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ABSTRACT

This study investigated the effect of initial instruction on the processes children use to solve basic addition and subtraction verbal problems. Prior to instruction and following a 2-month introductory unit on addition and subtraction, 43 first-grade children were individually tested on verbal problems representing different models of addition and subtraction. Prior to instruction, children's solution processes modeled the action or relationships described in the problem. Following instruction, they generally used a separating strategy for all subtraction problems. Although they could solve the problems, few children could coordinate their solutions with the arithmetic sentence they wrote representing the problem. (Author/MK)

The Effect of Instruction on First-Grade Children's Initial Solution Processes for Basic Addition and Subtraction Problems

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Abstract

This Study investigated the effect of initial instruction on the processes children use to solve basic addition and subtraction verbal problems. Prior to instruction and following a 2 month introductory unit on addition and subtraction, 43 first-grade children were individually tested on verbal problems representing different models of addition and subtraction. Prior to instruction children's solution processes modeled the action or relationships described in the problem. Following instruction they generally used a separating strategy for all subtraction problems. Although they could solve the problems, few children could coordinate their solutions with the arithmetic sentence they wrote representing the problem.

The Effect of Initial Instruction on First-Grade Children's Solution Processes for Basic Addition and Subtraction Problems

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The purpose of this study was to investigate the effects of initial instruction in addition and subtraction concepts and skills on the processes that children use to solve basic addition and subtraction verbal problems. In an earlier study (Carpenter, Hiebert, & Moser, in press), it was found that prior to formal instruction children are extremely successful in solving simple verbal problems. Although they did not have knowledge of basic facts or computational algorithms available to solve the problems arithmetically, children could represent the problems using cubes, fingers, or some internal representation and solve the problem using various counting strategies.

It is generally acknowledged that with older children solving verbal problems is an area of difficulty in mathematics instruction (c.f. Zweng, 1979). A basic question is why are the relatively sophisticated problem solving strategies of younger children that are based on the semantic structure of problems replaced by the superficial analysis of verbal problems found in many older children as they attempt to decide whether to add, subtract, multiply, or divide. Part of the answer to this question may lie in understanding the transition from informal knowledge of addition and subtraction to the formal symbolic systems of arithmetic. By comparing the processes that children use prior to instruction in addition and subtraction to the processes that they use after several months of formal instruction, this study attempts to gain a clearer picture of the early development of addition and subtraction concepts and skills as well as to provide some insights into children's problem solving abilities.

Background

Problem Structure

A major focus of this study is to investigate how children solve different types of addition and subtraction problems. In order to examine the effect of problem structure, it is necessary to characterize the major differences between different addition and subtraction problems. There are several approaches that previous research has taken to characterize verbal problems. One is to characterize problems in terms of syntax, vocabulary level, number of words in a problem, and so on (Jerman, 1973; Suppes, Loftus, & Jerman, 1969). A second approach differentiates between problems in terms of the open sentences they represent (Grouws, 1972; Rosenthal & Resnick, 1974; Lindvall & Ibarra, Note 1). This study is based upon a third alternative that considers the semantic structure of the problem (Moser, Note 2). This analysis is generally consistent with other analyses based on problem structure (Gibb, 1957; Vergnaud & Durand, 1976; Grenno, Note 3).

This analysis distinguishes between four semantically different classes of problems: Joining/Separating, Part-Part-Whole, Comparison, and Equalizing. For problems in the Joining/Separating class there is an initial quantity and some direct or implied action that causes a change in the quantity. For problems in the Part-Part-Whole class there is no action direct or implied. This class represents situations in which there are two quantities which may be considered individually or as parts of a whole. As the name implies, problems in the Comparison class involve the comparison of two quantities. This includes problems in which the difference between two given quantities is to be found and problems in which one of two quantities and the magnitude of the difference between them is given and the second quantity is the unknown. Equalizing problems share characteristics of both Joining/Separating and Comparison problems. There is implied action on one of two given sets, but a comparison is also involved.

There are two dimensions on which the four classes of problems differ. One major distinction is based on whether the problems describe action or static relationships. In Joining/Separating and Equalizing problems, there is direct or implied action in which one set is joined to or separated from another set. On the other hand, both Part-Part-Whole and Comparison problems involve relationships between quantities, and there is no action implied or direct. The second major distinction is based on set inclusion relationships. In both the Joining/Separating and Part-Part-Whole classes two of the entities involved in the problem are necessarily a subset of the third. In other words, either the unknown quantity is made up of the two given quantities or one of the given quantities is made up of the other given quantity and the unknown. For Comparison and Equalizing problems this is not the case.

By varying the unknown quantity or the nature of the action in the problems (joining or separating), both addition and subtraction operations can be represented by problems in each of the four classes.

Strategies

The initial problem solving study (Carpenter, Hiebert, & Moser, in press) identified a number of strategies that children use to solve addition and subtraction problems prior to instruction. Most of these strategies are based on counting and are similar to those found in the response latency studies of symbolic addition and subtraction (Groen & Parkman, 1972; Groen & Resnick, 1977; Suppes & Groen, 1967; Woods, Resnick, & Groen, 1975), although several additional strategies were identified as well. The basic addition strategies can be summarized as follows:

Counting All: The counting all strategy can be carried out using cubes or fingers as models, or by counting mentally. If cubes are used, both sets are represented, and then the union of the two sets is recounted beginning with one. If counting is done mentally or with fingers, the counting sequence begins with one and ends with the number representing the total of the two given quantities.

Counting On From First Number: In this strategy, the counting sequence begins either with the first given number in the problem or the successor of that number. Counting may be done mentally, or by using cubes or fingers to keep track of the number of steps counted.

Counting On From Larger Number: This is similar to the previous strategy except that the counting sequence begins with the larger given number or with the successor of that number.

Known Fact: The child gives an answer with the justification that is was the result of knowing some basic addition fact.

Heuristic: Heuristic strategies are employed to generate solutions from a small set of known basic facts. These strategies usually are based on doubles or numbers whose sum is 10. For example, to solve a problem representing 6 + 8 = ? a subject responds that 6 + 6 = 12 and 6 + 8 is just 2 more than 12. In another example involving 4 + 7 = ? a subject responds that 4 + 6 = 10 and 4 + 7 is just 1 more than 10.

Uncodable: A correct answer is provided but the interviewer is unable to determine what strategy a child is employing.

The Counting All strategy directly models the actions or relationships described in the addition problem. The Counting On strategies provide a more abstract representation of these relationships. There are parallel levels of abstraction for the subtraction strategies, but the subtraction strategies are a great deal more varied. Different subtraction strategies represent completely different interpretations of subtraction. The specific subtraction strategies can be summarized as follows:

Separating From: The child uses concrete objects or fingers to construct the larger given set and then takes away or separates, one at a time, a number of cubes or fingers equal to the smaller given number n in the problem. Counting the set of remaining cubes yields the answer.

Counting Down From: In a more abstract representation of the Separating From strategy, a child initiates a backwards counting sequence beginning with the given larger number m. The backwards counting sequence contains as many counting number words as the given smaller number. The last number uttered in the counting sequence is the answer.

Separating To: The Separating To strategy is similar to the Separating From strategy except that the separating continues until the smaller quantity is attained rather than until it has been removed. In the concrete case, after the larger set is counted out, the child removes cubes one at a time until the remainder is equal to the second given number of the problem. Counting the number of cubes removed gives the answer.

Counting Down To: A child initiates a backwards counting sequence beginning with the larger given number. The sequence ends with the smaller number. By keeping track of the number of counting words uttered in this sequence, either mentally or by using fingers or cubes, the child determines the answer to be the number of counting words uttered in the sequence.

Adding On: With concrete objects the child sets out a number of cubes equal to the smaller given number. The child then adds cubes to that set one at a time until the new collection is equal to the larger given number. Counting the number of cubes added on gives the answer.

Counting Up From Given: A child initiates a forward counting sequence beginning with the smaller given number. The sequence ends with the larger given number. Again, by using any of the available devices, the child keeps track of the number of counting words uttered in the sequence, and thereby determines the answer.

Matching: Matching is only feasible when concrete objects are available. The child puts out two sets of cubes, each set standing for one of the given numbers. The sets are then matched one-to-one. Counting the unmatched cubes gives the answer.

Strategies may be related to problems in several ways. A child might use the same strategy for all subtraction problems.

A second alternative is that a child might use different strategies depending on the relative size of the numbers in the problem (Woods et al., 1975). A third possibility is that the strategy would depend on the structure of the problem.

Certain of the strategies naturally model the action described in specific problems. The Separating problem is most clearly modeled by the Separating strategy. On the other hand, the implied joining action of the Joining (missing addend) problems is most closely modeled by the Adding On strategy. Comparison problems deal with relationships between sets rather than action. In this case the Matching strategy appears to provide the best model.

For the Part-Part-Whole and Equalizing problems the situation is more ambiguous. Since Part-Part-Whole problems have no implied action, neither the Separating nor Adding On strategies, which involve action, exactly model the given relationship between quantities. But since one of the given quantities is a subset of the other, there are not two distinct sets that can be matched.

For the Equalizing problems the situation is reversed. Since the Equalizing problems involve both a comparison and some implied action, two different strategies might be seen as appropriate. The addition Equalizing problems involve a comparison of two quantities and a decision of how much should be joined to the smaller quantity to make them equivalent. Thus, both the Matching or the Adding On strategies might be appropriate. For the subtracting Equalizing problems the implied action involves removing elements from the larger set until the two sets are equivalent. This action seems to be best modeled by the Separating To strategy while the Matching strategy is again appropriate for the comparison aspect of the problem.

In the initial study of children's solution processes (Carpenter et al., in press) it was found that prior to instruction in addition and subtraction children's strategies were based primarily on the structure of the problem. A major question to be investigated in this study is whether this general approach continues to operate after instruction or whether children begin to view subtraction as a single operation that has a single method of solution.

Purpose

The basic objective of this study was to investigate the effects of initial instruction in addition and subtraction concepts and skills on the processes that children use to solve simple addition and subtraction problems. The effect of instruction was measured in terms of (a) the changes that occured in the processes that children use to solve addition and subtraction problems as a result of instruction, (b) children's ability to write arithmetic sentences to represent different addition and subtraction problems, and (c) the specific effect of sentence writing on the processes that children use to solve addition and subtraction story problems.

Method

Procedures

Forty-three first-grade children were individually interviewed in February, before they had received any school instruction on addition and subtraction. The tasks for the interview consisted of 10 addition and subtraction problems. In May, following 2 months of instruction in addition and subtraction, the same subjects were readministered a subset of 6 of the same verbal problems. Several days later they were administered a set of parallel problems for which they were asked to write an arithmetic sentence before they solved the problem.

This study used individual interviews in order to be able to describe the process that children used to solve each of the problems. Interviews were conducted in a small room removed from the classroom. Each problem was read to the subjects by one of three experimenters. Problems were reread as often as necessary so that remembering the given numbers or relationships was not a factor. A set of red and white Unifix cubes was made available to the subjects. Subjects were encouraged to solve the problem without the cubes but were told to use the cubes to help them solve the problem if they needed them or were not sure of their answer. If subjects were having difficulty solving a problem without cubes they were reminded that they could use cubes to find the answer.

Essentially the same procedures were used for the problems for which subjects wrote arithmetic sentences. But for these problems subjects were also given a pencil and paper and instructed to write a number sentence before they solved the problem.

If subjects used cubes or fingers, their method of solution was often evident. In this case the experimenter coded the response and went on to the next problem. If the subject did not physically model the problem or it was not completely clear how a subject had arrived at a given answer, the subject was asked to describe how the answer was found. The experimenter continued questioning until it was clear what strategy the subject had used or it was clear that no explanation was forthcoming. In the February interview, problems were presented in one session that lasted 10 to 15 minutes. In the May interview, the sentence writing problems were administered several days after the other problems.

Tasks

Two basic types of addition problems and four types of subtraction problems were included in the study. One subtraction problem was selected from each of the four basic classes of problems that had been identified. Since the initial investigation had found no differences in the processes that children use to solve Joining and Part-Part-Whole addition problems only the Part-Part-Whole problem was included. Since Equalizing addition problems are somewhat awkward, they were also not used. The six verbal problems administered in the February and May interviews are presented in Table 1.

Insert Table 1 about here

To avoid any danger of contamination, a set of six identical problems were written for the sentence writing interview. The problems were identical except for a slight change in wording. For example, the Part-Part-Whole addition problem became:

> Some children were playing baseball. a were girls and b were boys. How many children were playing baseball altogether?

The number triples for the problems were selected to conform to the following specifications: (a) Each of the addends was greater than 2 and less than 10, (b) their sum was greater than 11 and less than 15, and (c) the absolute value of the difference between the two addends was greater than 1. These rules generated the following set of six triples: (3,9,12), (4,9,13), (5,7,12), (5,8,13), (5,9,14), (6,8,14). This number domain was selected because the numbers were small enough so that the problems could be reasonably modeled using concrete objects but were large enough so that it was unlikely that many children would have already learned the addition or subtraction combinations. It was also more likely that the children's strategies would be observable with numbers of this size than with smaller numbers. Doubles and near doubles were eliminated because it was hypothesized that children may operate differently with those combinations (cf. Groen & Parkman, 1972).

The number triples were equally distributed over the set of problems so that each number triple was paired with each problem either four or five times. Each subject received each number triple exactly once within the set of problems, but different subjects received different combinations for a given problem. For the addition problems, the smaller of the two addends was always presented first. For the subtraction problems, the larger of the two addends was always selected for the unknown. The order of the problems was randomized for each subject.

Subjects

The subjects for the study consisted of the 43 children in the two first-grade classes of a parochial school that draws students from a predominantly middle class area of Madison, Wisconsin. Mathematics instruction in both classes had consisted of topics 15 to 22 of the Developing Mathematical Processes (DMP) program (Romberg, Harvey, Moser, & Montgomery, 1974). At the time of testing in early February, only two arithmetic topics had been covered, Writing Numbers and Comparison Sentences. The other six topics deal with measurement and geometry. The topic of Comparison Sentences introduces the notion of a mathematical sentence, though it only deals with representing a static relation (equality) between two numbers. Thus, at the time the children were tested in February, no formal instruction in symbolic representation of addition and subtraction had been given. On the other hand, several lessons which included problem situations involving joining, separating, part-part-whole, and comparison had been presented. In those instances, modeling with objects to determine the solutions had been suggested.

By the time of the May interview, two instructional units on addition and subtraction were presented. The units required approximately two months of instruction and focused on the following objectives: writing number sentences of the form $a + b = \square$ or $a - b = \square$ to represent concrete and verbal problem situations and solving number sentences of the form $a + b = \square$ and $a - b = \square$ for sums between 0 and 10. The problem situations were of the joining, separating, comparison, and part-part-whole types. Several key features highlight these units. First, the children are strongly encouraged to use modeling behaviors by representing numbers with sets of physical objects. Second, various forms of counting are suggested. Finally, analysis of verbal problems is taught using a device that has the part-part-whole relationship as its basis. This device tends to highlight the inverse relationship of addition and subtraction.

Results

Change

The change in performance from the February interview to the May interview is summarized in Table 2. The table only includes problems given in both assessments and does not include the

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Insert Table 2 about here

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problems that required children to write arithmetic sentences. A response was coded as indicating a correct strategy if the method would have generated the correct answer provided that it was applied without error. In other words miscounting or forgetting one of the numbers in the problem did not invalidate a correct strategy.

Performance on the Part-Part-Whole problem in February left little room for improvement, but the five subjects who could not figure out an appropriate strategy at the time of the first interview were able to do so after several months of instruction in addition and subtraction. Children continued to have difficulty with the Comparison problem but there were substantial gains from the first interview.

There is an interesting shift in errors for the Comparison problem. Whereas at the time of the first interview most of the errors resulted from children immediately responding one of the numbers given in the problem, in the second interview errors were almost evenly divided between responding one of the given numbers and choosing the wrong operation. Operation errors involved the application of some strategy that would be appropriate for a subtraction problem.

Although earlier studies had suggested that children increasingly use Counting On strategies rather than Counting All (Groen & Parkman, 1972), there were no apparent changes in the strategies that children use to solve addition problems. Counting All was the primary strategy at both interviews. It should be noted that instruction only covered addition facts through 10, so the children had no experience with the number combinations used in this study and it is not surprising that few of them knew the arithmetic facts at the time of the May interview.

The change data for the subtraction problems are summarized in Table 3. Since there was no significant pattern of change in

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Insert Table 3 about here

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errors, this data was not included in order to simplify the table. For the four subtraction problems the wrong operation was never used in the first interview and was used only three times in the second. The corresponding figures for responding one of the given numbers are 10 and 12 respectively.

There was relatively little margin for improvement in the number of correct strategies, consequently there was relatively little change in performance. The Part-Part-Whole problem had an unusually high incidence of counting errors during the first interview and 13 additional subjects found a correct answer in the second interview. It does not appear, however, that there was an overall improvement in the accuracy with which children applied counting skills. This was the only problem which showed such an increase and it probably results from the uncharacteristically high number of counting errors for this particular problem during the first interview.

Although most children could identify an appropriate strategy to solve each problem at both interviews, there was a significant shift in the strategies they selected. At the time of the first interview most strategies were based on the structure of the problem and consequently varied from problem to problem. During the second interview most children consistently used a separating strategy for all four subtraction problems.

Sentence Writing

Sentence writing responses are summarized in Table 4. Responses were coded as correct if they were valid open number

Insert Table 4 about here

sentences or phrases that could be used to solve the problem or if they were true sentences involving the number triple in the problem. Examples of correct and incorrect responses are given in Table 5.

Insert Table 5 about here

In spite of the fact that children were consistently given directions to write the sentence before they solved the problem, about a fourth of the children generally solved the problem before they wrote the sentence. In many cases it appeared that they were unable to write the sentence before they knew the complete number triple. This accounts for the response 5 + 7 = 12 for the subtraction problem in Table 5.

The Part-Part-Whole addition problem and the Separating problem were most clearly associated with appropriate sentences.

There were only four incorrect sentences for either problem, all involving the incorrect operation. The Comparison subtraction problem and the Equalizing problem were the most difficult. In fact, 8 of the 15 correct responses to the Equalizing problem were addition sentences (5 + 7 = 12) written after the problem was solved. Three children responded in this way to the Comparison problem. Over half the children wrote invalid sentences for the Comparison and Equalizing problems. The most common incorrect response was writing an addition sentence involving the two numbers given in the problem. This accounted for 17 of the Comparison responses and 24 of the Equalizing responses. It is not surprising that the Equalizing problem would have a higher relative number of errors involving addition sentences than the Comparison problems because the implied action in the Equalizing problem is a joining action.

A number of children made the error of reversing the numbers in their subtraction sentence (5 - 12 = __). In both problems the smaller number is given first. A child may recognize that subtraction is required; but if they write the sentence with the numbers in the order in which they are given in the problem, the sentence is not correct. For the Separating problem this difficulty does not occur because the larger number is given first. Seven children wrote incorrect subtraction sentences for the Comparison problem and three did so on the Equalizing problem. There were no responses of this kind for the Separating problem.

Children's relative success in writing a sentence for the Comparison addition problem probably reflects a tendency to write addition sentences when in doubt rather than a recognition that the problem is represented by an addition sentence. Thirty children wrote appropriate sentences for the Comparison addition problem while only 20 used a strategy to solve the problem that would generate a correct solution. On the other hand only a third of the children wrote valid sentences for the Comparison subtraction or Equalizing problems while over three-fourths generated appropriate solutions. Although about half of the children wrote incorrect addition sentences for these two problems, only five children used a procedure that represented addition to solve them.

These results indicate that children's solutions were not seriously influenced by the sentence that they wrote to represent the problem. Further evidence for this conclusion is presented in Tables 6 and 7 in which performance on problems on which children did not write sentences is compared to performance on

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Insert Tables 6 and 7 about here

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problems that required children to write arithmetic sentences before they solved the problem. There is practically no difference between responses in any category. The only possible trend that can be observed is that there were fewer instances of children using a matching strategy for the Comparison and Equalizing problems. The numbers are so small that one should be cautious in attaching a great deal of significance to this result. However, the trend is consistent with children's shift from direct modeling of problem structure as a result of instruction on sentence writing.

Conclusions

Prior to instruction the general strategy that most children use to solve addition and subtraction problems is to directly model the action or relationships described in the problem. They have a number of different strategies for solving subtraction problems which represent distinct interpretations of subtraction. A completely developed concept of subtraction involves an integration of all these interpretations. It appears, however, that most children at this stage do not recognize that the different strategies are equivalent. They seem to regard them as distinct strategies that are used to solve different types of problems. The evidence for this conclusion is the uniformly close match of solution strategy to the structure of the problem.

Following initial instruction there is a distinct shift in the general approach children use to solve addition and subtraction problems. Rather than using a variety of strategies to solve different subtraction problems, most children begin to use a single strategy to solve all subtraction problems. It is not clear whether they clearly recognize the equivalence of the different strategies they were using earlier or whether their eclectic approach has been replaced by a single unified interpretation of subtraction.

Some caution should be exercised in interpreting these findings. This study did not include a Joining-missing addend problem, for which the Adding On strategy is extremely pronounced. A study that is currently in progress suggests that at this stage of instruction most children would probably continue to use an Adding On strategy to solve the problems. In other words, the shift to a single strategy probably is not complete at this point. The study currently in progress also suggests that the shift to a single dominant strategy occurs somewhat later than is indicated by the study reported in this paper.

A change that might be predicted that did not occur is a shift to more abstract strategies. For example, following instruction one might expect children to use the more efficient Counting On strategies rather than Counting All or to move from a Separating strategy to a Counting Down strategy. This study found a much lower incidence of Counting On strategies than were reported by Groen and Parkman (1972). One reason that this may not have occurred is that the presence of cubes tend to encourage the more complete representations. The study currently in progress indicates that when cubes are not available there is an increase in the use of more abstract strategies.

Instruction in addition appears to have caused some shift in the strategies that children use to solve addition and subtraction problems. Most children also learned to write addition and subtraction sentences to represent certain addition and subtraction problems. At this stage, however, very few children recognized that the arithmetic sentence was a mechanism that they might use to help them solve the problem. Once they had written a sentence most children appeared to ignore it and used the verbal problem to decide on a solution strategy. In fact, in spite of instructions to the contrary, about a fourth of the subjects would solve a problem before writing a sentence. In cases where children wrote an incorrect sentence but computed the correct answer, they would often complete the open sentence with their answer. Although there was occasional hesitation when they did this, none of them were able to resolve this conflict. The fact that the specific act of sentence writing did not influence children's solutions further supports the conclusion that children have not yet coordinated their problem solutions with the sentences they write to represent the problem.

These results indicate that following several months of instruction children have begun to shift from a concrete direct modeling approach to solving word problems to a more unified conception that incorporates symbolic representations of addition and subtraction problems. At this point, however, they are still in a transitional stage and have a long way to go to a completely developed concept of addition and subtraction.

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Table 1

Verbal Problems

Addition

Part-Part-Whole

Some children were ice-skating. a were girls and b were boys. How many children were skating together?

Comparison

Ralph has a pieces of gum. Jeff has b more pieces than Ralph. How many pieces of gum does Jeff have?

Subtraction

Separating

Leroy had a pieces of candy. He gave b pieces to Jenny. How many pieces of candy did he have left?

Part-Part-Whole

There are c children on the playground. a are boys and the rest are girls. How many girls are at the playground?

Comparison

Mark won a prizes at the fair. His sister Connie won c prizes. How many more prizes did Connie win than Mark?

Table 1 (continued)

Equalizing (+)
Joan picked a flowers. Bill picked c flowers. What could Joan do so she could have as many flowers as Bill? (Suggest, if necessary, that she pick some more.) How many more would she need to pick?

Table 2

Change in Addition Responses

Problem	Interview	Total Correct*		Strategy				Errors		
		Strategy	Answer	Count all	Count from first	Count from larger	Heuristic or fact	Given number	Wrong operation	No response or other error
Part-Part-Whole	1	38	37	22	3	9	3	1	1	3
	2	43	39	23	4	8	5	0	0	0
Comparison	1	12	10	7	1	1	2	23	3	5
	2	20	19	16	1	1	1	12	9	2

*Total N = 43 subjects.

Table 3

Change in Subtraction Responses

Problem	Interview	Total Correct* Strategy	Total Correct* Answer	Strategy: Separate	Strategy: Count down	Strategy: Add on	Strategy: Count up	Strategy: Match	Strategy: Heuristic or fact
Separate	1	39	31	19	9	0	0	4	5
	2	40	32	29	6	0	0	0	4
Part-Part-Whole	1	33	20	9	3	10	3	4	4
	2	38	33	25	0	3	0	1	6
Comparison	1	35	29	8	0	2	3	17	4
	2	35	31	22	2	0	2	7	2
Equalizing	1	39	30	9	0	5	1	15	6
	2	37	34	19	2	5	2	7	3

*Total N = 43 subjects.

Table 4

Summary of Sentence Writing

Problem	Number of correct sentences*	Number who wrote the sentence after solving the problem
Addition		
Part-Part-Whole	39	9
Comparison	30	8
Subtraction		
Separating	39	2
Part-Part-Whole	32	9
Comparison	19	12
Equalizing	15	15

*Total N = 43 subjects.

Table 5

Examples of Open Sentence Responses

Problem:	There are 12 children on the playground. Five are boys and the rest are girls. How many girls are at the playground?

Correct responses

12 - 5 = ___

12 - 5

5 + ___ = 12

12 - 5 = 7

5 + 7 = 12

Incorrect responses

5 - 12 = ___

12 + 5 = ___

12   5

Table 6

Comparison of Addition Problems With and Without Number Sentences

Problem	Write sentence	Total Correct*		Strategy				Wrong		
		Strategy	Answer	Count all	Count from first	Count from larger	Heuristic or fact	Given number	Wrong operation	No response or other error
Part-Part-Whole	No	43	39	23	4	8	5	0	0	0
	Yes	42	38	23	5	8	3	0	0	1
Comparison	No	20	19	16	1	1	1	12	9	2
	Yes	21	20	12	3	1	4	8	6	8

*Total N = 43 subjects.

Table 7

Comparison of Subtraction Problems With and Without Number Sentences

Problem	Write sentence	Total Correct*		Strategy					
		Strategy	Answer	Separate	Count down	Add on	Count up	Match	Heuristic or fact
Separate	No	40	32	26	6	0	0	0	4
	Yes	41	36	28	7	0	0	0	4
Part-Part-Whole	No	38	33	25	0	3	0	1	6
	Yes	38	31	26	4	1	4	0	2
Comparison	No	35	31	22	2	0	3	7	2
	Yes	35	27	21	2	2	4	3	3
Equalizing	No	37	34	19	2	5	2	7	3
	Yes	34	28	20	1	4	4	3	3

*Total N = 43 subjects.