questions about this pictograph:

a) What advantage can you see to using a pictograph in this situation instead of a bar graph?

b) Which of the following kinds of data would lend themselves to representation on a pictograph?

- High temperature on each day of the week
- The number of children who ride a bike, walk, and are driven to school
- The relative numbers of science, math, and social-studies books in the library
- The number of recyclable cans collected by each child in the class

5. Below are four sets of data presented in tables. For each set of data, construct a bar, line, circle, or pictograph to represent the data. Choose a different type of graph for each set of data. Base your choice on the appropriateness of the type of graph for the data.

Student	Books read so far this year
Sally	1
Mary	7
Bill	4
Don	3
Joe	2

Measured Diameter of Circle (in cm)	Measured Circumference of Circle (in cm)
1	3.1
1.5	4.71
3.25	10.4
4	12.8

Place of residence	Fraction of year spent
home	$\frac{2}{3}$
hotels	$\frac{1}{12}$
cottage	$\frac{1}{6}$
grand-mother's	$\frac{1}{12}$

Spelling test #	Johnny's score on test
1	50
2	90
3	33
4	79
5	97

6. Choose a grade level and one of your graphs in (5) and list several questions you would ask children at that level about the graph.

7. As a homework assignment, collect two examples each of bar, line, circle, and pictograph. These can be found in newspapers, magazines, reports, books, etc. A particular effort should be made to find examples that would be meaningful and interesting to children. Make up questions that you would ask a class about each of your graphs.

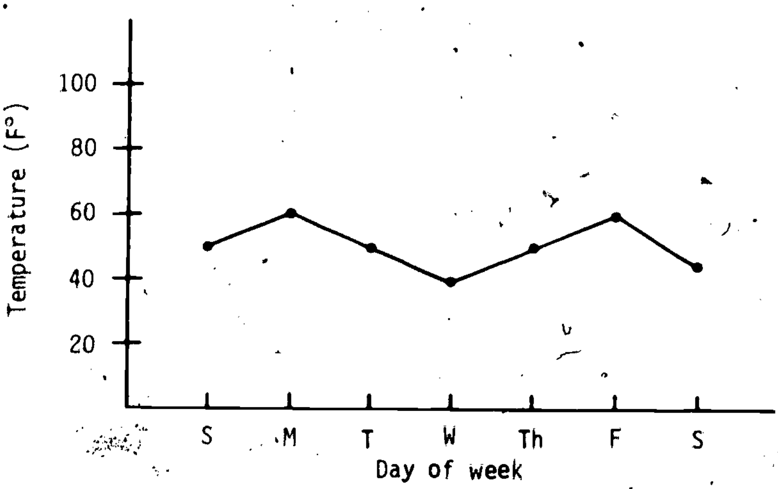
8. Graphing can be useful in many elementary school activities. Brainstorm several activities that would lend themselves to representation by bar, line, circle, and pictographs.

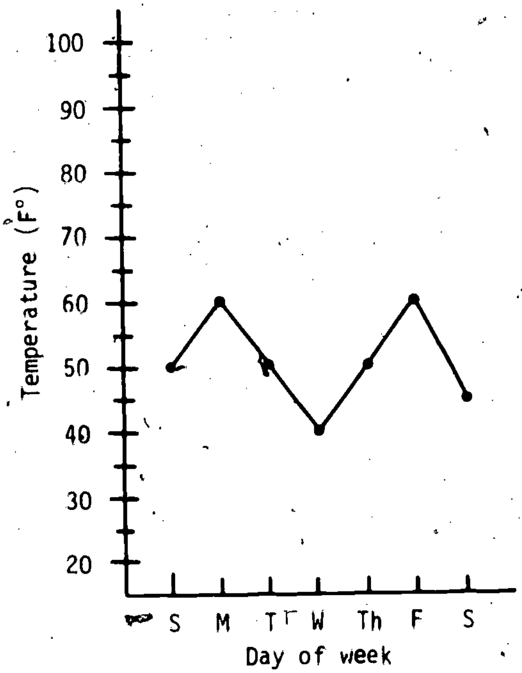
For additional practice with reading and constructing graphs, turn to the Appendix.

PART B: THE EFFECT OF SCALING

The message conveyed by a bar, line, or pictograph can be altered by a change in the scale of the graph. We present here a discussion of scaling, to be followed by some scaling activities for you.

The following two graphs present the same data on the same type of graph.





Both of these line graphs present the same data. Do they convey the same message? If you wanted to emphasize the variability of the weather, which graph would you use? Notice that, on the first graph, the days of the week on the horizontal axis are spread out and the numbers on the vertical axis are condensed. Both of these changes tend to minimize the appearance of variability in the data. In the second graph, the variability is emphasized by compressing the horizontal axis and extending the vertical axis.

DIRECTIONS:

9. Take some data. Choose opposing points of view or messages with respect to the data. Make two graphs of the data--one to support each of the opposing points of view. (You can use data you have collected; you can use data that is presented in a newspaper; or you can use some data from the Appendix.)

10. Collect examples of graphs from newspapers, magazines, reports, etc., and discuss the effect of scaling on the message conveyed. Did you find any situations where the data has been distorted by the choice of scale?

11. Briefly summarize the effects of scaling on the horizontal and vertical axes of bar, line, and pictographs.

ACTIVITY 2

DATA COLLECTION AND REPORT

FOCUS:

In Activity 1 you had experience reading and making four types of graphs of data. Graphs are constructed to picture information--to convey a message. In this activity, you will gain experience with picturing information contained in data. You will

- Choose a question or position,
- Collect data to answer the question or support the position,
- Decide on a message contained in the data,
- Choose a type of graph and a scale that will easily and clearly communicate the message,
- Construct a graph of the data,
- Present the graph to others for evaluation.

DISCUSSION:

You will be asked to carry out a small data-collection experiment and to make a graph of the data. Your instructor will make the assignment (Number 1 below) and will indicate when and how you need to have it completed (Number 2 below).

DIRECTIONS:

1. Assignment:

   a) Think of a question or position, and devise a data-collection experiment to shed light on the question or to support or refute the position. Use your imagination; almost any data-collection experiment would be fine; you might want to check your data-collection idea with your instructor. Some possible kinds of experiments are:

- Poll attitudes of classmates or dormmates toward a political issue.
- Determine the time it takes each of your favorite bugs to walk through a tube.
- Analyze the writing of different individuals in terms of the frequency of use of various parts of speech.

b) Collect your data (you may want to devise a convenient table or chart for collecting it).

c) Organize your data, and look for patterns or trends that support a point of view, position, or message.

d) Decide which types of graphs (bar, line, circle, or pictograph) are most suitable for presenting your data. You may use several different types of graphs, depending on the type of data and the message that you may want to convey. For bar, line, and pictographs, you need to choose a scale that will accurately communicate the message contained in your data.

e) Construct your graph(s), and prepare a report consisting of a description of your experiment and how you collected your data, the graph(s), and a description of any answers, messages, or conclusions that are warranted by the data. Your instructor may ask you to present your report to the class (Number 2 below) or to turn it in.

2. Students selected to present data reports should focus on how the data was collected, how it was organized, and why particular kinds of graphs were chosen in preference to others. Then, in a spirit of inquiry, not of criticism, the class should address itself to each of the following questions:

a) What was the message?

b) Did the data support the message?*

c) Could another kind of graph or a different scale have been used to better convey the message?

The class should pay particular attention to the general question of which kinds of graphs seem most appropriate for which kinds of data. After the reports have been made, the whole class, with the instructor's help, should summarize the discussion on this question. Filling in the following table might be of some help to that end.

	FEATURES OF GRAPHS	
	Particularly Good For	Not Particularly Good For
Circle Graph		
Line Graph		
Bar Graph		
Pictograph		

*An informal discussion of the adequacy of sampling procedures will be presented in the Probability and Statistics unit of the Mathematics-Methods Program.

ACTIVITY 3

GRAPHING WITH CHILDREN

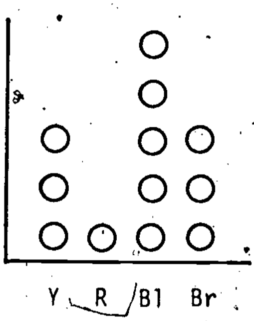
FOCUS:

An important objective of your work with this unit is to help you become more effective at doing graphing with children. In this activity you will prepare a graphing activity for children, and try it out if possible. You should note that graphing can be a part of many different elementary school activities.

DIRECTIONS:

1. Prepare a lesson plan for an elementary school activity that involves graphing. The lesson can have graphing as its objective, or graphing can be a tool to facilitate another objective. Be sure to specify the objectives of the lesson, the prerequisite skills required, and the grade level for which the lesson is intended. The following are some ideas for possible lesson topics.

   - Measure the circumference (C) and the diameter (d) of several circles and graph them on C-d axes. Can the children see a pattern? What would happen if you did a similar thing for squares (i.e., graph perimeter vs. length of diagonal)?
   - Have children cut out circles that are of the same color as their hair, and place them on a felt board--forming a pictograph.



- Take a passage in a book and graph the frequency of the various parts of speech, the occurrences of three-, four-, five-, and six-letter words, or some other linguistic attribute of the passage.
- Graph sunset hours for each day of a week. Look for a trend. (You may have some scaling problems here.)
- Graph the lengths of sweet potato and bean plants for each day of a month.
- Have each child represent on a graph the proportion of time spent in various activities during a day. Is it true that they work eight hours, sleep eight hours, and play eight hours?
- Graph the production of meat in various countries. Then graph relative populations, and compare. Can you guess which countries are exporters?

2. Try your activity with children. Be sure to provide the class with the materials that you need, and be sure to check that the children have the prerequisite skills. You may want to modify your lesson on the basis of the trial.

3. Briefly describe an elementary school graphing activity connected with each of the following subject areas.

   reading
   writing
   arithmetic
   social studies
   spelling
   science
   physical education
   art
   music

Section II

PICTURING LOCATIONS

The theme of picturing information is developed further in this section. The information to be pictured here has to do with location or position, and the basic tool for picturing will be the coordinate system. You are used to identifying locations using such phrases as "50 miles west of Detroit on Interstate 94," and "at the corner of Lexington Avenue and 42nd Street." As you know, these phrases refer to locations that can be pictured on a map. Coordinate systems are essential to picturing such locations. The most important aspect of such a system is that it provides a systematic means of assigning a name to each location.

Activity 4 is concerned with locating points using rectangular (Cartesian) coordinate systems. The activity involves information and skills that will be familiar to many readers. These readers can use the activity for recall and review.

Activity 5 investigates some other coordinate systems. In some cases you will be familiar with the system, but you may not be used to thinking about it as a coordinate system.

Activity 6 addresses itself to the problem of representing a three-dimensional world on two-dimensional maps. This is an historically important problem which admits of a variety of solutions, each particularly useful for a specific purpose. You should keep in mind throughout that the concept of a coordinate system is a useful organizer. It provides an umbrella under which many apparently dissimilar but actually related ideas may be collected.

Activity 7 brings the focus back to the elementary school classroom by asking you to analyze what is done in textbooks, and to outline graphing activities to be done with children.

MAJOR QUESTIONS

By the time you have completed Section II you should have answers to the following questions.

1. What is a definition of a coordinate system? Or at least, what are the important common attributes of all coordinate systems?
2. In what specific ways (outline at least three) can one use coordinate systems to effect a tie-in between mathematics and other elementary school topics?

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ACTIVITY 4

RECTANGULAR COORDINATE SYSTEMS

FOCUS:

This activity provides a brief opportunity to learn, relearn, or review skills with rectangular coordinate systems.

DISCUSSION:

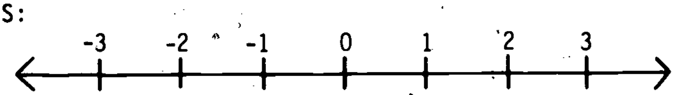
You have already used certain skills with rectangular coordinate systems in your work with bar, line, and pictographs in Section I. However, since rectangular coordinates are regularly taught at most levels in the elementary school, it seems worthwhile to confirm your understanding of them. This activity also provides a nice point of comparison and contrast with the next activity, in which other less commonly recognized coordinate systems are introduced.

MATERIALS:

Ruler and graph paper.

DIRECTIONS:

1.

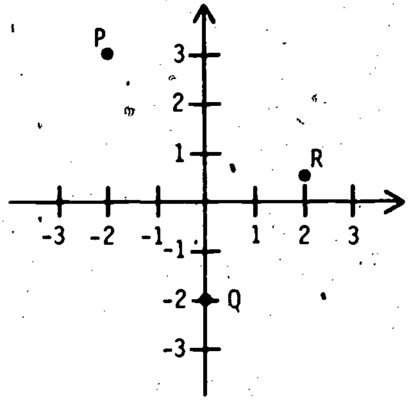


Above is a picture of what is called a <u>number line</u>. Every number has a location on the line, and every location on the line has a number that accurately describes it. The number line is one-dimensional, since a single number locates each point. Label as accurately as possible, using a ruler, the location described by each of the following numbers.

$$-3, \quad 1\tfrac{1}{2}, \quad 2.75, \quad -0.33, \quad 2$$

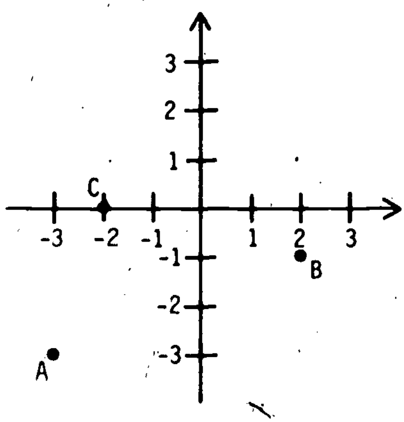
(Note that the number line proves to be a useful instructional aid with children. One can model such operations as addition and subtraction by "hopping" forward and backward on the number line. One can also embody distance in terms of lengths of intervals.)

2. Two perpendicular number lines form a two-dimensional rectangular coordinate system:



The point labeled P can be located by the ordered pair (-2,3) since its location corresponds to -2 on the horizontal number line or <u>axis</u>, and to 3 on the vertical axis. You should agree that Q is located by (0,-2) and R by $(2,\frac{1}{2})$. On the rectangular coordinate system below, label the point located by each of the following ordered pairs:

(1,1), (-1,1), (-1,-1), (1,-1), $(1,\sqrt{2})$, (3,5), (0,0)

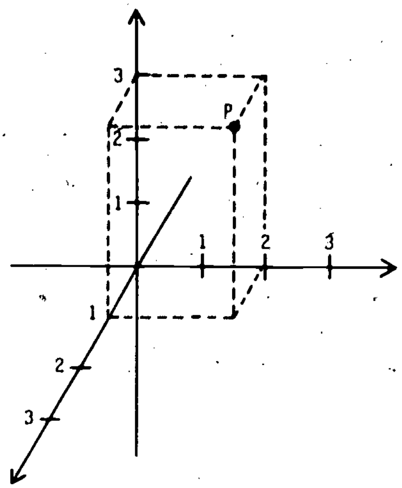


Also, write down ordered pairs which locate points A, B, and C.

3. The rectangular coordinates described in (2) are useful for locating points in a plane (two-dimensional space). This system is most appropriate for a flat sheet of paper or a flat chalkboard. Rectangular coordinates can also be used to locate points in three dimensions, using three number lines (coordinate axes). This is most important since we live in a physical world that can be thought of as three-dimensional.

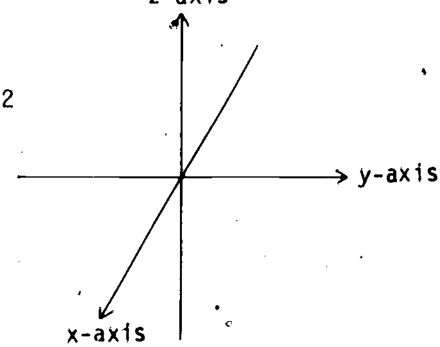
For example, the point P in Figure 1 below can be located by (1, 2, 3).

Figure 1



One way to describe the process of locating points in three dimensions is to label the three number lines so that they can be referred to. A common way is to call them the x-axis, y-axis, and z-axis.

Figure 2



The point P in Figure 1 is described by (1, 2, 3) since it is located one unit toward you in the direction of the x-axis, two units to the right in the direction of the y-axis, and three units up in the direction of the z-axis.

a) Find the coordinates (x, y, z) which describe the points Q and R in Figure 3.

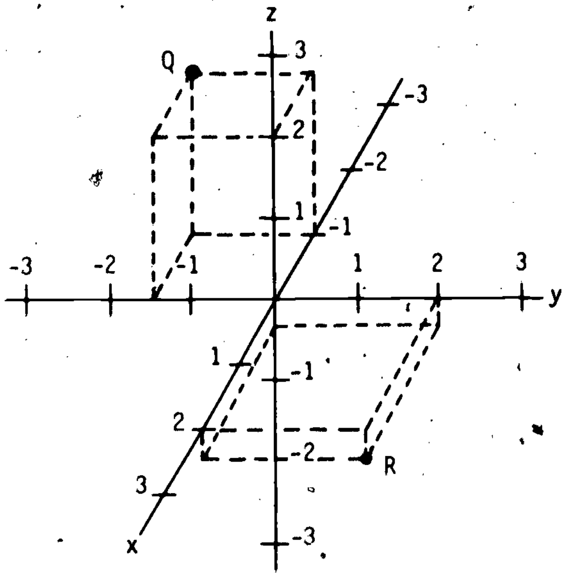


Figure 3

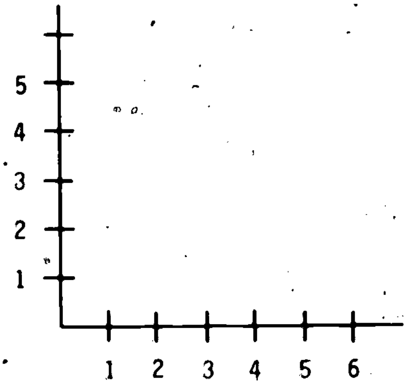
b) Draw a three-dimensional rectangular coordinate system, and locate the points that are described by:

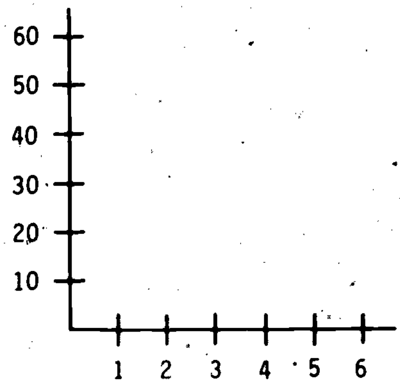
$(1,1,1)$, $(-1,2,1)$, and $(-\frac{1}{2},-\frac{1}{2},\frac{3}{4})$.

4. The corner of a typical room provides a convenient three-dimensional rectangular coordinate system. Discuss with classmates an activity that you might do with children to help them learn to locate points in the room by using rectangular coordinates.

5. Scaling was discussed in Activity 1. Scaling also enters into problems of locating points so that information is displayed

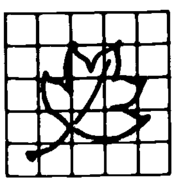
clearly and accurately. For example, which of the following two rectangular coordinate systems would be most convenient for picturing the following ordered pairs: (1,43), (2,57), (3,59), (4,63)? Why?

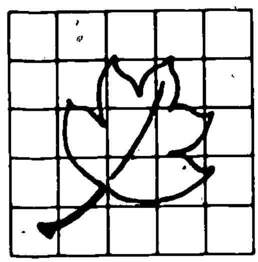




These pairs could represent the high temperatures on successive days, and locating them could be the first step in constructing a line graph.

6. In a different way, scaling can be a graphic aid.





Draw a small simple figure on one grid and then enlarge it by reproducing it with the aid of a larger grid. (This activity might be a good one to do at home.)

HISTORICAL HIGHLIGHT

René Descartes (1596-1650) and Pierre de Fermat (ca. 1601-1665) were the inventors of coordinate geometry. Coordinate geometry establishes a correspondence between points in the plane and ordered pairs of real numbers, with the latter normally plotted on a rectangular (Cartesian) coordinate system. Descartes, in addition to his contributions to mathematics, spent time and effort studying science, religion and philosophy. He culminated four years of study by writing a physical account of the universe, Le Monde. Just as he was preparing to have it published he learned of the fate of poor Galileo (1564-1642). In 1633 Galileo was found guilty of heresy for publishing his famous Discorsi, which supported the Copernican theory that the earth moves around the sun. Under threats of torture by the Inquisition, Galileo recanted and denounced his work. Since Le Monde contained arguments based on the truth of the Copernican theory, Descartes, also a devout Catholic, cancelled his publication plans for Le Monde.

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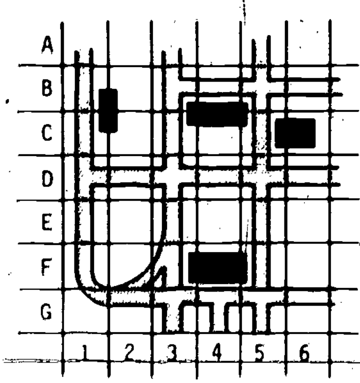
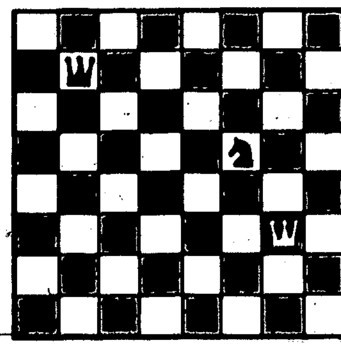
ACTIVITY 5

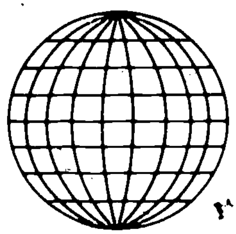
REAL-WORLD COORDINATE SYSTEMS

FOCUS:

In this activity, several coordinate systems for picturing locations will be introduced. Your experiences with them should make you aware of the large number of coordinate systems that exist and of the advantages of certain ones. There are many possibilities for using the ideas of this activity in lessons for children.







DIRECTIONS:

Discuss the following questions concerning coordinate systems.

1. If you were in Washington, D.C. and looking for the National Museum of Natural History (see Figure 1), when would you use the streets as your coordinate system, and when would you use the numbered and lettered grid lines? What are the advantages and disadvantages of each coordinate system?

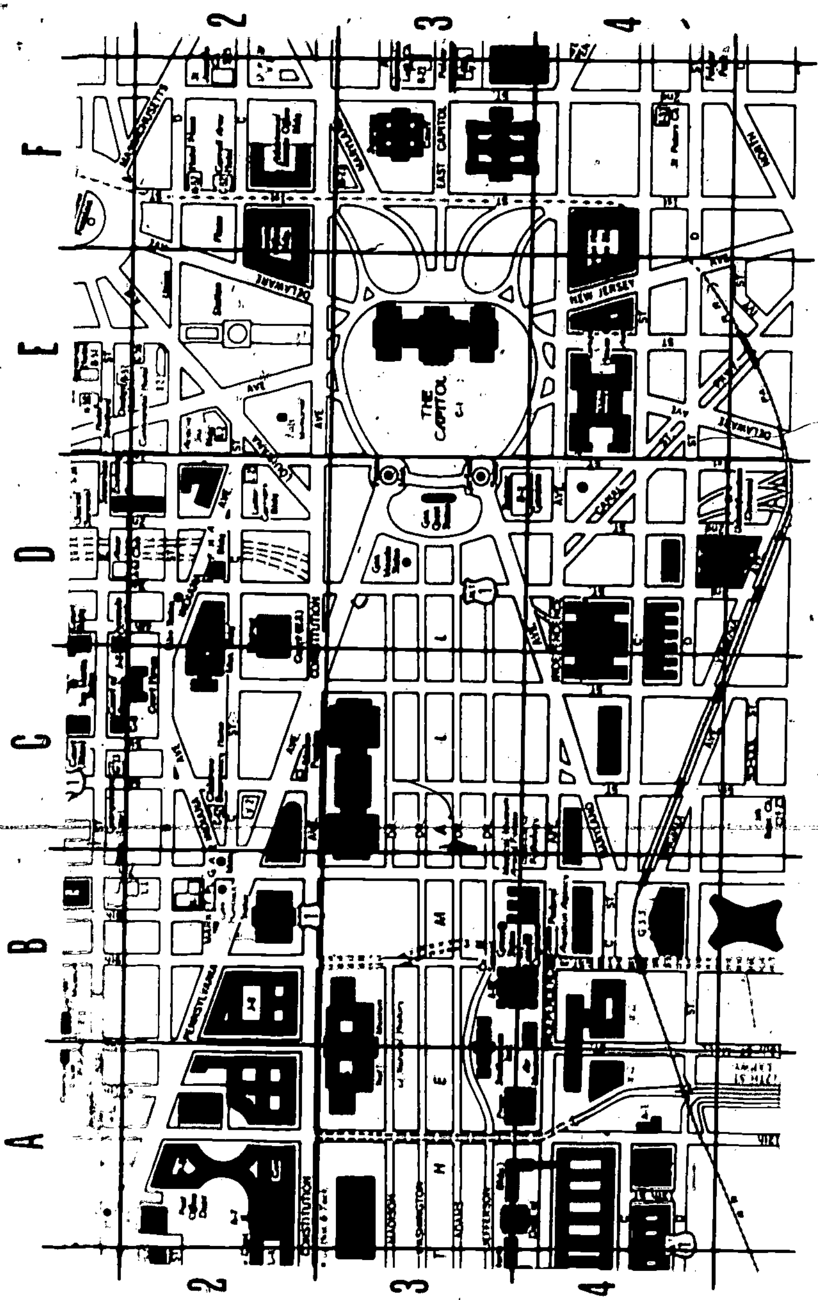
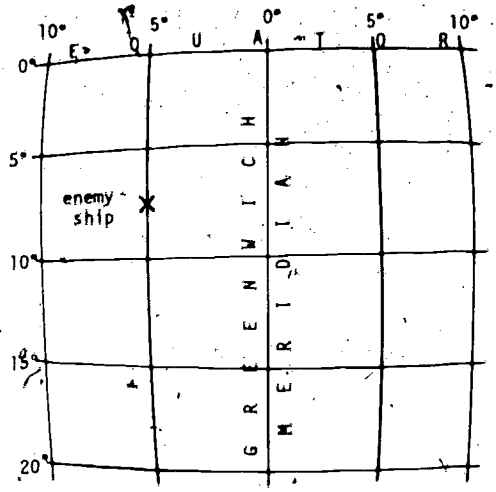

A
B
C
D
E
F
2
3
4
THE CAPITOL
EAST CAPITOL
NEW JERSEY
DELAWARE
CONSTITUTION
PENNSYLVANIA

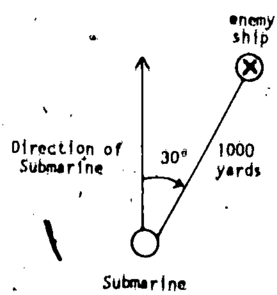
Figure 1

2. Suppose that a submarine commander is trying to destroy an enemy ship which is just out of sight in the middle of the ocean, and suppose that the commander does not know his own position. Which of the following two coordinate systems provides more useful information for the scouting plane to radio to the submarine commander?

a) The latitude-longitude position of the enemy ship:



b) A distance/bearing system centered in the submarine:



3. Suppose that an airplane pilot (who knew her/his position) had spotted the enemy ship discussed in problem (2). Which coordinate system should the pilot use in describing the location of the ship to the plane's home base?

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4. In what way is each of the following a coordinate system? Specifically, what coordinates are needed in each case?

   a) a state highway map

   b) the auditorium seating plan shown in Figure 2 below

   c) the index of a book

   d) the rows of seats in a classroom

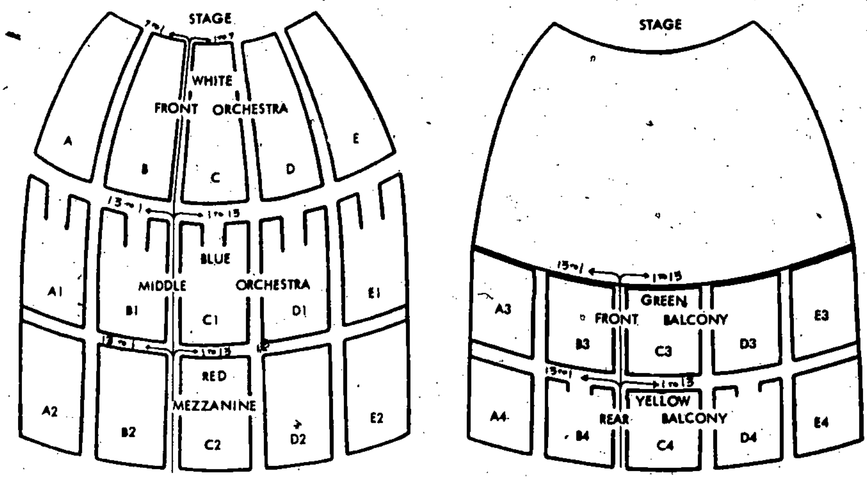


Figure 2

5. What coordinate systems can be used to describe the squares on a chessboard? What is the usual coordinate system? The table on the next page gives the plays of a chess game. Construct a rough chessboard and make the opening moves.

QUEEN'S PAWN OPENING

1.	N-QB3	P-Q4
2.	P-Q4	N-KB3
3.	B-N5	P-K3
4.	P-K4	PxP
5.	NxP	B-K2
6.	BxN	BxB
7.	N-KB3	O-O
8.	P-B3	P-QN3
9.	Q-B2	B-N2
10.	B-Q3	B-K2
11.	P-KR4	N-Q2
12.	N4-N5	P-N3
13.	NxRP	KxN
14.	P-R5	P-KB4
15.	PxPch	KxP
16.	Q-K2	BxN
17.	QxPch	R-B3
18.	BxPch	K-N2
19.	R-R7ch	K-B1
20.	R-R8ch	K-N2
21.	R-R7ch	K-B1
22.	Q-K3	B-Q4
23.	R-R8ch	B-N1
24.	B-R7	K-B2
25.	BxBch	K-N2
26.	Q-R3	N-B1
27.	B-N3	Q-Q3
28.	R-N8 mate.	

6. a) Outline an activity involving coordinate systems related to some elementary school subject besides mathematics.

   b) Do the lesson with children if you can.

ACTIVITY 6

THE MAPMAKER'S DILEMMA

FOCUS:

In this activity you will glimpse the problems involved in making flat maps of the spherical earth. This is clearly another aspect of picturing locations.

MATERIALS:

Globe; map of the United States, on which parallels of latitude are equally spaced; Mercator projection of North America (such projection maps are available in most atlases and in the backs of most large dictionaries).

DISCUSSION:

As the surface of the earth became better known, 16th-century map-makers wrestled with the problem of picturing on flat maps locations that are on the solid earth. The problem was solved in several ways, each way having its own inherent advantages and shortcomings. Maps in use today largely reflect these solutions.

DIRECTIONS:

Proceed through the following exercises.

1. Compare the map of the United States on which the parallels of latitude are equally spaced, with the United States on the globe. In what areas does distortion occur?
2. Suppose you had maps of Canada and of Mexico on which the parallels of latitude were equally spaced, which country would be most accurately represented? Why?

3. How does a Mercator projection attempt to resolve the distortion problem?

4. Find the shortest path between Halifax, Nova Scotia and San Francisco on the globe and on the Mercator projection. How do they compare? (Pick out a couple of cities along each route.)

5. Investigate in detail and prepare a report on several of the solutions of the mapping problem discussed in this section. Your report might include a discussion of cylindrical, conic, polar, polyconic, and stereographic projections. Comment on how an elementary teacher might identify some important mathematical ideas in a lesson on geography. Atlases and encyclopedias are possible sources of information.

ACTIVITY 7

COORDINATE SYSTEMS FOR CHILDREN

FOCUS:

Coordinate systems enter a child's life in the classroom and outside the classroom. In this activity you will consider the ways in which coordinate systems can relate to children. You will have an opportunity to plan a lesson on coordinate systems for children.

MATERIALS:

Several elementary school mathematics text series.

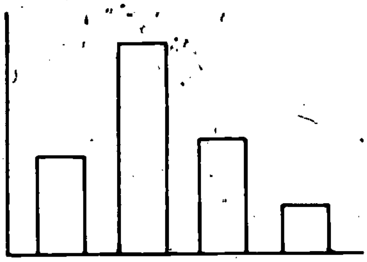
DIRECTIONS:

1. Brainstorm a list of ways in which coordinate systems could be used in conjunction with subjects other than mathematics in the elementary school curriculum.
2. List topics in the elementary mathematics curriculum where coordinate systems could play a part.
3. Thumb briefly through an elementary mathematics text series, to see how the use of coordinate systems there relates to your list in (2). Make any additions to your list.
4. Make a list of any additional ways in which coordinate systems might enter a child's life outside of the classroom.
5. Choose a grade level. Outline a coordinate-system activity for children at that level, which would relate coordinate systems to some activity or topic outside of the mathematics curriculum.

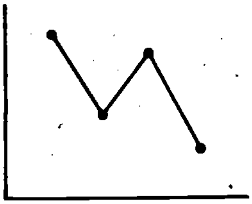
Section III

PICTURING RELATIONS

In Section I you had experience with the picturing of information that is contained in data. You used bar, line, circle, and picto-graphs to represent, possibly misrepresent, the information contained in numbers. In Section II you studied the picturing of locations using many different systems. One of the primary messages of that section was that these different systems can all be thought of as coordinate systems and are quite analogous to the rectangular coordinate systems with which you are familiar. In this section you will be working with a type of graph that is likely to be unfamiliar to you. Instead of having graphs that look like



or that look like



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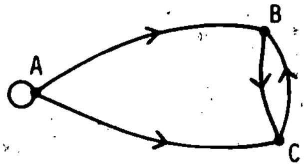
you will have graphs that look like



These graphs are called

digraphs (short for directed graphs), networks, and Papygrams,

and they picture relations. For example, an arrow in the above graph says "is related to;" so that the graph says that:

A is related to A
A is related to B
B is related to C
C is related to B
A is related to C.

The relations pictured by such graphs can be such familial relations as "is the sister of" and "is a blood relative of," or numerical relationships such as "is the square root of," and "shares a prime factor with" or the relationship can indicate a physical connection such as "lie on the same highway" and "can be hooked onto the same electric line." Picturing these relations with graphs can help to communicate the relation to others. It can also help to analyze the relationship as a step in solving some problem.

It is important to note that graphing relations is not a generally accepted part of the elementary school curriculum. Some pioneering work with relations for children has been done by the Papys (see Activity 8) and others, but there is no consensus as to the role of graphs of relations in the elementary curriculum. Relations are included here because of their mathematical importance, because graphing relations does seem to hold some real instructional potential for children, and because we thought you would find graphing relations both interesting and enjoyable.

In this section you will picture relations using digraphs and networks (Activity 8). You will analyze certain digraphs and networks (Activity 9). You will study relations represented as ordered pairs and the Papygrams associated with them (Activity 10). You will consider a particular kind of relation called an <u>equivalence relation</u> (Activity 11). Then you will consider some implications of all of this for the elementary school curriculum (Activity 12).

> Activity 10 involves analyzing certain digraphs and networks. This analysis will be challenging for most students, and should be started several days in advance of the day on which the analysis is due.

MAJOR QUESTIONS

1. Give examples of relations from arithmetic, geometry, biology, and geography.
2. Give examples of relations that would be meaningful to children. It may or may not be a good idea to bring the concept of relation explicitly into the elementary school curriculum. State and support your position on this issue.
3. Make up a problem for children which could be solved by analyzing the graph of a relation.
4. Make up an equivalence relation; verify that it satisfies the definition of an equivalence relation, and describe the equivalence classes for this equivalence relation.

ACTIVITY 8

PICTURING RELATIONS WITH DIGRAPHS AND NETWORKS

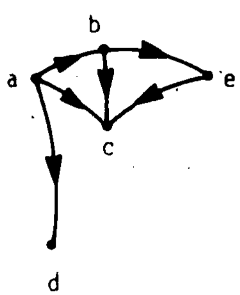
FOCUS:

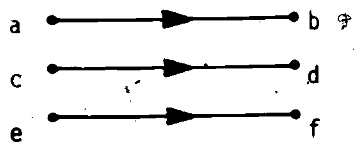
You will have an opportunity to picture relations as digraphs (directed graphs) and to analyze digraphs in order to understand the relations they picture. You will also have an opportunity to investigate relations which have a property called <u>symmetry</u>. They can be represented using networks (undirected graphs).

DISCUSSION:

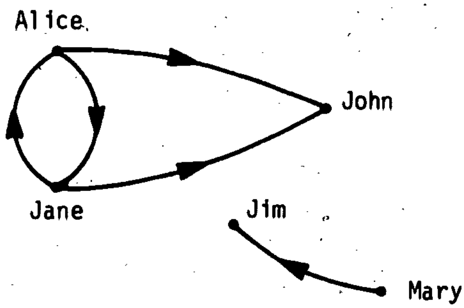
The real-world idea that something is related to something else can be expressed rather naturally in a pictorial fashion: Let a point denoted by "a" represent the first thing mentioned and let a point denoted by "b" represent the second thing. Draw an arrow from a to b to indicate that "a is related to b." The usual convention for drawing such a picture is:

A diagram such as this is an example of a digraph. Other examples of digraphs might have pictures such as:





For example, if the relation is "is the sister of," then the digraph

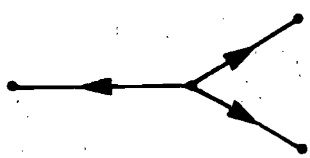


indicates that Alice and Jane are sisters and John is their brother, while Jim and Mary are brother and sister. This simple device for picturing information about relations proves to be effective and can lead to a clearer understanding of the information.

DIRECTIONS:

1. Suppose that the relation is "is the sister of," as in the discussion above.

   a) The following digraph pictures the brother-sister relationships on a school playground. Mark with an "M" those points which represent males, an "F" those points which represent females and a "U" those points which represent individuals of undetermined sex.



   b) Describe the information contained in the above digraph.

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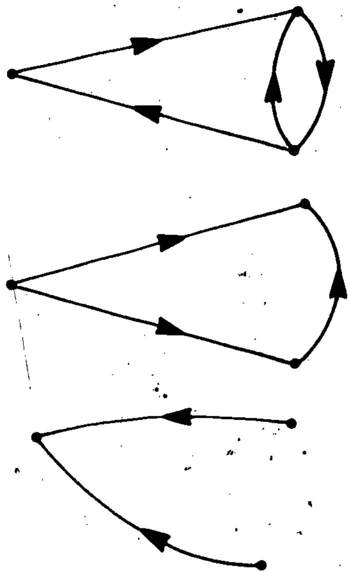
c) Something is wrong with each of the following digraphs. Indicate why they could not accurately and completely represent an "is the sister of" situation.



d) Another everyday instance of a relation is given on the next page by a section of a college catalog listing the courses in computer science. Prepare a pictorial representation of the relation "is a prerequisite to" defined on this set of courses. (Note: P means prerequisite.)

2. The Secretaries of State of the NATO countries were meeting in Brussels. The Belgian Secret Service was convinced that among the NATO staff members at the meeting there was a spy. As part of its evidence-collecting procedure, the secret service decided to carefully observe a cocktail party that was being held for the Secretaries of State and their staffs. They coded some of their observations onto the following digraph where a ⟶ b indicates that a initiated a conversation with b.

Computer Science

C201 Introduction to Computer Programming (3 cr.)
P: 2 years of high school mathematics or Mathematics M015 or M017. Computer programming and algorithms. Basic programming and program structure. Computer solution of problems. Different sections for students with various interests. Credit given for only one of C101 (dropped), C103 (dropped), C201, and C301. I Sem., II Sem., SS '73.

C202 Computers and Programming (4 cr.)
P: C201. Computer structure, machine language instruction execution, addressing techniques, digital representation of data. Computer systems organization, symbolic coding and assembly systems, systems and utility programs. Several projects to illustrate basic machine structure and programming techniques. Lecture and laboratory. I Sem., II Sem., SS '73.

C301 Fortran Programming (1 cr.)
Basic notions of computer programming. Debugging and verification of programs. Problems for programming and execution on computer. Credit given for only one of the following: C101 (dropped), C103 (dropped), C201, C301. I Sem., II Sem.

C311 Programming Languages (4 cr.)
P: C202. Systematic approach to programming languages. Relationships among languages, properties and features of languages, and the computer environment necessary to use languages. Lecture and laboratory. I Sem.

C321 Computer Organization (3 cr.)
P: C202. Organization, circuits, and logic design of digital computing systems. Course deals with the internal structure of computers. Some simple computers are designed. Experiments in basic computer circuitry are performed in the laboratory. A knowledge of electronics, while useful, is not a requirement. Lecture and laboratory. II Sem.

C343 Data Structures (4 cr.)
P: C202. Systematic study of data structures encountered in computing problems; structure and use of storage media; methods of representing structured data; and techniques for operation on data structures. Lecture and laboratory. I Sem.

C399 Individual Programming Laboratory (1-3 cr.)
P: C202. Student will design, program, verify, and document a special project assignment selected in consultation with his instructor. This course may be taken several times up to a maximum of 6 credits. Prior to enrolling a student must arrange for an instructor to supervise his course activity. I Sem., II Sem., SS '73.

C421 Advanced Computer Organization (3 cr.)
P: C321. Study of basic hardware design problems encountered in computers. Alternate solutions are compared, using a number of different computers as illustrations.

C431 Assemblers and Compilers I (3 cr.)
P: C311. P or concurrent: C343. Design and construction of bootstrap loaders, linking loaders, assemblers, macro expanders, and interpreters. I Sem.

C432 Assemblers and Compilers II (3 cr.)
P: C431. Compiler design and construction including scanners, parsers, code generation, and code optimization. II Sem.

C435 Operating Systems I (3 cr.)
P: C311 and C343. Software organization of computer systems which support a large community of users. Problems encountered in batch, multiprogramming, multiprocessing and time-sharing systems. Thorough study of an actual system. I Sem.

C436 Operating Systems II (3 cr.)
P: C435. Continuation of C435. Detailed study of more complex operating systems for time-sharing and multiprocessing. II Sem.

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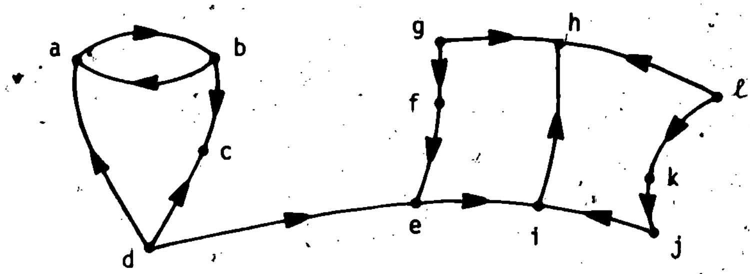
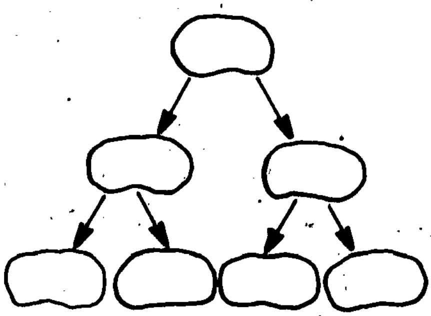


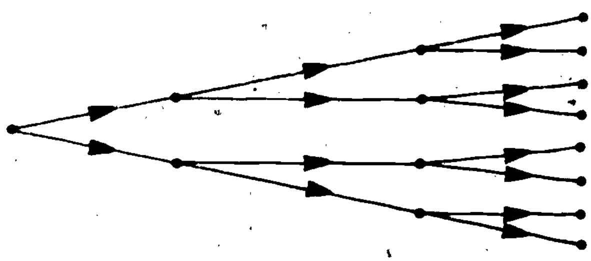
The secret service has found that spies tend to listen more and initiate fewer conversations. On the basis of this observation, who should the prime suspects be at the cocktail party?

3. a) The biological example of simple cell reproduction can be represented by the directed graph:



Think of a good way to write out the relation represented by this directed graph: "a ________________________ b."

NOTE: The diagram of cell reproduction above is an instance of a standard type of directed graph pictured below:

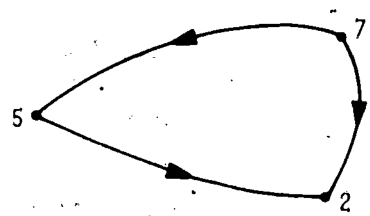


These are sometimes called <u>tree diagrams</u>. (The arrowheads are frequently omitted, and more than two "branches" can emanate from any "fork.")

b) Shifting to the study of languages, the chart on page 60 provides a nice example of a tree diagram. State the relation that is pictured in the chart.

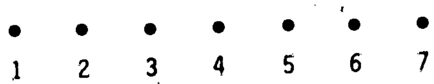
c) Describe two other relations that tree diagrams could be used to picture.

4. Some arithmetical relations that are nicely expressed as directed graphs are given here.

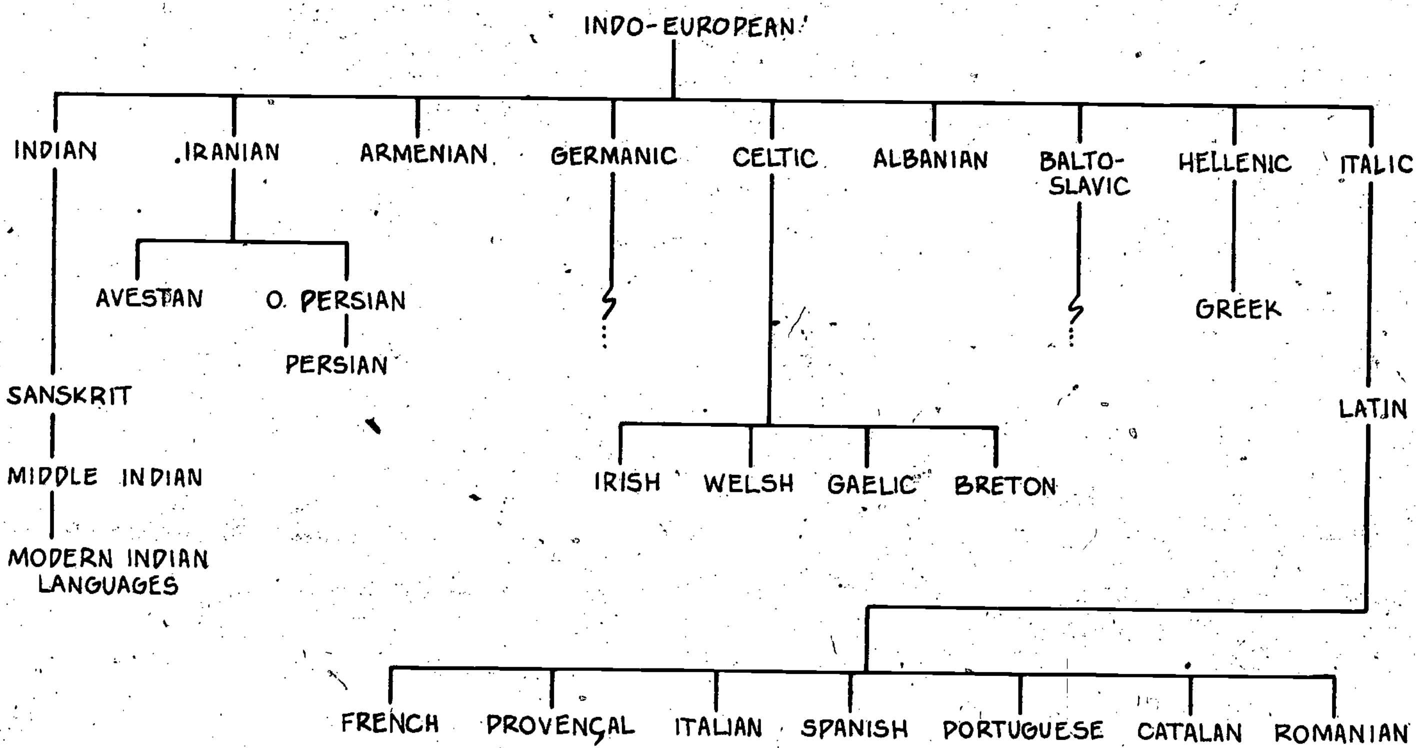


This is an illustration of the relation "is greater than" on the set $\{2, 5, 7\}$. For example, "5 is greater than 2."

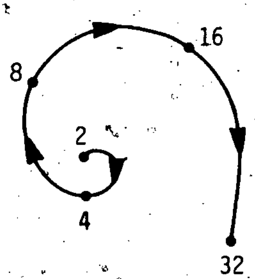
a) Graph the relation "is 2 less than" on the set below.

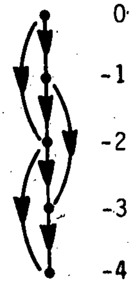


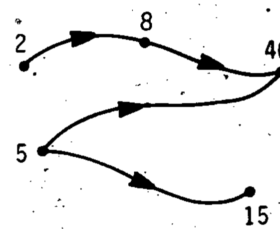
THE INDO-EUROPEAN FAMILY OF LANGUAGES



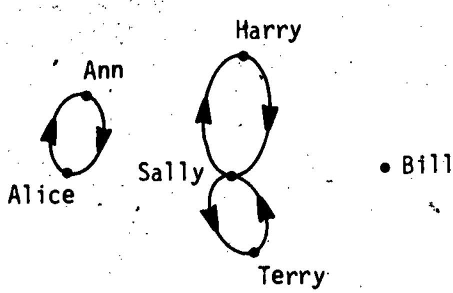
b) Can you determine the mathematical relations that are pictured in the following directed graphs?





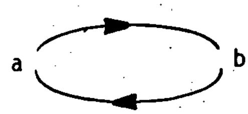


Comment: We need to stop for a moment and discuss the notion of symmetry in digraphs and the related notion of <u>networks</u>. For example, consider the following digraph of the relation "is a blood relation of."

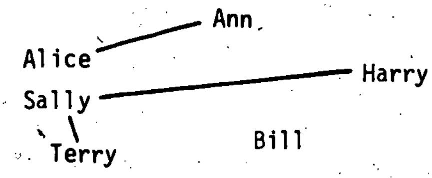


Notice that each time an arrow goes one way, there is one returning, i.e., whenever you have

you have



Any relation where this is true is called symmetric. Symmetric relations can be pictured more simply than can nonsymmetric ones. For example,



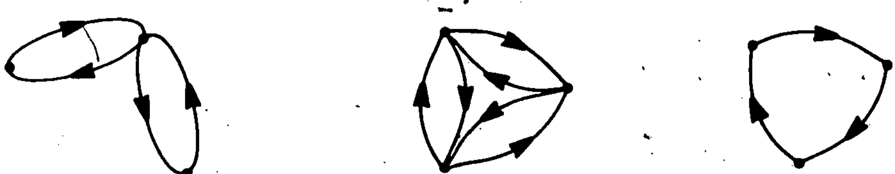
conveys the same information as the earlier digraph. In this case the single line means "are blood-related." A graph which pictures a relation that does not have an implicit direction is called a <u>network</u>. You will see, in the following exercises, that many interesting relations are symmetric and can be pictured by networks.

5. a) Analyze the digraphs in 1-4 to determine if any are symmetric. List any symmetric ones.

   b) Below is a list of relations. Check those that are symmetric.

      - lives in the same block as
      - is greater than or equal to
      - is equal to
      - plays on the same team as
      - played tennis against
      - is connected to
      - likes
      - shares a prime factor with

   c) Indicate which of the following digraphs are symmetric. Redraw each of the symmetric ones as a network.



6. a) London Paris

Dallas Rome

New York San Francisco

Fill in the above to make a network that pictures the relation "are connected by highways."

b) Using the numbers 6, 14, 12, 98, 42, 9 draw a network which pictures the relation "has the same prime factors as."

c) Make up a symmetric relation and picture it by means of a network. Make up and picture another symmetric relation which would have particular relevance for children.

ACTIVITY 9

ANALYSIS OF DIGRAPHS AND NETWORKS

FOCUS:

Digraphs and networks can be used to picture relations. Frequently it is desirable to go beyond picturing a relation to actually analyzing it or solving some problems concerning it. In this activity you will be asked to present your analysis and to discuss some general questions.

DISCUSSION:

The problems presented here do not require any special skills or techniques that have been developed in this unit, but they will require some thought. You should plan to start working on them well in advance of when they are due. These problems represent a sampling of some types of problems that one can encounter in the analysis of digraphs and networks.

DIRECTIONS:

You should work on the following problems at home or during free moments in class. You should do at least three of the problems. In general, the problems have several parts. If some of the parts seem hard, try lots of examples. In some cases, you may have to be satisfied with partial solutions.

1. Consider the task of scheduling a round robin tournament. In such a tournament, each team is to play every other team exactly once. We shall suppose that a team can play only once each day.

   a) What is the number of days necessary to have a round robin tournament with five teams? (Hint: You may find it helpful to consider the case of three teams first.)

   b) Find a schedule for a tournament with five teams.

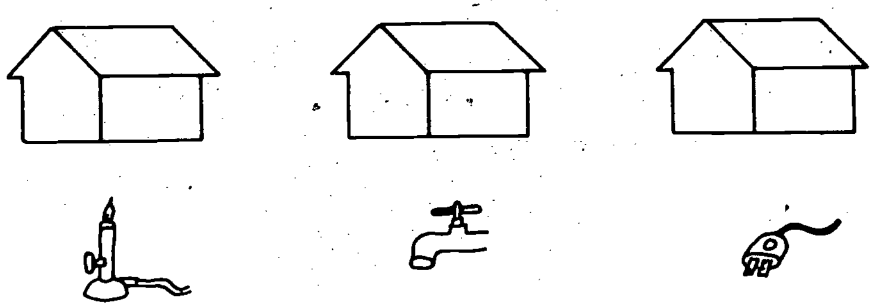
c) Use the information of parts (a) and (b) to answer the same questions for six teams.

d) Find a schedule for a tournament with seven teams.

You may find digraphs helpful in solving this problem.

2. Three houses are to be connected to gas, water, and electric utilities. The pipes and wires can go in any direction from the houses to the utilities, but they must not cross under, over, or through one another.

a) Can the connections be made?



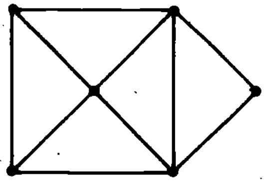
b) Suppose that the houses and utilities were located on the surface of a sphere. Could the connections be made then?

c) Suppose that the houses and utilities were located on the surface of a torus (doughnut). Could the connections be made then? Try it with a doughnut (unglazed).

d) Consider each of the three situations above in the case of four houses with three utilities and in the case of four houses with four utilities.

(You may find it helpful to draw networks. Note that each of the houses and the utilities can be represented by letters or numbers.)

3. Which of the following networks can you copy without taking your pencil from the paper and without going over any line twice? It is not necessary to finish where you start.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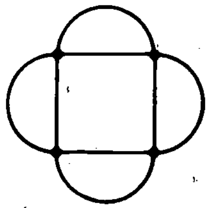
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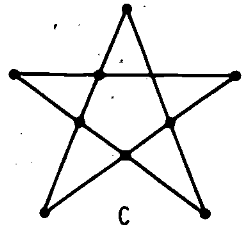
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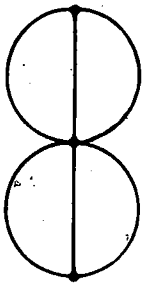
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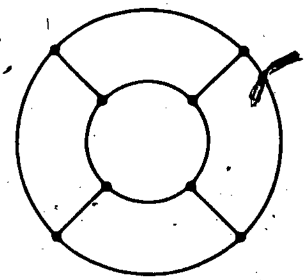
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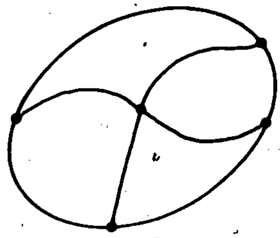
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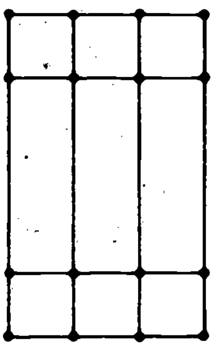
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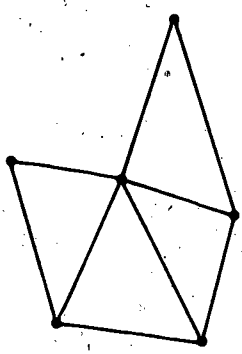
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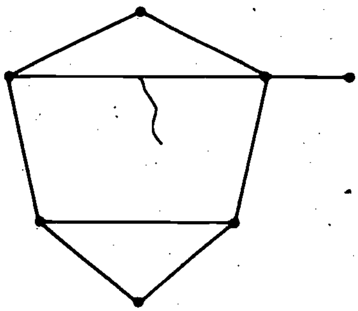
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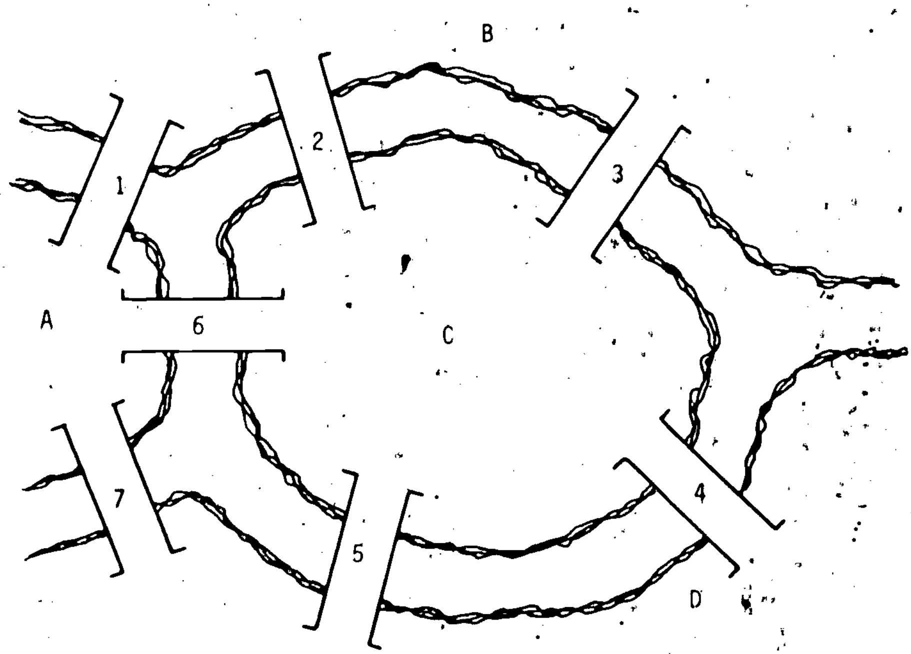
4. A network is said to be <u>Eulerian</u> if it is possible to trace a path, without lifting the pencil from the paper, which begins and ends at the same point and includes each line exactly once. By examining the above networks, try to find a condition under which a network is Eulerian. (<u>Hint</u>: Count the number of lines

meeting at each point and classify the points as to whether there are an odd or even number of lines connecting to them. Make a table listing the number of odd points and even points for each of the above networks.) The original problem which led to interest in questions of this sort is the following.

THE KÖNIGSBERG BRIDGE PROBLEM

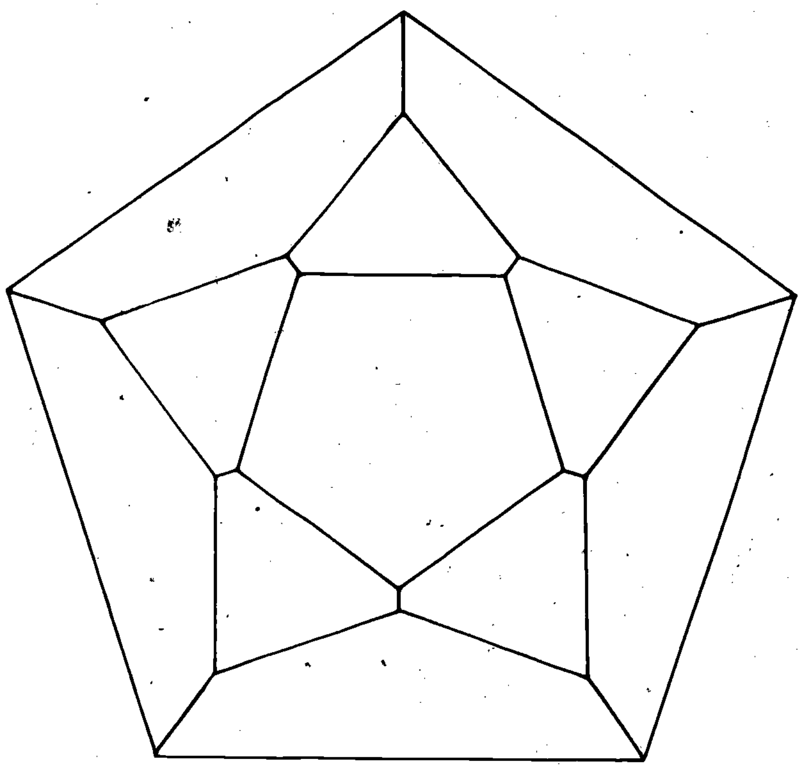
The city of Königsberg in East Prussia was located on the banks and on two islands of the river Pregel. The parts of the city were connected by seven bridges, as shown. On Sundays the citizens would



take a promenade around town. Is it possible to plan a promenade in such a manner that, starting from home, one can return there after having crossed each bridge once and only once?

This problem remained unsolved until the famous Swiss mathematician, Leonhard Euler, solved it using networks of the kind described above.

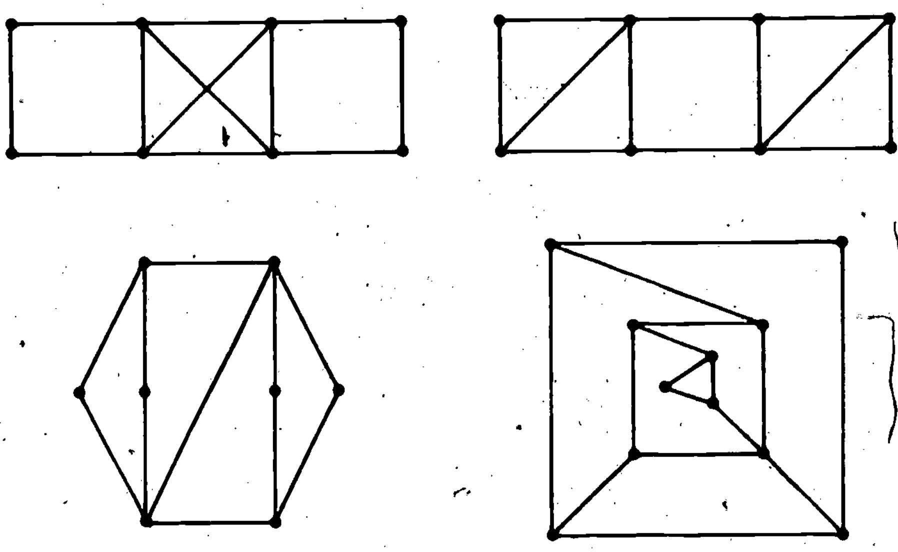
5. Sir William Hamilton invented a game which he descriptively entitled "Around the World." The game uses a regular solid dodecahedron each of whose vertices is identified with the name of a famous city. The object of the game is to travel "Around the World" by finding a path along the edges which begins and ends at the same city and passes through each city exactly once. The game was something less than a financial bonanza.



The illustration above is a planar representation of Hamilton's Traveller's Dodecahedron. (A dodecahedron is a three-dimensional figure with twelve pentagonally shaped faces. It has twenty vertices and thirty edges connecting these vertices.) The twenty corners of this planar figure correspond to the twenty vertices of the dodecahedron, and the thirty edges to the thirty edges. If we consider the region outside of the planar figure as being one face, our planar figure has a total of twelve faces, which correspond to those of the dodecahedron. Each of the twenty corners of the diagram represents a city to be visited, while the segments joining the corners represent roads connecting the cities.

a) Is it possible to plan the route for a complete trip around the cities, visiting each city exactly once, and returning in the end to the original home city? Travel along any road desired, but no more than once on each road. Ignore any roads not needed.

b) A network is said to be Hamiltonian if it is possible to trace a path which begins and ends at the same point and passes through each point exactly once. Note that one does not need to cover each line. Which of the following four networks are Hamiltonian?



6. Is it always true that in a group of six people there are three mutual acquaintances or three mutual nonacquaintances? Why?

7. Once you have completed your work on problems (1) through (6), discuss questions like the following with your classmates in a seminar setting.

   a) The Königsberg Bridge problem is a classical application of an Eulerian graph. What is a sufficient condition for a graph to be Eulerian? Can you identify some other applications of this idea?

   b) What interesting insights or ideas came to you while working on these problems?

   c) Give three real-world situations in which the question of whether a graph is Hamiltonian has business, economic, or social significance.

HISTORICAL HIGHLIGHT

Leonhard Euler (1707-1783), solver of the famous seven bridges of Königsburg problem, was the most prolific mathematician of all times. His writings, if collected, would fill over 80 large volumes--quite a feat, considering he was totally blind for his last seventeen years.

ACTIVITY 10

RELATIONS AS SETS OF ORDERED PAIRS

FOCUS:

Ordered pairs have many uses in mathematics, including locating points on a coordinate system, representing rational numbers, and representing relations. In this activity, the latter use will be investigated as an alternative to digraphs and networks for representing relations.

MATERIALS:

<u>Graphs and the Child</u> by Frédérique and Papy (Montreal: Algonquin Publishing Co., 1970).

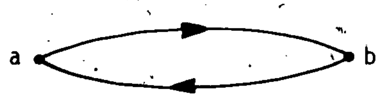
DISCUSSION:

The concept "<u>a</u> is related to <u>b</u>" can just as naturally be represented by the ordered pair notation (a,b) as by the directed graph $a \longrightarrow b$. One must remember that the order of the symbols within the parentheses is important and that in general $(a,b) \neq (b,a)$.

In fact, for future use in the unit we will consider a relation and a set of ordered pairs to be the same. The information contained in any directed graph may be expressed in a set of ordered pairs.

EXAMPLE

The information contained in the directed graph

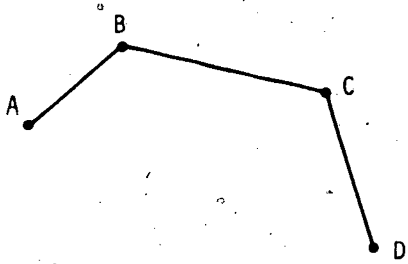


is the same as the information contained in the set of ordered pairs

$$\{(a,b), \ (b,a)\}$$

The braces { } indicate that the objects (a,b) and (b,a) are being considered as elements in the same set.

The network



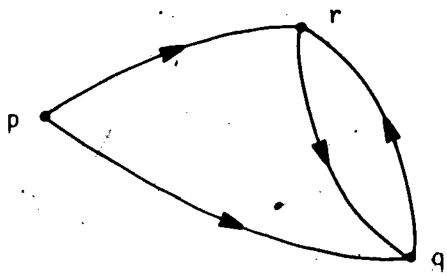
and the set

$\{(A,B,), (B,A), (B,C), (C,B), (C,D), (D,C)\}$

both represent the same relation.

DIRECTIONS:

1. a) Express the information contained in the following directed graph as a set of ordered pairs.



b) Convert the three directed graphs in Activity 8, Exercise 4(b), into sets of ordered pairs.

c) On the language tree of Activity 8, take the small part beginning with "Celtic" and convert it into a set of ordered pairs.

<u>Note</u>: Sets of ordered pairs can also be converted into directed graphs. For example, just think of all of the previous examples in reverse.

d) Convert the following relation into a directed graph.

$\{(1,2),\ (2,1),\ (2,3),\ (2,4),\ (3,1),\ (1,3)\}$

2. Discuss answers to the following questions:

a) For which kinds of relations do ordered pairs seem the most appropriate representation?

b) Given the relation "is the double of," how could you express it for <u>all</u> integers using directed graphs? How about using ordered pairs?

c) Can every set of ordered pairs be expressed as a directed graph? (For further insight into this question, you may want to glance at Exercise 3.)

3. Since every set of ordered pairs is a relation, the set $\{(a,a)\}$ must be a relation. Using the representation from Activity 9, we might represent it in this way:

a

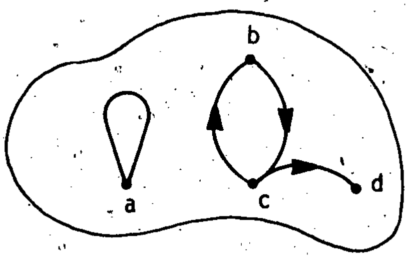
The arrowhead does not convey any additional information in this case so we write:

a

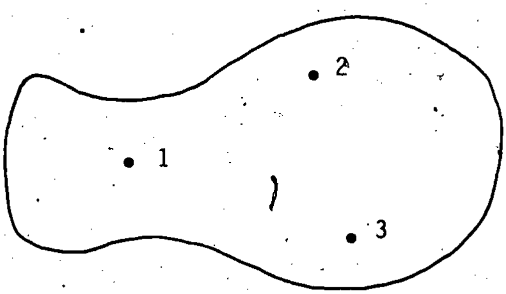
We have not given a technical definition of "digraph." However most technical definitions of digraphs do not allow such "loops," so we introduce the broader concept of a <u>Papygram</u> (after the Belgian mathematician and educator, Georges Papy, and his educator wife, Frédérique). Papygrams are like digraphs except that they do allow loops. Can you now see that any finite set of ordered pairs has a Papygram and conversely?

EXAMPLE

The relation $\{(a,a),\ (b,c),\ (c,b),\ (c,d)\}$ has the Papygram:

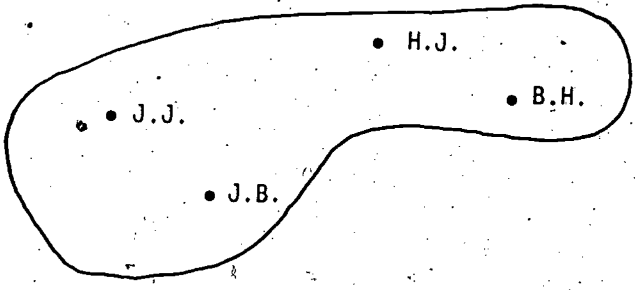


a) In the set below, draw the Papygram representing the relation "is less than or equal to."



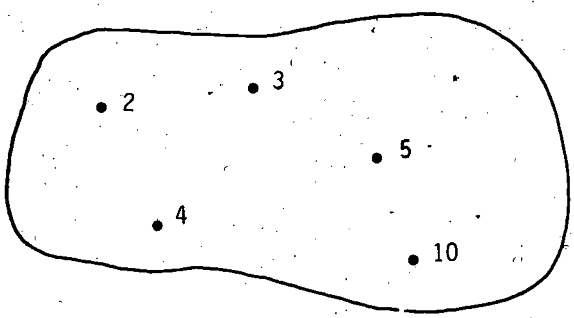
Also represent this relation as a set of ordered pairs.

b) In the set of people's initials below, draw a Papygram of the relation "has the same first initial as the last initial of."



Also represent this relation as a set of ordered pairs.

c) In the set below, draw a Papygram of the relation, "divides evenly into."



Also express the relation as a set of ordered pairs.

d) Repeat exercise 2(a) with the relation "is not greater than." What do you conclude from this exercise?

4. a) To gain insights into how the Papys use Papygrams with children, you should look through Graphs and the Child by Frédérique and Papy (Montreal: Algonquin Publishing Co., 1970).

   b) Outline a Papygram activity that you could do with children. Be sure to plan it for a specific grade level and to choose examples that would be particularly meaningful to children at that level. Also be sure to state your objectives for the lesson.

ACTIVITY 11

EQUIVALENCE RELATIONS

FOCUS:

You have been picturing and analyzing various rather general relations. In this activity you will investigate a particular kind of relation called an <u>equivalence relation</u> and you will identify the particular properties which characterize equivalence relations.*

DISCUSSION:

Simply stated, equivalence relations are those relations which are like the relation "equals." Precisely, we note the following three properties of "equals."

i. $a = a$ for any a.

ii. If $a = b$, then $b = a$.

iii. If $a = b$ and $b = c$, then $a = c$.

Property (ii) is just the <u>symmetry</u> property for relations that we discussed in Activity 8. It says that if <u>a</u> is related to <u>b</u> by the "equals" relation, then <u>b</u> is related to <u>a</u>.

Property (i) is called the <u>reflexive</u> property. It says that each <u>a</u> is related to itself by the relation "equals."

Property (iii) is called the <u>transitivity</u> property and says that if <u>a</u> is related to <u>b</u> and <u>b</u> is related to <u>c</u>, then <u>a</u> must also be related to <u>c</u>.

You may wonder if there is any interesting relation besides "equals" which is an equivalence relation. Let's check the relation "is the same color as."

*Equivalence relations are also discussed in Section III of the <u>Number Theory</u> unit of the Mathematics-Methods Program.

i. Each object is the same color as itself.

ii. If one object is the same color as another, the other is the same color as the one.

iii. If one object is the same color as a second and a second is the same color as a third, then the first is the same color as the third.

Since "is the same color as" is (i) reflexive, (ii) symmetric, and (iii) transitive, it is an equivalence relation. Yet it certainly is different from "equals" since quite different objects can have the same color.

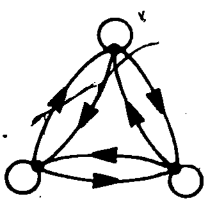
DIRECTIONS:

1. Below is a list of relations. Indicate those that are equivalence relations. For those that are not, show which of the properties (reflexive, symmetric, and transitive) they violate.

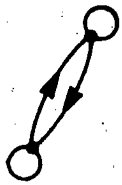
   - is less than
   - is less than or equal to
   - is a relative of
   - is a blood relative of
   - has the same prime factors as
   - has the same weight as
   - has the same teacher as
   - has different size than

2. Below are several Papygrams. Indicate which represent equivalence relations, and indicate why the ones that are not equivalence relations are not.

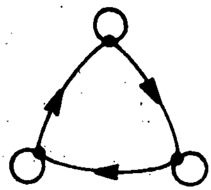
a)



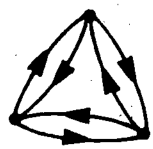
b)



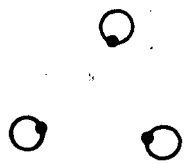
c)



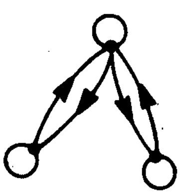
d)



e)



f)



3. Describe, in your own words, how you can tell by looking at the Papygram of a relation whether the relation is:

   a) reflexive

   b) symmetric

   c) transitive

4. One can say of an equivalence relation that it relates things that are the same in one respect (not necessarily in all respects).

   a) See if this description jibes with your results in (N).

   b) Give several instances where the word <u>equivalence</u> is used in everyday language. See if these uses jibe with the concept of equivalence relation.

5. Below are several sets of ordered pairs. Indicate those that represent equivalence relations. For those that do not, tell why not.

   a) $\{(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (1,3), (3,1), (3,3)\}$

   b) $\{(1,1), (1,2), (2,1), (2,2), (3,3)\}$

   c) $\{(1,1), (1,2), (2,3), (2,2), (3,1), (3,3)\}$

   d) $\{(1,2), (2,1), (2,3), (3,2), (3,1), (1,3)\}$

   e) $\{(1,1), (2,2), (3,3)\}$

   f) $\{(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3)\}$

6. Describe in your own words how you can tell, by looking at a set of ordered pairs, whether the relation which they represent is:

   a) reflexive

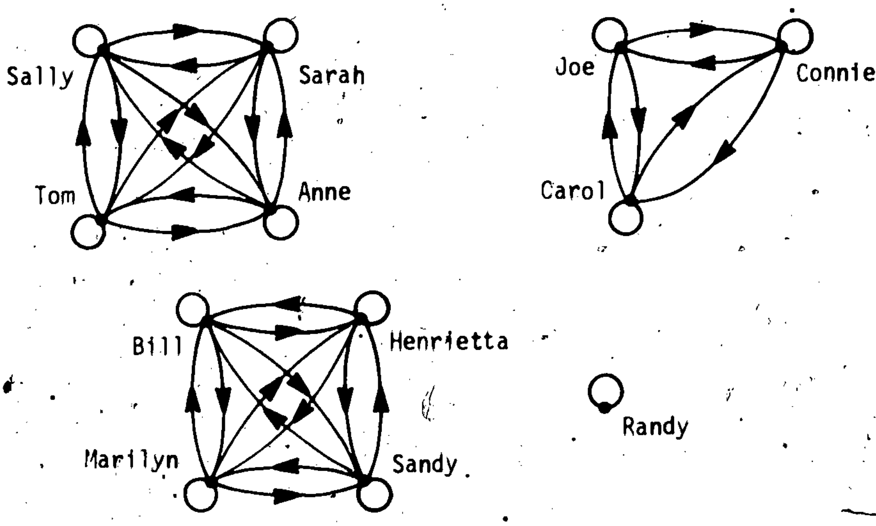
   b) symmetric

   c) transitive

7. Consider the relation "is the same age as" on the following set of people.

Name	Age
Sally	7
Joe	8
Bill	6
Tom	7
Sarah	7
Carol	8
Connie	8
Marilyn	6
Henrietta	6
Sandy	6
Anne	7
Randy	10

The Papygram of this relation looks like this:



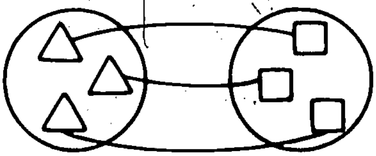
Notice that the people are separated into groups so that no arrows go between separate groups. These separate groups are called <u>equivalence classes</u>.

a) Draw a Papygram of the relation "has the same prime factors as" on the numbers 6, 12, 7, 14, 28, 36, and circle the separate equivalence classes.

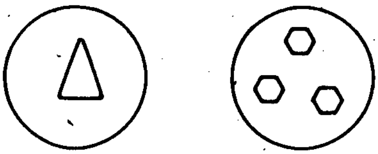
b) Verbally describe the equivalence classes of the relation "has the same color socks as" on the children in a classroom.

c) How many members are there in each equivalence class of the relation "equals"?

8. There is said to be a one-to-one correspondence between two sets if their elements can be paired or matched. For example,



there is a one-to-one correspondence between the two sets above, while there is not between the two sets below.



The relation defined between sets of objects by "there is a one-to-one correspondence between" is a very important one in early number work in the primary school.

a) Determine that this relation is an equivalence relation.

b) Describe the equivalence classes for this relation.

c) Some have advocated giving whole number names like "four" to the different equivalence classes of sets for the relation "there is a one-to-one correspondence between." Describe how that might be done. Do you think that there is any merit to doing it?

9. Attribute blocks have become fairly standard equipment in elementary classrooms.

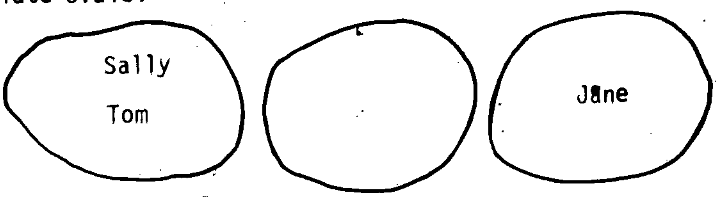
   a) Show that the same set of attribute blocks can be separated into different equivalence classes by different equivalence relations.

   b) Outline how you might introduce a child to equivalence relations using attribute blocks.

10. Children enjoy playing detective, and they can get the idea of equivalence classes from the following Guess My Relation detective game.

   i) The teacher thinks of an equivalence relation such as "has the same color hair as" and puts ovals on the board to represent the equivalence classes.

   ii) Then the teacher puts some names of students in the appropriate ovals.



   iii) Then the class guesses where additional names belong, with the teacher correcting the guesses. The first student who guesses the relation is "it" and gets to choose the next relation (probably in cahoots with the teacher).

   a) Play the Guess My Relation game with your classmates.

   b) Could the game be played with a nonequivalence relation?

ACTIVITY 12

SEMINAR ON DIGRAPHS, PAPYGRAMS AND NETWORKS IN THE CLASSROOM

FOCUS:

Digraphs, Papygrams, and networks are not part of the standard elementary curriculum. Some feel they should be; others fail to see that they have as high a priority as other content topics. In this activity you will consider what role they might have in work with children.

DISCUSSION:

The questions raised below could be the basis of a class discussion, a small group assignment to be done during class, or a homework assignment. It could also lead to a teaching assignment with children.

DIRECTIONS:

1. a) Familial relationships are often confusing to young children. Digraphs (in particular, tree diagrams) can be used to illustrate familial relationships. Make a digraph picturing the relationship "is a parent of" for your mother, father, aunts, uncles, cousins, and your grandparents.

   b) How would you work out such an activity with primary children? Outline the lesson. Magic markers on newsprint might yield a pleasing pattern.

2. List some relations (other than "is a parent of") that might be interesting and instructive to develop with children.

3. The Papys have made Papygrams an important feature of an entire elementary curriculum. Look in the following references to determine:

   - how they do it
   - what the advantages are in doing it

- what the disadvantages are in doing it
- how you feel about doing it

Frédérique and Papy. Graphs and the Child. Montreal: Algonquin Publishing, 1970.

Frédérique. Mathematics and the Child 1. New York: Cuisenaire Co. of America, 1971.

Section IV

PICTURING FUNCTIONS

The concept of a function is one of the most basic ones in all of mathematics. Its usefulness in everyday life is evidenced by expressions such as "gasoline mileage is a function of speed" and "the income tax you pay is a function of your income." In mathematics it serves as an organizer, which provides a convenient framework in which to view several mathematical ideas. Both uses will be illustrated in this section of the Graphs unit, and there will be occasional references to functions in other units of the Mathematics-Methods Program.

Activity 13 introduces functions as "input-output" systems and connects functions with the relations discussed in Section III. Several examples of the occurrence of functions in everyday situations are provided. The ways in which functions can be represented are described and compared. The activity concludes with a consideration of the distinction between functions and other relations.

The relationships that arise in many activities and experiments can be discussed in terms of functions. Six examples that illustrate this are contained in Activity 14. Experiments are to be performed, and the resulting data is to be viewed as defining a function. Representation questions are considered.

The function concept as a mathematical idea is developed further in Activity 15.

Certain kinds of functions are sufficiently important to merit their own names. A few of these are introduced in Activity 16. Also, the connection between functions and ratio/proportion is explored in this activity.

The last activity of the section is a seminar whose focus is on the ways in which functions enter into the elementary curriculum.

MAJOR QUESTIONS

1. The notion of a function is frequently referred to as one of the basic concepts of mathematics. Give three examples, other than those disucssed in this section, of instances in which mathematical topics can be presented using functions.
2. One can teach specific facts or general principles. Identify two strengths and two shortcomings of teaching a few general ideas versus teaching many specific facts. Give examples involving functions to illustrate your points.
3. List two advantages and two disadvantages of introducing functions notation and terminology in the elementary school.

ACTIVITY 13

FUNCTIONS AS DESCRIPTIONS OF INPUT-OUTPUT SYSTEMS

FOCUS:

The concept of a function will be developed as a means of describing situations in which an "input" yields a specific "output." Several methods of representing functions will be introduced. The activity has three parts:

Part A--The Function Concept,

Part B--Representations of Functions,

Part C--Functions as Special Kinds of Relations

Part A: The Function Concept

DISCUSSION:

It is common for certain types of athletic competition, e.g., swimming and track and field, to be organized so that children compete with others of approximately the same age. One might have age classifications as follows:

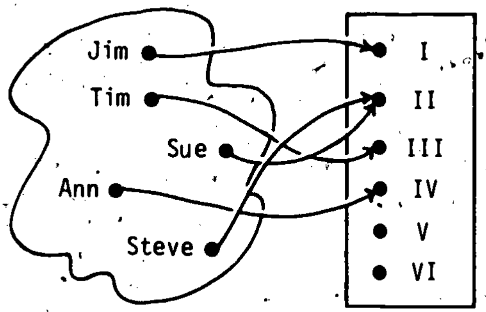
Age	Group
7-8	I
9-10	II
11-12	III
13-14	IV
15-16	V
over 16	VI

Using this table, we can assign each child (or input) exactly one group (or output) for competitive purposes. Participants in a summer track program might be divided up in this way.

For the purpose of generalization it is helpful to view this assignment as a correspondence which assigns a group to each child in a set. (The set might be a class or the participants in some athletic program.)

$$\text{child} \longrightarrow \text{group}$$

A specific instance might be represented as



One way of viewing a function is as

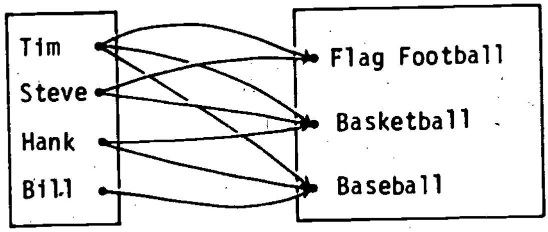
<u>a correspondence which assigns to each element of one set a unique element of another set.</u>

Other ways of viewing functions will be developed as we proceed.

It is useful to compare this example of a function with an example of a correspondence which is not a function. To this end, suppose there are four boys in a class who participate in athletics and who belong to teams as shown below

Tim	flag football, basketball, baseball
Steve	flag football, basketball
Hank	basketball, baseball
Bill	baseball

This correspondence of boys to the sports in which they participate can be represented as



It is clear that the correspondence

$$\text{Boy} \longrightarrow \text{Sport}$$

does not define a function since to each input (boy) there does not correspond a unique output (sport). The uniqueness of assignments is required for a correspondence to be a function.

DIRECTIONS:

1. Make a table showing the correspondence that assigns to each counting number between 2 and 12 its smallest factor different from 1. Does this correspondence define a function?

2. Make a table showing the correspondence that assigns to each counting number no larger than 10 a pair of whole numbers whose sum is that number. Does this correspondence define a function? (Does an input always result in the same output?)

3. Which of the following correspondences are functions? Why or why not?

   a) The correspondence that assigns to each school day the number of students in attendance at a certain elementary school;

   b) The correspondence that assigns to each student in a class his/her age;

   c) The correspondence that assigns to each letter a sufficient amount of postage;

   d) The correspondence that assigns to each counting number one of its factors;

e) The correspondence that assigns to each counting number 0 if it is even and 1 if it is odd;

f) The correspondence that assigns to each book on a shelf the number of pages it contains;

g) The correspondence that assigns to each boy in a class his favorite sport;

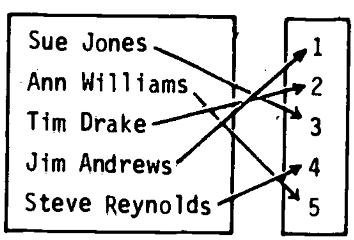
h) The correspondence that assigns to each pair of counting numbers their sum.

Part B: Representations of Functions

DISCUSSION:

There are many ways to represent function or input-output relationships; we identify five of them.

Mapping diagram. Two examples of this are given in Part A above. Another example is the association that assigns to a group of five students their place in an alphabetical list of their last names:



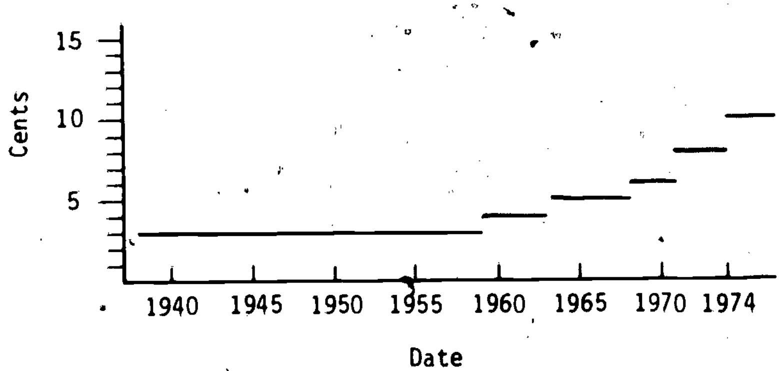
Tables or Sets of Ordered Pairs. The association that assigns to each day of a specific week the number of traffic accidents reported to the police in a certain city, can easily be represented in a table.

Day	Number of Accidents
Monday	12
Tuesday	15
Wednesday	27
Thursday	18
Friday	41
Saturday	53
Sunday	37

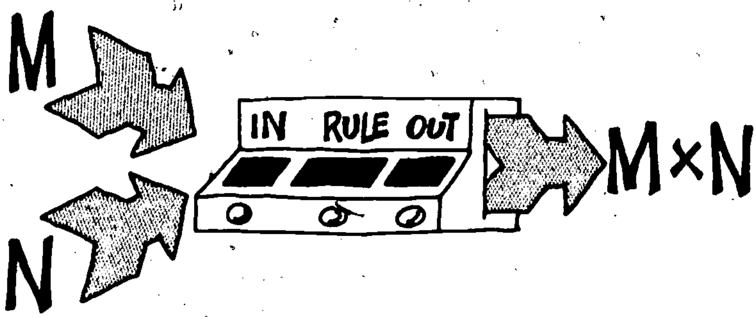
Another method of conveying the same information is through a set of ordered pairs. In each ordered pair the first element is the input (day of the week in this example) and the second element is the output (number of reported accidents).

$\{(M,12), (Tu,15), (W,27), (Th,18), (F,41), (Sa,53), (Su,37)\}$

Graphs on Cartesian Coordinate Systems. The association that assigns to each date from January 1940 to December 1974 the cost of mailing a one-ounce first-class letter within the United States, can be conveniently represented as:



Function Machines. The association that assigns to each pair of counting numbers their product can be pictured as:



Such representations are particularly common in elementary school textbooks, although they also appear elsewhere.

Formulas. We can represent the association that assigns to each counting number its double by

$$n \longrightarrow 2n,$$

and the association which assigns to each rectangle its area by

$$\text{rectangle} \longrightarrow \text{length} \times \text{width}.$$

DIRECTIONS:

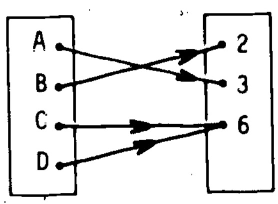
1. Which representation do you believe is especially appropriate for each of the functions of exercise (3) of Part A? Show the representation you have selected for four parts of (3).
2. Find examples of functions represented in each of these five ways in magazines or newspapers less than a month old (one example each). In each case explicitly identify the association that is represented.
3. Comment on the advantages and deficiencies of each representation.

Part C: Functions as Special Kinds of Relations

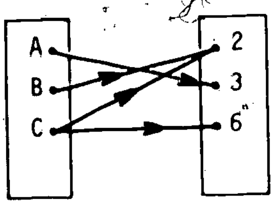
DISCUSSION:

A relation, like a function, can be represented in several ways. One representation is as a set of ordered pairs (see Activity 10). Recall that (a,b) means that "a is related to b." If we adopt the terminology of inputs and outputs, we might interpret this as meaning that input a results in output b. If a relation is such that to each first element (or input) there corresponds <u>exactly one</u> second element (or output), then the relation is a function.

The relation {(A,3), (B,2), (C,6), (D,6)} is a function, while the relation {(A,3), (B,2), (C,2), (C,6)} is not a function. The latter is not a function since both (C,2) and (C,6) belong to the relation. That is, C is a first element to which there corresponds more than one second element. The same test serves to determine whether a relation is a function when the representation of the relation is other than as a set of ordered pairs. For example, a relation is defined by each of Figures I and II below.

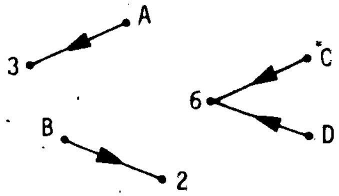


I

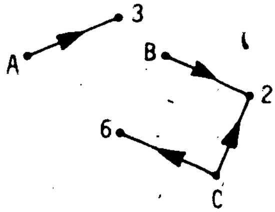


II

Relation I is a function while relation II is not. Relation I can also be represented as



where the arrows originate with inputs and terminate on outputs. Similarly Relation II can be represented as



Using this arrow representation, one can distinguish a function from a relation that is not a function by checking whether there is any input at which <u>two or more arrows originate</u>. If there is such an input, then the relation is not a function.

DIRECTIONS:

1. Determine which of the relations defined below are functions.
   a) The relation is defined on the set of counting numbers and "<u>a</u> is related to <u>b</u>" means that <u>b</u> is twice <u>a</u>.
   b) The relation is defined on the set of counting numbers and "<u>a</u> is related to <u>b</u>" means that <u>a</u> and <u>b</u> have a common factor.
   c) "<u>a</u> is related to <u>b</u>" means that date <u>b</u> is the birthday of student <u>a</u> from your class.
   d) "<u>a</u> is related to <u>b</u>" means that student <u>b</u> in your class has his/her birthday on date <u>a</u>.
2. A teacher refers to a function as a "reliable relation." Is this a reasonable term? In what sense?

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ACTIVITY 14

IDENTIFICATION OF FUNCTIONS THAT ARISE IN EXPERIMENTS

FOCUS:

Many activities, especially experiments and games, which are a standard part of the elementary curriculum, may be conveniently discussed in terms of functions. The relevant functions may not be at all obvious, and there may be several that include information about different aspects of the activity.

MATERIALS:

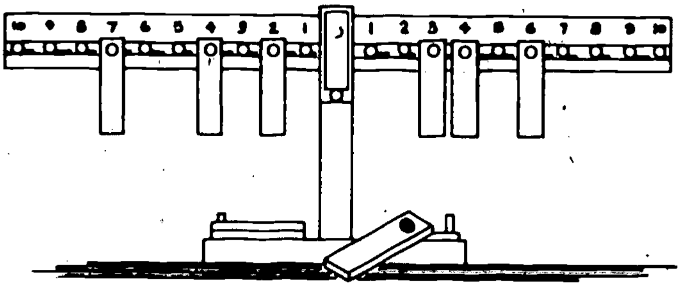
Balance beam, Cuisenaire rods, hinged mirror, graph paper, paper towels, metric ruler, jar, food coloring (optional), paper clip, scotch tape, guitar, protractor.

DIRECTIONS:

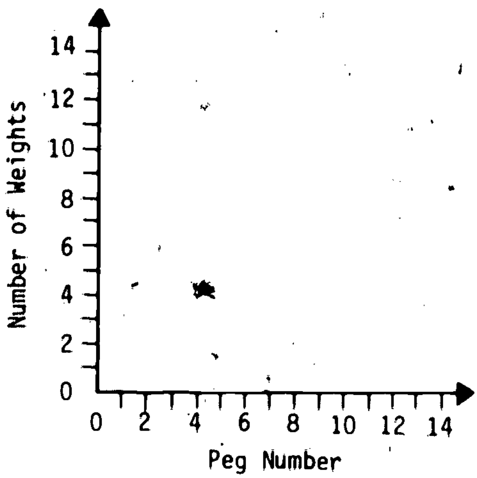
Six experiments are described below, and you will be asked to do a number of them. Before doing an experiment, read the questions that follow it. Then answer the questions as you proceed, or after you complete the experiment. It is important for you to keep the focus of the activity--the function concept--in mind. The experiments are interesting and it is easy to "miss the forest for the trees." Since not every student will have an opportunity to complete every experiment, there will be a wrap-up discussion and you should be prepared to summarize your results.

Experiment 1: Balance Beam

Place two weights on the "6" peg on the lefthand side of the balance beam. The experiment consists of finding out how many weights you need to place on the various pegs on the righthand side to make the beam balance. Of course you can easily guess that you can make the beam balance by placing two weights on the righthand "6" peg. But how many weights would you need to put on, say, the "1" peg to make the beam balance? Complete the table below and graph your data as you go along.



Peg Number	Number of Weights Needed for Balancing
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	



a. If you had a peg located $1\frac{1}{2}$ units to the right of the center of the balance, can you estimate (perhaps with the help of your graph) how many weights you would need to attach to that peg in order to balance the two weights hooked on the left "6" peg?

b) If your balance had a peg located 12 units to the right of center, how many weights would you need to put on that peg to make the beam balance?

c) Your present graph consists of isolated points. In what way would it make sense (i.e., be meaningful) to connect the dots of the graph with a smooth curve? Discuss whether the graph between two data points might really look like this:

rather than being a smooth curve like this:

d) Your graph shows a definite pattern. Have you been able to detect the numerical relationship that is behind the pattern? To balance the two weights that are located six units to the left of the center of the balance, how does the weight needed for balancing depend on the distance you are to the right of center? If W stands for the number of weights needed and D stands for the distance to the right of center, write an equation which expresses the relationship between W and D.

e) Finally double-check your equation and its graph by making some gross comparisons. If D becomes very large, what does your equation imply must happen to W? Does your graph show this? If D comes very close to zero, what does your equation imply must happen to W? Does your graph show this? Discuss whether your graph should eventually cross the vertical axis. What would crossing the vertical axis mean in terms of your experiment?

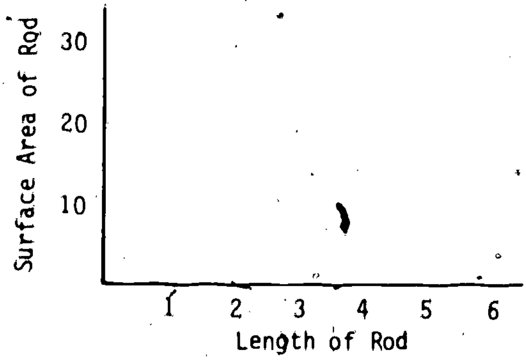
Experiment 2: Surface Area

Take one Cuisenaire rod of each of the following six colors: plain, red, light green, purple, yellow, and dark green.

Using the area of one face of the plain-colored cube as a unit, measure the surface area of each rod. For example, the cube itself has a surface area of 6 units, while the red rod has a surface area of 10 units. (If you wish, you can think of using the cube as a sort of ink stamp--in which case exactly 10 ink blots would cover the red rod.)

a) Complete the table and graph below.

Length of Rod		Surface Area of Rod
Plain	1	
Red	2	
Lt. Green	3	
Purple	4	
Yellow	5	
Dk. Green	6	



b) Discuss whether you see a pattern in your graph and whether you can obtain a precise equation relating the surface area of a rod to its length.

c) If you had a rod that was $1\frac{1}{2}$ units long (that is, halfway between the plain rod and the red rod), what would its surface area be? Does this follow the pattern observed in (b) above?

d) Would it make sense to draw a graph like the one above but with all the points connected by a smooth curve? Can you be sure that the curve between two data points could not be something like the following?

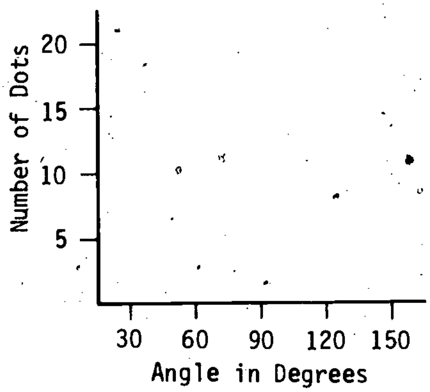
Would it be reasonable to include the point (0,2) on your graph? What would be the physical meaning of the point?

e. Notice that the horizontal and vertical scales on the graph are different. How does the graph change as you vary the vertical scale while keeping the horizontal scale fixed?

Experiment 3: Hinged Mirrors

a) Place your hinged mirrors on edge on the 100° angle shown on page 102. Look directly into the mirrors and count the number of black dots you see (including the original one). Record this number in the table below and plot the corresponding ordered pair. Now complete the table and plot the tabular values.

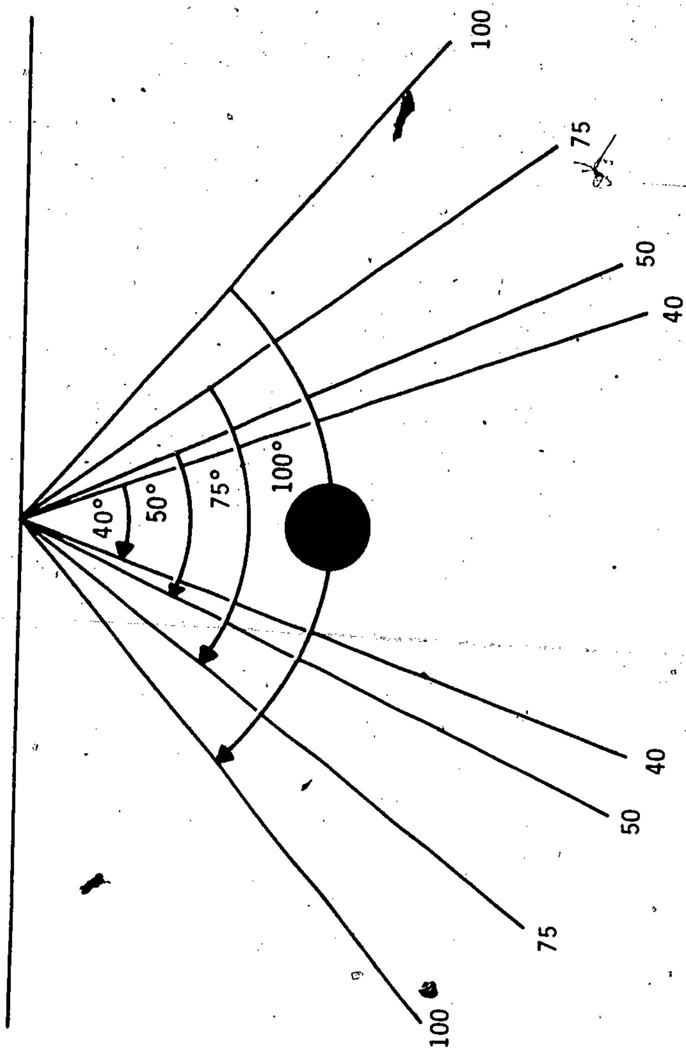
Angle	Number of Dots
100°	
75°	
50°	
40°	



b) Would it be reasonable to connect the data points with a smooth curve? What would the curve mean in terms of the experiment?

c) Complete the graph as best you can by using estimates of angles or by using a protractor.

d) Can you find a formula to describe the relation between the number of dots and the angle size?



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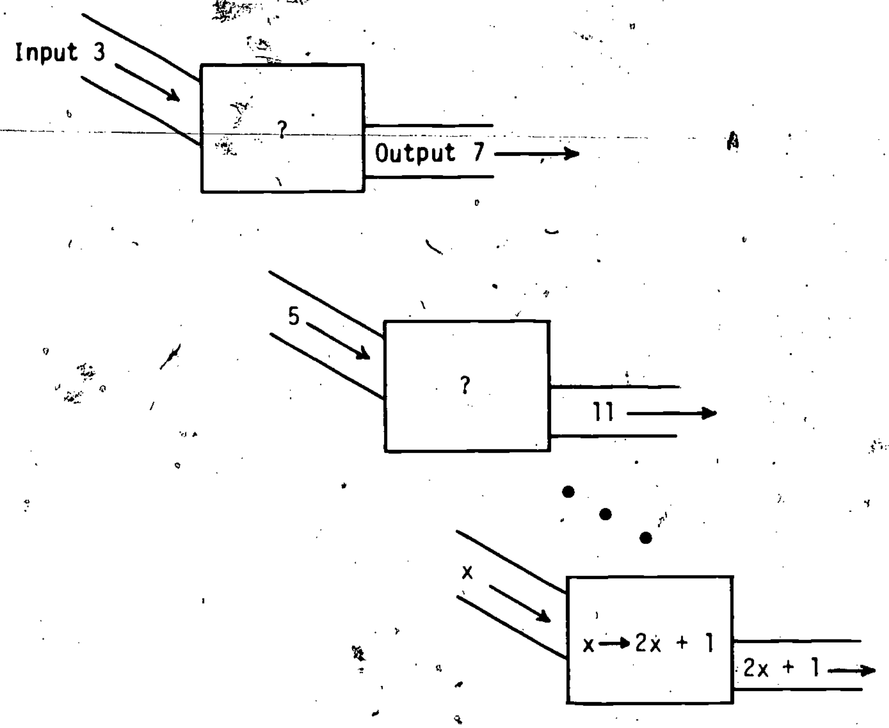
Experiment 4: The Function Machine

One way of presenting and representing a function that is particularly appropriate for elementary school students is by means of a function machine. Using such activities as a "Guess My Rule" game, the function machine can be fun and effective with children.

What's My Rule



a) Each player in your group thinks up a rule for the others to guess. In turn the rule-maker (machine) supplies outputs for inputs which are supplied by the guessers until they have guessed the rule. As the guessers are guessing, they should record inputs and outputs by means of either a table or a mapping diagram. For example,

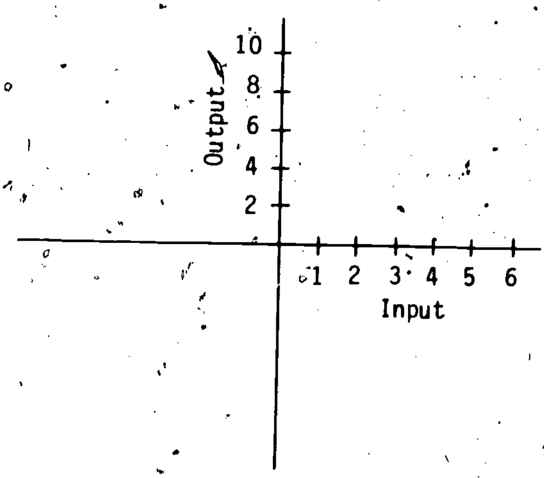
Input	Output
3	7
5	11
.	.
.	.
.	.
x	2x + 1

OR

$3 \longrightarrow 7$
$5 \longrightarrow 11$

$x \longrightarrow 2x + 1$

Once the rule has been guessed, plot the inputs versus the outputs on a Cartesian coordinate graph like the following:



If you decide to keep score in some way, you might give points for being able to guess what a specific output will be even if you can't guess the exact rule. Also, it might be appropriate to take off points for incorrect guesses.

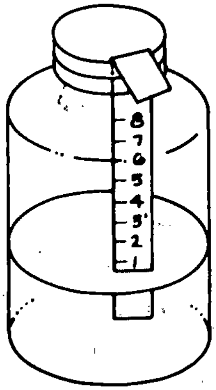
b) Function machines do appear in many current elementary mathematics texts. List several skills that could be developed using function machines.

Experiment 5: The Wick Experiment*

From a paper towel cut out a strip one inch wide by about eight to ten inches long. Draw a pencil line across the strip two or three centimeters from one end. Draw pencil lines one centimeter apart starting from this line. About ten lines will be sufficient. These lines should be numbered (also in pencil) with the first line starting at zero. Some care is needed in drawing and writing on the paper towel since it will tear quite easily.

Put some water in a jar. The experiment is easier if the water is colored with food coloring or a similar dye. Fasten a paper clip to the lower end of the paper strip in the jar so that the "zero" line is exactly even with the surface of the water. Fasten it at this height with scotch tape. On the record sheet, note the time this is done opposite the number zero. It is most convenient to do this exactly at an even minute to simplify later computations.

Record the time at which the water rises to each line. The water level is seen most easily if the jar is placed against a dark background with the light coming from behind the viewer. The time should be recorded to the nearest quarter of a minute. The water usually does not rise exactly evenly and there is often some uncertainty about the exact time when the water reaches a given line. One way of judging this is to record the time of the quarter-minute immediately before the first detectable water appears about the line (for this reason, it would be best to place the numbers below the lines).

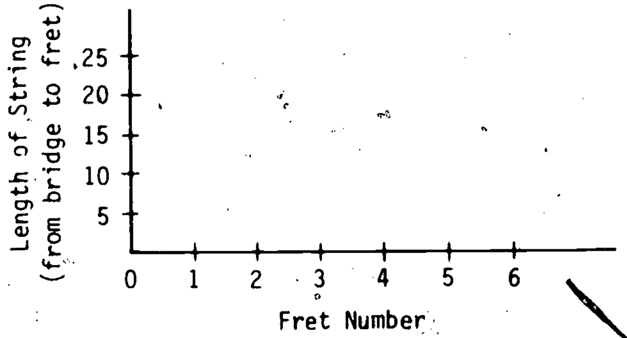


After all the data has been recorded, another column should be computed, showing the time from the start of the experiment required for the water to rise to the given level. A graph should then be prepared from this data.

*Taken from S.M.S.G. Mathematics Through Science, Part I: Measurement and Graphing, Teacher's Commentary (Pasadena, California: A. C. Vroman, Inc., 1963 and 1964), pp. 65-66.

Experiment 6: Guitars and Graphing

Take a guitar and record data relating the length of the vibrating portion of a string to the fret you are using. More specifically, number the frets from the bridge end of the guitar to the tuning end; you can start your numbering with either 0 or 1. Then choose a string and measure the length of that string from the bridge to each of the numbered frets. Thus, your graph will be on a grid like the following:



Try to decide what equation corresponds to the graph you have plotted. If you are musically inclined, try to determine why you obtain the graph and equation that you do. If you are mechanically inclined, you might try making your own stringed instrument by using the data you have obtained.

ACTIVITY 15

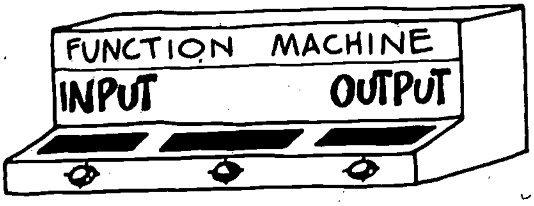
A CLOSER LOOK AT FUNCTIONS

FOCUS:

The development of the function concept is continued with the introduction of terminology and notation that will aid in describing functions.

DISCUSSION:

In the preceding activities of this section we have used the terminology of inputs and outputs. These words are especially appropriate when one visualizes a function as a machine, a common representation in elementary school textbooks. You should also be familiar with some other terms that are widely used in connection with functions.



The domain of definition of a function is the set of all possible inputs. If we view the function as a set of ordered pairs, then it is the set of all possible first elements in the ordered pairs.

The range of a function is the set of all possible outputs or, when the function is viewed as a set of ordered pairs, the set of all possible second elements.

EXAMPLES

- The function that associates with each student in a class his/her birthday, has as domain the set of students in the class and as range the set of dates that are birthdays for at least one student in the class. In the remainder of this unit we will use the letter B to denote this birthday function.

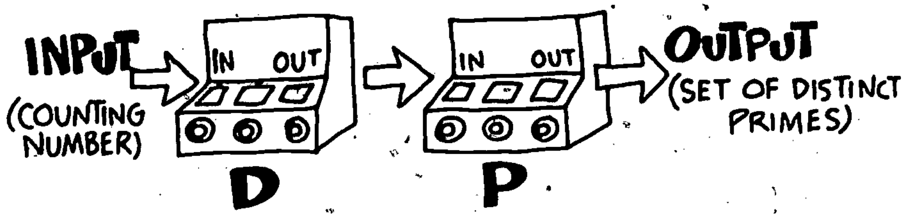
- The function that associates with each counting number its double has domain $\{1, 2, 3, \ldots\}$ and range $\{2, 4, 6, \ldots\}$. In the future we will refer to this function by D (for <u>doubler</u>).

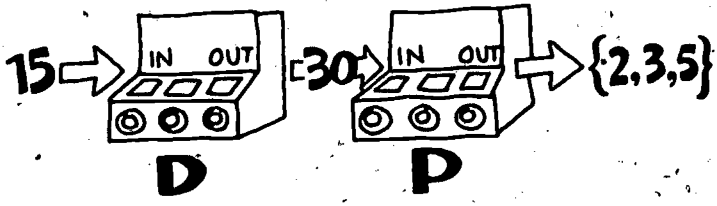
- The function that associates with each counting number the set of its distinct prime factors has domain $\{1, 2, 3, \ldots\}$, and its range is the set of all sets of distinct primes, including the empty set, i.e., the set of elements of the form $\emptyset$, $\{2\}$, $\{2, 3\}$, $\{2, 3, 5\}$, $\{5\}$, $\{5, 7\}$, ... We will refer to this function as P.

By definition, a function associates <u>exactly one</u> output with each input. If, in addition, each output is associated with <u>exactly one</u> input, then the function is said to be <u>one-to-one</u>. The doubling function D is a one-to-one function. The function P is not one-to-one. For example, 6 and 12 have the same output, $\{2, 3\}$. The function B may or may not be one-to-one depending on the class. It will be one-to-one if no two students have the same birthday; otherwise it is not one-to-one.

When actual machines are used in the manufacture of a product, it is frequently the case that there are several machines that process the product sequentially. That is, the output of one machine becomes the input for another. If we continue the analogy between functions and machines, then it is natural to consider those situations in which the output of one function provides the input for another. Of course, this is a restriction on the functions and it is not true, in general, that the output of one function is suitable as input for another. The technical requirement is that the set of outputs of the first machine must be contained in the set of possible inputs of the second.

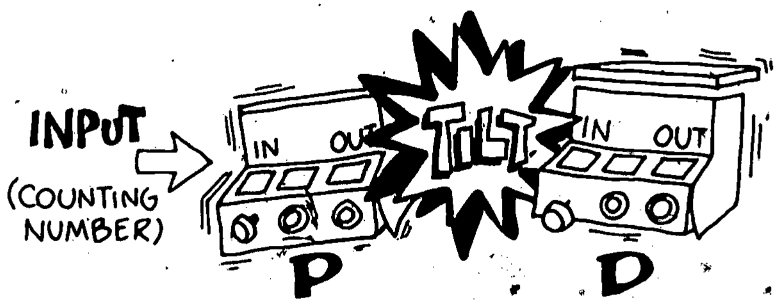
The function D is defined on the set of counting numbers and its sets of outputs (or range)--the set of even numbers--is contained in the domain of the function P. Therefore we can imagine a new function (or machine) consisting of D followed by P.





This new function will be denoted by P • D (read P composed with D). Notice the order; it is important that the function that operates first be written on the right.

Notice that the combination of the functions P and D, with the output of P considered as potential input for D, makes no sense. There are outputs of P --e.g., output {2, 3}, which results from input 12--that are not suitable as inputs for D.



Finally, we introduce the customary notation for functions. This notation is widely employed and provides a convenient shorthand for expressing functional relationships. For example, if D is the doubling function introduced above, then the output that results from input 3 is denoted by D(3), read "D of 3." Thus D(3) = 6. Pursuing the example of the function D a bit further, the output resulting from the input of a counting number n is written as $D(n)$. Thus $D(n) = 2n$. We often speak of applying the function D to the input n.

EXAMPLES

$D(12) = 24$

$P(3) = \{3\}$, $P(18) = \{2, 3\}$

$(P \cdot D)(9) = \{2, 3\}$

DIRECTIONS:

1. If we view a function as a set of ordered pairs, some of the following sets define functions and some do not. Identify those that do, and specify the domain and range in each of these cases.

   a) (3,6), (4,6), (5,6), (6,6), (7,10), (8,11)

   b) (3,-1), (4,-2), (5,-3), (6,-4)

   c) (3,0), (4,3), (5,6), (3,3), (2,-3), (1,-6)

   d) (a,e), (b,c), (e,c), (c,a)

2. For which functions f of exercise 1 is $f \cdot f$ defined? In each case in which $f \cdot f$ is defined, write out the set of ordered pairs corresponding to the function $f \cdot f$.

3. Give an example of two functions, f and g, for which $f \cdot g$ and $g \cdot f$ both make sense but are not the same function. That is, either the domains of definition are different, or there is an input that is assigned different outputs by $f \cdot g$ and $g \cdot f$.

Exercises 4 through 7 use the functions A, S, M, defined below.

- A is the function that associates with each counting number the sum of that number and 5.
- S is the function that associates with each integer the integer that is 3 less.
- M is the function that associates with each positive rational number $\frac{p}{q}$ (p and q counting numbers) the rational number $\frac{p^2}{q^2}$.

4. What is the range of the function A? of the function S?

   a) Does the composition $A \circ A$ make sense?

   b) If so, what is the range of $A \circ A$?

   c) What is $A \circ (A \circ A)(3)$?

5. Does the composition $A \circ S$ make sense? What about the composition $S \circ A$?

6. Determine which of the following values are defined. If a proposed expression makes no sense, explain why not.

   a) $S(-6)$ b) $M(2)$

   c) $A(-6)$ d) $(M \circ A)(3)$

   e) $S \circ S(17)$ f) $M \circ M(\frac{2}{3})$

7. Is it possible for $A \circ M(r)$ to make sense for a specific rational number r, even though $A \circ M$ cannot be defined?

8. A businessman has a supply of 5¢ and 8¢ stamps. What values of postage less than $1.00 can he make with these stamps? Relate this question to the problem of finding the range of a certain function. (<u>Hint</u>: If he uses as p and q, the numbers of 5¢ and 8¢ stamps, respectively, how much postage does he use? View the ordered pair (p,q) as an input.)

9. In terms of the domains and ranges of f and g, describe the condition that must be satisfied in order that $f \circ g$ be defined. This condition is discussed in terms of inputs and outputs on pages 108 and 109.

ACTIVITY 16

SOME SPECIAL FUNCTIONS AND THEIR GRAPHS

FOCUS:

There are a few classes of functions that arise frequently in the applications of mathematics. Three of these, linear, quadratic, and reciprocal functions, are discussed and graphed in this activity. In addition, the notions of ratio-proportion are connected with the concept of a linear function. The activity has five parts:

Part A: Graphs of Functions,
Part B: Linear Functions and Linear Processes,
Part C: Quadratic Functions,
Part D: Reciprocal Functions,
Part E: Linear Functions and Proportional Changes.

Part A: Graphs of Functions

DISCUSSION:

Since a function can be viewed as a set of ordered pairs, if both the domain and range are subsets of the real numbers, then, as we have seen earlier in this section, the function can be graphed on a Cartesian coordinate system.

EXAMPLES

The graph of the function defined by $\{(0,1), (\frac{1}{2}, \frac{3}{2}), (1,2), (\frac{3}{2}, \frac{3}{2}), (2,1), (\frac{5}{2}, \frac{1}{2})\}$ is shown at the right (Figure 1).

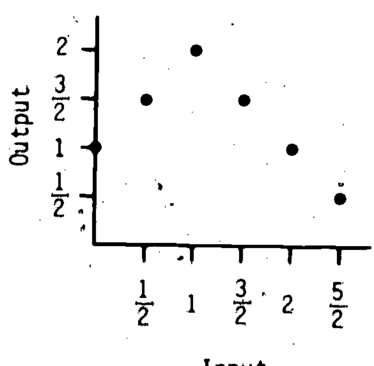


Figure 1

The graph of the function that associates with each real number between 1 and 2 (inclusive) the sum of 1 plus the square of that number is shown at the right (Figure 2). If we denote that function by f and write this in the usual notation, we write $f(x) = 1 + x^2$, $1 \leq x \leq 2$.

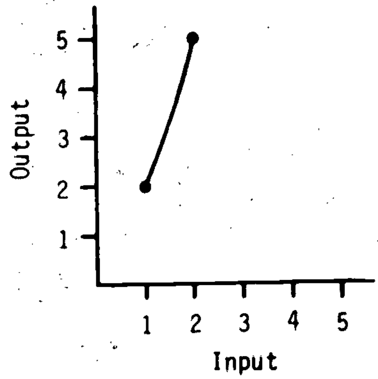


Figure 2

The fact that each input can be associated with exactly one output can be interpreted graphically as follows: If one constructs a vertical line through an input number, then it intersects the graph of the function exactly once. This remark provides an easy test to determine whether a graph is in fact the graph of a function. The graph in Figure 3 is not the graph of a function since, for example, input p does not correspond to a unique output.

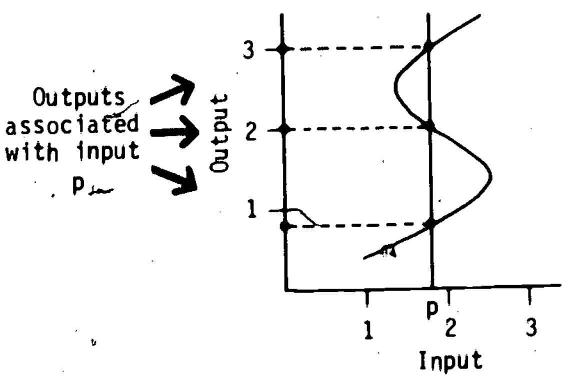


Figure 3

DIRECTIONS:

1. Graph the "postage function," i.e., the function that assigns to a real number x the cost of mailing a first-class letter weighing x ounces. Restrict x to be less than or equal to 8.

2. Graph the "income tax function," i.e., the function that assigns to a taxable income x (measured in dollars) the federal income tax due on that income. Use the information contained in the table below.

Individual Income Tax Table

Taxable Income (A)	Regular Tax on Amount in (A) (B)	Amount of Taxable Income in Excess of (A) but not in Excess of (C) is taxed at rate shown in (D) (C)	(D)
$ 0	$ 0	$ 500	14%
500	70	1,000	15%
1,000	145	1,500	16%
1,500	225	2,000	17%
2,000	310	4,000	19%
4,000	690	6,000	21%
6,000	1,110	8,000	24%
8,000	1,590	10,000	25%
10,000	2,090	12,000	27%
12,000	2,630	14,000	29%
14,000	3,210	16,000	31%
16,000	3,830	18,000	34%
18,000	4,510	20,000	36%
20,000	5,230	22,000	38%
22,000	5,990	26,000	40%

3. A table giving data on gasoline mileage as a function of speed is given below. Construct a graph of this "gasoline mileage function."

Car Speed (mph)	40	50	60	70	80	90
Gasoline Mileage (mpg)	20.4	19.1	17.7	16.1	14.0	12.0

Part B: Linear Functions and Linear Processes

DISCUSSION:

An airplane cruising at a steady 600 miles per hour will travel 300 miles in $\frac{1}{2}$ hour, 60 miles in $\frac{1}{10}$ hour, etc. If we take the number of hours traveled as input and the distance traveled at output, then we can define a function:

$$\text{Distance traveled} = 600 \times \text{number of hours.}$$

The graph of this function is easy to construct. Let us take inputs (number of hours traveled) to be $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, 2, 3. The outputs (distances) corresponding to these inputs are graphed in Figure 4. If you lay a ruler (or other straightedge) along these points, you will find that they all lie on a straight line. As a result we say that at constant speed the distance traveled is a <u>linear function</u> of time.

If we had plotted the points corresponding to all inputs less than or equal to 3, our graph would have appeared as in Figure 5.

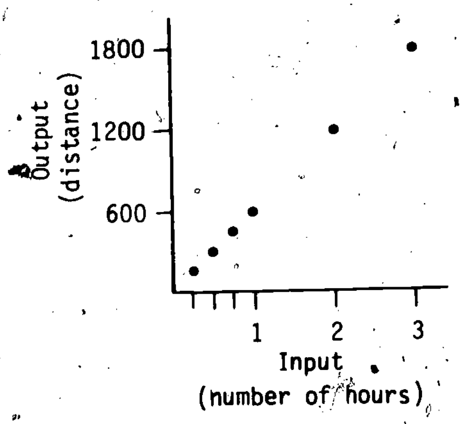


Figure 4

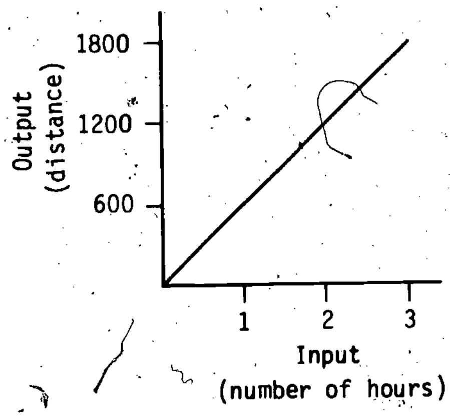


Figure 5

An important characteristic of many real-world situations is illustrated by this example. Observe that if the airplane makes a trip of t hours followed by a trip of T hours, then the distance traveled is the same as would have been traveled in a single trip of $(t + T)$ hours. Also, if it made a trip of cT hours, where c is a positive real number (e.g., $\frac{1}{2}T$ hours or $7T$ hours), then it travels c times the distance it would have traveled in T hours. A process that has these properties, such as travel at constant speed, is called a <u>linear process</u>.

A linear process is one in which the output of the input $(x + y)$ is the sum of the outputs of input x and input y and, for any real number r, the output for the input rx is r times the output of input x.

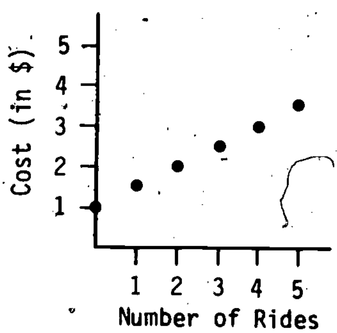
The idea of a change in input resulting in a proportional change in output is pursued further in Part E.

Usually the term "linear function" is used to describe a function whose graph on a standard coordinate system is a straight line or a subset of a straight line. This includes, in addition to the functions associated with linear processes, functions such as the one of the next example.

EXAMPLE

Suppose an amusement park has an admission charge of \$1 and a charge of \$.50 for each ride. The cost associated with 0 to 5 rides is shown in the table and graph below.

Number of rides	Cost
0	\$1.00
1	1.50
2	2.00
3	2.50
4	3.00
5	3.50



There is a formula that expresses the cost of a visit to the amusement park (in dollars) in terms of the number of rides. It is:

$$\text{Cost} = 1 + (\tfrac{1}{2} \times \text{number of rides}).$$

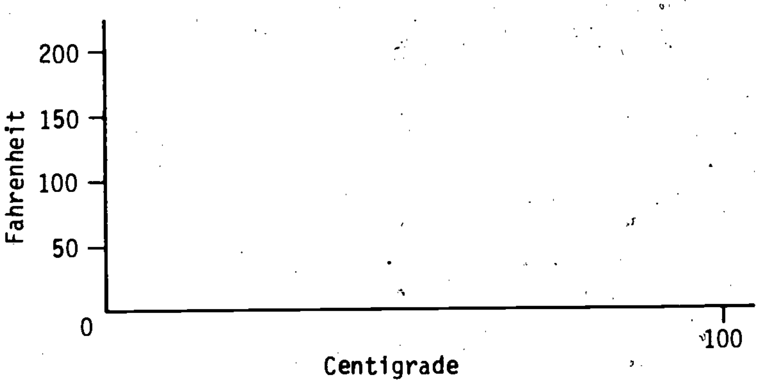
DIRECTIONS:

1. It takes 2 ounces of instant beverage powder and 1 gallon of water to make 1 batch of flavored drink. If mixing flavored drink is a linear process, use a graph to determine the amount of powder needed to mix 3 batches of drink, $\frac{1}{2}$ batch, m batches.

2. Graph the cost of a visit to the amusement park if the admission is \$2 and rides are \$.25 each. Are these prices better than those of the example? Does the answer to the question depend on how many rides are taken?

3. Any temperature can be measured either in centigrade degrees (C°) or Fahrenheit degrees (F°). There is a linear function relating the temperature measured in centigrade degrees to the temperature measured in Fahrenheit degrees. It has been agreed that water freezes at 0°C and 32°F and that water boils at 100°C and 212°F.

   a) Graph the linear function that assigns to each temperature measured in centigrade degrees the associated temperature measured in Fahrenheit degrees. Restrict attention to centigrade temperatures between 0 and 100°C.



   b) Use your graph to determine what Fahrenheit temperature corresponds to 40°C.

   c) As (b), for 20°C.

d) The temperatures in C° and F° are related by the formula

Temperature in F° = A + B x Temperature in C°.

Find A and B by substituting the temperature of the boiling and freezing points of water.

Part C: Quadratic Functions

DISCUSSION:

If a ball is dropped from rest, then the distance d (measured in feet) it falls in time t (measured in seconds) is given by:

$$d = 16t^2$$

This function is graphed for t between 0 and 2 in Figure 6.

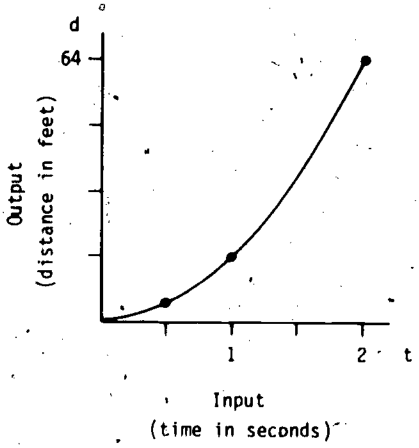


Figure 6

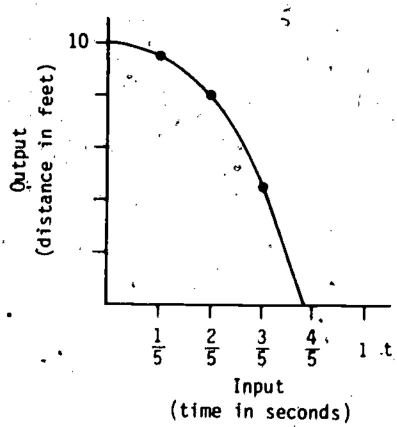


Figure 7

In this example, suppose that the ball is released at a height of 10 feet above the floor and its height above the floor is computed for each time t (= number of seconds after release) until it hits the floor. The graph of the function that assigns to each time t an output equal to the height of the ball above the floor, is given in Figure 7. The ball hits the floor approximately .79 seconds after it is released.

The functions that are graphed in Figures 6 and 7 are <u>quadratic functions</u>. The trajectory of a drop of water emitted by a garden hose is another example of the graph of a quadratic function. So is the cross section of the reflection in an automobile headlight. These varied examples give you an indication of the widespread occurrence of quadratic functions in the world around us.

DIRECTIONS:

1. Graph the function that assigns to each positive real number x the area of a rectangle whose sides are x and $3x$. Consider only values of x between $\frac{1}{2}$ and 3 (inclusive).

2. The area of a circular disk of radius r is πr^2. Graph the function that assigns to each real number r between 0 and 2 the area d, of a circular disk of radius r.

3. Graph the functions f, g, and h on the same coordinate system:

   - f assigns to each x the sum of 1 and x^2.
   - g assigns to each x the sum of 2 and x^2.
   - h assigns to each x the sum of -1 and x^2.

   Consider only values of x between -1 and +1 (inclusive). Describe the pattern displayed by these three graphs.

Part D: Reciprocal Functions

DISCUSSION:

What is the width of a rectangle whose length is 6 and whose area is 12? What if the length is 4 and the area is 12? What if the length is x and the area is 12? It is helpful to construct a table for this information:

Dimensions of a rectangle whose area is 12

Length	Width
6	$2 (= \frac{12}{6})$
4	$3 (= \frac{12}{4})$
⋮	
x	$\frac{12}{x}$

We say that the width is 12 times the <u>reciprocal</u> of the length x or, alternatively, that in rectangles of the same area the width is inversely proportional to the length.

DIRECTIONS:

1. A school has a need for 1200 pencils. The pencils can be purchased in packages of 12, 24, 48, 100 and 200 pencils. Define a function by assigning to each possible package size the number of packages the school would have to buy. Make a table summarizing the information contained in the function, and graph the function.

2. Graph the functions f, g, and h on the same coordinate system.
   - f assigns to each $x \neq 0$ the result of dividing 2 by x.
   - g assigns to each $x \neq 0$ the result of dividing 5 by x.
   - h assigns to each $x \neq 0$ the result of dividing $\frac{1}{2}$ by x.

   Consider only values of x between 0 and 5 inclusive. Describe the pattern displayed by these graphs.

Part E: Linear Functions and Proportional Changes

DISCUSSION:

A recipe for a fruitcake might call for 2 cups of fruit and $\frac{2}{3}$ cups of nuts. A cook who wants to double the recipe would use 4 cups of fruit and $1\frac{1}{3}$ cups of nuts; one who wanted to triple the recipe would use 6 cups of fruit and 2 cups of nuts. In general, the amount of fruit will be in a $2:\frac{2}{3}$ ratio to the amount of nuts, independent of the total amount of fruit and nuts used. To express this fact, we might write

$$\frac{\text{amount of fruit}}{\text{amount of nuts}} = \frac{2}{\frac{2}{3}} (=3)$$

$$\text{or amount of fruit} = 3 \times \text{amount of nuts},$$

and we might say that the amount of fruit is <u>proportional</u> to the amount of nuts and the <u>constant of proportionality</u> is 3.

EXAMPLE

Suppose that one Hong Kong dollar (1 \$ HK) is worth twenty cents in United States money (.20 \$ US). The cost of an item can be measured either in \$ HK or \$ US. Since it takes 5 \$ HK to have a value equal to 1 \$ US, we have the following

$$\text{cost in \$ HK} = 5 \times \text{cost in \$ US}$$
$$\text{or cost in \$ US} = \frac{1}{5} \times \text{cost in \$ HK}.$$

In mathematical terms, proportional changes are described by linear functions in which input 0 yields output 0--provided that 0 is a legitimate input. Such functions are often expressed by the equation $f(x) = kx$ where k is the constant of proportionality.

DIRECTIONS:

1. If the pound, £ (monetary unit in great Britain) is worth 2.25 \$ US, find the constants of proportionality A and B in the formulas

   cost of item in £ = A x (cost of item in \$ US).
   and cost of item in \$ US = B x (cost of item in £).

2. For a specific automobile, the cost of filling an empty gasoline tank is proportional to the price of gasoline. For a fixed price of gasoline, the cost of filling an empty gasoline tank is proportional to the size of the tank. Write these statements in mathematical form, and find constants of proportionality in each case. (Note that, in the former, the constant depends on the automobile.)

3. In a grocery store, find a commodity that comes in at least three sizes. (Laundry detergent and peanut butter frequently do.) Is the cost proportional to the size? Graph your data and comment on it from a consumer's point of view.

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ACTIVITY 17
SEMINAR

FOCUS:

Functions are studied explicitly in the upper levels of most elementary mathematics curricula, and they occur implicitly in the lower levels and in the discussions of other mathematical topics at the upper levels. Although there are pedagogical reasons for exercising selectivity in presenting the function concept as a mathematical idea, it may be the case that it can serve as a useful organizer for the teacher. This activity is concerned with identifying the occurrence of the function concept in elementary school mathematics curricula.

DIRECTIONS:

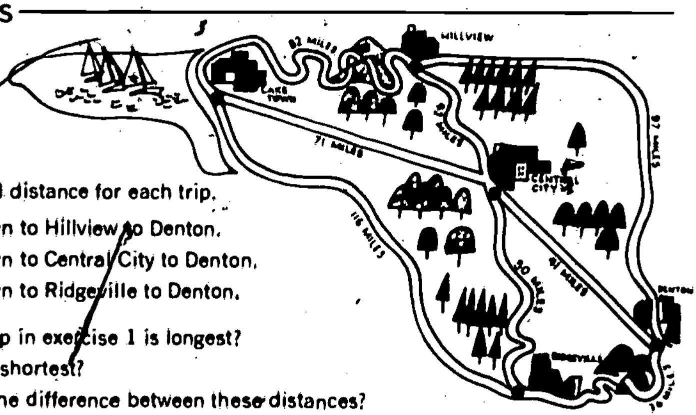
Each of the partial textbook pages reproduced below contains an activity or describes a situation that can be viewed as an example of a function. In each case

a) Identify the function.

b) Note its domain and range, and whether it is one-to-one.

c) Discuss how a teacher's awareness of the function concept might influence his/her presentation of the material.

Map problems



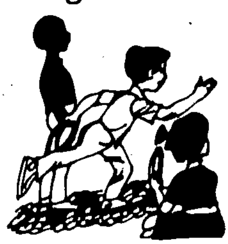
EXERCISES

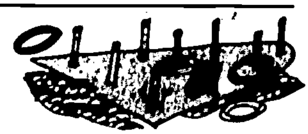
1. Give the total distance for each trip.
   [A] Lake Town to Hillview to Denton.
   [B] Lake Town to Central City to Denton.
   [C] Lake Town to Ridgeville to Denton.

2. [A] Which trip in exercise 1 is longest?
   [B] Which is shortest?
   [C] What is the difference between these distances?

3. How long is a trip that is 4 times as far as from Hillview to Denton?

Reading a chart





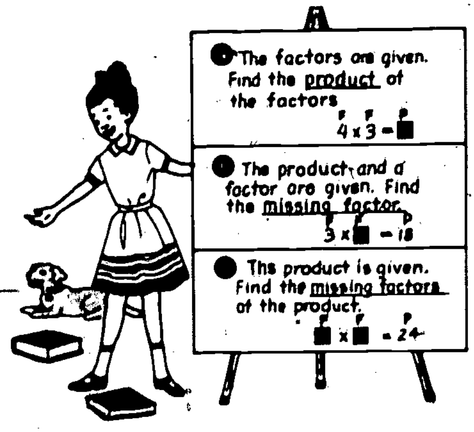
The numbers given in the chart are the children's scores for each game. For example, Dan scored 47 points in game 3.

	Ann	Bill	Carol	Dan	Ed	Fay
Game 6	31	71	42	54	34	36
Game 5	63	18	43	64	29	67
Game 4	50	76	40	56	67	58
Game 3	[redacted]	[redacted]	[redacted]	47	34	52
Game 2	34	26	58	[redacted]	54	23
Game 1	28	27	62	[redacted]	60	47

1. [A] What did Bill score in game 5?
   [B] What did Fay score in game 5?
   [C] What did Dan score in game 6?
   [D] What did Carol score in game 1?
   [E] What did Ann score in game 2?
   [F] What did Ed score in game 4?
   [G] What did Ed score in game 3?
   [H] What did Carol score in game 3?

Factors and products

Joan gave Sara these problems to solve. Sara solved problems A and B easily. When she tried problem C, she stopped and looked very puzzled. Can you explain why?



EXERCISES

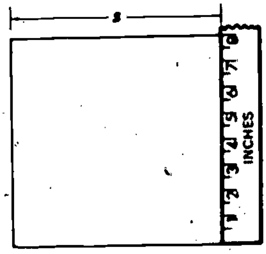
1. Give as many different pairs of numbers as you can that could serve as factors of 24.

2. List the equations that have no whole-number solution. Then find the missing factors in the other equations.

[A] ■ × 1 = 12	[G] ■ × 7 = 12	[M] ■ × 1 = 32
[B] ■ × 2 = 12	[H] ■ × 8 = 12	[N] ■ × 2 = 32
[C] ■ × 3 = 12	[I] ■ × 9 = 12	[O] ■ × 3 = 32
[D] ■ × 4 = 12	[J] ■ × 10 = 12	[P] ■ × 4 = 32
[E] ■ × 5 = 12	[K] ■ × 11 = 12	[Q] ■ × 5 = 32
[F] ■ × 6 = 12	[L] ■ × 12 = 12	[R] ■ × 6 = 32

Performance Objective: To use formulas to find perimeter

P means perimeter. s means side. l means length. w means width.



Perimeter of a square

$P = 8 + 8 + 8 + 8$

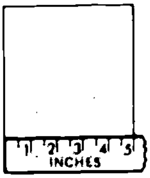
$P = 4 \times 8$

$P = 32$. Perimeter is 32 in.

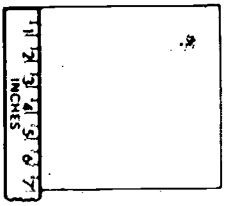
Formula: $P = 4 \times s$

Use the formula to find each perimeter.

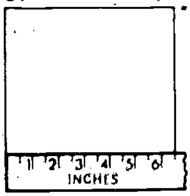
1.



2.



3.



Perimeter of a rectangle

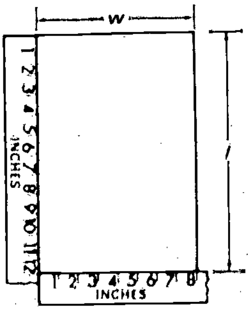
P 12 12 8 8

P (2 12) (2 8)

P 2 (12 8)

P 40 Perimeter is 40 in

Formula: $P = 2 \times (l + w)$



Performance Objective: To show that subtracting "undoes" adding

A is an "add 4" machine. When 6 goes in, what number comes out?

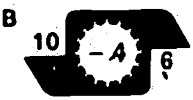
B is a "subtract 4" machine. When 10 goes in, what number comes out?

Is the number that came out of B the same number that went into A?

A

$6 + 4 = 10$

B

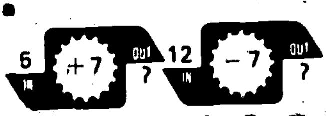


$10 - 4 = 6$

Subtracting a number "undoes" what was done by adding the number.

1. Look at the machines below. Then copy the sentences, putting in the missing numerals.

a



$5 + 7 = \square$ $\quad 12 - 7 = \square$

b

$8 + 9 = \square$ $\quad 17 - 9 = \square$

Copy each pair of sentences and put in the missing numerals.

	a	b
2.	$8 + 5 = \square;\ 13 - 5 = \triangle$	$9 + 9 = \square;\ 18 - 9 = \triangle$
3.	$6 + 9 = \square;\ 15 - 9 = \triangle$	$7 + 8 = \square;\ 15 - 8 = \triangle$
4.	$7 + 4 = \square;\ 11 - 4 = \triangle$	$3 + 9 = \square;\ 12 - 9 = \triangle$

APRENDIX

GRAPHING SELF-EVALUATION QUESTIONS

(Note: Questions 1 through 16 make reference to graphs 1 through 8, which are shown on pages 141 through 144.)

1. Referring to graphs 1 through 8, categorize each as a bar graph, line graph, circle graph, or pictograph. Summarize your results below by circling the graph numbers that apply.

Type of Graph	Graph Number							
Circle	1	2	3	4	5	6	7	8
Line	1	2	3	4	5	6	7	8
Bar	1	2	3	4	5	6	7	8
Pictograph	1	2	3	4	5	6	7	8

2. Referring to graph 1, rank the years listed (1970-1974), based on percentage of youths arrested for possession of drugs, from first (highest) to fifth (lowest).

	Year
First	______
Second	______
Third	______
Fourth	______
Fifth	______

3. Referring to graph 1, in the period from 1970 to 1974, determine whether the percentage of young people arrested for drug possession averaged: (Circle one of a, b, c, d.)

a) well below 10%

b) closer to 10% than 20%

c) closer to 20% than 10%

d) well above 20%

Don't just guess. Confirm your guess with measurement and computation.

4. Referring to graph 4, circle either true or false.

T F a) Alcoholic drinks were more in reported use than all of the other items combined.

T F b) Of the eight items shown, each of the two most heavily used items was reported as more heavily used than all of the other six items combined.

T F c) Smoking (marijuana and cigarettes combined) was more common than any single category.

5. Referring to graph 4, the table below was prepared by reading the data off the graph. Complete this table by filling in the boxes. Use your ruler to help make decimal approximations as accurately as you can.

Type of Drug	Percentage Reporting Use
Marijuana	About 2.4
L.S.D.	About .7
Alcoholic Drinks	About ☐
☐	About 1.0
Barbiturates	About ☐
☐	About 13.5
☐	About .2
Glue	About ☐

6. Referring to graph 5, between 1820 and 1970 population: (Circle one of a, b, c, d.)

a) increased about five-fold

b) decreased by at least one-half

c) increased about two-fold

d) increased about ten-fold

7. Referring to graph 6, complete the table below by reading the data off the graph. Make rough estimates for the fractions of each figure shown.

Countries	Iron Ore in Metric Tons
Canada	______
France	______
Australia	______
U.S.	______
U.S.S.R.	______

8. Refer to graph 2.

T F a) Deaths from motor vehicle accidents and drowning accounted for more than half of the deaths shown. True or false? (Circle one.)

T F b) Deaths from drowning accounted for more than one-third of the deaths shown. True or false? (Circle one.)

c) What two categories (i.e., sections of the circle), when combined, account for 64% of the deaths shown?

____

d) The total of the percentages of all deaths shown adds up to ______%.

9. Referring to graph 8, what is the largest expense shown on the graph? (Circle one of a, b, c.)

a) Entertainment

b) Tuition and Books

c) Room and Board

10. Referring to graph 8, name the category shown on the graph for which the least money was allocated:

11. Referring to graph 7, complete the following sentences.

a) During the period shown, bus ridership ranged from a high of ______ during the month of ________________ to a low of ______ during the month of ________________.

b) The longest period of increase of bus ridership was ______ months.

c) Since the month of August there has been a leveling off of bus ridership. In fact the largest change from one month to the next since August has been ______ riders.

12. Referring to graph 7, circle either true or false.

T F a) The bus ridership data has never sustained more than a two-month decline.

T F b) Bus ridership has been less stable since people have had to pay to ride.

T F c) During the period shown, bus ridership is more frequently above the average figure than below it.

13. Referring to graph 3, list the five types of sports listed for the 1962-63 season, from first (most participants) to fifth (least participants).

First ________________

Second ________________

Third ________________

Fourth ________________

Fifth ________________

14. Referring to graph 3, complete the following paragraph:

"From the 1962-63 season onward the following general pattern emerges: The number of participants in ____________ is a clear first, averaging about (circle one of the following choices) 1,200,000; 1,400,000; 1,600,000. The number of participants in _____________ is a clear second, averaging about (circle one) 800,000; 1,000,000; 1,200,000. Below second place the numbers of participants in _____________, _____________, and _____________ are more closely grouped together, in no case averaging more than (circle one) 200,000; 400,000; 600,000, or less than (circle one) 200,000; 400,000; 600,000."

15. Referring to graph 3, fill in the missing information.

    a) The number of tennis players and the number of joggers were closest together during the year _________.

    b) Of the years shown since 1961, the number of joggers was virtually the same as the number of _________ participants during the year _________.

    c) During the year 1956 there were about (circle one of the following choices) 2, 3, 4, 5 times as many tennis players as there were bowlers.

    d) Comparing the year 1961 to the year 1962, only the number of participants in _____________ showed a marked decrease. The number of participants in _____________ remained about the same, while the numbers of _____________, _____________, and _____________ showed increases.

16. Referring to graph 1, if you were to redraw graph 1 to occupy the same amount of space on the page that it currently occupies but in such a way as to make the relative highs and lows of the data more readable, you would do <u>best</u> to: (Circle one of a, b, c.)

    a) Put additional marks on the vertical scale so that it goes from 0% to 100% in steps of 5% instead of 10%.

    b) Make the vertical scale run from 0% to 70% in the same space it now takes to run from 0% to 100%.

    c) Make the vertical scale run from 0% to 30% in the same space it now takes to run from 0% to 100%.

17. Referring to the table on the fish harvest on the right, express the data to the nearest 10,000,000 metric tons. (Example: express 35,000,000 or 36,000,000 as 40,000,000.) Record your results on the table below.

APPROXIMATE ANNUAL FISH HARVEST IN METRIC TONS
(Top 6 countries in 1971)

Norway	2,838,000,000
U.S.S.R.	2,399,000,000
Denmark	1,388,000,000
Canada	1,143,000,000
Britain	1,110,000,000
U.S.	945,000,000

Fish Data, Accurate to the Nearest 10,000,000 Metric Tons

Norway	______________
U.S.S.R.	______________
Oenmark	______________
Canada	______________
Britain	______________
U.S.	______________

18. Suppose you were going to make a pictograph illustrating as closely as is practical the fish harvest data of problem 17. Suppose further that you were only allowed to use whole or half fish symbols.(for example  or ). Consider choices (i), (ii), or (iii) below:

i. Let  = 1,000,000,000 metric tons
ii. Let  = 100,000,000 metric tons
iii. Let  = 10,000,000 metric tons

Complete the following statements:

a) Under choice ii, Norway would have ______________ whole fish symbols and ______________ half fish symbols.

b) Under choice __________, Canada, Britain, and the United States would have the same number of whole fish symbols and no half fish symbols.

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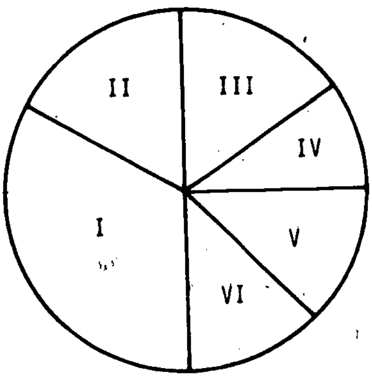
c) Of the three choices, choice ______ is the least practical for construction purposes, since the U.S.S.R. alone would require that ________ fish symbols be drawn.

d) Choice ____ would make construction of the pictograph easy. But it has the disadvantages that, just from looking at the fish symbols, it would not be possible to place each of the six nations precisely in order of size of fish harvest--something that it is possible to do by looking at the original table of data.

e) With choice ______, the pictograph would retain the precise order of the nations and would be a little tedious, but not outrageously difficult to construct.

19. Referring to the circle at the right, assuming that the entire circle represents 100% of a quantity, complete the table below by filling in the boxes with your best estimates.

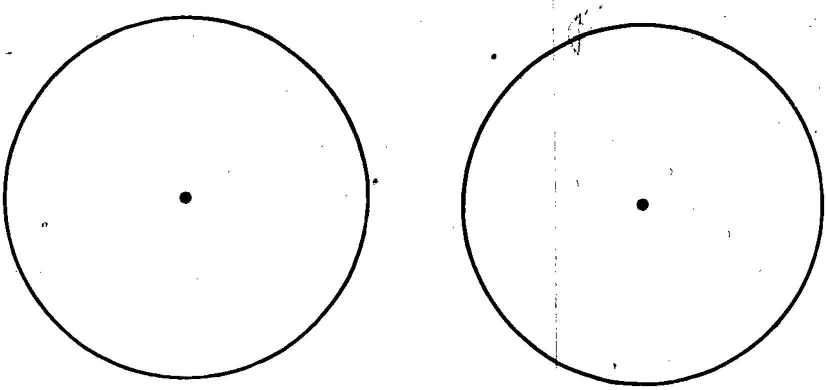


Percent	Sector Names
50%	Sectors I and II
	Sector I
	Sectors V and VI
12½%	
	Sectors I, II and III
75%	
	All except Sector VI

20. Divide one of the circles below into sectors determined by the following table:

Percent of Whole	Sector
40%	A
25%	B
20%	C
10%	D
5%	E

Two circles are given so that you may use one of them for scratch work if necessary. Try to make the best estimates of what parts of the whole circle you should allot for each percentage. Use a ruler or straightedge to draw the radii. Also, be sure to label each sector with its corresponding letter.



21. On the grid on the following page, construct an easily readable bar graph of the data given.

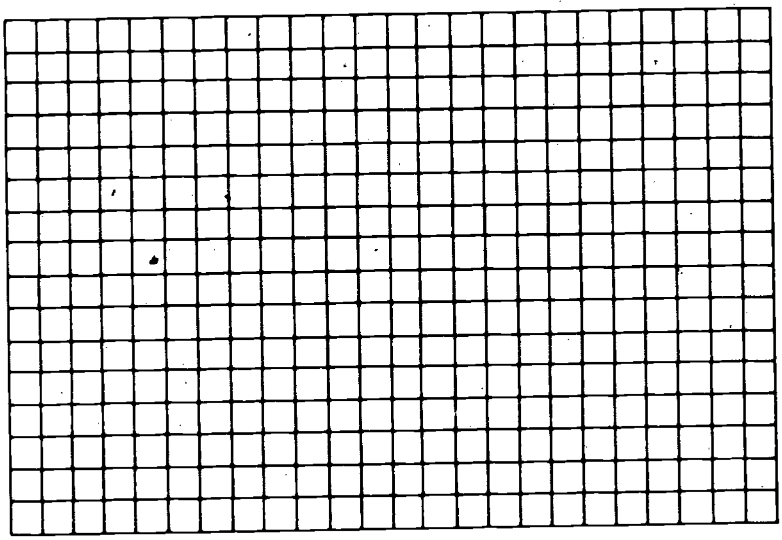
Title: Percent of Students Who Can Make Good Bar Graphs Before Taking This Course

(1964 through 1968)

Source: World Journal of Phony Statistics

Data:

Year	Percentage
1964	10%
1965	14%
1966	18%
1967	26%
1968	23%

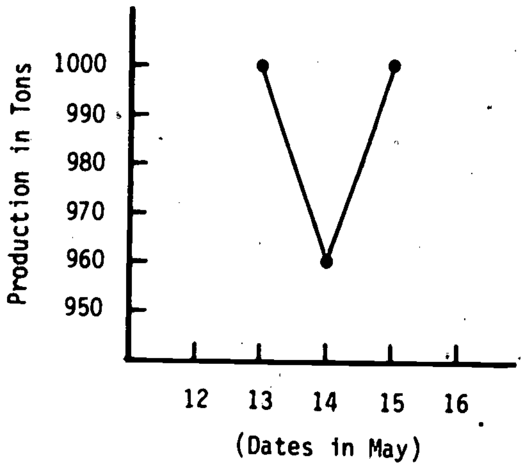


22. Statements (a) through (h) below describe changes that might be effected in the graph that is pictured below. Complete each statement by circling D, E, or N in the statements (a) through (h), depending on whether the change would tend to:

D. De-emphasize the daily change in production.

E. Emphasize the daily change in production.

N. Not change the emphasis of the daily change in production.



a) If you were to redraw the above with more space between each of the numbers on the horizontal scale, then it would tend to D E N the daily change in production.

b) If you were to redraw the above with less space between each of the numbers on the vertical scale, then it would tend to D E N the daily change in production.

c) If you were to redraw the above, using the same total length on the horizontal scale but equally spacing out the whole numbers from 0 to 16 instead of from 12 to 16, then it would tend to D E N the daily change in production.

d) If you were to refine the vertical scale by including numbers at 955, 965, 975, 985 and 995, then it would tend to D E N the daily change in production.

e) If you were to redraw the above, doubling the spacing between each of the numbers on both scales, then it would tend to D E N the daily change in production.

f) If you were to redraw the above, using the same total length on the vertical scale but equally spacing out the scale from 0 to 1000 instead of--as it is now--from 950 to 1000, then it would tend to D E N the daily change in production.

g) If you were to redraw the above, using a vertical scale running from 0 to 1000 in equal steps of 10 and using the same spacing as the present vertical scale, then it would tend to D E N the daily change in production.

h) If you were to do both (c) and (f), then it would tend to D E N the daily change in production.

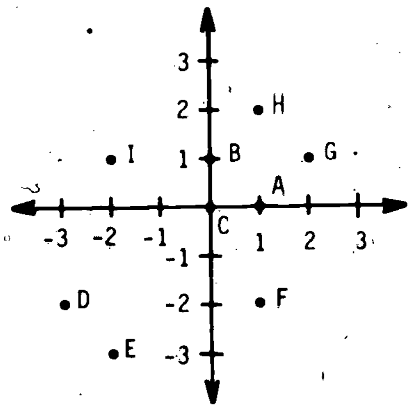
23. Each ordered pair below corresponds to a point on the given graph. Indicate the correct letters in the space below.

a) (1,-2) ______

b) (2,1) ______

c) (0,1) ______

d) (-3,-2) ______



24. Each of the letters below corresponds to a point on the graph in exercise 23. Indicate opposite each letter the ordered pair that locates that point.

A. ______________

B. ______________

C. ______________

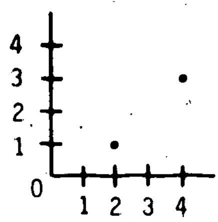
D. ______________

25. Three diagrams below contain the same information--but in different forms. List the three letters corresponding to those diagrams: _____, _____, _____.

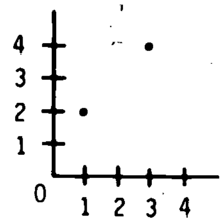
A.

1st coordinate	1	3
2nd coordinate	2	4

B.



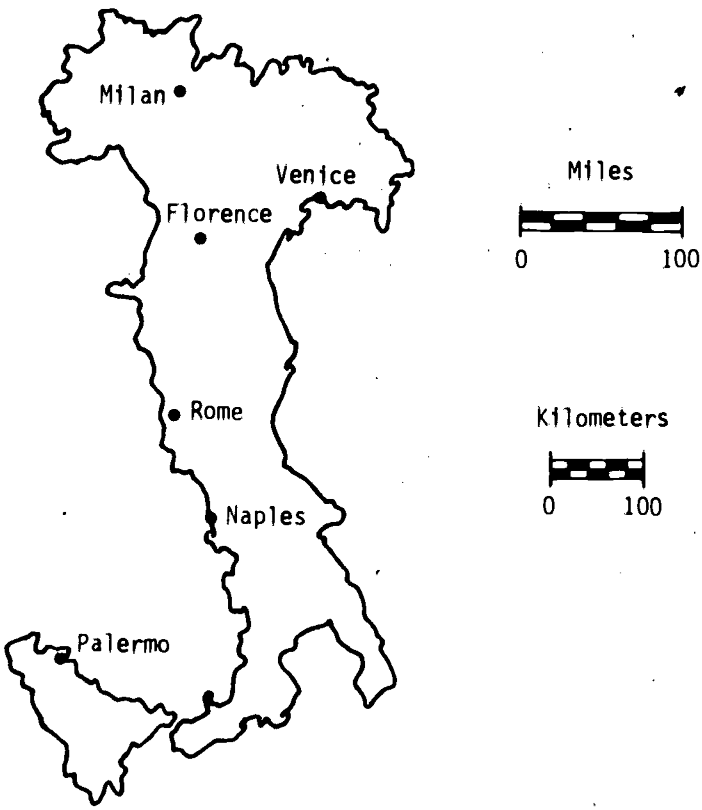
D.



C. (1,3) (2,4)

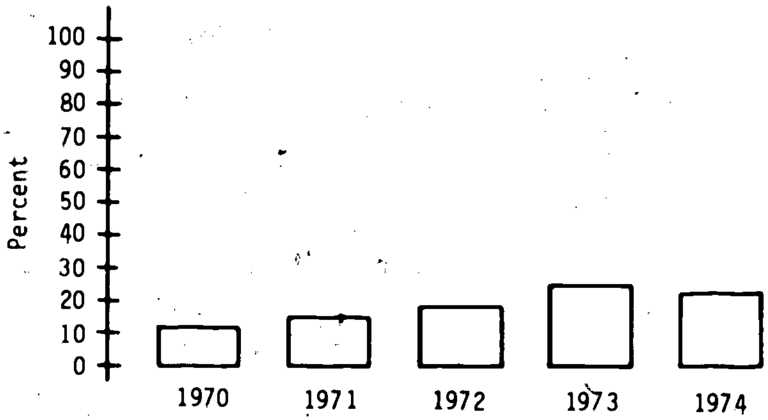
E. (1,2) (3,4)

26. By using the scale of distance on the map of Italy below, calculate the approximate distance between Rome and Palermo (Sicily).



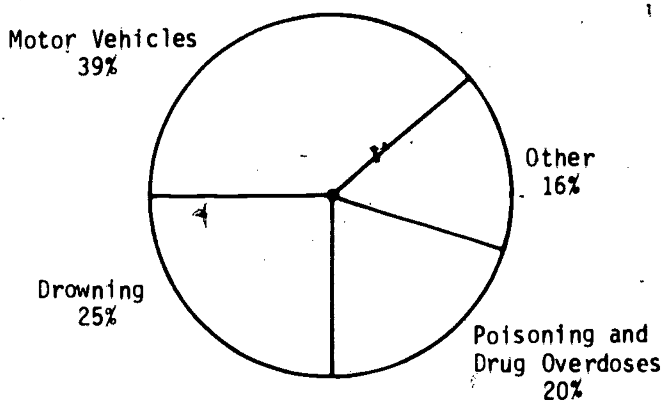
Graph 1

Percentage of Young People (Aged 12-21) Arrested for Possession of Drugs in a Town for the Period 1970-1974



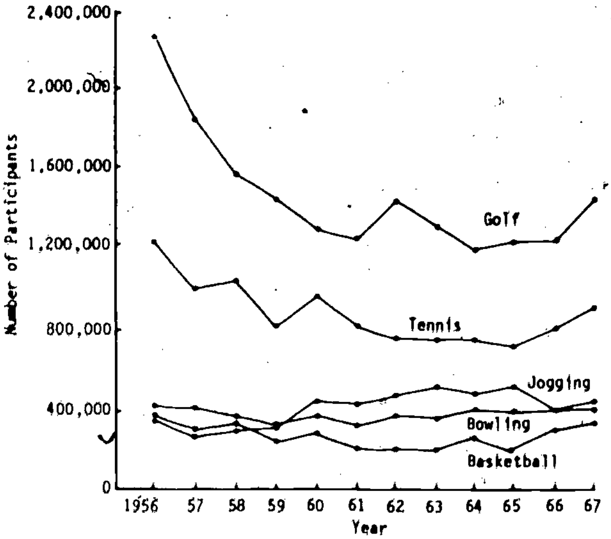
Graph 2

A Classroom Report on the Causes of Death From Accidents Among Young People (Aged 12-21)



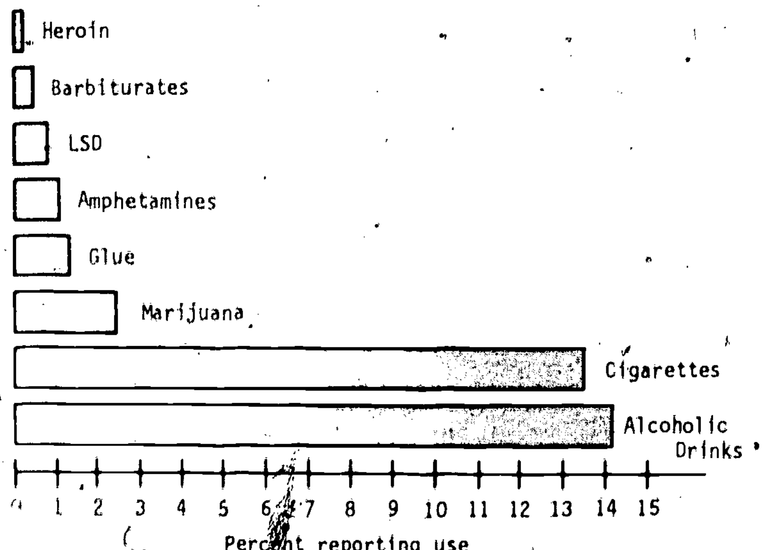
Graph 3

Data on Sports Participants in a State (1956-67)



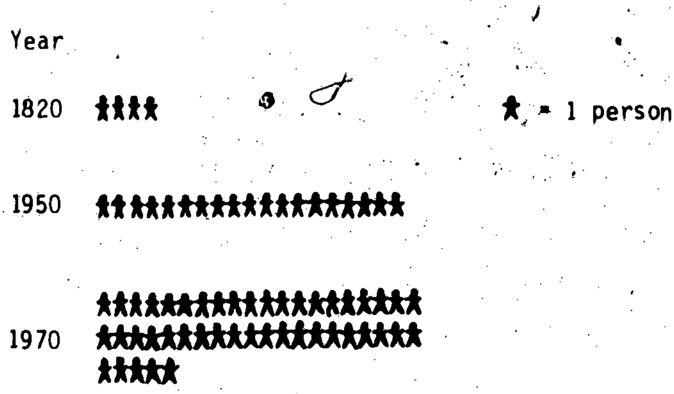
Graph 4

Self Report on Use of Drugs Among Seniors in a High School 1974



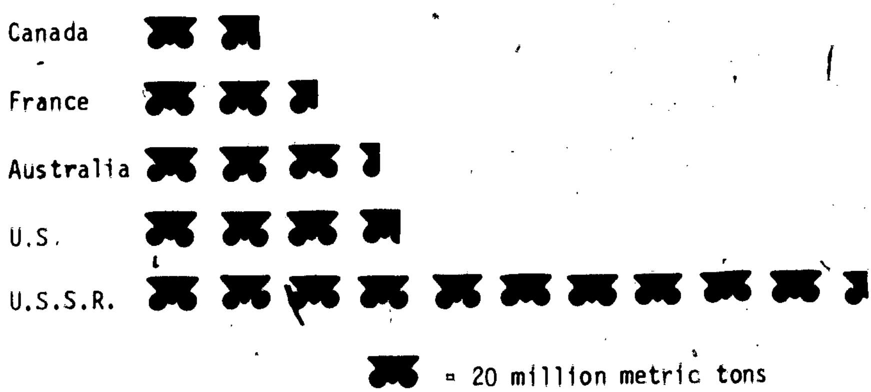
Graph 5

Increase in Population in a Small Town

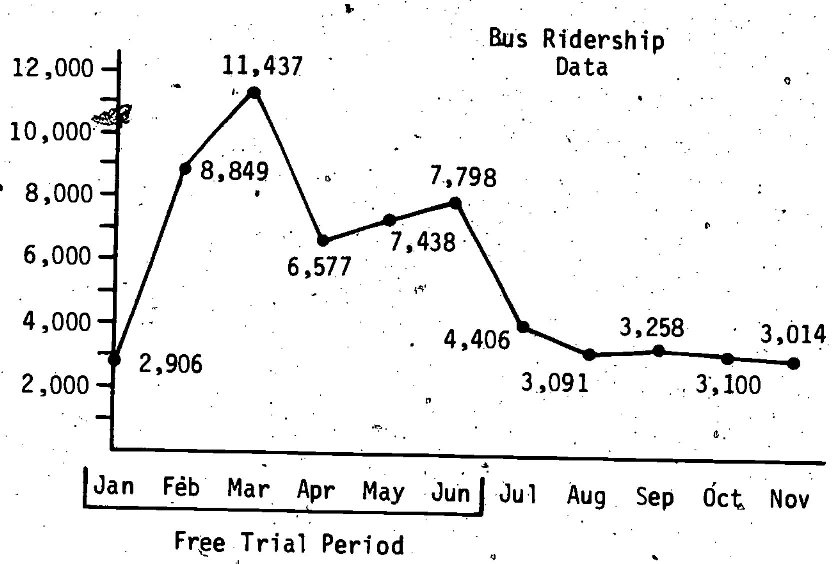


Graph 6

Iron Ore Production (1972)

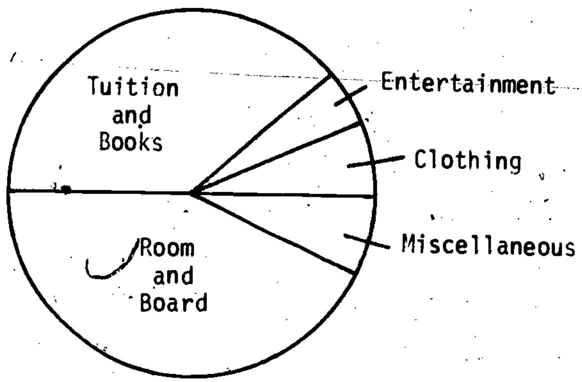


Graph 7



Graph 8

How One College Student Allocated His Budget



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SELF-EVALUATION FORM

Answers to the Questions

1. Type of Graph — Graph Numbers

Type of Graph								
Circle	1	2	3	4	5	6	7	8
Line	1	2	3	4	5	6	7	8
Bar	1	2	3	4	5	6	7	8
Pictograph	1	2	3	4	5	6	7	8

2. First 1973
   Second 1974
   Third 1972
   Fourth 1971
   Fifth 1970

3. c

4. a) False
   b) True
   c) True

5.

Type of Drug	Percentage Reporting Use
Marijuana	About 2.4
L.S.D.	About .7
Alcoholic Drinks	14.1, 14.2 are best; 14.3 acceptable
Amphetamines (Best choice, Glue acceptable)	About 1.0
Barbiturates	.3, .4 are best; .5 acceptable
Cigarettes	About 13.5
Heroin	About .2
Glue	1.2, 1.3, 1.4 acceptable

6. d

7.

Countries	Coal in Tons
Canada	Acceptable 30,000,000 to 40,000,000
France	Acceptable 50,000,000 to 60,000,000
Australia	Acceptable 60,000,000 to 65,000,000
U.S.	Acceptable 70,000,000 to 80,000,000
U.S.S.R.	Acceptable 200,000,000 to 210,000,000

8. a) True
   b) False
   c) Motor vehicles, drowning
   d) 100

9. c

10. Entertainment

11. a) 11,437; March
       3,014; November
    b) 2
    c) 167

12. a) True
    b) False
    c) False (average is between 5,000 and 6,000)

13. First: Golf
    Second: Tennis
    Third: Jogging
    Fourth: Bowling
    Fifth: Basketball

14. Golf; 1,400,000; tennis, 800,000; jogging, bowling, and basketball (those last three acceptable in any order); 600,000; 200,000.

15. a) 1965
    b) Bowling; 1966
    c) 3
    d) Tennis; basketball; golfers, bowlers, joggers (the last three sports persons acceptable in any order).

16. c

17.

Norway	2,840,000,000
U.S.S.R.	2,400,000,000
Denmark	1,390,000,000
Canada	1,140,000,000
Britain	1,110,000,000
U.S.	950,000,000

18. a) 28 whole; 1 half
    b) i
    c) iii; 240
    d) i
    e) ii

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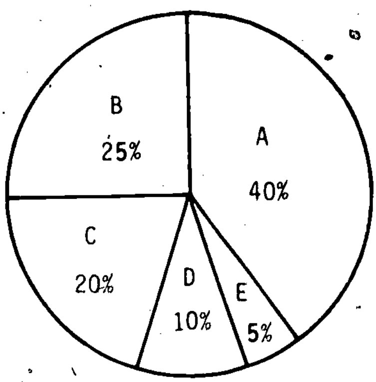
19.

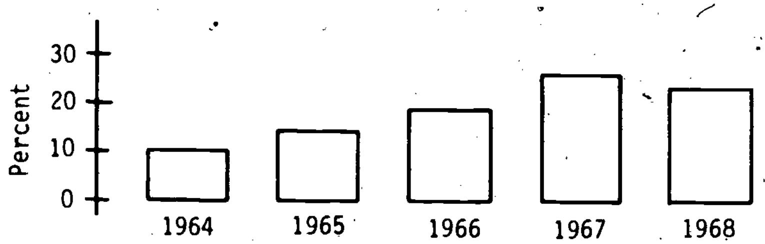
Percent	Sector Names
50%	Sectors I and II
$33\frac{1}{3}\%$ is best; 30% to 35% acceptable	Sector I
25%	Sectors V and VI
$12\frac{1}{2}\%$	Sector V or Sector VI is best
$66\frac{2}{3}\%$ is best; 65% to 70% acceptable	Sectors I, II and III
75%	Sectors I, II, III and IV or all except Sectors V and VI or Sectors I, II, V and VI or all except Sectors III and IV
$87\frac{1}{2}\%$ is best; 85% to 90% acceptable	All except Sector VI

20.



The size of your sectors should be very close to those shown. Obviously the order does not matter.

21. (Vertical numbering going up to 50% or less is acceptable. 30% shown is about the best. Any more than 50% is <u>not</u> acceptable.)



22. a) D

    b) D

    c) E

    d) N

    e) N

    f) D

    g) N

    h) D

23. a) F

    b) G

    c) B

    d) D

24. A. (1,0)

    B. (0,1)

    C. (0,0)

    D. (-3,-2)

25. A, D, E

26. About 200 miles or 320 kilometers

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REFERENCES

The view of graphs adopted here is much broader than the usual one, and consequently the reader is unlikely to find a single source which will serve as a reference for the entire unit. Standard textbooks on content and methods are likely to contain material on Cartesian coordinate systems and functions and their graphs. References for the remaining material are rather scattered.

There are many sources for raw data, both in tabular and graphical forms. The yearbooks of major encyclopedias are a good source, as are almanacs and atlases.

Some of the pedagogical issues associated with teaching graphs, coordinate systems, relations and functions in the elementary school are discussed in:

Fehr, H. F., and Phillips, J. M. Teaching Modern Mathematics in the Elementary School. 2d ed. Reading, Mass.: Addison-Wesley, 1972. (See especially Sections 14-6 and 14-7.)

National Council of Teachers of Mathematics. "Graphs, Relations and Functions" in NCTM Thirtieth Yearbook, More Topics in Mathematics for Elementary School Teachers. Washington, D. C: NCTM 1969. (See especially pp. 262-272 and 290-314.)

Swenson, E. J. Teaching Mathematics to Children. 2d ed. New York: The Macmillan Company, 1973. (See especially pp. 496-506.)

The use of data, graphs and maps in applied problems suitable for use in grades 7-10 is illustrated with ample pupil questions and exercises in:

Bell, Max S. Mathematical Uses and Models in Our Everyday World. Vol. XX of Studies in Mathematics. School Mathematics Study Group. Palo Alto, California: Leland Stanford Junior University, 1972.

Content books providing information on graphs of relations and abstract graphs are:

Ore, Oystein. Graphs and Their Uses. New York: Random House, 1963.

Wilson, Robin J. Introduction to Graph Theory. Edinburgh: Oliver and Boyd, 1972.

The Königsberg bridge problem referred to in Activity 9, Problem 4 is discussed in:

Euler, Leonhard. The Seven Bridges of Königsberg. Vol. I of The World of Mathematics. Ed. James R. Newman. New York: Simon and Shuster, 1956.

Sources which indicate how graphs of relations and abstract graphs may be used with children are:

Del Grande, M. A., et al. Math Book 1. Canada: Gage Educational Publishing Limited, 1971. (Chapter 1).

Frédérique and Papy. Graphs and the Child. Montreal, Canada: Algonquin Publishing Company, 1970.

REQUIRED MATERIALS

ACTIVITY	AUDIO-VISUAL AND OTHER RESOURCES	MANIPULATIVE AIDS
Overview	Slide-tape: "Graphs in the Elementary School," cassette recorder and projector. (Optional)	
1		Graph paper, ruler, compass, protractor.
4		Ruler, graph paper.
6		Globe, map of the U.S. with parallels of latitude which are equally spaced, Mercator projection of North America.
7	Several elementary school mathematics textbook series.	
10	Frédérique and Papy. Graphs and the Child. Montreal, Canada: Algonquin Publishing Co., 1970.	
11		Attribute blocks.

ACTIVITY	AUDIO-VISUAL AND OTHER RESOURCES	MANIPULATIVE AIDS
12	Frederique and Papy. Graphs and the Child. Montreal, Canada: Algonquin Publishing Co., 1970. Frederique. Mathematics and the Child 1. New York: Cuisenaire Company of America, 1971.	
14		Balance beam, Cuisenaire rods, hinged mirror, graph paper, protractor, paper towels, metric ruler, jar, food coloring (optional), paper clip, Scotch tape, guitar.

Continued from inside front cover

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. . . .

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This unit integrates the content and methods components of the mathematical training of prospective elementary school teachers. It focuses on an area of mathematics content and on the methods of teaching that content to children. The format of the unit promotes a small-group, activity approach to learning. The titles of other units are *Numeration*, *Addition and Subtraction*, *Multiplication and Division*, *Rational Numbers with Integers and Reals*, *Awareness Geometry*, *Transformational Geometry*, *Analysis of Shapes*, *Measurement*, *Number Theory*, *Probability and Statistics*, and *Experiences in Problem Solving*.

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