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ABSTRACT

This paper reports the results of the second of a series of collaborative studies examining how children acquire the skills to represent and solve verbal addition and subtraction problems. The purpose of this study was to identify the cognitive processing capabilities of a group of Tasmanian (Australian) children. Fifteen cognitive tests were administered to 122 4- to 8-year-old children enrolled at Sandy Bay Infant School in Hobart, Tasmania. The data derived from the tests were examined in four ways. First, responses were scored and distributions for the population were prepared. From this data it was evident that five of the tests failed to differentiate the children. Second, the relationship of the scores for the 10 remaining tests were examined and this analysis revealed one primary factor reflecting quantitative skills, and another reflecting qualitative transformations. Third, since the primary factor accounted for most of the variance, the hierarchical order of the tests was examined. No evidence of such an order was found. Finally, the test scores for the 10 differentating tests were related to four memory (M-space) tests given earlier to the same population. The pattern of responses on the cognitive tests for six groups identified via a cluster analysis of the M-space data demonstrated systematic differences between five of the six groups. (Author/MP)

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Technical Report 539

The Assessment of Children's Cognitive Processing Capabilities

by

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3

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Table of Contents

List of Tables

ERIC

List of Tables (Continued)

ERIC

List of Figures

Abstract

This paper reports the results of the second of a series of collaborative studies examining how young children acquire the skills to represent and solve verbal addition and subtraction problems. The purpose of this study was to identify the cognitive processing capabilities for a group of children. Fifteen cognitive tests were administered to 122 children of ages four to eight who were enrolled at Sandy Bay Infant School in Hobart, Tasmania.

The data derived from the tests were examined in four ways. First, responses were scored and distributions for the population (and for classes) were prepared. From this data it was evident that five of the tests failed to differentiate the children.

Second, the relationship of the scores for the ten tests were examined: The correlation of the test scores, while all positive, were not particularly high. A factor analysis of the test correlations revealed one primary factor which reflects quantitative skills influenced by the ability to count. A second, and less important factor was suggested which reflects qualitative transformations.

Third, since the first factor accounted for most of the variance, the hierarchical order of the tests was examined. No evidence of such an order was evident.

Finally, the relationship of test scores for these ten tests was related to the four memory (M-space) tests given earlier to the same population. (Romberg and Collis, 1980). A combined factor analysis revealed the same two dimensional structure with the memory tests loading on the first qualitative factor. The pattern of responses on the cognitive tests for the six groups identified via a cluster analysis of the M-space data demonstrated

systematic differences between five of the six groups.

In summary, simple correspondence appears to be the first cognitive processing capacity to develop and can be handled by children at M-space Level 1. This is followed by a qualitative correspondence capacity which involves understanding how correspondence between two sets is preserved or changed under varying circumstances and can be handled by children at M-space Level 2. Next, quantitative counting skills are developed and are related to M-space Level 3. This is followed by transitive and logical reasoning.

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This paper reports the results of the second of a series of related, collaborative studies examining how young school children acquire the skills needed to represent symbolically a variety of addition and subtraction situations. The evolution of children's performance on tasks requiring these skills must be related both to their cognitive abilities and to their engagement in instructional activities on related tasks. The specific purpose of this second study was to identify the cognitive processing capabilities of a group of children of ages 4 to 8. These capabilities are hypothesized to be critical factors in cognitive development. They also serve to identify individual differences which can be used to design instructional lessons.

The Collaborative Studies

This series of studies on children's acquisition of addition and subtraction skills is jointly funded by the Research Committee of the Graduate School at the University of Wisconsin, the University of Wisconsin Research and Development Center for Individualized Schooling and the University of Tasmania. The principal investigators of the studies brought different backgrounds and skills to this collaborative effort. The identification of cognitive abilities grows out of Professor Collis' extensive work in cognitive development (e.g., K.F. Collis and J.B. Biggs, Classroom Examples of Cognitive Development Phenomena: The SOLO Taxonomy, 1979). The classroom engagement ideas stem from Professor Romberg's research on teaching (e.g., T.A. Romberg, M. Small, and R. Carnahan, Research on Teaching from a Curricular Perspective, 1980).

The strategy adopted for the sequence of collaborative studies involves five steps:

1. identify M-space for a population of children of ages 4-8 (this step was reported in the first technical report on this study, Romberg and Collis, 1980);

2. identify "cognitive processing capabilities" for the same set of children;

3. from (1) and (2) identify a well-defined set of children with specific cognitive characteristics and assess their performance and the strategies they use when solving addition and subtraction problems;

4. from (3) identify a sample of children and observe their engagement in instructional activities on related tasks for three months;

5. repeatedly measure (on three occasions over a three month period) the sample's performance and the strategies they use with addition and subtraction problems.

These steps will enable us to relate performance (in terms of level of achievement and strategy adoption) at a given time to both the cognitive capability of the child and to the specific set of instructional activities that the child is engaged in. It will provide us the opportunity to consider questions of change in performance and strategy and their possible causes.

This Study

The reason for wanting a battery of tests which measure cognitive processes is rooted in the cognitive development theory of Jean Piaget (1964, 1970, 1974). For Piaget, cognitive development is embedded in the context of a developing human system. The development of cognition is inseparable from the growth of biological and psychological faculties. It is a broad-based process, generalizing to a wide variety of situations.

Piaget's position is summarized in the following statement: "I think that development explains learning, and this opinion is contrary to the widely held opinion that development is a sum of discrete learning experiences." (Piaget, 1964, p. 176). The phrase "development explains learning," implies that the outcome of a learning experience is in part accounted for by developmental capabilities. That is, learning potential is defined (or explained) to a large extent by developmental capacity.

With this in mind it should be apparent that to study the learning of a set of skills involving problem solving using addition and subtraction techniques, the subjects' learning potential should be ascertained. The battery of tests used in this study were derived in a large part from Fullerton's (1968) thesis directed by Professor Collis. Other tests were adapted from Romberg, Carpenter, and Moser (1978). Finally, some tests were prepared by Professor Collis for this study.

Piaget's Theory

The importance of Piaget's theory to learning can be clarified by describing his notions of assimilation and accommodation. For Piaget, development is motivated and controlled by the dynamic tension between these two processes. Assimilation is the incorporation of external stimuli into existing mental structures. Often the external stimuli need to be modified in order to "fit" the internal mental structures, and thereby become assimilated. Accommodation is the complementary process which involves the modification of mental structures to bring them "in line" with external reality. Thus, accommodation interjects a qualitatively new mental operation into the cognitive repertoire. Assimilation utilizes this operation in an ever-extending variety of situations to internalize incoming information. If the operation of assimilation becomes inadequate (i.e., the child is unable to make sense out of a novel stimulus), then mental restructuring (accommondation) occurs if the mismatch is not too great, and a slightly higher order mental operation is generated. In cyclic fashion this pattern repeats itself over and over. This description is oversimplified. In practice it is difficult to isolate a specific cycle and label the appropriate parts "assimilation" and "accommodation" since these processes are active on many fronts simultaneously.

For Piaget, learning and development involves the interaction of assimilation and accommodation. Since both assimilation and accommodation depend on the type of mental operation available, it follows that both are dependent on the stage of development the learner has attained.

Piaget also makes important distinctions between different types of learning or knowing. These distinctions provide the contrast between Piaget's theory and most other learning theories. Piaget makes one distinction between operative learning and figurative learning, and another between logical-mathematical knowledge and physical knowledge. These two distinctions are closely related (i.e., operative learning usually involves logical-mathematical knowledge) but they are not synonymous.

The distinction between operative and figurative learning is a distinction between learning about transformations and learning about states (Piaget, 1970). Operative learning generalizes across content, transfers to related problems, is invariably stable (i.e., is not based on recall), and is resistant to extinction; figurative learning is content specific, is subject to memory loss, and is susceptible to counter-suggestion.

The second distinction is between logical-mathematical knowledge and physical knowledge. The first results from acting on objects and discovering properties of the actions; the second results from acting on objects and discovering properties of the objects (Piaget, 1970). Logical-mathematical knowledge arises from deduction and is certifiable by logical reasoning; physical knowledge arises from induction and is verifiable by empirical test (Beilin, 1976).

Applied to the present study, Piaget's theory on learning and development, and his distinctions between figurative and operative learning and between logical-mathematical and physical knowledge have several implications. First, the theory clearly implies that children's developmental level constrains their ability to benefit from an instructional lesson. Thus, the need to identify several Piagetian measures of cognitive development which are logically related to addition and subtraction is apparent.

Fullerton's Study

In 1968 Fullerton developed a set of group tests (and parallel interview tests) designed to identify the developmental processes associated with number readiness. Wohlwill (1960) had identified a number of tasks which reflect children's notions of number at the earlier stages of development.

Using scalogram analysis Fullerton found evidence for the existence of three differentiated stages in the development of the concept of number. The first was an initial preconceptual stage in which the child responds to number in a purely perceptual manner; in the second stage the support necessary for the generalization between two equivalent stimuli is steadily reduced; and finally, in the third stage number is viewed abstractly and the relationships between individual numbers lead to an understanding of conservation of numbers and the coordination between ordinal and cardinal correspondence.

Fullerton was particularly interested in the tasks Wohlwill used with the first two stages. The tasks in the first stage involved different degrees of perceptual diversity but retained qualitative resemblance through the number dimension. The second stage used tasks reflecting functional relationships among numbers. Those relevant to this study are "extension" and "addition and subtraction." "Extension" involves the matching of sets and necessitates counting. Wohlwill's "addition and subtraction" tasks were designed to test the understanding of the relationship among adjacent numbers of the type, $x + 1 = y$ and $x - 1 = z$. These are of particular interest in this study.

Using Wohlwill's tasks and levels of number acquisition Fullerton found it was possible to predict where Smedslund's (1962) operations of addition and subtraction and Dodwell's (1960) number discrimination tasks fell in the developmental sequence, since both are related to sub-tasks included in Wohlwill's tasks scale. Smedslund's addition-subtraction task is designed to test whether a child understands that the addition to or subtraction from one of two identical sets leaves one set with more elements than the other and then understands that the inverse of the first operation restores the equality between the two sets. Wohlwill's task requires understanding of both the operation and the number scale. Consequently, Wohlwill's tasks place a greater cognitive load on the child than Smedlund's tasks.

These are the only tests directly related to addition and subtraction. However, Fullerton developed or adopted measures of other Piagetian logical tasks which develop during the age-range when initial mathematics instruction occurs. Some tasks, such as "elimination of perceptual cues" or "number discrimination" were considered as logical prerequisites to initial quantitative processes. Others were considered to signify that a child was in Wohlwill's third stage of conceptual development. Operations involved in this third stage include:

1. Conservation of quantity: the ability to conceive that a quantity, such as liquid, remains constant and is independent of differences of appearances that it may undergo.

2. Conservation of number: the ability to understand that the total value of a collection of objects remains constant, regardless of the changes that are introduced into the relationships of the elements among themselves.

3. Seriation: the ability to deal with series and understand the notion of ordinal correspondence.

4. Ordinal-cardinal correspondence: the ability to deal with a graduated series and the extent to which spoken enumeration integrates the cardinal and ordinal aspects of number.

5. Classification: the ability to understand the inclusion of partial

classes in a total class.

6. Additive composition of number: the ability to compose the numerical parts to make a whole.

7. Coordination of relations of equivalence: the ability to understand the transitive nature of relations, such that, if x = y and y = z then x = z.

Fullerton, however, had no real basis for predicting any order or structure among these logical tasks. He prepared 15 group tests, administered them to a population of students and grouped the test results using scalogram analysis. This procedure groups or orders scales in terms of level of difficulty. This analysis yielded an order for these tests illustrated in Figure 1.

In this diagram, all tests in the same block are not significantly different from each other, but they differ significantly from all items in other blocks. Those tests which have not been placed in blocks are associated with tests in the blocks preceding and succeeding them in the sequence. Thus, instead of the hypothesized three stage sequence in the development of the number concept, the results show six fairly differentiated stages linked together by intermediary number abilities.

In summary, Fullerton was able to develop a set of tests which were hierarchically ordered with respect to difficulty and ostensibly related to numerical capacity. Since the purpose of this study was to identify cognitive skills associated with the development of addition and subtraction skills it seems clear that several of Fullerton's tests would be useful.

Procedures

The Tests

Fifteen tests were selected or developed for this step in the set of

Group Tests	Test Titles
CRE	Coordination of Relations of Equivalence
CN	Conservation of Number
CQ	Conservation of Quantity
OCC	Ordinal Cardinal Correspondence
C	Classification
ASW	Addition-Subtraction (Wohlwill)
S	Seriation
CNW	Conservation of Number (Wohlwill)
ACN	Additive Composition of Number
ASS	Addition-Subtraction (Smedslund)
OC	Ordinal Correspondence
E	Extension
ND	Number Discrimination
A	Abstraction
EPC	Elimination of Perceptual Cues

Figure 1. Significant differentiated tests in the developmental sequence for group tests. (Fullerton, 1968, p. 49)

collaborative studies. In Table 1 the titles of the tests, source format, and number of items is given. Details of each test follows:

Extension (E). In this group test, developed by Fullerton, children

Table 1

Test Used to Assess Children's Cognitive Processing Capabilities

Title	Abbreviation	Source	Format	#of items
Extension	(E)	Fullerton	Group	12
Ordinal Correspondence	(OC)	Fullerton	Group	12
Conservation of Number (Wohlwill)	(CN-W)	Fullerton	Group	6
Addition-Subtraction (Wohlwill)	(AS-W)	Fullerton	Group	6
Addition-Subtraction (Smedslund)	(AS-S)	Fullerton	Group	6
Addition-Subtraction nonequivalent	(AS-N)	new	Group	6
Addition-Subtraction both equivalent	(AS-BE)	new	Group	6
Addition-Subtraction both inverse	(AS-BI)	new	Group	6
Addition-Subtraction both nonequivalent	(AS-BN)	new	Group	6
Transitivity	(T)	new	Group	6
Coordination of relations of equivalence	(CRE)	Fullerton	Group	6
Class Inclusion	(CI)	CS-1	Individual	2
Additive Composition of Number	(ACN)	Fullerton	Individual	3
Counting-On	(CO)	CS-1	Individual	9
Counting-Back	(CB)	CS-1	Individual	9

were to decide which of three "choice boxes" had the same number of dots as were in a "sample box." (See Figure 2). The term "extension" refers to the fact that the number sets extended beyond the usual level of subitization to a higher portion of the number scale. Six, seven and eight dots were shown in the choice boxes. A number of dots (6, 7 or 8) was also shown in the sample box but in a different configuration. Children were instructed to put a check in the space below the choice box they thought had the same number of

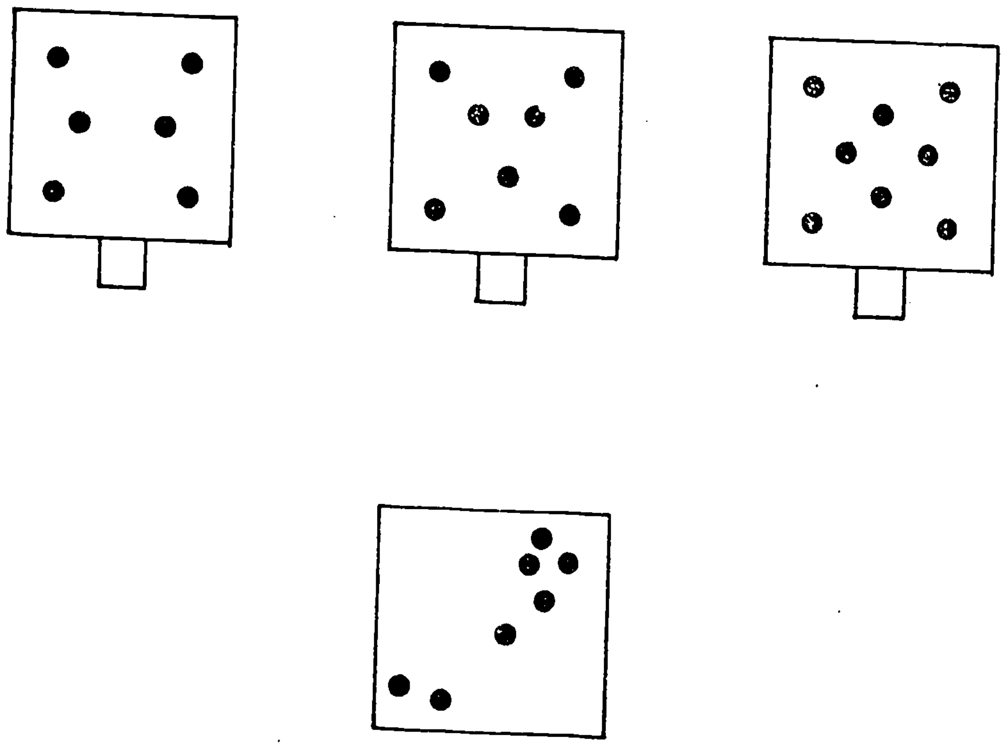

Figure 2. Sample item for the Extension Test.

dots as the sample box. A correct answer was interpreted as the child being able to set up a one-to-one correspondence between sets. The test contained 12 items, and scored in two ways: the count of correct responses (S-1) and a pass score (S-2) which was 10 out of 12. The protocol for this test appears in Appendix A.

Ordinal correspondence (OC). In this group test, also developed by Fullerton, the format for the items was similar to those in the Extension test. The sample box contained a set of nine narrow rectangular bars drawn in outline and arranged in order of increasing length. One bar in each set (cue bar) was a solid bar colored completely. This could be located in the second, third, or fourth position of the series of bars. (See Figure 3). The bars were

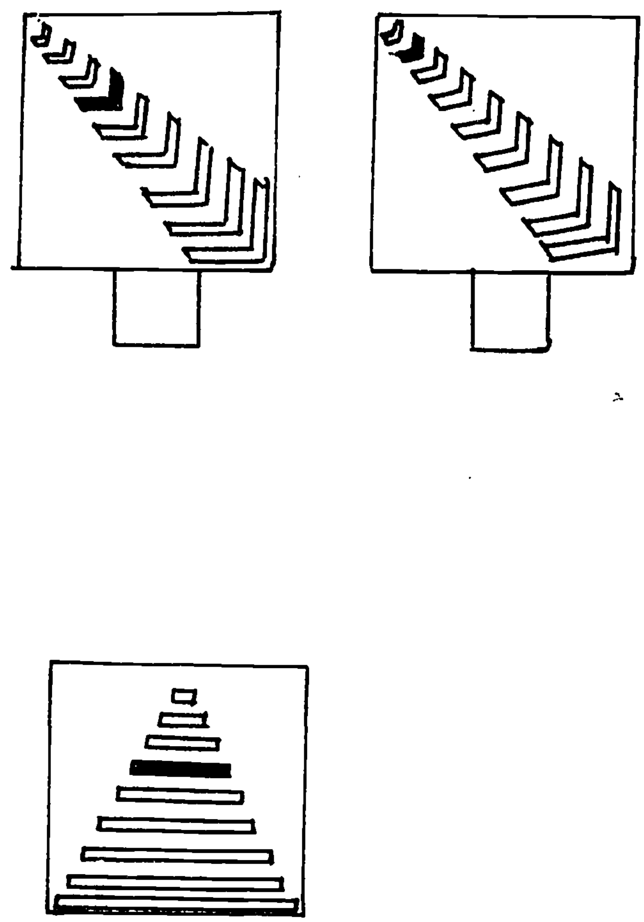

Figure 3. Sample item for the Ordinal Correspondence Test.

arranged either horizontally or vertically in the sample box so that they either increased in height from right to left (top to bottom), or vice versa. The configurations in the choice boxes varied, but were all the same kind on each page. They included rectangular bars, steps, arrowheads, circles, triangles, and curved bars, all of which were arranged in order of increasing size. In each choice box one element of the total configuration was solid. The position of the bar varied from choice box to choice box but the three positions, second, third, and fourth, were always represented in the choice boxes of each page. Children were instructed to decide which box at the top of the page matched the box at the bottom of the page. Again, a correct answer was interpreted as the child being able to establish a correspondence between sets. This test also contained 12 items. Both number correct and pass (10 out of 12) were scored. This test was administered with the Extension test and the protocol also appears in Appendix A.

Conservation of Number (Wohlwill)(CNW). This group test, also developed by Fullerton, was based on Wohlwill's earlier work (1960). In this test two horizontal rows of the three choice boxes appeared on each page. The first of the boxes in the top row contained six dots, the second seven and the third eight. The same number of dots was placed in the boxes in the bottom row, but the order of six, seven and eight dots was varied to guard against patterned response (see Figure 4). Children's first response was to the stimulus pattern presented to them in a box at the top of the page and their second response in a box at the bottom of the page. Following this a number of magnetized cardboard circles (six, seven, or eight) were placed on a magnetic board so that the pattern and number of circles duplicated exactly the dots in one of the choice boxes. The S was then told to put a cross in the space provided under the box at the top of the page which they thought went with the dots on the board. Since the purpose of this first choice was only to inform Ss of the choice box whose dots were in numerical correspondence with the set of circles,

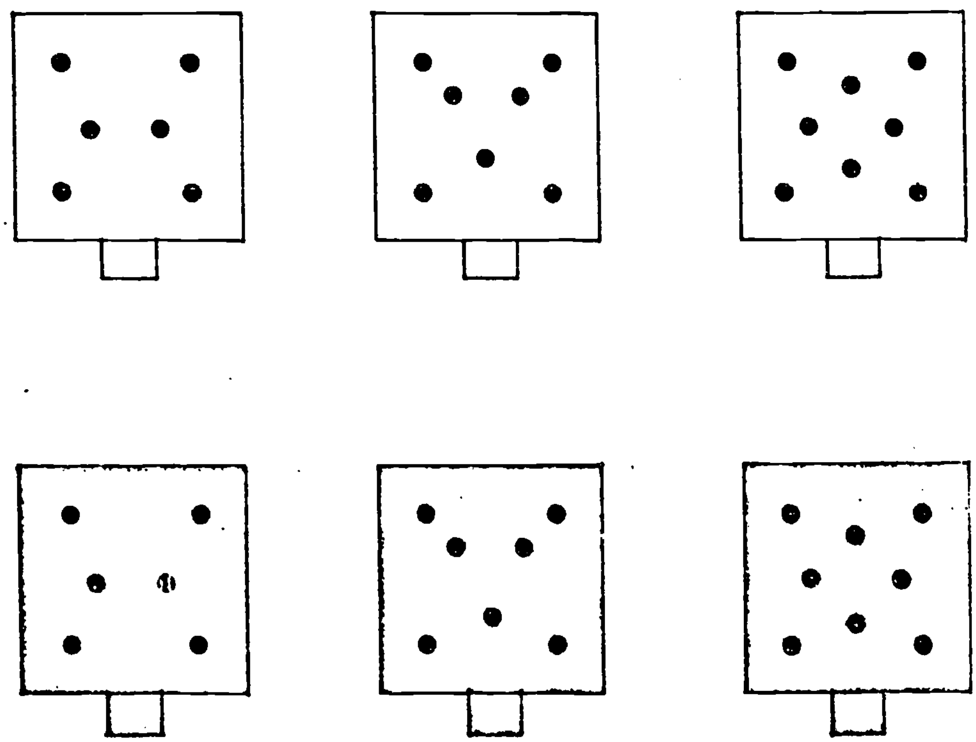

Figure 4. Sample item for the Conservation of Number (Wohlwill) Test.

this response was corrected. Next, the experimenter scrambled the circles by placing them in different positions on the board. This was done in full view of children and their attention was called to the rearranging of the circles. The children were then told to put a cross under the box at the bottom of the page which they thought went with the circles. A correct response is interpreted as the child being able to preserve one-to-one correspondence between sets after one set had been rearranged (i.e., being able to overcome perceptual distractions). To prevent counting the experimenter covered the rearranged circles with a large piece of cardboard, removing it periodically to let children see the spatial arrangement of the circles, but not long enough for them to count. Six items were given. Both number correct and a pass criterion of five correct responses were scored. The protocol used for this test appears in Appendix B.

Addition-Subtraction (Wohlwill)(AS-W). The items for this test group also developed by Fullerton were interspersed with those of the previous test (CNW) because of the similarity between the two tests. The only difference in procedure between these two tests was that for addition-subtraction a single circle was either added to or subtracted from the collection of circles in front of the children, immediately following the children's initial match. In either case, the experimenter alerted the children to the action taken. In this case a correct response is interpreted as the child being able to recognize that an increase or decrease in one of two sets in one-to-one correspondence means they are no longer in such correspondence and can establish a new one-to-one correspondence with another set. Six items were given, and both number correct and pass (5 out of 6) were scored. The protocol used also appears in Appendix B.

Addition-Subtraction (Smedslund)(AS-S). Fullerton developed this group test based on Smedslund's (1962) tasks. Pictures of two children named David and Wendy were used. The same number of magnetized cardboard circles were placed under both David and Wendy on a magnetic board. The children were asked if David and Wendy had the same number of marbles each to play with. They were asked:

> "If you think David has more than Wendy, put an 'X' on David. If you think Wendy has more than David, put an 'X' on Wendy. If you think they have the same number, put an 'X' in the circle between them." (See Figure 5).

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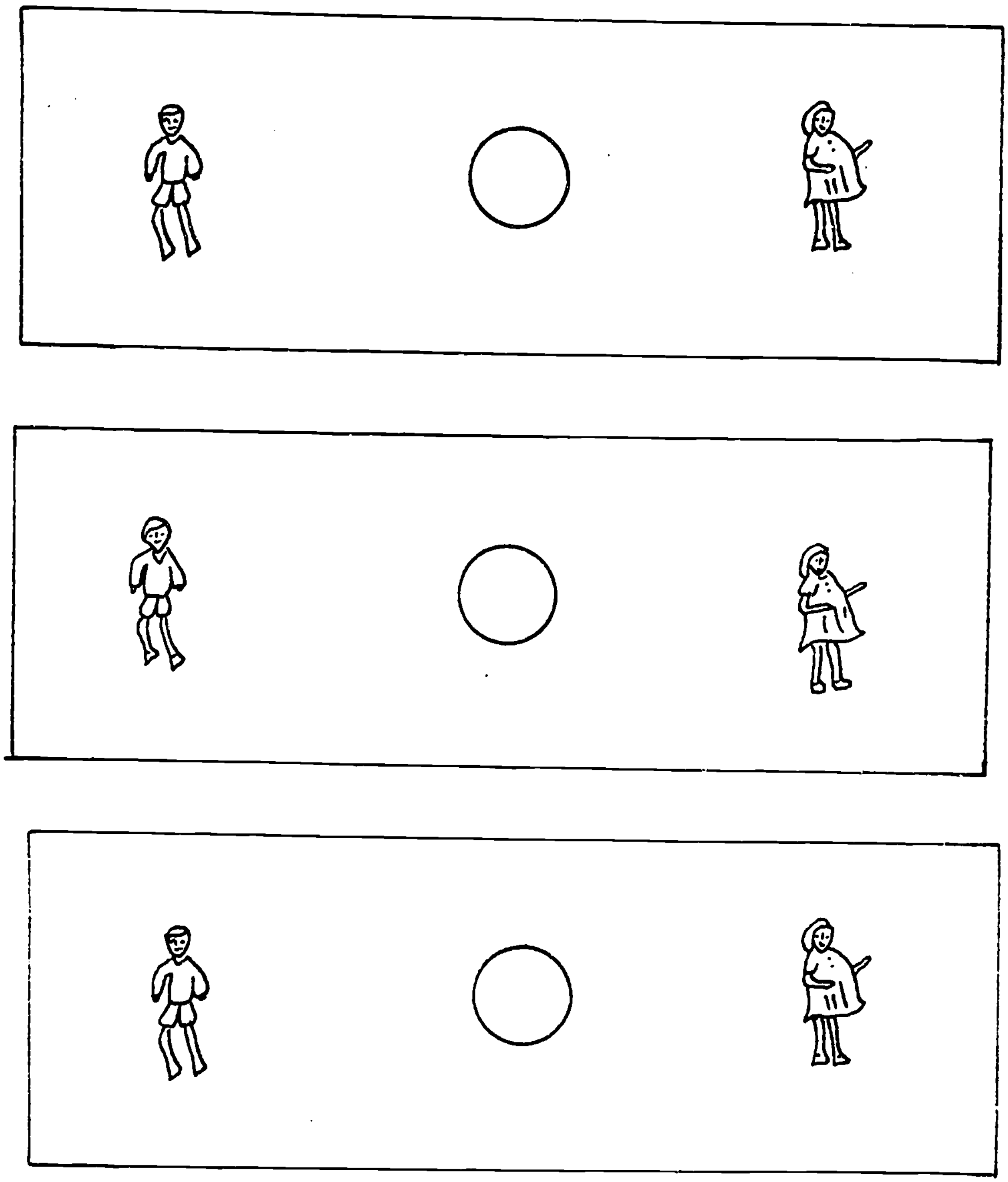

Figure 5. Sample item for the Addition-Subtraction (Smedslund) Test.

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Once equivalence between the two sets had been established the experimenter told the children that marbles were to be taken from (given to) David (Wendy). This was done. The standard question was repeated and children made their responses in the middle row. The operation was then reversed by the experimenter telling the children that marbles will be given to (taken back from) David (Wendy). This was done and the children made their responses to the standard question in the bottom row. Six items were given and again both total correct and pass-fail were recorded. The pass criterion was five correct operational sequences out of the possible six, where an operational sequence is defined as (+ -) or (- +). The protocol for this test appears in Appendix C.

Addition-Subtraction Nonequivalent (AS-N). This test, which follows the same sequence as the previous test AS-S, was newly created for this study. The only difference was the reverse operation involved giving back (taking from) a different number of marbles. For example, if 2 marbles had been given to David, 3 would be taken from him. The same standard questions were asked for six such items and both a total and pass-fail score (5 out of 6) were derived. The protocol also appears in Appendix C.

Addition-Subtraction Both Equivalent (AS-BE). This new test uses David and Wendy items like the previous two tests. However, in this case after equivalence is established both David's and Wendy's sets of marbles are changed an equivalent amount and both in the same directions. The standard question is then asked. Only this response is scored for six items. Both total score and pass-fail (5 out of 6) were recorded. This protocol appears in Appendix C.

Addition-Subtraction Both Nonequivalent (AS-BN). This test also involves six new items which differ from AS-BE only in that the changes to David's and Wendy's sets are nonequivalent. The test is scored in the same way and the protocols are in Appendix C.

Transitivity (T). We decided to develop this six item group test because the Coordination of Equivalence Relations Test (CRE) (described in the next section) requires a child to attend to both transitivity and a linear rearrangement of sets. This test was designed to assess just transitivity. Thus, it was assumed that T would be less complex than CRE. For each item the children were asked to first compare two sets of objects displayed on the magnet board (butterflies, rabbits, etc.) and decide which set had more objects or if they were the same. Their response was placed in the box at the top of the page. (See Figure 6). Then one set was removed. Another set was put in its place (frogs) and the children were asked to make the same decision and record it in the box in the middle of the page. Then the children were asked to compare the set which had been removed with the set that had been put in its place. An item was marked as correct only if all three responses on each page were correct. A correct response was interpreted as the child being able to preserve both equivalence and order relationships. A total score and a pass-fail score (5 out of 6) were recorded for each child. The protocol for this test appears in Appendix D.

Coordination of Relations of Equivalence Test (CRE). This six item group test was developed by Fullerton. The items are similar to those in the transitivity test except that the fixed set is also transformed (lengthened, shortened or heaped together). A correct response here was interpreted as the child being able to preserve equivalence relationships even after

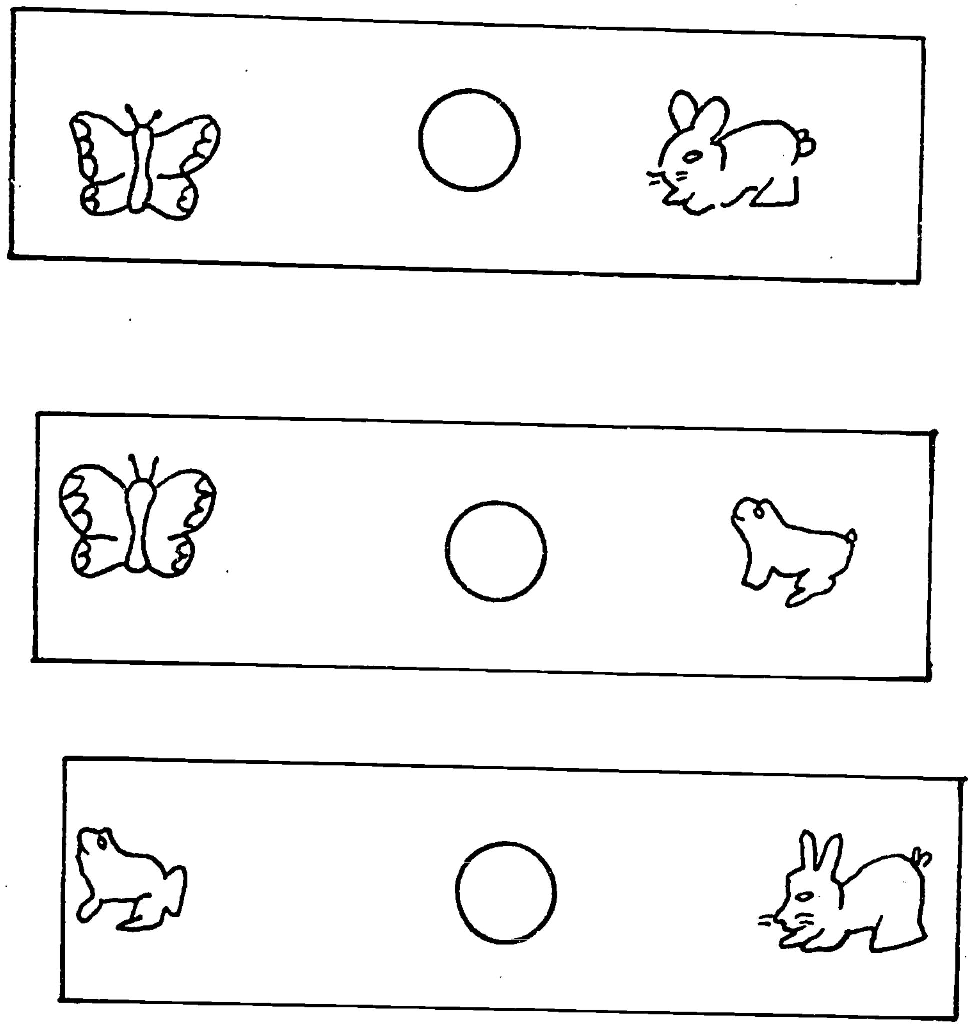

Figure 6. Sample item for the Transitivity Test.

rearrangement. The same scoring procedures were used and the protocol appears in Appendix D.

Class Inclusion. This individually administered test involving two items was developed for Coordinated Study #1 (Romberg, Carpenter and Moser, 1978). For the first item some plastic fruit (5 apples and 2 bananas) were placed in front of the child. The experimenter then said:

"Here are some pieces of fruit."

"What kind of fruit is this (pointing to the apples), and this (pointing to the bananas)?"

The experimenter then asked:

"Are there more apples or more pieces of fruit?"

After the child responded the experimenter asked:

"How can you tell?"

The second item followed the same pattern but used plastic blocks (3 white and 12 red). The number of correct responses was tallied. A correct response was interpreted as a child being able to logically subdivide a set into distinct subsets.

Additive Composition of Number (ACN). This individually administered test developed by Fullerton includes three items which ask children to respond to three quite different composition tasks. In the first task two sets of lollies were arranged in horizontal lines on a table in front of the child. The set closest to the child was divided into two subsets of 4 while the set nearest to the experimenter was divided into subsets of 7 and 1. Then the experimenter stated:

"Before school you ate 'these lollies' (pointing to one subset of 4) and I ate 'these lollies' (pointing to the subset of 1). After school you ate 'these lollies' (pointing to the subset of 4) and I ate 'these lollies' (pointing to the subset of 7). Did you eat more lollies or did we eat the same number of lollies or did I eat more lollies?"

For the second task two dolls were placed on a table, one with 14 lollies scattered around it, and the other 8 lollies around it. The experimenter

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then asked:

"Do the dolls have the same number of lollies to eat, or does one have more lollies than the other to eat?"

The child was then asked to move the lollies so that each of the dolls had the same number of lollies to eat. For the third question a pile of 18 lollies was placed between the two dolls. The child was then asked to share the lollies between the dolls so that they had the same number of lollies to eat. The number of correct responses was recorded. A correct response implies the child can establish an equivalence relationship by the common practice of "sharing" and preserve such a correspondence when distracting information is presented.

Counting-On (CO). This individually administered test was also developed for Coordinated Study #1. The test includes three items for each of the three levels of counting on. (Small number onto a number less than ten, a small number onto a number between ten and twenty, and a large number onto a number between ten and twenty). The typical question asked was "Could you start counting at 13 to find the number that is 4 more than 13?" Children were encouraged to count out loud. Children were marked as passing a level if 2 of 3 items were answered correctly. A pass-fail score was recorded for that level and the experimenter did not ask questions at a higher level. A total score was then recorded of the number of levels passed (0, 1, 2 or 3). The protocol for this test is in Appendix E.

Counting-Back (CB). This test is identical to format CO and was also developed for Coordinated Study #1. However, in this case the typical question asked was, "Could you count back starting at 15 to find the number that is 4 less than 15?" The same scoring procedure as CO was used and the protocol for this test also appears in Appendix E.

In summary, 15 tests were selected or prepared for this study. Since Fullerton had established an order for such tests in his study we hypothesized hierarchical order for the tests in this study (see Figure 7). The order indicated within the logical tests, the counting tests and the addition-subtraction tests seemed reasonable. The composite order, however, is only a guess which we decided to examine in this study.

The choice of these tests started with the consideration of Fullerton's two addition and subtraction tests. One was based on Smedslund's tasks (AS-S) and the other on Wohlwill's (AS-W). (See Figure 1). Fullerton found "Ordinal Correspondence" (OC) to be easier; AS-S, "Additive Composition of Number" (ACN) and Wohlwill's "Conservation of Number" (CN-W) were between AS-S and AS-W in difficulty and "Coordination of Relations of Equivalence" (CRE) more difficult than AS-W. Because of the large number of young children to be tested (see the section on population) an even easier test, "Extension" (E) was selected so that a baseline could be established.

Three interview tests being used in Coordinated Study #1 (Romberg, Carpente and Moser, 1978) were then selected. "Class Inclusion" (CI) which involves a standard logical task assumed to underlie the "part-part-whole" relationship involved in many addition and subtraction problems came next. "Counting-On" (CO) and "Counting-Back" (CB), which ascertain whether the child can handle these two strategies (often used by children to solve addition and subtraction problems), were obviously of value in this series of studies.

The Transitivity (T) test was then prepared to complement the CRE test.

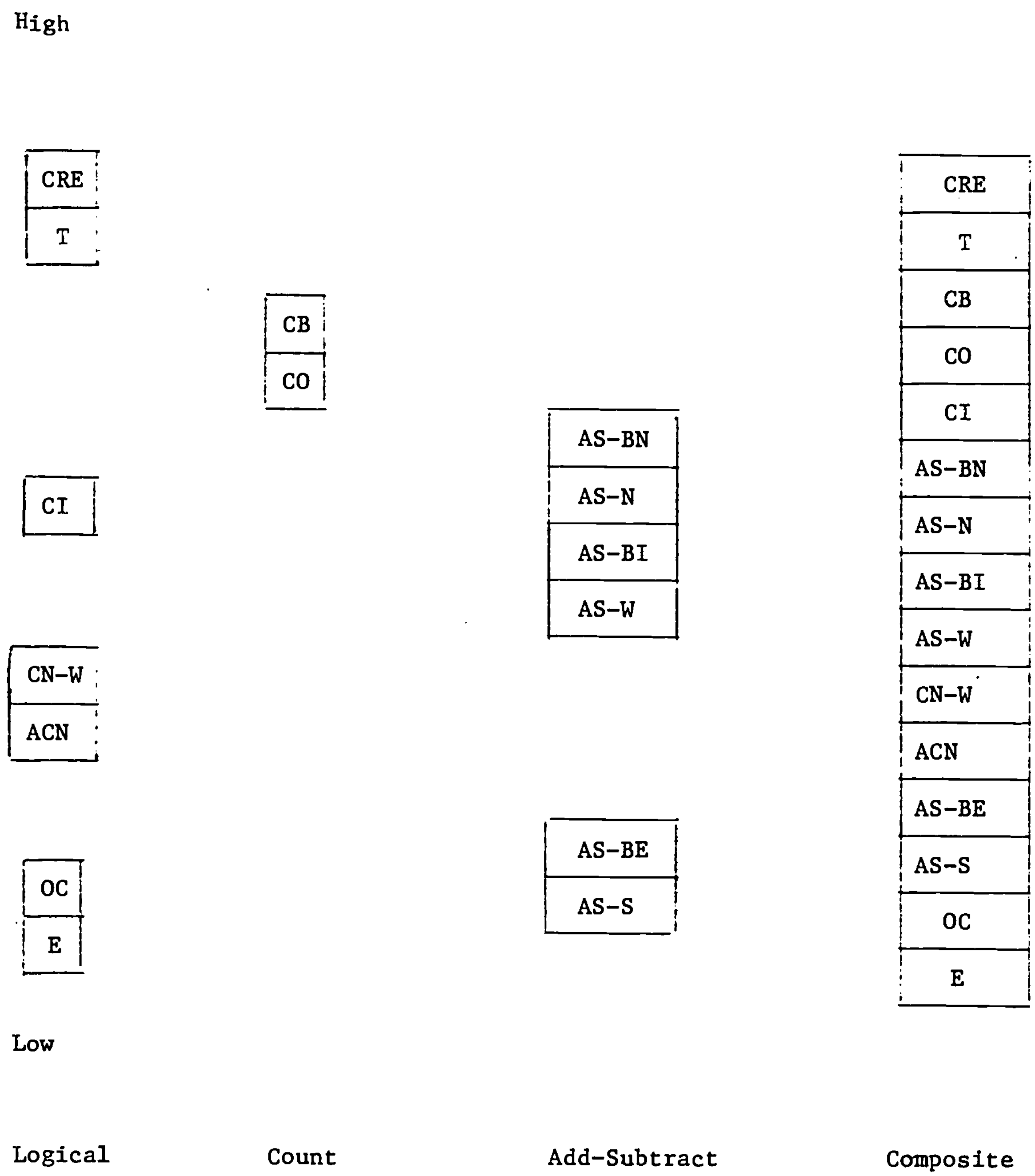

Figure 7. Hypothesized hierarchical order of the 15 cognitive process tests used in this study.

Finally, we decided to extend the types of items presented in the "Addition-Subtraction (Smedslund)" (AS-S) test.

The population

All of the children in Sandy Bay Infant School in Hobart, Tasmania constituted the sample for this study. The school is located on the Derwent River in a suburb of Hobart near the University of Tasmania. The community is middle to upper-middle class. The age range was from 4 years 9 months to 8 years 2 months. There were 65 boys and 57 girls (122 total) in six classrooms.[1] The distribution of the 122 boys and girls used in the study in each of the six classes is shown in Table 2. The six classes include two kindergarten classes[2] (one which met in the morning (K-AM) and one which met in the afternoon (K-PM),

Table 2

Number of Boys and Girls in Each of the Six Classes

	K-AM	K-PM	Prep	Gr 1	Gr 1/2	Gr 2
Boys	14	10	7	5	14	15
Girls	7	8	12	13	9	8
TOTAL	21	18	19	18	23	23

Note: 122 students in all were in the six classes.

a prep class, a grade 1 class, a grade 2 class, and a combination grade 1/2 class. The age distributions for each class are shown in Table 3.

Test Administration

Because of the large number of tests assumed to be hierarchically ordered to be administered to a variable age population two decisions were made to gather

Table 3

Range and Average Ages of Children in Each Class

Class	1 K-AM	2 K-PM	3 Prep	4 Gr 1	5 Gr 1/2	6 Gr 2
Youngest Ages	4-9	5-0	5-4	6-2	6-5	7-3
Oldest Ages	5-1	5-7	6-1	7-0	7-10	8-2
Average Age	4-11	5-4	5-9	6-7	7-3	7-8

Note: 4-9 means 4 years 9 months as of October 1, 1979.

the data more efficiently. First, the tests were separated into five sets to be administered at separate times. And second, we decided not to give all of the tests to all children. The organization of the tests and the rules for selecting who was to take which test are given in Table 4. The interview tests and Battery 2 were given to all children. If a child "passed" the two tests in Battery 2 (CN-W and AS-W), then both Battery 3 and Battery 4 were given and it was assumed that Battery 1 had been passed. However, if the child "failed" the tests, Battery 1 was administered and it was assumed that Battery 3 and 4 were also failed. The distribution of children in each class who took each battery is shown in Table 5.

The only question we had about this procedure was whether to give Battery 3 to all children or not since AS-S (the first test in the battery) was considered to be more difficult, and several young children had exhibited frustration when taking tests they could not do we decided to give Battery 3 only to

Table 4

Tests Included in Each Battery, Sequence of Administration and Rules for Who Was to Take Each Battery

Battery (tests)	Order	Rule
Interview (ACN, CI, CO, CB)	1	All Children
Battery 1 (E, OC)	3	Only if "failed either CN-W, AS-W in Battery 2
Battery 2 (CN-W, AS-W)	2	All Children
Battery 3 (AS-S, AS-N, AS-BE, AS,BI, AS-BN)	4	Only if "passed" both CN-W, AS-W in Battery 2
Battery 4 (T, CRE)	5	Only if "passed" both CN-W, AS-W in Battery 2

Table 5

Number of Children in Each Class Who Took Each Battery

Battery	Interview	1	2	3	4
Class					
K-AM	21	16	21	5	5
K-PM	18	9	18	9	9
Prep	19	10	19	9	9
Gr 1	18	5	18	13	13
Gr 1/2	23	6	23	17	17
Gr 2	23	2	23	21	21
Totals	122	48	122	74	74

children who had passed Battery 2. In retrospect this turned out not to be a wise decision, as we will explain later.

The administration of the tests was done by a research assistant and two experienced teachers who were hired for the project. All were trained before the testing. On the interview tests one assistant administered the Counting-On (CO) and Counting-Back (CB) tests, and a second administered the Class Inclusion (CI) and the Additive Composition of Number (ACN) tests. Children were randomly selected by their teacher to come to the interview room (the teachers' lounge). Each interviewer was in a corner of the room. Children were randomly assigned to an interviewer. Children took two tests on one day and the other two a day or two after. All interviewing was completed within six days. Shortly after the interviews were completed the group batteries were given. Battery 2 was given first to groups of 6 to 8 children at a time from each class. The graduate assistant presented the stimulus information for each test following a script and using a large magnet board. The other assistants observed the children to make sure they were on the correct page, responding in the right place and not copying from others (see the test protocols in Appendices A-E for administrative details for each test).

As soon as Battery 2 was completed it was scored and groups were identified for the next administrations. Battery 1 was given next, followed by Batteries 3 and 4. All testing was completed within four weeks.

The actual administration of the interview tests and Batteries 2 and 1 proceeded without incident. However, some problems occurred when administrating both Batteries 3 and 4. In the directions for AS-BI in Battery 3 there was a typographical error in the instructions. For the first item the numbers were +1 and +1 rather than +1 and -1. The test administrator did not notice the typo and gave the item using +1, +1. This "officially" would result in this item belonging in test AS-BE rather than in AS-BI. This resulted in AS-BE having 7 items and AS-BI having 5 items. However, since almost all the students got both the 7 items and 5 items correct, and because it would make data processing much less complicated we treated the tests as both having 6 items. A second problem in Battery 3 occurred with five grade 1 girls on the AS-N test. Evidence suggests that they shared the same incorrect answers.

Battery 4 was administered in November of 1979 after Professor Romberg, who had supervised the previous testing program, had returned to the United States. There was a mix-up in the test protocol. The graduate research assistant made some decisions about the items, sequence and instructions (see documented details in Appendix F). In all, the decisions about instructions did not affect the scoring of either of the two tests (T and CRE) in Battery 4. However, six children who were not supposed to be given Battery 4 (they had not passed the tests in Battery 2) nonetheless were given Battery 4. Although the scores were omitted from the general results, the actual scores are disconcerting (see Table 6). The "success" of these children certainly raises questions about the decision rules that were used and suggests a more detailed research analysis of the whole area purported to be covered by tests CN-W, AS-W, T, and CRE.

Results

Group test scores

In this section the scores for the children in each class and the total population for each test is presented for the group administered tests.

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Table 6

Scores for the Six Children Incorrectly Given Battery 4

Child	Battery 1		Battery 2		Battery 3	Battery 4	
	E	OC	CN-W	AS-W	N.A.	T	CRE
1	12	12	6	4*	N.A.	6	6
2	12	12	4*	6	N.A.	5	6
3	12	12	6	4*	N.A.	6	6
4	12	12	4*	6	N.A.	6	6
5	12	12	4*	5	N.A.	6	6
6	12	12	1*	6	N.A.	4*	6

Note: * = Not Passing, N.A. = Not attempted

Extension (E). The scores for this test appear in Table 7. Clearly this was an easy test. Only 5 of the 48 children who took it did not meet the "pass" criterion of 10 or more correct.

Ordinal Correspondence (OC). This test proved to be almost as easy as E for the 48 children who took Battery 1 (see Table 8). Only eight children "failed" this test. These two tests (E) and (OC) provided the baseline for the population as expected. Almost all children demonstrated that they were able to form one-to-one and order relationships.

Conservation of Number (Wohlwill)(CN-W). The scores for this test given to all children appear in Table 9. Although most children (83) passed the distribution of scores and the increase in mean scores across grades are consistent with the expectations for this population.

Table 7

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Extension Test

Score	NA	4	5	6	7	8	9	10	11	12	$\overline{X}$	SD
Class												
K-AM	5	1	2					4	5	4	9.81	2.66
K-PM	9					1			4	4	11.11	1.27
Prep	9					1		1	3	5	11.10	1.29
Gr 1	13							1		4	11.60	.89
Gr 1/2	17									6	12.00	.00
Gr 2	21									2	12.00	.00
Totals		1	2			2		6	12	25		

Table 8

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Ordinal Correspondence Test

Score	NA	2	3	4	5	6	7	8	9	10	11	12	$\overline{X}$	SD
Class														
K-AM	5	1				3			1	2	3	6	9.62	3.03
K-PM	9							1	1	1	2	4	10.78	1.48
Prep	9									1	2	7	11.60	.70
Gr 1	13			1							3	1	9.80	3.27
Gr 1/2	17									1		5	11.67	.82
Gr 2	21											2	12.00	.00
Totals		1		1		3		1	2	5	10	25		

Table 9

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Conservation of Number (Wohlwill) Test

Score	1	2	3	4	5	6	$\overline{X}$	SD
Class								
K-AM	2	5	4	3	3	4	3.57	1.69
K-PM	2	3	1	2	4	6	4.17	1.86
Prep		1	2	3	2	11	5.05	1.31
Gr 1			2	3	1	12	5.28	1.23
Gr 1/2	1			3	2	17	5.44	1.20
Gr 2	1			1	4	17	5.52	1.12
Totals	6	9	9	15	16	67		

Addition-Subtraction (Wohlwill)(AS-W). This test, also given to all children in Battery 2, proved to be somewhat easier than expected (see Table 10). The number who passed (90) and the class means were very similar to CN-W. This may imply that it is easier for children to comprehend how sets are changed than it is for them to see that sets remain unchanged in size when they are transformed on another attribute.

Addition-Subtraction (Smedslund)(AS-S). This was the first test in Battery 3. It was administered to the 74 children who passed both tests in Battery 2 (CN-W and AS-W). The test proved to be easier than expected since only eight children "failed" and no one got more than three of the six items wrong (see Table 11).

Table 10

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Addition-Subtraction (Wohlwill) Test

Score	0	1	2	3	4	5	6	$\bar{X}$	SD
Class									
K-AM		2	3	3	4	3	6	4.00	1.73
K-PM	1		2	2	2	7	4	4.28	1.67
Prep				3	6	2	8	4.79	1.18
Gr 1					1	5	12	5.61	.61
Gr 1/2				1	2	4	16	5.52	.85
Gr 2						4	19	5.82	.38
Totals	1	2	5	9	15	25	65		

Table 11

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Addition-Subtraction (Smedslund) Test

Score	NA	3	4	5	6	$\bar{X}$	SD
Class							
K-AM	16			2	3	5.60	.55
K-PM	9				9	6.00	.00
Prep	10		1	3	5	5.44	.73
Gr 1	5		1	8	4	5.23	.60
Gr 1/2	6	2	1	3	11	5.35	1.06
Gr 2	2	1	2	3	15	5.52	.87
Totals	48	3	5	19	47		

Addition-Subtraction Nonequivalent (AS-N). The scores for this test are shown in Table 12. It should have been much more difficult than AS-S but

Table 12

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Addition-Subtraction Nonequivalent Test

Score	NA	3	4	5	6	$\bar{X}$	SD
Class							
K-AM	16			1	4	5.80	.45
K-PM	9		1	1	7	5.67	.71
Prep	10		1	1	7	5.67	.71
Gr 1	5	1	5	2	5	4.85	1.07
Gr 1/2	6			1	16	5.94	.24
Gr 2	2		1		20	5.91	.44
Totals		1	8	6	59		

proved to be as easy. In fact if scores for the five grade 1 girls who made the exact same errors are not considered it was very easy for these students.

Addition-Subtraction Both Equivalent, Both Inverse and Both Nonequivalent. (AS-BE, AS-BI. and AS-BN). The scores for these tests given in Battery 3 appear in Tables 13, 14 and 15. Again these tests proved to be easier than anticipated. No one failed AS-BE, only five failed AS-BI, and three AS-BN.

The tests in Battery 3 all proved to be easier than anticipated. In fact the scores are too high to differentiate between students, and thus the data on these tests were not used in subsequent analyses.

Table 13

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Addition-Subtraction Both Equivalent Test

Score	NA	5	6	$\overline{X}$	SD
Class					
K-AM	16		5	6.00	.00
K-PM	9	3	6	5.67	.50
Prep	10		9	6.00	.00
Gr 1	5	1	12	5.92	.28
Gr 1/2	6		17	6.00	.00
Gr 2	2		21	6.00	.00
Totals		4	70		

Table 14

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Addition-Subtraction Both Inverse Test

Score	NA	2	3	4	5	6	$\overline{X}$	SD
Grade								
K-AM	16					5	6.00	.00
K-PM	9	1			1	7	5.44	1.33
Prep	10			1	3	5	5.44	.73
Gr 1	5	1				12	5.69	1.11
GR 1/2	6				2	15	5.88	.33
Gr 2				2	6	13	5.52	.68
Totals		2		3	12	57		

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Table 15

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Addition-Subtraction Both Nonequivalent Test

Score	NA	0	1	2	3	4	5	6	$\bar{X}$	SD
Grade										
K-AM	16							5	6.00	.00
K-PM	9	1						8	5.33	2.00
Prep	10						1	8	5.89	.33
Gr 1	5			1			1	11	5.61	1.12
Gr 1/2	6				1		1	15	5.77	.75
Gr 2	2						1	20	5.95	.22
Totals		1		1	1		4	67		

Transitivity (T). This test proved to be harder than anticipated since only 33 children passed. An examination of item responses indicated that the items on transitivity of order were more difficult than those on transitivity of equivalence. This suggests that in the future these should be measured with separate tests. The distribution of scores and increase in class means, however, exhibited the expected pattern. (See Table 16).

Coordination of Relations of Equivalence (CRE). The scores for this test appear in Table 17. The pattern of scores is as anticipated.

In summary, after examining the scores and distribtuions on each of the group administered tests we decided not to use any of the scores from Battery 3. For the remaining group administered tests in Batteries 1, 2 and 4, the

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Table 16

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Transitivity Test

Score	NA	0	1	2	3	4	5	6	$\bar{X}$	SD
K-AM	16	1		2	2				2.00	1.23
K-PM	9		2	2	3	1	1		2.67	1.32
Prep	10	1		2	5	1			2.56	1.13
Gr 1	5		1	4	1	4	3		3.31	1.38
Gr 1/2	6			2		2	4	9	5.06	1.35
Gr 2	2		1	1	1	2	5	11	5.00	1.45
Totals		2	4	13	12	10	13	20		

Table 17

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Coordination of Relations of Equivalence Test

Score	NA	1	2	3	4	5	6	$\bar{X}$	SD
K-AM	16	2	1	2				2.00	1.00
K-PM	9		4	5				2.56	.53
Prep	10		1	2	1	4	1	4.22	1.30
Gr 1	5		1		4	4	4	4.77	1.17
Gr 1/2	6				1	6	10	5.53	.62
Gr 2	2				2	3	16	5.67	.66
Totals		2	7	9	8	17	31		

number of children who "passed" each test is shown in Table 18. Recall that

Table 18

Number of Children for Each Class and for the Total Population Who Passed the Tests in Batteries 1, 2, and 4

	Battery 1			Battery 2		Battery 4	
	NA	E	OC	CN-W	AS-W	T	CRE
K-AM	21	18	16	7	9	0	0
K-PM	18	17	16	10	11	1	0
Prep	19	18	19	13	10	0	5
Gr 1	18	18	17	13	17	3	8
Gr 1/2	23	23	23	19	20	13	16
Gr 2	23	23	23	21	23	16	19
Totals	122	117	114	83	90	33	48

if a child "passed" the tests in Battery 2 we assumed that the tests in Battery 1 were also passed, while if the child "failed" either test in Battery 2, we assumed that the tests in Battery 4 were also "failed." The first assumption seems plausible, but the assumption for Battery 4 may not be correct (see the section on test administration).

Interview Test Scores

In this section the number of correct answers for children in each class are presented for the four tests administered by individual interviews.

Additive Composition of Number (ACN). The scores for this test appear in Table 19. The pattern of scores across classes was anticipated.

Table 19

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Additive Composition of Number Test

Score	0	1	2	3	X	SD
K-AM	7	14	2	2	.96	.84
K-PM	2	6	11	1	1.55	.76
Prep		5	13	3	1.91	.63
Gr 1	2		15	5	2.05	.79
Gr 1/2	1	2	5	16	2.50	.83
Gr 2			7	20	2.74	.45
Totals	12	27	53	47	1.97	.94

Class Inclusion (CI). These two items proved to be more difficult than expected since 96 of the 122 children got neither task correct. (See Table 20).

Counting-On (CO) and Counting-Back (CB). The levels of performance on these two counting tests appear in Table 21 and 22. As expected CO was easier than CB and the pattern of scores was as expected.

Relationship of Test Scores

In this section test scores for the population on the ten cognitive process tests which were individually administered or appeared in Batteries 1, 2, and 4 are examined. First, a factor analyses was carried out to establish the dimensionality of the tests. Second, the hierarchical order of the test was examined, and third, the scores were related to the working memory (M-space)

Table 20

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Class Inclusion Test

Score	0	1	2	$\overline{X}$	SD
K-AM	24	1		.04	.20
K-PM	17	2	1	.20	.52
Prep	17	3	1	.24	.54
Gr 1	16	2	4	.46	.80
Gr 1/2	12	5	7	.79	.88
Gr 2	10	2	15	1.19	.96
Totals	96	15	28	.51	.81

Table 21

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Counting-On Test

Level	0	1	2	3	$\overline{X}$	SD
K-AM	22	3			.12	.33
K-PM	13	4	3		.50	.76
Prep	13	2	6		.67	.91
Gr 1	5	8	5	4	1.36	1.05
Gr 1/2	2	5	2	15	2.25	1.07
Gr 2		1	2	24	2.85	.46
Totals	55	23	18	43	1.35	1.29

Table 22

Frequency of Correct Responses for Children in Each Class and for the Total Population for the Counting-Back Test

Level	0	1	2	3	$\bar{X}$	SD
K-AM	23	2			.08	.28
K-PM	18	2			.10	.31
Prep	13	6	1	1	.52	.81
Gr 1	10	7	4	1	.82	.91
Gr 1/2	4	4	8	8	1.83	1.09
Gr 2		3	5	19	2.59	.69
Totals	68	24	18	29	1.06	1.21

information previously gathered on the same population.

Structure of the Test Battery. Fullerton (1968) used scalogram analysis to organize the battery of tests he developed. He found tests which grouped together and established an order for the tests based on test difficulty. Unfortunately that methodology fails to establish the underlying dimensionality of the data matrix or the possible structure of the assumed hierarchy. A more satisfactory method is to first determine the dimensionality of the intercorrelations of the tests. If the matrix is unidimensional, then a hierarchy can be established using Guttman's (1954) "simplex" model.

The intercorrelations for the ten cognitive processing tests in the order shown in Figure 8 appear in Table 23. These correlations are all positive and

Table 23

Intercorrelations of the Ten Cognitive Processing Tests Presented in the Hypothesized Hierarchical Order

	E	OC	ACN	CN-W	AS-W	CI	CO	CB	T	CRG
E	1.00									
OC	.45	1.00								
ACN	.22	.25	1.00							
CN-W	.30	.32	.35	1.00						
AS-W	.35	.37	.48	.51	1.00					
CI	.13	.13	.32	.24	.28	1.00				
CO	.22	.28	.55	.43	.42	.44	1.00			
CB	.15	.21	.49	.40	.39	.45	.79	1.00		
T	.13	.16	.43	.42	.36	.47	.52	.61	1.00	
CRG	.17	.21	.51	.55	.48	.39	.58	.62	.68	1.00

of reasonable range. A factor analysis was performed on the correlation matrix presented in Table 23 for the eight of the ten tests across the total population. E and OC were not included since they were baseline tests. The model used was a multifactor solution model. All extractions were principal factor extractions with iteration estimates of commonalities, and the varimax rotation procedure was used. The data for this factor analysis appears in Table 24. A two factor solution was derived although the Eigenvalue for the first factor is considerably larger than that for the second factor. An examination of this factor matrix

Table 24

Factor Analysis for Eight Cognitive Process

Tests for the Total Population

	Factors	
	1	2
Eigenvalue	4.34	0.92
% variance	54.3	11.5
Raw(Rotated)Factor Matrix		
ACN	.64(.46)	.07(.45)
CN-W	.61(.25)	.35(.65)
AS-W	.60(.24)	.37(.66)
CI	.52(.49	-.13(.23)
CO	.80(.76)	-.22(.33)
CB	.83(.85)	-.33(.26)
T	.73(.60)	-.06(.41)
CRE	.81(.56)	.11(.59)

shows that the counting tests (CB, CO) load heaviest on the first factor followed by the tests in Battery 4, (T) and (CRE). This factor may reflect a mature level of counting skill. The four other tests also load on this factor but not to the same degree. At best we can say that it is probably a quantitative factor influenced by the ability to count. The second factor seems more qualitative, involving the ability to make comparisons and see transformations which can be done without having to count. In particular, the Wohlwill tests

(AS-W and CN-W) load heaviest on this factor while the two counting tests (CO and CB) load negatively. One test, Class Inclusion, does not load heavily on either factor. Since Class Inclusion involves logical reasoning and is the only nonquantitative test, this finding gives credence to the labels given to the first two factors.

In summary, the cognitive processing tests, contrary to expectations, do not measure a single dimension. Rather they measure two discernible dimensions, quantitative counting factor and a qualitative correspondence factor.

Hierarchical Order. Because the first factor accounted for a large proportion of the variance (54.3%) and all the tests loaded on that factor, the assumed hierarchical ordering of the tests was examined. Guttman's (1954) simplex procedure assumes that each more complex test requires everything previous tests require. This implies that the partial correlation between non-neighboring tests with the effects of an intermediate test partialed out is zero. Although this assumption is too restrictive in practice the general relationships between tests ordered on a single dimension can be examined by studying the matrix of test correlations. In practice one can usually determine if a set of tests is hierarchical by inspection since the correlation between "neighboring" tests should be higher than for "non-neighboring" tests because they contain more common components. In the correlation matrix for a hierarchical set the highest values should appear next to the diagonal and progressively get smaller as one moves away from the diagonal.

For this study the hypothesized order for all 15 tests administered was shown in Figure 7. Since five of the tests were eliminated the new hypothesized order appears in Figure 8.

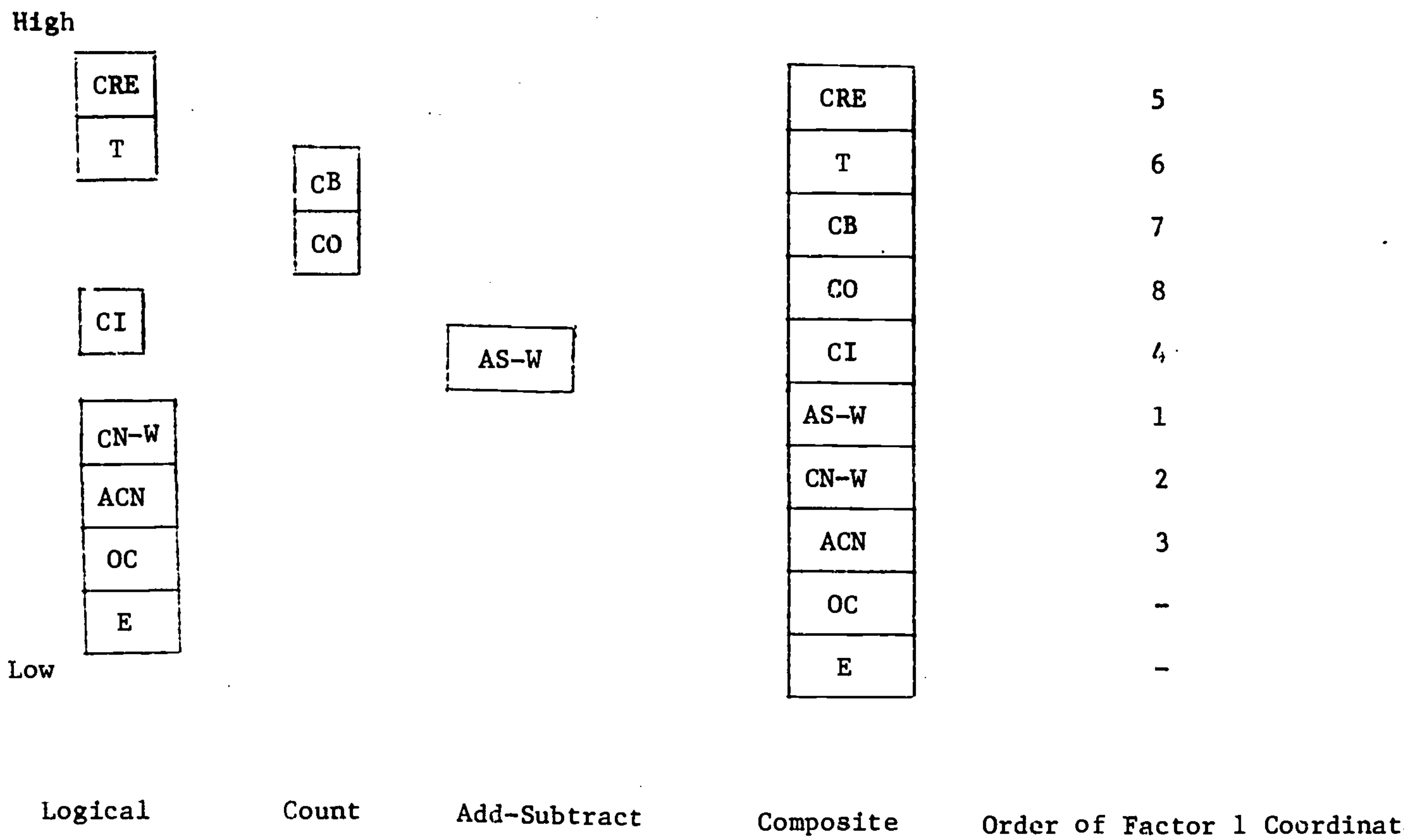

Figure 8. Hypothesized hierarchical order of the 10 cognitive processing tests.

The intercorrelations for the eight of the cognitive processing tests in the order shown in Figure 8 appears in Table 23. However, the tests are not in simplex order. For example, the correlations of Conservation of Number (Wohlwill)(CN-W) with the other tests fluctuate as one goes down the column rather than getting smaller.

This obvious lack of evidence for a hierarchical order could have been an artifact of the arbitrary manner in which the composite hypothesized order was formed, it was decided to examine the order of the five logical tests

(ACN, CN-W, CI, T, and CRE). If this correlation matrix was in simplex order, the other tests could then be joined to that ordered set empirically. The correlation matrix for those five tests appears in Table 25. Again, these tests do not form a simplex.

Table 25

Intercorrelations of Five Logical Tests

in the Predicted Hierarchical Order

	ACN	CN-W	CI	T	CRE
ACN	1.00				
CN-W	.35	1.00			
CI	.32	.24	1.00		
T	.43	.42	.47	1.00	
CRE	.51	.55	.39	.68	1.00

Finally, even the coordinates for the rotated first factor matrix show an order of loadings quite different from the hypothesized order (see Figure 8). In conclusion, no hierarchical ordering of these tests seems reasonable.

A Comparison of the Cognitive Processing Scores with M-space Scores

In the first technical report on this series of studies (Romberg and Collis, 1980) the same population was given four memory (M-space) tests. From the analysis of that data the children were grouped by cluster analysis into six discernible groups. In this section of this second paper the relationship of the children's memory capacity (M-space) scores and their cognitive processing scores is examined.

The correlations between the ten cognitive processing tests and the four memory tests appear in Table 26. The correlations range from .29 to .68

Table 26

Correlations of the Ten Cognitive Processing Tests and the Four M-space Tests

	Cognitive Processing Tests									
	E	OC	ACN	CN-W	AS-W	CI	CO	CB	T	CRE
space Tests										
[a]	.19	.27	.54	.32	.39	.43	.63	.61	.47	.53
[b]	.14	.17	.54	.44	.41	.45	.77	.79	.69	.63
[c]	.17	.24	.46	.32	.37	.48	.53	.55	.46	.47
S[d]	.31	.33	.48	.48	.50	.38	.61	.58	.55	.54

[a]Counting Span

[b]Digit Placement

[c]Mr. Cucui

[d]Backward Digit Span

if one disregards the "extension" (E) and "Ordinal Correspondence" (OC) tests. The higher correlations occur with both the counting tests (CO, CB). This is not surprising. The counting tests undoubtedly require a larger memory capacity than some of the other tests. However, there is no apparent significant variation in loadings of the different memory tests across the cognitive processing tests. This suggests that the positive correlation is along a single dimension.

To check this suggested unidimensional relationship a factor analysis was carried out in which the four M-space tests were added to eight of the cognitive processing tests (E and OC were omitted). The data for that factor analysis appears in Table 27. Again, as was the case with the factor analysis

Table 27

Factor Analyses for Eight Cognitive Process Tests

and the Four M-space Tests for the Total Population

	Factors	
	1	2
Eigenvalues	6.52	1.02
% variance	54.4	8.5
Raw (Rotated) Factor Matrix		
ACN	.65(.56)	.08(.36)
CN-W	.58(.40)	.41(.50)
AS-W	.59(.37)	.41(.60)
CI	.55(.56)	-.12(.12)
CO	.83(.78)	-.18(.28)
CB	.84(.85)	-.25(.15)
T	.73(.73)	-.01(.16)
CRE	.78(.70)	.16(.31)
CS	.71(.68)	-.13(.25)
DP	.86(.74)	-.19(.10)
MC	.63(.68)	-.09(.25)
BDS	.73(.62)	.13(.43)

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of the cognitive processing tests (see Table 24) two factors appeared. The two factors have the same structure as the two factors that appeared in the earlier analysis. The memory tests load on the first factor but not the second.

Next, because the memory tests all load on the first factor we decided to see if there was a pattern in their means for the six M-space groups of students formed in cluster analysis for the ten cognitive processing tests. The six groups (see Romberg and Collis, 1980) included 59, 38, 16, 11, 4, and 6 members. The test means for the ten cognitive process tests for these six groups are in Table 28 and the percentage correct for the six groups are in Table 29. A plot of the percent correct on each of the ten tests for the

Table 28

Means for the Six M-space Groups on

the Ten Cognitive Process Tests

Test	E	OC	AS-W	CN-W	ACN	CO	CB	CRE	T	CI
Group										
1	.90	.84	.46	.48	1.34	.28	.10	.06	.02	.12
2	1.00	1.00	.91	.71	2.40	1.57	1.06	.43	.14	.40
3	1.00	1.00	.87	.93	2.13	2.47	2.33	.73	.53	1.00
4	1.00	1.00	1.00	.90	2.60	2.80	2.60	.80	.90	1.00
5	1.00	1.00	1.00	1.00	3.00	2.75	2.25	1.00	1.00	1.75
6	1.00	1.00	1.00	1.00	2.80	1.80	2.60	1.00	.80	1.80

Table 29

Percent Correct for the Six M-space Groups

on the Ten Cognitive Process Tests

Test	E	OC	AS-W	CN-W	ACN	CO	CB	CRE	T	CI
Group										
1(1)	.90	.84	.46	.48	.45	.09	.03	.06	.02	.06
2(2)	1.00	1.00	.91	.71	.80	.52	.35	.43	.14	.20
3(2S+)	1.00	1.00	.87	.93	.71	.82	.78	.73	.53	.50
4(3S-)	1.00	1.00	1.00	.90	.87	.93	.87	.80	.90	.50
5(3S+)	1.00	1.00	1.00	1.00	1.00	.91	.75	1.00	1.00	.88
6(4S-)	1.00	1.00	1.00	1.00	.93	1.00	.87	1.00	.80	.90

students in each of these groups appears in Figure 9. Figure 10 better illustrates the patterns of differences. There are differences between the five of the six groups with respect to the four areas. Group 1 children with M-space level 1 are below the other groups in all four areas and are in general incapable of handling quantitative tasks or logical reasoning. They are capable of handling qualitative comparisons and transformations at a moderate level.

Group 2 children with M-space level 2 are also without specific quantitative and logical skills (although they performed considerably better than Group 1 on all the tests). They can handle qualitative correspondence at an acceptable level although they scored somewhat lower than the other groups on

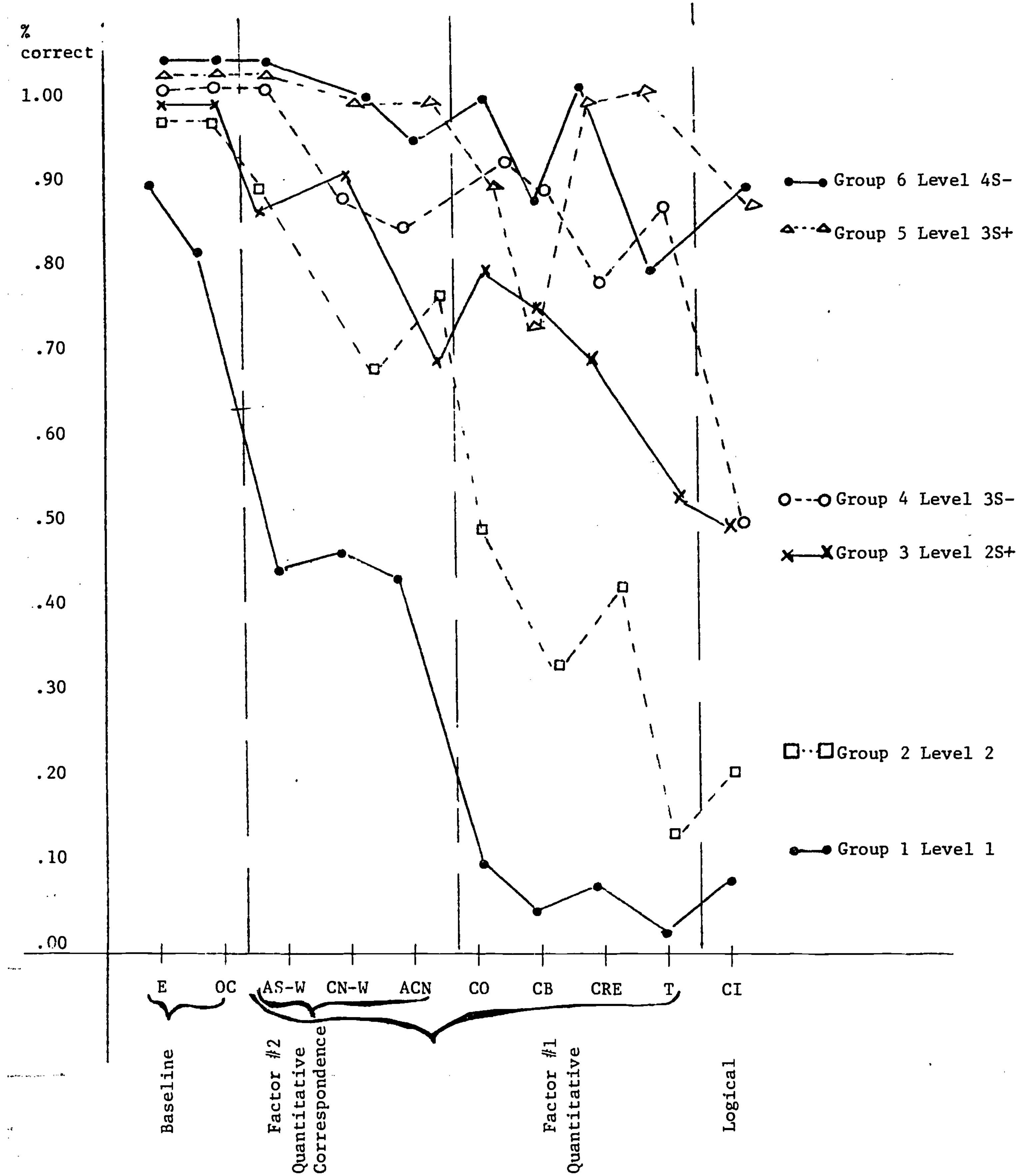

Figure 9. Pattern of scores (percent correct) for the six M-space groups on ten cognitive process tests grouped by factors.

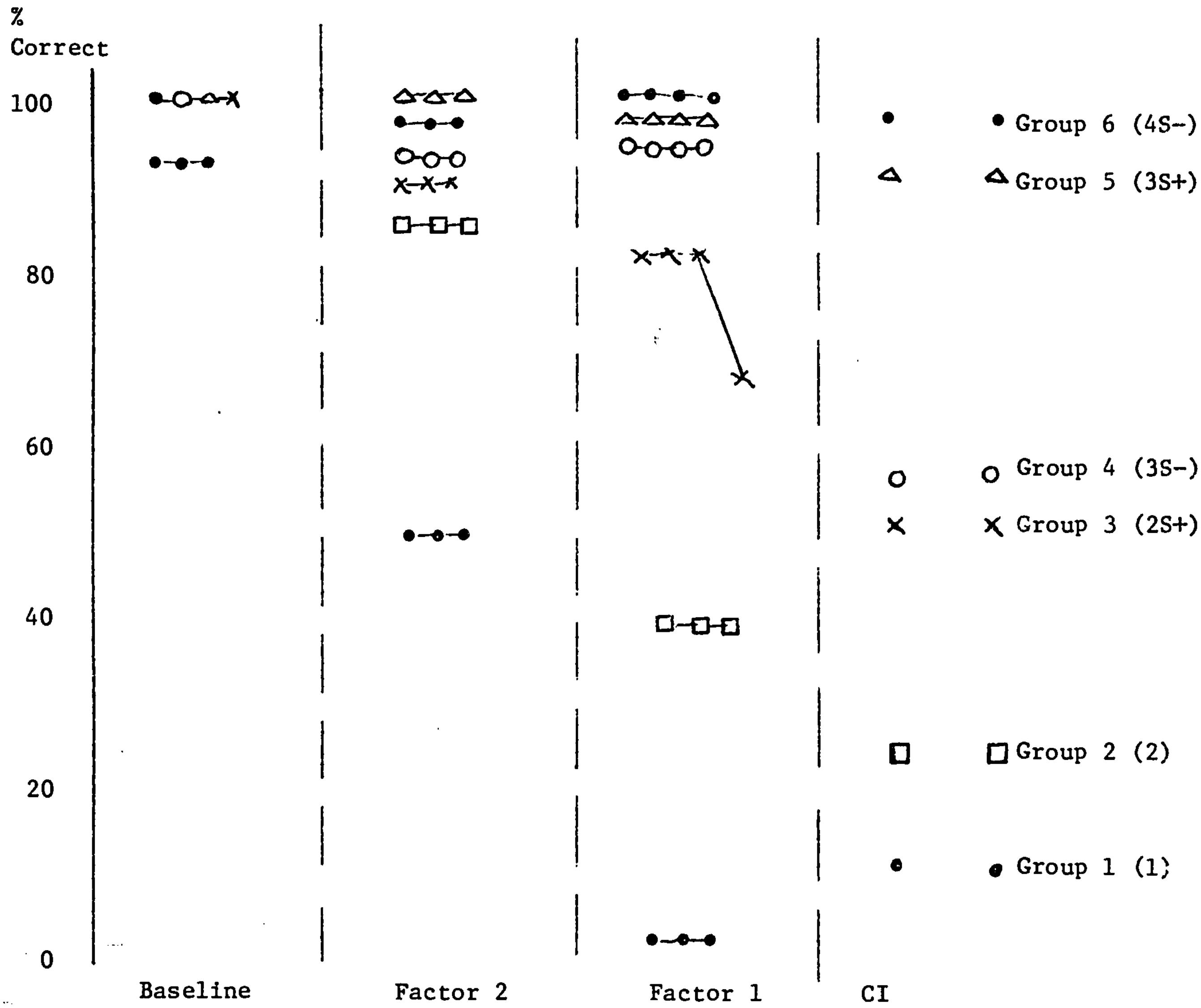

Figure 10. The general pattern of scores for the six M-space groups organized by factors.

the conservation of number test.

Group 3 children with M-space level 2S+ are high on qualitative correspondence, have developed the specific counting skills of counting-on and counting-back, but are inadequate in their use of those skills on the transitive reasoning test. They also are inadequate on logical reasoning although considerably better than Groups 1 or 2 on that test.

Group 4 children with M-space level 3S- are high on qualitative correspondence and all the quantitative tests, but inadequate on the logical reasoning test. In fact they differ significantly from Group 3 only on the Additive Composition Test and the Transitivity Test.

Groups 5 and 6 with M-space levels 3S+ and 4S- present similar profiles on these tests. They reach the ceiling on the qualitative correspondence tests, scoring a little higher than Groups 2, 3, and 4. They have very high scores on all the quantitative tests (like Group 4), and also are high on the logical reasoning test.

In summary, the following seven propositions are suggested by the data.

1. The global qualitative vs. quantitative distinction is constant through in both studies.

2. M-space level seems to be related to the development of other cognitive skills.

3. The developmental sequence appears to be comparison → qualitative correspondence → quantitative → logical operations.

4. An M-space level of 1 is enough for handling simple comparison tasks.

5. An M-space level of 2 is a prerequisite for the development of number skills but is enough for qualitative correspondence.

6. An M-space level of 3 seems necessary for complete success on quantitative tasks.

Conclusions

The purpose of this study was to identify the cognitive processing capabilities for a group of children ages 4-8 for which information on working memory capacity (M-space level) was already available. From this data we anticipated that we would be able to identify groups of children who have well-defined and different capabilities. The conclusion of this study is that this can be done. Using cluster analysis on the memory tests in the first study of this series we identified six groups of students with similar patterns of responses (Romberg and Collis, 1980). In this study we have demonstrated that the cognitive processing scores of five of those six groups differ systematically from each other.

Before coming to this conclusion we were able first to demonstrate that the battery of cognitive tests was not hierarchically ordered either in total or for the five logical processing tasks. From the results of a factor analysis we found the tests loaded on two factors: a quantitative factor that involves mature counting strategies and a qualitative correspondence factor. Also, one test (Class Inclusion), which involves logical reasoning, loaded on neither factor. The first dimension of the factor analysis of the cognitive processing tests indicates that the primary shift from one group to another for four groups identified is in terms of a child's development of mature counting strategies. In all, simple correspondence (both equivalence and order) appears to be the first cognitive processing capacity to develop. This is followed by a qualitative correspondence capacity which involves understanding how correspondence between two sets is preserved or changed under varying circumstances.

Next the quantitative skills of counting-on and counting-back develop, followed by their use in transitivity tasks. Finally, the capacity of logical reasoning develops.

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Footnotes

1. There were 142 children in the school in October of 1979. 139 participated in the first study (Collis and Romberg, 1980). Because of absences complete data on only 122 children was obtained for the 15 cognitive process tests.
2. In Australia kindergarten corresponds with nursery school age and prep corresponds with the U.S. kindergarten.

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APPENDIX A

Protocol for Battery 1

Extension (E)

Ordinal Correspondence (OC)

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BATTERY NO. # 1 (E,OC)

Pass out response booklets to each child.
Make sure each child has a pencil.

WRITE YOUR NAME IN THE SPACE

Check to see that each S has written his/her name legibly.

TURN TO THE PAGE WITH THE DOG ON IT. NOW LOOK AT THE MARBLES ON THE BOARD. WHICH OF THE BOXES SHOWS THE SAME NUMBER OF MARBLES. PUT AN "X" IN THE CORRECT CHOICE BOX.

Show cardboard set of boxes. Put 'x' in the correct choice box and say :

DID YOU PUT AN "X" IN THIS BOX?

Check to see if each child has responded correctly.

GOOD. NOW LET'S TRY ANOTHER ONE. TURN TO THE PAGE WITH A HORSE ON IT.

LOOK AT THE BOX AT THE BOTTOM OF THE PAGE. WHICH BOX AT THE TOP OF THE PAGE HAS THE SAME NUMBER OF MARBLES. WHEN YOU HAVE DECIDED PUT AN "X" IN THE CHOICE BOX.

Let children respond.

DID YOU MARK IT LIKE THIS?

Put "X" in the correct choice box on cardboard set of boxes. Check to see if all are correct.

GOOD. NOW LET US TRY SOME MORE.

For the following pages use the same sequence.

TURN TO THE PAGE ________

LOOK AT THE BOX AT THE BOTTOM OF THE PAGE. WHICH BOX AT THE TOP OF THE PAGE HAS THE SAME NUMBER OF MARBLES. PUT AN "X" IN THE CHOICE BOX.

Let children respond

Page	Figure
	star
	cow
	sheep
	pig
	horse
	chook
	deer
	wolf
	bird
	dog
	mouse
	ball

TURN TO THE PAGE WITH THE BIRD ON IT. THE NEXT SET OF PAGES ARE DIFFERENT.

NOW LOOK AT THE BOTTOM BOX. ONE OF THE BARS IS BLACK. THE REST ARE NOT. WHICH BOX AT THE TOP OF THE PAGE MATCHES IT? PUT AN "X" IN THE CHOICE BOX.

Let children respond.

DID YOU MARK IT LIKE THIS?

Put an "X" in the correct cardboard choice box. Check to see if all are correct.

GOOD. NOW LET'S DO SOME MORE.

For the following pages use the same sequence.

62 TURN TO THE PAGE WITH THE CAMEL ON IT.

LOOK AT THE BOX AT THE BOTTOM OF THE PAGE. WHICH BOX AT THE TOP OF THE PAGE MATCHES IT? PUT AN "X" IN THE CHOICE BOX.

Let children respond.

Page	Figure
	camel
	horse
	monkey
	sheep
	bear
	seal
	cow
	chook
	penguin
	woody woodpecker
	moose
	kiwi

APPENDIX B

Protocol for Battery 2

Conservation of Number (Wohlwill) (CN-W)

Addition-Subtraction (Wohlwill) (AS-W)

PROTOCOL

Battery No.2

Pass out response booklets to each child.
Make sure each child has a pencil.

WRITE YOUR NAME IN THE SPACE

Check to see that each child has written his/her name legibily.

TURN TO THE PAGE WITH THE CAT ON IT

PUT YOUR FINGER ON THE STAR. NOW LOOK AT THE MARBLES ON THE BOARD.

WHICH OF THE "BOXES" NEXT TO THE STAR (show 2 marbles in some array)
SHOWS THE SAME NUMBER OF MARBLES. (Show cardboard of star and 3 choices).
PUT AN "X" IN THE CHOICE BOX LIKE THIS.

Check to see if all responded.

GOOD. NOW LET'S TRY ANOTHER ONE. TURN TO THE PAGE WITH THE ROOSTER ON IT.
PUT YOUR FINGER ON THE STAR. NOW LOOK AT THE MARBLES ON THE BOARD.

Show 3 marbles in the same array.

WHICH OF THE BOXES NEXT TO THE STAR HAS THE SAME NUMBER OF MARBLES.
PUT AN "X" IN THE CHOICE BOX UNDER IT.

Let children respond.

DID YOU MARK IT LIKE THIS?

Put X in the correct choice box. Check to see if all are correct.

GOOD. NOW LET'S TRY ANOTHER ONE. TURN TO THE PAGE WITH THE SUN ON IT.
PUT YOUR FINGER ON THE STAR. NOW LOOK AT THE MARBLES ON THE BOARD.

Show 4 marbles in the same array.

PUT AN "X" IN THE CHOICE BOX UNDER THE BOX THAT HAS THE SAME NUMBER OF MARBLES.

Let children respond. Correct them if they are wrong.

NOW PUT YOUR FINGER ON THE BALL.
I AM GOING TO RE-ARRANGE THE MARBLES.

Mix the four marbles.

NOW PUT AN "X" IN THE CHOICE BOX UNDER THE BOX WITH THE SAME NUMBER OF MARBLES.

Let the children respond.

GOOD. NOW LET'S DO SOME MORE

TURN TO THE PAGE ________

PUT YOUR FINGER ON THE STAR
LOOK AT THE MARBLES ON THE BOARD ______
PUT "X" IN CHOICE BOX

Let children respond

PUT YOUR FINGER ON BALL

performs indicated action saying what is done. ________

NOW PUT "X" IN CHOICE BOX.

	OBJECT	NUMBER	ACTION
PAGE	rabbit	7	re-arrange
	bird	8	take 1 away
	wheelbarrow	6	put 1 with
	man	6	re-arrange
	pig	8	re-arrange
	tractor	7	take 1 away
	goat	6	put 1 with
	sheep	7	put 1 with
	woman	8	re-arrange
	mouse	7	take 1 away
	apples	6	re-arrange
	swan	8	re-arrange

APPENDIX C

Protocol for Battery 3

<u>Addition-Subtraction (Smedslund) (AS-S)</u>

<u>Addition-Subtraction Nonequivalent (AS-N)</u>

<u>Addition-Subtraction Both Equivalent (AS-BE)</u>

<u>Addition-Subtraction Both Inverse (AS-BI)</u>

<u>Addition-Subtraction Both Nonequivalent (AS-BN)</u>

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PROTOCOL

BATTERY # 3
AS-S-CR

Pass out response booklets to each child.
Make sure each child has a pencil.

WRITE YOUR NAME IN THE SPACE.

Check to see if each child has written his/her name legibly

TURN TO THE PAGE WITH STAR ON IT.

IN EACH PANEL THERE IS A PICTURE OF DAVID

Put out a large picture of David on one side

AND A PICTURE OF WENDY

Put out a large picture of Wendy on the other side of the board.

BOTH HAVE A SET OF MARBLES

Put seven marbles out for each like ∴∴·

DO THEY HAVE THE SAME NUMBER OF MARBLES? DO NOT SAY ANYTHING. LOOK AT T
TOP PANEL.

	Figure	Marbles	Wendy	David
Page	horse	7	-2, +2	______
	chicken	9	______	+4, -4
	ball	6	+3, -3	______
	penguin	7	______	-3, +3
	rabbit	8	______	-2, +2
	bird	6	+4, -4	______
	bear	9	+2, -1	______
	crocodile	7	______	-1, +3
	woody woodpecker	8	______	-3, +2
	deer	9	-1, +2	______
	hen	7	______	+2, -3
	geese	6	-2, +1	______

For the following pages use the same sequence

TURN TO PAGE ________

DAVID AND WENDY HAVE SOME MARBLES

Put out ________ marbles for each

IN THE TOP PANEL MARK WHO HAS THE MOST OR WHETHER THEY ARE THE SAME

Let children respond

NOW I AM GOING TO ____________________ AND ______________

IN THE BOTTOM PANEL MARK WHO HAS THE MOST OR WHETHER THEY ARE THE SAME

Show cardboard panel and place 'X' correctly. Check to see if all responses are correct.

NOW I AM GOING TO CHANGE THE MARBLES BY RE ARRANGING THEM.

NOW DO DAVID AND WENDY HAVE THE SAME NUMBER OF MARBLES?

MARK THE SECOND PANEL.

Let children respond

GOOD. NOW LET'S TRY SOME MORE.

For the following pages use the same sequence

TURN TO PAGE ______________

DAVID AND WENDY HAVE SOME MARBLES

put out ______ marbles for each

THE TOP PANEL MARK WHO HAS THE MOST OR WHETHER THEY ARE THE SAME

Let children respond

NOW I AM GOING TO ____________ IN THE MIDDLE PANEL MARK WHO HAS THE MOST OR WHETHER THEY ARE THE SAME

Let children respond

NOW I AM GOING TO ____________ IN THE BOTTOM PANEL MARK WHO HAS THE MOST OR WHETHER THEY ARE THE SAME

	Figure	Number	Wendy	David
Page	elephant	7	+1	+1
	platypus	8	+3	+3
	dolphin	6	-2	-2
	bat	9	-1	-1
	tiger	8	+2	+2
	tortoise	7	-3	-3
	parrot	9	+1	-1
	giraffe	6	+3	-3
	fox	7	-2	+2
	kangaroo	9	+2	-2
	blue whale	6	-1	+1
	koala	8	-3	+3
	polar bear	8	-1	+3
	walrus	6	+2	-1
	hippopotamus	9	+3	-2
	lion	7	-2	+3
	wolf	8	-1	+2
	rhinoceros	7	+3	-1

APPENDIX D

Protocol for Battery 4

Transitivity (T)

Coordination of Relations of Equivalence (CRE)

BATTERY # 4

Pass out the response booklets to each child. Make sure each child has a pencil.

WRITE YOUR NAME IN THE SPACE

Check to see that each child has written his/her name legibly.

TURN TO THE PAGE WITH THE ROOSTER ON IT.

NOW LOOK AT THE BOARD.

Display a set of five butterflies and four rabbits on cards placed in one-to-one correspondence.

ARE THERE MORE BUTTERFLIES OR RABBITS OR ARE THEY THE SAME. DO NOT SAY ANYTHING. IN THE PANEL AT THE TOP OF THE PAGE PUT AN "X" OVER THE BUTTERFLY IF YOU THINK THERE ARE MORE BUTTERFLIES THAN RABBITS. PUT AN "X" OVER THE RABBIT IF YOU THINK THERE ARE MORE RABBITS, AND PUT AN "X" IN THE CIRCLE IF YOU THINK THEY ARE THE SAME.

Let children respond.

DID YOU MARK IT LIKE THIS?

Display cardboard response sheet and mark it correctly.

NOW I AM GOING TO PUT THE RABBITS AT THE BOTTOM OF THE BOARD, AND PUT UP SOME FROGS.

Put up a set of five frogs in one-to-one correspondence with the butterflies.

IN THE PANEL IN THE MIDDLE OF THE PAGE PUT AN "X" OVER WHO HAS THE MOST, THE BUTTERFLIES OR THE FROGS OR IF YOU THINK THEY ARE THE SAME MARK THE CIRCLE.

Let children respond.

DID YOU MARK IT LIKE THIS?

Display cardboard response sheet and mark it correctly.

GOOD. NOW IN THE PANEL AT THE BOTTOM OF THE PAGE DECIDE WHO HAS THE MOST, THE FROGS OR THE RABBITS OR ARE THEY THE SAME.

Let children respond.

DID YOU MARK IT LIKE THIS?

Mark cardboard response sheet correctly.

GOOD. NOW LETS DO SOME MORE.

For the following pages use the same sequence.

TURN TO THE PAGE ______________

LOOK AT THESE SETS a)__________, and b)___________

DECIDE WHICH HAS MOST AND IN THE PANEL AT THE TOP OF THE PAGE PUT AN "X" OVER IT, OR IF THE SAME OVER THE CIRCLE.

Let children respond.

Move a)________or b)_________, and place c)___________ in one-to-one correspondence.

DECIDE WHICH HAS MOST AND IN THE PANEL IN THE MIDDLE OF THE PAGE PUT AN "X" OVER IT, OR IF THE SAME OVER THE CIRCLE.

Let children respond.

NOW DECIDE WHICH OF (a or b) AND (c) HAS MOST AND IN THE PANEL AT THE BOTTOM OF THE PAGE PUT AN "X" OVER IT, OR IF THE SAME OVER THE CIRCLE.

Let children respond.

1. (i) a) Five birds and, b) six flowers placed in one-to-one correspondence.
   (ii) Transformation: Birds lengthened.
   (iii) c) Seven spiders placed in one-to-one correspondence with the birds.
2. (i) a) Seven cats and, b) seven dogs placed in a one-to-one correspondence.
   (ii) Transformation: Dogs shortened.
   (iii) c) Seven cows placed in one-to-one correspondence with the dogs.
3. (i) a) Eight boats and, b) six fish placed in one-to-one correspondence.
   (ii) Transformation: Boats lengthened.
   (iii) c) Five ducks placed in one-to-one correspondence with the boats.
4. (i) a) Six rainhats and, b) six umbrellas placed in one-to-one correspondence.
   (ii) Transformation: Umbrellas shortened.
   (iii) c) Six raincoats placed in one-ton-one correspondence with the umbrellas.
5. (i) a) Seven tables and, b) six chairs placed in one-to-one correspondence.
   (ii) Transformation: Chairs heaped.
   (iii) c) Four vases placed in one-to-one correspondence with the chairs.
6. (i) a) Four trees and, b) six bushes placed in one-to-one correspondence.
   (ii) Transformation: Trees shortened.
   (iii) c) Eight flower pots placed in one-to-one correspondence with the trees.

7. (i) a) Seven blue flowers and, b) seven vases placed in one-to-one correspondence.

(ii) Transformation: Blue flowers lengthened.

(iii) c) Seven red flowers placed in one-to-one correspondence with the vases.

8. (i) a) Eight red eggs and, b) seven egg-cups placed in one-to-one correspondence.

(ii) Transformation: Red eggs shortened.

(iii) c) Seven blue eggs placed in one-to-one correspondence with the egg cups.

9. (i) a) Eight sailing-boats and, b) eight birds placed in one-to-one correspondence.

(ii) Transformation: Sailing boars lengthened.

(iii) c) Nine flower-pots placed in one-to-one correspondence with the birds.

10. (i) a) Nine butterflies and, b) nine rabbits placed in one-to-one correspondence.

(ii) Transformation: Rabbits shortened.

(iii) c) Nine frogs placed in one-to-one correspondence with the butterflies.

11. (i) a) Eight elephants and, b) eight umbrellas placed in one-to-one correspondence.

(ii) Transformation: Elephants heaped.

(iii) c) Eight pigs placed in one-to-one correspondence with the umbrellas.

12. (i) a) Eight ducks and, b) seven jugs placed in one-to-one correspondence.

(ii) Transformation: Ducks shortened.

(iii) c) Six fish placed in one-to-one correspondence with the jugs.

APPENDIX E

Count-On (CO)

Count-Back (CB)

Protocols

PROTOCOL

COUNTING ON

WE ARE GOING TO PLAY SOME COUNTING GAMES. CAN YOU COUNT ON FROM 5 TO 10?

Help S count correctly if necessary.

CAN YOU COUNT FROM 15 TO 20?

Help S count correctly if necessary. If S is unable to count from 15 to 20, Do Part A. If S is able proceed to Part B.

A. COULD YOU COUNT ON TO FIND THE NUMBER THAT IS 1 MORE THAN 5?

If S needs help say "6 COMES RIGHT AFTER 5 SO 6 IS 1 MORE THAN 5".

WHAT IS TWO MORE THAN 5?

If S needs help, say "6 IS ONE MORE THAN 5 SO 7 IS IS TWO MORE THAN 5".

NOW LET'S TRY THESE

A.1 COULD YOU START COUNTING AT 3 TO FIND THE NUMBER THAT IS 2 MORE THAN 3?

(If the procedure is not evident) COULD YOU COUNT OUT LOUD TO SHOW ME HOW YOU DID THAT?

A.2 COULD YOU START COUNTING AT 6 TO FIND THE NUMBER THAT IS 3 MORE THAN 6?

A.3 (Give this problem only if S has given one correct and one incorrect response.)

COULD YOU START COUNTING AT 4 TO FIND THE NUMBER THAT IS 5 MORE THAN 4?

If S answers 2 A problems correctly go to item B.1

B. GOOD. NOW COULD YOU COUNT ON LIKE THAT TO FIND THE NUMBER THAT IS 1 MORE THAN 15?

If S needs held, say "16 comes right after 15 so 16 is 1 more than 15."

WHAT IS 2 MORE THAN 15?

If S needs help, say "16 is 1 more than 15 so 17 is 2 more than 15".

NOW LET'S TRY THESE

B.1 COULD YOU START COUNTING AT 16 TO FIND THE NUMBER THAT IS 3 MORE THAN 16?

(If procedure is not evident) COULD YOU COUNT OUT LOUD TO SHOW ME HOW YOU DID THAT?

B.2 COULD YOU START COUNTING AT 13 TO FIND THE NUMBER THAT IS 4 MORE THAN 13?

B.3 (Give this problem only if S has given one correct and one incorrect response)

COULD YOU START COUNTING AT 15 TO FIND THE NUMBER THAT IS 3 MORE THAN 15?

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(Give C problems only if S has answered two B problems correctly.)

C.1 COULD YOU START COUNTING AT 15 TO FIND THE NUMBER THAT IS 7 MORE THAN 15?

C.2 COULD YOU START COUNTING AT 17 TO FIND THE NUMBER THAT IS 6 MORE THAN 17?

C.3 (Give this problem only if S has given one correct and one incorrect response.)
COULD YOU START COUNTING AT 18 TO FIND THE NUMBER THAT IS 7 MORE THAN 18?

PROTOCOL

COUNTING BACK

WE ARE GOING TO PLAY SOME MORE COUNTING GAMES. DO YOU KNOW HOW TO COUNT BACKWARDS? COULD YOU COUNT BACK STARTING AT 6?

Help the S count correctly if necessary.

CAN YOU COUNT BACK FROM 16?

Help S count correctly if necessary. If S is unable to count back from 16, Do Part A. If S is able to proceed to Part B.

A. COULD YOU COUNT BACK TO FIND THE NUMBER THAT IS ONE LESS THAN 6?

If S needs help, say "5 is 1 less than 6"

WHAT IS TWO LESS THAN 6?

If S needs help, say "5 is 1 less than 6 so 4 is less than 6".

NOW LET'S TRY THESE.

A.1 COULD YOU COUNT BACK STARTING AT 5 TO FIND THAT NUMBER THAT IS 2 LESS THAN 5?

(If procedure is not evident) COULD YOU COUNT OUT LOUD TO SHOW ME HOW YOU DID THAT?

A.2 COULD YOU COUNT BACK STARTING AT 7 TO FIND THE NUMBER THAT IS 3 LESS THAN 7?

A.3 Give this problem only if S has given one correct and one incorrect response). COULD YOU COUNT BACK STARTING AT 8 TO FIND THE NUMBER THAT IS 5 LESS THAN 8?

If S answers 2 A problems correctly go to item B.1.

B. NOW COULD YOU COUNT BACK TO FIND THE NUMBER THAT IS 1 LESS THAN 16?

If S needs help, say "15 is 1 less than 16".

WHAT IS 2 LESS THAN 16?

If S needs help, say "15 is 1 less than 16 so 14 is 2 less than 16".

NOW LET'S TRY THESE.

B.1 COULD YOU COUNT BACK STARTING AT 17 TO FIND THE NUMBER THAT IS 3 LESS THAN 17?

(If procedure is not evident) COULD YOU COUNT OUT LOUD TO SHOW ME HOW YOU DID THAT?

B.2 COULD YOU COUNT BACK STARTING AT 15 TO FIND THE NUMBER THAT IS 4 LESS THAN 15?

B.3 (Give this problem only if S has given one correct and one incorrect response)

COULD YOU COUNT BACK STARTING AT 18 TO FIND THE NUMBER THAT IS 3 LESS THAN 18?

(Give C problems only if S has answered two B problems correctly.)

C.1 COULD YOU COUNT BACK STARTING AT 19 TO FIND THE NUMBER THAT IS 6 LESS THAN 19?

C.2 COULD YOU COUNT BACK STARTING AT 18 TO FIND THE NUMBER THAT IS 7 LESS THAN 18?

C.3 (Give this problem only if S has given one correct and one incorrect response.)
COULD YOU COUNT BACK STARTING AT 17 TO FIND THE NUMBER THAT IS 6 LESS THAN 17?

APPENDIX F

Exact Protocol for Battery 4

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EXACT PROTOCOL USED - NOVEMBER, 1979
SANDY BAY INFANT SCHOOL, TASMANIA

1. Response booklets were passed out to each child - he/she was asked and/or helped to fill in their name. The children were tested in groups of approximately eight children from the one class, at any one time. For Battery 4, mostly whole classes were tested - Grade 1/2, Grade 1, and Grade 2, with the assistance of the teacher and usually one teacher aide and 1 student - we instructed any of these helpers to supervise only according to the exact instructions I stipulated initially to them e.g. no corrections to be made to the responses of any child on the third individual response box, occurring on pages in the first section of the testing i.e., the koala bear to parrot page inclusive.

2. After checking of name writing, the group was instructed to turn to the page with the rooster on it. The instructions for this practice session were followed precisely from the Protocol Battery 4 sheet (designed by Prof. Romberg).

3. After display of the correct response by the instructor after each of the individual responses were made, the subjects were asked to turn to the page with the koala bear on it.

4. To reduce time taken in preparation, the first display on each page (to which the individual responded in the first individual response box on each page) was prepared (out of sight of the children) on a second magnetic board (very similar in dimensions to the board described in the "Apparatus" section) by another person.

5. Once the subjects had been checked as being on the correct page, the instructor displayed the board bearing 7 black flowers (not blue, because of the complications with the colouring in each of the response sheets individually) in a row fairly close together along the top of the board. In the second row 7 vases were placed in 1:1 correspondence with the black flowers, and the subjects were asked by the instructor "Are there more black flowers or more vases or are there the same number of each?" They were asked not to say anything or copy each other, and mark in the panel at the top of the page an 'X' over which has the most, or if the same, over the circle.

6. After all the children responsed, the black flowers were then lengthened and 7 white flowers (instead of red) were put below in 1:1 correspondence with the bases. And the children were asked to respond by marking with an 'X' in the bottom box.

7. After the children had responded to this second box they were asked to "decide whether there are more black flowers or more white flowers or the same number of each?" They were asked to respond in the middle box of this page and quickly turn over to the next page (with the whale on it) after they had finished.
8. On the page with the whale on it, the subjects were presented with the board bearing 8 black eggs spread out in a row along the top, and 8 egg cups in 1:1 correspondence in a row below. The children were asked to mark a response with an 'X' in the top box on the page (exactly according to the instructions described above.
9. After responses had been made by all children the black eggs were shortened and 7 white eggs were put in a third row on the board in 1"1 correspondence with the egg cups.
10. The children were aksed (with the more specific instructions described above) to put an 'X' in the bottom box over which object had the most (egg cups or white eggs) or put an 'X' in the circle if they were the same.
11. In the third box (in the middle of the page) the children were asked to compare black eggs with white eggs.
12. On the page with kangaroo on it, 8 sailing boats were displayed closely together in a row at the top of the box. 8 birds were then put up in a second row in 1:1 correspondence with the sailing boats. The children were asked to make a comparison between these, in the top box on the page.
13. The sailing boats were lengthened on the board and 9 flowerports were put up in a third row on the board in 1:1 correspondence with the birds. In the bottom box the children were asked to make a comparison between the birds and the flowerpots.
14. In the middle box the children were asked to make a comparison between the sailing boats and flowerpots and then turn over the page.
15. On the page with the giraffe on it, 9 butterflies were spaced at the top of the board. 9 rabbits were put up in 1:1 correspondence with the butterflies and the children were asked to make a comparison between these, in the top box.
16. The rabbits were then shortened together. And nine frogs were put up in a third row placed in 1:1 correspondence with the butterflies. In the bottom box children were asked to make a comparison between the butterlies and the frogs.
17. In the middle box children were then asked to make a comparison between the rabbits and the frogs and then turn over the page.
18. On the page with the tortoise on it, 8 elephants were spaced out on the top

row of the board. 8 umbrellas were then put up in 1:1 correspondence with the elephants, in a second row on the board. (The children were asked to make a comparison by marking with an 'X' in the top box).

19. The elephants were then moved into a heap together. 8 pigs were then put up in a third row on the board in a 1:1 correspondence with the umbrellas. In the bottom box the children were asked to compare the umbrellas with the pigs.

20. In the middle box the children were then asked to make a comparison between the elephants and the pigs and quickly turn the page.

21. On the page with the parrot on it, 8 ducks were spaced out on the top row on the board. 7 jugs were then placed in a second row in 1:1 correspondence with the ducks. In the top box the children were asked to make a comparison between the ducks and the jugs.

22. The row of ducks was then shortened. And six fish were put up in a third row in 1:1 correspondence with the jugs.

23. In the bottom box the children were then asked to make a comparison between the jugs and the fish.

24. In the middle box the children were asked to make a comparison between the ducks and the fish and then turn back to the page with the star on it.

25. After the children were assisted to turn back to the page with the star on it, 5 birds were put up close together on a top row on the metallic board. 6 flowers were then put up in a second row on the board in 1:1 correspondence with the birds. The children were then asked to make a comparison between the birds and the flowers by deciding which had the most "and in the panel at the top of the page put an 'X' over it, or if the same, over the circle."

26. (These exact instructions were followed for each of the comparisons to be made from now on).

27. After the children had made a comparison the birds were then moved down to the bottom of the board and lengthened (see Prof. Collis about these).

28. The spiders were then put up in the middle row on the board in 1:1 correspondence with the flowers. The children using the bottom box were then asked to make a comparison between the flowers and the spiders.

29. In the middle box (with nothing further being moved on the board at this stage) the children were asked to make a comparison between the spiders, and the birds (and then quickly turn over the page).

30. On the page with wolf on it, 7 cats were spaced out on the top row on the board. 7 dogs were then put up in a second row in 1:1 correspondence with the cats. In the top box on the page, the children were asked to compare these 2 (according to the specific instructions).
31. After the children had responded the dogs were moved down to the bottom of the board and shortened. 7 cows were then put up in 1:1 correspondence with the cats and in the middle box on the page, the children were asked to compare these 2 (cats and cows).
32. In the bottom box on this page the children were asked to decide which of cows or dogs had the most and then turn over the page.
33. On the page with the walrus on it, 8 boats were put up close together in the top row on the board. 6 fish were put up 1:1 correspondence with the boats in the second row on the board. In the top box the children were asked to make a comparison between these two - boats and fish.
34. After responding, the boats were slid down to the bottom of the board and lengthened. 5 ducks were then put up in 1:1 correspondence with the fish. In the bottom box the children were asked to compare then the fish with the ducks.
35. In the middle box the children were asked to compare the ducks with the boats and turn over the page.
36. On the page with the lion on it, 6 rainhats were spaced out on the top row on the board. 6 umbrellas were then put up in 1:1 correspondence with the rainhats. The children were then asked to compare the rainhats with the umbrellas in the top box on this page.
37. After responding the umbrellas were moved down the bottom of the board and the row was shortened. 6 raincoats were then put up as a middle row in 1:1 correspondence with the rainhats. In the middle box in the page, the children were then asked to compare the rainhats with the raincoats.
38. In the bottom box of the page the children were asked to compare the raincoats with the umbrellas and turn over the page.
39. On the page with the hippopotomus on it, 7 tables were spaced out on the top row on the board. 6 chairs were then put up in 1:1 correspondence with the tables. In the top box on the page the children were asked to compare the tables with the chairs.
40. After responding, the chairs were slid down to the bottom of the board and heaped together.

41. 4 vases were then put up as a middle row in 1:1 correspondence with the tables. The children were then asked to compare in the middle box on the page, the tables with the vases.

42. In the bottom box on the page, the children were then asked to compare the vases with the chairs and asked to turn the page.

43. On the page with the polar bear on it, 4 trees were spaced out on the top row on the metallic board. 6 bushes were then placed in 1:1 correspondence with the trees as a second row on the board. In the top response box on the page the children were asked to compare the trees with the bushes.

44. The row of trees was then moved down to the bottom of the board and shortened. 8 flowerpots were then put up as a second row on the board in 1:1 correspondence with the bushes. The children were then asked to compare in the bottom box on the page the bushes with the flowerpots.

45. In the middle box the children were then asked to compare the flowerpots with the trees.

The children were then told they had completed the game, and were then thanked for their cooperation.

APPARATUS:-

The materials used in this project were a brown metallic board measuring approximately 5 foot across, by 2 foot down; and cut-out vyaline objects (n = 203) - the shapes of the objects to be displayed (representing each of the individual objects in the response boxes) were shaped and outlined using black felt pen traced on vyaline and glued onto small magnets in order to be easily moved around the board. In each case the vyaline objects were made as close as practicable to resemble the drawn object in each of the response boxes. Each shape measured approximately 1 1/2 inches across and 4 inches down. (3 samples of these enclosed in latest box set 19/11/79.)

Notes:-

/Problems

1. We found it very difficult as organizers to control all the copying that might have occurred during the period of testing - largely due to lack of space within the school for us to operate.

2. Most of the children especially older ones, found it very tempting to count the objects on the board, even though I constantly reminded them we did not want them to count but just to look.

3. Most of the children complained and showed signs of tiredness by the end of testing. Because of limited time however the whole test had to be administered in the one session.

4. I think it is very important that a gesture of thanks is made to this extremely tolerant school. They also expressed the desire to obtain a copy of the eventual report.

(K. Dedenczuk)

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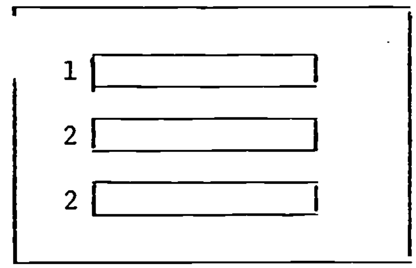
Order of presentation of response boxes, by page.



Object (identifying page)	Order of presentation (of Ind. box)
Rooster	1, 2, 3
Star	1, 3, 2
Wolf	1, 2, 3
Walrus	1, 3, 2
Lion	1, 2, 3
Hippopotomus	1, 2, 3
Polar Bear	1, 3, 2
Koala Bear	1, 3, 2
Whale	1, 3, 2
Kangaroo	1, 3, 2
Giraffe	1, 3, 2
Tortoise	1, 3, 2
Parrot	1, 3, 2

*These changes were made because of a mistake made by me, when originally drawing up response box on the pages.

The correct order of responding by each child on each of the pages was closely checked by the supervisors.

E.G.

1. Rooster (page)

The top response box (butterfly and rabbit) was presented first; then the middle box (butterfly and frog) was presented; then the bottom response box (frog and rabbit) on the page was presented.

2. Star (page)

The top response box (bearing bird and flower) on the page was presented first; then the bottom box on the page (bearing flower and spider) was presented to the subject; finally the middle box on this particular page (bearing bird and spider) was presented.