DOCUMENT RESUME

ED 209 093 SE 035 805

AUTHOR Blevins, Belinda; And Others
TITLE What Do Children Know About Addition and Subtraction?
PUB DATE Apr 81
NOTE 11p.; Paper presented at the Meeting of the Society for Research in Child Development (Boston, MA, April, 1981).

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Addition; Algorithms; Basic Skills; *Cognitive Development; Computation; Educational Research; Evaluation; Kindergarten; Learning Theories; *Mathematics Education; Mathematics Instruction; Models; *Preschool Children; *Preschool Education; Problem Solving; *Subtraction; Testing
IDENTIFIERS *Mathematics Education Research

ABSTRACT

The results of an investigation of the development of children's knowledge of addition and subtraction concepts before they start school are detailed. The purpose of the study was to test the predictions of the three-stage model about the distinctions between the last two stages. Twenty-four children participated in the investigation. None of these children had started first grade. The results support the existence of the qualitative and quantitative stages. The data indicate that: (1) Before children begin school and are given formal instruction on addition and subtraction problems, they can use the magnitude of the difference between two arrays and two transformations to make predictions about final numerosities; (2) A method of examining the relationship between understanding addition and subtraction and number conservation now exists; and (3) The distinction between qualitative and quantitative understanding allows for children to solve complex problems without relying on estimator solutions. (MP)

What Do Children Know About Addition and Subtraction?

Belinda Blevins, University of Texas; Patricia Goth Mace, Manchester, CT; Robert G. Cooper, Southwest Educational Development Laboratory; Prentice Starkey, University of Pennslyvania; Edward Leitner, Sunnyvale, CA

Paper presented to the meeting of the Society for Research in Child Development in Boston, April 1981.

What Do Children Know About Addition and Subtraction?

Today I will be presenting the results of an experiment which investigated the development of children's knowledge of addition and subtraction concepts before they start school. In previous work (Cooper, Starkey, Blevins, Goth, Leitner, Note 1) we have found that addition and subtraction concepts undergo developmental changes which are similar in nature to those found for number conservation. Our work has been organized around Gelman's (1972) distinction between estimators and operators. Estimators are procedures for determining the absolute or relative numerosities for one or two arrays. Counting and one-to-one correspondence are estimators. Operators are procedures for determining the effect of transformations on pre-determined numerosities. Addition and subtraction and conservation are all operators.

Much work on addition and subtraction has focused on the relationship of the counting estimator to addition and subtraction (Groen & Parkman, 1972; Groen & Resnick, 1977; Woods, Resnick, & Groen, 1975; Fuson, Note 2; Carpenter & Moser, Note 3). This type of research provides more information about the algorithms children use with the counting estimator than it does about children's understanding of addition and subtraction. The work that has been done on children's understanding of addition and subtraction operators suggests that before children start school they know that an addition means there is more than before, a subtraction means there is less than before, and that addition and subtration are inverse operations (Gelman, 1977; Brush, 1972; Smedslund, 1966a, 1966b, 1966c).

We have incorporated these early understandings into a model which proposes a systematic developmental sequence for addition and subtraction concepts (Cooper et al., Note 1). The model is focused on operator solutions to addition and subtraction problems which are dependent upon an integration of estimator and operator information. The model has three stages, the primitive stage, the qualitative stage, and the quantitative stage. The term stage is used to mean a coherent pattern of behavior. As of yet we do not know the nature of the transition between the stages in the model. The first stage is the primitive stage. In this stage children focus only on the effects of a transformation and believe an addition always means more and a subtraction always means less. This approach works in some cases, but not in others (see Figure 1). The next stage is the qualitative stage and in it children know that addition and subtraction are inverse operations. They pay attention to information about the difference between two arrays and the transformation, but they code such information qualitatively and not quantitatively. This means they will code two arrays as being equal or one array as being greater than or less than the other, but they do not quantify the magnitude of the difference. Children in this stage know that additions or subtractions to unequal arrays can lead to equality, but because they code such information relatively rather than absolutely they make errors on some addition and subtraction problems (see Figure 1). The last stage is the quantitative stage. In this stage children quantify the absolute difference between two arrays and the magnitude of the transformation (see Figure 1).

Our previous work has provided good support for a shift from a primitive understanding to a more advanced understanding. The purpose of this study was to test the predictions of the three-stage model about the distinctions between the last two stages. The last two stages differ from the primitive stage because in both children pay attention to the magnitude of the initial arrays as well as to the magnitude of the transformation. The difference is that the children in the qualitative stage code these magnitudes relatively while the children in the quantitative stage code the absolute difference between them. What has to develop before the quantitative stage is the ability to focus on numerical units. As defined here the quantitative stage is not dependent upon addition and subtraction number facts like 7 + 7 = 14. Rather quantitative problems can be solved by knowing the difference between one and two numerical units.

Because an understanding of addition and subtraction is relevant to understanding number conservation a second purpose of the study was to examine the relationship between number conservation and addition and subtraction. Several studies have examined this relationship because addition and subtraction are presumed to lead to or facilitate the development of number conservation (Wohwill, 1960; Gruen, 1965; Wallach, Wall, & Anderson, 1967). This hypothesized relationship has been most explicitly stated by Peill (1975) who predicts that children come to understand conservation through the use of the rule that amounts are unchanged if nothing is added or subtracted. Previous research has not carefully considered the difference between the levels of difficulty of the tasks used to assess number conservation and addition and subtraction. However, our three stage model provides a good criterion for distinguishing amoung children at different levels of reasoning about addition and subtraction. These levels can then be related to the different levels of reasoning about the number conservation task. We would argue that a qualitative understanding of addition and subtraction is necessary, but not sufficient, for number conservation. For both number conservation and qualitative addition and subtraction tasks children must be able to integrate information about the initial stages of the arrays and the transformation in order to make predictions about the final state.

Twenty-four children with a mean age of 6-3 participated in the study. None of the children had started first grade. Each child was given several tasks in the following order: a small numerosity addition/subtraction warmup trial, qualitative and quantitative addition and subtraction tasks, a small numerosity conservation task, and several large numerosity conservation trials. The children were given the small numerosity conservation trial to guard against response bias, since all previous trials were addition and subtraction trials which emphasized a change in numerosity. There were two linear conservation trials, one was an inequality trial and one was an equality trial, and two nonlinear conservation trials, one was an inequality trial and one was an equality trial. The children were asked for judgements and explanations. For the addition and subtraction task each child was given two qualitative tasks and two quantitative tasks. The structure of these tasks is in Figure 2. For each trial the experimenter presented the child with two cups of marbles and asserted that each cup had the same number of marbles. This procedure was adopted to prevent children from counting. Depending on whether the trial was a qualitative or quantitative trial the experimenter then added one or two marbles to a cup saying "watch what I do, I'm going to put one in this cup." Then the experimenter asked the child if both cups had the same number of marbles, if the cup on the right had more marbles, or if the cup on the left had more marbles. The order of the phrases in this question was systematically varied. This judgement had to be correct before the experimenter proceeded to make the first transformation. After the first transformation the experimenter reasked the same question. Then the experimenter performed the second transformation and asked the question again. The errors on the addition and subtraction tasks fell into predictable patterns so children were taking into account the differences in numerosity between the two arrays. This means that children believed the experimenter's initial assertion about the equality of the cups of marbles.

The results support the existence of the qualitative and quantitative stages.

Correct judgements on both phases of a trial were required before a trial was correct. As you can see in Table 1 children performed better on qualitative than quantitative trials. Performance on both qualitative and quantitative trials was better than chance. An analysis of errors made on the first phase of each trial revealed that of the eight errors on the quantitative trials, five were qualitative and three were primitive, and that the nine errors on the qualitative trials were all primitive. Performance on only the first phase of the trial was examined because when children missed the judgement after the second transformation it was not clear what they conceived the pretransformation relationship between the two arrays to be.

The conservation data were scored for correctness on both judgements and explanations. For those concerned about the coherence of the concept of number conservation it is encouraging to note that the linear and nonlinear trials were highly correlated, $r=.87$, $p < .05$; as were the equality and inequality trials, $r=.83$, $p < .05$. The relationship between addition and subtraction and conservation was examined by classifying children into one of the three addition and subtraction stages and into one of three categories for conservation. The conservation categories were determined by using a scoring system whose goal was to assign 0 points for nonconservation, one point for correct judgements without correct explanations, and two points for correct judgements and explanations. These results are shown in Table 2. As the level of addition and subtraction understanding got higher so did the level of conservation performance.

The addition of the distinction between the qualitative and quantitative stages expands our knowledge in three ways. First, it indicates that before children begin school and are given formal instruction on addition and subtraction problems they can use the magnitude of the difference between two arrays and two transformations to make predictions about final numerosities. Second, it provides us with a method of examining the relationship between understanding addition and subtraction and number conservation. Our data indicates that there is a linear trend between addition and subtraction understanding and conservation understanding. Most of the children who had correct judgements on the conservation trials were at least in the qualitatiatve stage. This suggests that qualitative changes in thought related to the conservation and addition and subtraction operators may be general characteristics of thought in number development. Third, the distinction between qualitative and quantitative understanding indicates that children can solve complex problems without relying on estimator solutions. Instead the correct solutions to these addition and subtraction problems rely on an integration of estimator and operator information. Our research points to the need for examining the interaction of estimators and operators with development.

One issue this research raises is what is the nature of the shift from the qualitative to quantitative stages? This shift could be a change from being able to deal with one unit of difference to being able to deal with two. If this were the case we would expect that future changes would follow a pattern of progressive arithmetization. The shift could also be a more general shift with the characteristic that children are now able to deal with any size difference between two arrays in terms of units. Our current research addresses this issue.

Reference Notes

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Figure 1

Predicted Response to a Trial by Three Addition/Subtraction Reasoning Levels

Trial Types

Levels of reasoning	A N vs N	B N vs N+1	C N vs N+2
	$\begin{matrix}8 \xrightarrow{+1} 9\\ 8 \longrightarrow 8\end{matrix}$	$\begin{matrix}7 \xrightarrow{+1} 8\\ 8 \longrightarrow 8\end{matrix}$	$\begin{matrix}7 \xrightarrow{+1} 8\\ 9 \longrightarrow 9\end{matrix}$
primitive	correct	(7+1) > 8	(7+1) > 9
qualitative	correct	correct	(7+1) = 9
quantitative	correct	correct	correct

Figure 2

Trial Structures Used in Addition/Subtraction Task

Qualitative Trials

$$\begin{matrix}N\\N+1\end{matrix} \xrightarrow{+1 \text{ (top)}} \begin{matrix}N+1\\N+1\end{matrix} \xrightarrow{+1 \text{ (bottom)}} \begin{matrix}N+1\\N+2\end{matrix}$$

$$\begin{matrix}N\\N+1\end{matrix} \xrightarrow{-1 \text{ (bottom)}} \begin{matrix}N\\N\end{matrix} \xrightarrow{+1 \text{ (bottom)}} \begin{matrix}N\\N+1\end{matrix}$$

Quantiative Trials

$$\begin{matrix}N\\N+2\end{matrix} \xrightarrow{+1 \text{ (top)}} \begin{matrix}N+1\\N+2\end{matrix} \xrightarrow{-1 \text{ (bottom)}} \begin{matrix}N+1\\N+1\end{matrix}$$

$$\begin{matrix}N\\N+2\end{matrix} \xrightarrow{-1 \text{ (bottom)}} \begin{matrix}N\\N+1\end{matrix} \xrightarrow{-1 \text{ (bottom)}} \begin{matrix}N\\N\end{matrix}$$

Table 1

Performance on Addition/Subtraction Tasks

Type of Trial	Mean % Correct
Qualitative	81%
Quantitative	63%

Table 2

Performance on Addition/Subtraction Tasks and Number Conservation Tasks

Addition/Subtraction Reasoning Levels	Conservation Performance (range 0 to 2)	Number of Subjects
primitive	.57	7
qualitative	1	6
quantitative	1.64	11