DOCUMENT RESUME

ED 187 587 SE 031 106

TITLE	The Arithmetic Project Course for Teachers - 6. Topics: Frame Equations, Midpoints, Some Wrong Answers, Absolute Value. Supplement: Ways to Find How Many.
INSTITUTION	Education Development Center, Inc., Newton, Mass.; Illinois Univ., Urbana.
SPONS AGENCY	Carnegie Corp. of New York, N.Y.; National Science Foundation, Washington, D.C.
PUB DATE	73
NOTE	18p.; For related documents, see SE 031 100-120. Contains occasional light type.
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Algorithms; Elementary Education; *Elementary School Mathematics; *Elementary School Teachers; Films; Grade 2; Inservice Education; *Inservice Teacher Education; Mathematics Curriculum; Mathematics Instruction; Mathematics Teachers; *Number Concepts; *Problem Sets; Teacher Education
IDENTIFIERS	*University of Illinois Arithmetic Project

ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains: exercises on frame equations, midpoints, some wrong answers, and absolute value; a summary of the problems in the film "Counting with Dots"; and the supplement. (MK)

THE ARITHMETIC PROJECT COURSE FOR TEACHERS

TOPICS: Frame Equations. Midpoints. Some Wrong Answers. Absolute Value.

FILM: Counting With Dots, Grade 2

SUPPLEMENT: Ways to Find How Many

6

NAME: .....

This booklet is part of a course for teachers produced by The Arithmetic Project in association with Education Development Center. Principal financial support has come from the Carnegie Corporation of New York, the University of Illinois, and the National Science Foundation.

The course is available from:

THE ARITHMETIC PROJECT
Education Development Center
55 Chapel Street
Newton, Massachusetts 02160

BOOK SIX

I.

1. $\square + \square + \square = 1{,}031 + \square$

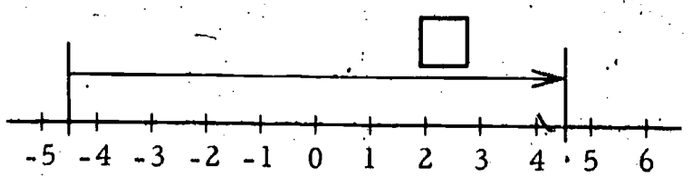
2. $\square + \square + \square + \square = \square + \square + \frac{1}{3}$

3. $3 + \square = 3$

4. $-401 + \square = 401$

5. $3 + \bigcirc + 2 = 3$

6. $-4\frac{1}{2} + \square = 4\frac{1}{2}$



7. $-3\frac{1}{2} + \square = 4\frac{1}{2}$

8. $-5\frac{1}{3} + \triangle = 3\frac{2}{3}$

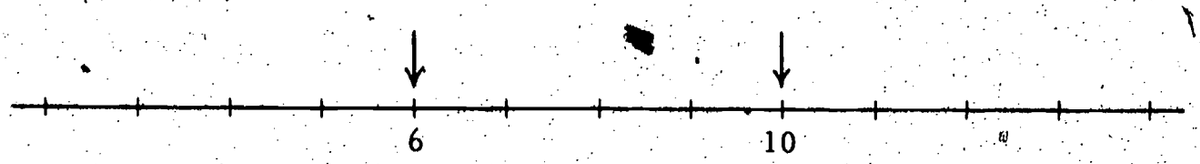
9. $-1\frac{7}{8} + \triangle = 2\frac{1}{8}$

10. $60 + 2 \times (\square + \square) = 9 + 4 \times \square + 51$

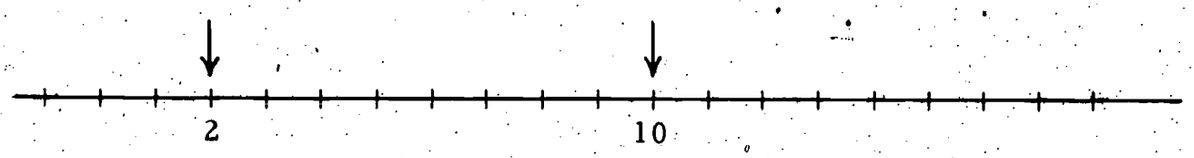
11. $52 + 2 \times (\square + \square) + 8 = 9 + 4 \times \square + 41$

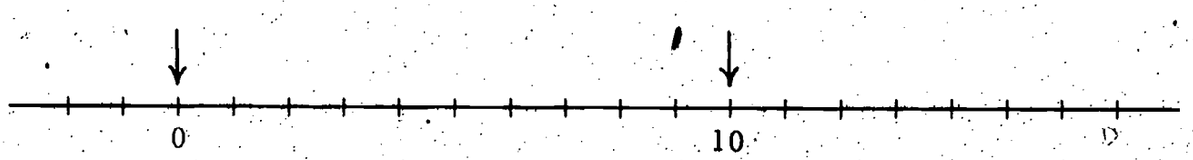
II.

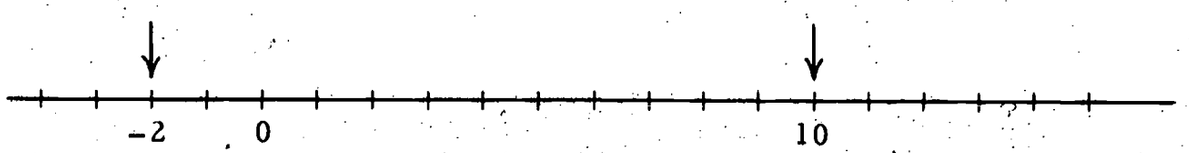
Find the point halfway between these two points:

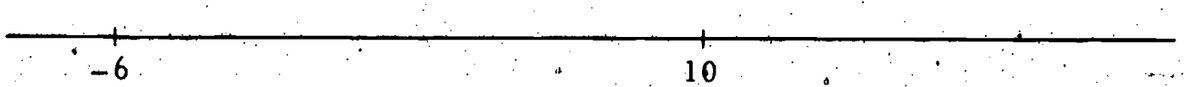
1. 

(For each problem, mark the halfway point on the number line, and tell what number it is.)

2. 

3. 

4. 

5. 

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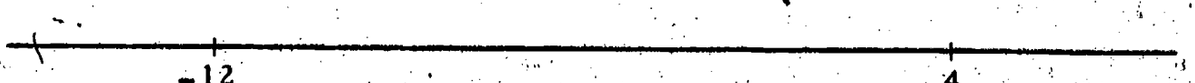
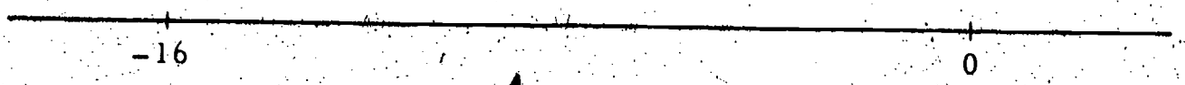
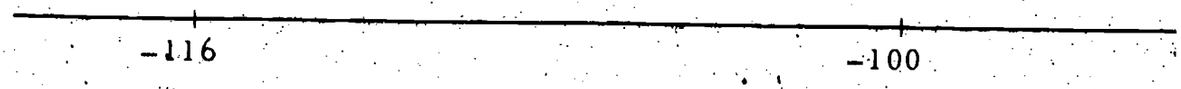
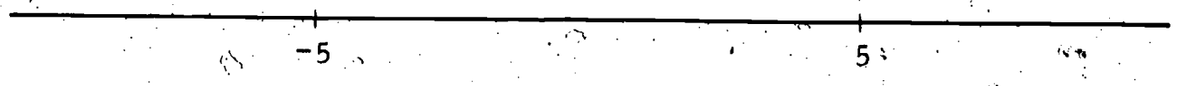
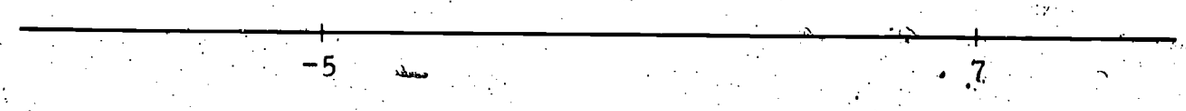
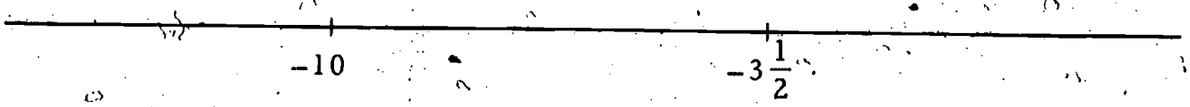
7. 

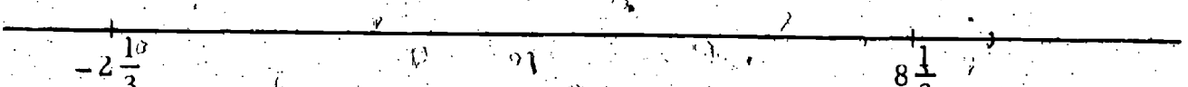
8. 

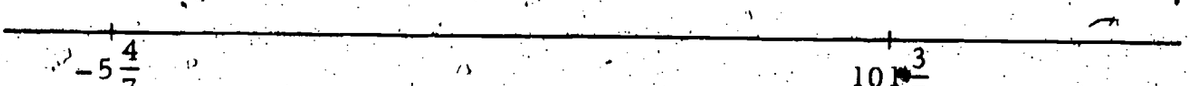
9. 

10. 

11. 

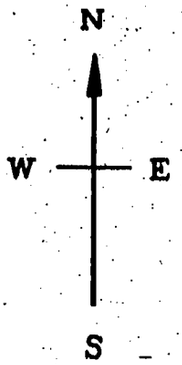
12. 

13. 

☆14. 

☆15. The most likely wrong answer to problem ☆14 is $53\frac{1}{2}$. Why might this be?

☆16.



(a) Point A goes west at 5 units per second, and at the same time point B goes west at 5 units per second. In what direction and how fast does the midpoint (halfway point) of AB move?

Midpoint moves _____ at _____ units per second.

(b) Point A goes west at 5 units per second.
Point B goes east at 5 units per second.

Midpoint moves _____ at _____ units per second.

(c) A goes west at 5 units per second.
B stands still.

Midpoint moves _____ at _____ units per second.

(d) A stands still.
The midpoint moves east $8\frac{1}{2}$ units per second.

B must be moving _____ at _____ units per second.

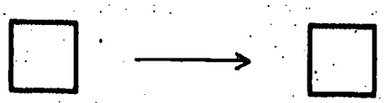
(e) A goes west at 5 units per second.
B goes east at 12 units per second.

Midpoint goes _____ at _____ units per second.

(f) If A stood still, what motion of point B would make the midpoint move in the same direction and at the same rate as the midpoint in problem (e)?

B must travel _____ at _____ units per second.

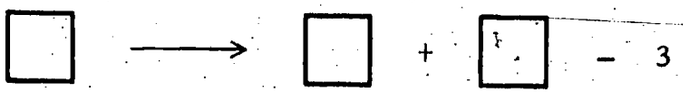
III. SOME WRONG ANSWERS

1. Rule:  + 5

Teacher: Start at 19. Make two jumps one right after the other. You land on what number?

Student: 21.

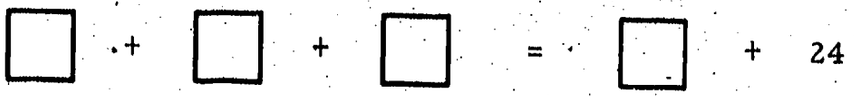
What was the student probably thinking?

2. Rule:  − 3

Teacher: Start at 4. (The teacher writes 4 in each box.) Where do you land?

Student: 9.

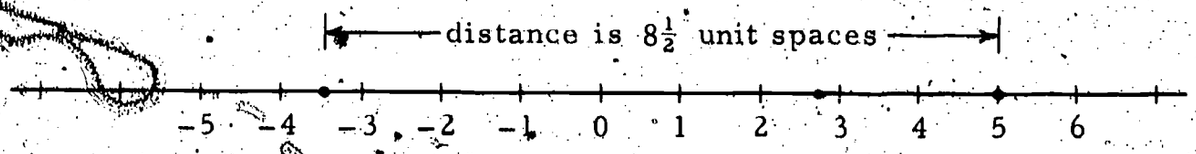
How did the student probably obtain this wrong answer?

3. Problem:  + 24

What is the most likely wrong answer for a student new to this kind of work? _____

IV. ABSOLUTE VALUE

Suppose you want to know the distance between 5 and $-3\frac{1}{2}$. You can plot these numbers on a number line and count the unit spaces between them.



But if you were working lots of such problems, you might soon observe that subtraction is helpful in finding distances on a number line. Notice:

$$5 - (-3\tfrac{1}{2}) = 8\tfrac{1}{2}$$

and $$-3\tfrac{1}{2} - 5 = -8\tfrac{1}{2}$$

In order to be sure that our distance is positive no matter which way we subtract, we will write:

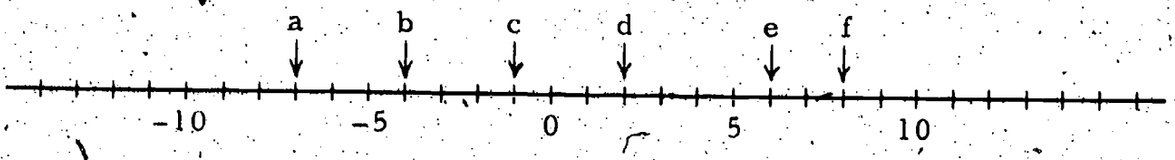
$$\left| 5 - (-3\tfrac{1}{2}) \right| = \left| 8\tfrac{1}{2} \right| = 8\tfrac{1}{2}$$

and $$\left| -3\tfrac{1}{2} - 5 \right| = \left| -8\tfrac{1}{2} \right| = 8\tfrac{1}{2}$$

The bars are called absolute value bars and $|\ 32\ |$ is read "the absolute value of 32".

The absolute value of a number gives its distance from 0. So $|\ -7\ | = 7$. What other number has an absolute value of 7 ? _____

$|d - e|$ is the distance between d and e, which the number line shows is 4. So $|d - e| = 4$.



Now try these:

1.	$\lvert d - c \rvert = 3$	☐ True	☐ False
2.	$\lvert c - d \rvert = -3$	☐ True	☐ False
3.	$\lvert c - b \rvert = 3$	☐ True	☐ False
4.	$\lvert a - c \rvert = 6$	☐ True	☐ False
5.	$\lvert f - d \rvert = 5$	☐ True	☐ False
6.	$\lvert e - f \rvert = \lvert f - e \rvert$	☐ True	☐ False

V.

1. $\lvert 3 - \frac{1}{2} \rvert =$ _____ (Your answer should be the number of spaces between 3 and $\frac{1}{2}$.)
2. $\lvert \frac{1}{2} - 3 \rvert =$ _____
3. $\lvert 3 - (-\frac{1}{2}) \rvert =$ _____
4. $\lvert 3 + \frac{1}{2} \rvert =$ _____
5. $\lvert 100 - 35 \rvert =$ _____
6. $\lvert 100 - (-35) \rvert =$ _____
7. $15 - \lvert -16 \rvert =$ _____
8. $\big\lvert \lvert -17 \rvert - \lvert -20 \rvert \big\rvert =$ _____

In the following problems, tell all the numbers that work:

9. $\square = |\square|$

10. $|\square| = |\,|\square|\,|$

11. $|\square - 3| = 5$

Note that viewed in terms of distance this says that the distance between $\square$ and 3 should be 5. So you want to find points on the line that are 5 units away from 3.

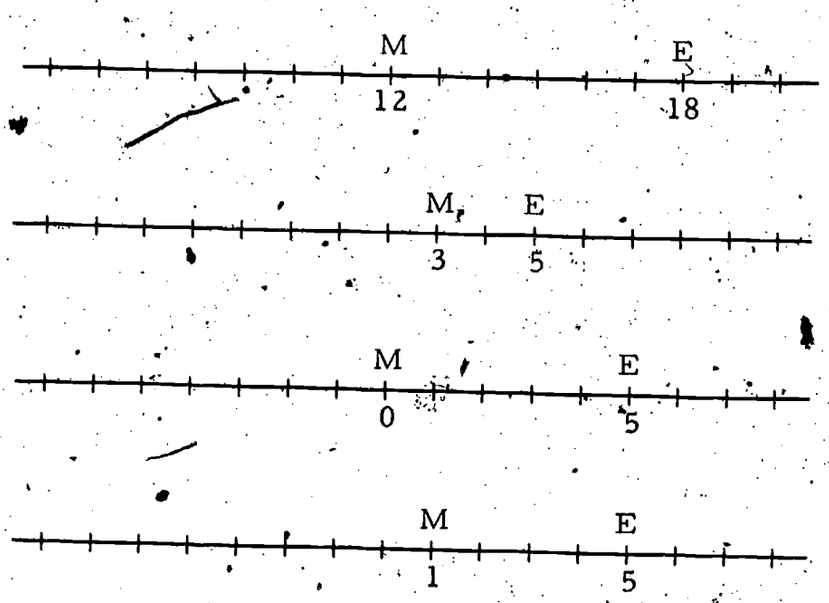
12. $|\square - 3| = \frac{1}{10}$

13. $|\square - 1{,}000| = \frac{1}{10}$

Epilogue: Using Midpoints to Introduce Negative Numbers.

Introducing negative numbers through the use of midpoints may be an appealing approach for teachers still hesitant to venture into the negative part of the line. The following are some questions which may help students discover negative numbers.

In the problems in this lesson two endpoints are given and the midpoint is to be found. Now here are problems showing one endpoint and the midpoint. In each case find the other endpoint.



When confronted with the last two questions, a pupil may be convinced that the other endpoint must lie on the left side of zero. He can even mark the place where it should be. What that point is named may temporarily be left up to the class. But now that negative numbers have been introduced, they can be used as starting and landing points for jumping rules, as numbers in lattices, and in other contexts.

Sometimes students will calculate the midpoints correctly, but will neglect to put the proper sign on them. This suggests some work with midpoints which might prove interesting to your class: If I know the two endpoints, how can I determine whether their midpoint is a positive or a negative number?

Endpoints	Midpoint: Positive or negative?
10 , $133\frac{1}{100}$	
$\frac{1}{15}$, $3{,}000$	
$\frac{1}{15}$, $\frac{1}{1{,}000}$	
0 , $\frac{1}{1{,}000}$	
-1 , $-\frac{1}{2}$	
$-33{,}333$, $-\frac{1}{4}$	

What if the endpoints are -15 and 15 ?

Now suppose the endpoints are $-5\frac{1}{2}$ and $146\frac{2}{3}$. You know the number halfway between them has to be _____ because _____

_____

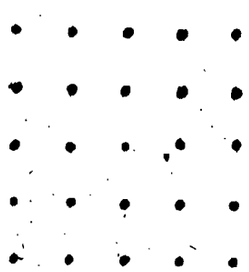
What if the endpoints are $-150\frac{2}{3}$ and $150\frac{1}{3}$? _____

_____

Summary of Problems in the Film "Counting With Dots"

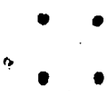
2nd Grade, James Russell Lowell School, Watertown, Massachusetts
Teacher: David A. Page

How many dots do you see?



Now how many? (Erases one; erases one more; erases bottom row of 4; erases another row; erases a column of 3.)

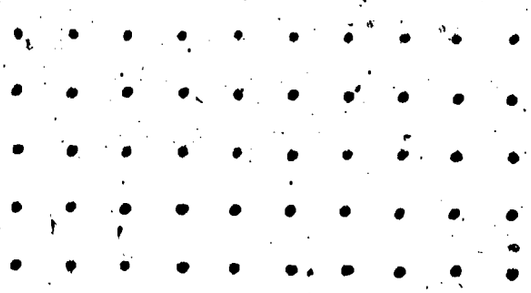
Now how many?



Now how many? (Erases all that are left.)

(0)

How many dots here?

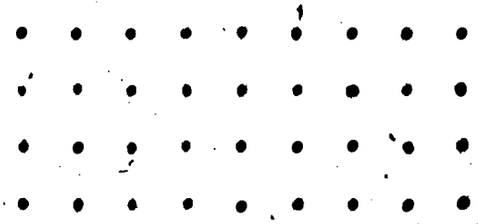


Now how many? (Erases 5 in the right column; erases bottom row.)

(36)

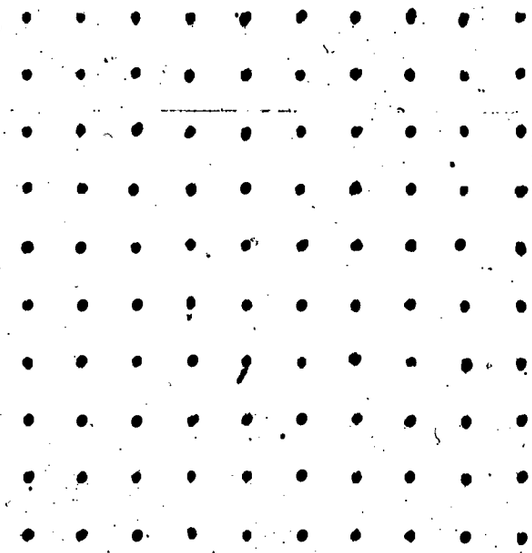
Convince us that it is 36.

"There are 9 in the top row and 9 in the second row. 9 and 9 is 18. The other two rows have 18 also. 18 + 18 = 36."

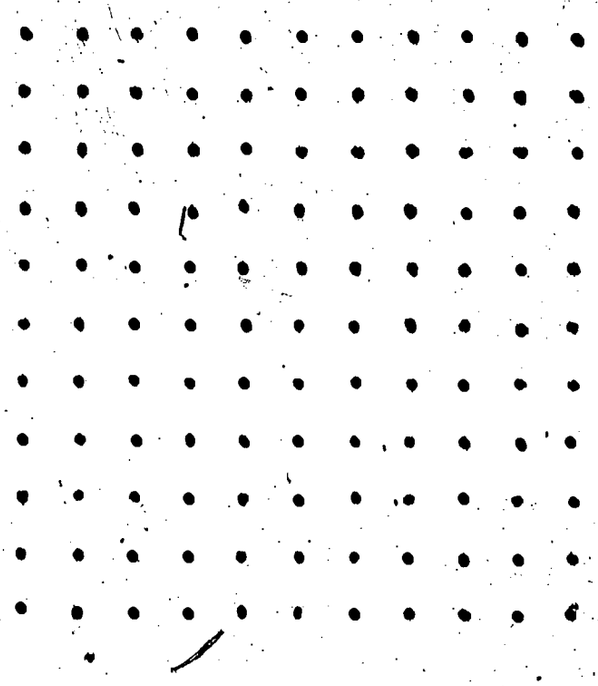


(Erases right-hand column, bottom row, other combinations of dots.)

Here is a new picture:



I'm going to add some more to this. (Adds a column of dots on the right and a row at the bottom, so that the picture is as appears at the top of the next page.)



What is the wrong answer I am expecting?

(100 is given)

That's a wrong answer but not the one I am thinking of.

What is the right answer?

(120 is given)

That is the wrong answer I was worried about.

How could you convince someone that it's 121 ?

What has he forgotten?

"They forgot one dot, the 121 st one." (Points to dot in lower right corner.)

If I take that one out, how many dots would there be?

Now, we will go back to the way it was before I added those on.

How many dots?

(100)

(Erases several rows and columns.)

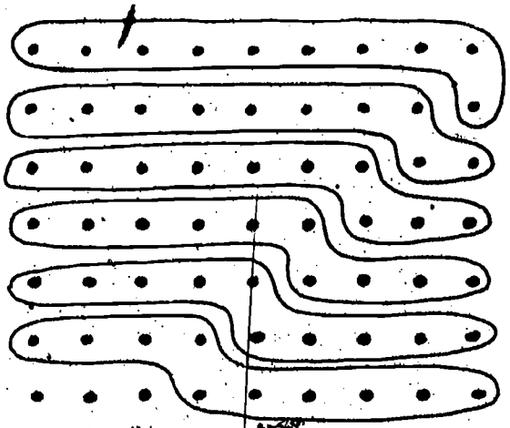
Are there more or less than 50 ?

More or less than 100 ?

Could you convince someone that it's 63 without going up to count them?

"There are 7 going down and 9 going across. 9 here and one make 10 and 8 here and two make 10 more...."

Teacher draws loops around the dots:

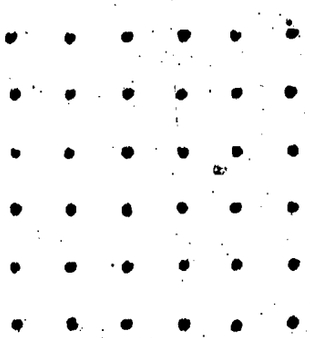


Now what does that tell you?

Other similar problems after more dots are erased.

The next day.

Behind this board I have some dots. I'm going to take the board away for a minute and put it back. Make a guess of how many dots there are.



(Guesses: 39; 25, 35, 36, 35)

I'll take it away again. What is it exactly?

(36)

Is there any way you could convince someone else?

"Two 6's are 12, and two others are 12, and two others are 12. Three 12's are 36."

Put a ring around one 12.

Someone else put a loop around another 12.

How many are there that aren't in any loop at all?

(12)

So how many altogether?