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ABSTRACT

In order to describe developments in children's conceptions of numbers and numerical relations, judgments of similarities between numbers were solicited from adults and from children in kindergarten and grades 3 and 6. A nonmetric multidimensional scaling analysis suggested that children gradually become sensitive to an expanding set of numerical relationships during this period, although even kindergartners appear to understand the importance of magnitude as a basis for judging similarity between numbers. A second study further probed the basis for kindergartners' number-similarity judgments, using the first 10 letters of the alphabet as stimuli and providing subjects with an explicit criterion for making judgments (distance between letters in the alphabet). These results suggest that the number-similarity judgments of kindergartners, and to a lesser extent of third-graders, are based upon counting distance. Results implicate the acquisition of numerical skills and operations such as counting, addition, and multiplication in a gradual broadening of the concept of number. (Author)

The child's representation of number: a multidimensional scaling analysis.

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Abstract

In order to describe developments in children's conceptions of numbers and numerical relations, judgments of similarities between numbers were solicited from children in grades kindergarten, three and six, as well as from adults. A nonmetric multidimensional scaling analysis suggested that children gradually become sensitive to an expanding set of numerical relationships during this period, although even kindergartners appear to understand the importance of magnitude as a basis for judging similarity between numbers. A second study further probed the basis for kindergartners' number-similarity judgments, using the first ten letters of the alphabet as stimuli and providing subjects with an explicit criteria for making judgments (distance between letters in the alphabet). These results suggest that the number-similarity judgments of kindergartners, and to a lesser extent of third-graders are based upon counting distance. Results implicate the acqusition of numerical skills and operations such as counting, addition and multiplication in a gradual broadening of the concept of number.

The child's representation of number: a multidimensional scaling analysis

Developmental research on children's understanding of numbers has focused on the comprehension of basic logical concepts such as one-to-one correspondence that can be used to define number. Children who demonstrate understanding of the concept of one-to-one correspondence (as in number-conservation tasks) are said to possess a "number concept", and further development is described as the learning of applications for this concept (Piaget (1965).

The possibility that learning new arithmetical operations alters children's conceptions of numbers has not been investigated. Development beyond the point at which children conserve number may consist merely of the learning of new applications for this number concept. Alternatively, mastering new operations may alter in some manner children's conception of what numbers are. The present study explored this issue by studying changes in the characteristics of numbers that children of various ages represent in making judgments of similarities between numbers.

Method

<u>Design</u>. Perhaps the simplest task for collecting judgments of similarities between numbers consists of presenting triads of numbers (e.g., 2-4-5) and asking subjects to pick the pairs of numbers that are most and least similar to each other. Different numerical relations can emerge in such judgments; in the example given, a person choosing on the basis of differences in magnitude would pick the pair

"4-5" as most similar, while another person might pick "2-4" as most similar due to the doubling relationship reflected in this pair.

Two balanced, incomplete subsets were formed from the 120 unique triads of the 10 single-digit integers (0-9), each consisting of 30 triads such that each pair of integers was presented twice and no triad appeared more than once. Adult subjects judged both sets of triads, while children saw only one.

Subjects. Subjects were 36 children from Philadelphia and an adjacent urban suburb, 12 (six boys and six girls) in each of grades kindergarten (mean age 5 years, 8 months), three (mean age 8 years, 9 months) and six (mean age 12 years, 3 months). Adult subjects were three males and three females from the graduate students and faculty of the psychology department of the University of Pennsylvania.

Apparatus. All numbers consisted of Chartpak, Futura Demi-Bold, 48 point press-on dry transfers. The following apparatus was employed for children subjects: The numbers zero through nine were placed on three cardboard wheels 12 cm in diameter. These three number-wheels were then placed on a plywood backing and covered with heavy paper so that one number at a time was visible in each of three 12 x 15 mm windows. The three numbers formed an equilateral triangle, with a line connecting each pair of numbers. Stimuli for adults were standard 3 x 5 in index cards each containing a row of three numbers.

Procedure. A brief pre-training task was given in which the children judged the similarity of triads from the set "Mother", "Father", "Sister" and "Brother." In order to avoid implying that answers should involve numerical magnitude on the main task, subjects were asked to determine which two were the "most closely related or the most similar to each other." Whichever pair the children picked, they were told that that was a good answer, but the experimenter added that they also could have picked another pair (naming one of the others), giving a reason for picking the other pair (e.g., "Because they're both adults" or "Because they're both boys or males").

Following this pre-training, subjects were presented with the experimental apparatus and told that the experimenter was interested in their opinions about what numbers are similar to each other. Subjects were presented with the first triad and told that the lines on the apparatus would show them three pairs of numbers, and they were asked to: a) name the three pairs of numbers, b) say which two numbers are the most closely related to each other (or the most similar), and c) say which two numbers are the least closely related to each other. On every fifth trial, children were asked to explain their answers and their responses were written down verbatim by the experimenter.

Results

The relation between similarity judgments made by the different age-groups was assessed using procedures developed by Hubert (1978, 1979). These procedures enable one to determine a conservative approximation of the probability of achieving a given concordance between two matrices based on permuting the rows and columns of the matrices being compared. The concordance statistic employed here corresponds to the Pearson product-moment correlation coefficient. A matrix of concordances between similarity judgments at different age-levels is presented in Table 1.

INSERT TABLE 1 ABOUT HERE.

While the judgments of our children subjects were related to each other, the judgments of adults showed a significant level of concordance only with the sixth-graders. Given the high degree of concordance between the kindergarten and third-grade data, these results suggest that there may be an adult pattern of number representation different from that of children (at least of kindergartners and third-graders), with sixth-graders perhaps demonstrating some combination of the two patterns.

Multidimensional scaling analysis. The proximity matrices were analyzed by the KYST-2A (Kruskal, Young and Seery, 1977,note 1) nonmetric multidimensional scaling (MDS) algorithm. This program produces a spatial representation of similarity data such that short distances between stimuli correspond to judgments that stimuli are highly similar, with more distant stimuli having been judged less closely related. Each similarity matrix was scaled from 12 initial configurations in addition to the "TORSCA" option that utilizes a pre-iteration procedure to find a favorable initial configuration prior to the actual scaling. Because of the small number of stimuli used in the present study, only two-dimensional solutions were obtained.

The MDS solution for the kindergarten judgments is presented in Figure 1, and a classification of their explanations is presented

INSERT FIGURE 1 ABOUT HERE.

in Table 2. Both sources of data indicate an emphasis on distance in

INSERT TABLE 2 ABOUT HERE.

the sequence of counting numbers as the basis for their judgments. All kindergartners referred to counting at least once to justify judgments, indicating that this is an ubiquitous basis for justifying numerical similarity judgments. The roughly horseshoe-shaped final configuration obtained for the kindergarten frequently indicates an underlying one-dimensional representation (Shepard, 1974), as is shown on the magnitude curve drawn on Figure 1. The disruption in this otherwise regular U-shaped configuration caused by the position of "0" is consistent with the confusion that some of the kindergarten subjects appeared to have over the proper place for zero. These results indicate that 5-year-old children are capable of making judgments about numbers which are presented to them as abstract symbols that correspond to an important numerical property (magnitude, or position in the counting sequence).

The MDS solution for third-graders' judgments is presented in Figure 2. The results for third-graders differed from the kindergarten

INSERT FIGURE 2 ABOUT HERE.

data in two regards. First of all, "0" was clearly located in its proper place in the numerical sequence, ending the confusion that some kindergartners had concerning its proper relation to the other integers. There was also some indication of sensitivity to the doubling relation implied in "3 + 3 = 6" and "2 + 2 = 4", although this doubling was described in terms of addition rather than multiplication. A classification of the explanations given by the third-graders is presented in Table 2, and it is notable for the increase in references to addition in contrast to the emphasis on counting of the kindergartners. Otherwise, the third grade data are most notable for their similarities with the kindergarten data, suggesting that a similar type of representation of number (at least as assessed by this task) would be consistent with both counting and integer addition. This is interesting in light of the research (Groen and Parkman, 1972) suggesting that addition in early elementary school children consists of a rapid counting process, with the exception of rapid addition of doubles such as "2 + 2" and "3 + 3." There is some indication in our results that such doubles are important relations for third-graders, as well as an indication of a general similarity between judgments explained in terms of counting distance and those explained with reference to addition.

Figure 3 contains the MDS solution for the sixth-graders, which is

INSERT FIGURE 3 ABOUT HERE.

notable for a substantial disruption of the systematic magnitude relation present in the kindergarten, and to a lesser extent, the third grade data. Although an axis can be drawn on which the integers can roughly project according to magnitude, it is evident that other criteria were also used by the sixth graders as they made their judgments. Sixth graders generally referred to oddness and evenness of stimuli as a basis for their judgments, as can be seen from Table 3. Multiplicative relations such as divisability by 3 were also cited in explanations and suggested by the scaling solution.

Analysis of the adult data is presented in Figure 4. The adult solution closely resembled results reported by Shepard, Kilpatric and

INSERT FIGURE 4 ABOUT HERE.

Cunningham (1975) for adults judging numbers as abstract concepts. This solution also resembles that obtained for the sixth-graders fairly closely, and provides a clear division into numbers that are even versus those that are odd, as well as sensitivity to magnitude and the suggestion that powers of two and multiples of three are features that influence adults' judgments concerning the similarity of numbers.

The present study suggests that the acquisition of new applications for numbers and new numerical operations is accompanied by the inclusion of increasingly more complex numerical relations in the concept of "number." The performance of the kindergartners appeared based almost exclusively upon the counting distance between numbers. This finding raises questions concerning the extent to which the kindergartners' judgment reflects a special sensitivity to counting distance for numbers, or could be extended to any well-learned ordered list. Accordingly, a second study was undertaken to explore this issue. Two main modifications were made in the second study. Instead of the numbers 0-9 the first ten capital letters (A-J) were used as stimuli, and instead of presenting the child with the task of choosing which two numbers are "most closely related", children were given a more explicit criterion upon which to base their judgments. Subjects were asked to determine from each triad which two letters are closest together, and which two farthest apart in the alphabet song.

Experiment 2.

Method

Apparatus. A letter-wheel apparatus identical in construction to that employed in the first study was constructed for the second study.

Subjects. Subjects were 12 kindergartners, 6 boys and 6 girls (mean age 6 years, 0 months) drawn from a private Philadelphia school. None of the subjects had participated in the first experiment.

Procedure. Subjects were asked if they knew the alphabet song, and requested to sing it. All subjects were able to sing the alphabet song without error. The procedure was otherwise identical to that employed in the first experiment, with the exceptions noted above.

Results.

Correlations of letter-similarity judgments with results of the first study are included in Table 1. These letter-similarity judgments are correlated with the number-similarity judgments of kindergarten, third- and sixth-grade subjects, suggesting that position within a well-known ordered series is a significant factor in number-similarity judgments for all of our groups of children.

Results of the MDS analysis of the kindergarten letter-similarity judgments are presented in Figure 5. This analysis yielded a roughly

INSERT FIGURE 5 ABOUT HERE.

horseshoe-shaped configuration similar to that obtained for the kindergarten number-similarity judgments (see Figure 1). The distortions from a regular horseshoe-shape contained in the KYST-2A output may indicate a tendency on the part of some of the kindergartners to show some local skewing of the alphabet, with accurate representation of the relations between, e.g., G and H, but less understanding of the relation between these letters and the remainder of the alphabet. Alternatively, these small violations from a smooth horshoe-shaped configuration may reflect chance fluctuations resulting from lesser familiarity with letters than with numbers on the part of kindergartners.

Discussion.

The present studies demonstrate three major points concerning the development of children's conceptions of number:

1) Five-year-old children are able to make consistent judgments about numbers presented as numerals reflecting a major feature of numbers (magnitude). Their judgments about relations between numbers when given no context resemble closely their judgments about distance between the members of another familiar ordered list, the alphabet, when given an explicit criteria on which to make their judgments. By the time childern enter school they have extended much of the early number knowledge demonstrated by Gelman and Gallistel (1978) into a consistent understanding of the magnitude relations implicit in numerical symbols.

2) The developmental changes in number similarity judgments suggest a gradual expansion of the concept of number and the set of numerical relations on which similarity judgments are based over the course of elementary school, and apparently continuing until adulthood. This gradual expansion of the concept of number during the course of elementary school complicates Piaget's characterization of number development as the acquisition in middle childhood of a "number concept" based on one-to-one correspondence. In terms of the present study, it may make more sense to speak of a multiplicity of "number concepts" with development consisting of the mastering of a gradually increasing set of such concepts as children become sensitive to new numerical relations.

3) Judgments and explanations of number similarity are related to numerical operations at all points in development. These numerical applications of counting, adding and multiplying appear to have a profound effect on the process of expanding children's conceptions of what numbers are. Work with young children (e.g., Gelman and Gallistel, 1978) has demonstrated that much of preschoolers' conceptual knowledge about numbers is manifested in procedures such as counting, and that these procedures are guided by tacit numerical principles. In the same manner in which preschoolers' knowledge of counting plays a major role in the way they think about number, acquisition of additional numerical procedures seems to play a continuing role in determining how children and adults represent number. Our findings suggest that the mundane arithmetical skills and procedures that Piaget (1941/1965: 29) dismissed as "merely verbal knowledge" may need to be given a more central place in determining the child's conception of number.

Reference notes.

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Table 1
Concordance between number-similarity judgments.[a]

	Kindergarten (numbers)	Kindergarten (letters)	Third-Grade (numbers)	Sixth-Grade (numbers)
Kindergarten (letters)	.934**			
Third-Grade (numbers)	.987**	.930**		
Sixth-Grade (numbers)	.776*	.679*	.814*	
Adults (numbers)	.435	.373	.465	.776*

[a]The concordance statistic is identical to the Pearson product-moment correlation statistic. Statistical significance was assessed following Hubert's (1979) generalized concordance procedures.

*p <.05

**p <.025

Table 2.

Explanations given for similarity judgments.[a]

Category	Example[b]	Kindergarten (numbers)	Kindergarten (letters)	Third-Grade (numbers)	Sixth-Grade (numbers)
Counting distance	"34 (from 134) because 3 comes right before 4."	12	12[c]	10	10
Addition	"14 (from 148) because 4 is only 3 more than 1."	0	0	8	5
Even/odd	"02 (from 027) because they're both even."	0	0	1	10
Divisability	"48 (from 148) because 4 x 2 = 8."	0	0	0	10
Other magnitude	"79 (from 679) because 7 & 9 are both big."	2	1[d]	2	2
Other or no reason	"25 (from 257) because they both go to 10 a lot, like 2 x 5 =10."	3	2	1	1

[a]Number of children in each group (out of 12) giving a particular justification at least once.

[b]Pair chosen as most similar (triad presented in parentheses) and explanation.

[c]Example: "AB (from ABF) because it goes A,B!"

[d]Example: "AC (from ACF) because A is in the beginning and C is, too."

Multidimensional Scaling Solutions

(dimensions and clusters are drawn to illustrate interpretations given in the text)

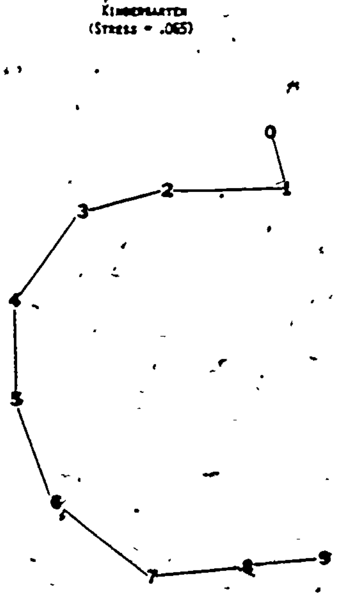

Figure 1

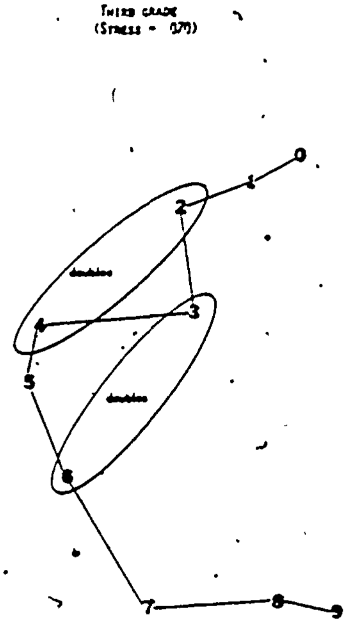

Figure 2

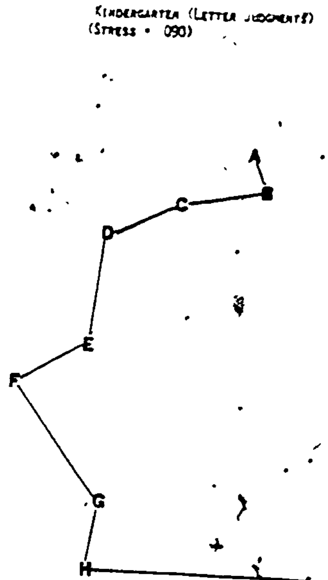

Figure 5

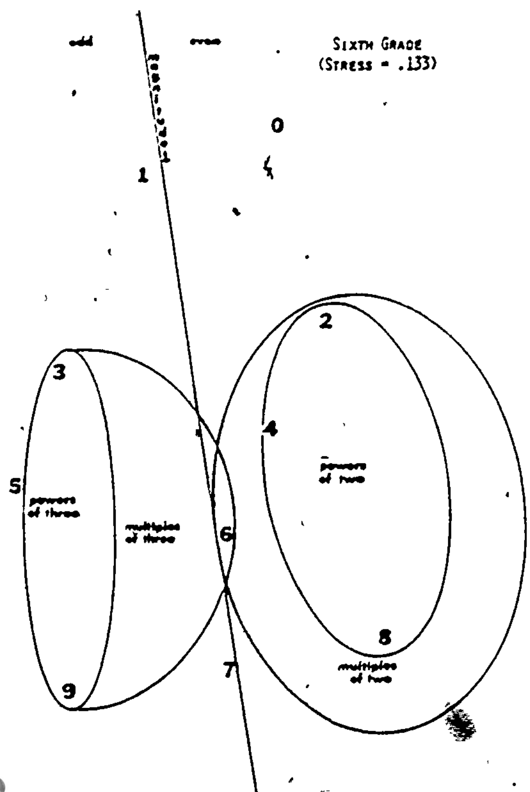

Figure 3

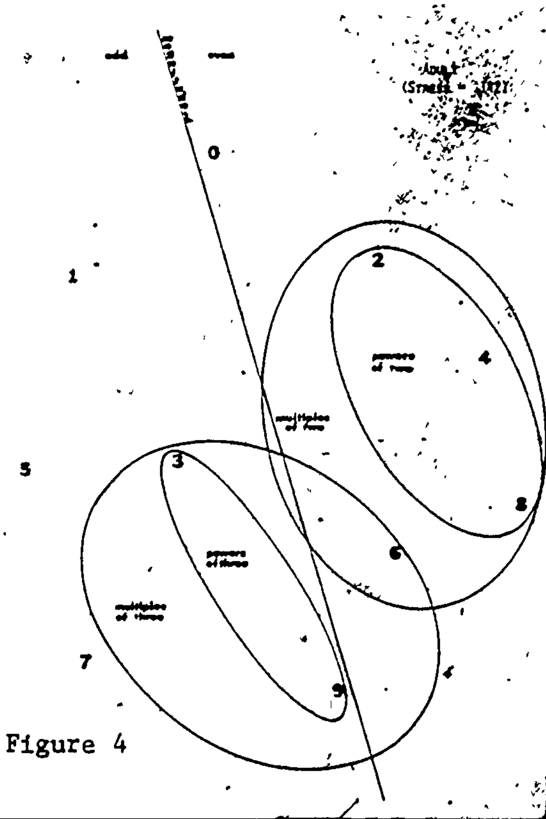

Figure 4