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ABSTRACT

The conceptualization of analysis of covariance (ANCOVA), as an analysis of variance (ANOVA) on the residual scores that are obtained when the dependent variable is regressed on the covariate, is mathematically incorrect. If residuals are obtained from the pooled within-groups regression coefficient, ANOVA on the residuals results in an inflated alpha-level. If the regression coefficient for the total sample combined into one group is used, ANOVA on the residuals yields an inappropriately conservative test. In either case, ANOVA of residuals fails to provide a correct test, because the significance test in ANCOVA requires consideration of both the pooled within-groups regression coefficient and the regression coefficient for the total sample combined into one group. It is recommended that the significance test of treatment effects in ANCOVA be conceptualized as a comparison of models whose parameters are estimated by the principle of least squares. (Author/RL)

# The Difference Between ANOVA of Residuals and ANCOVA

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Abstract

Analysis of covariance is often conceptualized as an analysis of variance of the residual scores that are obtained when the dependent variable is regressed on the covariate. Although this conceptualization is intuitively appealing, it is mathematically incorrect. If residuals are obtained from the pooled within-groups regression coefficient ($b_W$), an analysis of variance on the residuals results in an inflated $\alpha$-level. If the regression coefficient for the total sample combined into one group ($b_T$) is used, ANOVA on the residuals yields an inappropriately conservative test. In either case, analysis of variance of residuals fails to provide a correct test, because the significance test in analysis of covariance requires consideration of both $b_W$ and $b_T$, unlike analysis of residuals. It is recommended that the significance test of treatment effects in analysis of covariance be conceptualized as a comparison of models whose parameters are estimated by the principle of least squares.

Analysis of covariance (ANCOVA) tends to be one of the most misunderstood and misused statistical techniques, partly because is requires a synthesis of multiple regression/correlation (MRC) and analysis of variance (ANOVA) concepts. One commonly employed method of explaining how ANCOVA relates to MRC and ANOVA is to make use of the concept of a residual score. ANCOVA is presented as an ANOVA on the residual scores that are obtained when the dependent variable is regressed on the covariate. For example, Marascuilo states that "Covariance adjustment is equivalent to projecting the earned score [the dependent variable in his example] in a direction parallel to the regression line to the IQ score [the covariate in his example] defined at $\bar{X}$ [where $\bar{X}$ is the grand mean for the covariate]. This parallel projection is performed for all pairs of observations and an analysis of variance is then performed on the adjusted scores" (1971, p. 499).

This sort of explanation of ANCOVA can be found in many, if not most, of the standard sourcebooks on statistics used by social scientists. To cite only a few, Snedecor and Cochran state that "The analysis of covariance is essentially an analysis of variance of the quantity (Y-bX)" (1967, p. 424). Cohen and Cohen say that "The ACV [ANCOVA] involves the analysis of (the residuals of) Y when one or more other variables (the covariates) have been partialled out. . . . In ACV, the residual that is analyzed is $Y - \hat{Y}_A$ for each subject . . . in exactly the same way as Y itself is analyzed in AV [ANOVA]" (1975, p. 308). According to the SPSS manual, "Regression procedures are used to remove variation in the dependent variable due to one or more covariates, and a conventional analysis of variance is then performed on the 'corrected' scores" (Nie, Hull, Jenkins, Steinbrenner, and Bent, 1975, p. 409). Similar statements can also be

found in such sources as Dixon and Massey (1969, p. 223), Kerlinger and Pedhazur (1973, pp. 266-267), and Lindquist (1953, p. 318). However, it is well-known by these authors and others that ANCOVA and an analysis of variance conducted on the residuals (hereafter called ANORES) will be slightly different because one degree of freedom is used to estimate the slope of the regression line of the dependent variable on the covariate. Thus, without this adjustment in degrees of freedom, the error term for ANORES would have one additional but artificial degree of freedom. With this adjustment, it seems that ANCOVA and ANORES should be identical. In fact, however, they are not identical even after the degree of freedom adjustment. The commonly held belief that they are the same is a reflection of the lack of understanding of the ANCOVA model.

The current paper will employ the approach of comparing models using least squares (see, for example, Cramer, 1972) to explain why ANCOVA and ANORES are indeed different. Viewing ANCOVA as a comparison of models considerably illuminates the logic behind ANCOVA. A numerical example illustrating the general principles will also be presented.

The analysis of covariance test of group differences can be conceptualized as the comparison of the following two models:

$$\text{I.} \quad Y_{ij} = \mu + \alpha_j + \beta X_{ij} + \varepsilon_{ij} \qquad (1)$$

$$\text{II.} \quad Y_{ij} = \mu + \beta X_{ij} + \varepsilon_{ij} \qquad (2)$$

where $Y_{ij}$ is the score on the dependent variable of the ith subject in the jth group, $\mu$ is a grand mean parameter, $\alpha_j$ is a parameter indicating the effect of the jth treatment, $\beta$ is a regression coefficient, $X_{ij}$ is the score on the covariate for the ith subject in the

jth group, and $\varepsilon_{ij}$ is an error term for the ith subject in the jth group. The models are compared by using least squares to estimate each parameter in each model and then comparing the error sum of squares (SSE) for the two models. With the usual ANCOVA assumptions (see, for example, Glass, Peckham, and Sanders, 1972, or Elashoff, 1969), the following expression has a central F distribution with k-1 and N-k-1 degrees of freedom if the null hypothesis of no treatment effects is true (k is the number of treatment groups, and N is the total sample size):

$$F = \frac{(SSE\ (II) - SSE\ (I))/(k-1)}{SSE\ (I)/(N-k-1)} \qquad (3)$$

The model comparisons approach makes clear that the F test in ANCOVA is a function of the extent to which scores on the dependent variable can be more accurately predicted if group membership is known than if it is not, where prediction is performed in both models by using least squares. Of crucial importance later in our argument is the estimation of $\beta$ in models I and II. A common misconception is that since both models contain a $\beta$ parameter, the least squares estimate of $\beta$ in model I will be identical to the least squares estimate of $\beta$ in model II. However in model I the estimator is $b_W$, the pooled within-groups regression coefficient for Y regressed on X, while in model II the estimator is $b_T$, the regression coefficient for Y on X for the entire sample of observations combined into one group. In the absence of group membership parameters, optimal prediction is obtained by using $b_T$ as the slope coefficient; whereas when different intercepts for each group are allowed for by the introduction of group membership parameters

but a common slope of Y on X is assumed, optimal prediction is obtained by using $b_W$. It should be noted that rarely will $b_W = b_T$, principally because of sampling variability. Note also that even the corresponding $\beta$ parameters being estimated (i.e., $\beta$ in Equations 1 and 2, respectively) are themselves unequal unless the null hypothesis is true or assignment to groups is random. This can be seen by thinking of the ANCOVA models as regression models and realizing that the value of a parameter remains unchanged when another parameter is added to the model if and only if the second parameter equals zero or the variables associated with the parameters are uncorrelated. If the null hypothesis is true, the $\alpha_j$ parameters in model I will be zero and hence the $\beta$ parameters in models I and II will be equal. If assignment to groups is random, the X variable will be uncorrelated in the population with the $\alpha_j$ group-membership variables and hence the population regression coefficient associated with X will be the same in both models. Of course, even if one or both of these conditions are met with the result that the $\beta$'s are equal, the values of $b_W$ and $b_T$ will almost certainly differ because of sampling error.

Certain authors (e.g., Winer, 1971, pp. 763-764; see also Evans and Anastasio, 1968) have mistakenly stated that the equivalence of the two parameters is an assumption of the analysis of covariance. However, ANCOVA does not in general require equivalence, although interpretation of results is clearest when groups are randomly constituted and hence the two $\beta$ parameter values are equal.

This distinction between $b_W$ and $b_T$, as well as the introduction of $b_B$, the between-groups regression of $\bar{Y}_{.j}$ on $\bar{X}_{.j}$, often is difficult

to explain when relying on extensions of the ANOVA approach, and hence contributes to the confusion surrounding ANCOVA. In contrast, the model comparisons approach making the utilization of least squares explicit shows why it is necessary to define both $b_W$ and $b_T$ in order to discuss ANCOVA. However, $b_B$ is not a least-squares estimator of a parameter in any model, and seems to be of limited value, except when multilevels of unit of analysis are considered. For example, analyses might be conducted both at the level of students and classrooms, in which case $b_B$ may be of interest. For further discussion, see Burstein, Linn, and Capell (1978).

ANORES can also be conceptualized in terms of model comparisons. One virtue of this approach is that it necessitates explicit consideration of how the residuals are to be obtained, because either $b_W$ or $b_T$ could be used to define a residual score. Cohen and Cohen (1975, p. 308) have argued that $b_W$ should be used. However, because none of the other previously referenced sources have stated which coefficient should be used, we will examine both, beginning with $b_W$. The residual score for the ith subject in the jth group can then be written as $Y_{ij} - b_W X_{ij}$. And, the models compared by ANORES may then be written:

$$\text{III.} \quad Y_{ij} - b_W X_{ij} = \mu + \alpha_j + \varepsilon_{ij} \qquad (4)$$

$$\text{IV.} \quad Y_{ij} - b_W X_{ij} = \mu + \varepsilon_{ij} \qquad (5)$$

The significance test is again obtained by comparing the error sum of squares for the two models as follows:

$$F = \frac{(SSE\ (IV) - SSE\ (III))/(k-1)}{SSE\ (III)/(N-k-1)} \qquad (6)$$

The term N-k-1 appears as the denominator degrees of freedom because $\beta$ has been estimated to obtain the residual scores.

The relevant question at this point is how this F test relates to that in Equation 3 from ANCOVA, which translates to how the errors associated with the models compare. First, consider the relationship between models I and III. It can be shown that SSE(I) equals $SS_W$ for ANCOVA, i.e., the adjusted within-group sum of squares, which in turn equals (see Kirk, 1968, p. 461):

$$SS_W = \Sigma\Sigma(Y_{ij} - \bar{Y}_{.j})^2 - b_W^2\ \Sigma\Sigma(X_{ij} - \bar{X}_{.j})^2 . \qquad (7)$$

The error sum of squares for model III is given by $SS_W$ on the dependent variable $Y_{ij} - b_W X_{ij}$. Since

$$SSE(III) = \Sigma\Sigma(Y_{ij} - b_W X_{ij} - (\bar{Y}_{.j} - b_W \bar{X}_{.j}))^2 \qquad (8)$$

and

$$b_W = \frac{\Sigma\Sigma(X_{ij} - \bar{X}_{.j})\ (Y_{ij} - \bar{Y}_{.j})}{\Sigma\Sigma(X_{ij} - \bar{X}_{.j})^2} \qquad (9)$$

algebraic manipulation leads to

$$\begin{aligned} SSE(III) &= \Sigma\Sigma(Y_{ij} - \bar{Y}_{.j})^2 - b_W^2\ \Sigma\Sigma(X_{ij} - \bar{X}_{.j})^2 \qquad (10) \\ &= SSE(I). \end{aligned}$$

Consider next the relationship between model II and model IV. In model II, estimates for $\mu$ and $\beta$ are arrived at so as to minimize the error sum of squares for such a two parameter model. In model IV, however, $b_W$ is fixed, so that only $\mu$ is estimated through least squares. This estimate is given by

$$\hat{\mu} = \bar{Y}_{..} - b_W \bar{X}_{..} \qquad (11)$$

It must be the case that SSE(IV) $\geq$ SSE(II), because least squares for model II could always "choose" $\hat{\mu}$ to be as in (11) and $\hat{\beta} = b_W$, duplicating the estimates of model IV; otherwise, the estimates of

model II will differ and provide a better fit to the data, yielding a smaller error sum of squares, since this is precisely the goal of the least squares procedure. Reference to the formulas for the F test in ANCOVA (Equation 3) and ANORES (Equation 6) reveals that the observed F ratio for ANORES must be at least as large as the F for ANCOVA, since SSE(I) = SSE(III) but SSE(IV) $\geq$ SSE(II). The extent of the discrepancy will depend on the extent to which $b_W$ differs from $b_T$, since it can be shown that

$$SSE(IV) = SSE(II) + (b_W - b_T)^2 \Sigma\Sigma(X_{ij} - \bar{X}_{..})^2 \quad (12)$$

This relationship shows that the test given in Equation 6 is not a legitimate F test, because the sampling distribution of the statistic differs systematically from the sampling distribution of the proper test statistic. The reason that Equation 6 is inappropriate is that the numerator expression is not distributed as a chi-square random variable with k-1 degrees of freedom. This is obvious since SSE(IV) - SSE(III) is systematically larger than SSE(II) - SSE(I), which (when divided by $\sigma^2$) is distributed as a chi-square with k-1 degrees of freedom. Hence, ANORES using $b_W$ to obtain residual scores will lead to an inflated $\alpha$-level, and is certainly not equivalent to ANCOVA.

Although the use of $b_W$ to obtain residuals does not reproduce ANCOVA, it is also possible to perform ANORES using $b_T$ to obtain residuals. Perhaps it is this form of ANORES that previous authors have had in mind when they have written of the equivalence between ANCOVA and ANORES. It should be noted that this approach is often referred to as a residual gain analysis (e.g., Corder-Bolz, 1978).

With this method of forming residual scores, the dependent variable for the ith subject in the jth group is $Y_{ij} - b_T X_{ij}$. Models to be compared are then:

V. $$Y_{ij} - b_T X_{ij} = \mu + \alpha_j + \varepsilon_{ij} \qquad (13)$$

VI. $$Y_{ij} - b_T X_{ij} = \mu + \varepsilon_{ij} \qquad (14)$$

Once again, we might perform a significance test by comparing the error sum of squares for the two models as

$$F = \frac{(SSE(VI) - SSE(V))/(k-1)}{SSE(V)/(N-k-1)} . \qquad (15)$$

As before, N-k-1 appears in the denominator because $\beta$ has been estimated to obtain the residual scores. Now, models I, II, V, and VI must be compared. Models II and VI are identical because both include $b_T$ as the slope value and $\hat{\mu} = \bar{Y}_{..} - b_T\bar{X}_{..}$ in both cases. Hence,

$$SSE(VI) = SSE(II). \qquad (16)$$

Consider next the relationship between models I and V. In model I, estimates for $\mu$, $\alpha_j$, and $\beta$ are chosen so as to minimize the sum of squared errors, by the definition of least squares. In model V, least squares estimates are obtained for $\mu$ and $\alpha_j$ subject to the constraint that $\hat{\beta} = b_T$. However, $b_W$ is the least squares estimate and therefore must lead to a minimal sum of squared errors. Hence,

$$SSE(V) \geq SSE(I). \qquad (17)$$

Again, the extent of the difference in error sum of squares is related to the difference between $b_W$ and $b_T$. In particular,

$$SSE(V) = SSE(I) + (b_W - b_T)^2 \Sigma\Sigma(X_{ij} - \bar{X}_{.j})^2. \qquad (18)$$

Referring to Equations 3 and 14 shows that the "F value" obtained by using this approach to ANORES must result in a value that is less than or equal to the F obtained with ANCOVA, with equality holding only if $b_W = b_T$. Thus, this form of ANORES also fails to be equivalent to ANCOVA and fails to provide a valid F test, for much the same reason as did ANORES with $b_W$. With the $b_T$ approach however, neither the numerator nor the denominator have the chi square distributions necessary to make their ratio an F random variable.

Thus, claims that ANORES and ANCOVA are equivalent are false whichever approach to ANORES is employed. The fact that models I and III are equivalent and that II and VI are also equivalent suggests that it is possible to duplicate the ANCOVA test by examining residual scores. Specifically, the following test is equivalent to the ANCOVA test:

$$F = \frac{(SSE(VI) - SSE(III))/(k-1)}{SSE(III)/(N-k-1)} \qquad (19)$$

The crucial fact is that ANCOVA depends upon both $b_W$ and $b_T$, and consequently so must an equivalent analysis of residuals. It is insufficient to attend to only one regression coefficient. As stated previously, we believe that this is one of the least understood points concerning ANCOVA. Only by close inspection of model and the least squares principle does the logic underlying the regression coefficient parameter become clear.

A numerical example will demonstrate the theoretical argument concerning the relationship between ANCOVA and ANORES. Consider the hypothetical data given in Table 1. The error sum of squares for

the six models previously outlined are presented in Table 2 for these data, and Table 3 presents analysis of variance tables for ANCOVA and the two forms of ANORES. Results here verify that the

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Insert Tables 1, 2, 3, and 4 about here

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"F" obtained with ANORES using $b_W$ to form residuals is too large, while the "F" when $b_T$ is used is too small. This example illustrates that even when the two groups being compared have similar distributions on the covariate, the use of $b_W$ alone can lead to an incorrect conclusion of statistical significance at the .05 level. With different data, the use of $b_T$ alone might result in a failure to recognize an appropriately significant result. For example, by simply revising the data from the first example by subtracting 4 from each X score in group 2, the results of the analysis would be as shown in Table 4. Note that here while the appropriate ANCOVA yields a result that is significant at $p < .05$, results of the ANORES with $b_W$ are significant at $p < .025$, but the ANORES with $b_T$ is non-significant, $p > .10$. In fact, the F for ANORES-$b_T$ is less than half that for ANORES-$b_W$.

In addition, it is possible to duplicate the ANCOVA results by employing both $b_T$ and $b_W$ to form residual scores and then applying Equation 19. The reason this procedure works can be seen in Tables 3 and 4. ANORES with $b_W$ yields the correct adjusted $SS_W$, and ANORES with $b_T$ yields the correct adjusted $SS_T$. Thus, Equation 19 provides a ratio of the Adjusted Mean Square Between divided by the Adjusted Mean Square Within, as is desired.

Several authors of experimental design texts state that the

adjusted between-group sum of squares in ANCOVA is obtained by subtracting the $SS_W$ for residuals using $b_W$ from the $SS_T$ for residuals using $b_T$, instead of by direct calculation. However, the actual reason for the necessity of this approach has not been clearly elucidated by behavioral statisticians. For example, Lindquist states that "A different adjusted sum of squares for between-groups could be directly computed as $\sum_1^a (\bar{y}_i - b_W\bar{x}_i)^2$, using the within-groups regression coefficient. However, an adjusted sum of squares for between-groups thus computed would be inflated by sampling error in the estimate ($b_W$) of the regression coefficient employed, and would make the between-groups effect appear more significant than it really is" (1953, p. 323). Although Lindquist's conclusion is correct, the crux of the matter is not simply the nature of the $b_W$ estimate, but instead the use of $b_W$ alone instead of $b_W$ and $b_T$ together as detailed above.

In a somewhat similar vein, it has been stated (see, for example, Kirk, 1968) that $b_W$ should not be used to adjust both the numerator and the denominator of the F ratio because this would violate the independence condition necessary for an F ratio. While the numerator and denominator must indeed be independent, the current argument shows that this use of $b_W$ to obtain an adjusted sum of squares between groups also fails even to provide a numerator quantity that is distributed as a chi-square.

In addition to explaining why the use of $b_W$ alone results in an inflated and inappropriately distributed between-group sum of squares, the current approach also makes clear the different roles of $b_W$ and $b_T$. This distinction between $b_W$ and $b_T$ has been widely misinterpreted (see, for example, Cohen & Cohen, 1975). In contrast to Lindquist and Kirk, Cohen and Cohen err by recommending

use of $b_W$ alone, saying that to use $b_T$ would result in "removing from Y, in part, exactly what we mean to study" (1975, p. 308). Although use of $b_T$ does result in a lower adjusted $SS_B$ than if $b_W$ alone were used, use of $b_T$ in the restricted model should not be viewed as removing part of "what we mean to study." Rather, it gives the restricted model a fair chance in that it allows the estimate of the regression parameter to be an optimal, least squares estimate, as $b_W$ is in the full model.

In sum, although the concept of a residual score can be a useful pedagogical tool for explaining the logic of ANCOVA, it has typically not been utilized accurately. A correct $SS_B$ can be calculated by using residuals, but only by considering both $b_T$ and $b_W$, and hence at least implicitly considering <u>two</u> sets of residuals. In terms of the residual score models,

$$\text{Adjusted } SS_B = SSE(VI) - SSE(III). \qquad (20)$$

However, this eliminates much of the simplicity of the residual scores approach, since it requires a synthesis of ANORES using $b_T$ and ANORES using $b_W$. Instead of relying on ANORES to explain ANCOVA, an approach utilizing model comparisons and least squares clarifies the ANCOVA procedure and its underlying rationale.

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Table 1

Hypothetical Data to Illustrate

ANCOVA - ANORES Relationship

| Group 1 | | Group 2 | |
|---|---|---|---|
| X | Y | X | Y |
| 100 | 100 | 100 | 105 |
| 95 | 98 | 90 | 109 |
| 105 | 102 | 95 | 104 |
| 110 | 106 | 105 | 112 |
| 105 | 101 | 95 | 100 |
| 90 | 103 | 100 | 104 |

Table 2

ANCOVA and ANORES Models and Associated

Error Sums of Squares

| | Model | SSE |
|---|---|---|
| I. | $Y_{ij} = \mu + \alpha_j + \beta X_{ij} + \varepsilon_{ij}$ | 110.3 |
| II. | $Y_{ij} = \mu + \beta X_{ij} + \varepsilon_{ij}$ | 170.7 |
| III. | $Y_{ij} - b_W X_{ij} = \mu + \alpha_j + \varepsilon_{ij}$ | 110.3 |
| IV. | $Y_{ij} - b_W X_{ij} = \mu + \varepsilon_{ij}$ | 175.7 |
| V. | $Y_{ij} - b_T X_{ij} = \mu + \alpha_j + \varepsilon_{ij}$ | 114.9 |
| VI. | $Y_{ij} - b_T X_{ij} = \mu + \varepsilon_{ij}$ | 170.7 |

Table 3

Analysis of Variance Tables

| | Source | SS | df | MS | F |
|---|---|---|---|---|---|
| ANCOVA | Between | 60.4 | 1 | 60.4 | 4.9 |
| | Within | 110.3 | 9 | 12.3 | |
| | Total | 170.7 | 10 | | |
| ANORES with $b_W$[a] | Between | 65.3 | 1 | 65.3 | 5.3* |
| | Within | 110.3 | 9[b] | 12.3 | |
| | Total | 175.7 | 10 | | |
| ANORES with $b_T$[c] | Between | 55.8 | 1 | 55.8 | 4.4 |
| | Within | 114.9 | 9[b] | 12.8 | |
| | Total | 170.7 | 10 | | |

[a] $b_W$ = 0.20 for these data

[b] $df_W$ = 9 because of the estimation of β in forming the residual

[c] $b_T$ = 0.09 for these data

*$p < .05$

Table 4

Analysis of Variance Tables for Revised Example

| | Source | SS | df | MS | F |
|---|---|---|---|---|---|
| ANCOVA | Between | 64.3 | 1 | 64.3 | 5.2** |
| | Within | 110.3 | 9 | 12.3 | |
| | Total | 174.7 | | | |
| ANORES with $b_W$ [a] | Between | 89.7 | 1 | 89.7 | 7.3*** |
| | Within | 110.3 | 9 [b] | 12.3 | |
| | Total | 200.0 | 10 | | |
| ANORES with $b_T$ [c] | Between | 46.1 | 1 | 46.1 | 3.2* |
| | Within | 128.5 | 9 [b] | 14.3 | |
| | Total | 174.7 | 10 | | |

[a] $b_W$ = 0.20 for these data

[b] $df_W$ = 9 because of the estimation of $\beta$ in forming the residual

[c] $b_T$ = -0.01 for these data

*** $p < .025$

** $p < .05$

* $p > .10$