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ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains exercises on frames and number line jumping rules, a summary of the problems in the film "A First Class With Number Line Rules and Lower Brackets," and the supplement "Answers to Common Questions About the Course." (MK)

THE ARITHMETIC PROJECT COURSE FOR TEACHERS

TOPIC: Introduction to Frames and Number Line Jumping Rules.

FILM: A First Class With Number Line Rules and Lower Brackets, Grade 5

SUPPLEMENT: Answers to Common Questions About the Course

1

NAME: ....

This booklet is part of a course for teachers produced by The Arithmetic Project in association with Education Development Center. Principal financial support has come from the Carnegie Corporation of New York, the University of Illinois, and the National Science Foundation.

The course is available from:

THE ARITHMETIC PROJECT
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Newton, Massachusetts 02160

BOOK ONE

This is a course in mathematics probably unlike anything you have done before. Its aim is to teach you some kinds of mathematics and mathematical ideas that you can use successfully with your students.

To learn mathematics one needs to wrestle with it. In this institute you will be working sequences of problems, many of them similar to those you might use with your class. Also, you will be watching the teaching of classes of children of various grades and backgrounds. Watching classes being taught (even when all of them are not entirely successful) will help you learn both the subject matter and a variety of ways it can be pursued in the classroom.

In doing this work, try as much as possible to put yourself in the place of the child. You may know a routine way to approach this problem:

$$3 \times \square + 7 = 19$$

You might subtract 7 from both sides of the equation:

$$3 \times \square = 12$$

and then divide both sides by 3 :

$$\frac{3 \times \square}{3} = \frac{12}{3}$$

and finally simplify:

$$\frac{\cancel{3} \times \square}{\cancel{3}} = 4$$

Try to avoid this temptation for now. The procedure you have used is perfectly valid, but it is one of many things a child can have the pleasure of uncovering for himself if it has not already been given to him arbitrarily. Even if he doesn't find a general rule himself, he will ultimately appreciate such a rule because he has worked with the problem and found answers by other methods. Nearly all problems in mathematics can be approached in more than one way. Put what you know of algebra out of your mind for the time being.

Consider yourself a second or third grader with this same problem:

$$3 \times \square + 7 = 19$$

Happily, the question is not what rule to apply first. What you want to know is, what number can I put in the box to make it work? Try some things. Suppose you think: "Well, if I put 100 in the box, that makes it more than 300 on that side, and way too much. Even 10 in the box is too much. But, 0 or 1 in the box doesn't make the left-hand side big enough; it's only 7 or 10, and I want 19. [Notice how much you have already learned from these quick trials about the answer to this problem.] I will try 5. 3×5 is 15. Add 7. That is 22, which is just 3 away from 19. Too high by 3! I'll use one less than 5: 4. Now it works."

Of course, a child's thoughts, or yours, would probably not occur exactly in such words (and many thoughts don't occur in words at all), but some semblance of these ideas would be far closer to the way a child would solve this problem than the method of subtracting 7 from both sides and dividing by 3.

Questions may arise in your mind as to what mathematicians call some of the things we are doing; where a sequence of problems is travelling; whether other routes are as good. You will wonder if we advocate that you write your own textbook and if we recommend only the styles of teaching we show you.

Hold these questions in abeyance if you can. Many will be answered as we proceed, and in other cases the questions will change. A three-year-old enjoys learning to talk even though no one gives him reasons for it. Teachers who have attended previous institutes sometimes observe that if we had honestly answered some of the questions they had in mind at the outset, they wouldn't have believed us.

If it turns out that you enjoy wrestling with a certain problem, you may take it as a hint that there is some mathematics lurking in it, even if we have not said so. In fact, we shall try to avoid during the early stages using verbal generalizations and mathematical terminology where they will not help you start working with children. Too often, words without meaning to children get into the classroom ahead of—or in place of—the ideas they name, and these interfere with the mathematics. Speaking the name of an idea is too readily mistaken for understanding it. A child can best learn what commutativity is from exploring places where things commute and where things don't. When the idea has begun to take substance, learning its name is useful—and natural.

* * *

For many people, the early problems in a topic will be too easy. They are intended to suggest ways of introducing the material to children. Be patient: in each area more challenging problems lie ahead.

I. FRAME EQUATIONS

The following equations contain frames of only one shape. When two or more frames of the same shape occur in one equation, the same number must be used to fill each frame.

Find all the numbers which make these equations true, preferably by methods which might be used by children.

Example: $\boxed{9\frac{1}{2}} + \boxed{9\frac{1}{2}} = 19$

1. $\boxed{} + 3 = 10$

2. $\boxed{} + 7 = 10$

3. $\boxed{} + 10 = 10$

4. $\boxed{} + 12 \quad 1,012$

5. $\boxed{} + 12 = 1,011$

6. $\boxed{} + \boxed{} + \boxed{} = 99$

7. $\Diamond + \Diamond + \Diamond = 100$

8. $101 = \Diamond + \Diamond + \Diamond$

7

9. $\square + \square + \square = -99$

10. $\square + \square + \square = -34$

11. $\square + \square = 18$

12. $\triangle + \triangle = \triangle + 18$

13. $\triangle + \triangle = \triangle + 19$

14. $\triangle + \triangle = \triangle + 137\frac{1}{2}$

15. $\triangle + \triangle = \triangle + (-84)$

16. $\bigcirc + \bigcirc + \bigcirc = \bigcirc + 18$

17. $\bigcirc + \bigcirc + \bigcirc = \bigcirc + 18 + \bigcirc$

18. $2 \cdot \square = \square \times \square$

19. $1 + (2 \times \square) + 2 = \square + 3 + \square$

20. For which problem(s) above does more than one number work?

____

21. For which problem(s) above do <u>all</u> numbers work?

____

3

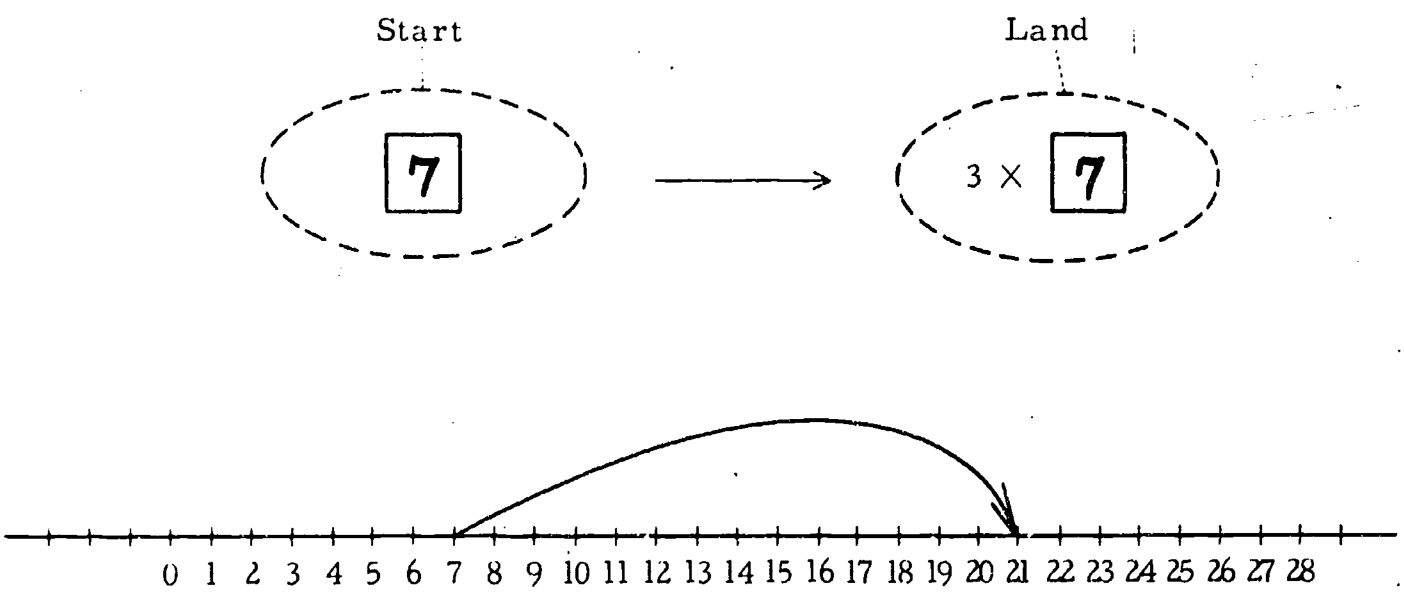
1. Here is a jumping rule:

$$\boxed{} \longrightarrow 3 \times \boxed{}$$

Read it as, "Go from box to three times box." The rule tells you how to make jumps along the number line.* For example, suppose we start at 7. We put 7 in each box:

$$\boxed{7} \longrightarrow 3 \times \boxed{7}$$

The rule tells us that if we start at 7 we land at 3×7 or 21.



Notice that the arrow shows the <u>direction</u> of the jump.

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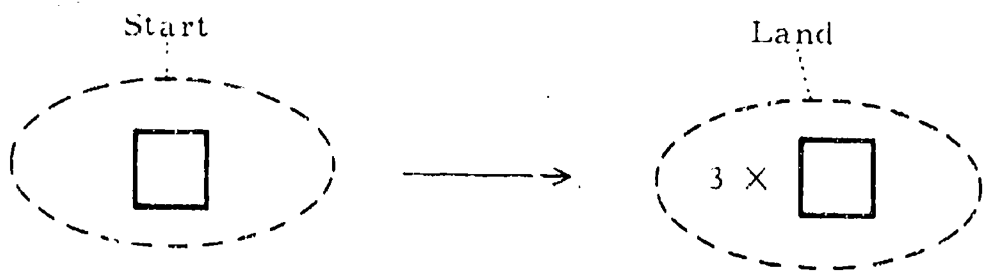
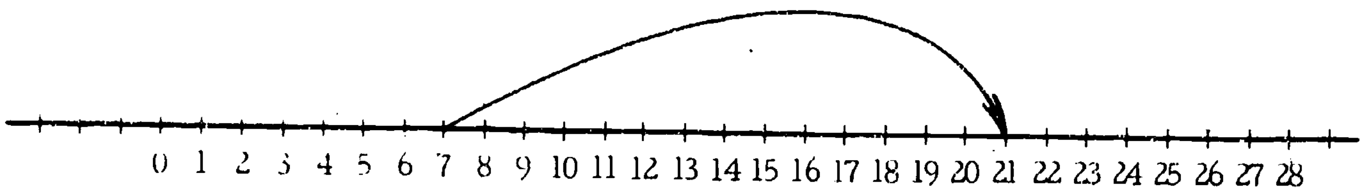
* For younger children, you may want at first to write this rule as $\square \rightarrow \square + \square + \square$

Continue using this rule for all the problems on this page:





(a) Make another jump, this time starting at 4. Where do you land?

____

(b) Draw the jump n the number line above.

(c) Start at 1. Landing point: __________ Draw the jump above.

(d) Start at 5. Land: __________ Draw the jump.

(e) Where can you start in order to land on 27? __________

Draw the jump.

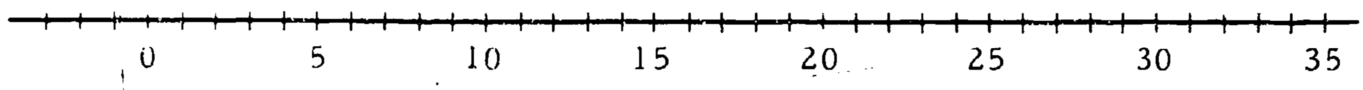
(f) Start at 0. Land: __________

(g) Start at $11\frac{1}{2}$. Land: __________

(h) The jump from 4 to 12 is eight spaces long. Still using the above rule, how many spaces long is the jump from 7? __________ Where can you start so that the length of the jump (between the starting point and the landing point) is 10 spaces? __________ 20 spaces? __________ 105 spaces? __________

2. Here is another rule:

$$\square \longrightarrow \square + \square + \square + \square$$



On the line above, draw the following jumps. Remember to show the direction of each jump.*

(a)	Start:	6	Land:	________	Distance traveled:	________
(b)	Start:	2	Land:	________	Distance:	________
(c)	Start:	________	Land:	32	Distance:	________
(d)	Start:	________	Land:	________	Distance:	12
(e)	Start:	________	Land:	________	Distance:	0

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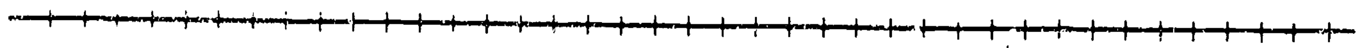
*Drawing jumps on the board helps you and your class to see the characteristics of different rules. Some teachers have their students draw jumps also, but too much of this can be tedious.

3. New rule: $\square \longrightarrow \square + 5$



(a) Draw a few jumps.

(b) Where must you start in order to land on 13 ? __________ To land on 700 ? __________ To land on 0 ? __________

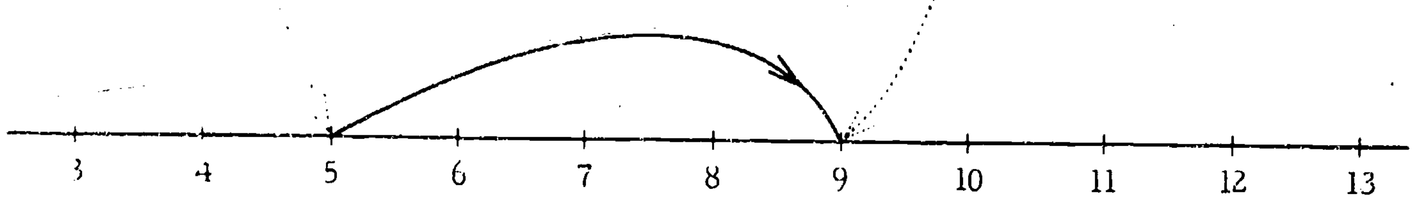
4. Jumps with the rule above always went to the right. Give two rules whose jumps always go to the left:

(a) $\square \longrightarrow$

(b) $\square \longrightarrow$

5. Another new rule:

$$\square \longrightarrow \square + \square + \square - 6$$

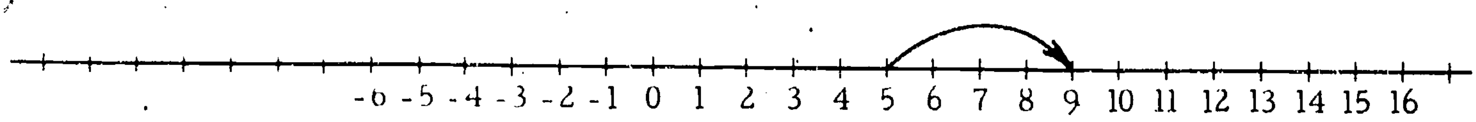


(a) Start at 5 . Take one jump. Where do you land? 9

This jump has been drawn on the number line above.

Spaces traveled? __________

Continuing with the rule $\square \longrightarrow \square + \square + \square - 6$, show all jumps for (b) through (e) on the number line below:



(b) Start at 7 . Make one jump. Where do you land? __________ Draw this jump on the number line above. Number of spaces traveled?

____

(c) Start at −1 . Make one jump. Where do you land? __________

(The answer is neither 3 nor −3.[†]) Draw this jump on the number line.

☆(d) Where can you start to get a jump shorter than any yet drawn on the line? __________

☆(e) Where can you start to land at 0 with this rule? __________ Draw the jump. Number of spaces traveled? __________

☆(f) Where can you start so that you travel 0 spaces with this rule?

____

[†] Those who would like a brief refresher in computing with negative numbers should see the next page.

☆Throughout this institute, problems marked with a star (☆) are wholly optional. They are included for those who may want an additional challenge.

Here are some examples of correct computation with negative numbers:

$$10 - 13 = -3$$

$$0 - 6 = -6$$

$$3 + (-1) = 2$$

$$-5 + (-6) = -11$$

$$17 + (-20) = -3$$

$$-20 + 17 = -3$$

$$-20 - 17 = -37$$

$$-20 - (-17) = -3$$

$$10 + 5 - 5 + 10 = 20$$

Later in the course you will receive more detailed suggestions for handling and teaching computations of this kind.

Epilogue

For the first 19 questions of this written lesson you were solving equations. A number which makes an equation come out right is called a root of that equation. The equations which you worked with here and the equations you will solve during the rest of this course will mostly have just one root. However, an equation may not have any roots. (Example: $\square + 3 + \square = 1 + \square + 5 + \square$.) There are some equations where every number works, such as problem 19 on page 5 of this written lesson. Finally, there are equations where only a few numbers work. It is not particularly difficult to find the two numbers which work in problem 18 on page 5. However, if you make up an equation at random and if it contains multiplication of a frame by itself, it may be extremely difficult to find the roots of the equation.

One can devote a lot of time learning to solve equations. When we work with equations we will usually be concerned with equations that are fairly easy to solve.

* * *

If the equation in problem 8 made you uncomfortable, you should work more often with the equal sign in different locations. If with no preparation you ask your class to solve the equation

$$\square + \square = \square + 15$$

you will probably see uniform agreement that 5 is the answer. Your students will look at the equation and read it as

$$\square + \square + \square = 15$$

because all the equations they have worked with until now have always had one number ("the answer") following the equal sign.

* * *

Be sure to understand our caution not to show students methods <u>you</u> may have learned for solving equations. These methods are probably very good ones. We want students to work on simple equations with methods they invent for themselves before having methods handed to them. In time formal methods which are beyond student capabilities of invention will be given or closely hinted at.

* * *

Working with number line jumping rules will make up a large portion of this course. You will see them again frequently. They are a convenient way of introducing an elementary study of what mathematicians call "functions". Later on in the course you will receive a supplement connecting jumping rules and mathematical functions.

* * *

Sometimes teachers ask why we bother to write the frame for the starting number each time we write a number line jumping rule. The reason is that we want to stress that two numbers are important in a jump: the starting number and the landing number. Also, we shall have occasion to use jumping rules like this one:

$$\longrightarrow \quad 7$$

This is a simple rule but an important one. If the frame and arrow were not written, we would no longer have a rule. We would have just the number seven. Also, although we will not have occasion to use them much in this course, you should be ready to see jumping rules such as

$$3 \times \quad \longrightarrow \quad \frac{\quad + 5}{2}$$

where the left side involves more than a single frame.

Summary of Problems In the Film

"A First Class With Number Line Rules and Lower Brackets"

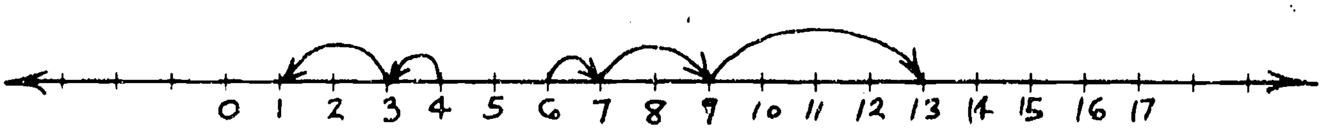
5th Grade, Daniel S. Gleason School, Medford, Massachusetts
Teacher: Lee Osburn

This is a jumping rule: $\underset{\text{Start}}{\square} \longrightarrow \underbrace{2 \times \square - 5}_{\text{Land}}$.

If you start at 10, where do you land?	(15)
Start at 6. Land?	(7)
Start at 100. Land?	
What is 200 − 5 ?	(Wrong answer of 105)
Teacher writes on board: 200 − 1 =	(Wrong answers of 119, 210)
20 − 1 =	(19)
30 − 1 =	
50 − 1 =	
100 − 1 =	
Now, how about this one? 200 − 1 =	(199)
200 − 2 =	
200 − 5 =	(195)

One way to keep track of starting and landing is to draw a picture. (See below.)

Where do you land if you start at 4 ?	(3)
Another jump from 3.	(1)
Start at 0.	(Answers of 0, 5, and "Can't do it with 5.")
All right, for now we can't do it with 5.	
Start at 6. Where do you land ?	(7)
Start at 7. Land?	(9)
Start at 9.	(13)



Start at 13.

Let's look at a different rule: $\square \longrightarrow \square + 3\frac{1}{2}$

Pick a starting place. (5)

Start at 5, where do you land? $(8\frac{1}{2})$

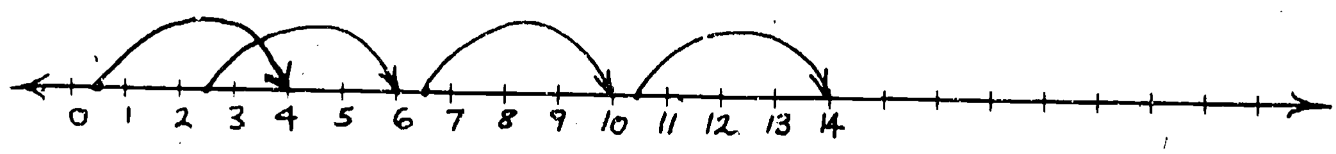
Start at $8\frac{1}{2}$. Land?

One more jump from 12. $(15\frac{1}{2})$

Where can I start with this rule and land on an even number? $(\frac{1}{2})$

Where do we land, starting at $\frac{1}{2}$? (4)

Where else can I start? (Answers of $2\frac{1}{2}$, $6\frac{1}{2}$, $10\frac{1}{2}$)



Tell me all of the places to start so that you land on an even number.

(Answers of $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$)

Where do we land, starting at $5\frac{1}{2}$? (9)

What about $1\frac{1}{2}$? (5)

Who can tell now all of the places to start?

($\frac{1}{2}$, $2\frac{1}{2}$, $4\frac{1}{2}$, $6\frac{1}{2}$, $8\frac{1}{2}$, $10\frac{1}{2}$, $12\frac{1}{2}$, $14\frac{1}{2}$, $16\frac{1}{2}$)

Can someone describe all these numbers?

New Rule: $\square \longrightarrow \square + 7$

Where can we start to land on an even number? (Students suggest 1, 3, 5)

Give me a starting number bigger than 100 so that we land on an even number. (101, 103, 105)

Describe all the numbers that land on an even number. (Odd numbers)

New Rule: $\square \longrightarrow \square + \square$

Start some place so that you land on an odd number.

(Answers of 3, 8, 1, "can't do it", $3\frac{1}{2}$)

Where do we land, starting at $3\frac{1}{2}$? (7)

Where else can we start? ($4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$, $9\frac{1}{2}$)

A starting place larger than 100? ($102\frac{1}{2}$, $101\frac{1}{2}$)

Tell all the numbers that we can start on to land on an odd number with that rule.

($1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{1}{2}$, $9\frac{1}{2}$, $10\frac{1}{2}$)

Is there some way you can describe all those numbers?

(Every number between 1 and 10 is used)

What about $102\frac{1}{2}$? (All numbers with $\frac{1}{2}$ added to it)

These are true: $\left\lfloor 8\frac{1}{3} \right\rfloor = 8$, $\left\lfloor 11\frac{5}{6} \right\rfloor = 11$

$\left\lfloor 15\frac{17}{23} \right\rfloor =$ (15)

$\left\lfloor 29\frac{99}{100} \right\rfloor =$ (29)

$\left\lfloor 506\frac{3}{4} \right\rfloor =$

Writes on board: $\left\lceil 8\frac{1}{3} \right\rceil = 9$

$\left\lceil 11\frac{5}{6} \right\rceil =$ (12)

$\left\lceil 15\frac{17}{23} \right\rceil =$ (16)

$\left\lceil 29\frac{99}{100} \right\rceil =$

$\left\lceil 506\frac{3}{4} \right\rceil =$

$\lfloor 9 \rfloor =$ (9)

$\lceil 9 \rceil =$ (Wrong answer of 10)

You might think that it would work that way, but it doesn't.

(9)

$\left\lfloor \frac{2}{5} \right\rfloor =$ (Wrong answers of $\frac{2}{5}$, $\frac{1}{5}$, $\frac{7}{5}$)

Let me give you a different problem first; we will come back to that one.

$\left\lfloor 4\frac{2}{5} \right\rfloor =$ (4)

$\left\lfloor 3\frac{2}{5} \right\rfloor =$ (3)

$\left\lfloor 2\frac{2}{5} \right\rfloor =$

$\left\lfloor 1\frac{2}{5} \right\rfloor =$ (1)

$\left\lfloor \frac{2}{5} \right\rfloor =$ (0)

$\left\lfloor \frac{2}{5} \right\rfloor =$ (0)

$\left\lceil \frac{2}{5} \right\rceil =$ (1)

$$\left\lfloor \frac{2}{5} \right\rfloor + \left\lfloor \frac{2}{5} \right\rfloor + \left\lfloor \frac{2}{5} \right\rfloor + \left\lfloor \frac{2}{5} \right\rfloor + \left\lfloor \frac{2}{5} \right\rfloor + \left\lfloor \frac{2}{5} \right\rfloor = \qquad (0)$$

$$\left\lfloor \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} \right\rfloor = \qquad (2)$$

(Wrong answers of 6, $\frac{4}{5}$, $\frac{12}{5}$, 1, 0, 10)

$$\left\lceil \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} \right\rceil = \qquad (3)$$

(Wrong answers of 5, 12)

Supplement

Answers to Common Questions About the Course

The Arithmetic Project course includes a number of supplements to the written lessons. Some deal with the mathematics behind what you will be doing. Others provide extensions of the various topics you will meet in the written lessons—things you may want to try in the classroom when you have become familiar with the material in the lessons. Some supplements are simply outlines of classes in which topics of interest were introduced. These supplements may give you some further ideas about how to get started with a topic.

In this supplement you will find answers to some of the questions commonly asked early in an institute.

Q: Do you recommend as much oral arithmetic as is shown in this film?

A: The Project does not recommend any particular style of teaching. It offers various mathematical topics and ideas that can be adapted to an individual teacher's style and to local preferences and procedures. Teachers who are successfully using such techniques as team teaching or pupil-team learning are naturally the best judges of how to adapt Project ideas for their own use.

In their day-to-day classes, Project teachers regularly give more written work than is shown in the films. This activity is minimized in the films because it is considerably less interesting to watch.

Q: By the end of the class in the film only a few children seemed to know what was going on. What do you do about the others? (This is sometimes asked in the form "What do you do about. . . ? She doesn't know her number facts.")

A: Normally the teacher would work much more than a single hour with this topic, and on successive days other children would begin to see what was happening.

Different children may excel at different topics. Also, the observer (and teacher) can never be absolutely sure at any given moment that a child is really not paying attention or understanding an idea. In a number of the classes you will see, a child who is apparently paying no attention at all will soon afterwards give the correct answer to a difficult problem or "wrap up" the subject with a general method for solving such problems. In other instances a child who is having trouble answering a question may really be trying to answer an entirely different question.

Q: How much do the cameras and lights affect the children?

A: Classes usually come to the studio for filming several days at a stretch. After the first day the children tend to look at the cameras no more often than they might look out the window of an ordinary classroom. The cameras are in plain sight, but extraneous movements and other distractions are kept to a minimum. (The adult teacher, however, is sometimes quite bothered by the cameras.)

Q: The film never says what is important about the lesson, nor what the mathematical ideas are. Does the course explain this to teachers?

A: The written materials of the course do discuss the mathematics to some extent, but technical terminology is held to a minimum in the course. The danger in emphasizing terminology is that the names for things tend to get carried into the classroom sooner than (and sometimes instead of) the ideas they stand for.

It is important to remember that teachers and children can see and appreciate the mathematics without having the technical language for it beforehand. One's own "feeling" for what is interesting should be relied on at the outset.

Q: Do you have a separate course for the earlier grades?

A: None of the Project's topics is identified with one particular grade. This is partly because the abilities at a given grade vary widely from one locality to another, and even within a school system. Equally important, the Project has found that most topics rich enough to be worth developing at some particular grade level are also rich enough to be adaptable for each elementary grade, and beyond. Number line rules, of simpler kinds (at first), can be interesting to the third grades and first grades. Written with frames or letter variables, jumping rules provide problems that would challenge a top high school class.

Q: How can a school system use the Arithmetic Project course?

A: The Project is not developing textbooks for children that must be adopted, but rather a course for teachers involving a number of topics that teachers can learn to work with successfully. The extent to which a teacher uses Project topics in the classroom would depend on his interests and those of his students. Teachers who complete the course might be able to devote a third or more of their arithmetic class time to pursuing Project materials. This fraction would increase with each year's experience in the classroom.

Many good teachers and good school systems now use texts, workbooks, problems of their own devising, and other materials to put together their own "curriculum" in terms of the particular needs of their classes. The Project's course is intended to add substantially to such resources. In time the Project hopes to have a sufficiently rich battery of topics to allow a teacher profitably to devote most of his time to working with its materials.

* * *

The work of the University of Illinois Arithmetic Project has been made possible principally by grants from the Carnegie Corporation of New York and the National Science Foundation, with additional support from the University of Illinois and the Ford Foundation. The Project was formed in 1958 at the University of Illinois, and moved in 1963 to Education Development Center (then Educational Services Incorporated) to complete production of a course for teachers as well as to continue its work of developing and refining mathematical topics for the elementary school classroom. The present course embodies the richest and most successful of the many ideas developed by the Project; course materials were extensively tried and repeatedly revised through a sequence of seventeen institutes for teachers in Massachusetts and Illinois school systems.