

DOCUMENT RESUME

ED 190 890

CE 026 575

TITLE Military Curricula for Vocational & Technical Education. Basic Electricity and Electronics Individualized Learning System. CANTRAC A-100-0010. Module Thirteen. Series AC RLC Circuits and Resonance. Study Booklet.

INSTITUTION Chief of Naval Education and Training Support, Pensacola, Fla.: Ohio State Univ., Columbus. National Center for Research in Vocational Education.

REPORT NO NAVEDTRA-34258-13

PUB DATE Mar 77

NOTE 101p.: For related documents see CE 026 560-593.

EDRS PRICE MF01/PC05 Plus Postage.

DESCRIPTORS \*Electric circuits: \*Electricity: \*Electronics: Individualized Instruction: Learning Activities: Learning Modules: Postsecondary Education: Programed Instruction: \*Technical Education

IDENTIFIERS \*Alternating Current: Military Curriculum Project: \*Resonance (Physics)

ABSTRACT

This individualized learning module on series alternating current resistive-inductive-capacitive circuits and resonance is one in a series of modules for a course in basic electricity and electronics. The course is one of a number of military-developed curriculum packages selected for adaptation to vocational instructional and curriculum development in a civilian setting. Four lessons are included in the module: (1) Solving Resistive-Inductive-Capacitive Circuits, (2) Resonant Frequency in Series Circuits, (3) Conditions of Series Resonance, and (4) Experiments with Series Resonance. Each lesson follows a typical format including a lesson overview, a list of study resources, the lesson content, a programmed instruction section, and a lesson summary. (Progress checks are provided for each lesson in a separate document, CE 026 562.) (LRA)

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### MILITARY CURRICULUM MATERIALS

The military-developed curriculum materials in this course package were selected by the National Center for Research in Vocational Education Military Curriculum Project for dissemination to the six regional Curriculum Coordination Centers and other instructional materials agencies. The purpose of disseminating these courses was to make curriculum materials developed by the military more accessible to vocational educators in the civilian setting.

The course materials were acquired, evaluated by project staff and practitioners in the field, and prepared for dissemination. Materials which were specific to the military were deleted, copyrighted materials were either omitted or approval for their use was obtained. These course packages contain curriculum resource materials which can be adapted to support vocational instruction and curriculum development.

## Military Curriculum Materials Dissemination Is . . .

an activity to increase the accessibility of military-developed curriculum materials to vocational and technical educators.

This project, funded by the U.S. Office of Education, includes the identification and acquisition of curriculum materials in print form from the Coast Guard, Air Force, Army, Marine Corps and Navy.

Access to military curriculum materials is provided through a "Joint Memorandum of Understanding" between the U.S. Office of Education and the Department of Defense.

The acquired materials are reviewed by staff and subject matter specialists, and courses deemed applicable to vocational and technical education are selected for dissemination.

The National Center for Research in Vocational Education is the U.S. Office of Education's designated representative to acquire the materials and conduct the project activities.

### Project Staff:

Wesley E. Budke, Ph.D., Director  
National Center Clearinghouse  
Shirley A. Chase, Ph.D.  
Project Director

## What Materials Are Available?

One hundred twenty courses on microfiche (thirteen in paper form) and descriptions of each have been provided to the vocational Curriculum Coordination Centers and other instructional materials agencies for dissemination.

Course materials include programmed instruction, curriculum outlines, instructor guides, student workbooks and technical manuals.

The 120 courses represent the following sixteen vocational subject areas:

Agriculture	Food Service
Aviation	Health
Building & Construction	Heating & Air Conditioning
Trades	Machine Shop
Clerical Occupations	Management & Supervision
Communications	Meteorology & Navigation
Drafting	Photography
Electronics	Public Service
Engine Mechanics	

The number of courses and the subject areas represented will expand as additional materials with application to vocational and technical education are identified and selected for dissemination.

## How Can These Materials Be Obtained?

Contact the Curriculum Coordination Center in your region for information on obtaining materials (e.g., availability and cost). They will respond to your request directly or refer you to an instructional materials agency closer to you.

### CURRICULUM COORDINATION CENTERS

**EAST CENTRAL**  
Rebecca S. Douglass  
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100 North First Street  
Springfield, IL 62777  
217/782-0759

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## The National Center Mission Statement

The National Center for Research in Vocational Education's mission is to increase the ability of diverse agencies, institutions, and organizations to solve educational problems relating to individual career planning, preparation, and progression. The National Center fulfills its mission by:

- Generating knowledge through research
- Developing educational programs and products
- Evaluating individual program needs and outcomes
- Installing educational programs and products
- Operating information systems and services
- Conducting leadership development and training programs

### FOR FURTHER INFORMATION ABOUT Military Curriculum Materials

#### WRITE OR CALL

Program Information Office  
The National Center for Research in Vocational  
Education

6 The Ohio State University  
1960 Kenny Road, Columbus, Ohio 43210  
Telephone: 614/486-3655 or Toll Free 800/  
848-4815 within the continental U.S.  
(except Ohio)



## Military Curriculum Materials for Vocational and Technical Education

Information and Field  
Services Division

The National Center for Research  
in Vocational Education



O V E R V I E W  
MODULE THIRTEEN  
SERIES AC RLC CIRCUITS AND RESONANCE

In this module you will combine RL and RC circuits and learn some of the phenomena which result.

For you to more easily learn the above, this module has been divided into the following four lessons:

Lesson I.	Solving RLC Circuits . . . . .
Lesson II.	Resonant Frequency in Series Circuits . . . . .
Lesson III.	Conditions of Series Resonance . . . . .
Lesson IV.	Experiments with Series Resonance . . . . .

TURN TO THE FOLLOWING PAGE AND BEGIN LESSON I.

BASIC ELECTRICITY AND ELECTRONICS  
INDIVIDUALIZED LEARNING SYSTEM



MODULE THIRTEEN  
LESSON I

Solving RLC Circuits

Study Booklet

OVERVIEW

LESSON I

Solving RLC Circuits

In this lesson you will study and learn about the following:

- the impedance triangle
- voltage drops
- other circuit quantities
- quality of a coil
- deriving the formula for  $Q$
- the value of  $Q$
- effective or AC resistance
- skin effect
- proximity effect

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON I

Solving RLC Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

- NAVPERS 934DQA-1b "Basic Electricity, Alternating Current."  
Fundamentals of Electronics. Bureau of Naval Personnel.  
Washington, D. C.: U. S. Government Printing Office, 1965.

AUDIO-VISUAL:

- Sound-Slide Presentation - "Solving for Total Impedance and Total Voltage."

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

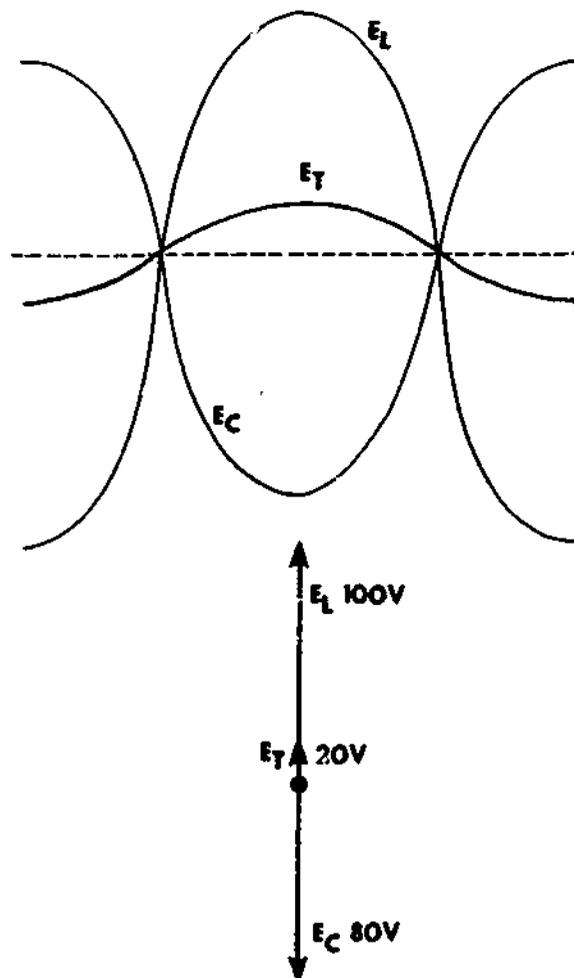
NARRATIVE  
LESSON 1

Series RLC Circuits

Voltages in Series

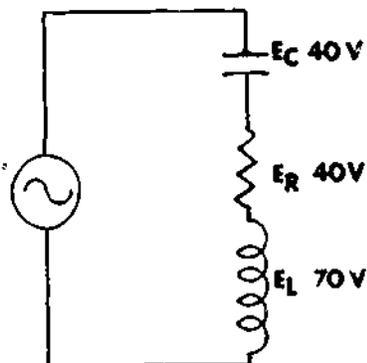
The voltage drops in a series RLC circuit have, of course, phase differences. Because of this, it is necessary to use instantaneous values and graphs or vectors to find the total values in the circuit.

When using vectors, current is used as the reference, for it is the value which has the same phase (is common) in all parts of the circuit. The drop across the resistance is in phase with the current and so is represented with a vector in the standard position. The capacitor voltage drop lags circuit current by  $90^\circ$  and the voltage across the inductor leads current by  $90^\circ$ . Combining either the instantaneous values or the vectors for  $E_C$  and  $E_L$  reduces the value of the larger one. In other words, you must subtract the smaller value from the larger.



The diagrams on the preceding page show, graphically and by vectors, the addition of 100 volts  $E_L$  and 80 volts  $E_C$ . They show that the voltages are in direct opposition, so that the resultant voltage (the voltage which affects electron movement) is the difference between  $E_L$  and  $E_C$ . This circuit, then acts just like a circuit with 20 volts dropped across a single inductor.

Rectangular notation provides a convenient method for finding the total voltage in a series RLC circuit like this one:

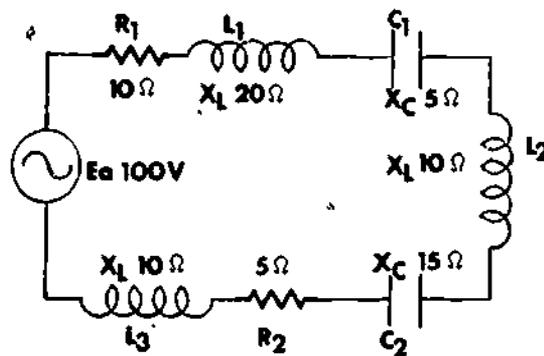


Total voltage is the vector sum of the voltage drops, i. e.,

$$\begin{aligned} E_T &= E_R + jE_L - jE_C \\ &= 40 \text{ v} + j70 \text{ v} - j40 \text{ v} \\ &= 40 \text{ v} + j30 \text{ v} \end{aligned}$$

### Impedance

Once again voltage and impedance are directly related, so the impedance can be found in the same way as total voltage. In the circuit diagram on the following page, the values of resistance and reactance are written in rectangular form then added.

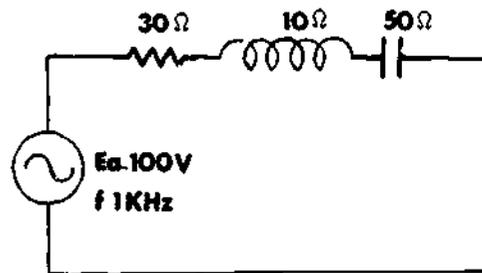


R1	$10 + j0$
L1	$0 + j20$
C1	$0 - j5$
L2	$0 + j10$
C2	$0 - j15$
R2	$5 + j0$
L3	$\frac{0 + j10}{15 + j20}$

The circuit impedance is  $15 + j20$  ohms or, in polar form,  $25 \angle 53.1^\circ$  ohms. Current is therefore 4 amperes, and the phase angle is  $-53.1^\circ$ . So far as the overall effect on the source is concerned, this circuit could be replaced with a 15 ohm resistor in series with an inductor having a reactance of 20 ohms, but the individual components will develop much greater voltage than the source provides to the circuit because of the cancellation between  $E_L$  and  $E_C$ . For example,  $E_{L1}$  is 80 volts,  $E_{L2}$  is 40 volts, and  $E_{L3}$  is 40 volts for a total of 160 volts dropped across the inductors. This increase in voltage can sometimes be useful, and you will learn more about it later.

Now work the following problems for practice with these concepts.

Solve the RLC circuit shown.



$$P_t = \underline{\hspace{2cm}}$$

$$P_a = \underline{\hspace{2cm}}$$

$$\angle \theta = \underline{\hspace{2cm}}$$

$$PF = \underline{\hspace{2cm}}$$

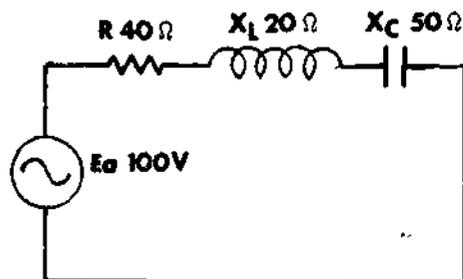
By using the formula  $P_t = I^2 R$ , we find  $P_t = 120 \text{ w}$ .

By the formula  $P_a = E \times I$ ,  $P_a = 200 \text{ va}$ .

You know that  $\angle \theta = -53.1^\circ$ , and PF is equal to the  $\cos \angle \theta$  or 0.6.

Solve this RLC circuit.

Draw the Vector Diagrams



$$Z_T = \underline{\hspace{2cm}}$$

$$I_T = \underline{\hspace{2cm}}$$

$$E_R = \underline{\hspace{2cm}}$$

$$E_L = \underline{\hspace{2cm}}$$

$$E_C = \underline{\hspace{2cm}}$$

$$P_t = \underline{\hspace{2cm}}$$

$$P_a = \underline{\hspace{2cm}}$$

$$PF = \underline{\hspace{2cm}}$$

$$\angle \theta = \underline{\hspace{2cm}}$$

$$Z_T = 40 \Omega - j30 \Omega \text{ or } 50 \Omega \angle -36.9^\circ; I_T = 2 \text{ a}; E_R = 80 \text{ v};$$

$$E_L = 40 \text{ v}; E_C = 100 \text{ v}; P_t = 160 \text{ w}; P_a = 200 \text{ va}; PF = 0.8;$$

$$\angle \theta = -36.9^\circ$$

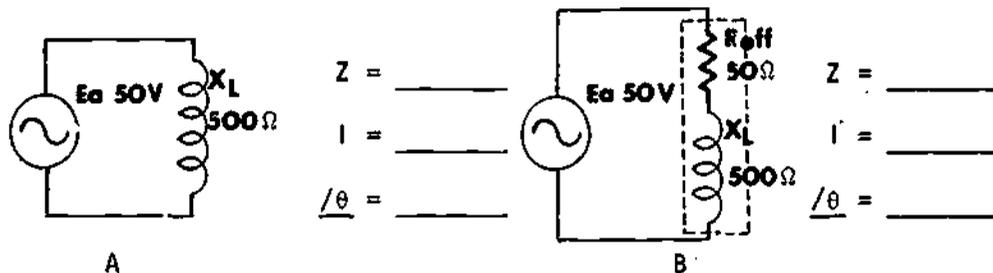
Figure of Merit

The figure of merit ( $Q$ ) of a reactor is a measure of how close the reactor comes to having all reactance and no resistance. For example, a coil with a high  $Q$  is one with very little resistance in its wiring. This means that very little power is dissipated in the coil and nearly all the power delivered to the coil is stored and later returned to the source.  $Q$  is defined as the ratio of power stored to power

dissipated in the coil  $\frac{P_x}{P_t}$ . Since the reactive (stored) power is found by  $I^2 X_L$  and true (dissipated) power by  $I^2 R$ , this can be written as  $Q = \frac{I^2 X_L}{I^2 R}$ . Cancellation of  $I^2$  in the fraction leaves  $Q = \frac{X_L}{R}$ , a more useful equation for most practical uses.

The figure of merit is seldom used with capacitors, for their construction normally keeps their internal resistance so low that it can be ignored. This is not true of inductors; however, because the wires used to make coils usually have significant amounts of resistance, and the  $Q$  of the coil is affected. In most RLC circuits, the coil contains nearly all the resistance of the circuit, so the  $Q$  of the coil can be used as the circuit  $Q$ .

A coil with a  $Q$  of ten or greater is considered a high  $Q$  coil. This is because, in most circuits at least, a resistance one-tenth or less of the reactance will have so little effect on the circuit that it can be ignored. Compare these coil values for a pure inductive reactance of 500 ohms with those for a coil containing 500 ohms of inductive reactance in series with 50 ohms of resistance. (NOTE: The dotted line around the inductor and  $R_{eff}$  indicate that both are contained in the coil.)



A.  $Z = 500 \Omega$

$I = 100 \text{ ma}$

$\theta = 90^\circ$

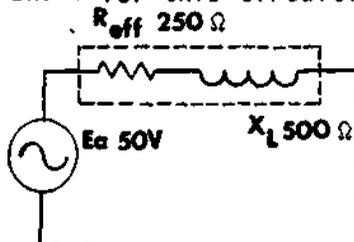
B.  $Z = 502.3$

$I = 99.5 \text{ ma}$

$\theta = 84.3^\circ$

The differences between these two circuits are smaller than you could detect with normal test equipment and can be considered identical.

Find  $Q$ ,  $\theta$ ,  $Z$  and  $I$  for this circuit.



$$Q = 2; \theta = 63.4^\circ; Z = 559 \Omega; I = 89.4 \text{ ma}$$

The circuit above illustrates a low  $Q$ -coil. Compare these values to those for the ideal coil and high  $Q$  coil you worked out earlier. As a rule of thumb, if the coil  $Q$  is 10 or more (reactance is at least 10 times the resistance), ignore the resistance and treat the coil as a pure inductance. If  $Q$  is less than 10, include the resistance in your calculations.

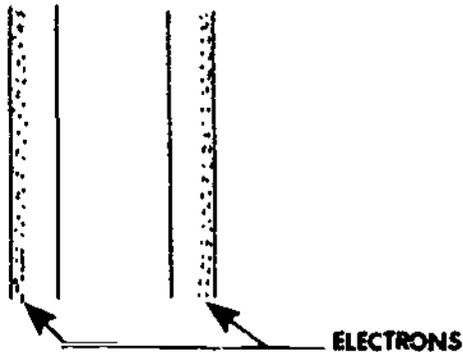
The resistance we have discussed so far has been a constant value unaffected by any condition within a circuit. Unfortunately, this is not exactly the case, and frequency of the applied voltage often affects circuit resistance. When the  $Q$  of a circuit is figured, the resistance used must be the actual resistance of the circuit at the applied frequency. This is called the AC resistance or effective resistance. The  $Q$  of a

circuit is also defined as  $\frac{X_L}{R}$  but in this case  $R$  includes all the resistance in the circuit, and not just the AC resistance of the coil. Unless otherwise specified, when dealing with series circuits  $Q$  means the  $Q$  of the circuit rather than the  $Q$  of the coil.

AC resistance is the result of two actions in a wire carrying an alternating current. Skin effect results from self induction inside the conductor which causes electrons to crowd away from the center of the wire toward its outer surfaces. This effectively reduces the cross section of the wire, increasing its resistance.

At fairly high frequencies, no electrons travel near the center of the wire, so hollow tubing can be used to save weight and expense in building a circuit without any loss of efficiency

Proximity effect is the second major factor of AC resistance. The proximity effect occurs when wires carrying AC are placed side by side. The electrons in each wire tend to move away from the adjacent wire like this:



This further reduces the cross-sectional area available for current flow and increases the wire's resistance.

Both skin effect and proximity effect cause resistance to increase with frequency. Since both  $X_L$  and  $R$  increase as frequency increases, the  $Q$  of a circuit will remain nearly constant over a broad frequency range.

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AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

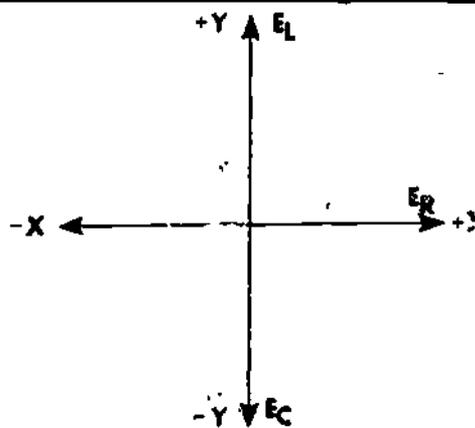
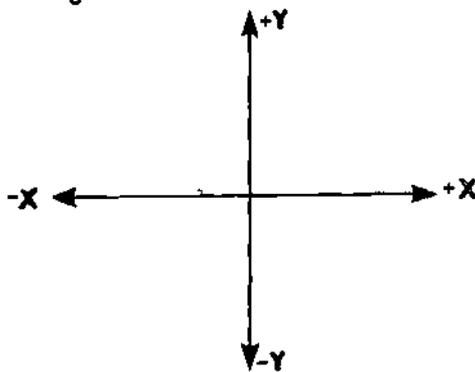
PROGRAMMED INSTRUCTION  
LESSON 1

Solving RLC Circuits

TEST FRAMES ARE 11 and 31. AS BEFORE, GO FIRST TO TEST FRAME 11 AND SEE IF YOU CAN ANSWER ALL THE QUESTIONS THERE. FOLLOW THE DIRECTIONS GIVEN AFTER THE TEST FRAME.

1. Recall that  $\underline{E}$  and  $\underline{I}$  are in phase in a resistive circuit,  $\underline{E}$  leads  $\underline{I}$  in an inductive circuit, and  $\underline{I}$  leads  $\underline{E}$  in a capacitive circuit.

Plot  $E_L$ ,  $E_R$ , and  $E_C$  on the graph shown below.



2. Since the voltage drop across  $X_L$  leads the current by  $90^\circ$ ,  $X_L$  is plotted upward in the  $+j$  position on an impedance triangle.

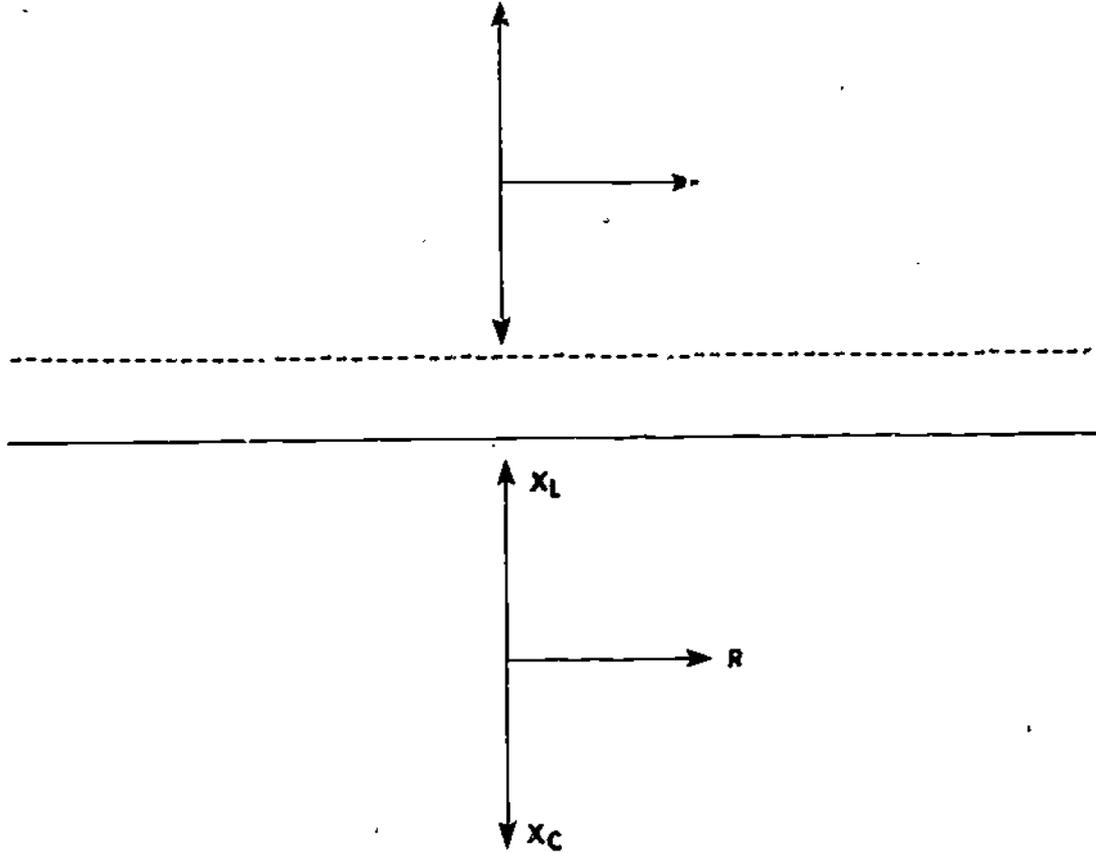


Since the voltage drop across  $X_C$  lags the current by  $90^\circ$ ,  $X_C$  is plotted \_\_\_\_\_ or in the \_\_\_\_\_ position.

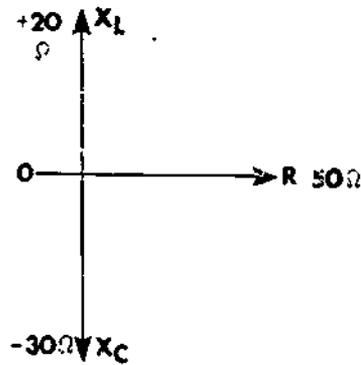
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 \_\_\_\_\_  
 \_\_\_\_\_  
 (downward;  $-j$ )

3. Since  $\underline{E}$  and  $\underline{I}$  are in phase through a resistance,  $\underline{R}$  is plotted in the standard vector position.

On the following graph, indicate where the values of  $X_L$ , and  $X_C$ , and  $\underline{R}$  are plotted by labeling the respective vectors.



4. Since  $X_L$  is plotted opposite  $X_C$ , they can be added algebraically.

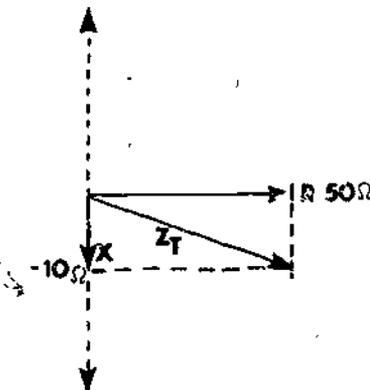


Draw a vector indicating the resultant value.

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5. Recall from Module Twelve that the j operator is simply another tool for solving AC resistive circuit problems. A +j indicates a 90° counterclockwise rotation or an  $X_L$  quantity; a -j indicates a 90° clockwise rotation or an \_\_\_\_\_ quantity.

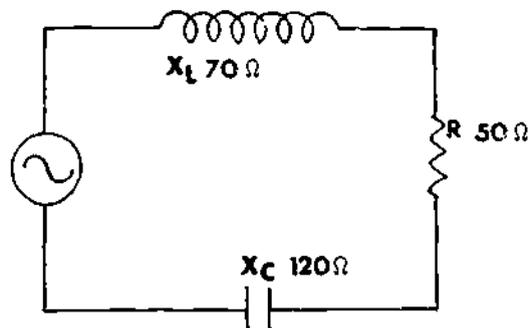
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( $X_C$ )

6. To express a series RLC circuit in rectangular notation you must express each component in rectangular notation and algebraically add.



The impedance of this circuit is:

$$R \quad 50 + j0$$

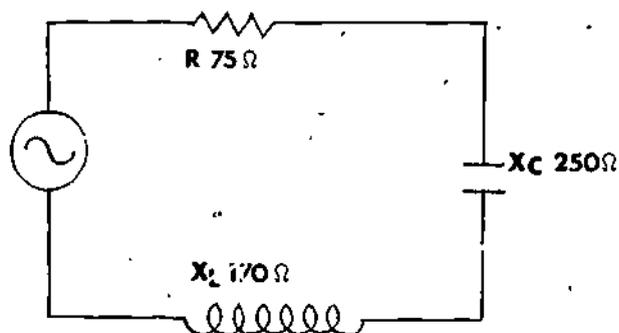
$$X_L \quad 0 + j70$$

$$X_C \quad 0 - j120$$

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$$\text{TOTAL} \quad 50 - j50$$

Express the impedance of the following circuit in rectangular notation.



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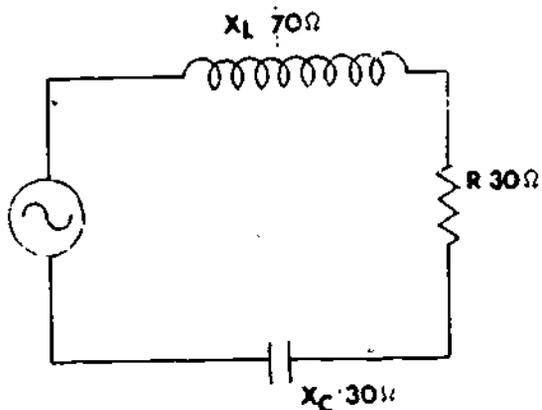


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$$\begin{array}{r} (75 + j0 \\ 0 + j170 \\ 0 - j250 \\ \hline 75 - j80) \end{array}$$

7. Algebraic addition enables us to simplify the circuit, leaving either a simple RL or RC circuit to solve.

Solve the following for all values indicated.

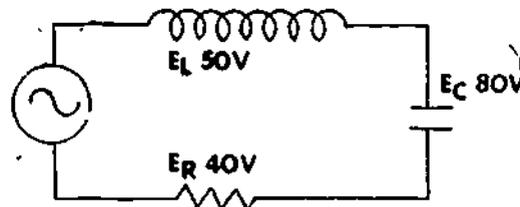


- a.  $\angle \theta$  = \_\_\_\_\_  
 b.  $Z_T$  = \_\_\_\_\_  
 c. sine = \_\_\_\_\_  
 d. cosine = \_\_\_\_\_  
 e. TAN = \_\_\_\_\_

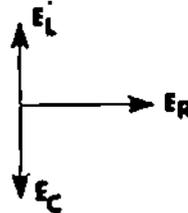
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(a.  $53.1^\circ$ ; b.  $30 + j40$  or  $50 \Omega$ ; c.  $0.7997$ ; d.  $0.6004$ ; e.  $1.3319$ )

8. Solving a series RLC circuit for voltage values is accomplished in the same way as solving for impedance.



- a. Draw the vector diagram.



- b. Express the voltage across each component in rectangular notation.

$$E_R = 40 \text{ v} + j0$$

$$E_L = 0 + j50 \text{ v}$$

$$E_C = 0 - j80 \text{ v}$$

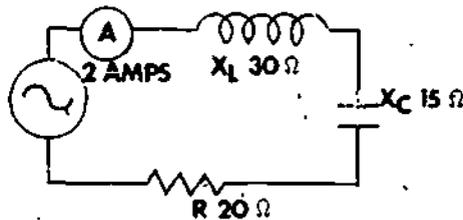
- c. Algebraically add  $40 \text{ v} - j30 \text{ v}$

- d. Determine the TAN  $\theta$   $0.75$

- e. Determine  $\theta$   $-36.9^\circ$

- f. Determine  $E_a$   $50 \text{ v}$

Solve for  $E_a$ .



$$E_a = \underline{\hspace{2cm}}$$

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$$(50 \text{ v} / 36.9^\circ)$$


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9. Recall that only resistance dissipates true power.

The formula for true power is:

\_\_\_ a.  $P_t = EI$ .

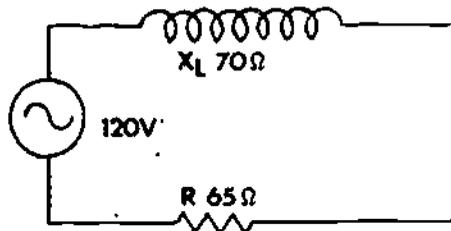
\_\_\_ b.  $P_t = IR$ .

\_\_\_ c.  $P_t = I^2 R$ .

\_\_\_ d.  $P_t = \frac{E}{R}$ .

-----  
 \_\_\_\_\_  
 (c)

10. Use of the formula  $P = E_a I_T$  results in apparent power. However, recall that in a reactive circuit true power may be determined by the formula  $P_t = E_a \times I_T \times \cos \theta$ .

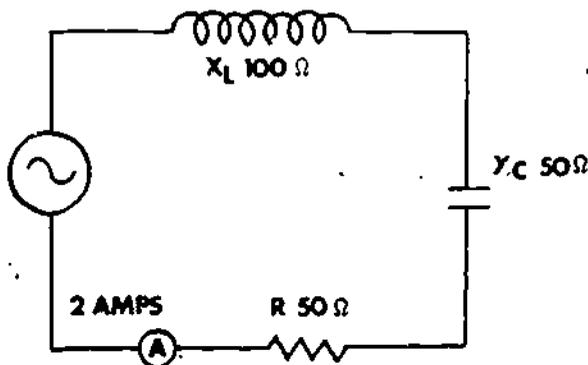


Find  $P_a$  \_\_\_\_\_

Find  $P_t$  \_\_\_\_\_

-----  
 \_\_\_\_\_  
 (150 va; 102 w)

11. Solve.



a.  $E_a =$  \_\_\_\_\_

b.  $\theta =$  \_\_\_\_\_

c.  $Z_T =$  \_\_\_\_\_

d.  $P_a =$  \_\_\_\_\_

e.  $P_t =$  \_\_\_\_\_

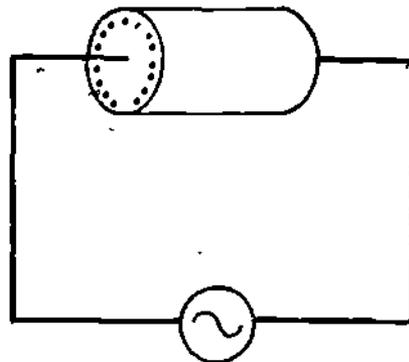
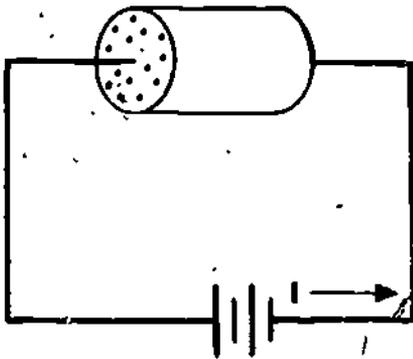
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(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)



(We have said in the past that frequency has no effect upon resistance; however, we need to qualify this statement a bit. Frequency does have a slight effect on resistance.)

13. Recall that alternating current causes flux lines to expand and collapse around a conductor. These flux lines induce a voltage into the conductor. The induced voltage opposes the current and tends to decrease it. This induced voltage is greater at the center of the conductor where the flux lines are concentrated. The effect of this is to force the electrons to move towards the skin of the conductor.



By forcing current to flow along the skin of the conductor (skin effect), the cross-sectional area of the conductor is effectively

\_\_\_\_\_

-----

(reduced or decreased)

14. When the cross-sectional area of a given conductor decreases, what happens to the resistance of the conductor?

\_\_\_\_\_

-----

(It increases)

15. Increasing frequency increases the skin effect. Thus, the higher the frequency, the \_\_\_\_\_ the circuit resistance.

\_\_\_\_\_

-----

(higher or greater)

16. Because of skin effect, hollow conductors are sometimes used to reduce weight and cost. In a high-frequency circuit a reduction in weight and cost may be obtained by using \_\_\_\_\_ conductors.

-----

\_\_\_\_\_

(hollow)

\_\_\_\_\_

17. There is another factor which causes circuit resistance to increase with frequency. This is called the proximity effect. The proximity effect is also caused by the magnetic field around a conductor. This effect, however, is caused in adjacent conductors.

The proximity effect causes resistance to \_\_\_\_\_ when frequency decreases.

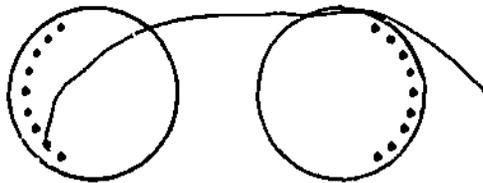
-----

\_\_\_\_\_

(decrease)

\_\_\_\_\_

18. Electrons in conductors placed side by side are forced away from the parts of the conductors nearest each other. Here is an end view of two wires showing electron distribution resulting from both skin effect and proximity effect:



Proximity effect is similar to skin effect in that the effective cross-sectional area of the conductor is \_\_\_\_\_.

-----

\_\_\_\_\_

(reduced)

\_\_\_\_\_

19. Proximity effect causes the resistance of the conductor within a coil to increase as frequency \_\_\_\_\_.
- 
- \_\_\_\_\_
- (increases)
- \_\_\_\_\_

20. The method used to reduce the problem of proximity effect is to space the turns of the windings farther apart.

This method reduces the amount of voltage induced, thus reducing the increase in \_\_\_\_\_.

-----  
 \_\_\_\_\_  
 (resistance)

21. Proximity effect and skin effect cause a conductor to offer more resistance to AC than to DC. Because of this, we call the resistance offered to AC the effective or AC resistance, abbreviated  $R_{eff}$  or  $R_{ac}$ .

The resistance any inductor offers to AC is somewhat greater than that offered to DC, and it is called the \_\_\_\_\_ resistance.

-----  
 \_\_\_\_\_  
 (effective or AC)

22. The purpose for discussing the effective resistance of a coil is to discover its effect on the ability of a coil to store energy.

The greater the resistance, the more power \_\_\_\_\_.

-----  
 \_\_\_\_\_  
 (lost or dissipated)

23. The ratio of the amount of energy stored in an inductor to the amount of energy lost in the same period of time indicates the quality of a coil. The quality of a coil is also referred to as the figure of merit of a coil and is abbreviated Q.

The Q of a coil may be indicated as:  $Q = \frac{P}{X}$

-----  
 \_\_\_\_\_  
 (P<sub>t</sub>)

24. Using the formula  $Q = \frac{P_x}{P_t}$ , and substituting the equivalent values for  $P_x$  and  $P_t$ , results in  $Q = \frac{I^2 X_L}{I^2 R_{ac}}$ .

(Note that  $R_{ac}$  must be used to determine the correct value of true power.)

Deleting the common factors in the numerator and denominator result in the formula for the Q of a coil:

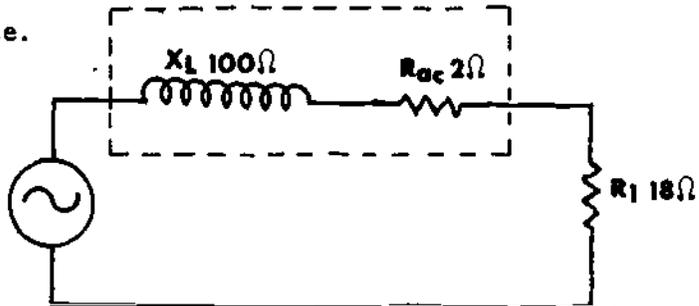
$Q = \underline{\hspace{2cm}}$

-----  
 \_\_\_\_\_  
 $\frac{X_L}{R_{ac}}$

25. Since both  $X_L$  and  $R_{ac}$  are directly proportional to frequency, how is Q affected by a change in frequency?

-----  
 -----  
 \_\_\_\_\_  
 (It is not appreciably affected.)

26. The Q of a series circuit and the Q of a coil are determined in the same way:  $Q = \frac{X_L}{R_{ac}}$ . The only difference between the two is the value of resistance.



(Note: The dotted line around the inductor and  $R_{ac}$  indicate that both are contained in the coil.)

- a. Determine the Q of the coil. \_\_\_\_\_
- b. Determine the Q of the circuit. \_\_\_\_\_

-----  
 \_\_\_\_\_  
 (a. Q of the coil = 50; b. Q of the circuit = 5)

27. Unless otherwise specified, when dealing with series circuits,  $Q$  means the  $Q$  of the circuit rather than the  $Q$  of the coil.

The  $Q$  of a series circuit is determined by using the values of the  $X_L$  of the coil and \_\_\_\_\_

-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (total effective circuit resistance)

28. When  $Q$  is 10 or greater, the coil is considered to be a high- $Q$  coil. A coil with a  $Q$  of less than 10 is a low- $Q$  coil. In solving high- $Q$  circuits, the resistance is normally disregarded and only the reactance is considered.

When solving a circuit with a  $Q$  of 2, the resistance is \_\_\_\_\_

-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (considered or important)

29. You may have recognized that the formulas for the tangent and  $Q$  are the same.

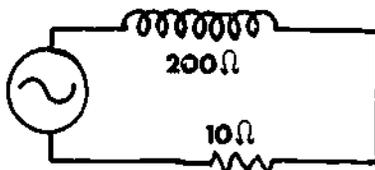
What is the  $\angle\theta$  for a circuit with a  $Q$  of 10?

-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (approximately  $84.3^\circ$ )

30. You can see that  $\angle\theta$  is very near  $90^\circ$ ; therefore, the circuit is almost purely reactive. This is the reason the resistance in a circuit with a  $Q$  of 10 or more is disregarded.

-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (Go to the next frame.)

31. Solve for  $Z_T$ :



$Z_T =$  \_\_\_\_\_

-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (THIS IS A TEST FRAME. COMPARE YOUR ANSWER WITH THE CORRECT ANSWER GIVEN AT THE TOP OF THE NEXT PAGE.)

---

ANSWERS - TEST FRAME 31

200  $\Omega$  for all practical purposes

---

IF YOUR ANSWER IS INCORRECT, GO BACK TO FRAME 12 AND TAKE THE PROGRAMMED SEQUENCE.

IF YOUR ANSWER IS CORRECT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY  
LESSON 1

Series RLC Circuits

In this lesson you have learned how to solve series RLC (combination resistive-inductive-capacitive) circuits.

Voltage leads current by 90 electrical degrees in a purely inductive circuit, lags current by 90 electrical degrees in a purely capacitive circuit, and is in phase with current in a purely resistive circuit. Since the individual voltages are out of phase, vectors must be used to solve for total voltage.

Impedance must also be solved vectorially because the reactive components (inductors and capacitors) vary in the opposition to circuit current as the applied frequency is changed. Circuit current is used as the reference in all series circuits because it is common to all parts of the circuit. Since  $E$  and  $I$  are in phase across a resistive component,  $E_R$  and  $I_R$  are plotted in the standard vector position. Kirchhoff's Voltage Law, "Sum of the voltage drops around a circuit must equal source voltage," still applies, and the instantaneous and vector sums of the voltage drops across the components equal the applied voltage, although the individual voltages across the capacitor and the inductor may be many times the voltage applied.

A coil has a figure of merit called the quality (Q) of the coil. The  $Q$  of a coil is  $X_L$  divided by the effective resistance of the coil. The effective resistance ( $R_{eff}$  or  $R_{ac}$ ) varies with frequency as does  $X_L$ ; therefore,  $Q$  does not vary appreciably as frequency changes. The variation of  $R_{eff}$  caused by frequency is a result of two factors - the skin effect and the proximity effect.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

BASIC ELECTRICITY AND-ELECTRONICS  
INDIVIDUALIZED LEARNING SYSTEM



MODULE THIRTEEN

LESSON I I

Series AC Circuits at Resonance

Study Booklet

OVERVIEW  
Lesson 11

Series AC Circuits at Resonance

In this lesson you will study and learn about the following:

- resonant frequency
- circuit analysis at  $f_0$
- voltage gain
- solving for resonant frequency

TURN TO THE FOLLOWING PAGE AND BEGIN LESSON 11.

List of Study Resources

Lesson 11

Series AC Circuits at Resonance

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

- NAVPERS 93400A-1b "Basic Electricity, Alternating Current."  
Fundamentals of Electronics. Bureau of Naval Personnel.  
Washington, D.C.: U.S. Government Printing Office, 1965.
- Powers of Ten Program.

AUDIO VISUAL:

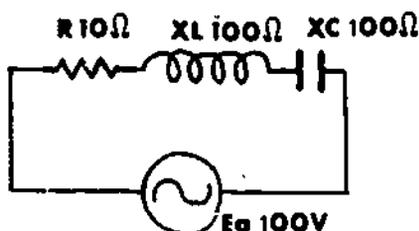
- Slide/Sound Presentation - "Factors Affecting Resonant Frequency."

• YOU MAY NOW STUDY ANY, OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE  
LESSON 11

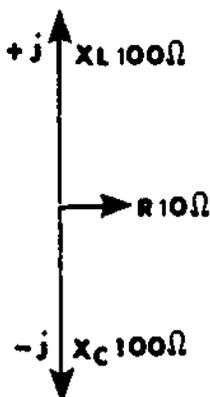
Series AC Circuits at Resonance

Resonant Frequency



Looking at this series RLC circuit, we see that the values of  $X_L$  and  $X_C$  both equal 100 ohms. Let's draw the impedance vector diagram to compute  $Z_T$ , and see what happens.

Current is the common reference and, as always in a series circuit, the resistive values are plotted in the standard position with current.



The  $X_L$  vector is rotated  $90^\circ$  counter-clockwise, the  $X_C$  vector is rotated  $90^\circ$  clockwise. Since the  $X_L$  and  $X_C$  vectors are equal and in opposite directions, they tend to cancel each other.

$$X_L = +j100 \Omega$$

$$X_C = -j100 \Omega$$

$$\text{Reactance} = 0 \Omega$$

For all practical purposes the circuit at this frequency has only one kind of opposition -- resistance.

Resistance is the only thing that limits current, and therefore, the source sees the circuit as being purely resistive. In a purely resistive circuit, all quantities are in phase and the vector representation is  $R = 10 \Omega$ . The phase angle is 0.

This circuit condition, in which  $X_L = X_C$  and  $I$  is limited only by  $R$ , exists at the resonant frequency. For every value of  $L$  and  $C$  in a series circuit, there is a single frequency which causes  $X_L$  to equal  $X_C$ . That frequency is the resonant frequency of the circuit -- abbreviated  $f_0$ .

At  $f_0$ , these conditions exist in a series RCL circuit:

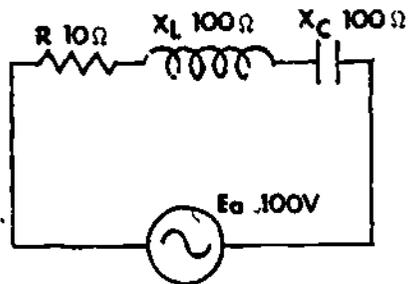
$$X_L = X_C$$

$$Z_T \text{ is minimum (} R \text{ only)}$$

$$I_T \text{ is maximum (limited only by } R)$$

Analyzing the Circuit at  $f_0$ 

Let's look at the circuit again to determine other factors about it. Since at  $f_0$  the circuit appears purely resistive,  $\theta = 0$ , PF is 1, and  $Z_T = 10 \Omega$ .



Solve the above circuit.

$$I_T = \underline{\hspace{2cm}}$$

$$P_t = \underline{\hspace{2cm}}$$

$$P_a = \underline{\hspace{2cm}}$$

$$E_R = \underline{\hspace{2cm}}$$

By Ohm's Law,  $I_T = 10 \text{ a}$ ;  $P_t = 1000w$ ;  $P_a = 1000 \text{ va}$ ;  $E_R = 100 \text{ v}$ .

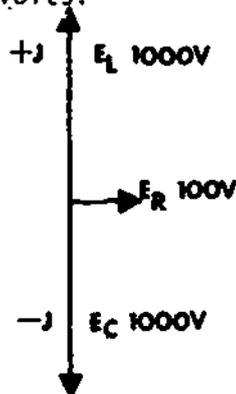
Now you may be assuming that since the full applied voltage of 100 volts is dropped across the resistor, there is no voltage drop across the coil or the capacitor, but this is not the case.

Voltage Drops at  $f_0$ 

Remember that in AC series circuits which have resistive and reactive components, we must compute the total voltage drops vectorially.

Therefore, with 10 amps current through the coil,  $E_L = I \times X_L$   
 or  $E_L = 10 \text{ a} \times 100 \Omega$   
 $E_L = 1000 \text{ v}$

We know that  $X_C$  is equal to  $X_L$ ; therefore,  $E_C$  also equals 1000 volts.



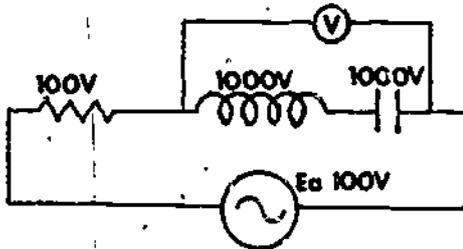
We can plot the voltage vectors this way.

$$E_L = +j1000 \text{ v}$$

$$E_C = -j1000 \text{ v}$$

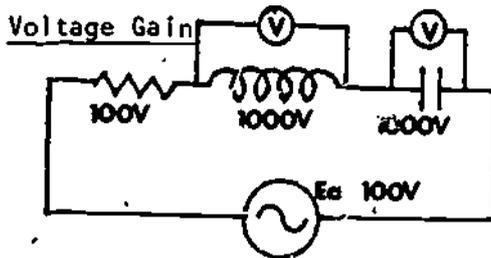
$$\text{Total } \underline{V} = 0 \text{ v}$$

Because these voltages are  $180^\circ$  out of phase, they cancel each other, leaving the full applied voltage of 100 volts dropped across the resistance.



If we connect a voltmeter across both reactive components as shown, the meter indicates 0 volts.

This is true because  $E_L$  and  $E_C$  are equal and opposite and cancel each other.



If, however, we connect meters across the individual components as shown here, the meter reads 1000 volts in each case. This means we can tap off 1000 volts across either the inductor or the capacitor. Notice that the source voltage is only 100 volts, but at  $f_0$  the circuit is capable of providing a greater voltage than the amount supplied by the source.

This increase of voltage is called voltage gain.

Solving for Resonant Frequency

We have said that for every value of  $L$  and  $C$ , a frequency exists which causes  $X_L$  to equal  $X_C$ . This is the resonant frequency.

The formula for finding  $f_o$  is derived from the formulas for  $X_C$  and  $X_L$ .

$$\text{At } f_o: X_L = X_C$$

$$\text{Substituting: } 2\pi f_o L = \frac{1}{2\pi f_o C}$$

$$(2\pi f_o C) 2\pi f_o L = 1$$

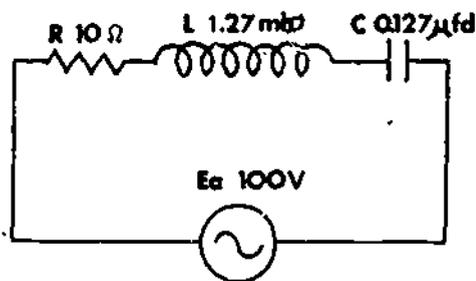
$$4\pi^2 f_o^2 LC = 1$$

$$\text{Solving for } f_o^2: f_o^2 = \frac{1}{4\pi^2 LC}$$

$$\text{Taking the square root of both sides: } f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$\text{Simplifying, } f_o = \frac{0.159}{\sqrt{LC}}$$

Using this equation, let's find  $f_o$  for this circuit.



$$f_o = \frac{0.159}{\sqrt{LC}}$$

$$f_o = \frac{0.159}{\sqrt{(1.27 \times 10^{-3}) (0.127 \times 10^{-6})}}$$

$$f_o = \frac{0.159}{\sqrt{(1.27 \times 10^{-3}) (1.27 \times 10^{-7})}}$$

$$f_o = \frac{0.159}{1.27 \times 10^{-5}}$$

$$f_o = 12.5 \times 10^3 \text{ Hz or } 12.5 \text{ KHz}$$

(Note: If you do not understand the procedure for extracting the square root of a number containing a power of ten, refer to the Powers of Ten Program in the reference library.)

---

**Practice:**Solve for  $f_o$  when:

1.  $L = 100 \text{ mh}$

$C = 10 \text{ } \mu\text{f}$

$f_o = \underline{\hspace{2cm}}$

2.  $L = 20 \text{ mh}$

$C = 50 \text{ pf}$

$f_o = \underline{\hspace{2cm}}$

3.  $L = 3 \text{ mh}$

$C = 120 \text{ } \mu\text{f}$

$f_o = \underline{\hspace{2cm}}$ 

---

Check answers, and procedures if necessary, on following pages.

$$1. \text{ Solution: } f_o = \frac{0.159}{\sqrt{LC}}$$

$$f_o = \frac{0.159}{\sqrt{(100 \times 10^{-3}) (10 \times 10^{-6})}}$$

$$f_o = \frac{0.159}{\sqrt{(100 \times 10^{-3}) (1 \times 10^{-5})}}$$

$$f_o = \frac{0.159}{\sqrt{100 \times 10^{-8}}}$$

$$f_o = \frac{0.159}{10 \times 10^{-4}}$$

$$f_o = 0.159 \text{ KHz or } 159 \text{ Hz}$$

$$2. \text{ Solution: } f_o = \frac{0.159}{\sqrt{(20 \times 10^{-3}) (50 \times 10^{-12})}}$$

$$f_o = \frac{0.159}{\sqrt{(20 \times 10^{-3}) (5 \times 10^{-11})}}$$

$$f_o = \frac{0.159}{\sqrt{100 \times 10^{-14}}}$$

$$f_o = \frac{0.159}{10 \times 10^{-7}}$$

$$f_o = 0.159 \text{ MHz or } 159 \text{ KHz}$$

$$3. \text{ Solution: } f_o = \frac{0.159}{\sqrt{(3 \times 10^{-3}) (120 \times 10^{-6})}}$$

$$f_o = \frac{0.159}{\sqrt{(3 \times 10^{-3}) (12 \times 10^{-5})}}$$

$$f_o = \frac{0.159}{\sqrt{36 \times 10^{-8}}}$$

$$f_o = \frac{0.159}{6 \times 10^{-4}}$$

$$f_o = 0.265 \text{ KHz or } 265 \text{ Hz}$$

---

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION  
LESSON 11

Series AC Circuits at Resonance

THERE ARE NO TEST FRAMES IN THIS PROGRAMMED SEQUENCE.

1. Both  $X_L$  and  $X_C$  within a series RLC circuit are affected by the frequency of the applied voltage.

(Note: Unless  $R$  is specified to be effective or AC resistance, it is considered not to be affected by frequency.)

Changing the frequency of the applied voltage causes:

- \_\_\_ a.  $L$  and  $C$  to change and  $R$  to remain the same.  
 \_\_\_ b.  $X_L$  and  $R$  to change and  $C$  to remain the same.  
 \_\_\_ c.  $X_L$  and  $X_C$  to change and  $R$  to remain the same.  
 \_\_\_ d.  $X_C$  and  $R$  to change and  $L$  to remain the same.

-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (c)

2. If the frequency applied to an RLC series circuit is increased,  $X_L$  increases and  $X_C$  decreases.

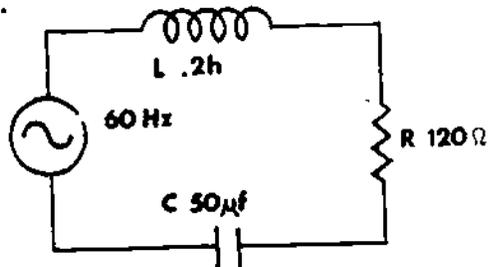
$X_L$  is \_\_\_\_\_ proportional, and  $X_C$  is \_\_\_\_\_ proportional to frequency.

-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (directly; inversely)

3. As frequency varies, the values of  $X_L$  and  $X_C$  change.

Solve for  $X_L$  and  $X_C$  in the circuit below using the applied frequency of 60 Hz in circuit A and 120 Hz in circuit B.

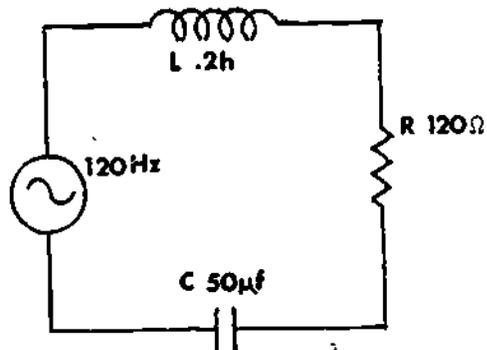
A.



$$X_L = \underline{\hspace{2cm}}$$

$$X_C = \underline{\hspace{2cm}}$$

B.



$$X_L = \underline{\hspace{2cm}}$$

$$X_C = \underline{\hspace{2cm}}$$

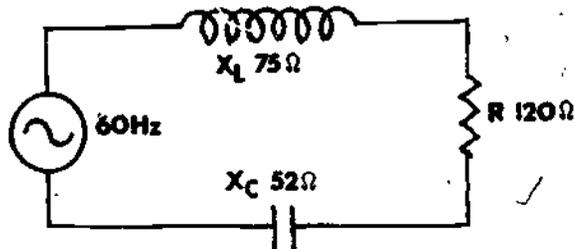
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(A.  $X_L = 75 \Omega$ ,  $X_C = 53 \Omega$ ; B.  $X_L = 150 \Omega$ ,  $X_C = 26.5 \Omega$ )

---

4. Since frequency affects  $X_L$  and  $X_C$ , it also has an effect on total circuit impedance.

Solve for  $Z_T$  and  $\angle\theta$  in the circuit below.



$$Z_T = \underline{\hspace{2cm}}$$

$$\angle\theta = \underline{\hspace{2cm}}$$

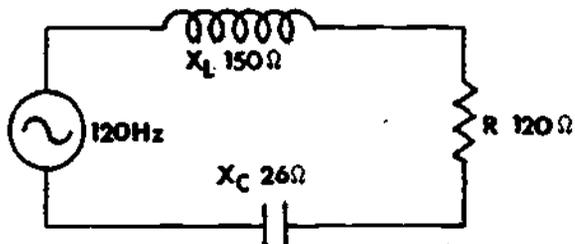
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$$(Z_T = 123 \Omega; \angle\theta = 10.8^\circ)$$


---

5. A change in the applied frequency results in a corresponding change in  $Z_T$  and the circuit phase angle.

Solve the  $Z_T$  and  $\angle\theta$  in the circuit below.



$$Z_T = \underline{\hspace{2cm}}$$

$$\angle\theta = \underline{\hspace{2cm}}$$

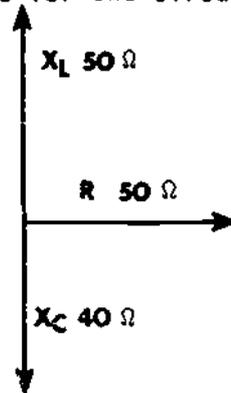
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$$(Z_T = 172.5 \Omega; \angle\theta = 45.9^\circ)$$


---

6. When a frequency applied to a series RLC circuit causes  $X_L$  to equal  $X_C$ , the frequency is known as the resonant frequency for that circuit.

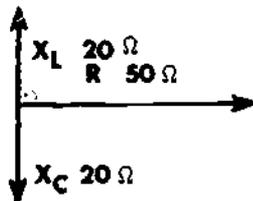
In the circuit represented by the vector diagram below, does frequency increase or decrease for the circuit to become resonant? \_\_\_\_\_



(decrease)

7. Remember,  $X_L$  increases as frequency increases;  $X_C$  increases as frequency decreases.

The vector diagram below represents a series RLC circuit at resonance. Decreasing frequency has what effect on the circuit?

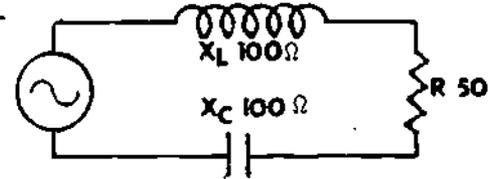


- \_\_\_ a.  $R \uparrow X_C \uparrow X_L \uparrow$   
 \_\_\_ b.  $R \uparrow X_C \uparrow X_L \uparrow$   
 \_\_\_ c.  $R \uparrow X_C \uparrow X_L \uparrow$   
 \_\_\_ d. (none of the above)

(c)

8. In a series RLC circuit at resonance,  $E_L$  cancels  $E_C$  and  $X_L$  and  $X_C$  no longer affect the total circuit values.

What component represents total impedance of the circuit below when it is at resonance? \_\_\_\_\_



-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (resistor)

9. The resultant of a vector diagram representing an RLC series circuit at resonance is resistance only.

At resonance, the circuit appears purely \_\_\_\_\_ to the source.

-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (resistive)

10. The formula used to find the resonant frequency for a given value of L and C is derived from the fact that, at resonance,  $X_L = X_C$ .

$$X_L = X_C$$

Substituting Formula  $2\pi f_o L = \frac{1}{2\pi f_o C}$

Cross Multiplying  $4\pi^2 f_o^2 LC = 1$

Isolating Frequency  $\frac{4\pi^2 f_o^2 LC}{4\pi^2 LC} = \frac{1}{4\pi^2 LC}$

Resulting in:  $f_o^2 = \frac{1}{4\pi^2 LC}$

Take Square Root of Both Sides:

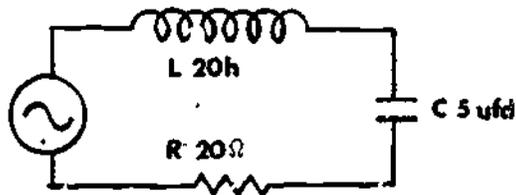
$$\sqrt{f_o^2} = \frac{\sqrt{1}}{\sqrt{4\pi^2 LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

Determine the resonant frequency of a circuit when  $L = 50 \text{ mh}$   
and  $C = 5 \text{ } \mu\text{f}$ .  $f_o = \underline{\hspace{2cm}}$

(318 Hz)

11. Solve for  $f_o$  in the circuit shown.



$f_o = \underline{\hspace{2cm}}$

$$f_o = \frac{0.159}{\sqrt{LC}}$$

$$f_o = \frac{0.159}{\sqrt{20 \times 5 \times 10^{-6}}}$$

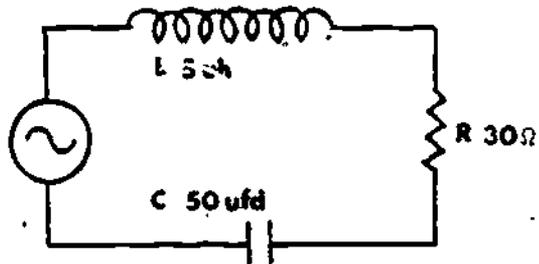
$$f_o = \frac{0.159}{\sqrt{100 \times 10^{-6}}}$$

$$f_o = \frac{0.159}{10 \times 10^{-3}}$$

$$f_o = 0.0159 \times 10^3 \text{ or } 15.9 \text{ Hz}$$

(NOTE: If you do not understand how to extract the square root of numbers including powers of ten, refer to the Powers of Ten Program in the reference library.)

12. Solve for  $f_o$  in this circuit.



$f_o =$  \_\_\_\_\_

-----  
 \_\_\_\_\_  
 ( $f_o = 10 \text{ KHz}$ )  
 \_\_\_\_\_

13. At resonance,  $X_L$  and  $X_C$  are of \_\_\_\_\_ value.

-----  
 \_\_\_\_\_  
 (equal)  
 \_\_\_\_\_

14. Also under these conditions, the circuit impedance is \_\_\_\_\_

-----  
 \_\_\_\_\_  
 (minimum)  
 \_\_\_\_\_

15. Under resonant conditions, the circuit impedance is equal to circuit \_\_\_\_\_

-----  
 \_\_\_\_\_  
 (resistance)  
 \_\_\_\_\_

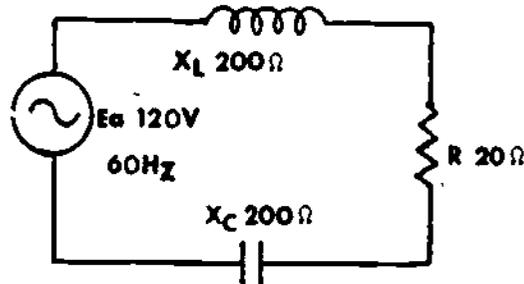
16. Since at resonance  $Z_T$  is equal to  $R$ , the circuit phase angle at resonance is \_\_\_\_\_

-----  
 \_\_\_\_\_  
 (zero)  
 \_\_\_\_\_

17. Let's review the characteristics of a series RLC circuit at resonance.
- The values of  $X_L$  and  $X_C$  are \_\_\_\_\_.
  - $Z_T$  of the circuit is equal to the circuit \_\_\_\_\_.
  - Circuit current is \_\_\_\_\_.
  - Phase angle between  $E$  and  $I$  is \_\_\_\_\_.

(a. equal; b. resistance; c. maximum; d. zero)

18. We know that the circuit shown below is operating at resonance because  $X_C$  is equal to \_\_\_\_\_.



( $X_L$ )

19. Since the circuit is operating at resonance,  $Z_T$  is equal to the \_\_\_\_\_. By Ohm's Law, current is \_\_\_\_\_ amps.

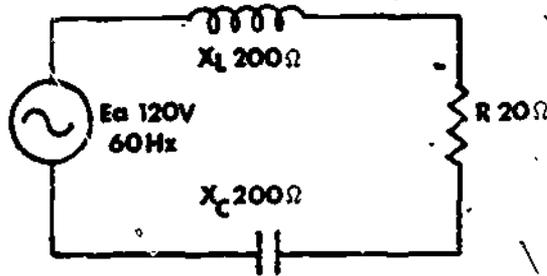
(resistance, 6)

20. Using Ohm's Law, we can solve for the voltage drop across each component.

The voltage drop across the resistor is \_\_\_\_\_ volts.

The voltage drop across the coil is \_\_\_\_\_ volts.

The voltage drop across the capacitor is \_\_\_\_\_ volts.




---

(120; 1200; 1200)

---

21. The source voltage is only 120 volts, yet  $E_L$  and  $E_C$  are 1200 volts each. This voltage gain is characteristic of series RLC circuits at resonance.

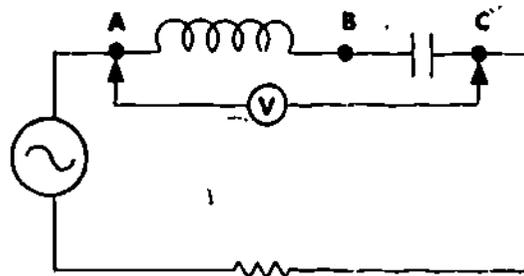
How is it possible to have a voltage drop across a component greater than the applied voltage? \_\_\_\_\_

---

(The canceling effect of the opposing voltages across the capacitor and the inductor makes it possible.)

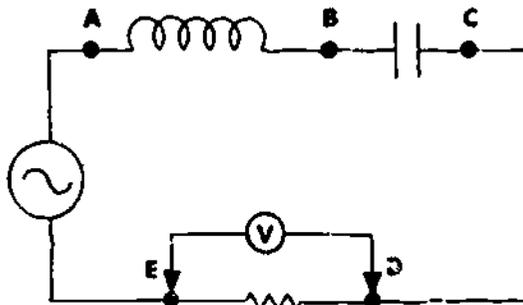
---

22. If a voltmeter is connected across point A to C in this series RLC circuit operating at resonance, what is the reading?



(zero)

23. If the voltmeter is connected from points D to E in the same circuit, the reading is equal to \_\_\_\_\_.



(source voltage)

YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY  
LESSON 11

Series AC Circuits at Resonance

In this lesson, you have learned that when the value of  $X_L$  is equal to the value of  $X_C$ , a series RLC circuit is operating at a condition known as resonance. For any combination of values of  $L$  and  $C$ , there is a frequency which produces a value of  $X_L$  that is exactly equal to the value of  $X_C$ . The symbol for resonant frequency is  $f_o$ .

You will find that the voltage drops across the reactive components at resonance are also exactly equal, and that they are  $180^\circ$  out of phase; thus, they effectively cancel each other, and the entire source voltage is dropped across the circuit resistance.

At resonance, the total impedance of the circuit is equal to the value of  $R$  since  $X_L$  cancels  $X_C$ . Circuit current is maximum at resonance and limited only by the value of  $R$ . If the applied frequency is changed from the resonant frequency,  $Z_T$  increases and  $I_T$  decreases.

The formula for  $f_o$  is  $f_o = \frac{1}{2\pi\sqrt{LC}}$   $\frac{0.159}{\sqrt{LC}}$ . From the formula, you

can see that if the value of  $L$  or  $C$  is changed, the resonant frequency changes. An increase of  $L$  or  $C$  causes a decrease in  $f_o$ .

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.



BASIC ELECTRICITY AND ELECTRONICS  
INDIVIDUALIZED LEARNING SYSTEM



MODULE THIRTEEN  
LESSON III

Resonance In Series AC Circuits

Study Booklet

OVERVIEW  
LESSON III

Resonance in Series AC Circuits

In this lesson you will study and learn about the following:

- current and impedance curves at  $f_o$
- circuit behavior above  $f_o$
- circuit behavior below  $f_o$
- bandwidth
- effects of  $Q$  on bandwidth
- practical applications

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES  
ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON III

Resonance in Series AC Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

Lesson Narrative  
Programmed Instruction  
Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400a-1b "Basic Electricity, Alternating Current."  
Fundamentals of Electronics. Bureau of Naval Personnel.  
Washington, D.C.: U.S. Government Printing Office, 1965.

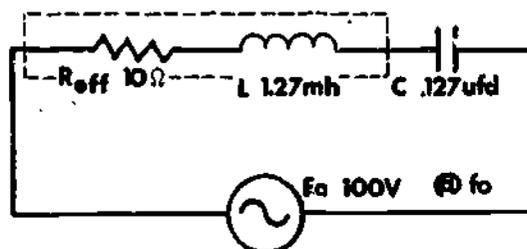
YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE  
LESSON III

Resonance in Series AC Circuits

In this lesson we will analyze the behavior of a single series RLC circuit as three different frequencies are consecutively applied: resonant frequency, a frequency above  $f_0$ , and a frequency below  $f_0$ .

This is the circuit we analyzed in the previous lesson.



We know that as this circuit is operating at  $f_0$ , these conditions exist:

$$I_T = \text{maximum}$$

$$Z_T = \text{minimum (R only)}$$

$$X_L = X_C$$

$$E_L = E_C$$

By the formula  $f_0 = \frac{0.159}{\sqrt{LC}}$ , we can determine that  $f_0$  is 12.5 KHz.

To find the value of  $X_L$ , we can use the formula for  $X_L$  and use  $f_0$  for  $f$ :

$$X_L = 2\pi f_0 L$$

$$X_L = 98.55 \text{ (approximately 100 ohms)}$$

at  $f_0$ ,  $X_L = X_C$ ; therefore,

$$X_C = 100 \text{ ohms}$$

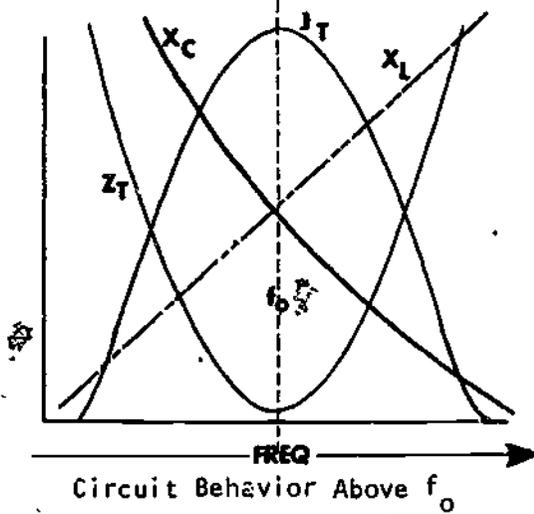
$X_L = +j100$ , and  $X_C = -j100$ ; their reactances oppose each other, since they are equal and they cancel each other. This leaves  $Z_T$  at a minimum with the resistance of 10 ohms being the only impedance.

Because  $Z_T$  is minimum,  $I$  is maximum at  $f_0$  and equal to 10 amps. We know that the source sees a purely resistive circuit. Then  $E$  and  $I$  are in phase and  $\theta$  is 0. The power factor is 1.

By  $P_t = I^2 R$ , true power equal 1000 w ( $10^2 \times 10 \Omega$ ). In a purely resistive circuit, true power equals apparent power; therefore,  $P_a = 1000$  va. We can prove this by the formula  $P = E \times I$  ( $100 \text{ v} \times 10 \text{ a} = 1000 \text{ va}$ ).

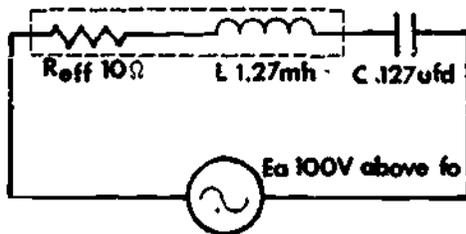
### Current and Impedance Curves at $f_0$

Recall that earlier you saw curves representing the relationship of voltage and frequency for both RL circuits and RC circuits. A similar graph of current shows some relationships in a series RLC circuit.



At the peak of the current curve (maximum current) we have resonant frequency. The lines that are inversely proportional to  $I$  represent  $Z_T$ , so when  $I$  is maximum,  $Z_T$  is at its minimum.

Now for a better understanding of what resonance means, we will analyze the same circuit with the applied frequency above resonance.



Assume we have increased the frequency from 12.5 KHz to 13.125 KHz.

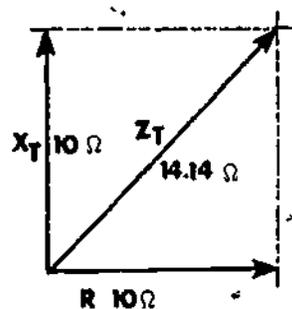
When  $f$  increases, what happens to:

$X_L$  \_\_\_\_\_ (Indicate with arrows.)

$X_C$  \_\_\_\_\_

ANSWERS:  $X_L \uparrow$ ;  $X_C \downarrow$

$X_L$  increases to  $105 \Omega$ ;  $X_C$  decreases to  $95 \Omega$ . The formulas prove this to you if you are in doubt:  $X_L = 2\pi fL$ ,  $X_C = \frac{0.159}{fC}$ .



By algebraically adding, we can determine the value and direction of the reactive vector of the impedance vector diagram.

$$X_L = +j105 \Omega$$

$$X_C = -j95 \Omega$$

$$\text{reactive vector} = +j10 \Omega$$

Now the source effectively sees a circuit containing only  $R$  and  $X_L$ . Notice that  $R$  and the effective  $X_L$  are equal.

What frequency condition exists when  $R$

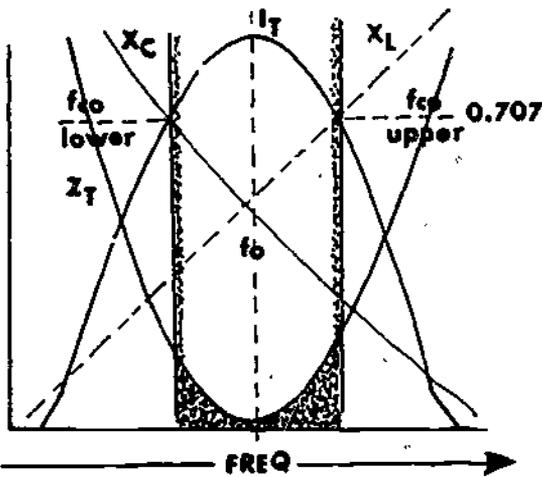
and effective  $X_L$  are equal? \_\_\_\_\_

What is  $\theta$ ? \_\_\_\_\_

When  $R = X_L$ , we have the frequency cutoff point ( $f_{co}$ ). Since the resistance and reactance are equal,  $\theta$  is  $45^\circ$ .

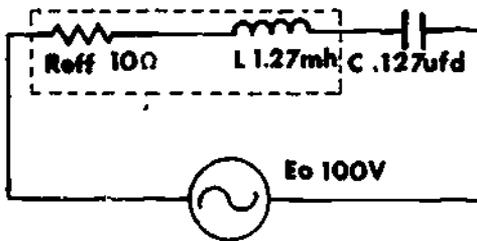
At  $f_{co}$ , we are at the half-power point, so  $P$  is 500 watts, or half of what it was in the purely resistive circuit at  $f_0$ .

When frequency reaches a point above resonance where  $X_L$  and  $R$  are equal, we call this the high-frequency cutoff point.



On the current and impedance curve, the high  $f_{co}$  point is shown on the high side of  $f_0$ . Observing the curve shows that at the upper  $f_{co}$   $Z_T$  has increased and  $I_T$  has decreased.

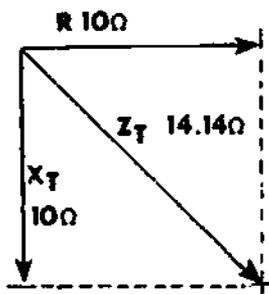
Circuit Behavior Below  $f_0$



Now we will change to a frequency below the resonant frequency. We will use 11.875 KHz as the frequency this time.

We know that when  $f$  decreases,  $X_L$  decreases and  $X_C$  increases. By using the formulas for  $X_L$  and  $X_C$ , we determine that  $X_L = 95 \Omega$  and  $X_C = 105 \Omega$ . Notice these values are just the reverse of the values for  $X_L$  and  $X_C$  in the circuit operating above  $f_0$ .

To find  $Z_T$ , we need to use vectors. The reactance vector is determined by algebraically adding.



$$X_C = -j105 \Omega$$

$$X_L = +j95 \Omega; \text{ therefore,}$$

$$X_T = -j10 \Omega$$

and we draw the vector in the  $-j$  position.

Since the resistance and effective reactance are equal,  $\theta$  is  $-45^\circ$ , and  $Z_T = 14.14 \Omega$ .

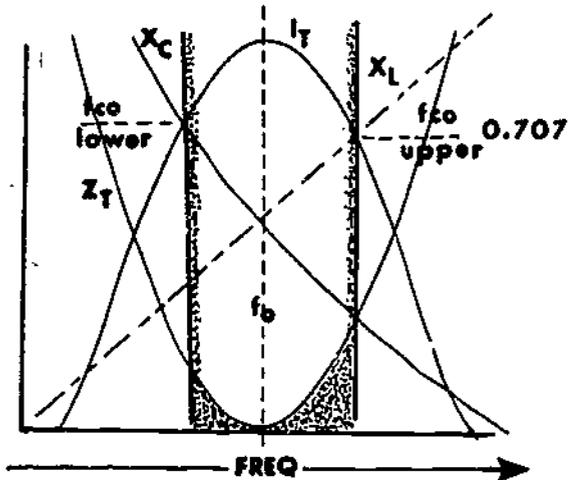
When effective  $X_C = R$ , we know that we have reached a  $f_{co}$  point -- in this case the lower  $f_{co}$  because frequency is below resonant frequency.

We know that at  $f_{co}$ :

$I$  is 70.7% of maximum and is 7.07 amps.

$P_t$  is half of maximum or 500 watts.

At low frequency cutoff, the source sees an RC circuit.



On the current and impedance curves, the half-power point on the  $I$  curve for RLC circuits indicates the lower  $f_{co}$ .

Here again you see that at low  $f_{co}$ ,  $I_T$  has decreased.

Conclusions

at  $f_0$

$Z_T = \text{minimum}$

$I_T = \text{maximum}$

$\angle\theta = 0$

upper  $f_{co}$

$Z_T \uparrow$

$I_T \downarrow$

$\angle\theta = 45^\circ$

lower  $f_{co}$

$Z_T \uparrow$

$I_T \downarrow$

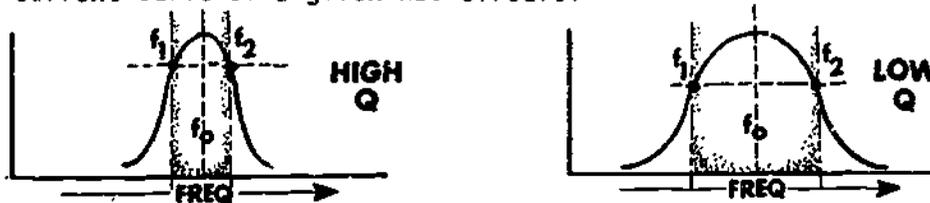
$\angle\theta = -45^\circ$

1. Above or below  $f_0$ ,  $Z_T$  will                     .  
(+ or -)
2. Above or below  $f_0$ ,  $I_T$  will                     .  
(+ or -)
3. Above  $f_0$ , the circuit will appear:
  - a. purely resistive.
  - b. RL.
  - c. RC.
  - d. RLC.
4. Below  $f_0$ , the circuit will appear:
  - a. purely resistive.
  - b. RL.
  - c. RC.
  - d. RLC.

Answers: 1. + ; 2. + ; 3. b ; 4. c

### Bandwidth

The curves below indicate the effect of  $Q$  on the shape of the current curve of a given RLC circuit.

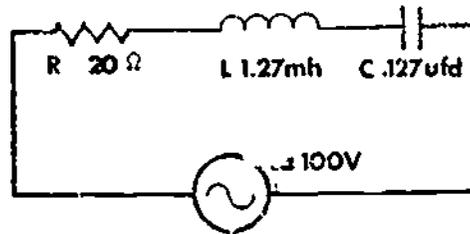


The shaded areas on these curves represent the distance between the upper  $f_2$  and lower  $f_1$ . This distance is called the bandwidth of an RLC circuit - abbreviated BW. Bandwidth is the range of frequencies that a circuit passes with little loss.

Effects of Q on Bandwidth

You know that the  $Q$  of a series circuit is determined by the value of  $X_L$  and of the effective  $R$ .  $Q = \frac{X_L}{R_{\text{eff}}}$ .

If we were to change  $R$  in our circuit from  $10 \Omega$  to  $20 \Omega$ , what would happen to  $Q$ ? \_\_\_\_\_

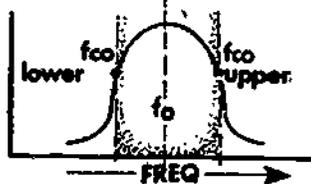


$Q$  would decrease because more resistance had been added and this decreased the amount of power available to be stored in the coil.

Recall that  $Q$  does not change much with a change in frequency. If it were possible to change the effective resistance of the circuit, keeping all other values constant, the following would occur.

If the effective resistance increased from 10 ohms to 20 ohms then it would take a higher frequency for  $X_L$  to equal the 20 ohms of  $R$ . Similarly, we would have to lower frequency more to have  $X_C$  equal the 20 ohms of  $R$ . The result would be an increased bandwidth, and the resonance curve would look like the one below.

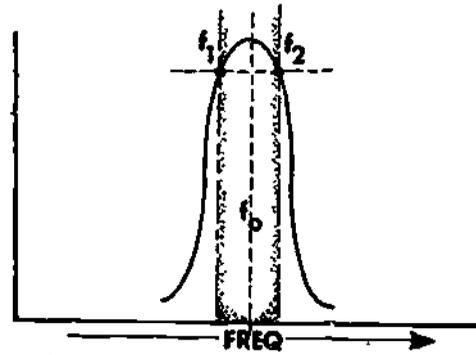
(Note: The maximum current value also decreased.)



The spread between upper  $f_{co}$  and lower  $f_{co}$  increased, upper  $f_{co}$  minus lower  $f_{co}$  equals BW; therefore, bandwidth increased.

The circuit conditions at  $f_{co}$  are described in Module 12, Lesson VI.

If the effective resistance of our circuit was decreased from 10 ohms to 1 ohm, this would increase  $Q$ , and decrease bandwidth. Here  $I_{\max}$  would increase tenfold.



Therefore, if:  $Q \uparrow$  then  $BW \downarrow$

$Q \downarrow$  then  $BW \uparrow$

$Q$  and  $BW$  are:

- a. directly proportional.  
 b. inversely proportional.

They are inversely proportional.

A second method for computing bandwidth is available if you do not know the upper  $f_{co}$  and lower  $f_{co}$ .

$$BW = \frac{f_0}{Q}$$

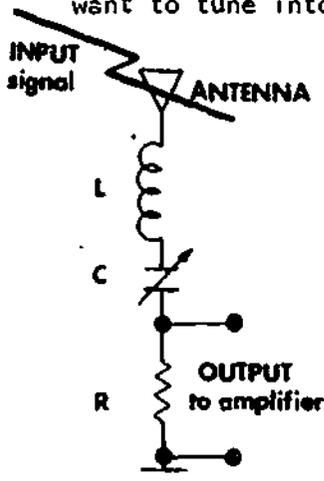
In our circuit, where resonant frequency is 12.5 KHz and  $Q$  is 10, use the formula above to solve for bandwidth.  $BW =$  \_\_\_\_\_

Observe that you found  $BW = 1.25$  KHz just as when you subtracted lower  $f_{co}$  from upper  $f_{co}$ .

A Practical Application

We cannot say that a high  $Q$  is necessarily good, or that a low  $Q$  is necessarily bad. It depends on the application and on how wide a range of frequencies is desired. If we want a wide range of frequencies, we use a lower  $Q$ . Conversely, if we want a narrow bandwidth, we use a higher  $Q$  circuit.

For example, let's say you turn on your car radio. You know you want to tune into 1250 KHz on the dial to hear "The Latest News."



The car antenna has a multitude of frequencies to pick up, but you only want 1250 KHz so you can hear "The Latest News," and you want to hear it loud and clear without garble from stations on other frequencies.

You turn the tuning knob to 1250 KHz on the radio dial. This varies the amount of capacitance in the circuit, so that the RLC circuit resonates at 1250 KHz.

Because you do not want to hear the stations at 1350 KHz or 1150 KHz along with "The Latest News," your radio needs a narrow bandwidth which passes only the frequencies you desire.

---

The tuning circuit of your radio should have:

- \_\_\_ a. a high  $Q$ .
  - \_\_\_ b. a low  $Q$ .
- 

A high- $Q$  coil passes only a narrow range of frequencies on either side of  $f_0$ ; therefore, a good radio has a high- $Q$  circuit.

The ability of a circuit to tune a narrow band of frequencies is called selectivity. A circuit with high selectivity has a high Q and narrow bandwidth.

In reality, we seldom see a series circuit used in the radio application just described. At  $f_0$  in a series RLC circuit, current is very high; therefore, circuit components have to be large, and expensive to handle the great amount of power. Generally, a parallel circuit is used for such an application and we'll soon be learning about parallel resonant circuits.

Now check yourself on the following questions.

- 
1. If Q is decreased, will selectivity increase or decrease?  
\_\_\_\_\_
  2. If Q is decreased, will BW increase or decrease? \_\_\_\_\_
  3. The relationship between Q and  $Z_T$  at  $f_0$  is:
    - \_\_\_ a. inversely proportional.
    - \_\_\_ b. directly proportional.
  4. What is the circuit phase angle at  $f_0$ ? \_\_\_\_\_
  5. If you want to design a circuit to pass a frequency range of 950 Hz to 1050 Hz, what must Q be? \_\_\_\_\_
- 

Answers:

1. decrease
2. increase
3. a
4. zero
5.  $Q = 10$

In a circuit which has:

upper  $f_{co}$  of 1050 Hz

lower  $f_{co}$  of 950 Hz,

the BW is 100 Hz

If  $Q = \frac{f_o}{BW}$ , then to find  $f_o$ , we simply take half the BW and subtract it from the upper  $f_{co}$ :

1050 Hz

- 50 Hz (1/2 BW)

1000 Hz =  $f_c$

Then:

$$Q = \frac{f_o}{BW}$$

$$Q = \frac{1000}{100} = 10$$

---

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

## PROGRAMMED INSTRUCTION

## LESSON 111

Resonance in Series AC Circuits

TEST FRAMES ARE 4, 6, 15, 17 AND 26. AS BEFORE, GO FIRST TO TEST FRAME 4 AND SEE IF YOU CAN ANSWER ALL THE QUESTIONS THERE. FOLLOW THE DIRECTIONS GIVEN AFTER THE TEST FRAME.

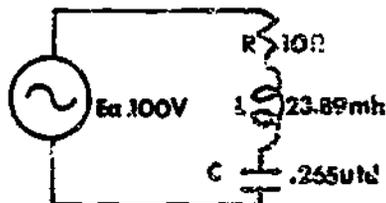
1. Once again, in a series RLC circuit operating at its resonant frequency,  $X_C$  and  $X_L$  are equal. At resonance,  $Z_T$  is:

- \_\_\_ a. maximum  
\_\_\_ b. minimum

-----  
\_\_\_\_\_  
(b)

2. Because the effects of  $X_L$  and  $X_C$  cancel at resonance,  $Z$  is at its minimum value and equal to the value of resistance in the RLC circuit.

What is  $I_T$  (circuit is operating at resonance)?

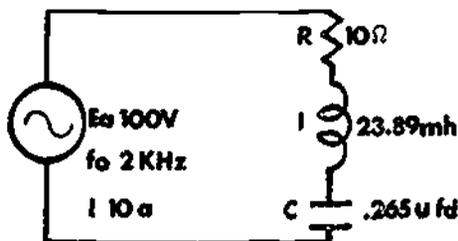


$I_T =$  \_\_\_\_\_

-----  
\_\_\_\_\_  
(10 amps)

3. In a series RLC circuit at resonance,  $X_L = X_C$ ; and since current is common, this results in  $E_C = E_L$ . To compute  $E_C$  and  $E_L$ , it is necessary to find either  $X_L$  or  $X_C$ , and then multiply this value by  $I_T$ .

Find  $E_C$  and  $E_L$ .

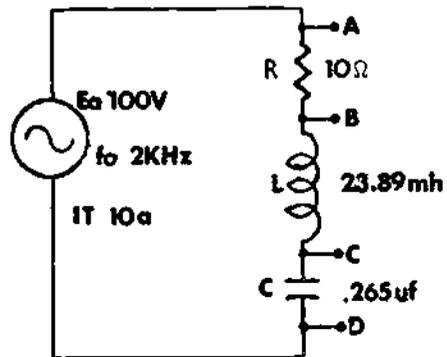


$E_C =$  \_\_\_\_\_

$E_L =$  \_\_\_\_\_

-----  
\_\_\_\_\_  
(3000 volts; 3000 volts)

4. What does a voltmeter read when connected between the following points?



A-B: \_\_\_\_\_

B-C: \_\_\_\_\_

C-D: \_\_\_\_\_

B-D: \_\_\_\_\_

A-D: \_\_\_\_\_

---

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

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ANSWERS - TEST FRAME 4

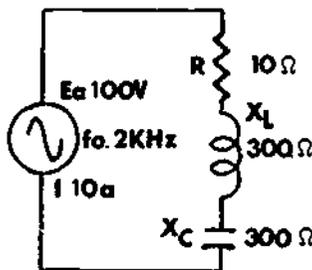
A-B = 100 volts; B-C = 3000 volts; C-D = 3000 volts;

B-D = 0 volts; A-D = 100 volts

IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 6. OTHERWISE, GO BACK TO FRAME 1 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 4 AGAIN.

5. Because the reactances effectively cancel at the resonant frequency and the circuit impedance is equal to resistance, the source sees a purely resistive circuit.  $E_a$  and  $I_T$  are in phase.

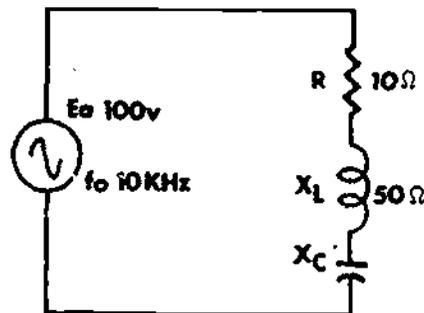
Solve for  $\angle\theta$  and PF.



$\angle\theta =$  \_\_\_\_\_  
 PF = \_\_\_\_\_

( $\angle\theta = 0^\circ$ ; PF = 1)

6. Solve for the following values. (Circuit is at resonance.)



- a.  $X_C$  \_\_\_\_\_  
 b.  $E_R$  \_\_\_\_\_  
 c.  $\angle\theta$  \_\_\_\_\_  
 d. PF \_\_\_\_\_  
 e. Q \_\_\_\_\_  
 f.  $Z_T$  \_\_\_\_\_

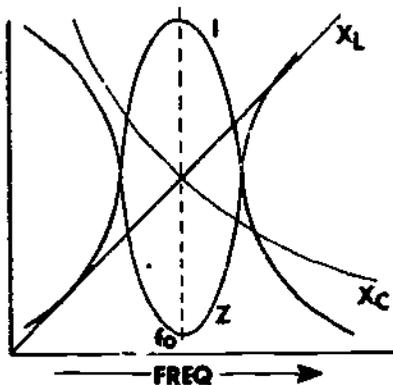
(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN AT THE TOP OF THE NEXT PAGE.)

## ANSWERS - TEST FRAME 6

a.  $50 \Omega$ ; b.  $100 \text{ v}$ ; c.  $0^\circ$ ; d.  $1$ ; e.  $5$ ; f.  $10 \Omega$

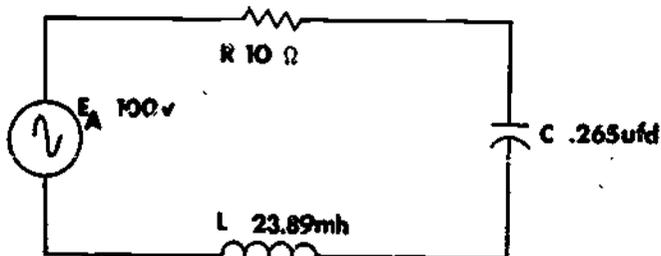
IF ALL YOUR ANSWERS MATCH THE CORRECT ANSWERS, YOU MAY GO TO TEST FRAME 15. OTHERWISE, GO BACK TO FRAME 5 and TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 6 AGAIN.

7. The relationship between  $I$  and  $Z$  can be seen quite well when plotted on a graph.



As you can see from the graph, as frequency increases,  $X_C$  starts to decrease and  $X_L$  starts to increase. The canceling effect between the two becomes greater and greater until  $X_L = X_C$  (at  $f_o$ ). At this point  $Z$  is minimum and  $I$  is maximum. If frequency is increased above this point,  $X_L$  becomes greater than  $X_C$  and  $E_L$  is no longer completely cancelled by  $E_C$ , so  $E_R$  decreases and  $I$  decreases.

8. By using the formula to determine  $F_o$ , we calculate that  $F_o$  for the circuit below is  $2 \text{ KHz}$  and that at  $F_o$ ,  $I = 10 \text{ a}$   
 $Z = 10 \Omega$ .



If frequency is increased by  $30 \text{ Hz}$  the balance between the reactances is upset.  $X_L$  increases to  $305 \text{ ohms}$  and  $X_C$  decreases to  $295 \text{ ohms}$ .

Compute  $\underline{Z}$  and  $\underline{I}$  at the increased frequency.

$$Z = \underline{\hspace{2cm}}$$

$$I = \underline{\hspace{2cm}}$$

---



---

(14.14  $\Omega$ ; 7.07 a)

9. At the increased frequency,  $\underline{Z}$  is no longer equal to  $R$ , but once again is a combination of resistance and reactance.

At frequencies above resonance, the circuit appears:

- a. resistive.  
 b. inductive-resistive.  
 c. capacitive-resistive.

---



---

(b) inductive-resistive

10. At a frequency of 2030 Hz,  $X_L$  is 10 ohms greater than  $X_C$ , so impedance equals  $R + jX_L - jX_C$  or  $10 + j10$ . Notice that  $R = X_L - X_C$ . You've seen this condition before. It's known as

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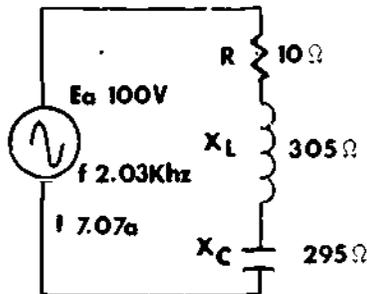


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( $f_{co}$  or cutoff frequency)

11. The circuit is now operating at  $f$  on the high or inductive side of resonance. As the source  $E_s$  sees it, the circuit now consists solely of 10 ohms of R and 10 ohms of  $X_L$ , even though  $X_L$  actually represents total reactance ( $X_L - X_C$ ). The conditions described in Module Twelve, Lesson VI, for cutoff now exists (series RL circuits).

Solve for the values indicated.



- a.  $\theta =$  \_\_\_\_\_  
 b.  $E_L =$  \_\_\_\_\_  
 c.  $E_R =$  \_\_\_\_\_  
 d.  $P_t =$  \_\_\_\_\_

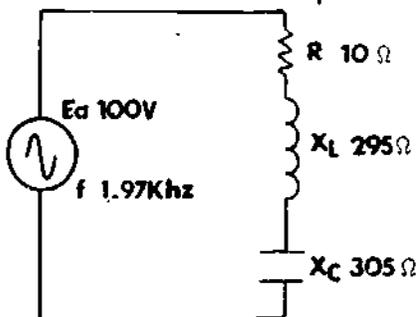
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(a.  $45^\circ$ ; b. 2156.35v; c. 70.7 volts; d. 500 watts)

---

12. Frequency is now reduced to 30 Hz below  $f_0$ . As this is done, Z decreases to its minimum value (10 ohms,  $Z=R$  at  $f_0$ ), and then increases as frequency goes below  $f_0$  to 1.97 KHZ.  $X_C$  is now 305 ohms while  $X_L$  is 295 ohms.  $X_C$  is 10 ohms greater than  $X_L$ .

Solve for Z and  $I_T$  at this new frequency.



- Z = \_\_\_\_\_  
 I = \_\_\_\_\_

---

(Z = 14.14Ω    I = 7.07 amps)

---

13. Notice that  $Z$  and  $I$  are the same value at 30 Hz below resonance (1.97 KHz) as they did at 30 Hz above  $f_0$ . The difference lies in that the circuit appears capacitive and the phase angle is negative.

Which statement is correct?

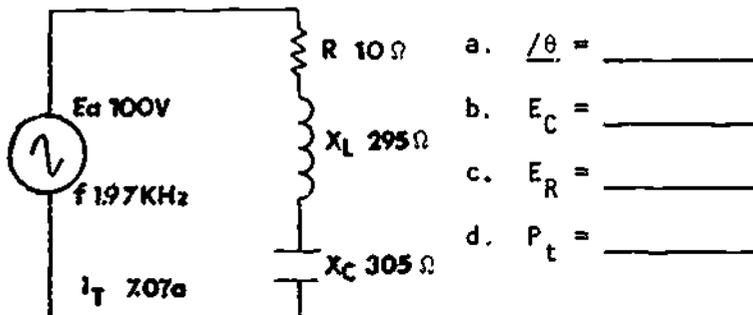
1. Impedance increases as frequency increases above  $f_0$ .
2. Impedance increases as frequency decreases below  $f_0$ .

- \_\_\_ a. 1  
 \_\_\_ b. 2  
 \_\_\_ c. both  
 \_\_\_ d. neither

-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (c) both

14. The circuit now appears resistive-capacitive with 10 ohms resistance and 10 ohms capacitive reactance. Once again resistance equals reactance and the circuit is at cutoff on the low or capacitive side of resonance.

Solve for the following values.



-----  
 \_\_\_\_\_  
 \_\_\_\_\_  
 (a.  $-45^\circ$ ; b. 2156.35 volts; c. 70.7 volts; d. 500 watts)

15. A series RLC circuit below  $f_0$  appears to be:

- a. inductive-resistive.
  - b. resistive-capacitive.
  - c. purely resistive.
- 

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(THIS IS A TEST FRAME. COMPARE YOUR ANSWER WITH THE CORRECT ANSWER GIVEN AT THE TOP OF THE NEXT PAGE.)

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ANSWER - TEST FRAME 15

b

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IF YOUR ANSWER MATCHES THE CORRECT ANSWER, YOU MAY GO TO TEST FRAME 17. OTHERWISE, GO BACK TO FRAME 7 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 15 AGAIN.

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16. The extent of the range of frequencies between the upper and lower cutoff frequencies is known as the bandwidth (BW) of the circuit.

Determine the bandwidth of a circuit whose upper  $f_{co}$  is 1520 Hz and lower  $f_{co}$  is 1480 Hz.

- a. 1520 Hz  
 b. 20 Hz  
 c. 1480 Hz  
 d. 40 Hz

---

(d) 40 Hz

---

17. Determine BW if:

upper  $f_{co} = 13.125$  KHz, and

lower  $f_{co} = 11.875$  KHz.

BW = \_\_\_\_\_

---



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(THIS IS A TEST FRAME. COMPARE YOUR ANSWER WITH THE ANSWER GIVEN ON THE TOP OF THE NEXT PAGE.)

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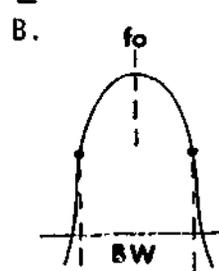
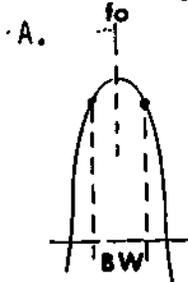
ANSWER - TEST FRAME 17 .

1.250 KHz

IF YOUR ANSWER MATCHES THE CORRECT ANSWER, YOU MAY GO TO TEST FRAME 26. OTHERWISE, GO BACK TO FRAME 16 AND TAKE THE PROGRAMMED SEQUENCE BEFORE TAKING TEST FRAME 17 AGAIN.

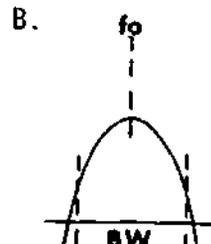
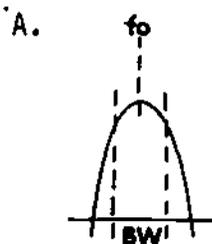
18.  $Q$  is a prime factor in determining the bandwidth of a circuit. The higher the  $Q$ , the more narrow the bandwidth.

Which drawing depicts a high- $Q$  circuit?



(A)

19. A narrow BW indicates a high- $Q$  or a highly selective circuit. The bandwidth indicates the range of frequencies a circuit will pass with the output taken across the resistor.



Which circuit represented in the above graphs passes the wider range of frequencies? \_\_\_\_\_ Which graph has the poorer selectivity? \_\_\_\_\_

(B; B)

20. Bandwidth is inversely proportional to  $Q$ . As circuit  $Q$  increases, BW \_\_\_\_\_

(decreases)

21. To increase the  $Q$  of the circuit, the effective resistance ( $R_{eff}$ ) must be decreased. As we decrease the  $R_{eff}$  value of the circuit, BW \_\_\_\_\_

(decreases)

22. Since decreasing  $R_{eff}$  decreases BW and increases circuit  $Q$ , BW is \_\_\_\_\_ proportional to  $R_{eff}$  and \_\_\_\_\_ proportional to  $Q$ .

(directly, inversely)

23. To determine the BW of a circuit, you may use the formula

$$BW = \frac{f}{Q}$$

The BW of a circuit with a  $Q$  of 100 and an  $f_o$  of 12.5 KHz is \_\_\_\_\_ Hz.

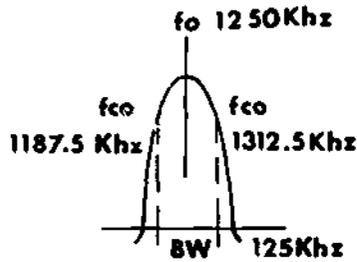
(125 Hz)

24. A radio circuit with a BW of 125Khz receives a signal from a station 62.5Khz above  $f_o$  to 62.5Khz below  $f_o$ .

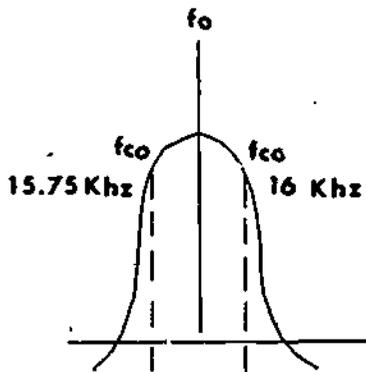
This circuit adjusted to a  $f_o$  of 1250 KHz passes a band of frequencies from 1187.5 KHz to \_\_\_\_\_ KHz.

(1312.5)

25. The circuit discussed in the previous frame shown on a resonance curve graph looks like this:



What is the  $f_0$  of the circuit represented by the graph below?



$f_0 =$  \_\_\_\_\_

(15.875KHz)

26. Fill in the following spaces.

1. A series RLC circuit, in which  $X_L = X_C$ , has total reactance of \_\_\_\_\_.

2. If there is zero reactance at resonance, then circuit impedance is equal to the circuit \_\_\_\_\_.

3. The equation for finding the resonant frequency is

$$f_o = \frac{1}{2\pi \underline{\quad}}$$

4. In a series RLC circuit at resonance, the phase angle between source voltage and current is \_\_\_\_\_.

5. If the  $Q$  of a series RLC circuit is increased, the resonance curve is \_\_\_\_\_; bandwidth is \_\_\_\_\_; and selectivity is \_\_\_\_\_.

steeper/flatter                      wider/narrower  
higher/lower

---

(THIS IS A TEST FRAME. COMPARE YOUR ANSWERS WITH THE CORRECT ANSWERS GIVEN ON THE TOP OF THE NEXT PAGE.)

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ANSWERS - TEST FRAME 26

1. zero; 2. resistance; 3.  $\sqrt{LC}$ ; 4. zero;
  5. steeper, narrower, higher
- 

IF ANY OF YOUR ANSWERS IN INCORRECT, GO BACK TO FRAME 18 AND TAKE THE PROGRAMMED SEQUENCE.

IF YOUR ANSWERS ARE CORRECT, YOU MAY TAKE THE PROGRESS CHECK OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

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SUMMARY  
LESSON III

Resonance in Series AC Circuits

You have already studied resonant circuits. You know that resonance occurs when  $X_L$  equals  $X_C$  in RCL series circuits.

In this lesson, we will take a resonant circuit and vary the frequency above resonance and below resonance to observe the effects of the circuit.

Recall that a circuit at resonance offers minimum impedance to current flow -- total impedance is equal to the value of circuit resistance, so maximum current flows. If frequency is increased above resonance,  $X_L$  increases and total impedance increases as it is the vector sum of  $X_L$  and  $R$ . Circuit current decreases as the frequency increases, and the circuit becomes more inductive.

If we go the other direction from resonance and decrease frequency, the value of  $X_C$  increases, again increasing total impedance and decreasing circuit current. The more the frequency is decreased, the more capacitive reactance increases and the more capacitive the circuit appears. The series RCL circuit has a usable output across the resistive component through a central frequency range. The extent of this range is dependent upon the quality ( $Q$ ) of the circuit. The lower end of this range is called the low-frequency cutoff point and the upper or high end is called the high-frequency cutoff frequency. The range of frequencies between the low  $f_{co}$  point and the high  $f_{co}$  point is called the circuit bandwidth. Resonant frequency is the center frequency. This range of frequencies that provides a usable output explains why you can hear your favorite radio station around the exact station frequency. When you tune the station in loud and clear, you have adjusted the circuit to resonate at the station frequency. As you adjust above or below this point, the reception gets progressively worse.

The bandwidth of the circuit is dependent upon the circuit  $Q$ .

Remember,  $Q$  is determined by the value of  $\frac{X_L}{R_{eff}}$ .  $f_o$  is determined by  $\frac{1}{2\pi\sqrt{LC}}$ . Bandwidth (BW) is determined by dividing  $f_o$  by the circuit  $Q$ ,  $BW = \frac{f_o}{Q}$ .

---

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

BASIC ELECTRICITY AND ELECTRONICS  
INDIVIDUALIZED LEARNING SYSTEM



MODULE THIRTEEN  
LESSON IV

Experiments with Resonance in a Series R<sub>L</sub> Circuit

Study Booklet

## OVERVIEW

## LESSON IV

Experiments With Resonance in a Series RCL Circuit

In this lesson you will perform experiments with a series RCL circuit, using various equipment to see the effects of resonance that you have learned in the previous lessons.

You will use:

- an audio signal generator.
- NEAT board 6.
- a multimeter.

You will:

- determine  $f_0$ .
- read voltage drops at  $f_0$ .
- see the effects of varying capacitance.

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON IV

Experiments With Resonance in a Series RCL Circuit

Since this lesson consists of experiments, there is only the narrative.  
There are no other study resources and no progress check.

TURN THE PAGE AND-BEGIN-THE NARRATIVE.

## NARRATIVE

## LESSON IV

Experiments With Resonance in a Series RCL Circuit

Now that you understand the theory of resonance in an AC series RCL circuit, you will have the opportunity to see these concepts at work.

For this lesson, go to the materials center and get any of the following which you do not have in your carrel.

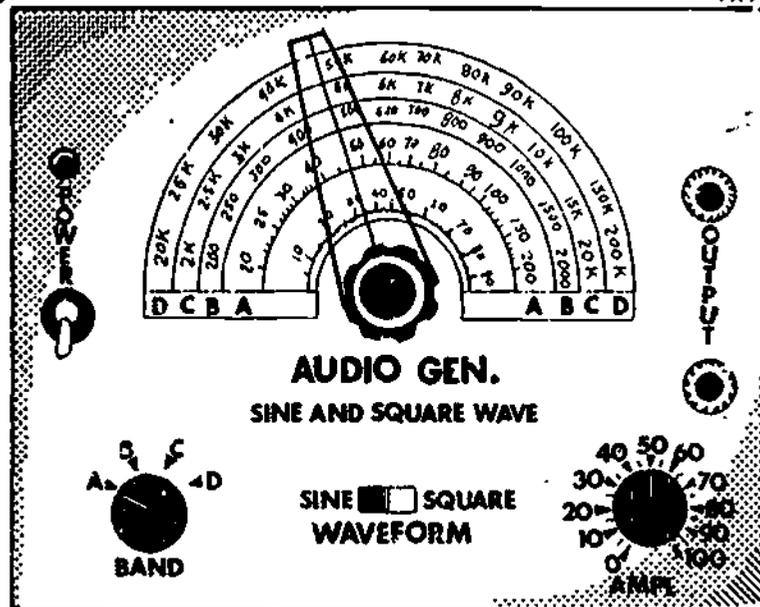
1. an audio signal generator
2. NEAT Board 6 for series resonance
3. a multimeter
4. two sets of test leads

The Audio Signal Generator

The signal generator plugs into your AC wall source and supplies to the NEAT board a controlled signal at any frequency which you determine.

This signal generator can supply a range of frequencies between 20 and 20,000 hertz by generating a sine wave. It is also capable of generating square waves  in a frequency range of 60 to 30,000 hertz. The entire frequency spectrum is covered in four ranges for maximum accuracy and readability. For our use in this lesson we will be concerned with only the C scale -- the 2 KHz to 20 KHz range. (Observe the range scale in the center of the generator.)

The output voltage is obtained from the two terminal jacks labeled OUTPUT and located on the right side of the front panel. Study the illustration.



Listed below are the controls of the audio signal generator. Locate these controls on the generator.

1. Power on-off toggle switch (left side of the front panel)
2. Output jacks (right side)
3. Band selector (lower left) to set at A, B, C or D
4. Amplitude Knob (lower right) determines amplitude of output voltage
5. Waveform switch (center bottom) selects sine or square wave
6. Frequency control (center, below range dial) selects output frequency.

One word of caution. DO NOT SHORT OUTPUT TERMINALS. Shorting the output terminals would result in a blown fuse and your signal generator would not operate. (At this point, set the amplitude knob to 0, plug the signal generator into the AC outlet, and turn on the power switch. This is to allow warm-up of the generator while you read the rest of the instructions.)

#### The NEAT Board

This NEAT board is designed for experimental purposes, and has a series AC circuit built into it. As indicated by the schematic on the front panel, this NEAT board contains a coil and a capacitor. It also has a resistor that can be put in series with the resistive components, depending on the position of S-603.

Notice there are two capacitors, one of a fixed value, and the other a variable capacitor with a knob to vary the amount of

capacitance. The position of S-604 determines which capacitor is included in the circuit at a particular time.

Also observe that S-602 will need to be open if the ammeter is connected in the circuit at the location shown on the schematic. If the ammeter is not in the circuit at that point, S-602 will need to be closed to complete the circuit.

### Experiment 1 - Determining $f_o$

Follow these instructions step by step.

1. Connect one set of test leads to the output jacks of the signal generator. The bottom jack is common (ground).
2. Insert the other end of the leads into terminal points (T) for input to the neat board at T<sub>p</sub> 601-A and T<sub>p</sub> 602-A (common).
3. Turn band selector switch on generator to C position.
4. Turn frequency control fully counterclockwise.
5. Set waveform switch at SINE.
6. Set up the neat board as follows:
  - S-601 to ON position (↑)
  - S-602 to OFF position (↑) (open)
  - S-604 to position A (↑) so that the fixed capacitor is in the circuit
  - S-603 to position A (↑) so there is no resistor in the circuit
7. Set up your multimeter to read AC voltage on the 50-volt scale.
8. Insert meter leads in multimeter and in neat board in T<sub>p</sub> 601-B and T<sub>p</sub> 602-C.
9. Now turn the amplitude knob on signal generator until you get a 12-volt reading on the meter. Leave the amplitude knob set in that position.

10. Disconnect the meter leads from the neat board.
11. Set meter to measure DC on the 100 ma range and insert the meter between T 603 (-) and T 604 (+) (NOTE: A rectifier in the neat board changes AC to DC, so the ammeter will measure DC; polarity must be observed.)
12. Draw a schematic of the neat board circuit as it is now set up -- including the ammeter.

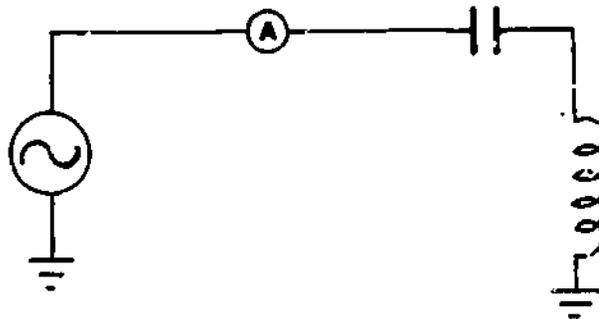
13. Slowly turn the frequency knob clockwise on the signal generator and observe the current reading on the meter.

Determine from the meter reading at what frequency resonance is reached. Record it. You will be reading  $f$  on the C scale.

$$f_o = \underline{\hspace{2cm}}$$

Answers for Experiment 1

(12). Schematic

(13)  $f_o$  is approximately 4.3 KHz when  $I$  is maximum.Experiment 2 -- Reading Voltage Drops at  $f_o$ 

1. Change S-603 to position B. This puts a resistor in series in the circuit.

Observe the  $I$  drops when  $R$  is added.

2. Change S-603 back to position A. You have not changed frequency; it is still at  $f_o$ .
3. Remove ammeter from circuit. (Remember to observe all safety precautions.)
4. Set S-602 to ON position to close circuit.
5. Set meter to read AC voltage on 250 volt scale.
6. Insert meter probes in  $T_p$  604 -  $T_p$  605 to read the voltage drop across the capacitor. Record.  

$$E_C = \underline{\hspace{2cm}}$$
7. Now move probes to  $T_p$  605 -  $T_p$  606. Read the voltage drop across the inductor.  

$$E_L = \underline{\hspace{2cm}}$$

Recall that at  $f_o$ ,  $E_C$  and  $E_L$  are approximately equal.

8. Remove voltmeter probes.

Answers to Experiment 2

(6). Approximately 80 v

(7). Approximately 80 v

Experiment 3 -- Varying Capacitance

1. Set up neat board as follows.

Turn S-602 to OFF (open circuit)

Turn S-604 to B position.

Now the variable capacitor is in the circuit, but the fixed capacitor is not.

Adjust C-601 (variable capacitor knob) to the far left. The capacitor is now at its maximum value.

2. Set up ammeter to read DC current, 100 ma range.

3. Insert meter probes between  $T_p$  603 -  $T_p$  604 (check for proper polarity).

4. Because capacitance has been varied, the circuit will no longer be at  $f_0$ . Now slowly turn frequency knob on signal generator back and forth until you find  $f_0$  as indicated by the ammeter reading.

5. Now decrease the value of capacitance by turning C-601 to the mid-scale mark (straight up).

Observe that as capacitance decreases, current:

a. increases.

b. decreases.

As capacitance decreases  $X_C$ :

a. increases.

b. decreases.

$X_C$  and  $X_L$  are no longer equal, so the circuit is not at  $f_0$ .  
 $X_C$  is greater than  $X_L$  now.

6. Again turn the frequency knob on the signal generator until  $X_C$  and  $X_L$  are equal.

$$f_o = \underline{\hspace{2cm}}$$

7. Turn amplitude dial to 0.
  8. Turn off audio generator power.
  9. Disconnect all leads.
  10. Return the borrowed equipment to the materials center.
- 

#### Answers to Experiment 3

- (4).  $f_o$  = approximately 14 KHz
- (5). As  $C$  decreases,  $I$  decreases.  
As  $C$  decreases,  $X_C$  increases.
- (6).  $f_o$  = approximately 20 KHz

AT THIS POINT SEE YOUR LEARNING SUPERVISOR FOR FURTHER INSTRUCTIONS.

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	0.9998	0.9996	0.9994	0.9992	0.9990	0.9988	0.9986	0.9984	0.9982
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9883	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2273	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2318	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2307	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
14	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
15	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2847
16	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9566
	tan	0.2867	0.2886	0.2905	0.2924	0.2942	0.2962	0.2981	0.3000	0.3019	0.3038
17	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
21	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.9996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
26	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5045	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
28	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
29	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
30	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
31	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
32	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
33	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
34	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
35	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
36	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
37	sin	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
	cos	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7616	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
38	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
39	sin	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
40	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
	cos	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
41	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678
	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0 0°	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
42	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
	cos	0.7421	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
43	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
44	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
45	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
49	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
53	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
55	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
deg	function	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90

deg	func-tion	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°
56	sin	0. 8290	0. 8300	0. 8310	0. 8320	0. 8329	0. 8339	0. 8348	0. 8358	0. 8368	0. 8377
	cos	0. 5592	0. 5577	0. 5563	0. 5548	0. 5534	0. 5519	0. 5505	0. 5470	0. 5476	0. 5461
	tan	1. 4826	1. 4882	1. 4938	1. 4994	1. 5051	1. 5108	1. 5166	1. 5224	1. 5282	1. 5340
57	sin	0. 8387	0. 8396	0. 8406	0. 8415	0. 8425	0. 8434	0. 8443	0. 8453	0. 8462	0. 8471
	cos	0. 5446	0. 5432	0. 5417	0. 5402	0. 5388	0. 5373	0. 5358	0. 5344	0. 5329	0. 5314
	tan	1. 5399	1. 5458	1. 5517	1. 5577	1. 5637	1. 5697	1. 5757	1. 5818	1. 5880	1. 5941
58	sin	0. 8480	0. 8490	0. 8499	0. 8508	0. 8517	0. 8526	0. 8536	0. 8545	0. 8554	0. 8563
	cos	0. 5299	0. 5284	0. 5270	0. 5255	0. 5240	0. 5225	0. 5210	0. 5195	0. 5180	0. 5165
	tan	1. 6003	1. 6066	1. 6128	1. 6191	1. 6255	1. 6319	1. 6383	1. 6447	1. 6512	1. 6577
59	sin	0. 8572	0. 8581	0. 8590	0. 8599	0. 8607	0. 8616	0. 8625	0. 8634	0. 8643	0. 8652
	cos	0. 5150	0. 5135	0. 5120	0. 5105	0. 5090	0. 5075	0. 5060	0. 5045	0. 5030	0. 5015
	tan	1. 6643	1. 6709	1. 6775	1. 6842	1. 6909	1. 6977	1. 7045	1. 7113	1. 7182	1. 7251
60	sin	0. 8660	0. 8669	0. 8678	0. 8686	0. 8695	0. 8704	0. 8712	0. 8721	0. 8729	0. 8738
	cos	0. 5000	0. 4985	0. 4970	0. 4955	0. 4939	0. 4924	0. 4909	0. 4894	0. 4879	0. 4863
	tan	1. 7321	1. 7391	1. 7461	1. 7532	1. 7603	1. 7675	1. 7747	1. 7820	1. 7893	1. 7966
61	sin	0. 8746	0. 8755	0. 8763	0. 8771	0. 8780	0. 8788	0. 8796	0. 8805	0. 8813	0. 8821
	cos	0. 4848	0. 4833	0. 4818	0. 4802	0. 4787	0. 4772	0. 4756	0. 4741	0. 4726	0. 4710
	tan	1. 8040	1. 8115	1. 8190	1. 8265	1. 8341	1. 8418	1. 8495	1. 8572	1. 8650	1. 8728
62	sin	0. 8829	0. 8838	0. 8846	0. 8854	0. 8862	0. 8870	0. 8878	0. 8886	0. 8894	0. 8902
	cos	0. 4695	0. 4679	0. 4664	0. 4648	0. 4633	0. 4617	0. 4602	0. 4586	0. 4571	0. 4555
	tan	1. 8807	1. 8887	1. 8967	1. 9047	1. 9128	1. 9210	1. 9292	1. 9375	1. 9458	1. 9542
63	sin	0. 8910	0. 8918	0. 8926	0. 8934	0. 8942	0. 8949	0. 8957	0. 8965	0. 8973	0. 8980
	cos	0. 4540	0. 4524	0. 4509	0. 4493	0. 4478	0. 4462	0. 4446	0. 4431	0. 4415	0. 4399
	tan	1. 9626	1. 9711	1. 9797	1. 9883	1. 9970	2. 0057	2. 0145	2. 0233	2. 0323	2. 0413
64	sin	0. 8988	0. 8996	0. 9003	0. 9011	0. 9018	0. 9026	0. 9033	0. 9041	0. 9048	0. 9056
	cos	0. 4384	0. 4368	0. 4352	0. 4337	0. 4321	0. 4305	0. 4289	0. 4274	0. 4258	0. 4242
	tan	2. 0503	2. 0594	2. 0686	2. 0778	2. 0872	2. 0965	2. 1060	2. 1155	2. 1251	2. 1348
65	sin	0. 9063	0. 9070	0. 9078	0. 9085	0. 9092	0. 9100	0. 9107	0. 9114	0. 9121	0. 9128
	cos	0. 4226	0. 4210	0. 4195	0. 4179	0. 4163	0. 4147	0. 4131	0. 4115	0. 4099	0. 4083
	tan	2. 1445	2. 1543	2. 1642	2. 1742	2. 1842	2. 1943	2. 2045	2. 2148	2. 2251	2. 2355
66	sin	0. 9135	0. 9143	0. 9150	0. 9157	0. 9164	0. 9171	0. 9178	0. 9184	0. 9191	0. 9198
	cos	0. 4067	0. 4051	0. 4035	0. 4019	0. 4003	0. 3987	0. 3971	0. 3955	0. 3939	0. 3923
	tan	2. 2460	2. 2566	2. 2673	2. 2781	2. 2889	2. 2998	2. 3109	2. 3220	2. 3332	2. 3445
67	sin	0. 9205	0. 9212	0. 9219	0. 9225	0. 9232	0. 9239	0. 9245	0. 9252	0. 9259	0. 9265
	cos	0. 3907	0. 3891	0. 3875	0. 3859	0. 3843	0. 3827	0. 3811	0. 3795	0. 3778	0. 3762
	tan	2. 3559	2. 3673	2. 3787	2. 3906	2. 4023	2. 4142	2. 4262	2. 4383	2. 4504	2. 4627
68	sin	0. 9272	0. 9278	0. 9285	0. 9291	0. 9298	0. 9304	0. 9311	0. 9317	0. 9323	0. 9330
	cos	0. 3746	0. 3730	0. 3714	0. 3697	0. 3681	0. 3665	0. 3649	0. 3633	0. 3616	0. 3600
	tan	2. 4751	2. 4876	2. 5002	2. 5129	2. 5257	2. 5386	2. 5517	2. 5649	2. 5782	2. 5916
69	sin	0. 9336	0. 9342	0. 9348	0. 9354	0. 9361	0. 9367	0. 9373	0. 9379	0. 9385	0. 9391
	cos	0. 3584	0. 3567	0. 3551	0. 3535	0. 3518	0. 3502	0. 3486	0. 3469	0. 3453	0. 3437
	tan	2. 6051	2. 6187	2. 6325	2. 6464	2. 6605	2. 6746	2. 6889	2. 7034	2. 7179	2. 7325
deg	func-tion	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°



deg	func- tion	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°
70	sin	0. 9397	0. 9403	0. 9409	0. 9415	0. 9421	0. 9426	0. 9432	0. 9438	0. 9444	0. 9449
	cos	0. 3420	0. 3404	0. 3387	0. 3371	0. 3355	0. 3338	0. 3322	0. 3305	0. 3289	0. 3272
	tan	2. 7475	2. 7625	2. 7776	2. 7929	2. 8083	2. 8239	2. 8397	2. 8556	2. 8716	2. 8878
71	sin	0. 9455	0. 9461	0. 9466	0. 9472	0. 9478	0. 9483	0. 9489	0. 9494	0. 9500	0. 9505
	cos	0. 3256	0. 3239	0. 3223	0. 3206	0. 3190	0. 3173	0. 3156	0. 3140	0. 3123	0. 3107
	tan	2. 9042	2. 9208	2. 9375	2. 9544	2. 9714	2. 9887	3. 0061	3. 0237	3. 0415	3. 0595
72	sin	0. 9511	0. 9516	0. 9521	0. 9527	0. 9532	0. 9537	0. 9542	0. 9548	0. 9553	0. 9558
	cos	0. 3090	0. 3074	0. 3057	0. 3040	0. 3024	0. 3007	0. 2990	0. 2974	0. 2957	0. 2940
	tan	3. 0777	3. 0961	3. 1146	3. 1334	3. 1524	3. 1716	3. 1910	3. 2106	3. 2305	3. 2506
73	sin	0. 9563	0. 9568	0. 9573	0. 9578	0. 9583	0. 9588	0. 9593	0. 9598	0. 9603	0. 9608
	cos	0. 2924	0. 2907	0. 2890	0. 2874	0. 2857	0. 2840	0. 2823	0. 2807	0. 2790	0. 2773
	tan	3. 2709	3. 2914	3. 3122	3. 3332	3. 3544	3. 3759	3. 3977	3. 4197	3. 4420	3. 4646
74	sin	0. 9613	0. 9617	0. 9622	0. 9627	0. 9632	0. 9636	0. 9641	0. 9646	0. 9650	0. 9655
	cos	0. 2756	0. 2740	0. 2723	0. 2706	0. 2689	0. 2672	0. 2656	0. 2639	0. 2622	0. 2605
	tan	3. 4874	3. 5105	3. 5339	3. 5576	3. 5816	3. 6059	3. 6305	3. 6554	3. 6806	3. 7062
75	sin	0. 9659	0. 9664	0. 9668	0. 9673	0. 9677	0. 9681	0. 9686	0. 9690	0. 9694	0. 9699
	cos	0. 2588	0. 2571	0. 2554	0. 2538	0. 2521	0. 2504	0. 2487	0. 2470	0. 2453	0. 2436
	tan	3. 7321	3. 7583	3. 7848	3. 8118	3. 8391	3. 8667	3. 8947	3. 9232	3. 9520	3. 9812
76	sin	0. 9703	0. 9707	0. 9711	0. 9715	0. 9720	0. 9724	0. 9728	0. 9732	0. 9736	0. 9740
	cos	0. 2419	0. 2402	0. 2385	0. 2368	0. 2351	0. 2334	0. 2317	0. 2300	0. 2284	0. 2267
	tan	4. 0108	4. 0408	4. 0713	4. 1022	4. 1335	4. 1653	4. 1976	4. 2303	4. 2635	4. 2972
77	sin	0. 9744	0. 9748	0. 9751	0. 9755	0. 9759	0. 9763	0. 9767	0. 9770	0. 9774	0. 9778
	cos	0. 2250	0. 2232	0. 2215	0. 2198	0. 2181	0. 2164	0. 2147	0. 2130	0. 2113	0. 2096
	tan	4. 3315	4. 3662	4. 4015	4. 4374	4. 4737	4. 5107	4. 5483	4. 5864	4. 6252	4. 6646
78	sin	0. 9781	0. 9785	0. 9789	0. 9792	0. 9796	0. 9799	0. 9803	0. 9806	0. 9810	0. 9813
	cos	0. 2079	0. 2062	0. 2045	0. 2028	0. 2011	0. 1994	0. 1977	0. 1959	0. 1942	0. 1925
	tan	4. 7046	4. 7453	4. 7867	4. 8288	4. 8716	4. 9152	4. 9594	5. 0045	5. 0504	5. 0970
79	sin	0. 9816	0. 9820	0. 9823	0. 9826	0. 9829	0. 9833	0. 9836	0. 9839	0. 9842	0. 9845
	cos	0. 1908	0. 1891	0. 1874	0. 1857	0. 1840	0. 1822	0. 1805	0. 1788	0. 1771	0. 1754
	tan	5. 1446	5. 1929	5. 2422	5. 2924	5. 3435	5. 3955	5. 4486	5. 5026	5. 5578	5. 6140
80	sin	0. 9848	0. 9851	0. 9854	0. 9857	0. 9860	0. 9863	0. 9866	0. 9869	0. 9871	0. 9874
	cos	0. 1736	0. 1719	0. 1702	0. 1685	0. 1668	0. 1650	0. 1633	0. 1616	0. 1599	0. 1582
	tan	5. 6713	5. 7297	5. 7894	5. 8502	5. 9124	5. 9758	6. 0405	6. 1066	6. 1742	6. 2432
81	sin	0. 9877	0. 9880	0. 9882	0. 9885	0. 9888	0. 9890	0. 9893	0. 9895	0. 9898	0. 9900
	cos	0. 1564	0. 1547	0. 1530	0. 1513	0. 1495	0. 1478	0. 1461	0. 1444	0. 1426	0. 1409
	tan	6. 3138	6. 3859	6. 4596	6. 5350	6. 6122	6. 6912	6. 7720	6. 8548	6. 9395	7. 0264
82	sin	0. 9903	0. 9905	0. 9907	0. 9910	0. 9912	0. 9914	0. 9917	0. 9919	0. 9921	0. 9923
	cos	0. 1392	0. 1374	0. 1357	0. 1340	0. 1323	0. 1305	0. 1288	0. 1271	0. 1253	0. 1236
	tan	7. 1554	7. 2066	7. 3002	7. 3962	7. 4947	7. 5958	7. 6996	7. 8062	7. 9158	8. 0285
83	sin	0. 9925	0. 9928	0. 9930	0. 9932	0. 9934	0. 9936	0. 9938	0. 9940	0. 9942	0. 9943
	cos	0. 1219	0. 1201	0. 1184	0. 1167	0. 1149	0. 1132	0. 1115	0. 1097	0. 1080	0. 1063
	tan	8. 1443	8. 2636	8. 3863	8. 5126	8. 6427	8. 7769	8. 9152	9. 0579	9. 2052	9. 3572
deg	func- tion	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°