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ABSTRACT

This area package emphasizes three facets: (1) the concept of area as a covering; (2) the square unit; and (3) formula development. There are two enrichment activities included. The first requires the aid of a programmable calculator or computer. (Author/MK)

TOPICAL MODULE FOR USE

IN A

MATHEMATICS LABORATORY SETTING

TOPIC: AREA

by

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A Publication
of
University of Denver
Mathematics Laboratory
Regional Center for
Pre-College Mathematics

Dr. Ruth I. Hoffman, Director

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TEACHER'S GUIDE

Area

This module may be used in a variety of ways.

1. It may be used with a pretest as suggested in the teacher's guide.
2. Groups of students may be assigned to various experiments according to past performance.
3. The class may be divided so each group has a group leader who has the ability to help the other students.
4. Teachers may use a group-method of their own choosing or the entire class.

MATERIALS

- Contained in this Module:

1. One overlay with three networks. (SW #7)
2. One square-grid overlay. (SW #9)
3. One tangram set. (SW #5 and #6)

- Teachers must provide:

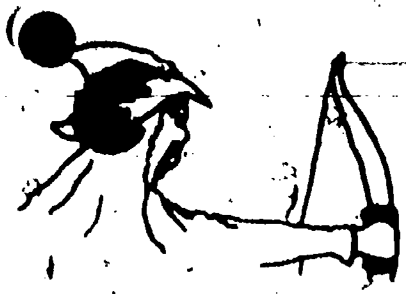
1. One pair of scissors (SW #1, 4, 14 and 15)
2. One geoboard and rubberbands. (SW #3, 8, 10, 12, and 13)

- Optional (Teacher provides)

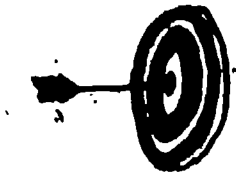
1. Computer (Enrichment #1)

TIME SCHEDULE

During field testing, this module took an average of ________ fifty-minute class periods to complete.



OBJECTIVES



- Given manipulatives such as a geoboard and tangrams, the student will develop the notion of covering as related to the concept of area.

- Given data, a student will develop formulas for the area of a square, rectangle, and triangle.

OVERVIEW

The area package emphasizes three facets - (1) the concept of area as a covering (cards 1-5), (2) the square unit (cards 6-8), and (3) formula developemnt (cards 9-15). There are two enrichment activities included. The first requires the aid of a programmable calculator or a computer.

TESTING PROCEDURE

I. Pretest

A. The pretest checks the behavior entry level of the students to determine if they need additional preparation before beginning work on this module.

B. Students who are unsuccessful on the pretest may have difficulty working this module, and they may need additional help.

II. Posttest

A. The posttest checks to see if the student is able to successfully complete the objectives established for this module.

Outline

Card 1

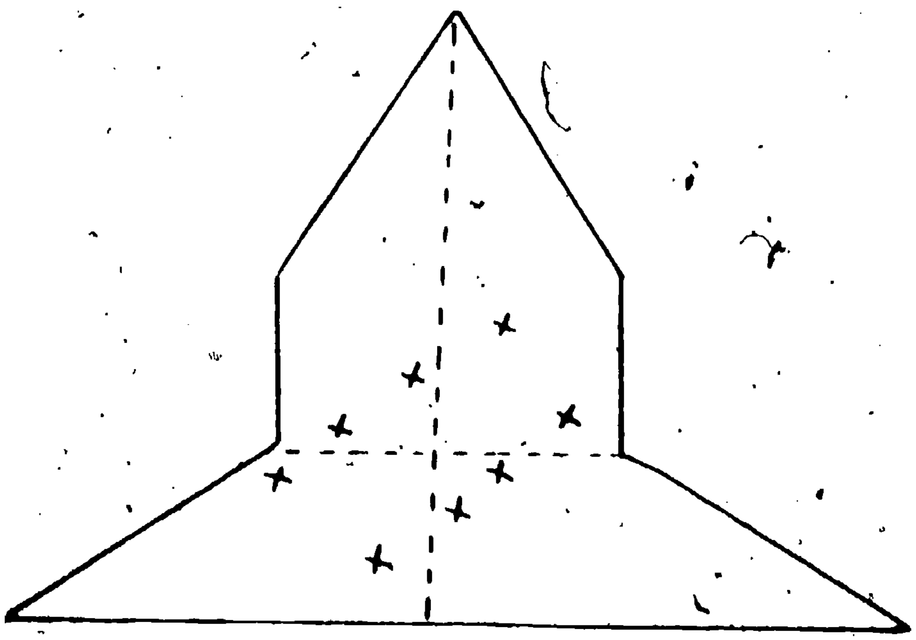
A. Teaching suggestions

1. Figures are equal in area if they are congruent.
2. Students may need several copies of 1A in order to explore freely.

B. Materials

1. Scissors
2. Several copies of 1A.

C. Solution



Card 2

A. Teaching suggestions

1. Figures with the same shape are different if the space they "cover" (area) is different.
2. Figure can be cut to show congruence similar to Card 1.

B. Materials

1. It may be an advantage to enlarge the figure on Card 2 and display in front of the class.
2. Copies of 2A to be completed by the student.

C. Solution

1. 10 squares -- 4 different-sized squares
2. 20 triangles -- 4 different-sized triangles.

III. Card 3

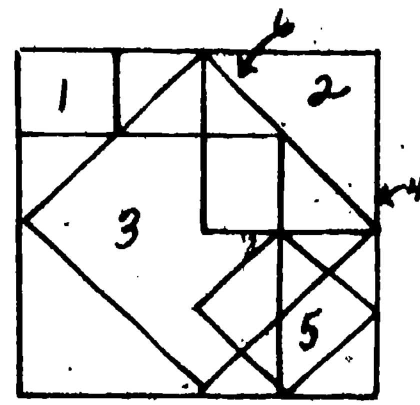
A. Teaching suggestions

1. Begin introducing the term "area".
2. Variation in size may be determined by the number of pegs (nails enclosed by the square) without introducing unit measure.

B. Materials

1. Instruction sheet
2. Geoboard and rubber bands.

C. Solution



IV. Card 4

A. Teaching suggestions

1. Figures need not be congruent to be equal in area.

2. Students will need several copies of 4A in order to develop congruent figures.

B. Materials

1. Scissors

2. Several copies of 4A.

C. Solution

All five figures have the same area.

V. Card 5

A. Teaching suggestions

1. Congruence is a sufficient, but not a necessary condition for equal area.

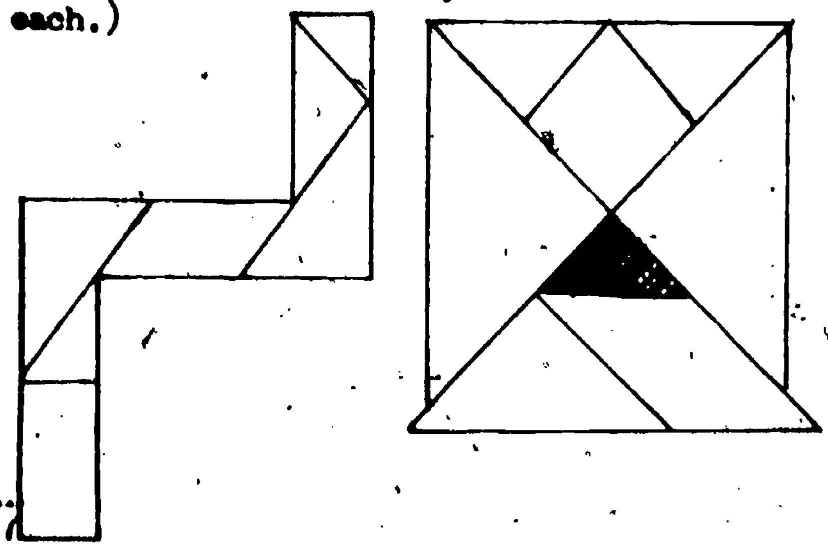
2. Supply each student with a tangram set.

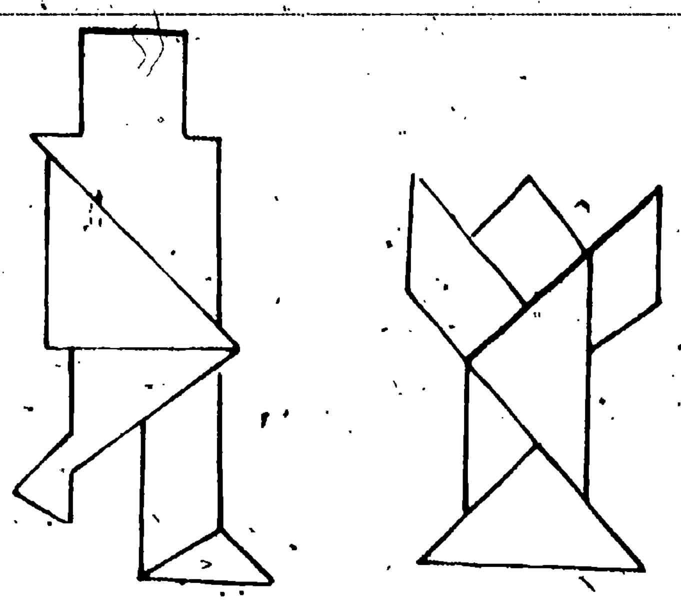
B. Materials

1. Copies of $5A_1$ and $5A_2$.

2. Set of tangrams.

C. Solution (there are many solutions for each.)





VI. Card 6

A. Teaching suggestions

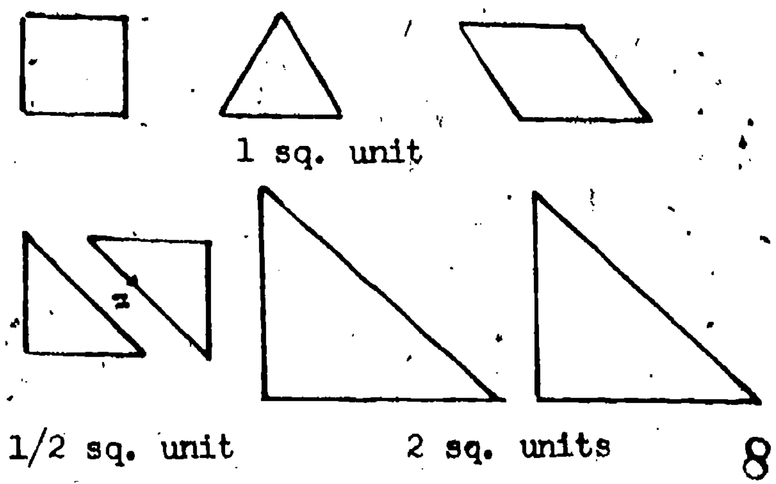
1. Assigns a number value to measures of area.

2. Suggest finding the area of the two small triangles first, then use these (plus the square when necessary) to find the area of the larger pieces.

B. Materials - set of tangrams.

C. Solution

1.



2. Area of figures on $5A_1$ and $5A_2$ is 8 sq. units.

VII. Card 7

A. Teaching suggestions

1. Any consistent square unit can be used for measuring area.

2. Answers will not be exact.

B. Materials

1. Copies of 7A and 7B.

2. Network overlay.

C. Solutions are in surface units (sur.)

1.

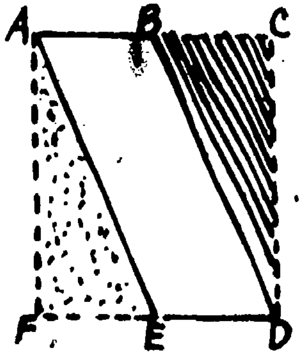
Fig.	Network 1	Network 2	Network 3
A	51 sur. units	15 sur. units	13 sur. units
B	77 sur. units	21 sur. units	19 sur. units
C	93 sur. units	26 sur. units	23 sur. units
D	57 sur. units	17 sur. units	15 sur. units

2. Even if the numbers vary, the size of the figure remains constant.

VIII. Card 8

A. Teaching suggestions

1. Demonstrate the subtractive method.



The entire rectangle A C D F contain six square units.

The shaded portion Δ B C D contains 1/2 of the area of rectangle B C D E.

Since the area of rectangle B C D E is 3 sq. units, B C D = 1 1/2 sq. units.

The same is true for rectangle A B E F and A E F.

Therefore, the area of A B D E equals

$6 - (1\frac{1}{2} + 1\frac{1}{2}) = 6 - 3 = 3$ sq. units.

2. Suggest small group work.

3. Can be done without geoboard as a paper and pencil exercise.

B. Materials

1. Copies of $8A_1$, $8A_2$, 8B, and 8C.

2. Geoboard and rubber bands.

C. Solution

1.	2 sq. units	8.	3 sq. units
2.	$\frac{1}{2}$ sq. units	9.	7 sq. units
3.	2 sq. units	10.	3 sq. units
4.	$1\frac{1}{2}$ sq. units	11.	7 sq. units
5.	2 sq. units	12.	1 sq. unit
6.	4 sq. units	13.	$5\frac{1}{2}$ sq. units
7.	3 sq. units	14.	2 sq. units

IX. Card 9

A. Teaching suggestions

1. Cards 9-15 are designed to aid the students in discovery of the parameters important in writing area formulas.

2. Can be done as a classroom exercise.

B. Materials

1. Copies of 9A and 9B.
2. Square acetate network.

C. Solution

1. The length of the side changes (student may respond "perimeter",) radius (or diameter or circumference), sides, and sides or base and height.
2. Each square is $\frac{25}{4} = 6\frac{1}{4}$ sq. units (approx. 6).
3. Each side measures 3 units.
4. When side is doubled, area increases by a factor of 4.
5. If the area is doubled, the side length increases by a measure of 2.

X. Card 10

A. Teaching suggestions

1. If using the acetate is cumbersome, merely assign a value of one square unit to the smallest square on the geoboard.
2. Encourage students to look at the relationships.

B. Materials

1. Copies of 10A.
2. Geoboard and rubber bands.

C. Solution

1. As S increases, A increases.
2. $A = 4 \cdot S$.

XI. Card 11

A. Teaching suggestions

1. Group discussion of formulas.
2. Discuss the difference between constants and variables.
3. 11A is a checkpoint to tell if the student is making the correct generalization.

B. Solutions

1. The area of a square is a product of a side times a side (or a side squared.)
2. $A = S^2$.
3. The area of a rectangle is a product of the length times the width.
4. $A = l \cdot w$ (or $A = b \cdot h$).

XII. Card 12

A. Teaching suggestions

1. Discussion of right triangles.
2. The area of a right triangle is one-half the enclosing rectangle.

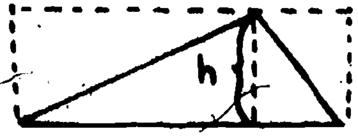
B. Material

1. Geoboard and rubber bands.

XIII. Card 13

A. Teaching suggestions

1. Discussion of height of a triangle.
2. Relate to height of a right triangle.
3. Compare with two rectangles



B. Material - Geoboard

C. Solution

1. $A = \frac{1}{2} b \cdot h.$

2. The area of a triangle is 1/2 of an enclosing rectangle with the same base and height.

XIV. Card 14

A. Teaching suggestions

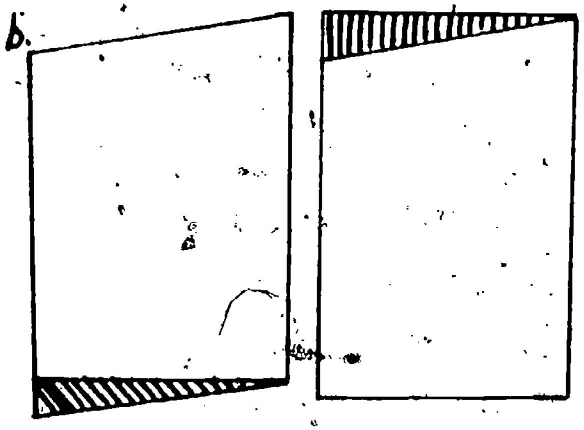
1. Define a parallelogram.

2. Compare the parallelogram to a rectangle.

B. Solution

1. a

b.



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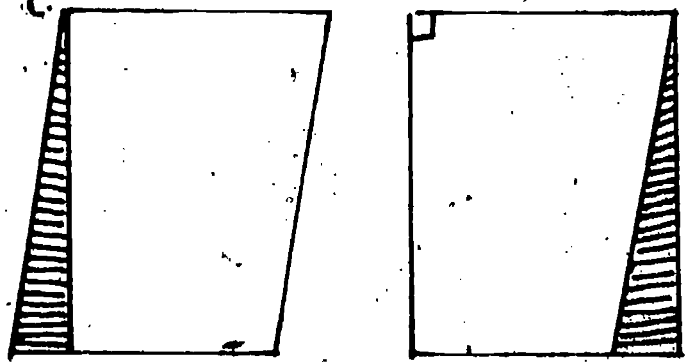


C. Materials - Scissors

XV. Card 15

A. Teaching suggestions

1. Stages

a. Complete the triangles as parallelograms on a transparency for overhead use.

b. Sketch lines converting parallelograms into a rectangle.

2. Students can contribute more examples.

B. Solution

1. $A_{\square} = b \cdot h$

2. $A_{\Delta} = \frac{1}{2} \cdot b \cdot h.$

XVI. Enrichment I

A. Teaching suggestions

1. Can be used without a computer by having students do the calculations, following the flowchart.

2. Designed as a small group activity.

B. Solutions

<u>Square</u>

1. Double area - side increases by the multiple $\sqrt{2}$.

2. Triple area - side increases by the multiple $\sqrt{3}$.

3. Reduce area by one-half - side is reduced by multiple $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$.

4. $A = s^2$.

Circle

1. Double the area - radius increases by the multiple $\sqrt{2}$.
2. Triple the area - radius increases by the multiple $\sqrt{3}$.
3. Reduce area by one-half - radius is reduced by the multiple $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$.
4. $A = \pi r^2$.

Right Triangle

1. Double area - double height.
2. Triple area - triple height.
3. Reduce area by one-half - reduce height by one-half.
4. $A = \frac{1}{2} b \cdot h$.

XVII. Enrichment II

A. Teaching suggestions

1. Can be used in conjunction with a graphing module.
2. For a class exercise, make a transparency of E-IIA for use on the overhead projector.

B. Solution

Curve 1		Curve 2		Curve 3	
L	A	L	A	L	A
1	1	1	3.1	1	5
2	4	2	12.6	2	10
3	9	3	28.3	3	15
4	16	4	50.2	7	35
5	25	2.5	19.6	4	20
6	36	5	78.5	2.5	12.5
7	49	8	201	3.5	17.5
8	64	11	379.9	6	30
4.5	20.25				
6.5	42.25				
5.5	30				
2.5	6				
$A = L^2$ Square		$A = L^2$ Circle		Rectangle Triangle	

Answer Key - Area

Pretest

1. a. 90 sq. inches b. 11 inches
2. a. 9 sq. inches b. 12 sq. inches
   c. 24 sq. feet

Posttest

1. a. 9 sq, units b. 4 sq. units
   c. 5 sq. units d. 8 sq. units
   e. 9 sq. units
2. a. 25 sq. inches b. 6 sq. inches
   c. 6 sq. feet d. 153.86 sq. inches
3. 1,695.60 sq. inches.

Pretest

20 points for each answer

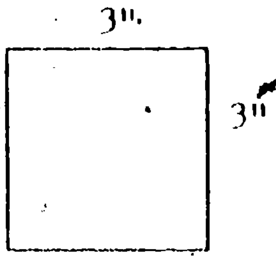
1. Simplify the following expressions.

   a. 6 in. x 15 in. =

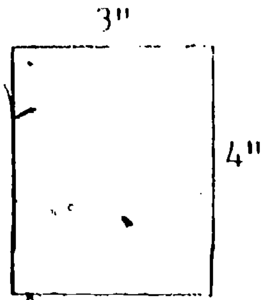
   b. 5 1/2" + 3" + 2 1/2" =

2. Find the area of the following figures.

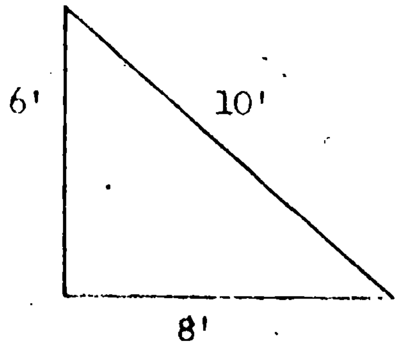
   a.



   b.



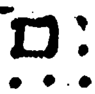
   c.

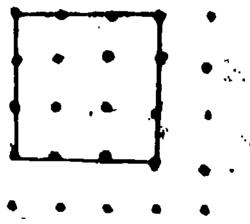


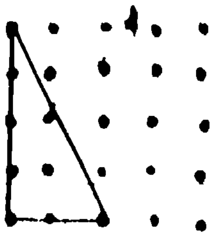
Posttest

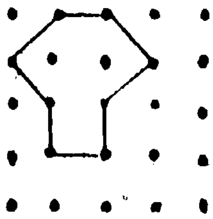
10 points for each answer

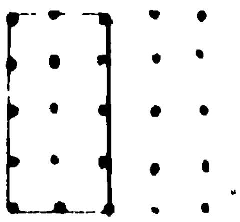
1. Find the area for each of the following figures:

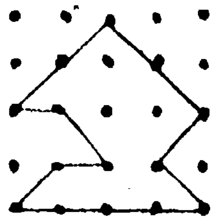
Given:  is one square unit.

a. 

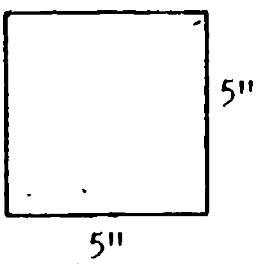
b. 

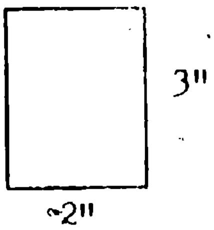
c. 

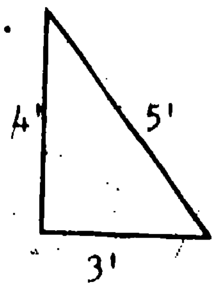
d. 

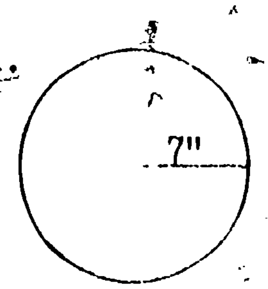
e. 

2. Find the area of each of the following figures.

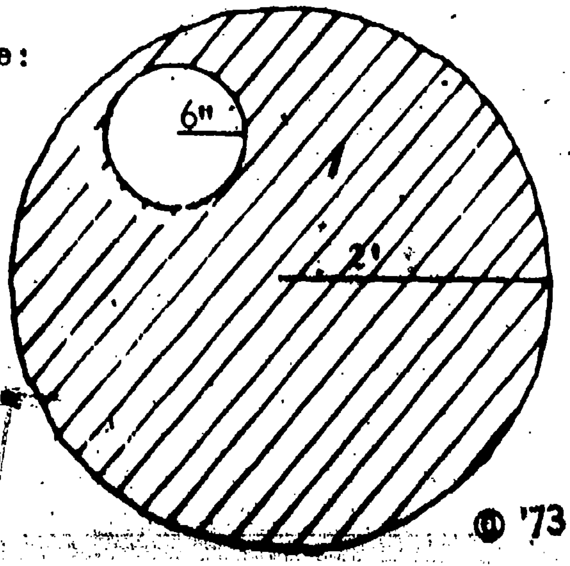
a. 

b. 

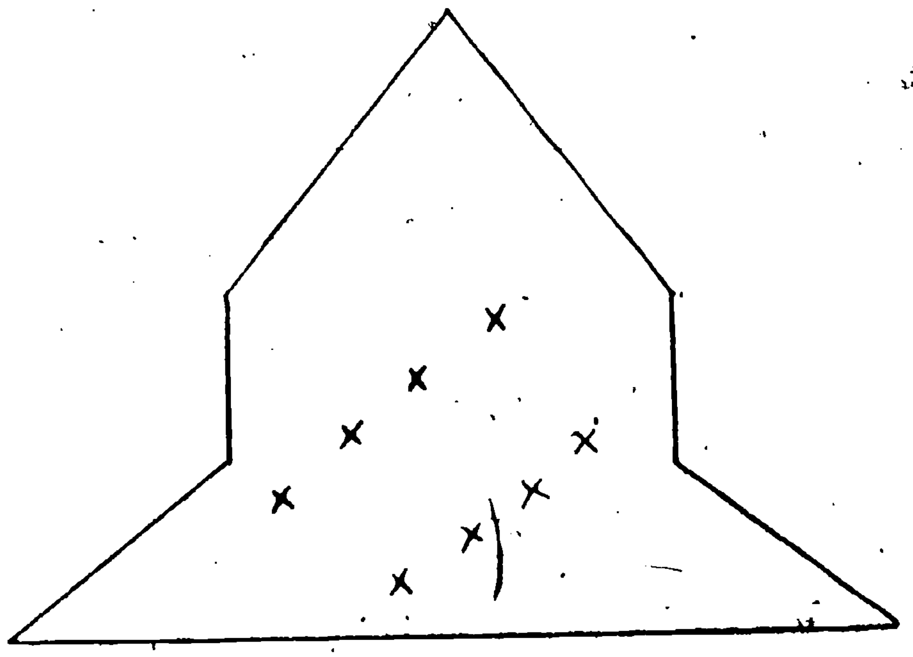
c. 

d. 

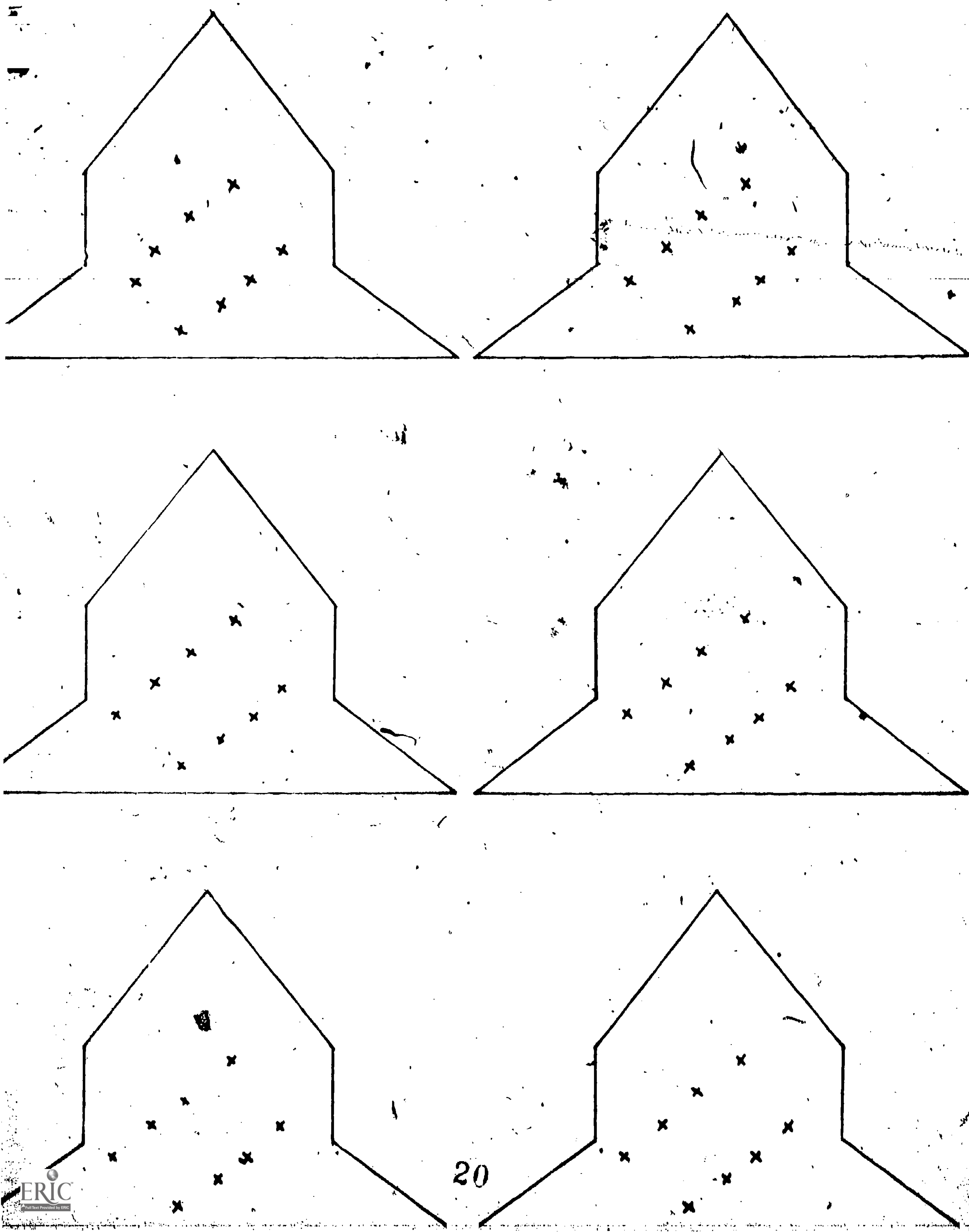
3. Find the area of the shaded figure:



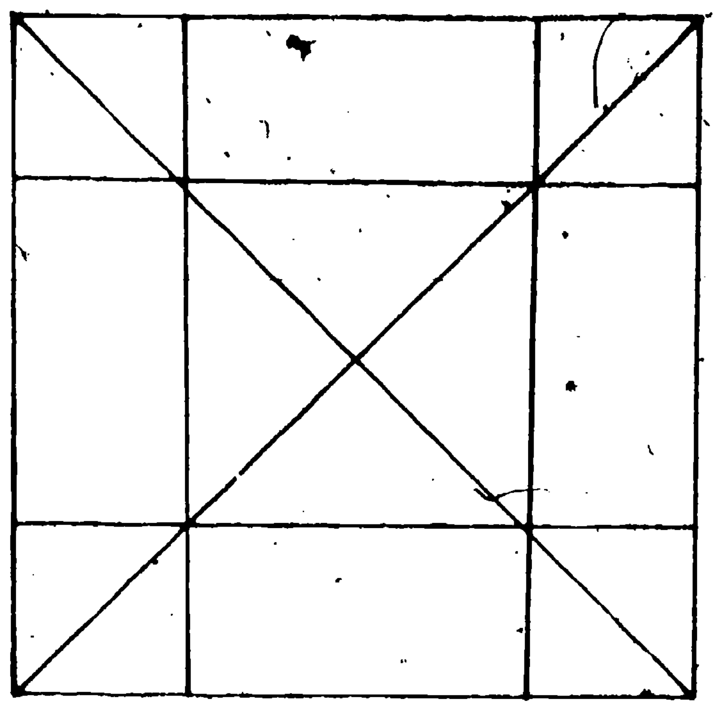
Examine the figure below. Using worksheet 1A and a pair of scissors, divide the figure into four sections each of which contains two x's and all of which are the same size and shape.



How can you tell if they are the same?



Refer to the figure below to answer the questions on 2A.



1. How many squares can you find?

2. How many different sized squares?
   Draw each

3. How many triangles can you find?

4. How many different sized triangles?
   Draw each

Construct as many different sized squares on the geoboard as you can.

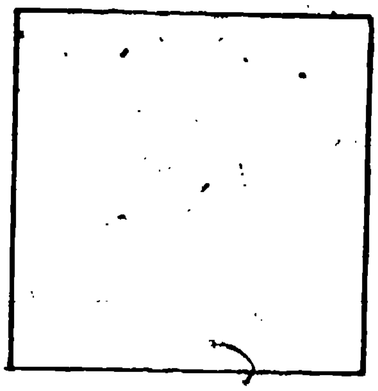
How can you determine if they are different sized?

What factor(s) determines if one square is larger than another?

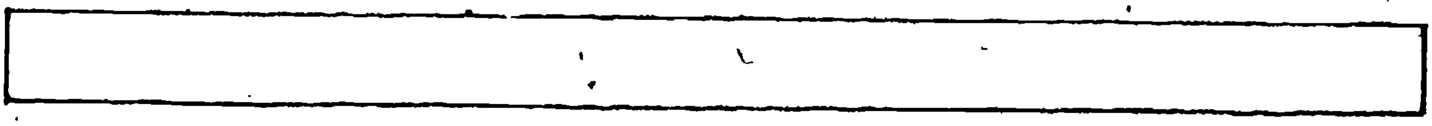
Determine which figure on worksheet 4A is the largest (has the greatest area) and which is the smallest (has the least amount of area).

Use scissors to cut the figures and overlap in order to compare the sizes.

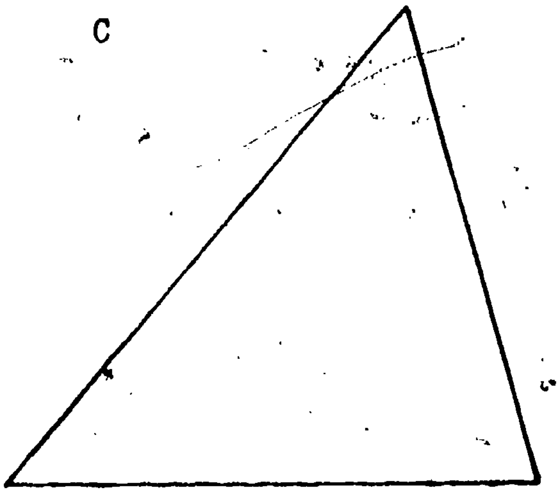
A



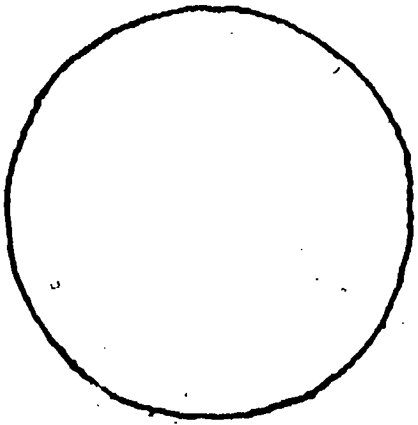
B



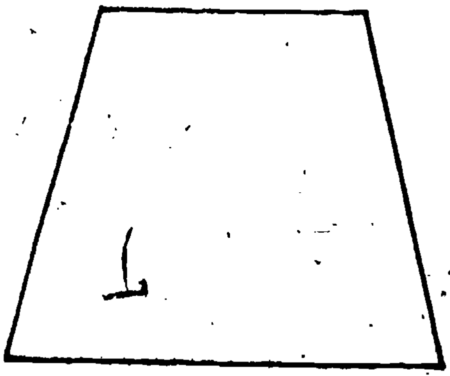
C



D



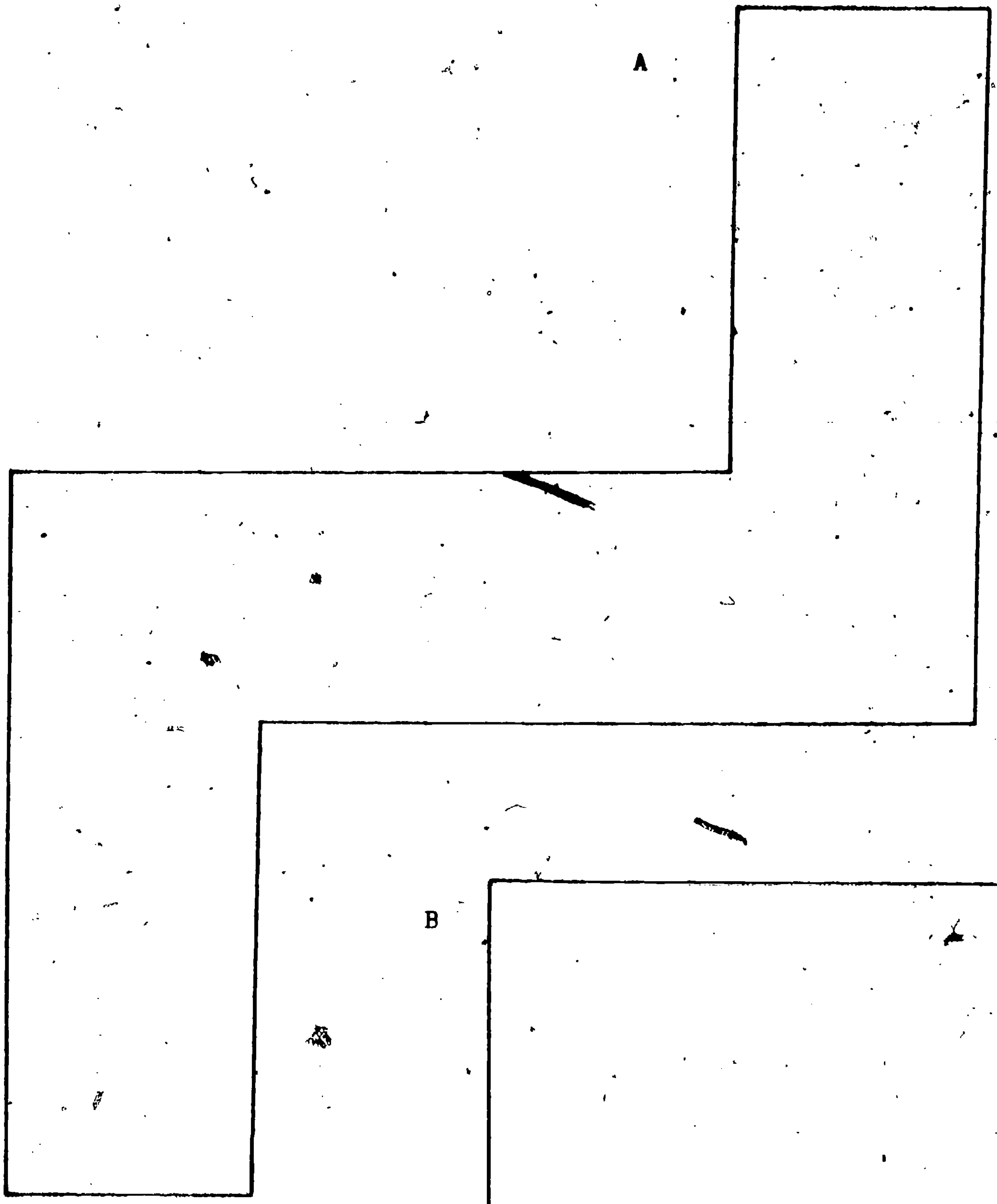
E



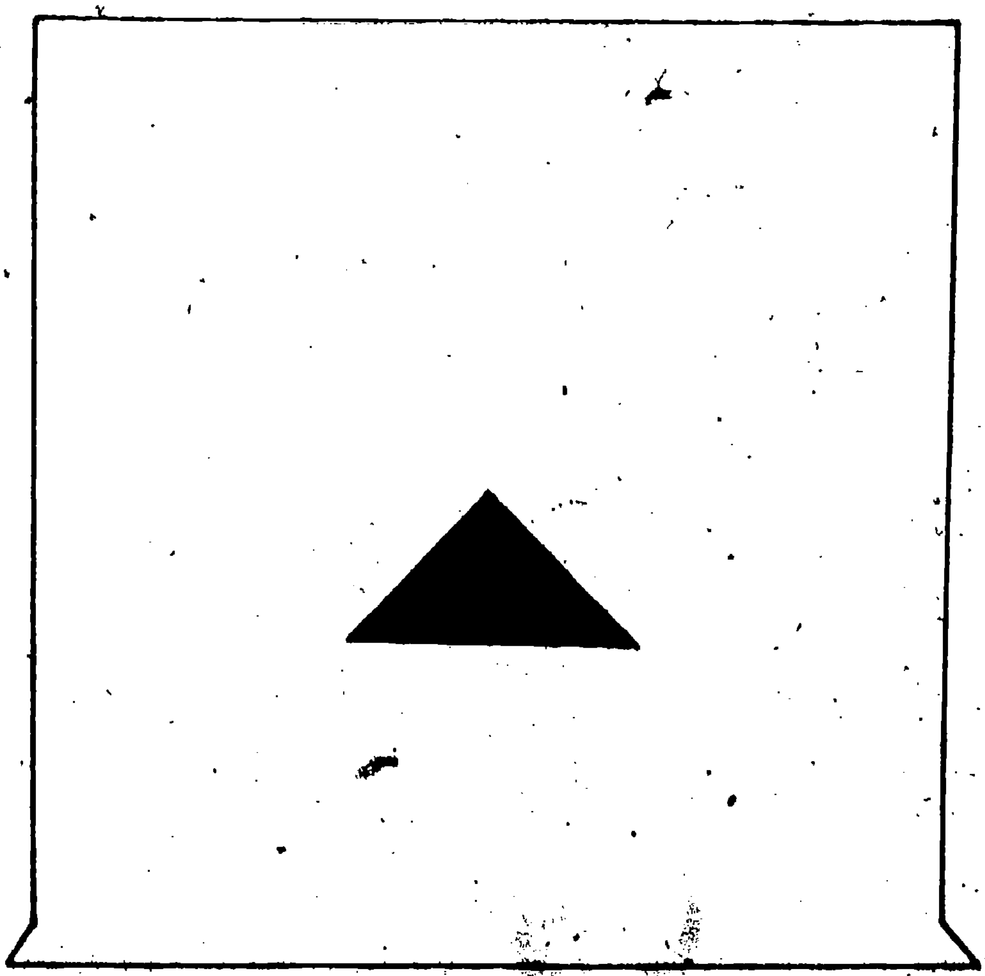
Using all seven tangram pieces, arrange them to fit the patterns on $5A_1$ and $5A_2$.

Which figures have the greatest area?

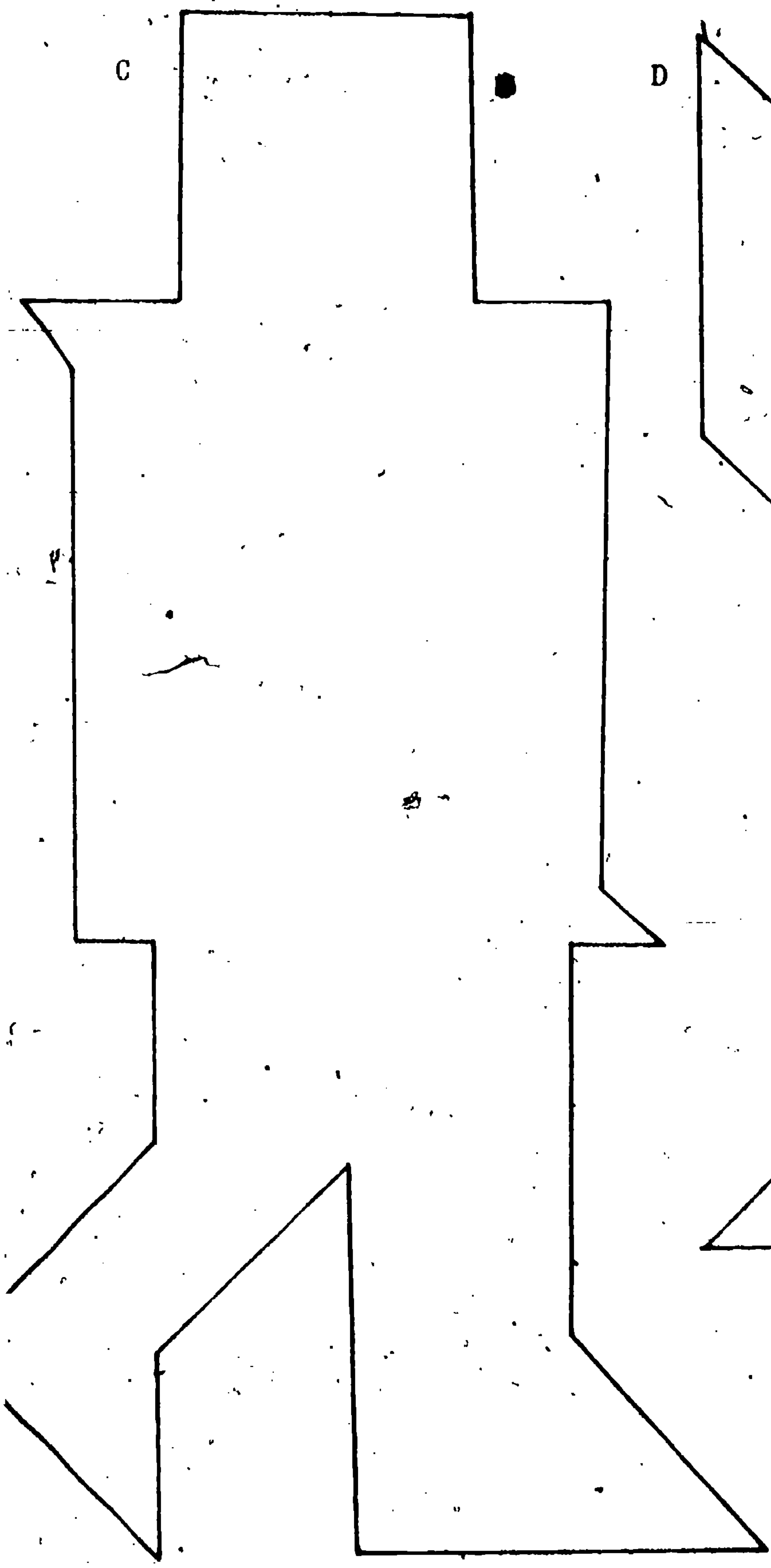
A



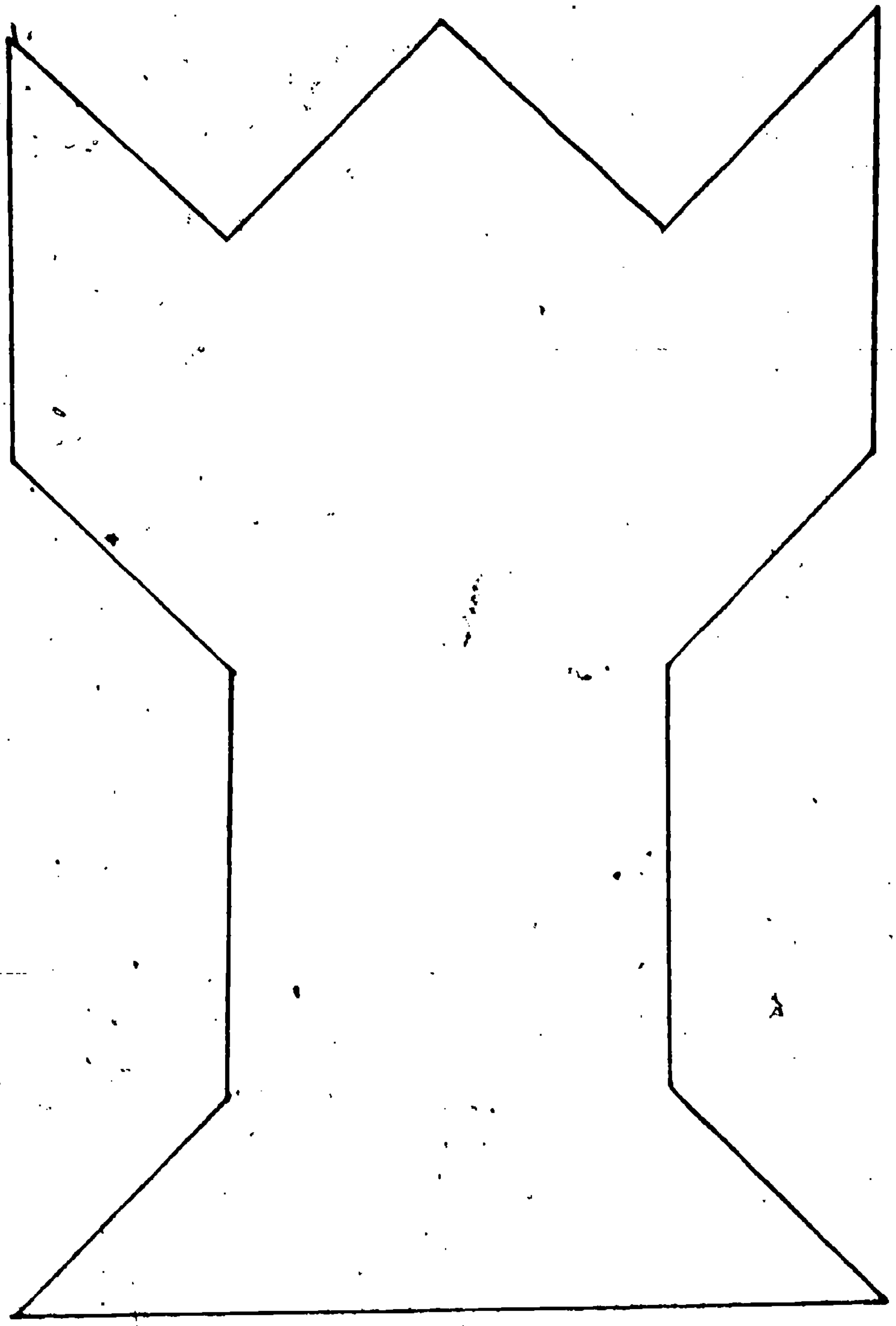
B



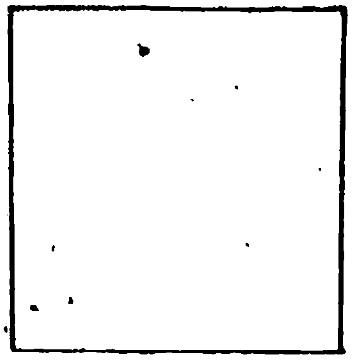
C



D



If the square in the tangram set represents one square unit, find the areas of the other six pieces.



What is the area of each figure of $5A_1$ and $5A_2$?

Any unit of coverage can be used for area. It is not necessary that it is square. Examine the overlay with network 1, 2 and 3. Any of these grids could be used to measure area with a unit of area consisting of a single closed figure in the network.

Using 7A, draw a unit of area for each of the networks.

Look at the figures on 7B. Which is largest? Measure each using the network and complete 7A.

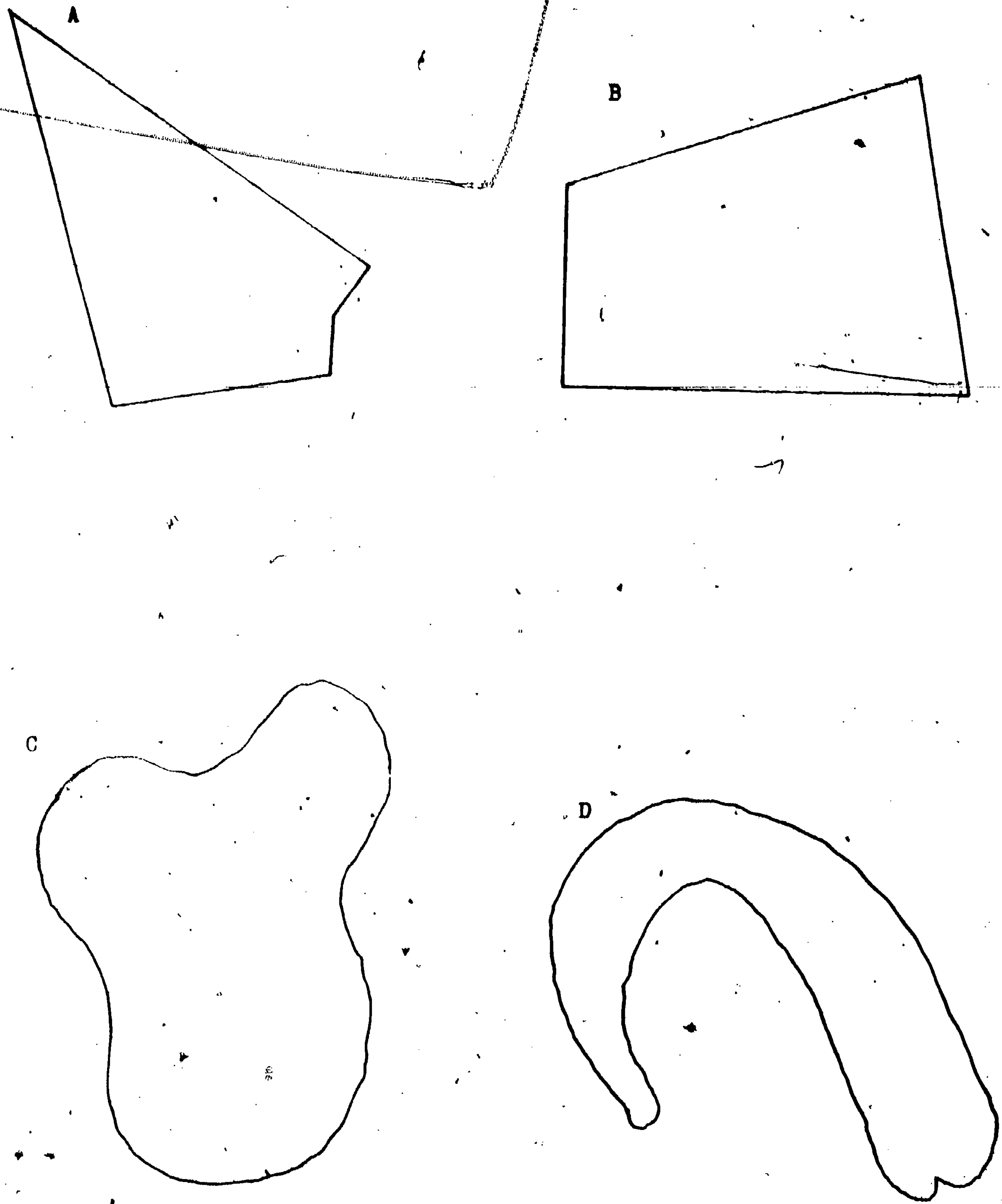
Units of area

Network 1 Network 2 Network 3

Area

Figure	Network 1	Network 2	Network 3
A			
B			
C			
D			

What remains constant in the relationship?


A
B
C
D

Construct the figures shown on $8A_1$ and $8A_2$ on the geoboard. If we let the smallest square that can be formed on the geoboard be one square unit, find the area of each.

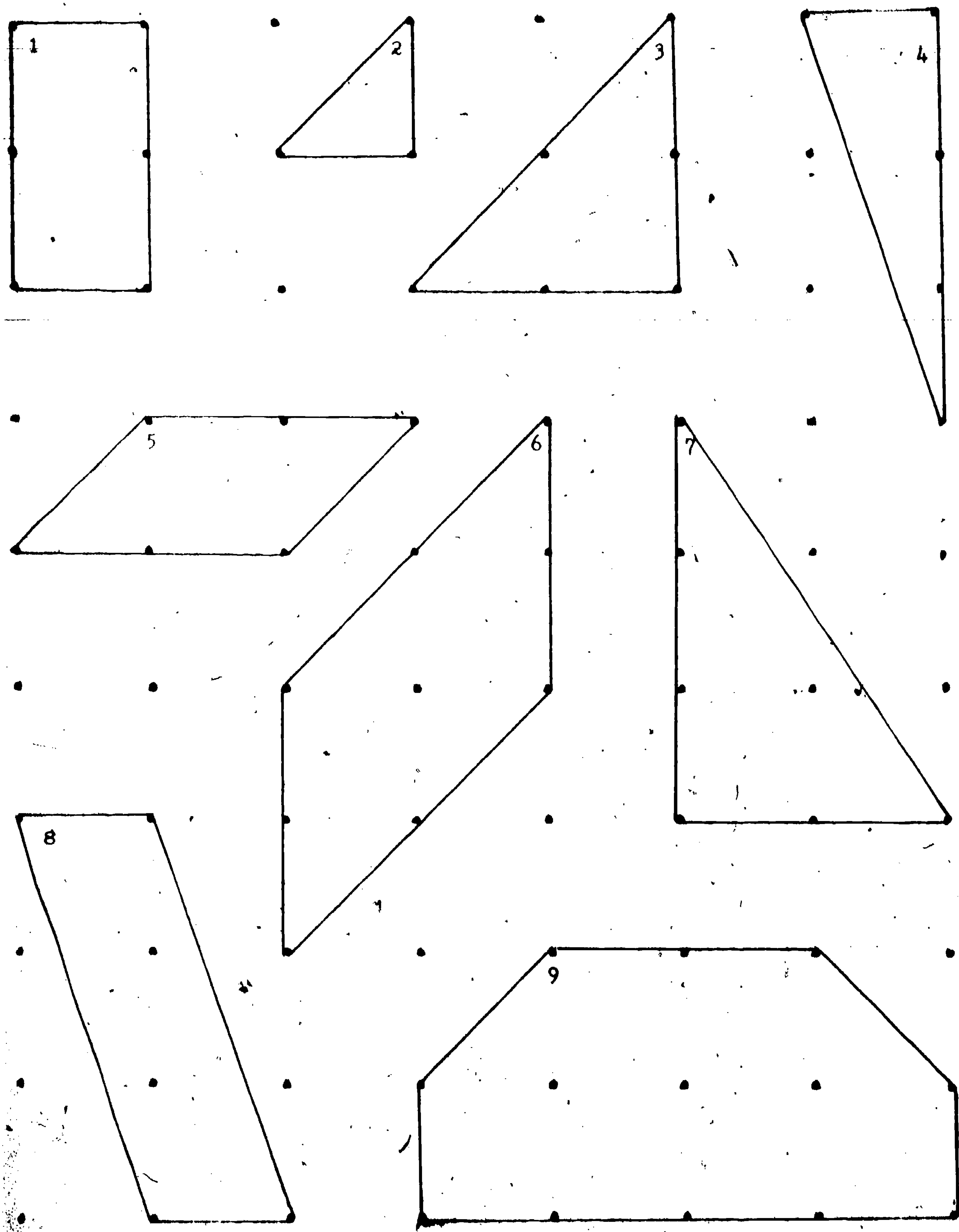
Record your findings on 8B.

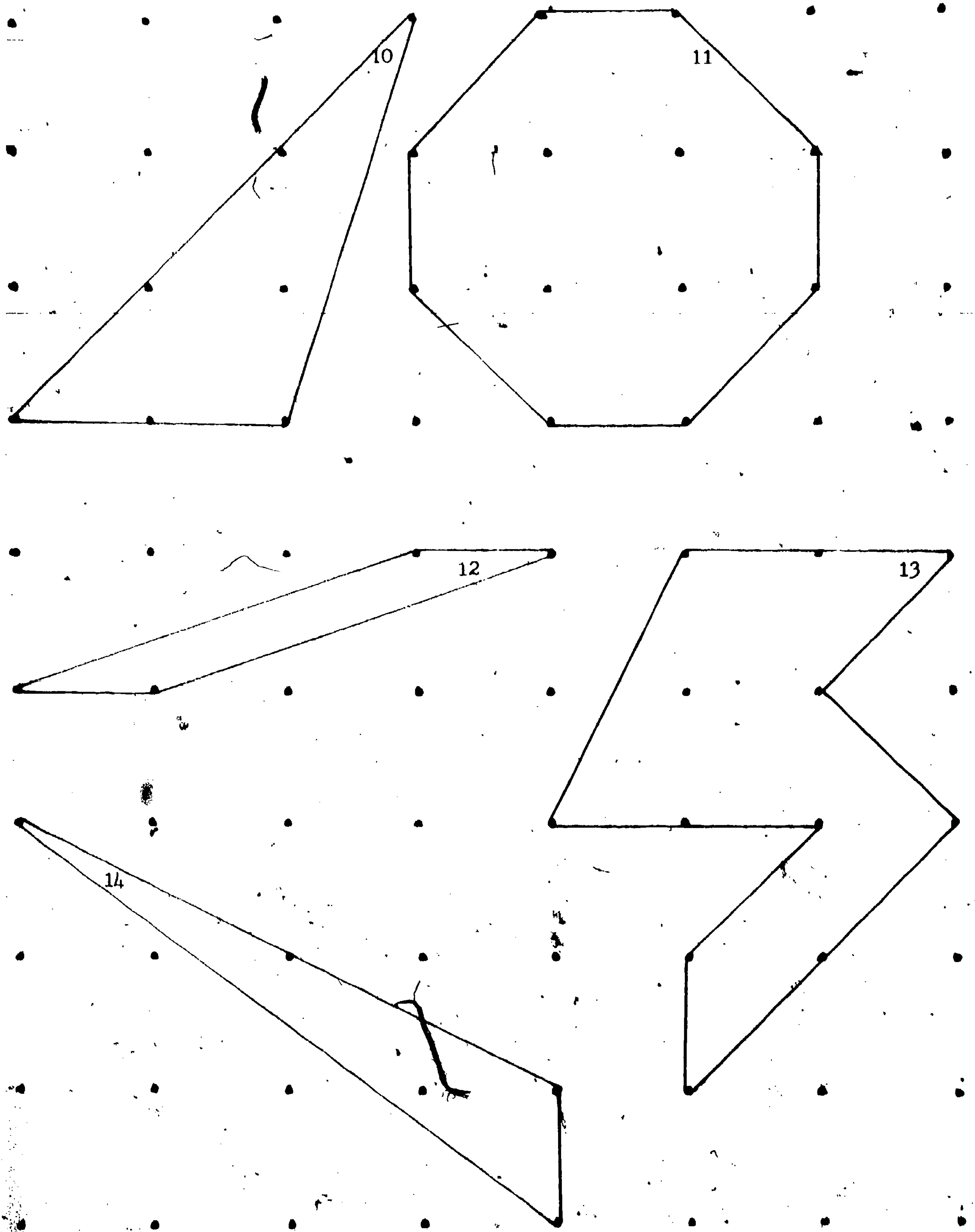
Construct your own figures, represent them on the dot paper, 8C, number each, and record their areas on 8B.

Challenge a friend to find the area of the figures you have constructed.

What if the smallest square of the geoboard represented two square units, how would the entries on 8B change?

What if the smallest square represented half a square unit, how would this affect the entries on 8B?


1
2
3
4
5
6
7
8
9


10
11
12
13
14

	Area		Area
		21	
		22	
		23	
		24	
		25	
		26	
		27	
		28	
		29	
		30	
		31	
		32	
		33	
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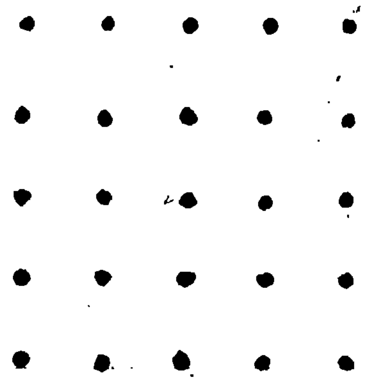
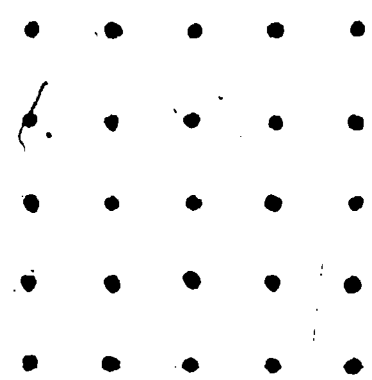
		35	
		36	
		37	
		38	
		39	
		40	



1



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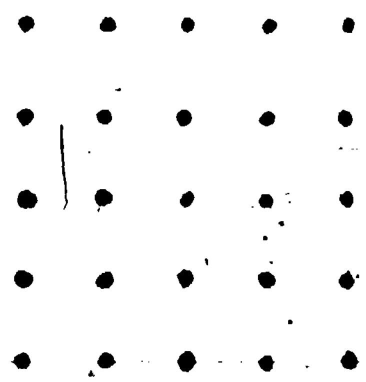

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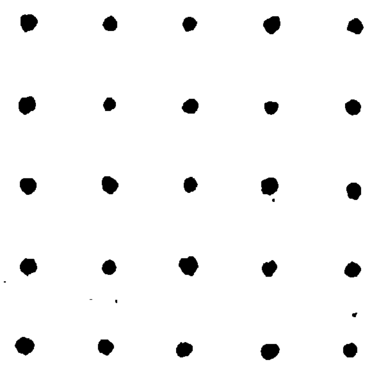

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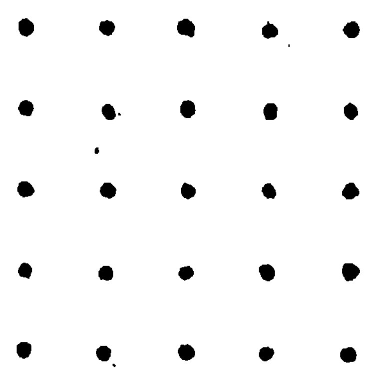

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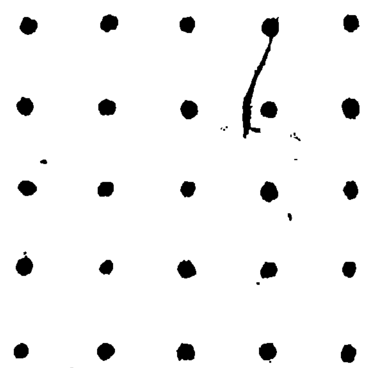

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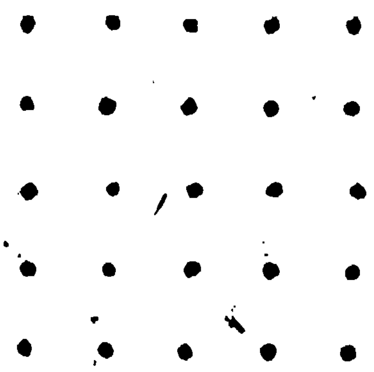

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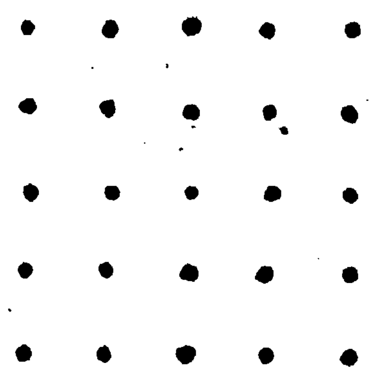

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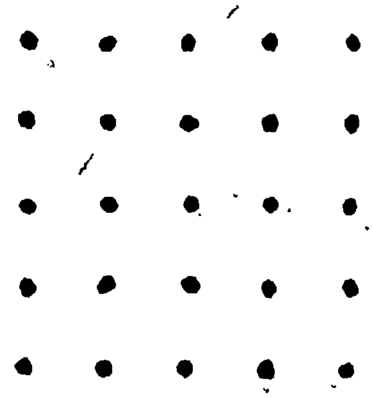

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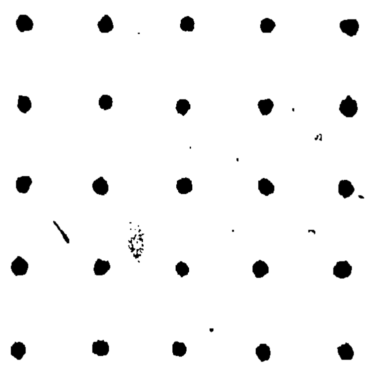

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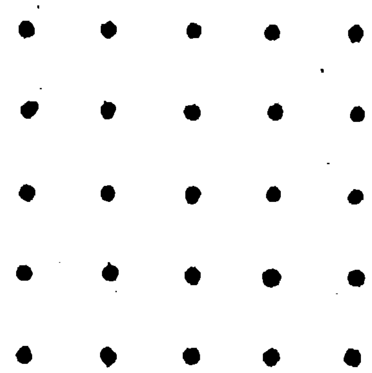
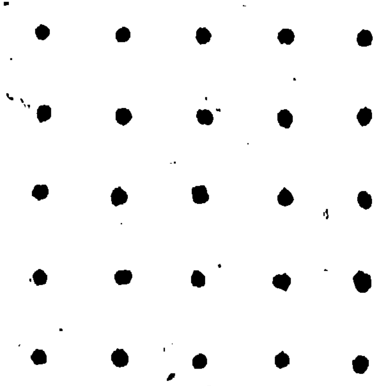
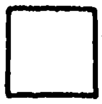


11



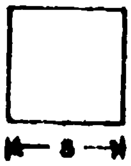
12

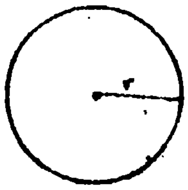
The objective of 9-15 is to help you understand how a formula (mathematical sentence) is developed. The figures discussed are 1) square, 2) rectangle, 3) triangle, 4) parallelogram. Note carefully what factors (variables) need to be considered and the relationship of the variables.

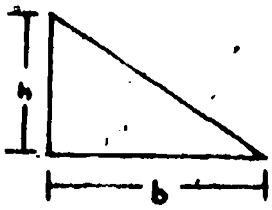
Our basic tool for area is called a square unit.  It is a square whose side is equal to one unit.

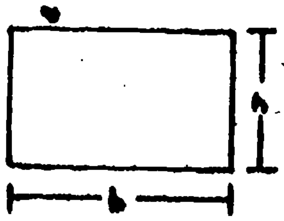
Complete worksheets 9A and 9B.

Examine the following

a) 

b) 

c) 

d) 

Draw similar figures each of which has a larger area. (s = side, r = radius, h = height, b = base) (Similar figures have the same shape, but may differ in size.)

a) b) c) d)

Which dimension(s) did you change in each?

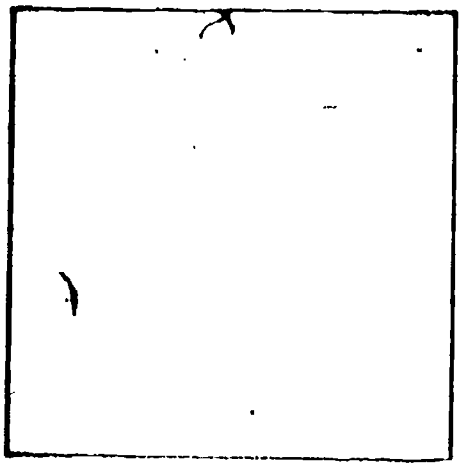
a) ______________________ c) ______________________

b) ______________________ d) ______________________

Could you have changed others and still increased the area?

a) ______________________ c) ______________________

b) ______________________ d) ______________________



The area of this square is 25 square units. (Use the grid as an overlay to check the area.)

Divide the square into four equal squares. What is the area of each square? ____________

Draw a square that contains nine square units. How long is each side? ____________

What happens to the area when you double the length of a side? (Look at the grid.)

What happens to the length of a side when you double the area?

Construct several squares on the geoboard. Measure s (side) and A (area) of each using the grid.

What happens to A as s increases?

Construct several rectangles on the geoboard.

Measure b (base), h (height) and A (area) with the grid.

Do you see a relationship?

Complete the chart on 10A.

Square

Area	Side	

Rectangle

Area	Length	Width

$P_s = 4 \times s$ is a formula which represents the perimeter of a square. It indicates that the perimeter of a square can be found by multiplying the constant, 4, times the length of the side, s.

A formula is a generalization we can use to solve problems quickly.

Complete 11A.

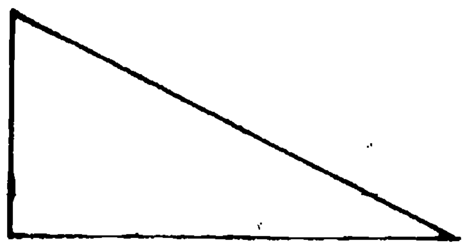
Write a sentence that describes the relationship between the sides and area of a square as expressed in table 10A.

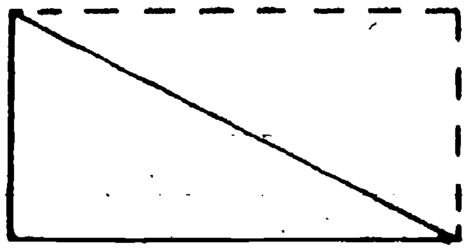
Write a mathematical sentence (formula) for area of a square.

Write a sentence that describes the relationship between the area of a rectangle and its sides as expressed in table 10A.

Write a formula for the area of a rectangle.

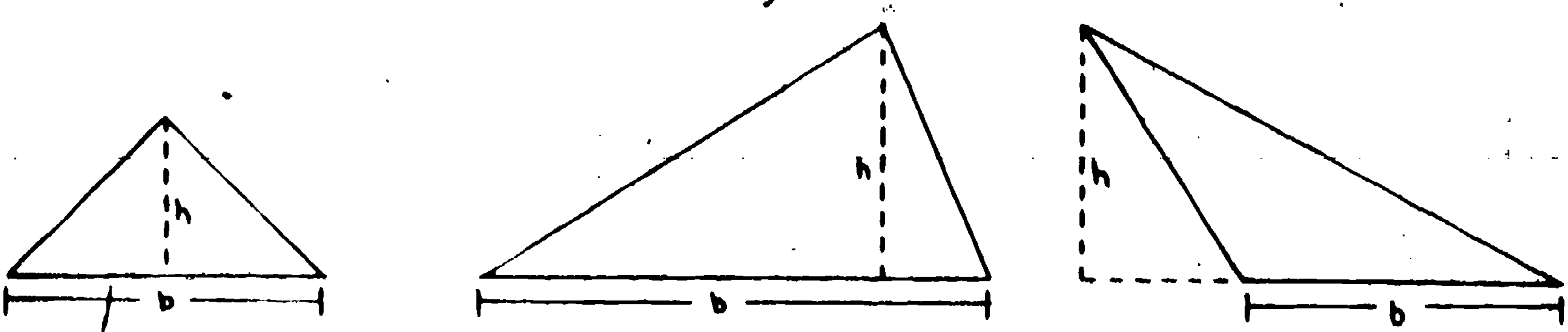
A right triangle is a triangle that contains a 90° (right) angle. Construct several right triangles on the geoboard. Make a rectangle by overlaying each right triangle with another rubber band, for example:





Do you see a relationship between the areas?

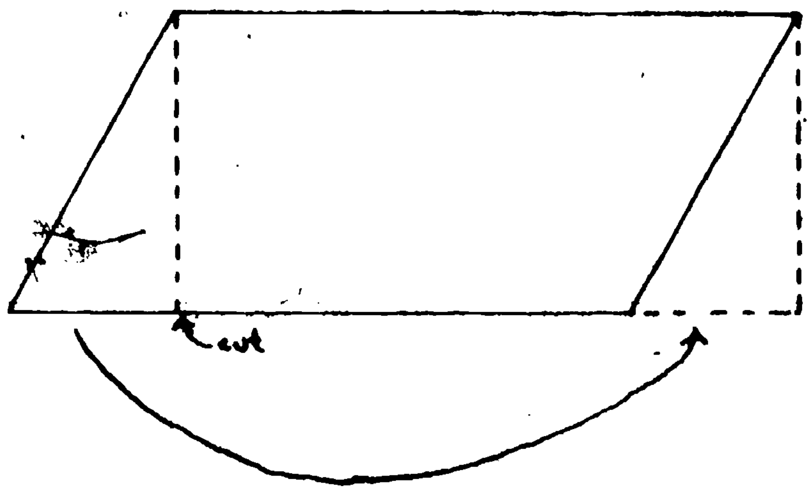
Not all triangles are right triangles, but the height can always be found by dropping a perpendicular (line forming a 90° angle) from the apex to the base.



Construct some triangles on the geoboard which are not right triangles. Can you find the height?

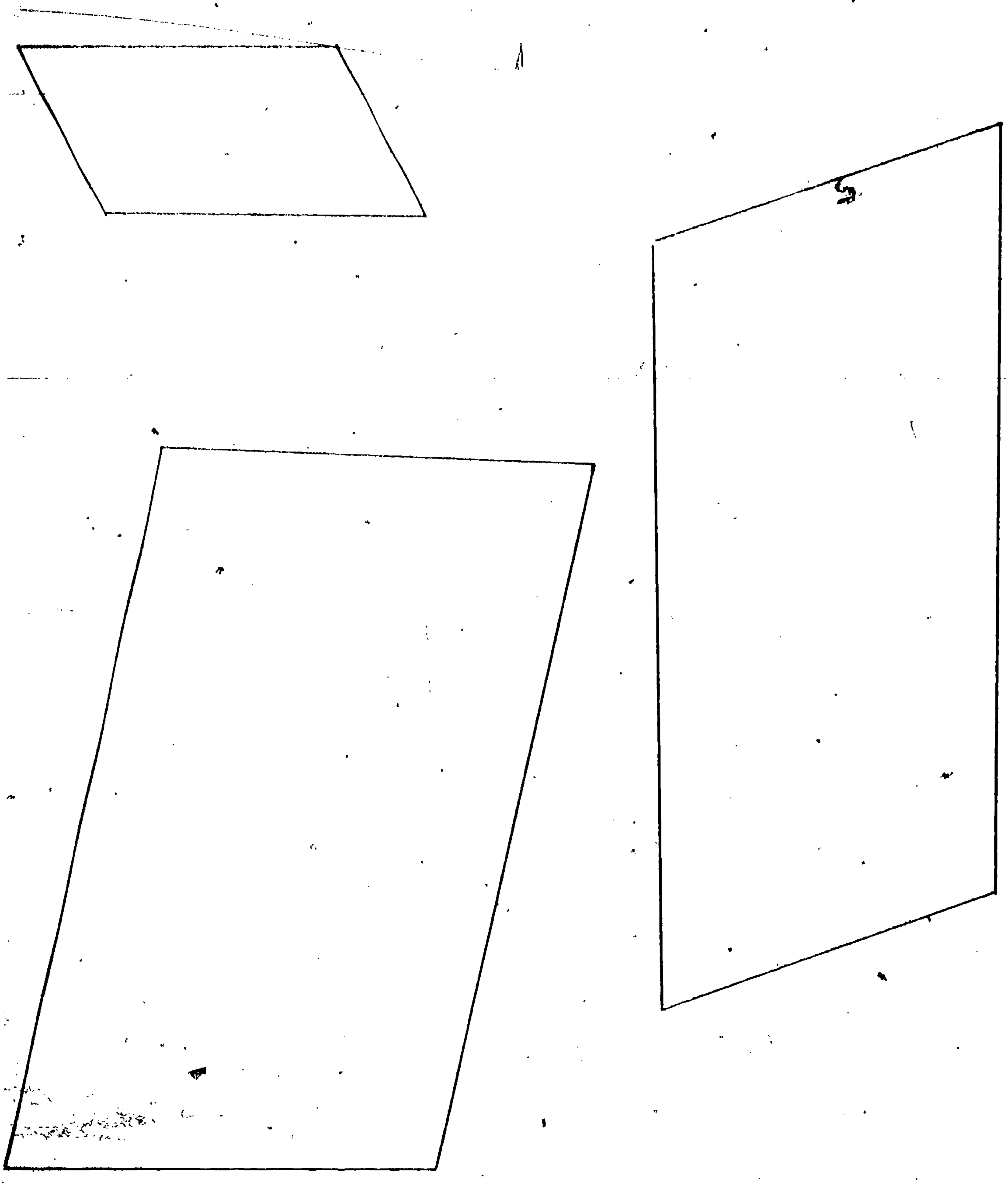
Can you write a formula for the area of a triangle?

Using a pair of scissors make each parallelogram (four sided figure with the opposite side parallel) on 14A into a rectangle. For example:



Can you find the height (perpendicular distance to the base) of each?

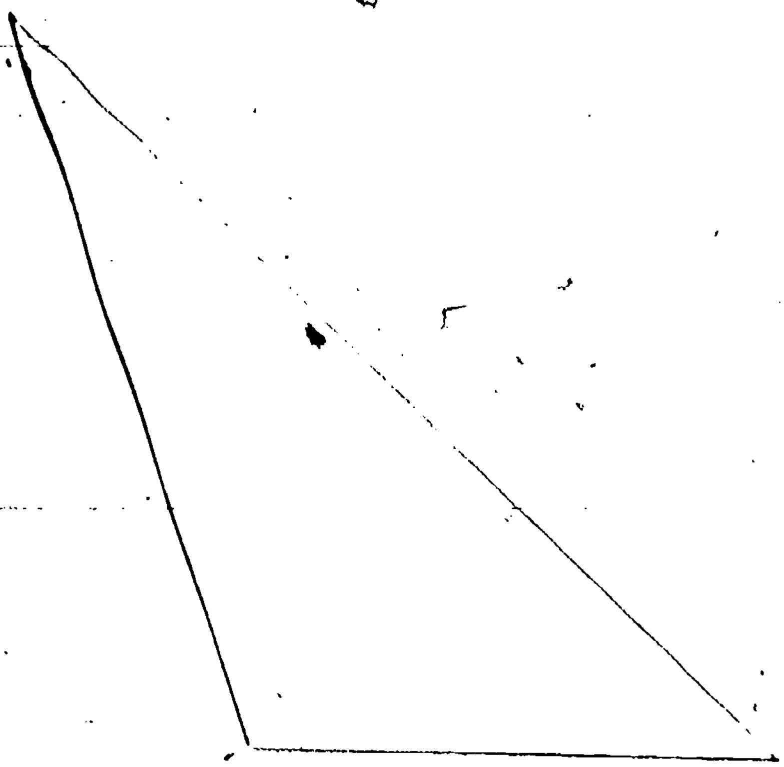
Can you write a formula for its area?

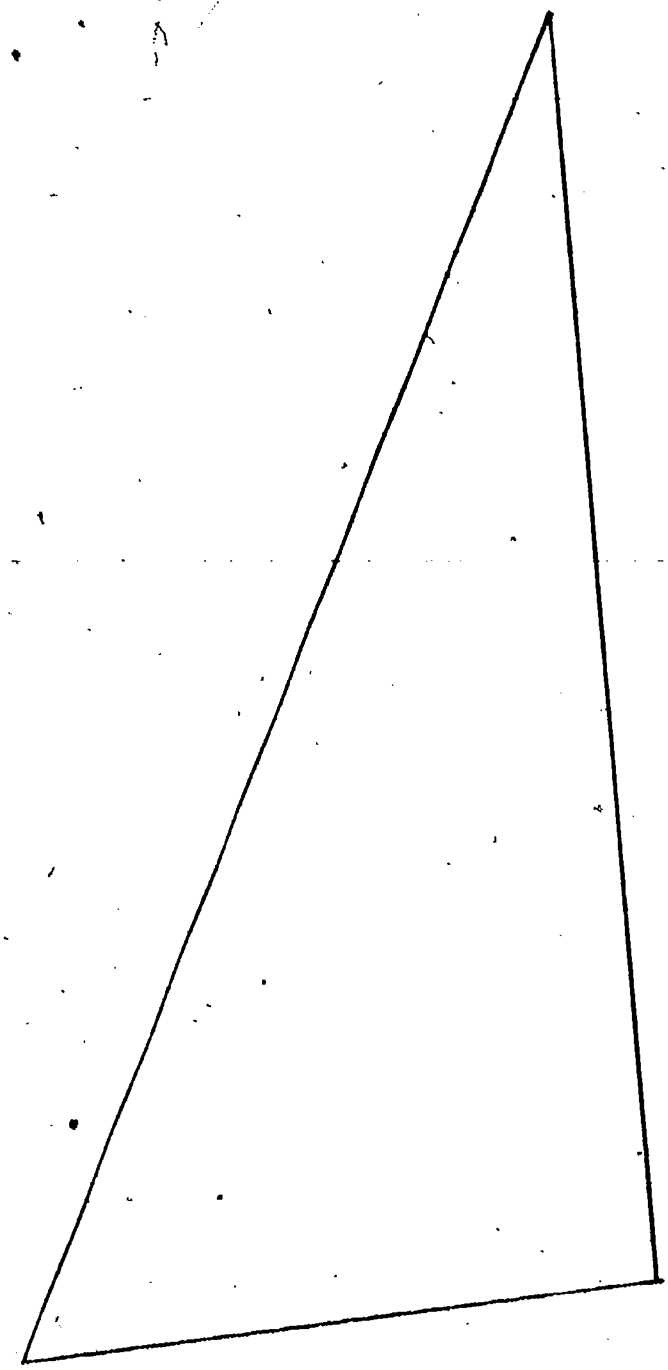


Add lines to each triangle on 15A to make a parallelogram. Use scissors to cut each parallelogram to form a rectangle.

Write a formula for the area of a parallelogram on 15B.

Write a formula for the area of a triangle on 15B.





What factors affect the area of

a) a square

b) a rectangle

c) a triangle

d) a parallelogram

Write a formula for the area of

a) a square

b) a rectangle

c) a triangle

d). a parallelogram

Use the supplied program for your computer to fill in the worksheet E18 and answer the questions.

You may choose any or all of the figures.

SQUARE

What happens to the length of a side if we double the area?

What happens to the length of a side if we triple the area?

What happens to the length of a side if we reduce the area by half?

Can you write the formula for the area of a square?

CIRCLE

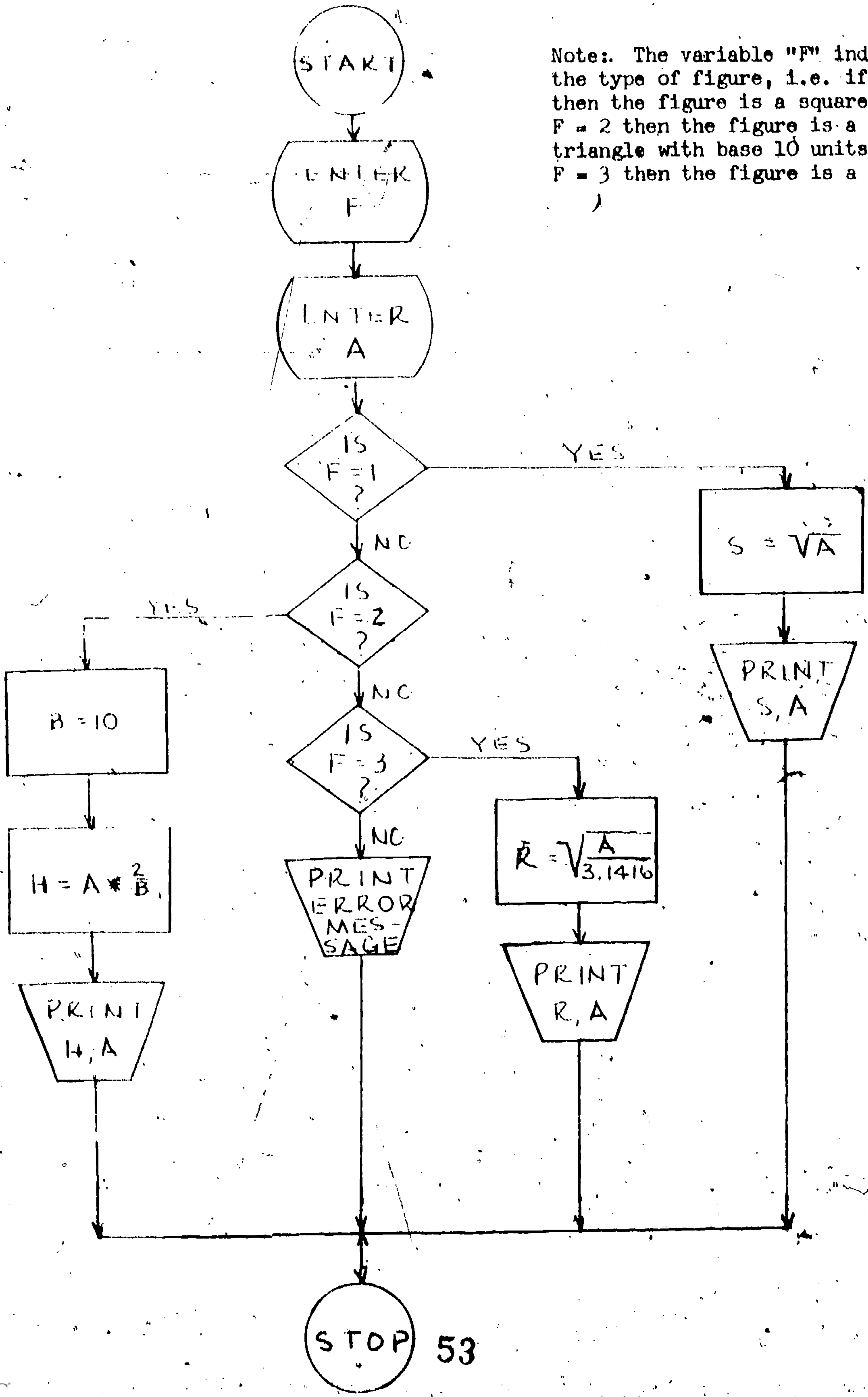
What happens to the length of the radius if we double the area?

What happens to the length of the radius if we triple the area?

What happens to the length of the radius if we reduce the area by half?

Can you write the formula for the area of a circle?

Note:. The variable "F" indicates the type of figure, i.e. if F = 1 then the figure is a square, if F = 2 then the figure is a right triangle with base 10 units, if F = 3 then the figure is a circle.



	SQUARE			TRIANGLE				CIRCLE	
	AREA	BASE		AREA	HEIGHT	BASE		AREA	RADIUS
	Sq. Units	Units		Sq. Units	Units	Units		Sq. Units	Units
	(All units to nearest tenth)								
			1				1		
			2				2		
			3				3		
			4				4		
			5				5		
			6				6		
7			7				7		
8			8				8		
9			9				9		
10			10				10		
11	49		11	2			11		
12			12	10			12		
13	4		13	10			13		
14			14				14		
15	20		15	7.5			15		

Use the supplied computer program to answer the following questions.

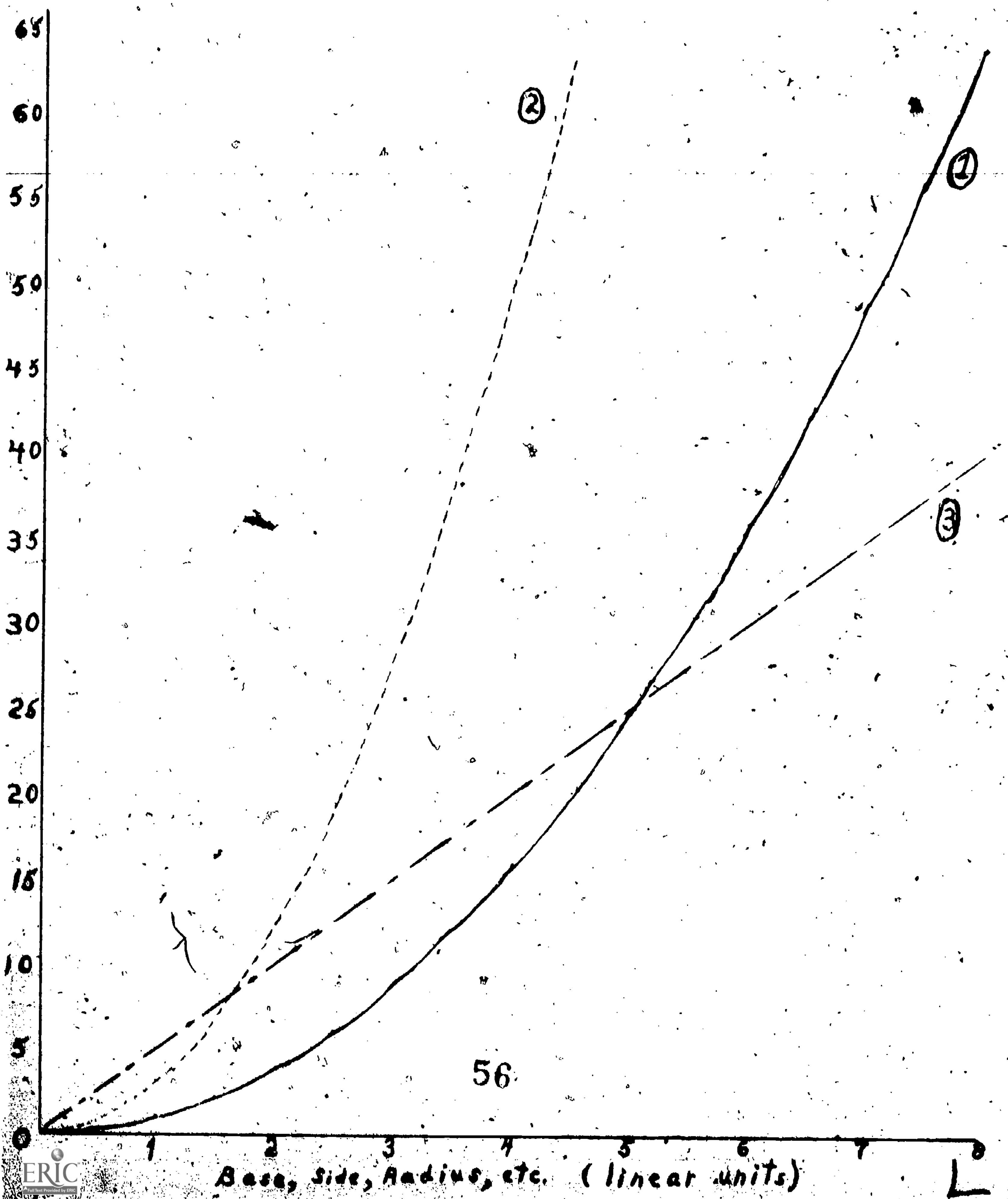
RIGHT TRIANGLE

What happens to the height if we double the area?

What happens to the height if we triple the area?

What happens to the height if we reduce the area by half?

Can you write the formula for the area of a triangle?



65
60
55
50
45
40
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
2
1
3
Base, Side, Radius, etc. (linear units)
L

GRAPHING

(Nearest Tenth)

	Curve 1		Curve 2		Curve 3	
A.	L	A	L	A	L	A
	1		1		1	
	2		2		2	
	3		3		3	
	4		4			35
	5			19.6		20
	6			7.1		40
	7		STOP!		2.5	
	8		5		3.5	
	4.5		8			30
	6.5			379.9		
		30				
		6				
B.	Formula A =		A =		A = 10x	
C.	Figure					

D. Formula	A = 5x
E. Figure	

By studying your completed table for Curve 1, perhaps you can write the formula which relates L and A. This has been begun in part B (on E-IIB) below your table. (Stop reading and try it!) Hint: if you are puzzled, think of L as a number in your table under "L". What happens to that number to get the area listed under "A"?

For what figure does the formula give us the area? List the name as part C on your worksheet.

Now consider curve 2.
Proceed as in the previous card: complete the table (part A) under "Curve 2" on worksheet E-IIB.

Stop at the "STOP!" position and study your table. As before, try to complete the formula (B) relating A and L. Do this before reading further! Hint: note that Curves 1 and 2 are similar looking. So the formulas will look similar, except that a number is included for the formula for Curve 2.

For what geometric figure does the formula give the area? What is L then in terms of the figure? Complete part C for Curve 2. Now complete table 2.

Look at Curve 3 of sheet E-IIA. Complete the table (part A) for Curve 3.

We are going to assume that one linear dimension of the figure, for which Curve 3 gives the area, is 10. By studying your table complete the formula (part B) for Curve 3. CAREFUL: the pairs of numbers (L,A) from your table must work in (satisfy) the formula.

Name a figure (part C) for which the formula will give the area.

Now suppose the fixed linear dimension is 5, not 10. Complete the formula, part D, so that the pairs (L,A) for Curve 3 satisfy that formula. Name a figure for which the formula would give the area (part C).

Why do Curves 1 and 3 meet where they do?

Look at the table for Curve 1 (worksheet E-IIB). The linear measure 2 (L column) is halfway between 1 and 2. Is the area (A column) for 2 halfway between the area for 1 and 3?

Try this for Curve 2 and Curve 3.

In the L column for Curve 1, 6 is twice 3. Is the area for 6 twice that of the area for 3? Try for Curves 2 and 3.

Can you make any general statements about these comparisons?