DOCUMENT RESUME

ED 194 337 SE 033 164

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TITLE	Cognitive Development and Performance on Verbal Addition and Subtraction Problems.
INSTITUTION	Wisconsin Univ., Madison. Research and Development Center for Individualized Schooling.
SPONS AGENCY	National Inst. of Education (DHEW), Washington, D.C.
REPORT NO	WRDCIS-TR-560
PUB DATE	Aug 80
GRANT	OB-NIE-G-80-0117
NOTE	59p.; Report from the Project on Studies in Mathematics.
EDRS PRICE	MF01/PC03 Plus Postage.
DESCRIPTORS	Addition; *Cognitive Development; Cognitive Processes; Elementary Education; *Elementary School Mathematics; *Grade 1; *Learning Theories; Logical Thinking; Mathematics Instruction; *Problem Solving; Subtraction
IDENTIFIERS	*Mathematics Education Research; Piagetian Stages; *Piagetian Tasks

ABSTRACT

This study investigated the relationships between Piagetian logical reasoning abilities with an information processing capacity, and first-grade children's performance on verbal addition and subtraction problems. An analysis of simple arithmetic problems indicated that several reasoning abilities identified by Piaget are needed to achieve operational understanding of addition and subtraction. The abilities studied were number conservation, class inclusion, and transitive inference. The arithmetic problems used were varied systematically in semantic structure, number size, and the availability of objects to aid the solution process. Statistically significant differences between the developmental groups were found for some of the cognitive variables on some problem types. However, no clear pattern emerged to suggest that a particular cognitive ability was especially important for solving a specific problem type or using a specific solution strategy. None of the cognitive abilities was absolutely required to solve any of the problem types or to use a given strategy. The report indicates that the use of these studied cognitive tasks as readiness variables for arithmetic instruction should be questioned. The study also suggests that mathematics problem solving ability may need to be reassessed for each instructional topic. (MP)

Technical Report No. 560

COGNITIVE DEVELOPMENT AND PERFORMANCE ON VERBAL ADDITION AND SUBTRACTION PROBLEMS

by

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Report from the Project on
Studies in Mathematics

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August 1980

Published by the Wisconsin Research and Development Center for Individualized Schooling. The project presented or reported herein was performed pursuant to a grant from the National Institute of Education, Department of Health, Education, and Welfare. However, the opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education, and no official endorsement by the National Institute of Education should be inferred.

Center Grant No. OB-NIE-G-80-0117

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The Wisconsin Research and Development Center is supported with funds from the National Institute of Education and the University of Wisconsin.

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Table of Contents

List of Tables

List of Figures

Abstract

This study investigated the relationships between Piagetian logical reasoning abilities and an information processing capacity, and first-grade children's performance on verbal addition and subtraction problems. The logical reasoning abilities of interest were number conservation, class inclusion, and transitive inference. Information processing capacity was considered to be the number of information pieces that could be simultaneously processed in working memory and was measured by backward digit span. The arithmetic problems varied systematically in semantic structure, number size, and availability of objects to aid the solution process.

Statistically significant differences between the developmental groups were found for some of the cognitive variables on some problem types, but no clear pattern emerged to suggest that a particular cognitive ability was especially important for solving a specific problem type or using a specific solution strategy. Multiple regression analyses showed that backward digit span was the best predictor of performance accuracy and of the frequency with which advanced solution strategies were used. Transitive reasoning and class inclusion rarely made significant contributions to the regression models. The R^2's for all models were statistically significant, and generally ranged from .10 to .20.

None of the cognitive abilities was required to solve any of the arithmetic problem types or to use a given solution strategy. Some children who had not yet developed a particular ability solved at least some of the problems of each type, and occasionally applied the more advanced strategies to do so. This calls into question the use of these cognitive tasks as readiness variables for arithmetic instruction.

Cognitive Development and Performance on Verbal Addition and Subtraction Problems

As any first-grade teacher can attest, beginning school children differ in their ability to solve basic mathematics problems. In particular, some children advance their arithmetic skills with apparent ease, while others struggle to grasp certain concepts, and even with prolonged instruction experience some confusion. The purpose of this study was to examine one of the factors that may account for individual differences in the acquisition of arithmetic concepts and skills -- the presence of more general cognitive developmental abilities that may be prerequisites for mastering the arithmetic operations. Specifically. the study was designed to investigate the relationship between several Piagetian abilities and an information processing capacity, and first-grade children's performance on verbal addition and subtraction problems.

Background

A logical analysis of simple arithmetic problems suggests that several logical reasoning abilities identified by Piaget (1952) are needed to achieve an operational understanding of addition and subtraction. The first of these, number conservation, represents an ability to distinguish those transformations that are relevant for number (adding objects to, or removing them from, a set) from those that are irrelevant for number (rearranging the objects in a set). Many of the strategies used to solve simple addition and subtraction problems involve transformations which presuppose this ability. Second, certain part-whole addition and subtraction problems deal with superordinate and subordinate relationships that are very similar to those found in the Piagetian class inclusion task. In fact, it is reasonable to believe that the class inclusion ability is needed to understand the part-whole relationships in a problem such as "There are m children on the playground; n of them are boys and the rest are girls; how many girls are on the playground?" Transitive reasoning represents a third ability which may be important for children's acquisition of addition and subtraction skills. Since many arithmetic problems require an application of simple ordering principles, transitivity may be linked to success on certain addition and subtraction problems.

In addition to examining the role of these Piagetian reasoning abilities in young children's arithmetic problem solving, the present study also investigated the effect of information processing capacity on arithmetic performance. Case (1975, 1978a) argues convincingly that often children possess all of the individual skills needed for a particular task and still fail the task. The reason for this failure may be children's limited capacity to deal with all of the incoming information and to integrate the skills which they possess. While it is not yet possible to specify the information processing demands of individual arithmetic problems, it is reasonable to assume that certain solution strategies require more processing capacity and therefore are available only to children who are more developmentally advanced.

In spite of the logical reasons for believing that these four cognitive variables may affect children's ability to solve basic arithmetic problems, the relevant empirical data are inconclusive. The evidence most often cited to support the potential relationships between Piagetian reasoning abilities and children's arithmetic performance in school is the frequently reported significant correlation between performance on the Piagetian tasks and arithmetic achievement (see Carpenter, 1980). Although these results suggest a global relationship between cognitive development and arithmetic problem solving, they provide little insight into the relationships between specific developmental abilities and particular arithmetic concepts and skills.

A few studies do provide some information on the role of particular cognitive abilities in children's arithmetic performance. With respect to number conservation, Steffe (1970) and LeBlanc (Note 1) found that conservation performance was a significant predictor of first grade children's addition and subtraction skills respectively, with low conservation scores associated with especially poor arithmetic scores. These results suggest that conservation may need to be at least partially developed before children can master addition and subtraction concepts. However, the results of several other studies indicate that such a conclusion is questionable, at best. Pennington (1977) reports that, while number conservers performed significantly better than nonconservers on many school-type arithmetic tasks, a substantial number of nonconservers performed successfully on at least some problems. Furthermore, Woodward (1978) and Sohns (1974) found that number conservation ability was related to children's performance on some addition and subtraction problems but not on others. Finally, Steffe and Johnson (1971) discovered that number conservers outperformed nonconservers on only one out of four kinds of verbal addition and subtraction problems. It appears from these results that different types of arithmetic problems may make different demands on conservation ability. No consistent pattern has emerged from these studies, however, that might indicate which type of problems require conservation and which do not.

A partial answer to this question may be emerging from a series of teaching experiments designed to study the effect of number conservation on learning to solve various arithmetic tasks. Mpiangu and Gentile (1975) instructed number conservers and nonconservers in several routine calculation tasks which required simple counting skills. After finding that the nonconserving children gained as much from instruction as their conserving peers, the authors conclude that conservation is not required to solve arithmetic problems. However, the results of a study by Steffe, Spikes, and Hirstein (Note 2) suggest that this conclusion may overgeneralize the data. Steffe et al., using a static form of the number conservation task, found several qualitative differences between the performances of conservers and nonconservers after rather extensive arithmetic instruction. While conservation had no apparent effect on children's ability to perform routine calculations, it had a significant effect on more difficult tasks, such as missing addend problems. Conserving children were able to learn advanced counting strategies (e.g., counting on and counting back) and use these to solve the more difficult problems; nonconserving children mastered only rote forms of counting and consequently were less successful in the more complex problem situations.

In summary, it appears that different kinds of addition and subtraction problems make different demands on number conservation. While conservation is not required to master simple skill tasks, it may affect performance on problems that require logical reasoning or genuine problem solving behavior. Steffe et al. conclude that conservation is needed to acquire more advanced solution strategies which in turn can be used to solve novel and difficult problems.

The effects of class inclusion and transitivity on arithmetic performance have been less well researched. Sohns (1974) found no significant correlations between class inclusion ability and first-, second-, and third-grade children's performance on a subtraction computation test. Steffe et al. also report nonsignificant relationships between class inclusion and a variety of counting and number skills. However, Howlett (1974) reports that class inclusion ability did relate to first grade children's ability to solve missing addend problems and suggests that the more advanced developmental children used different strategies to solve the problems. Taken together, the results of these studies paint a picture similar to that for number conservation. Different kinds of arithmetic tasks make different demands on class inclusion ability, and its effect on performance may be mediated through its influence on the types of strategies available to children at different developmental levels.

Although transitivity has a less obvious logical connection with addition and subtraction, Brainerd (1979) argues that it represents a reasoning ability which is psychologically more basic than conservation or class inclusion for the acquisition of arithmetic concepts. Brainerd's data support this contention but it is not clear whether the results reflect children's competence or are simply an artifact of the rather narrow set of tasks used to assess these concepts. For example, arithmetic concepts are frequently measured with a paper-and-pencil number fact test. Further research is needed to clarify the role of transitive reasoning in young children's understanding of addition and subtraction operations. It should be noted that several other investigators have studied the emergence of preschool children's initial understanding of addition and subtraction concepts, and the relationship of this understanding to Piagetian cognitive abilities. However these studies will not be reviewed here since the focus of this study is on school-related arithmetic tasks.

Very little evidence exists on the relationship between information processing capacity and arithmetic problem solving. Whimbey, Fischhof, and Silikowitz (1969) used a common test of processing capacity, backward digit span, and found significant correlations among college students between capacity and performance on a mental additions test. With regard to young children, Case (1978b) describes an instructional program designed to teach missing addend problems to kindergarten children by reducing the information processing demand at each step. Although apparently the program was successful, it provides little insight into the nature of the relationship between processing capacity and ability to solve arithmetic problems. The present study was one of the first to explore the effect of information processing capacity on young children's ability to solve arithmetic problems.

Placed in the context of this brief review, the present study can be viewed as an exploratory investigation of the relationship between information processing capacity and young children's arithmetic problem solving, and an extension of previous work on the role of logical reasoning abilities in these problem solving processes. The study moved beyond a comparison of right and wrong answers to an analysis of the solution processes used on the arithmetic problems by children at different developmental levels. The aim of the study was to determine whether any of the three Piagetian abilities, or a minimum level of processing capacity, were prerequisites for solving verbal addition and subtraction problems, or were needed to acquire more advanced solution strategies. Verbal addition and subtraction problems were used in order to elicit the application of problem solving strategies rather than the recitation of rote skills.

Method

The present study is one of a series of studies designed to examine the development of basic addition and subtraction concepts in young children (Carpenter & Moser, Note 3). The principal focus of these studies is on the strategies children use to solve addition and subtraction problems, and on problem, individual, and classroom characteristics which might affect these strategies. This study describes the relationship between the individual characteristics selected for inclusion and children's arithmetic performance.

Subjects

The sample consisted of 149 first-grade children from three elementary schools. The schools draw from predominantly middle-class neighborhoods in a midwestern city. All schools used a modified version of Developing Mathematical Processes (Romberg, Harvey, Moser, & Montgomery, 1974) for their instructional program. At the time of testing, in January, the children had received several lessons on solving different kinds of verbal arithmetic problems. In these lessons, modeling the problem situation with objects was suggested as an appropriate solution strategy.

Cognitive Tasks

The cognitive tasks were selected to measure children's reasoning ability in number conservation, class inclusion, and transitivity situations, and to assess their information processing capacity. Each child received two forms of the number conservation, class inclusion, and length transitivity tasks. Both forms of each task used conventional task formats. The first form of the conservation task involved spreading out the seven markers in one of two rows and asking the child if there were still the same number in each row; the second form involved grouping the seven markers in one row into small sets of three and four markers and asking a similar question. The forms of class inclusion were identical except that in the first form the superordinate set consisted of 12 red and white blocks and the subordinate set consisted of nine red blocks; in the second form the superordinate set included seven pieces of fruit and the subordinate set was made up of five apples. The two forms of length transitivity differed in that one of the forms included a Mueller-Lyer illusion and the other form did not.

The response to each form of a task was scored as correct (1) or incorrect (0). Consequently each child received a score of 0, 1, or 2 for each of the three logical reasoning abilities.

A backward digit span task was used to measure information processing capacity. Backward digit span appears to have a good deal of face validity since processing capacity is considered to be the amount of information which can be held in mind while solving a problem, and backward digit span measures the number of information pieces which can be remembered while operating on them. There is also empirical precedent for using backward digit span as a measure of processing capacity (Case, 1974, 1977; Lawson, 1976).

The backward digit span task used in this study consisted of 10 two-digit, 10 three-digit, and 10 four-digit trials. Testing began with the two-digit trials and moved to the three- and four-digit trials. Testing was terminated when the child missed three consecutive items or made five or more errors in a set of 10 trials. Performance was scored by assigning a full point for each set of 10 trials completed successfully (six consecutive correct responses) plus a fractional point for the number of correct responses in those sets which showed neither complete mastery (six consecutive correct responses) nor complete failure (three consecutive errors). As an example, a child who responded correctly to six consecutive two-digit items and five out of 10 three-digit items (without three consecutive errors) received a score of 1.5.

For data analysis purposes, four levels of backward digit span were established: 0-1.4, 1.5-1.9, 2.0-2.4, 2.5-3.0. Corresponding to each of these levels children were assigned a processing capacity score of 0, 1, 2, or 3.

Arithmetic Problems

Verbal addition and subtraction problems differ in the action or relationships which they describe. Two orthogonal dimensions can be used to characterize the different problem types used in this study (see Moser, Note 4). The first is a distinction between problems which describe some form of action and those which present a static relationship between the objects in question. The second dimension focuses on set relationships and distinguishes problems in which one group of objects is an implied subset of the second group from problems in which the described sets are disjoint. Within each problem type arising from these distinctions, several different problems can be generated by placing the unknown quantity in a different "position" and by describing an increase or a decrease in the quantity in question.

For this study, six different verbal arithmetic problems were selected from this large set of problem types. Table 1 gives representative problems of each type selected along with the labels, for future reference, assigned to each problem type. The problems are placed within the 2 x 2 matrix generated by crossing the two dimensions described above.

These six problem types were selected because: (a) they typify problems commonly included in elementary school textbooks; (b) they include the three

Table 1

Representative Addition and Subtraction Problems

	action	static
set inclusion	Joining (Addition). Wally had 3 pennies. His father gave him 5 more pennies. How many pennies did Wally have altogether? Joining (Subtraction). Kathy has 5 pencils. How many more pencils does she have to put with them so she has 8 pencils altogether? Separating (Subtraction). Tim had 8 candies. He gave 3 candies to Martha. How many candies did Tim have left?	Part-Part-Whole(Addition). Sara has 5 red cars. She also has 3 blue cars. How many cars does Sara have altogether? Part-Part-Whole (Subtraction) There are 8 children on the playground. Five are boys and the rest are girls. How many girls are on the playground?
no set inclusion	Equalizing. (No problems were included from this cell).	Comparison (Subtraction). Joe has 3 balloons. His sister Connie has 8 balloons. How many more balloons does Connie have than Joe?

basic types associated with subtraction; (c) they were problems that first-grade children would most likely be able to solve; and (d) an earlier study (Carpenter, Hiebert, & Moser, in press) indicated that they would elicit a variety of solution strategies. Each problem type was presented under four different conditions resulting from crossing two variables -- number size and availability of physical objects which could be used to help solve the problem. Therefore a total of 6 x 4 or 24 problems were presented to each child. Two sets of number triples were used. The sum of addends in the smaller set were between 5 and 9, and the sum of those in the larger set were between 11 and 16. The assignment of number triples to problem types involved a six-by-six Latin square design resulting in six sets of six problem tasks, each of which was uniformly and randomly distributed across subjects. For the verbal problems with a missing addend structure, the unknown number was always presented as the second of the two addends.

The strategy used to solve each problem was recorded, and the final response was scored as correct or incorrect. Therefore, two measures were collected for each problem -- the accuracy of the solution and the strategy used to generate the solution.

Individual Interviews

Each child was tested three times in an individual interview setting. The first interview consisted of the 12 arithmetic problems with smaller numbers, the second interview consisted of the 12 large-number arithmetic problems, and the third interview contained the cognitive ability tasks. Each interview lasted about 15 minutes. The first two interviews were terminated early if the child showed a complete lack of understanding or failed to initiate a solution strategy on several successive problems. Arithmetic problems that were not administered were scored as incorrect.

The order in which the arithmetic problems were administered was the following (refer to Table 1 for problem names): Joining (Addition), Separating (Subtraction), Part-Part-Whole (Subtraction), Part-Part-Whole (Addition), Comparison (Subtraction), and Joining (Subtraction). The same order was used within each of the four sets of problems. The six problems in which manipulative objects were available always preceded the six problems in which the objects were absent. The same order was used for all subjects since a previous study (Carpenter, et al., in press), using similar problems, found no significant order effects.

The procedures used in the first two interviews could be considered a form of naturalistic observation. If the solution strategy used on a particular problem was apparent, no follow-up questions were posed. If the strategy was not apparent, the child was asked to describe how the answer was found. The interviewer continued questioning until the child's strategy was apparent or until it was clear that no explanation was forthcoming. All interviewers followed a standardized routine for questioning children and coding responses. Interviewers were trained to a minimum .90 level of intra and intercorder reliability.

The cognitive tasks were presented in the following order: length transitivity (no Müeller-Lyer illusion), class inclusion (blocks), backward digit span, number conservation (one row spread), length transitivity (Müeller-Lyer illusion), class inclusion (fruit), and number conservation (one row grouped). In the Piagetian tasks children were asked to provide explanations for their responses but only their final judgments were scored. (Two counting tasks were included in this interview, following the second length transitivity task, but the results of these tasks are not contained in this report.)

Addition and Subtraction Strategies

Based on other studies (Carpenter, et al., in press; Carpenter & Moser, Note 3) potential addition and subtraction strategies were identified. These are strategies that, if applied accurately to solve the arithmetic problems, will yield the correct answer. A brief description of the strategies follows; for a complete description of the strategies see Carpenter and Moser (Note 3).

For the addition strategies let m + n = t where m is the smaller of the two addends.

Counting All. Counting sequence begins with 1 and ends with t, either with or without the aid of objects to model the sets.

Counting On from First (Smaller) Number. Counting sequence begins with m or m + 1, involves n increments, and ends with t.

Counting On from Larger Number. Counting sequence begins with n or n + 1, involves m increments, and ends with t.

Number Fact. Response based on recall of that particular number fact.

Heuristic. Solution generated from a related number fact or property of the number system. For example, "4 + 7 is 11 because 4 and 6 are 10 and 1 more is 11."

Based on a logical analysis, the final three strategies in this list appear to be more advanced that the first two strategies. They require a decision to use a more efficient strategy and/or more advanced knowledge of the number system. These strategies are of particular interest since one hypothesis investigated in this study was whether certain developmental abilities are needed to acquire them.

The subtraction strategies are appropriate for solving problems represented by the number sentences $n - m = d$ or $m + d = n$ where d is the unknown quantity. Since there are more problem types for subtraction than addition there are also a greater number of appropriate solution strategies. The following list includes a brief description of each; again, for a more complete description see Carpenter and Moser (Note 3).

Separating From. Set n is modeled with objects; m objects are removed. Remaining objects are counted 1 to d.

Separating To. Set n is modeled with objects. Some objects are removed until m are left. Removed objects are counted 1 to d.

Adding On. Set m is modeled with objects. Additional objects are added on until collection equals n. Objects added on are counted 1 to d.

Matching. Both n and m are modeled with objects and then matched one-to-one. Unmatched cubes are counted 1 to d.

Counting Down From. Backwards counting sequence begins with n or $n - 1$, involves m decrements, and ends with d.

Counting Down To. Backwards counting sequence begins with n or $n - 1$ and involves as many decrements as necessary to end with m. The number of decrements equals d.

Counting Up From Given. Forward counting sequence begins with m and involves as many increments as necessary to end with n. The number of increments equals d.

Number Fact. (Same as for addition)

Heuristic. (Same as for addition)

As with addition, some of the subtraction strategies seem to be more advanced than others. In general, the strategies which do not depend upon the use of objects -- the last five strategies in the above list -- demonstrate a more efficient and sophisticated method of solution. The acquisition of these strategies may require a certain minimum level of cognitive development. The results to follow include an analysis of this question.

Results

The aim of this study was to examine the relationship between several specially selected cognitive variables and children's arithmetic performance. In particular, the intent was to determine whether any of the cognitive variables serve as prerequisites for successful performance on various addition and subtraction verbal problems, or are needed to acquire certain solution strategies. These questions will be addressed by presenting first the results with respect to accuracy of arithmetic performance, and then the results on solution strategies.

Accuracy of Arithmetic Performance

The relationship of each cognitive variable with the accuracy of children's addition and subtraction solutions was examined initially by comparing the performance of the developmental groups. Children were classified as low, transitional, or high for each Piagetian ability based on their score of 0, 1, or 2 on each pair of tasks. Four groups were formed with respect to information processing capacity based on the scoring system for backward digit span described earlier. With respect to the dependent variables, performance was summed across the four arithmetic tasks of similar problem type. This generated an accuracy score of 0-4 for each subject on each of the six problem types.

Mean scores and standard deviations for each developmental group are presented in Table 2. The relative performance of the developmental groups within each cognitive ability is pictured in Figures 1-4. The graphs shown in the figures, and the accompanying means in Table 2, indicate that, in general, the more advanced developmental children performed more accurately than their less cognitively developed peers. This informal observation apparently

Table 2

Mean Number of Correct Responses (out of 4) and

Standard Deviations of the Developmental Groups on Problems of Different Type

Cognitive ability	Developmental level	Number of subjects	Arithmetic problem type					
			Joining addition	Part-Part-Whole addition	Joining subtraction	Part-Part-Whole subtraction	Comparison subtraction	Separating subtraction
Number conservation	0	16	2.00(1.37)	1.75(1.44)	1.63(1.63)	1.19(1.28)	1.00(1.26)	1.06(1.18)
	1	14	2.36(1.60)	2.57(1.79)	2.07(1.44)	.93(1.07)	1.71(1.14)	1.93(1.33)
	2	119	2.84(1.10)	2.86(1.11)	2.69(1.23)	1.86(1.44)	1.87(1.50)	2.23(1.17)
Class inclusion	0	39	2.44(1.33)	2.33(1.34)	2.00(1.30)	1.23(1.31)	1.44(1.39)	1.92(1.26
	1	30	2.67(1.32)	2.73(1.31)	2.63(1.35)	1.23(1.33)	1.60(1.50)	1.67(1.30)
	2	80	2.85(1.09)	2.89(1.18)	2.73(1.30)	2.11(1.40)	1.99(1.47)	2.30(1.16)
Transitive reasoning	0	22	2.50(1.26)	2.55(1.50)	2.18(1.33)	1.05(1.21)	1.64(1.56)	1.55(1.37)
	1	52	2.65(1.28)	2.58(1.29)	2.35(1.38)	1.52(1.45)	1.50(1.38)	1.88(1.23)
	2	75	2.80(1.15)	2.85(1.17)	2.73(1.29)	2.03(1.39)	1.99(1.48)	2.36(1.13)
Backward digit span	0	51	2.25(1.38)	2.12(1.38)	1.86(1.46)	1.12(1.28)	1.47(1.46)	1.65(1.21)
	1	45	2.84(1.09)	2.73(1.10)	2.73(1.18)	1.78(1.35)	1.56(1.36)	2.24(1.19)
	2	40	2.88(1.07)	3.20(1.04)	2.77(1.17)	1.93(1.42)	1.93(1.53)	2.33(1.29)
	3	13	3.46(.66)	3.46(.97)	3.54(.66)	3.08(1.12)	3.15(.80)	2.38(.96)

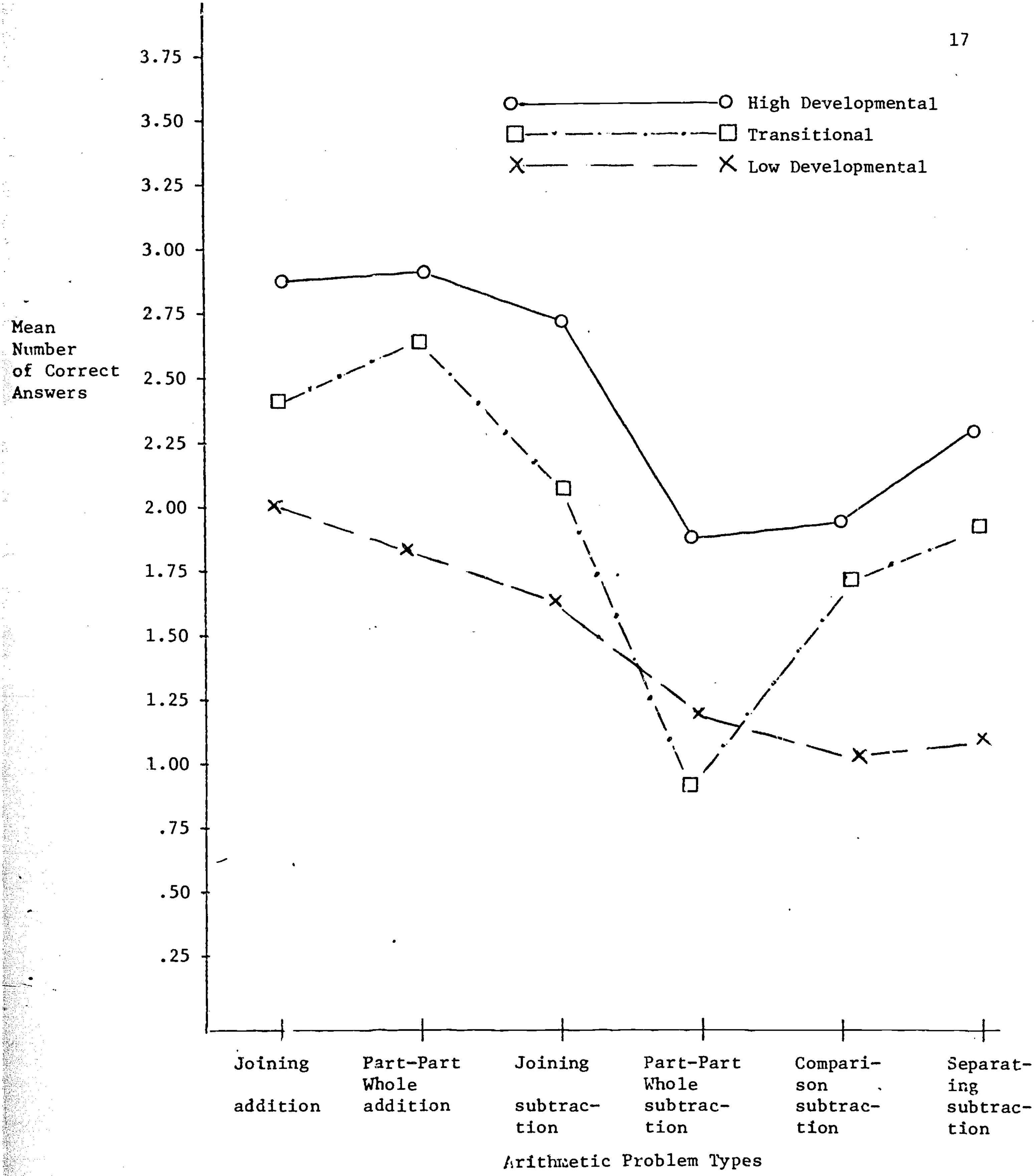

Figure 1. Performance of number conservation ability groups.

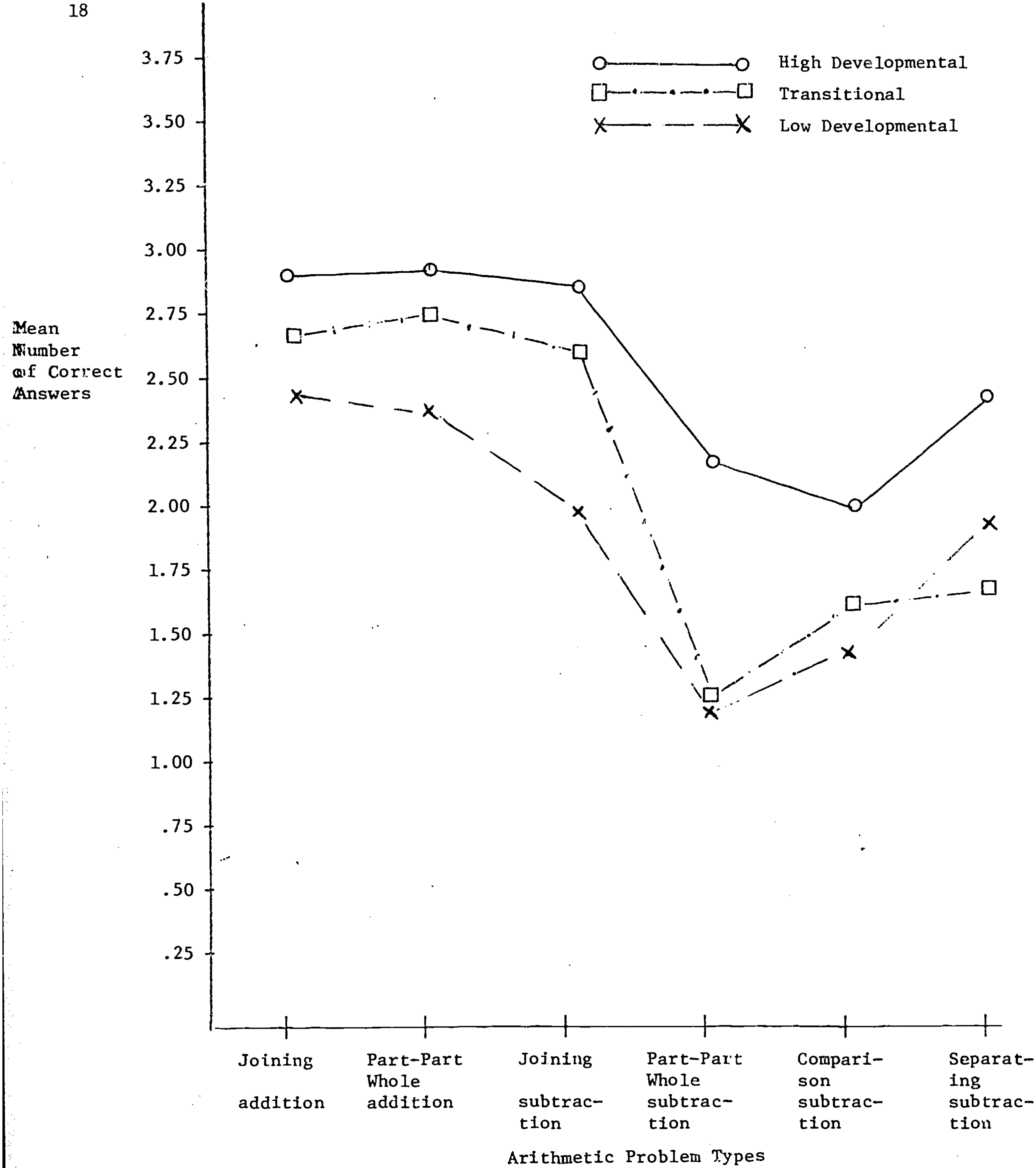

Figure 2. Performance of class inclusion ability groups.

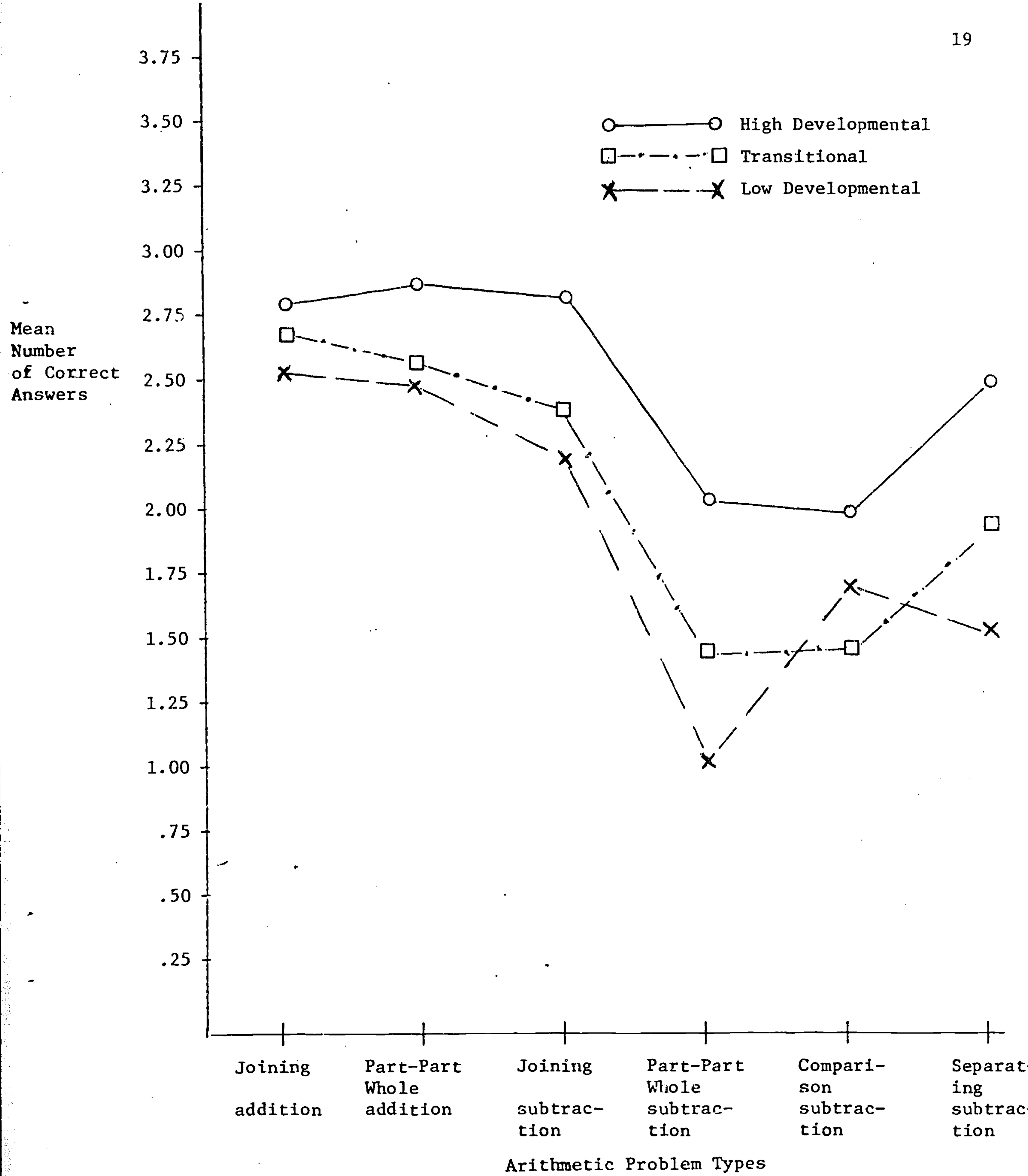

Figure 3. Performance of the transitive reasoning ability groups.

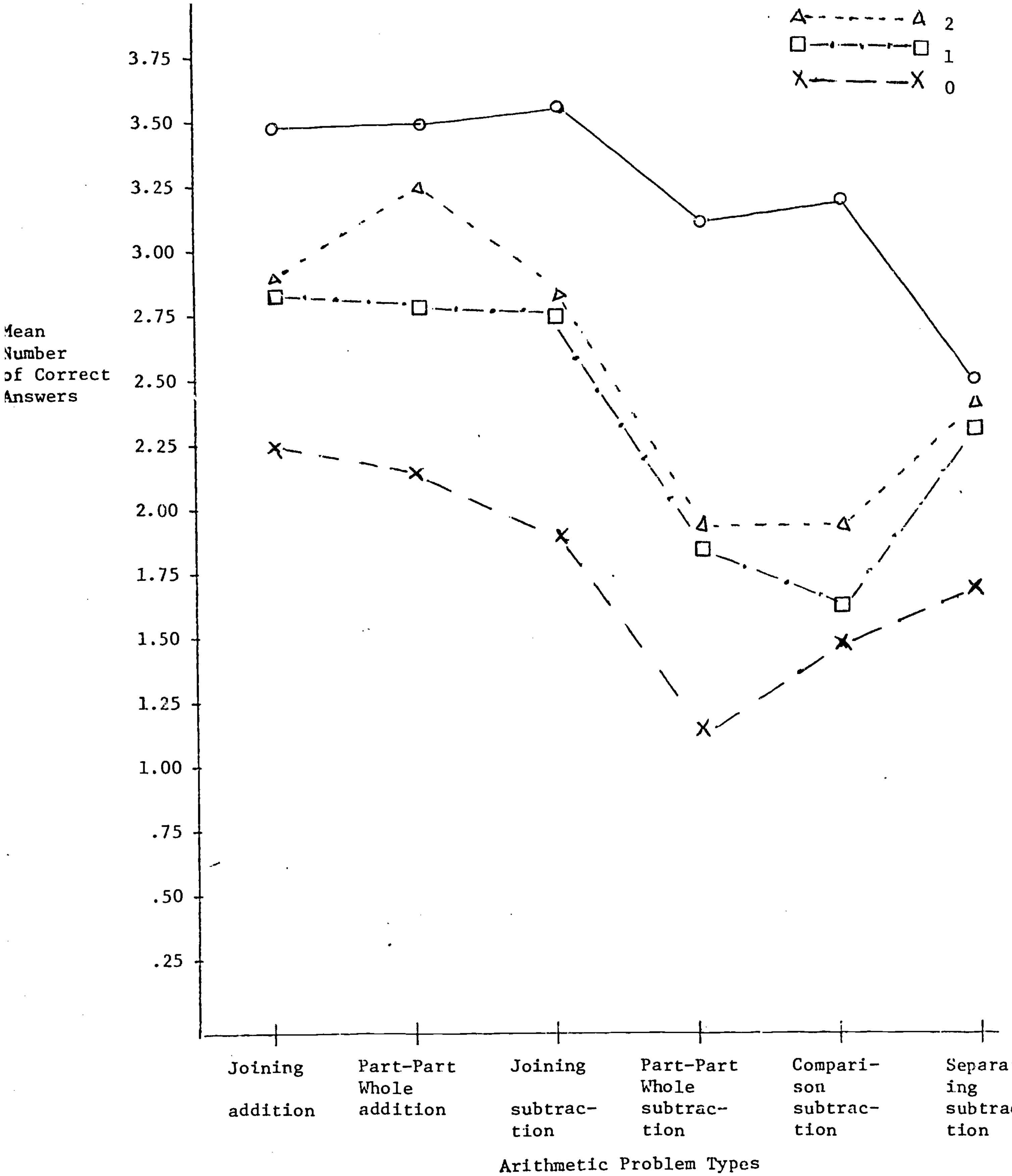

Figure 4. Performance of the backward digit span groups.

holds true for each of the cognitive abilities and for [illegible] of the problem types. The question is whether these differences are real differences, i.e., statistically significant, and whether any of the cognitive abilities are prerequisites for successful performance.

The second of these questions can be answered by looking at the means of the lowest developmental groups. The lowest mean score of 1.00 indicates that on the average, children who did not yet possess a particular cognitive ability still mastered at least one of the four tasks of each problem type. Apparently, none of the four cognitive abilities included in this study are required to solve these addition and subtraction problems.

What about statistical significance? To compare the overall performance of the developmental groups, a multivariate analysis of variance was run for each cognitive variable across all six problem types. The means used for these analyses are shown in Table 2. A summary of the multivariate analyses, and the follow-up univariate analyses for each problem type, is contained in Table 3. The analysis procedures were appropriate for the unequal cell sizes evident in Table 2. Based on the multivariate analyses, when all six problem types are considered simultaneously, only class inclusion and backward digit span sorted children into developmental groups that differed significantly in their arithmetic performance. The univariate analyses provide a clue with regard to which problem type(s) contributed to this overall difference. It is particularly interesting to note that the class inclusion ability groups differed significantly only on the Part-Part-Whole (subtraction) problem, the problem type which, from a logical perspective, is related most closely to class inclusion. Backward digit span, on the other hand, had significant

Table 3

Summary of Multivariate and Univariate Analyses on Problem Type for Each Cognitive Variable

Cognitive ability	Multivariate analysis		Univariate analyses for each problem type											
			Joining addition		Part-Part-Whole addition		Joining subtraction		Part-Part-Whole subtraction		Comparison subtraction		Separating subtraction	
	df	F value	df	F value	df	F value	df	F value	df	F value	df	F value	df	F value
Number conservation	12,282	1.96	2,146	4.2[illegible]	[illegible]146	5.86[a]	2,146	5.64[a]	2,146	4.07	2,146	2.56	2,146	6.87[a]
Class inclusion	12,282	2.49[a]	2,146	1.56	2,146	2.58	2,146	4.16	2,146	7.76[a]	2,146	2.12	2,146	3.36
Transitive reasoning	12,282	1.55	2,146	.59	2,146	.96	2,146	2.12	2,146	4.99[a]	2,146	1.81	2,146	4.87[a]
Backward digit span	18,396	3.00[a]	3,145	4.85[a]	3,145	8.35	3,145	8.63[a]	3,145	8.39[a]	3,145	5.49[a]	3,145	3.28

[a] $p < .01$

discriminatory power on almost all problem types.

A perusal of the univariate analyses results shown in Table 3 suggests that different problem types make different demands on the various cognitive abilities. That is, the power with which a specific cognitive variable is able to predict arithmetic performance appears to depend upon the particular problem type under consideration. It is also possible that a combination of developmental abilities may best explain performance on the arithmetic tasks. For example, children may need to develop class inclusion ability and a high information processing capacity before they can master all forms of the Part-Part-Whole problem. In order to address these issues and check which cognitive variables best predict accuracy of solution on each problem type, a series of six multiple regressions were run, one for each problem type. The regression procedure, a forward stepwise process, first selected the best single predictor and then in each succeeding step added the variable from those remaining that contributed most to the existing model in terms of added predictive power. Correlation coefficients between predictor and criterion variables are given in Table 4. The regression models, presented in Table 5, include only those predictors which added a significant portion ($\alpha = .01$) of predictive power to the model. The predictors are listed in the equation in the order in which they were entered into the model.

As seen in Table 5, the regression model for each problem type explained a statistically significant amount of variation in performance on those problems, with R^2's ranging from .07 for the Comparison (subtraction) problems to .23 for the Part-Part-Whole (subtraction) problems. The single best predictor for five of the six problem types was the information processing

Table 4

Correlations Between Performance on Cognitive Tasks

and Solution Accuracy on Arithmetic Problems

	NC	CI	TR	BDS	J01	PPW1	J02	PPW2	CO	SE
Number conservation (NC)	1.00									
Class inclusion (CI)	.13	1.00								
Transitive reasoning (TR)	.33	.19	1.00							
Backward digit span (BDS)	.16	.14	.04	1.00						
Joining addition (J01)	.23	.15	.09	.26	1.00					
Part-Part-Whole addition (PPW1)	.27	.18	.11	.37	.74	1.00				
Joining subtraction (J02)	.27	.22	.16	.37	.77	.74	1.00			
Part-Part-Whole subtraction (PPW2)	.20	.29	.25	.36	.49	.53	.57	1.00		
Comparison subtraction (CO)	.18	.17	.12	.27	.52	.55	.57	.54	1.00	
Separating subtraction (SE)	.29	.15	.25	.25	.52	.60	.61	.45	.48	1.00

Table 5

Regression Models that Best Predict Performance

Accuracy on the Arithmetic Problem Types

Problem types	Regression model[a]	R^2	Significance level $p<$
Joining addition	y=1.26+.51(BDS)+.36(NC)	.11	.01
Part-Part-Whole addition	y=.75+.78(BDS)+.41(NC)	.18	.01
Joining addition	y=.46+.81(BDS)+.43(NC)	.18	.01
Part-Part-Whole addition	y=.65+.85(BDS)+.40(TR)+ .33(CI)	.23	.01
Comparison subtraction	y=.61+.71(BDS)	.07	.01
Separating subtraction	y=.52+.48(NC)+.46(BDS)	.09	.01

[a]Abbreviations used in the equations are: NC-number conservation; CI-class inclusion; TR-transitive reasoning; BDS-backward digit span.

variable--backward digit span. Number conservation was the best single predictor in the remaining case and was included as the second variable in three other equations. Transitive reasoning and class inclusion appeared in only one regression model -- that for the Part-Part-Whole (subtraction) problem type.

A final analysis of the accuracy scores focused on the conditions under which the problems were administered rather than on the semantic structure of the problems. Piagetian tasks like conservation, class inclusion, and transitivity involve relationships between physical objects. It may be that the ability to deal with these relationships is more closely related to solving problems that involve manipulating objects than to solving problems where no objects are available. Information processing capacity, on the one hand, may be more critical for problems where objects are absent than for problems where objects can be used as memory aids. Processing capacity may be especially important when the numbers are larger and children are less likely to use quickly accessible number facts or other automatized routines.

Means and standard deviations for the developmental groups on the problems given under different conditions are shown in Table 6. A summary of the univariate analyses for each cognitive ability on each problem condition is given in Table 7. Number conservation and backward digit span are the most successful variables in terms of sorting children into groups that differ significantly in accuracy of responses. However, the effect of neither seems to be dependent on the particular condition. Number conservation is a significant factor in problems where physical objects were present <u>and</u> where they were absent. Backward digit span is a significant classification variable

Table 6

Mean Number of Correct Responses and Standard Deviations of the Developmental Groups on Problems Given Under Different Conditions

Developmental level	Number of subjects	Arithmetic problem condition: Physical objects available (12 problems)	Physical objects not available (12 problems)	Small numbers (12 problems)	Large numbers (12 problems)	Physical objects not available <u>and</u> large numbers (6 problems)
0	16	5.25(4.01)	3.38(3.38)	5.19(3.95)	3.43(3.50)	1.19(1.60)
1	14	6.71(3.65)	4.86(3.57)	6.64(3.88)	4.93(3.54)	1.93(1.86)
2	119	8.06(2.88)	6.29(3.42)	8.68(2.91)	5.67(3.49)	2.15(2.05)
0	39	6.67(3.45)	4.69(3.30)	6.92(3.48)	4.44(3.25)	1.49(1.81)
1	30	7.23(3.19)	5.30(3.45)	7.30(3.37)	5.23(3.30)	1.90(1.83)
2	80	8.25(2.97)	6.61(3.54)	9.00(2.99)	5.86(3.71)	2.34(2.11)
0	22	6.73(3.37)	4.73(3.63)	6.68(3.76)	4.77(3.37)	1.73(1.79)
1	52	7.17(3.47)	5.31(3.42)	7.54(3.43)	4.94(3.51)	1.77(1.86)
2	75	8.21(2.86)	6.55(3.49)	8.93(2.90)	5.83(3.60)	2.29(2.14)
0	51	6.31(3.79)	4.16(3.46)	6.45(3.74)	4.02(3.56)	1.31(1.85)
1	45	7.96(2.55)	5.93(2.97)	8.56(2.70)	5.33(3.05)	2.00(1.78)
2	40	8.17(2.81)	6.85(3.52)	8.78(2.82)	6.25(3.60)	2.53(2.15)
3	30	10.00(1.41)	9.08(2.29)	11.08(1.19)	8.00(2.74)	3.39(1.85)

<u>Note</u>: Standard deviations are given in parentheses.

Table 7

Summary of Analyses of Variance on Problem

Condition for Each Cognitive Variable

ive y	Univariate analyses for each problem condition									
	Physical objects available		Physical objects not available		Small numbers		Large numbers		Physical objects not available _and_ large numbers	
	_df_	_F_ value	_df_	_F_ value	_df_	_F_ value	_df_	_F_ value	_df_	_F_ value
nservation	2,146	6.52[a]	2,146	5.75[a]	2,146	10.53[a]	2,146	3.00	2,146	1.67
lusion	2,146	3.63	2,146	4.51[a]	2,146	6.75[a]	2,146	2.18	2,146	2.50
e reasoning	2,146	2.71	2,146	3.27	2,146	5.42[a]	2,146	1.32	2,146	1.35
digit span	3,145	6.44[a]	3,145	10.16	3,145	10.08[a]	3,145	6.29[a]	3,145	5.44[a]

in all problem conditions.

The relative strength of number conservation and backward digit span as predictors of solution accuracy is confirmed by the multiple regression analyses. Table 8 shows the regression models for each problem condition. Class inclusion and transitive reasoning did not make a significant contribution to any of the models.

While statistically significant relationships are evident, the cognitive abilities are not prerequisites for solving verbal arithmetic problems given under a particular condition. The mean scores in Table 6 indicate that at least some children who had not yet developed a particular cognitive ability were able to solve some problems given under each condition. In this sense the analyses with respect to condition yield the same results as those for problem type. Differences between developmental groups are significant for some cognitive variables on problems given under certain conditions. Although no clear patterns are evident to link specific abilities with specific conditions, backward digit span is the most consistently significant variable. However, none of the cognitive abilities appear to be essential for solving a problem given under any of the conditions.

Solution Strategies

One of the primary concerns of this study was to determine whether children of different developmental status used different strategies to solve the arithmetic problems. A question of particular interest was whether any of the cognitive abilities are needed to acquire more advanced solution strategies. Tables 9 and 10 show the frequency with which appropriate solution strategies were used by an "average" student from each developmental group. For the eight

Table 8

Regression Models that Best Predict Performance

Accuracy on the Arithmetic Problems Given Under Each Condition

Problem condition	Regression models[a]	R^2	Significance level: $p <$
Physical objects available	y=2.89+1.69(BDS)+1.16(NC)	.16	.01
Physical objects not available	y=.02+2.40(BDS)+1.13(NC)	.21	.01
Small numbers	y=2.16+2.09(BDS)+1.51(NC)	.24	.01
Large numbers	y=1.83+2.17(BDS)	.11	.01
Physical objects not available <u>and</u> large numbers	y=.20+1.12(BDS)	.09	.01

[a] Abbreviations used in the equations are=BDS - backward digit span; NC - number conservation.

Table 9

Mean Number of Appropriate Strategies Used by

Each Developmental Group on the 8 Addition Problems

Cognitive ability	Developmental level	Strategy				
		Count All	Count On From First Number	Count On From Larger Number [a]	Number Fact [a]	Heuristic [a]
Number conservation	0	2.69	.50	.50	.38	.13
	1	2.79	.50	1.07	.71	.29
	2	3.24	.76	.96	.77	.36
Class inclusion	0	3.51	.51	.46	.51	.21
	1	3.40	.30	.77	.90	.20
	2	2.85	.96	1.20	.76	.44
Transitive reasoning	0	3.59	.59	.68	.73	.09
	1	3.52	.75	.88	.29	.12
	2	2.73	.72	1.01	1.03	.55
Backward digit span	0	3.14	.45	.61	.35	.18
	1	3.38	.80	.87	.67	.36
	2	2.85	1.00	1.20	1.03	.40
	3	3.15	.54	1.46	1.46	.62

[a]Advanced strategy

Table 10

Mean Number of Strategies Used by Each

Developmental Group on the 16 Subtraction Problems

Cognitive ability	Developmental level	Separating From	Separating From	Adding On	Matching	Counting Down From[a]	Counting Down To[a]	Counting Up From Given[a]	Number Fact	Heuristic[a]
Number conservation	0	2.38	.06	1.31	.50	.06	0	.69	.19	.13
	1	2.86	.07	1.14	.79	.57	0	1.57	.57	.21
	2	3.35	.05	1.73	.57	.17	.03	1.66	1.15	.44
Class inclusion	0	3.05	.08	1.38	.54	.21	0	1.13	.56	.33
	1	2.93	0	2.03	.67	.20	0	1.10	.73	.23
	2	3.38	.06	1.60	.58	.19	.04	1.91	1.35	.46
Transitive reasoning	0	2.59	0	1.55	.55	.32	0	1.32	.55	.14
	1	3.38	.06	1.67	.77	.02	0	1.12	.50	.15
	2	3.25	.07	1.63	.47	.28	.04	1.91	1.47	.61
Backward digit span	0	2.75	0	1.43	.63	.06	0	.84	.65	.22
	1	3.71	.13	2.11	.58	.18	0	1.44	.60	.40
	2	3.22	.03	1.33	.48	.33	.05	2.13	1.53	.45
	3	3.15	.08	1.69	.77	.38	.08	2.85	2.08	77

[a]Advanced strategy

addition problems, shown in Table 9, it is clear that none of the cognitive abilities were prerequisites for acquiring a particular strategy. That is, at least some low developmental children, with respect to each cognitive ability, used a given strategy.

Several other patterns are evident from these data. First, in most cases the low developmental children used appropriate strategies less frequently than their more advanced peers. This is consistent with their less accurate performance reported above. Second, the relative frequency with which the various strategies were used is similar for each developmental group. For example, the strategy used most often by nonconservers was Count All; the strategy used least often was of a heuristic type. This same pattern also holds true for number conservers.

Similar observations can be made from the data for the 16 subtraction problems shown in Table 10. In general, none of the cognitive abilities appeared to be a prerequisite for acquiring a given strategy. The only strategy which the low developmental children did not use was Counting Down To. However, this strategy was used so infrequently by any developmental group that it is impossible to draw any conclusions from this result. As was true for the addition problems, the low developmental children used most of the appropriate subtraction strategies less frequently than the higher developmental children. However, the distribution of use across strategies was again similar for groups of different developmental level.

A question of special interest in this study concerned the effect of developmental status on the use of more advanced solution strategies. The data relevant for this question are summarized in Table 11. A pattern of

Table 11

Mean Number of Advanced Strategies

Used by Each Developmental Group

Cognitive ability	Develop- mental level	Addition problems (8)	Subtraction problems (16)	All problems (24)	
				$\bar{X}$	SD
Number conservation	0	1.00	1.06	2.06	2.65
	1	2.07	2.93	5.00	6.66
	2	2.09	3.44	5.53	6.08
Class inclusion	0	1.18	2.23	3.41	4.71
	1	1.87	2.27	4.13	5.73
	2	2.40	3.90	6.30	6.33
Transitive reasoning	0	1.50	2.32	3.82	3.92
	1	1.29	1.79	3.08	4.02
	2	2.59	4.31	6.90	6.96
Backward digit span	0	1.14	1.76	2.90	4.15
	1	1.89	2.62	4.51	5.45
	2	2.63	4.48	7.11	6.77
	3	3.54	6.15	9.69	6.87

responses similar to that found in Tables 9 and 10 is evident here. While the low developmental children, with respect to each cognitive ability, did not apply advanced strategies as frequently as the higher developmental children, they apparently did use these strategies on some problems. Analyses of variance showed that the total mean differences were statistically significant (at the .01 level) for only two of the cognitive variables: transitive reasoning, $F(2,146)=7.57$; and backward digit span, $F(3,145)=7.44$. For transitive reasoning it is interesting to note that the transitional group used advanced strategies less frequently than the low developmental group. However, post hoc comparisons showed this difference to be statistically nonsignificant ($\alpha = .05$).

A forward stepwise multiple regression procedure, similar to that described earlier, was used to establish the relative contribution of each cognitive variable in predicting the use of advanced strategies. Backward digit span was the best single predictor of the frequency with which advanced strategies were used. The regression model y=3.37+3.59(backward digit span)+1.98 (transitive reasoning) was significant at the .01 level, accounting for 17% of variation. Number conservation and class inclusion did not explain a significant ($\alpha = .01$) portion of the remaining variance and were not included in the model.

An important descriptive statistic shown in Table 11, which may account for the nonsignificant effect of number conservation and class inclusion on the use of advanced strategies, is the high within group variance. The large standard deviations indicate that categorizing children according to developmental level, on any of these cognitive variables, did not yield a homogeneous sorting with respect to the use of more sophisticated solution strategies. While the tendency to use advanced strategies generally increased with development, there were many low developmental children who apparently used these strategies and many high developmental children who did not.

Discussion

The aim of this study was to examine the relationship between addition and subtraction problem solving performance and more general cognitive abilities that might be prerequisites. From an instructional point of view, the question of whether the ability to solve problems or apply certain strategies is tied to the development of basic cognitive abilities is an important one. There are potentially different instructional implications if certain problem solving processes are closely linked to underlying cognitive abilities whose development is difficult to accelerate than if this is not the case.

The results reported here indicate that the answer to this question, as with many questions in human learning and development, is more complicated than one would like. The graphs in Figures 1-4 show a consistent effect of cognitive development on problem solving performance. In all cases, the children who possessed a particular cognitive ability performed better than those who did not, although the mean differences across all tasks reached statistical significance only for two of the four cognitive variables. These results suggest that the absence of at least some of the cognitive abilities have a limiting effect on children's acquisition or deployment of accurate solution strategies.

The results do not suggest, however, an immediate application of the cognitive variables as readiness indicators in an instructional setting. It is quite clear that the cognitive abilities were not prerequisites for successfully solving the arithmetic problems, of whatever type, or under whatever condition. Furthermore, the regression analyses showed that the most parsimonious models usually accounted for 10% to 20% of the variance in performance on the various sets of problems. Although this was sufficient to reach statistical significance, it is not clear whether it is sufficient to be educationally significant.

The information on solution strategies supports these reservations expressed about using the cognitive tasks as readiness variables for arithmetic instruction. The fact that at least some low developmental children not only solved the problems correctly, but did so with a variety of appropriate strategies, argues against interpreting their performance as an application of rote skills. By using the more advanced strategies such as counting on, counting back, and some heuristic forms, these children demonstrated that a clear understanding of the problem situation and the acquisition of an efficient solution process does not depend upon the development of these particular cognitive abilites.

This finding is problematic for Piaget and those sympathetic with Piaget's position. They would not deny that preoperational children can learn simple number skills that require some form of symbol manipulation, but they would argue that such manipulations lack meaning for the child. Consequently, children who have not yet developed the logical reasoning ability of, for example, conservation, would not be able to apply their number skills to solve novel problems. However, the strategies used by the preoperational children in this study indicate that such children can and do acquire strategies similar to those of their developmentally advanced peers. Furthermore they are able to apply them successfully to solve a variety of problems.

Of the four cognitive variables included in this study, the one measuring information processing capacity was the variable most consistently related to arithmetic performance. It is intuitively appealing to characterize children's potential problem solving limitations in information processing terms. However, even this variable accounted for a relatively small percentage of variation in performance accuracy and in use of advanced strategies. Nevertheless, given the exploratory nature of this study with respect to processing capacity, and the consistently strong showing of this variable relative to the other three, further research should examine more carefully its role in the acquisition of arithmetic skills.

It is clear that first grade children differ in their ability to solve arithmetic problems. Large variations were found in performance accuracy on each of the problem types and in the use of advanced strategies. However, the cognitive variables employed in this study do not seem to fully capture the reasons for these differences. The large variances within the developmental groups, particularly with respect to the use of advanced strategies, indicate that none of the cognitive variables were very successful in sorting the children into homogeneous groups with respect to problem solving ability. Based on the correlations shown in Table 4, it appears that it may be better to deal with individual differences in performance directly rather than to hope for uncovering a more general trait which will predict performance on a wide range of tasks. The fact that the correlations within the set of six arithmetic problem types were substantially higher than those between the cognitive variables and the arithmetic problems indicates that ability to solve a set of arithmetic problems is best measured by performance on a similar arithmetic problem. This implies, of course, that mathematics problem solving ability may need to be reassessed for each instructional topic. Although this approach lacks the potential generalizability which makes the search for underlying cognitive prerequisites so appealing, it is the approach which is currently suggested by the data.

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