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ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains exercises on the use of lower brackets, a summery of the problems in the film "Lower and Upper Brackets," and the supplement. (MK)

THE ARITHMETIC PROJECT COURSE FOR TEACHERS

TOPIC: Lower Brackets.

FILM: Lower and Upper Brackets, Grade 4

SUPPLEMENT: Using Centimeter Blocks to Introduce Prime Numbers to a Third Grade

NAME: ....

7

This booklet is part of a course for teachers produced by The Arithmetic Project in association with Education Development Center. Principal financial support has come from the Carnegie Corporation of New York, the University of Illinois, and the National Science Foundation.

The course is available from:

THE ARITHMETIC PROJECT
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LOWER BRACKETS

Although many people feel that lower brackets is most valuable after a class has had other experience with fractions, some teachers find it useful also during the early stages of introducing fractions to children.

Notice that lower brackets is not "rounding off". (In teaching children you will probably not have to give this warning—they are likely not to have the preconception that it might be rounding off!)

Here are some true equations:

$\left\lfloor 5\frac{1}{2} \right\rfloor = 5 \qquad \left\lfloor \frac{1}{10} \right\rfloor = 0 \qquad \left\lfloor \frac{20}{3} \right\rfloor = 6$

$\left\lfloor 5\frac{9}{10} \right\rfloor = 5 \qquad \left\lfloor 1{,}029 \right\rfloor = 1{,}029 \qquad \left\lfloor \frac{21}{7} \right\rfloor = 3$

Now do these:

1. $\left\lfloor 2\frac{6}{11} \right\rfloor =$ __________

2. $\left\lfloor \frac{100}{99} \right\rfloor =$ __________

3. $\left\lfloor 7\frac{1}{100} + 2\frac{1}{100} \right\rfloor =$ __________

4. $\left\lfloor \frac{3}{4} \right\rfloor =$ __________

5. $\left\lfloor 3\frac{1}{5} + 2\frac{4}{5} \right\rfloor$ = __________

6. $\left\lfloor 3\frac{1}{5} \right\rfloor$ = __________

7. $\left\lfloor 2\frac{4}{5} \right\rfloor$ = __________

8. $\left\lfloor 3\frac{1}{5} \right\rfloor + \left\lfloor 2\frac{4}{5} \right\rfloor$ = __________

Note: Your answers for problem 5 and problem 8 should be different.

9. $\left\lfloor \left\lfloor \left\lfloor 147\frac{1}{2} \right\rfloor \right\rfloor \right\rfloor$ = __________

10. $\left\lfloor \left\lfloor 3\frac{1}{5} \right\rfloor + \left\lfloor 2\frac{4}{5} \right\rfloor \right\rfloor$ = __________

11. $\left\lfloor \frac{4,263,849}{4,263,859} \right\rfloor$ = __________

12. $\left\lfloor \frac{9,163,237}{4,263,859} \right\rfloor$ = __________

13. $13\frac{7}{8} - \left\lfloor 13\frac{7}{8} \right\rfloor =$ ________

14. $\square - \left\lfloor 13\frac{7}{8} \right\rfloor = 10\frac{7}{8}$

15. $\square - \left\lfloor 13\frac{97}{100} \right\rfloor = 10\frac{7}{8}$

16. $\left\lfloor 1\frac{6}{7} + 1\frac{10}{11} + 1\frac{97}{100} \right\rfloor =$ ________

17. $\left\lfloor 1\frac{6}{7} \right\rfloor + \left\lfloor 1\frac{10}{11} \right\rfloor + \left\lfloor 1\frac{97}{100} \right\rfloor =$ ________

18. $\left\lfloor 3\frac{10}{13} + 2\frac{19}{20} + 10\frac{96}{97} + \frac{199}{200} \right\rfloor =$ ________

19. $20\frac{3}{5} + \left\lfloor 20\frac{3}{5} \right\rfloor =$ ________

20.* $\square + \left\lfloor \square \right\rfloor = 64\frac{1}{2}$

21. $\square + \left\lfloor \square \right\rfloor = 64\frac{9}{10}$

22. $\square + \left\lfloor \square \right\rfloor = 64\frac{9{,}999}{10{,}000}$

23. $\square + \left\lfloor \square \right\rfloor = 65$

24. $\dfrac{\left\lfloor 3\frac{1}{3} \right\rfloor \times \left\lfloor 4\frac{2}{7} \right\rfloor}{5} =$ ________

* Remember, frames of the same shape in an equation require the same number. E.g., $\boxed{6} + \boxed{6} = 12$

25. $\left\lfloor \dfrac{\left\lfloor 2 \times 3\frac{1}{2} \right\rfloor}{\left\lfloor 2\frac{1}{3} \right\rfloor} \right\rfloor$ = ______

26. $\left\lfloor \dfrac{\left\lfloor 2 \times 5\frac{3}{4} \right\rfloor}{\left\lfloor 1\frac{1}{2} + 1\frac{3}{4} \right\rfloor} \right\rfloor$ = ______

27. $\left\lfloor \dfrac{\left\lfloor 2 \right\rfloor \times \left\lfloor 5\frac{3}{4} \right\rfloor}{\left\lfloor 1\frac{1}{2} \right\rfloor + \left\lfloor 1\frac{3}{4} \right\rfloor} \right\rfloor$ = ______

28. $\left\lfloor \dfrac{235}{100} \right\rfloor$ = ______

29. $\left\lfloor 235 \div 100 \right\rfloor \times 100$ = ______

30. $\left\lfloor 353 \div 100 \right\rfloor \times 100$ = ______

31. $\left\lfloor 6353 \div 100 \right\rfloor \times 100$ = ______

In the problems below, what numbers work? If in doubt, try some.

☆32. $\lfloor \square \div 2 \rfloor = 5$

☆33. $\square \div 2 > \lfloor \square \div 2 \rfloor$

(Note: ">" means "is greater than"; for example, $3 > 2$, and $\frac{16}{3} > 5$, and $-1 > -15$.)

In each of the following expressions use one pair of lower brackets. Place the brackets so that you will get the largest possible answer. Include your answers.

34. $\frac{2}{3} + \frac{2}{3}$

35. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3}$

36. $\frac{1}{2} + \frac{1}{2} + \frac{49}{100}$

37. $\frac{1}{3} + \frac{1}{3} + \frac{33}{100}$

38. $\frac{1}{3} + \frac{1}{3} + \frac{34}{100}$

39. $\frac{1}{2} + \frac{3}{8} + \frac{5}{8}$

40. $\frac{3}{8} + \frac{1}{2} + \frac{5}{8}$

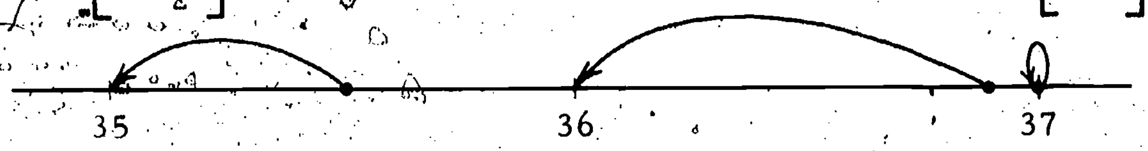
41. $\frac{1}{2} + \frac{5}{8} + \frac{1}{8}$

42. $\frac{3}{4} + \frac{3}{16} + \frac{2}{16} + \frac{1}{2} + \frac{1}{4}$

Epilogue

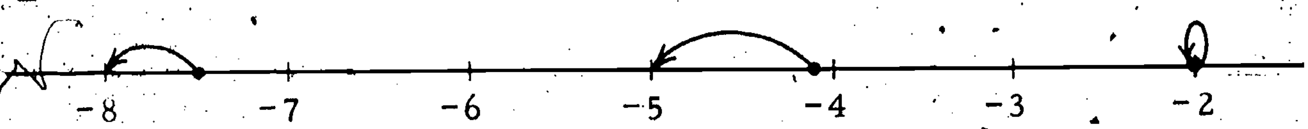
Students who know about number lines often view "lower bracketing" as follows: "If you are on a whole number, stay there. If you are between whole numbers, move to the left until you get to a whole number."

$\left\lfloor 35\frac{1}{2} \right\rfloor = 35$ $\left\lfloor 36\frac{9}{10} \right\rfloor = 36$ $\left\lfloor 37 \right\rfloor = 37$

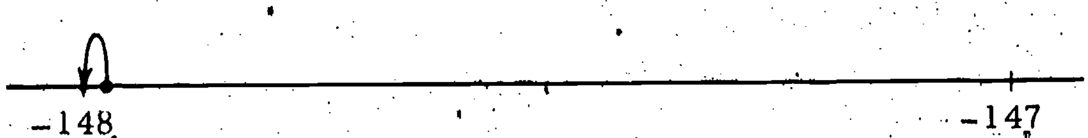


This notion is true for lower brackets with negative numbers also. You still stay on integers and move left when you are between them:

$\left\lfloor -7\frac{1}{2} \right\rfloor = -8$ $\left\lfloor -4\frac{1}{10} \right\rfloor = -5$ $\left\lfloor -2 \right\rfloor = -2$



$$\left\lfloor -147\frac{97}{100} \right\rfloor = -148$$



This conception of moving down the number line will be of great value to you when you give lower brackets problems involving negative numbers. Students usually expect that the answer to $\lfloor -5\frac{2}{3} \rfloor$ will be -5. The number line is probably the best way to lead them to the correct answer, -6.

We recommend strongly, however, that you use lower brackets with positive numbers for quite some time before using them with negative numbers.

* * *

Sometimes adults object to a statement such as

$$\left\lfloor 3\frac{3}{4} \right\rfloor = 3$$

feeling that: "You just can't throw away fractions!" Or: "My students have enough trouble with fractions anyway. If I let them just drop fractions, well... !"

You are not just throwing away fractions. You are following the accepted rule for a specific mathematical function. This function can be fascinating to play with, and it also has practical uses which we haven't dealt with yet.

We want students not to cringe in horror if they see a problem such as $15\frac{8}{17} + 6\frac{9}{11}$, even though the computation would be messy. At least they should be able to examine this problem and say, "Well, it could be done with enough time and care, and I know the answer will be between 22 and 23." This is summed up by

$$\left\lfloor 15\frac{8}{17} + 6\frac{9}{11} \right\rfloor = 22$$

Summary of Problems in the Film "Lower and Upper Brackets"

4th Grade, Ballard School, District #63, Niles, Illinois
Teacher: Mrs. Carol Daniel

While teaching a fourth grade at the Ballard School, Mrs. Daniel attended a twenty-week in-service institute using Project materials. The year after this institute Mrs. Daniel aided in giving an institute for the other teachers in District #63. This film was edited from a videotape made for the latter institute.

$\left\lfloor 4\frac{1}{4} \right\rfloor = 4$

$\left\lfloor 8\frac{1}{2} \right\rfloor = 8$

$\left\lfloor 25\frac{16}{17} \right\rfloor = 25$

$\left\lfloor 100\frac{78}{98} \right\rfloor =$ (100)

$\left\lfloor 76\frac{54}{55} \right\rfloor =$

$\left\lfloor 1,287,687\frac{9}{10} \right\rfloor =$

What does it ($\lfloor \quad \rfloor$) do to a number? ("If there's a fraction it cuts it off.")

$\lfloor 106 \rfloor =$

$\lfloor 94 \rfloor =$

$\left\lfloor \frac{3}{4} \right\rfloor =$ (0)

$\left\lfloor \frac{7}{8} \right\rfloor =$

$\left\lfloor \frac{9}{16} \right\rfloor =$

Give me some other number that I could put in brackets to give an answer of 0. ($\frac{4}{16}$, $\frac{31}{32}$, $\frac{1{,}080}{17}$)

$\left\lfloor \frac{1{,}080}{17} \right\rfloor$ ("It's more than 1, because the top number is more than the bottom number.")

Fix the bottom number. (Answers given: 26 , 1,081)

$\left\lfloor \frac{1}{2} + \frac{1}{2} \right\rfloor =$ (1)

$\left\lfloor 2\frac{1}{2} + 3\frac{1}{2} + 4 \right\rfloor =$ (10)

$\left\lfloor \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \right\rfloor =$ (Answers given: 7, $7\frac{1}{2}$, 1)

$\left\lfloor \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \right\rfloor + \frac{1}{5} =$ (0, $\frac{1}{5}$)

$\left\lfloor \frac{1}{5} \right\rfloor + \left\lfloor \frac{1}{5} \right\rfloor + \left\lfloor \frac{1}{5} \right\rfloor =$ (0)

$\left\lfloor \frac{4}{4} \right\rfloor =$ (0 is given.)

This much would be zero:

$\left\lfloor \frac{3}{4} \right\rfloor = 0$

($\frac{1}{4}$ is given.)

("1, because $\frac{4}{4} = 1$. The brackets is only for the extra.")

$\left\lfloor \frac{5}{4} \right\rfloor =$ (0 is given.)

("1. There's still extras in that one so you have to take the extras out. You just get 1.")

$\left\lfloor \frac{6}{4} \right\rfloor =$	(1)
$\left\lfloor \frac{\square}{4} \right\rfloor = 2$	(8)
I wonder if that's the only answer,	(2 is given.)
	(9, 10, 11)
$\left\lfloor \frac{\square}{4} \right\rfloor = 3$	(12, 13, 14, 15)
How about 16 ?	(No.)
$\left\lfloor 10\frac{1}{8} \right\rfloor =$	
$\left\lfloor \left\lfloor \left\lfloor 10\frac{1}{8} \right\rfloor \right\rfloor \right\rfloor =$	(Answers given: 30, 13, 10)
$\left\lfloor \left\lfloor 10\frac{1}{8} \right\rfloor \right\rfloor + \frac{1}{2}$	(10, $10\frac{1}{2}$)
$\left\lfloor 200 + \left\lfloor \left\lfloor 10\frac{1}{8} \right\rfloor \right\rfloor + \frac{1}{3} + \frac{1}{3} \right\rfloor =$	(210)

For the next two problems, which one gives the smaller number for an answer?

$$\left\lfloor 4\frac{1}{2} + \frac{5}{8} + 10\frac{1}{8} \right\rfloor =$$

$$4\frac{1}{2} + \left\lfloor \frac{5}{8} \right\rfloor + 10\frac{1}{8} =$$

(The top one, the bottom one)

(The answers are computed.)

$\left\lceil 3\frac{1}{5} \right\rceil =$	($\frac{1}{5}$, $3\frac{1}{5}$, are given.)
Teacher writes: $\left\lceil 3\frac{1}{5} \right\rceil = 4$	
$\left\lceil 3\frac{1}{10} \right\rceil$	(4)
$\left\lceil 48\frac{7}{8} \right\rceil$	(49)
$\left\lceil 57 \right\rceil$	(Answers given: 6, 56, 58, 57)
$\left\lceil 69\frac{99}{100} \right\rceil$	(69, 70)
$\left\lceil \frac{3}{3} \right\rceil$	
$\left\lceil \frac{4}{3} \right\rceil$	(1 and 2 are given.)
$\left\lceil \frac{5}{3} \right\rceil$	(2)
$\left\lceil \frac{7}{3} \right\rceil$	(3)
$\left\lceil 4\frac{1}{5} \right\rceil - 4 =$	(1)
$\left\lceil 4\frac{1}{5} \right\rceil - \left\lfloor 4\frac{1}{5} \right\rfloor =$	($\frac{4}{5}$, 1)
$\left\lceil 781,968\frac{3}{4} \right\rceil - \left\lfloor 781,968\frac{3}{4} \right\rfloor =$	(1)
How did you do it so fast?	("Even though the number is long, the same rule still exists as it did in the problem that you just did before.")

$$\lceil \square \rceil - \lfloor \square \rfloor = 0$$

("Could it be any number?"

$\lceil \boxed{2} \rceil - \lfloor \boxed{1} \rfloor = 0$ is given, then rejected.

1 and 0 are given.,

"Put the same number that doesn't have a fraction into the two boxes."

2 below zero is given.)

I think that we finally agree that any whole number works as long as it doesn't have a fraction.

$$\lceil \square \rceil - \lfloor \square \rfloor = 2$$

Supplement

Using Centimeter Blocks to Introduce Prime Numbers to a Third Grade

This is a description of seven consecutive classes during which a third grade was introduced to centimeter blocks* and began to pursue the notion of primeness of numbers.

The first two days' work is included for the benefit of people who would like to know just how the blocks had been introduced to the class. Those who are familiar with using blocks may want to start on page 15, where the teacher begins work with primes.

The teacher was Miss Jennifer Abraham; the class was a third grade at the Browne School, Watertown, Massachusetts. The observer was Mrs. Sally Agro.

First Day

Teacher: Untie your bags. Without looking inside, reach into the bag and pick out the smallest block you can find.

Most children picked out the white block quickly after the first few held theirs up in the air.

Teacher: Now find a block in your bag which is twice as long as the white one. Hold it up in the air. You may measure to see if the block you are holding is the same length as two white ones. Find one which is three times as long as the white one. Now get the one that is four times the length of the white one. Continue building the staircase which we have started. You may make it as high as you can build it.

Now you can put extra blocks together to get higher than 10. Hold up the block that is as long as 10 white ones.

Take the 10 block and find how many ways you can build the 10 block using only two blocks each time.

*Centimeter blocks of the sort used in these classes are available from the Cuisenaire Company of America, Inc., 12 Church St., New Rochelle, N. Y., or from South West Imports, Ltd., P. O. Box 4071, Sta. D., Vancouver, B.C., Canada.

The children worked at this for a while. Then they compiled the results on the blackboard:

1 and 9	(a white and a blue)
2 and 8	(a red and a brown)
3 and 7	(a light green and a black)
4 and 6	(a pink and a dark green)
5 and 5	(a yellow and a yellow)

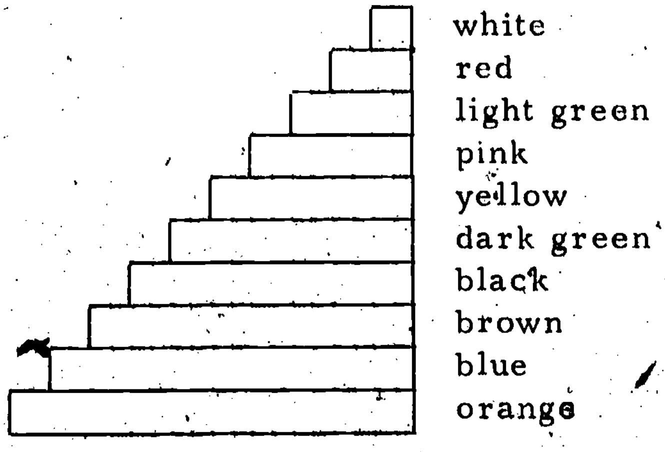
A few children offered suggestions such as 8 and 2 (a brown and a red). The others pointed out that a brown and a red was really the same thing as a red and a brown, which was already listed.

Second Day

The class again looked for the smallest block and the next-smallest block in their bags. They then built a staircase. The teacher asked the class to do things such as "Hold up the block that is as long as 6 white blocks."

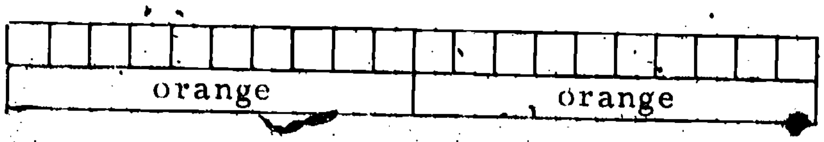
The class was given a problem: "Take a blue block, a dark green block, and a yellow block and make a train. How many white blocks long is the train?" Common answers were 20, 17, and 22. The teacher asked for other ways to measure besides with the white blocks. Class suggestions were "two yellow blocks and an orange in the middle" and "five pink blocks". The teacher asked, "What would be the smallest number of blocks we could use to measure?"

The next problem was: "How many white blocks would it take to measure a blue block, a black, and an orange?" The class volunteered the following ways to measure: 5 yellows, 1 white; 2 oranges, 1 dark green; 5 light greens, 1 pink, 3 reds, and 1 white.



Third Day

Teacher: Take out two orange blocks and line them up to make a train. Now let's build some more trains that are this long, but each time you do it, use only blocks that are all the same color. How many ways can you do it? One way we can do it is with 20 white blocks. What other ways are there?



The children were all very busy with the blocks. They worked at this problem for about 12 minutes. Many children tried to build 20 with every different color.

Sally: 4 yellows.

Diana: 5 pinks.

Susan: 10 reds.

Liz: 20 whites.

Terry: 6 light greens.

Teacher: How many found 6 light greens?

The class tested this and decided that it didn't work.

This list was put on the board:

Building 20

4 yellows	4 fives
5 pinks	5 fours
10 reds	10 twos
2 oranges	2 tens

Teacher: Now let's see how many ways we can build 24 using only blocks of the same color. What shall we use to measure?

Scott: We could use 2 oranges and a pink to measure.

This time the children took about 8 to 10 minutes to work the problem at their desks.

This chart was then put on the board:

<u>Building 24</u>

6 pinks	6 fours
4 dark greens	4 sixes
3 browns	3 eights
12 reds	12 twos
8 light greens	8 threes
24 whites	24 ones

Liz: If we had a twelve block, we could build 24 with 2 twelve blocks.

Teacher: Now, how many ways can we build 12?

David: We can measure it with an orange and a red.

They worked about five minutes at their desks on this problem.

<u>Building 12</u>

6 reds	6 twos
4 light greens	4 threes
12 whites	12 ones
3 pinks	3 fours
2 dark greens	2 sixes

Teacher: What is the biggest block you can use, following our rule, and still build 12?

David: 6, because $6 + 6 = 12$, and $7 + 7 = 14$, and that's too big.

Liz: But if we had a twelve block, that would be the biggest block we could use to build 12.

Fourth Day

The class began by building 8 as many ways as possible. Penny quickly found that 4 reds or 8 whites would do it. Here is the table that was recorded on the board:

Building 8

2 pinks	2 fours
8 whites	8 ones
4 reds	4 twos
1 brown	1 eight

Then the class worked at building 16. Suggestions for ways to measure 16 were "a ten and a six" and "two eights". By this time the children were very quick to find most possibilities. The class compiled the following results on the board:

Building 16

16 whites	16 ones
8 reds	8 twos
4 pinks	4 fours
2 browns	2 eights

Teacher: Are there any others that make 16?

Bill: If there was a 16 block, we could build it with one 16.

Paul: An orange and a dark green.

Linda: You have to have them all the same color.

Teacher: Build 17, still following our rule. How could we measure it?

Caroline: Two eights and a one, or a ten and a seven.

The children worked for several minutes on this problem. They tried lots of blocks. The blocks they tried first were the ones near half of 17 (eights and nines), or the threes and fours. After many unsuccessful trials by themselves, they began to have great interest in what their neighbors were doing. They were quite puzzled. Finally the teacher suggested that they share their answers on the board. They offered "17 whites" and "a 17 block if there were such a block," and then several announced, "That's all." The teacher asked why these were the only ways. Why was 17 acting this way? A child suggested that we can build 17 only two ways because it is an odd number. The class generally agreed.

Teacher: Okay, let's build 15, using our rule. Fifteen is odd, too, so it ought to act the way 17 did.

The children worked at this for a few minutes. The observer was impressed with how rapidly Penny was working the problems. She had found 5 light greens and 3 yellows within half a minute. Answers were shared on the board.

Building 15

3 yellows	3 fives
5 light greens	5 threes
15 whites	15 ones

A few children decided immediately that 15 would act the way 17 did, and so they hadn't bothered to try building it any other way. When the first suggestion came on the board, they looked startled and promptly checked it at their seats. Then they started working furiously to catch up. One child suggested that 17 can't be broken evenly, but 15 can. The teacher suggested that they try to predict whether other numbers can or cannot be broken into equal parts.

Numbers that can be broken into equal parts

8	16
10	12
15	18
20	

But when 21 was proposed, the class disagreed as to whether or not it could be broken into equal parts.

Teacher: Let's build 21 and see.

They worked at this and came up with these solutions:

Building 21

21 whites	21 ones
3 blacks	3 sevens
7 light greens	7 threes

Teacher: So there are lots of ways to build 21. Tomorrow we will see if there are any other numbers that act the way 17 did. If you have time tonight to think about it, see if you can find some numbers that we should try out tomorrow.

Fifth Day

During this class period the children tried to find other numbers that acted like 17. They built 7, and the suggestion immediately came forth that perhaps all numbers that ended with 7 were the ones that you can build only with ones and with themselves. They tested this theory by trying to build 27 with blocks other than ones or an imaginary 27 block. They found that they could build it with threes and with nines, so this theory was abandoned. Then they tried 11. At this point the teacher introduced the term "prime"—it is a number that one can build only with white blocks or with "itself" if one has such a block.* Then they sorted the numbers from 3 to 10 into primes and non-primes.

*The standard definition of a prime says that a positive whole number is prime if and only if it has exactly two positive whole numbers as factors (itself, and 1). If you exclude 1, and are allowed to imagine centimeter blocks longer than the orange block, the definition given by the teacher is quite adequate as a beginning. For example, 121 is not prime, and all you need is a train of eleven imaginary 11-centimeter blocks to show this—not too great a problem for a third grade that has gone this far.

Sixth Day

The class first built 10 very quickly. Then the teacher proposed building 36. A student suggested measuring 36 with 3 tens and 1 six. Much confusion arose from this problem, mainly because the children didn't have enough of any one color block to make it as long as 36 whites. One interesting result, however, was that they were forced to improvise. Some took only a few of one color, and, having checked to see that it would work if they had enough blocks, left the unfinished train as their answer. Others felt the need to get something that was 36 long and completed rows with another color, saying things such as, "I don't have enough threes to make 36, so I'm filling up my 3-row with sixes, because each of them is like 2 threes." Others decided to share blocks and build things together. The teacher heard many comments such as, "You give me all your twos, and I'll build us a 2-row; and I'll give you my fours, and you try to build us a 4-row." After about five minutes, they compiled their answers on the board.

Building 36

9 pinks	9 fours
4 blues	4 nines
18 reds	18 twos
12 light greens	12 threes
36 whites	36 ones
1 ∿∿∿	1 thirty-six*
6 dark greens	6 sixes
2 ≈	2 eighteens
3 ≋	3 twelves

Teacher: Now, let's build 13, still using our rule.

Building 13

Susan: 13 whites 13 ones

Bill: The end.

Teacher: The end?

Bill: Yes, the end. There are no more ways.

Tommy: It's a prime.

David: But a 13 block would do it.

Joanne: Maybe 13 is like 17? Maybe it's all of the odd teens that are prime.

*A child decided that ∿∿ (a squiggle) should stand for the color of the imaginary thirty-six block.

John:	If there was a $6\frac{1}{2}$ block, you could use them.
Teacher:	No fair using fractional blocks.
Scott:	Maybe all the odds are primes.

There was general confusion for a moment. It seemed that by this time the children really wanted to impose a pattern on the primes and felt frustrated by the unpredictability of these numbers. The teacher suggested that they try other numbers. They tried 5 and decided it was prime. The class decided to list on the board the prime numbers they knew about.

Primes

13
5
17
7

Someone suggested 12, but the class disagreed. Someone suggested that perhaps all odd teens were prime. These were put on the board: 13, 15, 17, 19. John said 11 is a teen, too, even though it doesn't sound like it, so 11 was added to the list. Then Joanne became confused and started naming even teens, too. The class caught her mistake. Caroline offered some odd numbers that were less than 10: 1, 3, 7, 5, 9. By this time there were lots of numbers on the board under the "prime" list.

Primes

13	15	1
5	19	3
17	11	9
7		

The teacher asked if they were sure that all of these numbers were prime and could be build only with ones and themselves. Everyone agreed, but suddenly Marie jumped up and exclaimed, "15 isn't a prime because it can be built with 3 yellows." The other children told her she was wrong and set out to prove it. Suddenly they found out she was right. They then tried building 15 other ways. (It is interesting to note that they had already built 15 a few days before.)

Building 15

3 yellows	3 fives
5 light greens	5 threes
15 whites	15 ones

Donald:	That's all.
Marie:	1 "squiggle" 1 fifteen
Liz:	We have to take 15 off the list.
Dicky:	Take 9 off the list, too.
Teacher:	Are you sure that 11 and 19 are primes?

There was general agreement.

Scott:	There's an even number that's prime — 2.
Bill:	Two is the only even number that's prime.
Teacher:	Are there any other even numbers that are prime?
Caroline:	19 has to come off the list because 3 dark greens [sixes] are 19.

The class disagreed. Someone helped her to recount, and she saw the mistake.

Marie:	Isn't 12 a prime?
Terry:	Two sixes make 12.
John:	Isn't 4 an even prime?
Tommy:	No, you can build it with 2 twos.
Teacher:	Is 8 prime or not prime?
Children:	Not prime.

They offered only 4 twos and 2 fours as proof. By this time some students saw that they didn't have to keep saying "whites" or "itself".

John:	I think Scott is right that 2 is the only even prime.
Scott:	I know I'm right because if 4 isn't prime, than no other number above 4 can be prime. [He meant "no other even number. . ."].

Seventh Day

The class built 20 first. They measured with 2 tens. They found the following ways to build 20: 5 pinks, 20 whites, 4 yellows, 10 reds, 2 oranges, and 1 ∿. (∿ stands for the twenty block.)

Then the teacher suggested 21. One of the students decided that 21 was a prime and didn't test anything. The teacher urged him to try some of the blocks just to be sure. The class listed these ways on the board: 3 blacks, 7 light greens, 21 whites. Caroline said, "Two elevens if there were elevens." Scott pointed out that 11 + 11 = 22. Other suggestions were 1 twenty-one block and 2 ten-and-one-half blocks.

Next the class worked on building 19. They found 19 whites and 1 "nineteen block." The class agreed that 19 is a prime.

A worksheet on prime numbers was given out to the children. After they finished it, the class checked their papers. Some of the numbers that they listed as primes were 17, 19, 5, 1, 3, 11, 2. The worksheet is reproduced on page 24.

(To follow this up, a class may wish to make a permanent list of all the primes under 50 or under 100.)

During the seven day period, the class was given a quiz on using blocks, a copy of which appears on pages 25 and 26.

Name: ____________

Write "prime" beside the numbers that are prime. You may test with your blocks if you need to:

12 ____________

9 ____________

13 ____________

25 ____________

100 ____________

7 ____________

Name two other prime numbers:

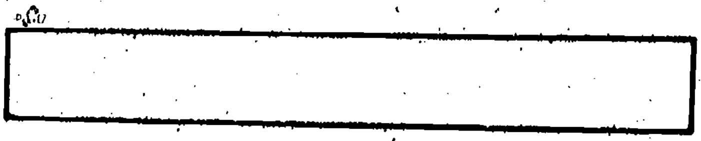
________ and ________.

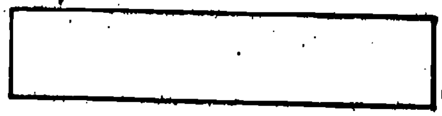
Name: ____________

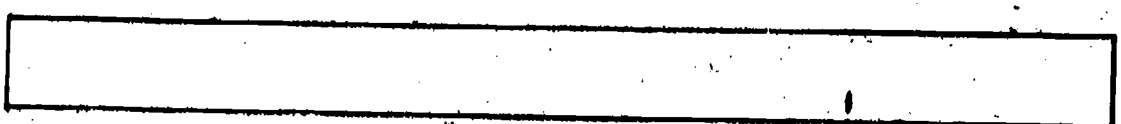
Using Blocks

This is a "1" block: 

What blocks do you think these are?

①  ______

②  ______

③  ______

What blocks did you use to measure this one? ____________________

Questions:

① If, you have a blue block, a green block, and a yellow block, how many white blocks would this be? ________

② How long is a black block, a brown block, and a white block all together? ________

③ How many white blocks are in 15 red ones? ______
How many orange blocks are in 15 red ones? ______

What are 3 ways of building 18 using the same color blocks each time?

1.

2.

3.

* Following our rule, what is the biggest block you can use to build 18? ________