DOCUMENT RESUME

ED 201 524 SE 034 925

AUTHOR	Lindvall, C. Mauritz; Ibarra, Cheryl Gibbons
TITLE	The Development of Problem-Solving Capabilities in Kindergarten and First-Grade Children.
INSTITUTION	Pittsburgh Univ., Pa. Learning Research and Development Center.
SPONS AGENCY	National Inst. of Education (DHEW), Washington, D.C.
REPORT NO	LRDC-1979/21
PUB DATE	79
NOTE	21p.; Contains occasional small print which may not reproduce well.
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Arithmetic; Cognitive Development; *Cognitive Processes; Educational Research; *Elementary School Mathematics; *Grade 1; Learning Processes; Learning Theories; *Mathematical Models; Models; Primary Education; *Problem Solving
IDENTIFIERS	Heuristics; *Mathematics Education Research

ABSTRACT

This study investigated a theoretical structure which attempts to explain the problem-solving strategies used by first-grade students who are successful in solving simple arithmetic story problems. The structure used describes the problem-solving procedure as consisting of a three-stage model building process: (1) comprehending the story, (2) building a general conceptual model, and (3) generating a mathematical model. Results, based on a study of the performance of 81 students, appear to support the proposed theoretical structure in that essentially all students who can give the correct answer to a problem are able to develop a general model of the problem. (Author)

LEARNING RESEARCH AND DEVELOPMENT CENTER

1979/21

THE DEVELOPMENT OF PROBLEM-SOLVING CAPABILITIES IN KINDERGARTEN AND FIRST-GRADE CHILDREN

C. MAURITZ LINDVALL AND CHERYL GIBBONS IBARRA

University of Pittsburgh

THE DEVELOPMENT OF PROBLEM-SOLVING CAPABILITIES
IN KINDERGARTEN AND FIRST-GRADE CHILDREN

C. Mauritz Lindvall and Cheryl Gibbons Ibarra

Learning Research and Development Center

University of Pittsburgh

1979

The research reported in this paper was supported by the Learning Research and Development Center, funded in part as a research and development center by the National Institute of Education (NIE), United States Department of Health, Education, and Welfare. The opinions expressed do not necessarily reflect the position or policy of NIE, and no official endorsement should be inferred.

Abstract

This study investigated a theoretical structure which attempts to explain the problem-solving strategies used by first-grade students who are successful in solving simple arithmetic story problems. The structure used describes the problem-solving procedure as consisting of a three-stage model building process: (a) comprehending the story, (b) building a general conceptual model, and (c) generating a mathematical model. Results, based on a study of the performance of 81 students, appear to support the proposed theoretical structure in that essentially all students who can give the correct answer to a problem are able to develop a general model of the problem.

THE DEVELOPMENT OF PROBLEM-SOLVING CAPABILITIES IN KINDERGARTEN AND FIRST-GRADE CHILDREN

C. Mauritz Lindvall and Cheryl Gibbons Ibarra

Learning Research and Development Center
University of Pittsburgh

One of the more obvious settings for investigating the development of problem-solving capabilities in young children is in the efforts of kindergarten and first-grade pupils to solve simple arithmetic story problems. By studying children who are just entering kindergarten, one can see how pupils who have no formal command of the arithmetic operations of addition and subtraction attempt to analyze and solve the problems posed by such stories. Then by examining the efforts of first-grade children after they have been introduced to addition and subtraction, it is possible to study what is involved in children's early attempts to associate simple "mathematical models" to the solution of such quantitative problems. Working with children at these grade levels who are acquiring capabilities in solving problems of this general type provides some insight concerning possible stages in the development of such problem-solving abilities. This was the purpose of the study reported here.

The study of factors related to the success that elementary school pupils have in solving story problems has been a focus of many investigators. Some of this research has investigated the effect of various characteristics of the story (Beardslee & Jerman, 1973; Linville, 1976; Loftus & Suppes, 1972; Rosenthal & Resnick, 1974; Steffe, 1967) and has served to identify those qualities that appear to be related to pupil performance. Other studies (Balow, 1964; Henney, 1970; Knifong & Holtan, 1976a, 1976b) have been concerned with the pupil abilities that are used in solving such problems and have investigated such things as reading ability and computational ability. All of these studies have certain implications for the planning of instruction and for the diagnosis of individual pupil difficulties.

Other investigators, studying the procedures and processes used by students, have produced results that bear more directly on the focus of the present research. For example, one group of studies has focused on the effect of processes used by children and studied the extent to which pupil use of manipulative aids has led to greater success in problem solving. Four of these studies, conducted with first-grade children (Bolduc, 1970; LeBlanc, 1968; Steffe, 1966, 1970; Steffe & Johnson, 1971), indicate that, in general, children are more successful in solving problems when they can use manipulative aids. Evidently such aids permit students to develop a meaningful physical "model" of a story. Problem solution also appears to be aided when the manner in which the problem is presented includes some type of concrete representation in addition to the verbal description (Gibb, 1956). This would appear to indicate that young children are better able to visualize or model the story when the presentation includes some aid to this process. All of these studies on the processes that children use in solving story problems suggest that the more successful students probably represent the essential elements of the story through some meaningful model. That is, they approach these stories as a true problem-solving activity.

As a result of the above view of the results of past research and because of some recent investigations that they have carried out, the present writers feel that if primary grade pupils are to acquire capabilities that will increase the likelihood that they will continue to be successful problem solvers in later grades, it is important that the procedures they use in the primary grades be based on a complete understanding of the problems rather than on the application of some memorized rules which may work with most simple problems but which are of no value with more complex problems. It is further suggested here that some understanding of what these meaningful solution procedures might be can be obtained by analyzing the processes used by successful primary grade problem solvers in terms of some of the components that have been found to be significant in the problem-solving behavior of adults. This general approach has previously been suggested by Heller and Greeno (1978). Although some mathematics educators suggest that the solution of most arithmetic problems is a matter of merely applying the proper algorithm, Heller and Greeno (1978) have presented a strong case for the point that even apparently simple arithmetic problems present most of the characteristics of a real problem situation, particularly for the relatively naive student.

Newell and Simon (1972) have developed a rather complete theory of human problem solving derived largely from their study of how adults solve certain types of problems. Their theory as to how humans carry out the planning involved in problem solving suggests the following four steps:

> 1. Abstracting by omitting certain details of the original objects and operators.
> 2. Forming the corresponding problem in the abstract problem space.
> 3. When the abstract problem has been solved, using its solution to provide a plan for solving the original problem.
> 4. Translating the plan back into the original problem space and executing it. (Newell & Simon, 1972, p. 429)

In a recent study of problem solving in physics, Simon and Simon (1978) report that an expert problem solver examines the problem, then attempts to translate it into some type of physical representation before deriving the mathematical expression that can be used to solve it. Similarly, the work of Larkin (1977) suggests that the successful problem solver analyzes the problem in some qualitative form before representing it mathematically. The work of all of these investigators indicates that effective problem solvers make use of an intermediate stage between their initial comprehension of the problem and their development of a mathematical equation for solving it. In this stage they abstract the essential information and develop some type of conceptual representation of the problem, mental or physical. From their review of past work on problem solving and their own work with arithmetic word problems, Heller and Greeno (1978) describe the place of this intermediate stage as follows:

> Word problems can be grossly characterized as well-structured problems of transformation; definite initial and goal states are both given, and there exists a set of possible operators for moving toward solution. However, mediating representational and initial elaboration processes are required to transform word problem texts into problem spaces suitable for solution with understanding. . . . Equations are evoked or constructed and instantiated on the basis of these analytical processes. (p. 16)

In attempting to analyze what is involved when a first-grade student learns to generate an appropriate addition or subtraction sentence from a simple story problem and then proceeds to solve the sentence, the writers have chosen to employ a structure that is an adaptation of what has been proposed in these recent studies. This structure is derived from a "modeling" frame of reference and outlines the problem-solving process in terms of three stages.

Stage 1. The substantive problem is identified and examined.

Stage 2. The essentials of the substantive problem are abstracted and the problem is formulated in terms of some simplified general conceptual model (i.e., a representation involving abstracted general concepts).

Stage 3. The general model is reformulated as a mathematical model that can be used in problem solution.

In the case of the primary grade pupil writing a simple number sentence for a given story problem, these three stages may be exemplified as follows:

Stage 1. The child comprehends the story problem after reading it or hearing it read. The comprehension involved at this stage need only be that evidenced by the ability to repeat the story in a paraphrased form and would be identified as the "translation" level of the Bloom Taxonomy (Bloom, 1956).

Stage 2. The story problem is reformulated in the child's mind (or in some overt representation) in terms of a general conceptual model (e.g., "This is a problem where someone had a certain number of things and then got some more so I have to find how many there are if I put those two amounts together"). Child may use fingers, blocks, etc., at this stage.

Stage 3. The child uses this general model to write an appropriate number sentence (e.g., "3 + 4 = 7"). This sentence is actually a simple mathematical model appropriate for this problem.

The diagram in Figure 1 is intended to show the relationship of these three stages and also to indicate the solution procedure that may be employed if children have the capabilities represented by each stage.

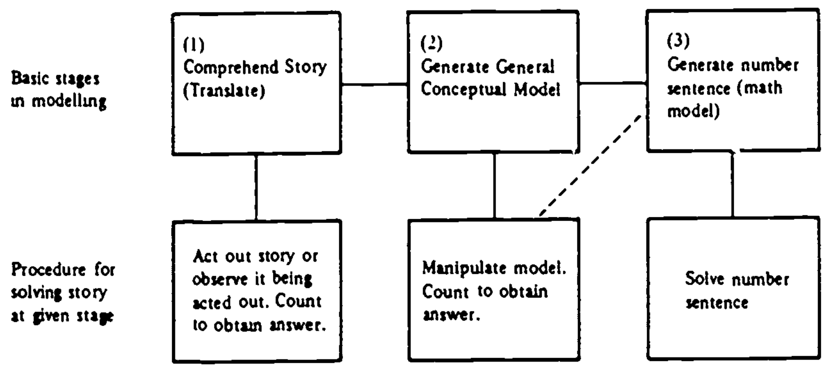

Figure 1. Suggested stages used by students in developing appropriate number sentence (or mathematical model) for a story problem showing the problem-solving procedure that could be used at each stage.

Previous research conducted by the writers (Ibarra & Lindvall, 1979) indicates that kindergarten children who have had no formal introduction to the use of simple addition and subtraction sentences can operate at either Stages 1 or 2 in the proposed three-stage model to obtain the correct answer for simple story problems. However, this earlier research showed that the proportion (.75) of students who could solve a story when it was acted out for them (that is, when both the sets described and any operations on these sets were demonstrated by the tester) was much greater than the proportion (.42) of students who could solve it when they had to generate their own models. This was taken to mean that being able to generate a model for a story represented a higher level of ability than being able to solve the story when it was acted out for the pupil. This appears to indicate the reality of Stages 1 and 2 as representing stages in increasing competency in problem solving. To further investigate the three-stage model, it then appeared useful to examine the relationship of Stage 2 to Stage 3. This was the purpose of the study reported in the present paper.

An examination of the diagram in Figure 1 will show that being able to generate a general conceptual model can result in a solution of the problem by manipulating that model directly or can lead to the generation of a number sentence which can then be solved, in turn, to find the answer to the story. This indicates that being able to generate a general conceptual model for a story is prerequisite both to solving the problem by using the model and to writing the appropriate number sentence for the story. Actually, with stories of the degree of simplicity used in the first grade, being able to generate a general conceptual model of the story almost automatically results in its solution. This should mean that most pupils who can generate the model will solve the story. That is, there should be a strong positive correlation between these two abilities. Also, it has been found (Steffe, Hirstein, & Spikes, 1976) that at this level most students who can write a number sentence for a story first solve the story by some other means and then write the number sentence. This suggests that the ability to solve the story should be found to be a prerequisite to writing a number sentence for it. The dashed line in Figure 1 is intended to indicate this relationship.

Quite obviously, the relationships indicated in Figure 1 suggest that pupils can solve a story problem without generating a number sentence.

The Problem

The relationships implied by Figure 1 and discussed above may be summarized in the following hypotheses.

1. There should be a strong positive relationship between pupil ability to generate a general conceptual model for a story and the ability to solve the story.
2. The ability to generate a general conceptual model for a story is prerequisite to the ability to write a number sentence for this story.
3. The ability to solve a story of the simple type used in the introduction of the basic concepts of addition and subtraction is prerequisite to writing an appropriate number sentence for the story.
4. The ability to write a number sentence for a simple story is not prerequisite to the ability to solve it.

Method

Subjects

The hypotheses defined above were investigated by using an individual interview procedure to obtain data on pupil capabilities from a sample of 81 first- and second-grade pupils. The sample consisted of approximately 20 pupils from a school in each of four different types of communities: (a) industrial, (b) suburban, (c) rural, and (d) a university laboratory school. In three of these schools the pupils involved were first graders tested in April, while in one school the pupils were second graders tested in October. The curricula in these schools were such that all pupils had received instruction in the writing of addition and subtraction sentences and on the related number facts.

Since results revealed no major differences among pupils from these four schools in terms of the variables of interest in this study, results from all schools were combined into one sample of 81 students for the analysis presented in this paper.

Procedure

In this study each pupil was tested on six different arithmetic story problems, each of which could be represented by an addition or subtraction sentence with a specific term missing. These stories, together with the number sentence which most closely parallels the structure of each story, are shown in Figure 2.

	Story Problems	Number Sentence for Story
1.	Jenny ate 3 cookies. Amy ate 4 cookies. How many cookies did they eat altogether?	$a + b = \square$
2.	Jim worked 8 problems. Andy worked 2 problems less than Jim. How many problems did Andy work?	$a - b = \square$
3.	Jane has 8 cookies. She gave some cookies away and only has 3 cookies left. How many cookies did she give away?	$a - \square = c$
4.	For her birthday, Sue got 4 books from her parents. Sue's grandmother also gave her some books. In all, Sue got 6 books. How many books did Sue's grandmother give her?	$a + \square = c$
5	Bill has some cars. His friend gave him 2 more cars. Now Bill has 5 cars. How many cars did Bill have in the beginning?	$\square + b = c$
6.	Joe has some comic books. He gave 5 of them to Sam. Now Joe has 4 comic books left. How many comic books did Joe have in the beginning?	$\square - b = c$

Figure 2. Story problems used in the study with the number sentence which most closely parallels the structure of each story

With each problem the story was read to the student and he or she was asked: (a) to give the answer, (b) to write a number sentence that could be used to represent the story and to find the answer, and (c) to use counting cubes to explain the story. For each of these three tasks, the pupil performance was judged as "pass" or "fail." The criterion for passing the first task was merely that of giving the correct answer. To earn a "pass" on writing the number sentence for story problems 1 and 2, as shown in Figure 2, the pupil was expected to write the sentence in the form shown in the right hand column of the Figure with the proper numeral inserted in each of the three positions. For story problems 3 through 6 a "pass" was given if the pupil either wrote the sentence in the form shown in Figure 2 or the form which showed the missing term as the "answer" (e.g., for story 3 either $4 + 2 = 6$ or $6 - 4 = 2$ was accepted as pass). On the third task, the pupil was expected to use the counting cubes to show each of the two given sets and the set representing the missing term and to explain the relationships among the sets in order to receive a "pass."

To investigate the four hypotheses posed for this study, the pass-fail results for each possible pairing of the three variables were examined through the use of simple two-by-two contingency tables. These are presented in Tables 1-3.

Results

Table 1 provides data showing the relationship of ability to solve the story problem to the ability to build or generate a model for the problem using the counting cubes provided. A contingency table is shown for each of the six story types. These data appear to support the hypothesis that there is a strong positive relationship between these two abilities. A possible explanation for the fact that some students could solve the story even though they could not build the model as called for in this testing situation may be that these students are able to use some other type of model. The performance of the few students who could build the model but did not arrive at the correct solution may have resulted from relatively mechanical errors such as incorrect counting.

Table 1

Contingency Tables Showing Relationship of Ability to Build a Model for a Story Problem to Ability to Solve It for each of the Six Types of Stories.

		Solve Story			Solve Story			Solve Story	
		F	P		F	P		F	P
Build Model	P	1	58	P	2	50	P	4	65
	F	16	6	F	24	5	F	9	3
		$a + b = \square$			$a - b = \square$			$a - \square = c$	
		$r_{phi_1} = .78$			$r_{phi_2} = .81$			$r_{phi_3} = 67$	
		F	P		F	P		F	P
Build Model	P	2	59	P	0	57	P	6	49
	F	18	2	F	19	5	F	25	1
		$a + \square = c$			$\square + b = c$			$\square - b = c$	
		$r_{phi_4} = 87$			$r_{phi_5} = .85$			$r_{phi_6} = 82$	

Hypotheses 2, 3, and 4, as formulated for this study, are concerned with prerequisite relationships rather than correlations. In studying prerequisite relationships through the use of contingency tables, such as those presented in Tables 2 and 3, the essential data to be examined are the frequencies in certain cells rather than the phi coefficients. For this reason the following discussion of Tables 2 and 3 focuses on the relevant cell frequencies, and the phi coefficients presented with each contingency table are provided merely for descriptive purposes.

The results in Table 2 provide data for investigating the hypothesis that ability to build a model for a story is a prerequisite to being able to write a number sentence for it. The r latively small number of students who failed in building the model but were able to write the number sentence provides support for the hypothesis. Again, these few students may have been able to use some model other than the one required in the test. The fact that a certain number of students could not write the sentence even though they could build the model may merely mean that these students have not yet learned to write such sentences.

Table 2

Contingency Tables Showing Relationship to Ability to Build a Model for a Story Problem to Ability to Write a Number Sentence for It.

Write Number Sentence	Build Model F	Build Model P
P	6	46
F	16	13

$a + b = \square$

$r_{phi_1} = .45$

Write Number Sentence	Build Model F	Build Model P
P	0	46
F	29	6

$a - b = \square$

$r_{phi_2} = .86$

Write Number Sentence	Build Model F	Build Model P
P	3	64
F	9	5

$a - \square = c$

$r_{phi_3} = 63$

Write Number Sentence	F	P
P	1	56
F	19	5

$a + \square = c$

$r_{phi_4} = 82$

Write Number Sentence	F	P
P	4	51
F	20	6

$\square + b = c$

$r_{phi_5} = 68$

Write Number Sentence	F	P
P	5	47
F	21	8

$\square - b = c$

$r_{phi_6} = 65$

Table 3 also provides evidence concerning prerequisite relationships. It supports the hypothesis that, at this level, being able to solve the story is probably a prerequisite to being able to write a number sentence for it. This would support the findings of other researchers (Steffe, Hirstein, & Spikes, 1976) that when young children are asked to write a number sentence in solving a story problem, they first solve the story and then write the number sentence. The Table 3 contingency tables also show that it is not necessary to be able to write a number sentence for a story to be able to solve the problem posed in the story. Some students evidently solve the story using the Stage 2 capabilities.

Table 3

Contingency Tables Showing Relationship of Ability to Write a Number Sentence for a Story Problem to Ability to Solve It.

		Solve Story			Solve Story			Solve Story	
		F	P		F	P		F	P
Write Number Sentence	P	5	47	P	2	44	P	4	63
	F	12	17	F	24	11	F	9	5
		a + b = □			a − b = □			a − □ = c	
		r_{phi_1} = .37			r_{phi_2} = 68			r_{phi_3} = 60	
		F	P		F	P		F	P
Write Number Sentence	P	1	56	P	1	54	P	7	45
	F	19	5	F	18	8	F	24	5
		a + □ = c			□ + b = c			□ − b = c	
		r_{phi_4} = 82			r_{phi_5} = .74			r_{phi_6} = .68	

Discussion

This study investigated a simple theoretical structure which attempts to explain the problem-solving strategies employed by first-grade students in solving simple arithmetic story problems. This structure, outlined in Figure 1, describes the problem-solving procedure as consisting of a three-stage model building process: (a) comprehending the story, (b) building a general conceptual model, and (c) generating a mathematical model (the appropriate number sentence). This simple three-stage structure corresponds to that proposed by several investigators (Heller & Greeno, 1978; Larkin, 1977; Newell & Simon, 1972; Simon & Simon, 1978) working with a variety of types of problems.

In general, this three-stage description of the problem-solving process when applied to the grade levels and the content that was the focus of this study can be considered as identifying three levels at which pupils may understand a story and at which they might work out a solution. Since previous studies by the writers (Ibarra & Lindvall, 1979) had shown that kindergarten pupils who were unable to solve a story problem under other conditions were able to solve it when they were permitted to act it out or had it acted out for them (i.e., were able to solve it using Stage 1 procedures), the present study focused on Stages 2 and 3.

The results of the study indicate that the students who are successful in solving stories of this type (when acting it out is not an available strategy) are able to generate some type of general conceptual model. This is taken to mean that successful problem solvers abstract the general and essential elements of a story and then proceed to organize and manipulate these elements, either mentally or physically. For example, we might consider the simple story "Tom had three apples. His brother Bill had two apples. How many apples did the two brothers have altogether?" Here the successful problem solver abstracts the essential ideas that we have three "things" in one place and two "things" in another place and that to find "how many altogether" one must count the total in these two sets. This person then has no trouble in modeling the problem by building two sets of blocks of appropriate size, by using the fingers on each hand to represent the two sets, or by using some other countable objects. This type of representation, or model, can then be used to solve the story directly or it can be used as the basis for writing an appropriate number sentence.

The study also supports the idea that pupils who can write a number sentence for a story are also able to develop a general conceptual model and to solve the story using such a model. That is, with this level of content the students appear to be able to write the number sentence for a story only when they are already able to solve it by using a general conceptual model. It seems rather clear that the typical first-grade student does not formulate the incomplete number sentence as a step in problem solution and then find the answer to the story by solving this number sentence. The number sentence is only a way of modeling a solution process that has already been completed. Actually using the number sentence to solve a problem is a capability that is probably acquired at some higher grade level where problems become more complex (or involve larger numbers) and the use of an arithmetic operation is an essential aid to solution.

In general, the results of this study imply that teaching pupils how to solve arithmetic story problems should always involve the intermediate step of making certain that the pupils can translate the story into some type of meaningful general conceptual model before they attempt to write a number sentence for it. That is, they should be taught to model the essential elements of the story through some type of physical representation that will contribute to their understanding of the problem and clarify the process needed for solution. Placing an emphasis on this intermediate step should mean that students would not only become more proficient in solving these arithmetic story problems but should also increase the likelihood that they would acquire a general problem-solving strategy that appears to be characteristic of effective adult problem solvers.

References

Balow, I. H. Reading and computation ability as determinants of problem solving. The Arithmetic Teacher, 1964, 11(2), 18-22.

Beardslee, E. C., & Jerman, M. E. Linguistic variables in verbal arithmetic problems. University Park, Pa.: Pennsylvania State University, 1973. (ERIC Document Reproduction Service No. ED 073 926)

Bloom, B. S. (Ed.). A taxonomy of educational objectives, Handbook 1, Cognitive domain. New York: Longmans, Green, 1956.

Bolduc, E. J., Jr. A factoral study of the effects of three variables on the ability of first-grade children to solve arithmetic addition problems (Doctoral dissertation, University of Tennessee, 1969). Dissertation Abstracts International, 1970, 30, 3358A.

Gibb, E. G. Children's thinking in the process of subtraction. Journal of Experimental Education, 1956, 25, 71-80.

Ginsburg, H. Learning difficulties in children's arithmetic: A clinical cognitive approach. In A. Osborne & D. A. Bradbard (Eds.), Models for learning mathematics, papers from a research workshop, 1976. (ERIC Document Reproduction Service No. ED 128 226)

Heller, J. I., & Greeno, J. G. Information processing analyses of mathematical problem solving. Paper presented at the National Science Foundation Seminar on Applied Problem Solving, Northwestern University, Evanston, Illinois, March 1978.

Henney, M. Improving mathematics verbal problem solving ability through reading instruction. Paper presented at the conference of International Reading Association, Anaheim, California, May 1970. (ERIC Document Reproduction Service No. ED 044 243)

Ibarra, C. G., & Lindvall, C. M. An investigation of factors associated with children's comprehension of simple story problems involving addition and subtraction prior to formal instruction on these operations. Paper presented at the National Council of Teachers of Mathematics Meeting, Boston, Mass., 1979.

Knifong, J. D., & Holtan, B. D. An analysis of children's written solutions to word problems. Journal for Research in Mathematics Education, 1976, 7(2), 106-112. (a)

Knifong, J. D., & Holtan, B. D. A search for reading difficulties among erred word problems. Morgantown: West Virginia University, 1976. (b)

Larkin, J. H. Skilled problem solving in physics: A hierarchical planning model. Unpublished manuscript. Berkeley, Calif.: University of California, 1977.

LeBlanc, J. F. The performance of first grade children in four levels of conservation of numerousness and three I. Q. groups when solving arithmetic subtraction problems (Doctoral dissertation, University of Wisconsin, 1968). Dissertation Abstracts, 1968, 29, 67A-68A.

Lindvall, C. M., & Ibarra, C. G. The relationship of mode of presentation and of school/community differences to the ability of kindergarten children to comprehend simple story problems. Paper presented at the meeting of the American Educational Research Association, San Francisco, California, April 1979.

Linville, W. J. Syntax, vocabulary, and the verbal arithmetic problem. School Science and Mathematics, 1976, 76(2), 152-158.

Loftus, E. F., & Suppes, P. Structural variables that determine problem-solving difficulty in computer-assisted instruction. Journal of Educational Psychology, 1972, 63(6), 531-542.

Newell, A., & Simon, H. A. Human problem solving. Englewood Cliffs, N.J.: Prentice-Hall, 1972.

Rosenthal, D. J., & Resnick, L. B. Children's solution processes in arithmetic word problems. Journal of Educational Psychology, 1974, 66(6), 817-825.

Simon, D. P., & Simon, H. A. Individual differences in solving physics problems. In R. S. Siegler (Ed.), Children's thinking: What develops? Hillsdale, N. J.: Lawrence Erlbaum Associates, 1978.

Steffe, L. P. The performance of first grade children in four levels of conservation of numerousness and three IQ groups when solving arithmetic addition problems (Technical Report No. 14). Madison, Wisconsin: Research and Development Center for Learning and Re-education, 1966.

Steffe, L. P. The effects of two variables on the problem-solving abilities of first grade children (Technical Report No. 21). Madison, Wisconsin: Wisconsin Research and Development Center for Cognitive Learning, 1967.

Steffe, L. P. Differential performance of first-grade children when solving arithmetic addition problems. Journal for Research in Mathematics Education, 1970, 1(3), 144-161.

Steffe, L. P., Hirstein, J. J., & Spikes, W. C. Quantitative comparisons and class inclusion as readiness variables for learning first grade arithmetic content. Athens: University of Georgia, 1976.

Steffe, L. P., & Johnson, D. C. Problem solving performances of first grade children. Journal for Research in Mathematics Education, 1971, 2(1), 50-64.