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ABSTRACT

A relatively simple procedure for modeling periodic components in time series data is presented in this paper, along with an example of the procedure's use with communication data. Similar to multiple regression analysis, the described procedure has four steps that are based on information about periodic waves and their components, how to create models of wave components by using least squares estimations of amplitude and phase, the estimation of frequency, an algorithm for computing estimates in multiple component models, significance testing with these estimates, and harmonic analysis. The paper reports on the use of the procedure on data concerning news stories about the Concorde supersonic transport in the Washington, D. C., mass media, noting that the procedure's adjustments to hypotheses about intervals between Concorde stories helped predict cycles in the data that accounted for large amounts of variance. The paper concludes that as communication data becomes increasingly time based, procedures such as the one described should become more prominent. (RL)

PERIODIC COMPONENTS IN COMMUNICATION DATA: MODELS AND HYPOTHESIS TESTING

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MODELS AND HYPOTHESIS TESTING

INTRODUCTION

This paper will concern itself with some solutions to a common problem which arises in handling time series data: the modeling and testing of hypotheses concerning periodic rises and falls in the level of a single variable measured over time. With communication researchers increasingly using over-time data to represent dynamic communication processes (e.g. Krull and Paulson, 1977; Arundale, 1977), discussions of alternative techniques for describing and testing hypotheses concerning cyclic repetition in data are needed.

Communication data is particularly likely to exhibit a periodic nature. Mass media coverage of a controversial subject often appears for a while, tapers off, then again reappears, in a cyclic pattern. Audience preference for particular television content is often cyclic, as the periodic reappearance and disappearance of Westerns, comedies, etc., testifies. The total amount of news a newspaper carries is cyclic over a week, because of factors like slow news days over the weekend, the large news hole in Thursday papers created by supermarket advertising, and other environmental variables.

There are two general reasons to create a model of a cyclically changing time series of communication data. First, the cyclic change might be theoretically important. An *priori* hypothesis predicting the frequency and/or the amplitude of change over time could be set up and statistically tested. This is the analog of predicting a linear relationship between two sets of static data, and testing the significance of a fitted regression line.

The second reason to create a periodic model is to describe, in simplified terms, an extensive data set. This is parallel to using linear regression to *a posteriori* describe the relationship between two static variables.

There are two general techniques by which models of time series can be created. The *time domain* techniques, based on linear regression concepts and methods for the most part (cf. Box and Jenkins, 1970; Ostrom, 1978), yield linear models in which variable values at one point in time predict subsequent values. Time domain modeling is the more prevalent technique in communication research, but it is somewhat more difficult to use in certain situations than the alternative method, *frequency domain* modeling.

Frequency domain models fit periodic sinusoid (sine and cosine) functions to the observed data in much the same way linear regression fits straight line functions to data. Multiple functions can be additively fitted to produce complex variations over time. In contrast to time domain models, which produce prediction equations which are a function of prior variable values, frequency domain models give functional relationships between the variable and a fixed frequency component.

PERIODIC WAVES

Any pattern of values of a variable which repeats regularly over time is a periodic wave. A simple example is the cosine wave

$$y = f(t) = R \cos(wt + p)$$

R is the maximum amplitude of the wave, w is the frequency in radians per unit time, and p is the phase angle with respect to the origin. This phase angle is also expressed in radians. If p is zero, the maximum value of y will fall at t=0; otherwise, the time origin will intersect the periodic curve at some other point. The <u>period</u> of this wave, or the time it takes to complete a single complete cycle, is 2 pi / w.

It can be shown that any set of data can be represented by a series (possibly infinite) of sinusoid functions similar to the above (Kreyszig, 1972). This remarkably general statement, which applies not only to data which is clearly periodic over time, but to <u>all</u> functions (even ones with discontinuities, like step or impulse functions), is generally termed a Fourier series representation of a function. It is named for Joseph Fourier (1768-1830) who was instrumental in developing what is now generally called Fourier analysis, i.e., the representation of complex dynamic functions as a series of sinusoid (sine and cosine) functions.

Example of Components of a Periodic Wave

Consider the example periodic wave plotted in Figure 1. It consists of 100 time points of a square wave with period 50, or two complete cycles. Since one cycle takes a period of 50 time units, and corresponds to 2 pi radians, the fundamental frequency of this wave is 2 pi / 50 = .12566 radians per time unit.

Fourier procedures (discussed later in this paper) show that a wave such as this can be represented by odd harmonics of the fundamental frequency. Let us remove these components one at a time, look at the residuals, and gradually build up a model of the square wave. Figure 2 shows the original data with a sinusoid of the same fundamental frequency fit to the data under a least squares restriction. Note that this fundamental fit is a reasonably good approximation of the data (the F-value for explained variance is over 450!) by itself.

Figure 3 plots the residuals when the fundamental frequency is removed. Figure 4 shows the third harmonic (w = 6 pi / 50) fit to the residuals, and Figure 5 shows the residuals after both the fundamental and third harmonic are removed. The fifth harmonic (w = 10 pi /50) is fit to these residuals in Figure 6, and the final set of residuals plotted in Figure 7.

Figure 8 illustrates the point to be made here. When only the first three periodic components of the square wave are combined, we have a very good approximation of the original data. Addition of further components will improve the fit, although each will contribute proportionally less to the goodness of fit.

It is difficult to model this data with time domain procedures, because of the discontinuities in the data, yet frequency domain analysis will produce this simple model:

$$y = \sum_{k=1,3,5} [A_k \cos(kwt) + B_k \sin(kwt)]$$

In this data set the coefficients A and B must be estimated to give a least squares fit, and in a realistic situation w must also be estimated. The derivation of these procedures, due primarily to Bloomfield (1976) and Kreyszig (1972) will next be addressed.

CREATING PERIODIC MODELS

Least-Squares Estimations of Amplitude and Phase

Consider the time series data

$$x = [x_1, x_2, x_3, \ldots, x_N]$$

We can model any periodic component in the data as

$$x_t = mu + R \cos(wt + p) + e_t$$

Where x_t is any data point in the series
mu is the mean of the series
R is the maximum amplitude
w is the frequency in radians per unit time
p is the phase angle in radians
t is time units
e_t is residual error

Residual error terms are assumed to be uncorrelated, and all discrete observations of x are taken at equal time intervals. For a series of discrete data points the sum of squares of the residual values must be minimized. To facilitate this, the equation is rewritten by trigonometric identity as

$$x_t = mu + A\cos(wt) + B\sin(wt) + e_t$$

Where $A = R\cos(p)$
$B = -R\sin(p)$

The function to be minimized is then

$$SS(mu,A,B,w) = \mathrm{SIGMA}_{t=1}^{N} \left[(x_t - mu - A\cos(wt) - B\sin(wt))^2 \right]$$

The procedure for minimizing this function for any w is to set the partial derivatives of SS with respect to mu, A and B equal to zero and solve for the estimated values of each parameter. Bloomfield (1976) provides a very readable presentation of this derivation. For any fixed w, the least squares estimates for mu, A, and B are approximately given by

$$\hat{X} = 1/N\ \mathrm{SIGMA}\ [x_t]$$

$$\hat{A} = 2/N\ \mathrm{SIGMA}\ [(x_t - \hat{X})\cos(wt)]$$

$$\hat{B} = 2/N\ \mathrm{SIGMA}\ [(x_t - \hat{X})\sin(wt)]$$

The exact solutions for $\hat{A}$ and $\hat{B}$ are given in Bloomfield, and are computationally difficult. The above approximations are generally accurate to at least four significant digits. This is more than sufficient for most applications. The estimate of the mean, $\hat{X}$, is the simple arithmetic mean of the series. In these and all further formulae, SIGMA will be assumed to cover the range 1 to N if the range is unspecified.

__Estimating w__

$\hat{A}$ and $\hat{B}$ have additional properties which make them useful in estimating the w which will best fit the data. The amplitude R of the fitted periodic component is estimated by

$$R(w)^2 = A(w)^2 + B(w)^2$$

The frequency is optimized when the magnitude of the fitted component is maximum. The squared magnitude is usually referred to as the periodogram, although strictly speaking the true periodogram values differ by a scale factor of N / (8 pi).

The amplitude of the periodogram at frequencies near the strongest periodic component describes an inverted-U, as in Figure 9, which is taken from Bloomfield. It is simple to find the value for w which maximizes the amplitude in the neighborhood of w by numerical methods such as Newton's (cf. Conte, 1965). One must begin near the actual frequency, however, as most numerical methods will detect only local maxima. For example if one attempted to optimize the frequency at a starting point of w=.23 in Figure 9, Newton's Method would produce a maximum of less than 10 near the frequency of .23 radians, when the actual maximum is about 110 near w=.2175.

Multiple Components

Estimates of A, B, and w can be found for models made up of more than one periodic component. The following algorithm was used to compute estimates for multiple component models in this paper:.

1) Set initial estimates for A, B and w

2) Remove these components by subtracting out each periodic function in turn, i.e., remove each subsequent periodic component from the residuals left after removing the prior component.

3) Compute the mean of the corrected (residual) series and remove this mean from the residuals.

4) Estimate new values of A, B, and w for each component; if the new w differs from the previous value by more than some criterion value, go to 2 and repeat.

5) When no w value for any of the components changes by more than the criterion value in a complete cycle, the optimization is complete.

The virtue of this procedure is that it treats each periodic component separately. The importance of this can be seen by inspecting the periodgram in Figure 9. Note the side-lobes in the amplitude produced by the single strong periodic component. In a multiple component model, these side-lobes may interfere with detection of primary periodic components. However, since each component is removed before considering the next, this "leakage" is of no consequence in the parameter estimation or optimization process.

Significance Testing

After a model of periodic components is fit to observed data, it is appropriate to ask if it explains more variance than chance alone would provide. Since all parameters were estimated under least squares constraints, it is easy to carry out an analysis of variance for the specified periodic model.

The sums of squares are partitioned as

$$SS_x = SS_{model} + SS_{residual}$$

Where $$SS_{resid} = \mathrm{SIGMA}_{t=1}^{N}[(x_t - \hat{X} - \mathrm{SIGMA}_{k=1}^{M}[A_k \cos(wt) + B_k \sin(wt)])^2]$$

N is the number of time points
M is the number of components in the model
All other variables are as before

From this partitioning, the appropriate F-test is evident:

$$MS_{model} = SS_{model} / M \text{ and}$$

$$MS_{residual} = SS_{residual} / (N-M-1)$$

$$F = MS_{model} / MS_{residual}$$ with M and N-M-1 degrees of freedom

Each component in the model introduces one degree of freedom, as two constants (A and B) are being estimated, just as the estimation of intercept and slope (two constants) in linear regression introduces a degree of freedom for each variable fit to the data. The error degrees of freedom follow from this.

By using appropriate F-tests it is possible to determine if single components explain significant amounts of variance, test the overall significance of a multiple component model, or see if addition of another single component explains significant additional variance.

To test a single component model, the sums of squares are obtained directly from the parameter estimation process, and an F-ratio with 1 and N-2 d.f. is computed. Multiple component models first undergo the parameter optimization process and at its completion have F-ratios with M and N-M-1 d.f. computed from the sums of squares from all components of the optimized model. This F-test gives a significance level for variance explained by all periodic components taken together.

To test the significance of adding an additional component to an existing model, an incremental F-ratio is computed as

$$F_{inc} = (SS_{\text{with new component}} - SS_{\text{without component}}) / [(SS_{resid} / (N-M-1)]$$

Degrees of freedom are 1 and N-M-1. (See Kelly, 1970 or Nie, et al., 1975, for a discussion of this procedure, in the context of linear regression.)

An additional statistic of interest can be computed from the sums of squares. This statistic is the equivalent of the Multiple R-Square of regression or Eta-Square of analysis of variance. It will be called E-Square here to avoid confusion with R-Square which represents the squared amplitude of a periodic wave. E-Square is interpretable as the analog of Multiple R-Square, i.e., the percentage of total variance explained by the fitted model. It is computed by

$$R^2 = SS_{model} / SS_x$$

HARMONIC ANALYSIS

A critical question arises in considering the discussion of the previous section, namely, how does one determine the number of periodic components and their starting values? The answer depends on whether one is testing a hypothesis about the presence of components or is attempting to describe a set of data. If the former is the case, there is no problem; presumably the theory being tested supplies the description of the components. However, if one is attempting _a posteriori_ description, some means of detecting strong periodic components is needed.

The first technique which seems obvious is to plot the data and look for periodicities. If there are very strong periodicities, this is an adequate procedure. However, the human eye is not a sensitive pattern detector, at least when dealing with data plots. It is almost impossible, for example, to visually detect statistically significant linear relationships with correlations of less than .50. Yet detection of a periodic component which explained 10% of the variance in a time series might be considered quite important.

Fourier analysis provides a systematic procedure for detecting periodic components. A full discussion of Fourier analysis is far beyond the scope of the paper. Only a conceptual outline is included here. The interested reader may consult a number of intermediate mathematics texts, such as Kreyszig (1972) or Wylie (1960) for the proofs of the statements made here.

If a set of frequencies are chosen so that

$$w_j = 2 \text{ pi } j / N \qquad 0 \le j \le N/2$$

it can be shown that the sinusoids corresponding to these frequencies are orthogonal, and hence independent. Only frequencies corresponding to 0 to pi radians need be considered, as higher frequencies are indistinguishable from their lower harmonics. This phenomenon is called "aliasing", and it implies that the highest frequency detectable in a time series has a period of 2t, i.e. a frequency of w = (2 pi) / (2 t) = pi / t radians per time unit.

We can define the sine and cosine coefficients corresponding to the Fourier frequencies as

$$A_0 = 1/N \; \underset{t=0}{\overset{N-1}{\text{SIGMA}}} \; [\, x_t \,] = \bar{x}$$

$$A_j = 2/N \; \underset{t=0}{\overset{N-1}{\text{SIGMA}}} \; [x_t \cos(w_j t)\,]$$

$$B_j = 2/N \; \underset{t=0}{\overset{N-1}{\text{SIGMA}}} \; [x_t \sin(w_j t)\,]$$

$$A_{N/2} = 1/N \sum_{t=0}^{N-1} [(-1)^t x_t] \quad \text{if } N \text{ is even}$$

$$= 0 \quad \text{if } N \text{ is odd}$$

If N is even, the last data point corresponds to a Fourier frequency of pi radians, at which the cosine term will be either positive or negative unity. If n is odd, the last data point does not correspond to a Fourier frequency, and thus there is no cosine coefficient.

Note that the cosine and sine coefficients correspond to the values derived for model fitting in the last section of this paper. They have one important additional property, however: they are orthogonal, and hence can be linearly superimposed without any interaction among components. Any time series can thus be represented as a series of summations of Fourier coefficients

$$x_t = A_0 + \sum_{j=1}^{N/2-1} [A_j \cos(w_j t) + B_j \sin(w_j t))] + (-1)^t A_{N/2}$$

The A and B coefficients can be computed at each Fourier frequency by evaluating the above sums. The resulting set of coefficients is called the Discrete Fourier Transform of the time series. Conceptually, it describes a series of periodic components which, when added together, produce a continuous function which passes through each point in the discrete time series. In practice, the A and B coefficients are computed by more efficient methods than the evaluation of sums. The "Fast Fourier Transform" developed by Tukey and others (Tukey, 1967) uses mathematical identities to simplify the computation process, while producing the same results.

Having established that a time series can be represented by a series of independent periodic components, we are in a position to detect important periodicities in a data set by observing the magnitudes of the Fourier transform coefficients at the Fourier frequencies. These amplitudes, as before, are the square root of the sum of the squared sine and cosine (B and A) coefficients.

Figure 10 shows the plot of the amplitude squared of each Fourier frequency for the square wave example. Note the very strong peak at w = .16. This is close to the actual fundamental frequency of .13. If the periodicty was not visually evident, it still could be detected by choosing the strongest Fourier component, then optimizing the w, as discussed in the previous section.

It might occur to the reader that fitting a model of k components would simply involve choosing the k strongest Fourier components and using them as starting points for the previously outlined model fitting procedure. Unfortunately, the "leakage" problem evident in Figure 9 still exists. Strong periodic components produce side-lobes in the Fourier periodogram which may mask weaker independent components.

Bloomfield (1976) and others have devoted much effort to outlining data smoothing procedures which will dampen the leakage effect. These are very useful if one is restricted to "one-pass" data analysis, such as in processing real-time signals. However, communication data is usually not subject to this restriction, so step-wise procedures which isolate a single component, then remove its effects are possible. Since one component at a time is processed, leakage is irrelevant.

A PROCEDURE FOR DETECTING AND MODELING PERIODICITIES

The following algorithm draws upon harmonic analysis to detect periodicities in data, and model parameter estimation techniques to fit these periodicities, in a least squares fashion to the time series data.

1) Compute the discrete Fourier transform, and choose as a starting point the frequency which produces the largest amplitude.

2) Optimize the frequency to produce the greatest amplitude in the neighborhood of the starting frequency, and obtain the A and B estimates for this frequency. If the F-test for explained variance is non-significant, this frequency may be the spurious result of other periodicities in the data, or there may be no significant periodic components.

3) Remove the components from the original data, using the A and B estimates. Repeat steps 1-3 until all desired components are removed. If no component gives significant F-values, one may conclude that there are no periodic components which are distinguishable from noise. Components which give significant explanation of variance will be used in the final

model. Removing components may be continued until further components do not produce significant F-values for explained variance in the residuals, until the additional explained variance is less than a criterion value, or until a maximum number of components have been extracted.

4) Choose the frequencies which gave individually significant F-values as starting estimates in building a multi-component model. The frequencies of the optimal fit will not in general be identical to the frequencies obtained from step-wise harmonic analysis and single-component model fitting, as the set of frequencies obtained from the above steps will not be orthogonal. The final frequencies and A and B coefficients from the model fitting procedure may be tested for significance by a final F-ratio computed for the entire multi-component model.

The procedure outlined above is similar to step-wise regression. The strongest component is isolated first, its parameters estimated, the variance explained by it removed, then the next strongest component chosen, etc. The same warnings usually associated with step-wise regression also apply to this procedure. The order in which the components are chosen will affect the relative variance explained by subsequent components; and any component may have been chosen over another as a result of chance variation in the data.

The procedure outlined above is not the only one possible. An alternative to be explored involves using only Fourier frequencies in models, rather than estimating optimal frequencies. This would capitalize on their orthogonal properties and allow one to speak of the components separately, just as one speaks of orthogonal beta weights in regression. However, using only Fourier frequencies magnifies the complexity of the resulting models, as a number of Fourier components may be necessary to describe a periodicity which does not occur at a Fourier frequency.

AN EXAMPLE OF HYPOTHESIS TESTING AND MODEL BUILDING

The following is an example of the procedures outlined above applied to a real communication data set. The data used were gathered as part of a mass media monitoring project. The details of the project and the data gathering procedures can be found elsewhere (Watt, 1977) and will not be discussed here.

The time series plotted in Figure 11a-d represents the daily prominence of stories about the Concorde supersonic transport carried in Washington, D.C. mass media, including network television. The data shows no obvious periodicities.

Testing an A Priori Hypothesis

It was proposed that mass media coverage is subject to three major cycles:

1) A story "wear-out" cycle of two days period. This is an result of the event-orientation of the media. Any single announcement or event will be prominent only for a short period of time and will result in short bursts of coverage of high frequency.

2) A "weekend" cycle of seven days duration. Fewer newsworthy events occur on the weekend, especially when a major portion of the news events concern governmental announcements, as was the case in the Concorde trials. There is also less breaking news printed on weekends, due to normal staff days-off in the news organizations.

3) An "issue" cycle of about 30 days duration. A continuing controversy will exhibit recurrent coverage. For the issue to remain on the public agenda, some coverage once a month is hypothesized.

It must be noted that these are very stringent hypotheses. They are the equivalent of specifying the slope of a regression line in linear regression. It is more realistic to specify approximate frequencies of hypothesized components and then optimize the function fit within the neighborhood of the frequencies, similar to the regression procedure of adjusting the slope for maximum explained variance.

Table I contains the results of the model fit. The three components were first fit to the data independently to test the hypothesis that each component was present in the data at greater than chance levels. As the table indicates, only the

30 day cycle approached significance. The optimized frequency gave a 29 day period. This component gave the greatest E-Square value of almost .01, equivalent to a Multiple R of .10.

The lower portion of Table I shows the full test of the three component hypothesis. As would be expected, the explained variance is improved only marginally over that of the 29 day component alone. The significance of the F-value is less than .10, but greater than .05.

If the alpha level for rejection of the null hypothesis was set at .05, as seems normally to be the case, we would fail to reject the null statement that there are no periodicities of period 30, 7, and 2 days in the data. If the level was set at .10, we would conclude that the longer cycle of nearly 30 days was present, indicating an "issue-oriented" coverage pattern. In any case, only about 1% of the variance is explained by the hypothesized component(s).

Fitting an A Posteriori Model

The procedures for detecting and modeling periodicities outlined in the previous section were carried out on the Washington data set, in an attempt to create a descriptive model of the data. A discrete Fourier transform of the original data was first obtained.

The full transform is shown in Table II for illustrative purposes. The computational procedure used to obtain the transform utilizes complex variable algebra, so the transform is expressed in real and imaginary components, rather than sine and cosine coefficients. The squared amplitude can be obtained from these coefficients, however, and it corresponds to that obtained from A and B coefficients. Since the amplitude and frequency values are the important ones in model building, there is no need to go into the details of the complex algebraic procedure except to state that the jth Fourier transform term, J, expressed in real and imaginary terms is related to A and B as defined previously by

$$J_j = 1/2 (A_j - iB_j)$$

Where J is the Fourier term
i is the imaginary operator
(square root of -1)
A and B are the cosine and sine coefficients

Note that the strongest component in the Fourier transform occurs at w = .5154, which corresponds to a period of about 12 days. The amplitude squared of this component is .5093, corresponding to a maximum amplitude of about .71.

This component was entered as the starting point for frequency optimization. The results of this single component model fit are shown in the first entry of Table III. Surprisingly, the explained variance was not significant. Apparently the actual periodicity in the data was far enough from the Fourier frequency that the optimization process converged on a side-lobe frequency, rather than the actual frequency. This is further indicated by the estimated maximum squared amplitude of the fitted function (the sum of the squares of the A and B estimates). This value is only .39, considerably below the .51 found in the Fourier transform.

This side-lobe component was removed from the data, and the residuals again Fourier transformed. Suspicion that the first frequency was spurious was confirmed when again $w = .5154$ emerged as the strongest Fourier component, with a squared magnitude of .47. Since the spurious frequency had been removed, the frequency estimation procedure this time converged to $w = .5032$ with a squared amplitude of .57, higher than the Fourier amplitude. This frequency also explained much more than chance variation in the data, as the second entry in Table III shows (over 2% of the variance, $p = .003$). The period corresponding to this frequency is about 12 1/2 days.

This component was removed from the data (which had previously had the spurious component removed), and the whole process repeated. Table III summarizes results of the first five components. For the purposes of this example, the model was arbitrarily limited to five components. However, significant F-ratios for single components could still be obtained after 10 components were isolated.

In order to test the final five component model, all the frequencies obtained above were entered in the frequency optimization process as starting points. The final model is summarized at the bottom of Table III. As it indicates, the model fit is quite good, with an R-Square value of .18, equivalent to a Multiple R of over .40. If an additional five components were added to the model, indications are that about 30% of the variance in the time series could have been explained with the resulting 10 component model.

It should be noted that the sums of squares are computed about the mean for the fitted function, rather than about the mean for the data, so the total sums of squares may be different for different models. But the sums of squares due to function fit varies accordingly, as removal of the constant mean constitutes a linear transformation, and thus the F-values are identical.

The final model consists of the addition of periodic components of period 12.52, 20.43, 15.21, 8.68, and 683 days. The values predicted by this model are plotted with the original data in Figure 12a-d. The effect of each component can be seen in the plot. The eight day component produces rapid fluctions which are superimposed on the 12 to 20 day components. This complex relationship gives several peaks in the predictive function near peaks in the observed data. The 683 day component goes only through half a cycle in the data. It is responsible for the rising trend in the predictive function at either end of the plot. Since these were periods of peak coverage, it accounts for large amounts of explained variance.

CONCLUDING REMARKS

A relatively simple procedure for modeling periodic components in time series data was presented, with an example application to communication data. The mathematical procedures outlined have been available for 150 years, and have been in use in engineering applications for over 30 years. As communication data increasingly becomes time-based, procedures such as these should become more common.

A number of topics have not been addressed in this paper. All time series discussed here are assumed to be equally spaced in time, and have Gaussian error terms (uncorrelated error). Neither of these assumptions are necessary, with modification of the estimation procedures (Bloomfield, 1976). A primary topic not touched upon is the use of harmonic models to detect relationships between variables in multivariate time series. This is a quite important extension for individuals interested in testing hypotheses about covariation (and possibly causation) between two variables measured over time, and as such deserves immediate attention.

Finally, it is evident that periodic modeling can only be carried out with the aid of a computer program, as the data manipulation is extensive. In this it is not unlike many common statistical procedures such as multiple regression, factor analysis, etc. The computer program used to analyse and plot the data in this paper is available, along with a documentation and users' manual, from the author for a small reproduction fee.

TABLE I

ANALYSIS OF VARIANCE FOR HYPOTHESIS TEST OF THREE COMPONENT MODEL
WASHINGTON DATA

Single Component Model Fits

Start Frequency	Final Frequency	A	B	E^2	F	Sig of F
3.14159	3.14124	-.40982	-5.4516	.003	.992	.320
.89760	.89642	.18096	.2886	.006	2.116	.147
.20944	.21652	.41413	.0219	.009	3.167	.076

Degress of Freedom for F-ratios are 1 and 364

Three Component Model

INITIAL VALUES ARE -

COMPONENT	FREQUENCY (RADIANS)	COSINE COEFFICIENTS	SINE COEFFICIENTS
1	3.1415901	0.0	0.0
2	0.8975971	0.0	0.0
3	0.2094393	0.0	0.0

FINAL VALUES ARE -

COMPONENT	FREQUENCY (RADIANS)	COSINE COEFFICIENTS	SINE COEFFICIENTS
1	3.1425810	-0.334536E+00	0.148439E+01
2	0.8965008	0.180508E+00	0.285941E+00
3	0.2165423	0.411908E+00	0.227290E-01

FITTED CONSTANT (MEAN) IS 0.694916E+00
TOTAL SUM OF SQUARES IS 0.367834E+04
SUM OF SQUARES DUE TO FUNCTION FIT IS 0.632766E+02
RESIDUAL SUM OF SQUARES IS 0.361507E+04
MEAN SQUARE DUE TO FUNCTION FIT IS 0.210922E+02
MEAN SQUARE ERROR IS 0.998637E+01
F = 2.1121 WITH 3 AND 362 DEG. OF FREEDOM
SIGNIFICANCE OF F-RATIO IS 0.098388

E-SQUARE = .0172 E = .131

TABLE II a

DISCRETE FOURIER TRANSFORM

FREQUENCY	PERIOD	REAL	IMAGINARY	AMPLITUDE SQ
0.0123	512.0000	0.0000	0.0	0.0000
0.0245	256.0000	0.1000	0.1753	0.3335
0.0368	170.6667	0.1917	0.0003	0.2976
0.0491	128.0000	0.1719	-0.0305	0.2234
0.0614	102.4000	0.1746	-0.0003	0.2346
0.0736	85.3333	0.1133	0.0296	0.1080
0.0859	73.1429	0.1522	-0.0879	0.2417
0.0982	64.0000	0.1977	-0.0091	0.3066
0.1104	56.8889	0.0506	0.0134	0.0215
0.1227	51.2000	0.1849	-0.0667	0.3022
0.1350	46.5454	0.1126	-0.0030	0.0993
0.1473	42.5667	0.1637	-0.0100	0.2263
0.1595	39.3846	0.1267	-0.0463	0.1425
0.1718	36.5714	0.1046	-0.0562	0.1104
0.1841	34.1333	0.1955	-0.0553	0.3200
0.1963	32.0000	0.0047	0.0160	0.0022
0.2086	30.1176	0.1893	-0.0952	0.3486
0.2209	28.4444	0.0537	0.0293	0.0394
0.2332	26.9474	0.1245	-0.0409	0.1346
0.2454	25.6000	0.1233	0.0390	0.1428
0.2577	24.3810	0.1379	-0.0858	0.1487
0.2700	23.2727	0.1769	0.0645	0.2774
0.2823	22.2609	0.0872	-0.0758	0.1345
0.2945	21.3333	0.2131	-0.0019	0.3554
0.3068	20.4800	0.0505	-0.0412	0.0333
0.3191	19.6923	0.2481	-0.0349	0.4760
0.3313	18.9630	0.0744	-0.0515	0.0642
0.3436	18.2857	0.1410	-0.0352	0.1653
0.3559	17.6552	0.1597	-0.0757	0.2445
0.3682	17.0667	0.0987	-0.0087	0.0768
0.3804	16.5161	0.1632	-0.0470	0.2257
0.3927	16.0000	0.1143	-0.0542	0.1253
0.4050	15.5152	0.2236	-0.0736	0.4338
0.4172	15.0588	0.0897	-0.0514	0.0822
0.4295	14.6286	0.1293	-0.1152	0.2347
0.4418	14.2222	0.1637	-0.1042	0.3103
0.4541	13.8378	0.0669	-0.0469	0.0521
0.4663	13.4737	0.1228	-0.0971	0.1919
0.4786	13.1282	0.0630	-0.[illegible]235	0.1505
0.4909	12.8000	0.1936	-0.0528	0.3151
0.5031	12.4878	-0.0785	-0.0307	0.0556
0.5154	12.1905	0.2104	-0.1442	0.5093
0.5277	11.9070	0.0510	0.0111	0.0213
0.5400	11.6364	0.1081	-0.0329	0.1000
0.5522	11.3778	0.1048	-0.1168	0.1928
0.5645	11.1304	0.1347	-0.0780	0.1902
0.5768	10.8936	0.0914	-0.0173	0.0663
0.5890	10.6667	0.0359	-0.1230	0.1284
0.6013	10.4490	0.1664	-0.0679	0.2527
0.6136	10.2400	0.0350	-0.0251	0.0150
0.6259	10.0392	0.1127	-0.0593	0.1269
0.6381	9.8462	0.0950	-0.1029	0.1535
0.6504	9.6604	0.0914	-0.0018	0.0659
0.6527	9.4815	0.1334	-0.0640	0.1819
0.6750	9.3091	0.0710	-0.0593	0.0669
0.[illegible]72	9.1429	0.1898	-0.0869	0.3411
[illegible]35	8.9825	0.0926	-0.0377	0.0783
0.7119	8.8276	0.1530	-0.1622	0.3893
0.7240	8.6780	0.1700	-0.0652	0.0716
0.7363	8.5333	0.1423	-0.1420	0.3164

TABLE IIb

0.7609	9.2591	0.0344	-0.1350	0.1529
0.7731	9.1271	0.1679	-0.0762	0.2660
0.7854	8.0000	-0.0052	-0.1149	0.1036
0.7977	7.8769	0.1834	-0.0951	0.2316
0.8099	7.7576	0.0352	-0.1149	0.1129
0.8222	7.5419	0.0955	-0.0354	0.1295
0.8345	7.5294	0.0553	-0.1362	0.1631
0.8467	7.4203	0.0735	-0.0750	0.0863
0.8590	7.3143	0.0451	-0.1213	0.1317
0.8713	7.2110	0.0523	-0.1224	0.1186
0.8835	7.1111	0.0545	-0.0767	0.0693
0.8958	7.0137	-0.0083	-0.0808	0.0517
0.9081	6.9189	0.0704	-0.0987	0.1151
0.9204	6.8257	0.0484	-0.0470	0.0357
0.9327	6.7358	0.0400	-0.0229	0.0166
0.9449	6.6494	0.0888	-0.1494	0.2423
0.9572	6.5681	0.0275	-0.0318	0.0138
0.9695	6.4910	0.1436	-0.0516	0.1822
0.9817	6.4000	-0.0509	-0.1296	0.1517
0.9940	6.3210	0.1848	-0.0753	0.3117
1.0063	6.2430	-0.0036	-0.0395	0.0123
1.0186	6.1687	0.0891	-0.1295	0.1835
1.0309	6.0952	0.0259	-0.0911	0.0703
1.0431	6.0235	0.1261	-0.0463	0.1412
1.0554	5.9535	0.0129	-0.0730	0.0430
1.0675	5.8851	0.0690	-0.1128	0.1379
1.0798	5.8182	0.1631	-0.0606	0.2370
1.0922	5.7528	0.0036	-0.0956	0.0716
1.1045	5.6889	0.1268	-0.1302	0.2585
1.1167	5.6264	0.0470	-0.1443	0.1803
1.1290	5.5652	0.0986	-0.0697	0.0938
1.1413	5.5054	0.0080	-0.1800	0.2540
1.1536	5.4468	0.0458	-0.0869	0.0755
1.1653	5.3895	0.0745	-0.1349	0.1859
1.1781	5.3333	-0.0205	-0.0985	0.0792
1.1904	5.2783	0.0363	-0.1174	0.1182
1.2025	5.2245	0.0366	-0.0823	0.0636
1.2149	5.1717	0.0436	-0.1230	0.1334
1.2272	5.1200	0.0025	-0.0401	0.0126
1.2395	5.0693	0.0560	-0.1845	0.2909
1.2517	5.0196	0.0599	-0.0560	0.0525
1.2640	4.9709	-0.0663	-0.1164	0.1604
1.2763	4.9231	0.1074	-0.1074	0.1806
1.2885	4.8762	-0.0623	-0.1040	0.1151
1.3008	4.8302	0.0662	-0.0483	0.0525
1.3131	4.7850	-0.0273	-0.1240	0.1263
1.3254	4.7407	0.0471	-0.0453	0.0334
1.3376	4.6972	0.0159	-0.0829	0.0557
1.3499	4.6545	0.0386	-0.0714	0.0516
1.3622	4.6126	0.0676	-0.0710	0.0752
1.3744	4.5714	-0.0009	-0.1264	0.1250
1.3867	4.5310	0.0944	-0.0712	0.1093
1.3990	4.4912	0.0031	-0.1079	0.0913
1.4113	4.4522	0.0235	-0.1136	0.1053
1.4235	4.4138	0.0577	-0.1345	0.1677
1.4358	4.3761	-0.0056	-0.0453	0.0163
1.4481	4.3390	0.0173	-0.1750	0.2420
1.4603	4.3025	-0.0224	-0.0518	0.0250
1.4726	4.2667	0.1171	-0.1138	0.2088
1.4849	4.2314	-0.1055	-0.0758	0.1321
1.4972	4.1967	0.1025	-0.1512	0.2611
1.5094	4.1626	-0.0030	-0.0814	0.0520
1.5217	4.1290	-0.0081	-0.1026	0.0839
1.5340	4.0960	-0.0226	-0.1302	0.1367
1.5463	4.0635	0.0431	-0.1001	0.1261

TABLE IIc

[illegible]	[illegible]	-0.0542	-0.0704	0.0434
1.5710	4.0000	-0.0702	-0.1316	0.1741
1.5833	3.9600	0.0511	-0.0749	0.0644
1.5953	3.9345	-0.0639	-0.0721	0.0633
1.6076	3.9088	0.0021	-0.0525	0.0216
1.6199	3.8798	-0.0139	-0.1155	0.1077
1.6322	3.8436	0.0486	-0.0432	0.0331
1.6444	3.8213	-0.1392	-0.1018	0.0910
1.6567	3.7926	0.0004	-0.0602	0.0293
1.6690	3.7647	0.0436	-0.1223	0.1319
1.6812	3.7372	-0.0392	-0.0538	0.0341
1.6935	3.7101	0.0066	-0.0743	0.0436
1.7059	3.6835	0.0049	-0.1268	0.1261
1.7181	3.6571	-0.0176	-0.0494	0.0215
1.7303	3.6312	0.0184	-0.0400	0.0152
1.7426	3.6056	-0.0242	-0.1321	0.1411
1.7549	3.5804	0.0807	-0.0535	0.0734
1.7671	3.5556	-0.0564	-0.0767	0.0739
1.7794	3.5310	0.0714	-0.1255	0.1632
1.7917	3.5069	-0.0429	-0.1071	0.1043
1.8040	3.4830	0.0493	-0.0681	0.0553
1.8162	3.4595	-0.0516	-0.1522	0.2021
1.8285	3.4362	0.0209	-0.0951	0.0742
1.8403	3.4133	-0.0416	-0.1053	0.1003
1.8530	3.3907	-0.0424	-0.1106	0.1098
1.8653	3.3694	-0.0033	-0.0761	0.0454
1.8775	3.3464	-0.0671	-0.1001	0.1137
1.8899	3.3247	0.0204	-0.0595	0.0313
1.9021	3.3032	-0.0409	-0.1384	0.1630
1.9144	3.2921	-0.0434	-0.0476	0.0325
1.9267	3.2511	-0.0084	-0.1247	0.1223
1.9399	3.2405	-0.0646	-0.0678	0.0688
1.9512	3.2201	-0.0006	-0.0983	0.0765
1.9635	3.2000	-0.1013	-0.0564	0.1052
1.9759	3.1801	0.0312	-0.1288	0.1366
1.9880	3.1605	-0.1196	-0.0217	0.1157
2.0003	3.1411	-0.0133	-0.0764	0.0470
2.0126	3.1220	-0.0351	-0.0677	0.0455
2.0249	3.1030	-0.0333	-0.0322	0.0168
2.0371	3.0843	-0.0133	-0.0665	0.0360
2.0494	3.0659	-0.0402	-0.0851	0.0693
2.0617	3.0476	0.0202	-0.0514	0.0239
2.0739	3.0296	-0.0836	-0.0763	0.1032
2.0862	3.0118	0.0369	-0.0726	0.0519
2.0985	2.9942	-0.0758	-0.1057	0.1324
2.1109	2.9767	-0.0143	-0.0245	0.0063
2.1230	2.9595	-0.0411	-0.1141	0.1237
2.1353	2.9425	-0.0348	-0.0358	0.0135
2.1476	2.9257	-0.0393	-0.0983	0.0876
2.1599	2.9091	-0.0192	-0.0318	0.0105
2.1721	2.8927	-0.0535	-0.1156	0.1269
2.1844	2.8764	-0.0304	-0.0389	0.0191
2.1967	2.8603	-0.0215	-0.0721	0.0443
2.2099	2.8444	-0.0401	-0.0971	0.0864
2.2212	2.8287	-0.0679	-0.0667	0.0739
2.2335	2.8132	0.0024	-0.0819	0.0526
2.2457	2.7978	-0.1318	-0.0665	0.1699
2.2580	2.7826	0.0047	-0.0597	0.0281
2.2703	2.7676	-0.0844	-0.0500	0.0752
2.2826	2.7527	-0.0185	-0.0508	0.0279
2.2949	2.7380	-0.0747	-0.1078	0.1335
2.3071	2.7234	-0.0313	-0.0205	0.0110
2.3194	2.7090	-0.0649	-0.0770	0.0803
2.3316	2.6947	-0.0433	-0.0601	0.0429
2.3439	2.6916	-0.0356	-0.0188	0.0127
2.3562	2.6667	-0.0621	-0.1018	0.1110

TABLE IId

2.3685	2.6528	-0.0306	-0.0456	0.0236
2.3807	2.6392	-0.0769	-0.0569	0.0710
2.3930	2.6256	-0.0511	-0.0112	0.0212
2.4053	2.6122	-0.0549	-0.1210	0.1393
2.4175	2.5990	-0.0459	0.0774	0.0633
2.4298	2.5859	-0.0118	-0.1434	0.1619
2.4421	2.5729	-0.0331	-0.0028	0.0770
2.4544	2.5600	0.0353	-0.0718	0.0501
2.4666	2.5473	-0.1094	-0.0289	0.0995
2.4789	2.5347	-0.0131	-0.0967	0.0751
2.4912	2.5222	-0.0434	-0.0266	0.0203
2.5035	2.5098	-0.0571	-0.0629	0.0564
2.5157	2.4976	-0.0671	-0.0841	0.0916
2.5280	2.4854	-0.0611	-0.0393	0.0413
2.5403	2.4734	-0.0618	-0.0527	0.0517
2.5525	2.4615	-0.0977	-0.0326	0.0830
2.5649	2.4498	-0.0215	-0.0470	0.0206
2.5771	2.4381	-0.1009	-0.0393	0.0918
2.5894	2.4265	-0.0046	-0.0153	0.0720
2.6015	2.4151	-0.1194	-0.0973	0.1857
2.6139	2.4038	-0.0228	0.0125	0.0053
2.6262	2.3925	-0.0746	-0.0775	0.0907
2.6384	2.3814	-0.0950	-0.0198	0.0596
2.6507	2.3704	-0.0693	-0.0345	0.0416
2.6630	2.3594	-0.0294	-0.0364	0.0171
2.6753	2.3486	-0.1367	-0.0325	0.1546
2.6875	2.3379	-0.0164	-0.0158	0.0041
2.6998	2.3273	-0.0935	-0.0204	0.0792
2.7121	2.3167	-0.0424	-0.0324	0.0223
2.7243	2.3063	-0.0960	0.0381	0.0835
2.7366	2.2960	0.0176	-0.0795	0.0519
2.7489	2.2857	-0.1146	0.0210	0.1362
2.7612	2.2756	-0.0052	-0.0413	0.0135
2.7734	2.2655	-0.0861	-0.0512	0.0785
2.7857	2.2555	-0.0477	0.0277	0.0238
2.7980	2.2456	-0.0598	-0.0667	0.0628
2.8103	2.2358	-0.0339	0.0161	0.0110
2.8225	2.2261	-0.0579	-0.0501	0.0459
2.8348	2.2164	-0.0643	-0.0446	0.0479
2.8471	2.2069	-0.0408	0.0076	0.0135
2.8593	2.1974	-0.0847	-0.0578	0.0823
2.8716	2.1880	-0.0457	0.0121	0.0175
2.8839	2.1787	-0.0388	-0.0653	0.0451
2.8962	2.1695	-0.0557	0.0295	0.0311
2.9084	2.1603	-0.0969	-0.1411	0.2292
2.9207	2.1513	-0.0390	0.0816	0.0634
2.9330	2.1423	-0.0854	-0.0891	0.1192
2.9452	2.1333	-0.1015	0.0157	0.0826
2.9575	2.1245	-0.0024	-0.0252	0.0050
2.9698	2.1157	-0.1009	-0.0102	0.0806
2.9821	2.1070	-0.0777	-0.0378	0.0588
2.9943	2.0984	-0.0429	-0.0110	0.0154
3.0066	2.0898	-0.0838	0.0035	0.0632
3.0189	2.0813	-0.0685	-0.0340	0.0458
3.0311	2.0729	-0.0633	0.0043	0.0377
3.0434	2.0645	-0.0375	-0.0180	0.0136
3.0557	2.0562	-0.1293	-0.0316	0.1386
3.0680	2.0480	-0.0352	0.0228	0.0138
3.0802	2.0398	-0.1137	-0.0391	0.1078
3.0925	2.0317	-0.0489	0.0551	0.0424
3.1048	2.0237	-0.1058	-0.0318	0.0951
3.1170	2.0157	-0.0127	0.0381	0.0126
3.1293	2.0078	-0.0872	-0.0140	0.0612
3.1416	2.0000	-0.0616	0.0048	0.0299

TABLE III

ANALYSIS OF VARIANCE FOR FIVE COMPONENT FITTED MODEL
WASHINGTON DATA

Single Component Model Fits

Start Frequency	Final Frequency	A	B	E^2	F	Sig of F
.51540	.52175	.34422	.10124	.006	2.347	.126
.51540	.50328	.55060	.40989	.024	8.840	.003
.31910	.30754	.65429	.16693	.023	8.707	.003
.40500	.41336	.49421	.24951	.013	4.699	.031
.71160	.72361	.40900	.37539	.016	6.073	.014
.02450	.00924	.19756	-1.54000	.112	45.706	.000

Degress of Freedom for F-ratios are 1 and 364

Full Five Component Model

INITIAL VALUES ARE -

COMPONENT	FREQUENCY (RADIANS)	COSINE COEFFICIENTS	SINE COEFFICIENTS
1	0.5032803	0.0	0.0
2	0.3075374	0.0	0.0
3	0.4133582	0.0	0.0
4	0.7236307	0.0	0.0
5	0.0092419	0.0	0.0

FINAL VALUES ARE -

COMPONENT	FREQUENCY (RADIANS)	COSINE COEFFICIENTS	SINE COEFFICIENTS
1	0.5016992	0.624748E+00	0.239774E+00
2	0.3076050	0.653865E+00	0.157052E+00
3	0.4132021	0.439358E+00	0.237236E+00
4	0.7231592	0.406224E+00	0.365453E+00
5	0.0092050	0.190464E+00	-0.157298E+01

FITTED CONSTANT (MEAN) IS 0.160342E+01
TOTAL SUM OF SQUARES IS 0.398156E+04
SUM OF SQUARES DUE TO FUNCTION FIT IS 0.706653E+03
RESIDUAL SUM OF SQUARES IS 0.327491E+04
MEAN SQUARE DUE TO FUNCTION FIT IS 0.141331E+03
MEAN SQUARE ERROR IS 0.909696E+01
F = 15.5360 WITH 5 AND 360 DEG. OF FREEDOM
SIGNIFICANCE OF F-RATIO IS 0.000012

E-SQUARE = .1775 E = .421

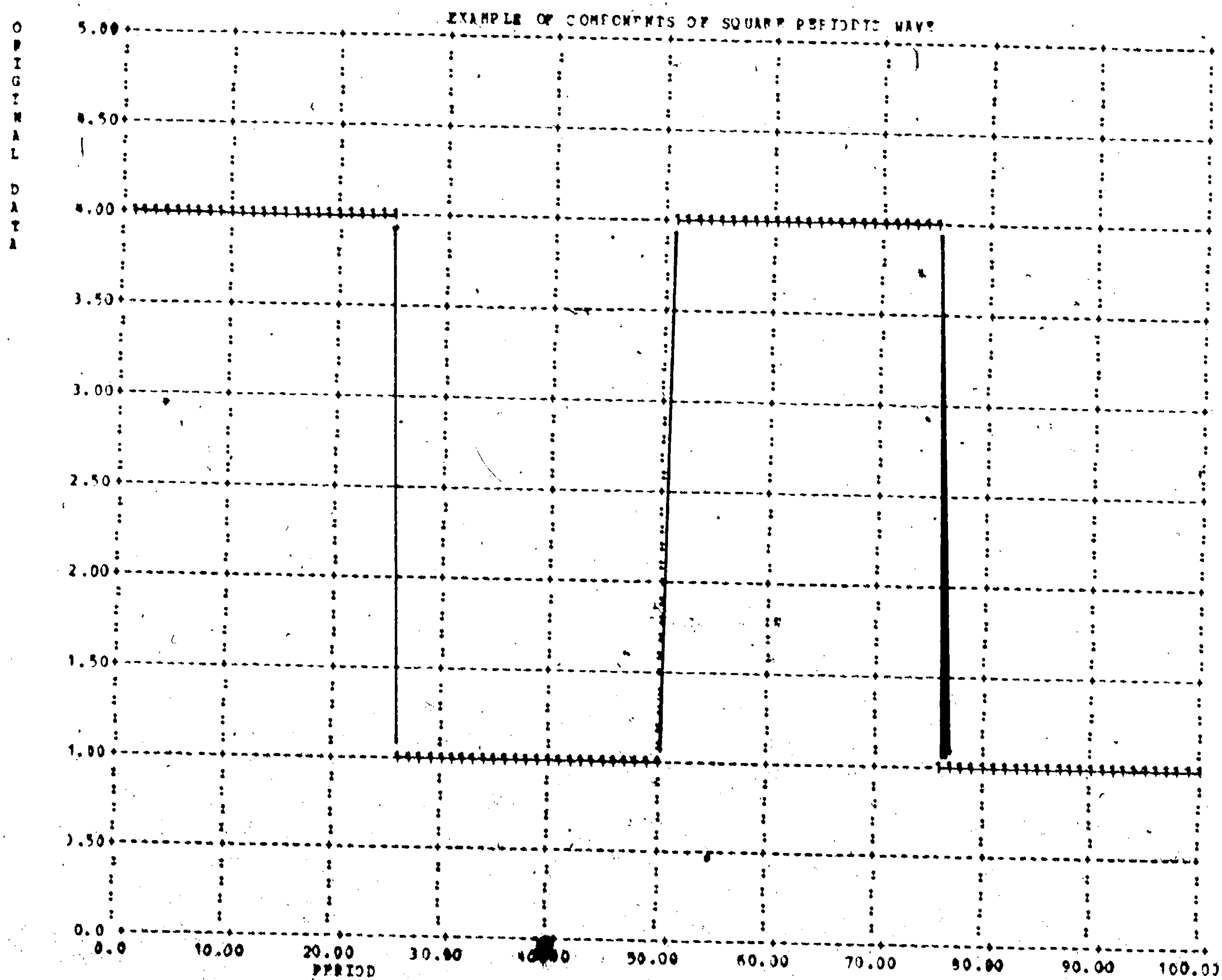
EXAMPLE OF COMPONENTS OF SQUARE PERIODIC WAVE
ORIGINAL DATA
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.0
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
PERIOD

FIGURE 1

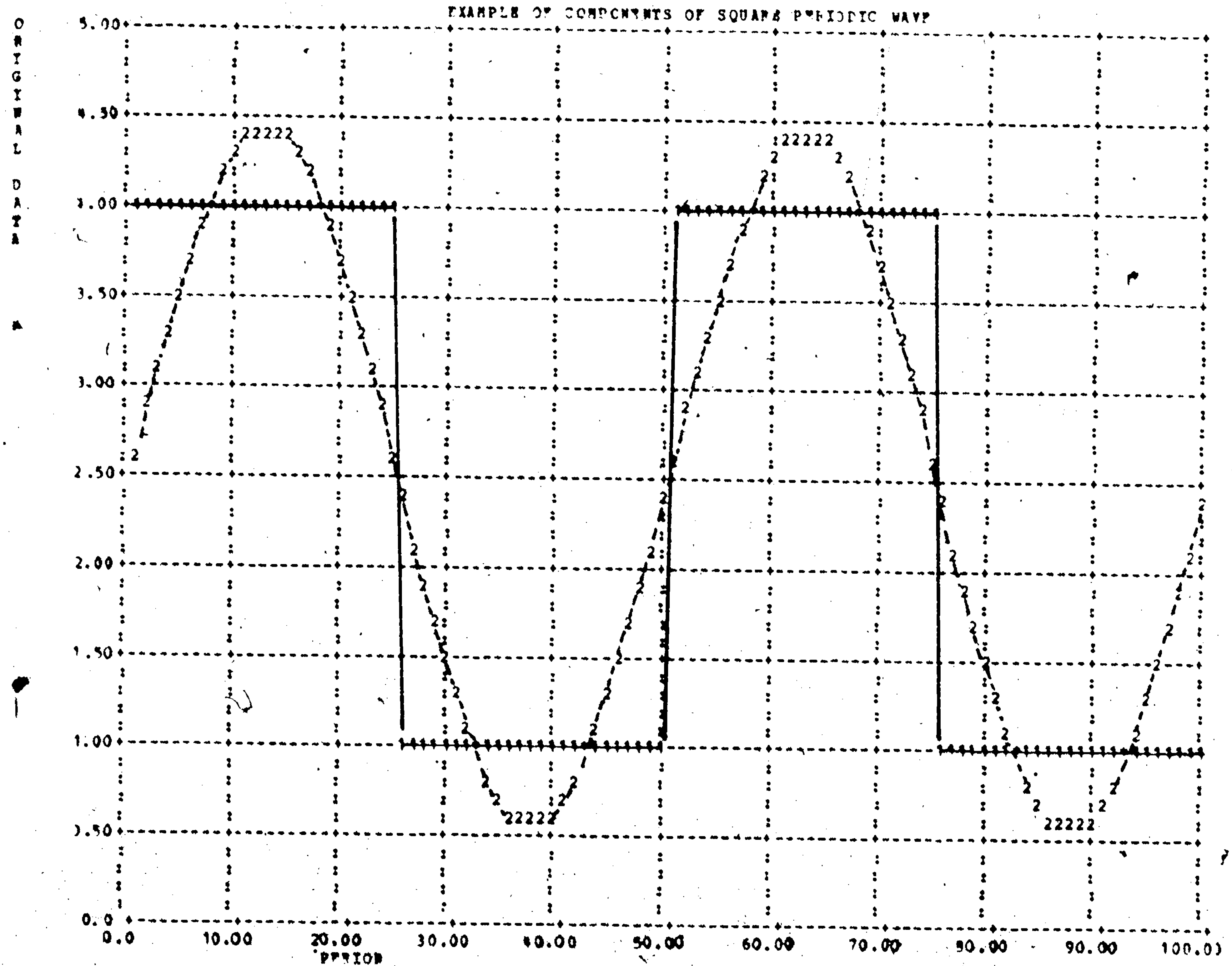
EXAMPLE OF COMPONENTS OF SQUARE PERIODIC WAVE
ORIGINAL DATA
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.0
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
PERIOD

FIGURE 2

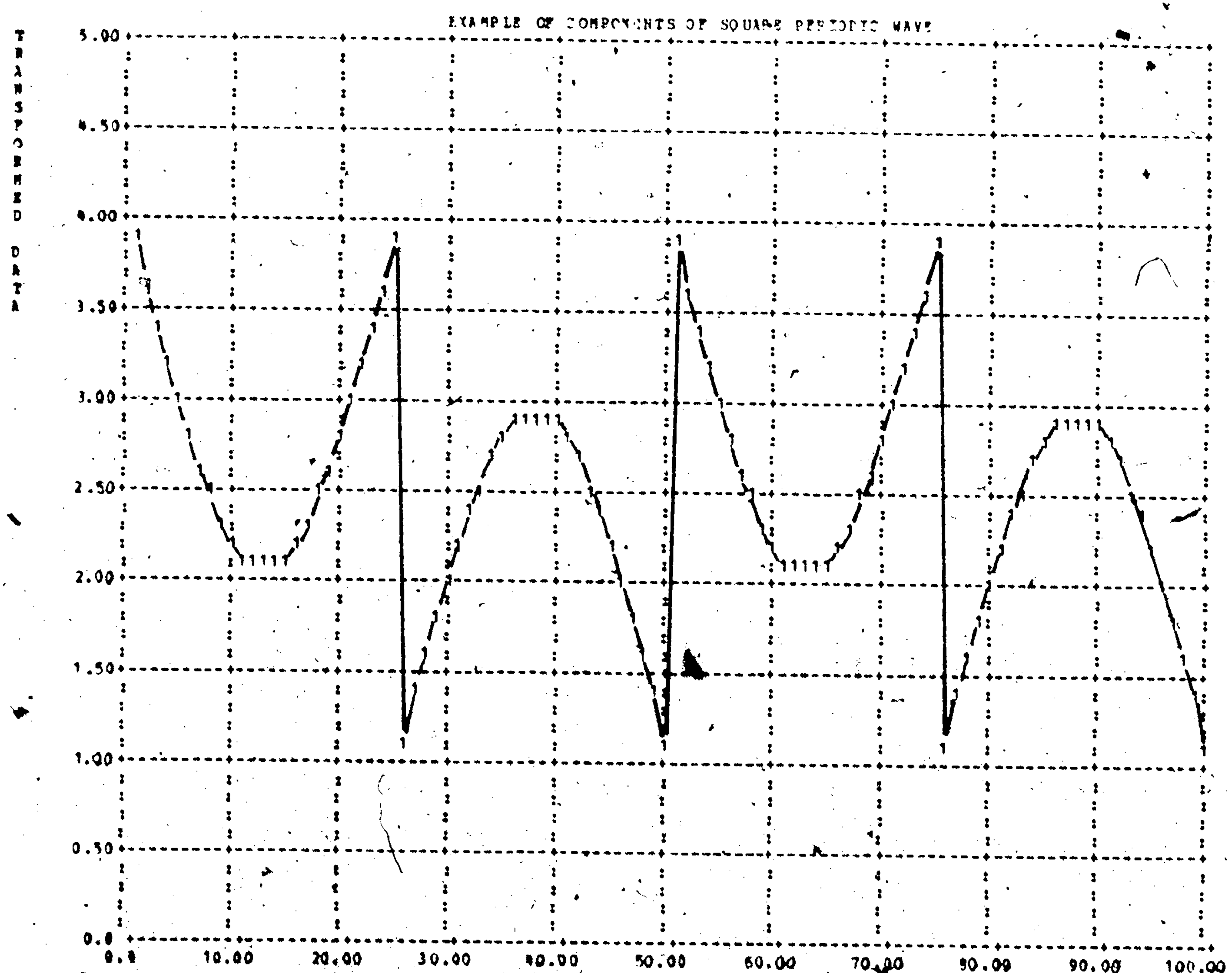
EXAMPLE OF COMPONENTS OF SQUARE PERIODIC WAVE
TRANSFORMED DATA
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.0
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00

FIGURE 3

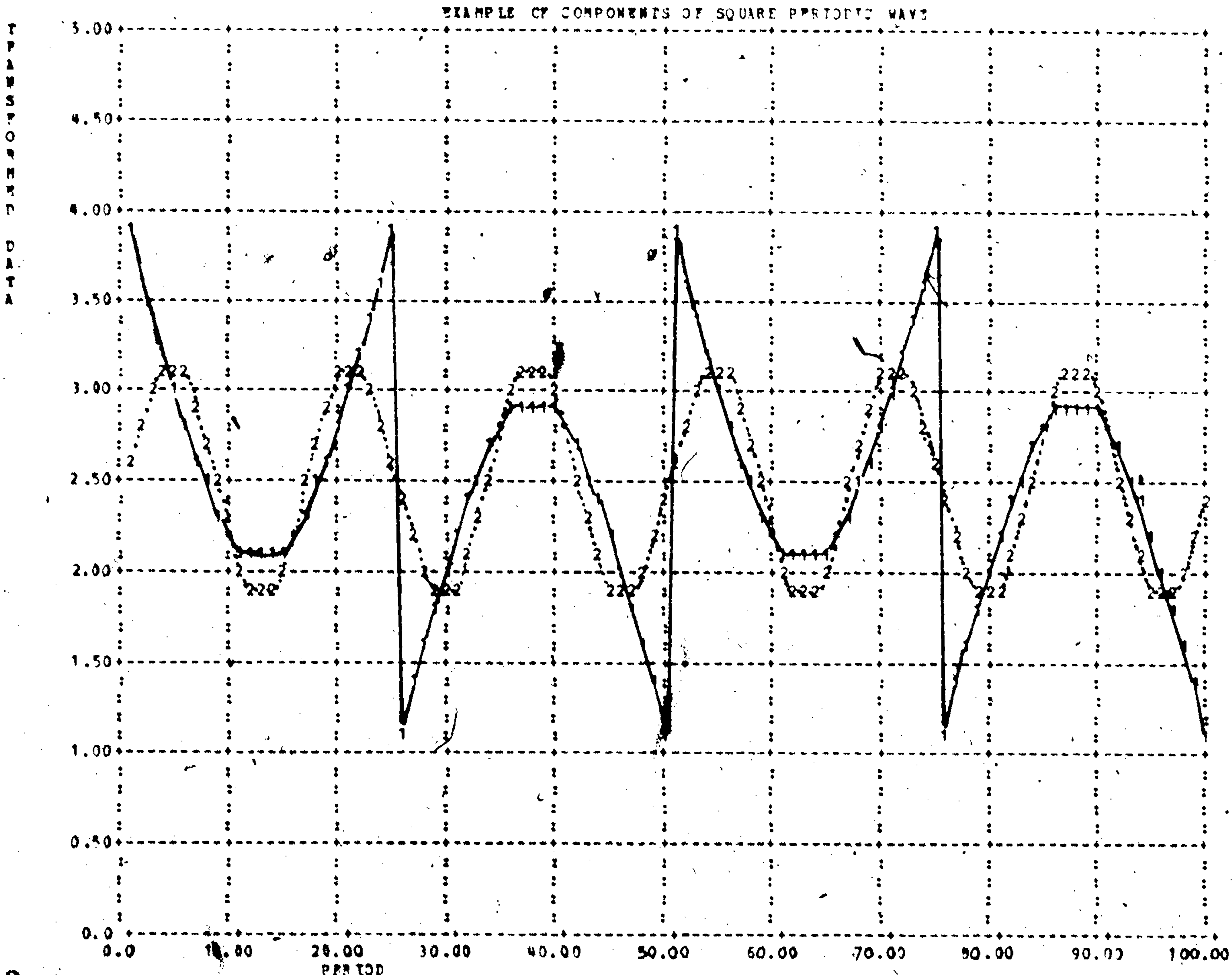
EXAMPLE OF COMPONENTS OF SQUARE PERIODIC WAVE
TRANSFORMED DATA
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.0
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
PERIOD

FIGURE 4

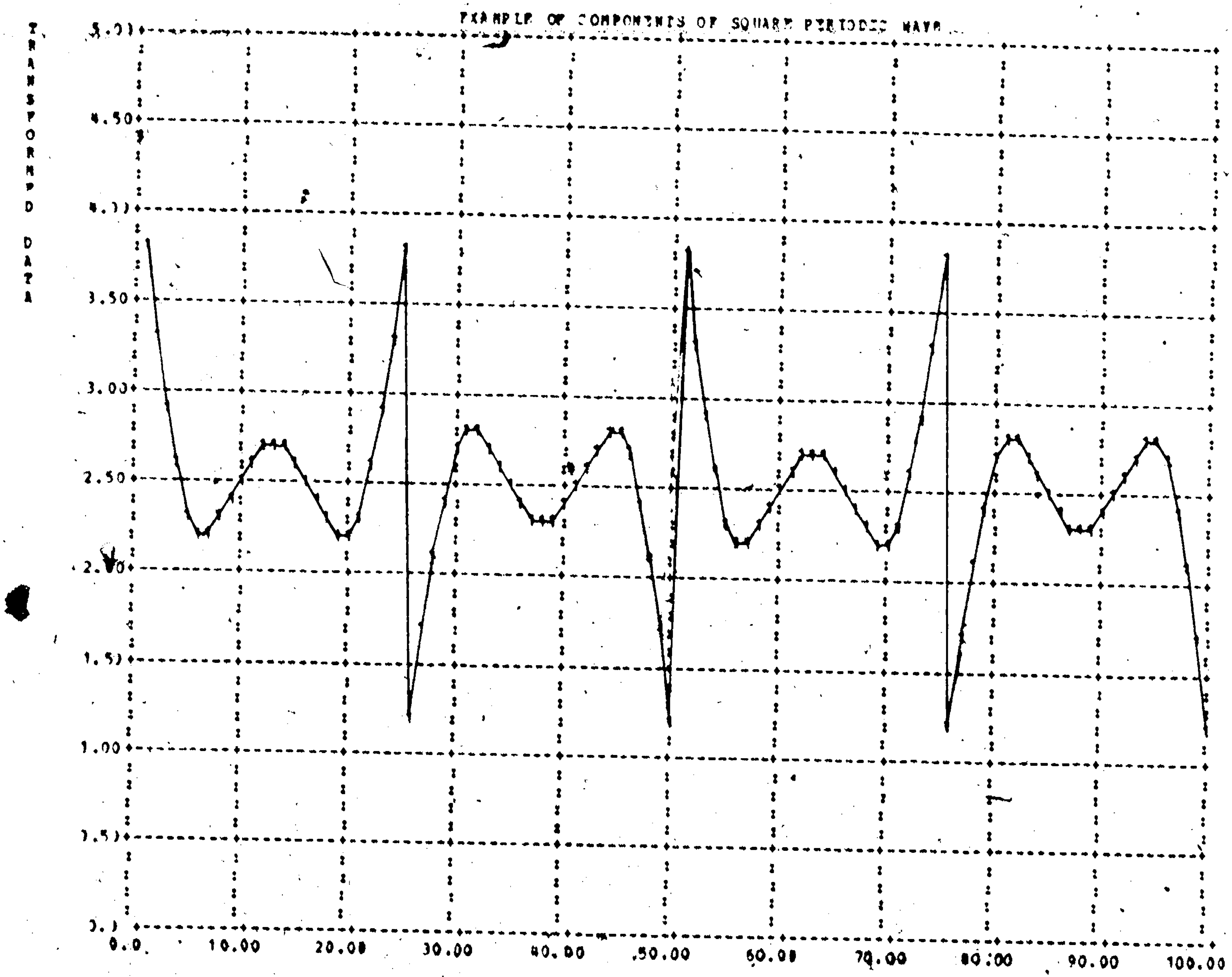
EXAMPLE OF COMPONENTS OF SQUARE PERIODIC WAVE
TRANSFORMED DATA
4.50
3.50
3.00
2.50
1.00
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00

FIGURE 5

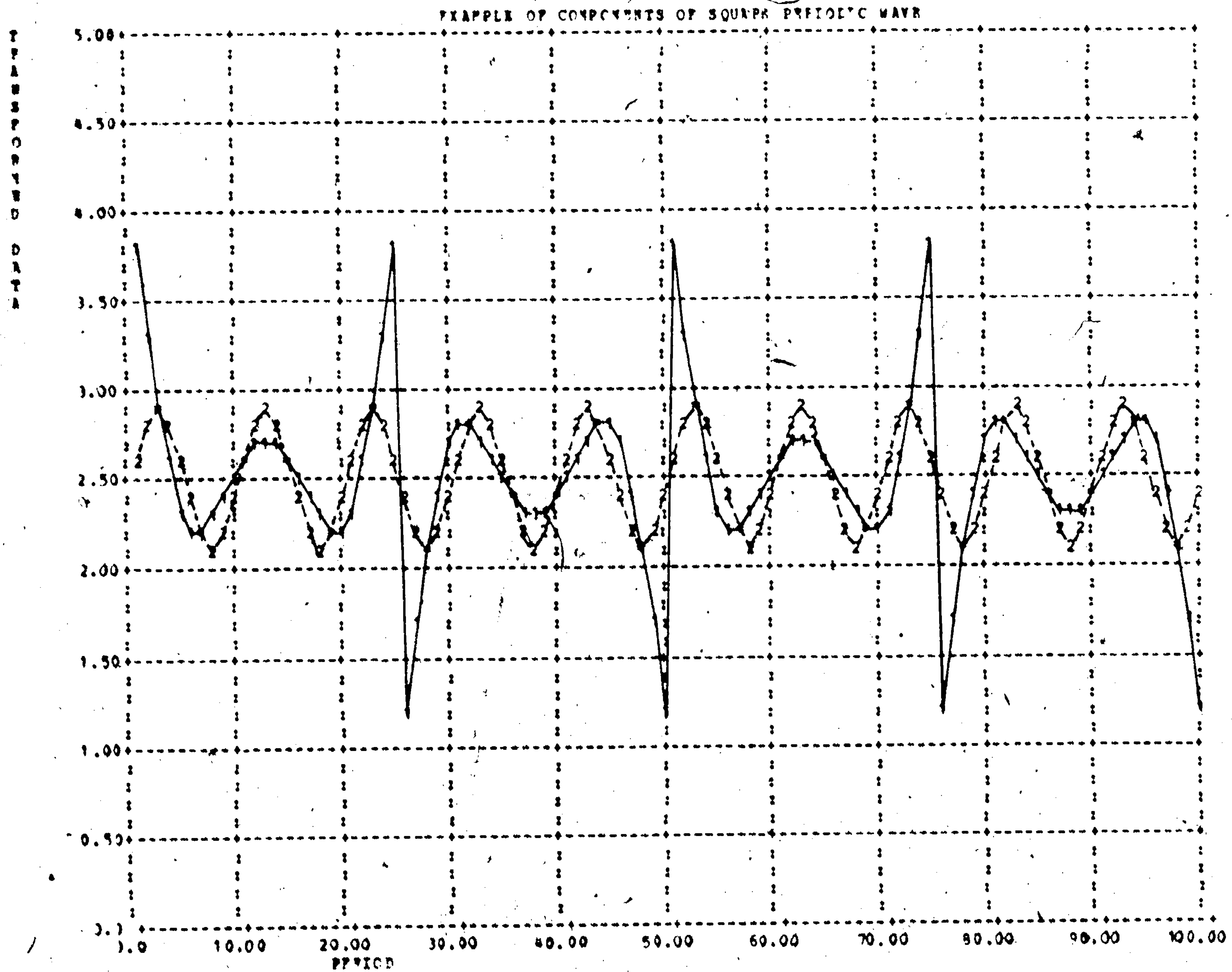
EXAMPLE OF COMPONENTS OF SQUARE PERIODIC WAVE
TRANSFORMED DATA
PERIOD
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00

FIGURE 6

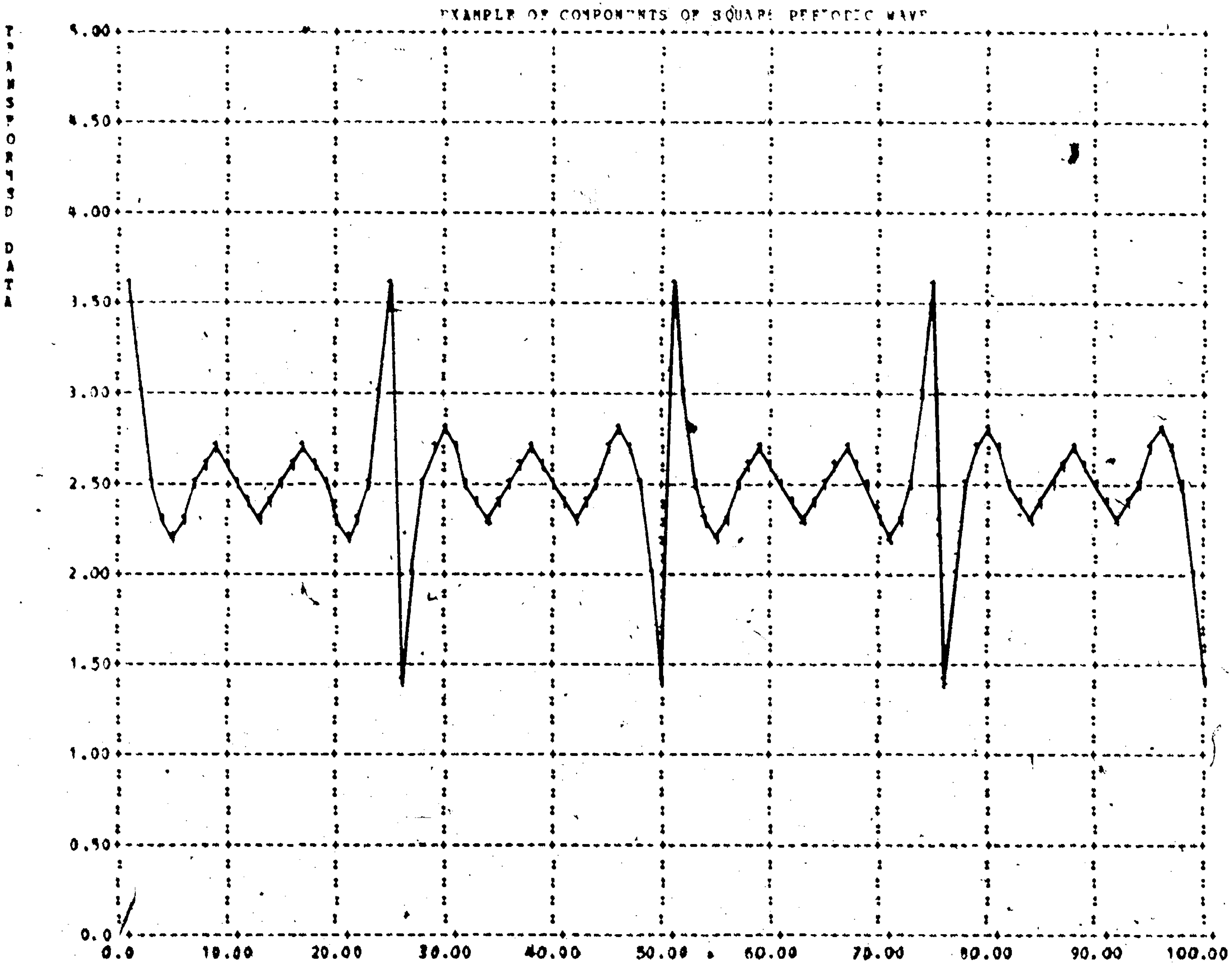
EXAMPLE OF COMPONENTS OF SQUARE PERIODIC WAVE
TRANSFORMED DATA
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.0
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00

FIGURE 7

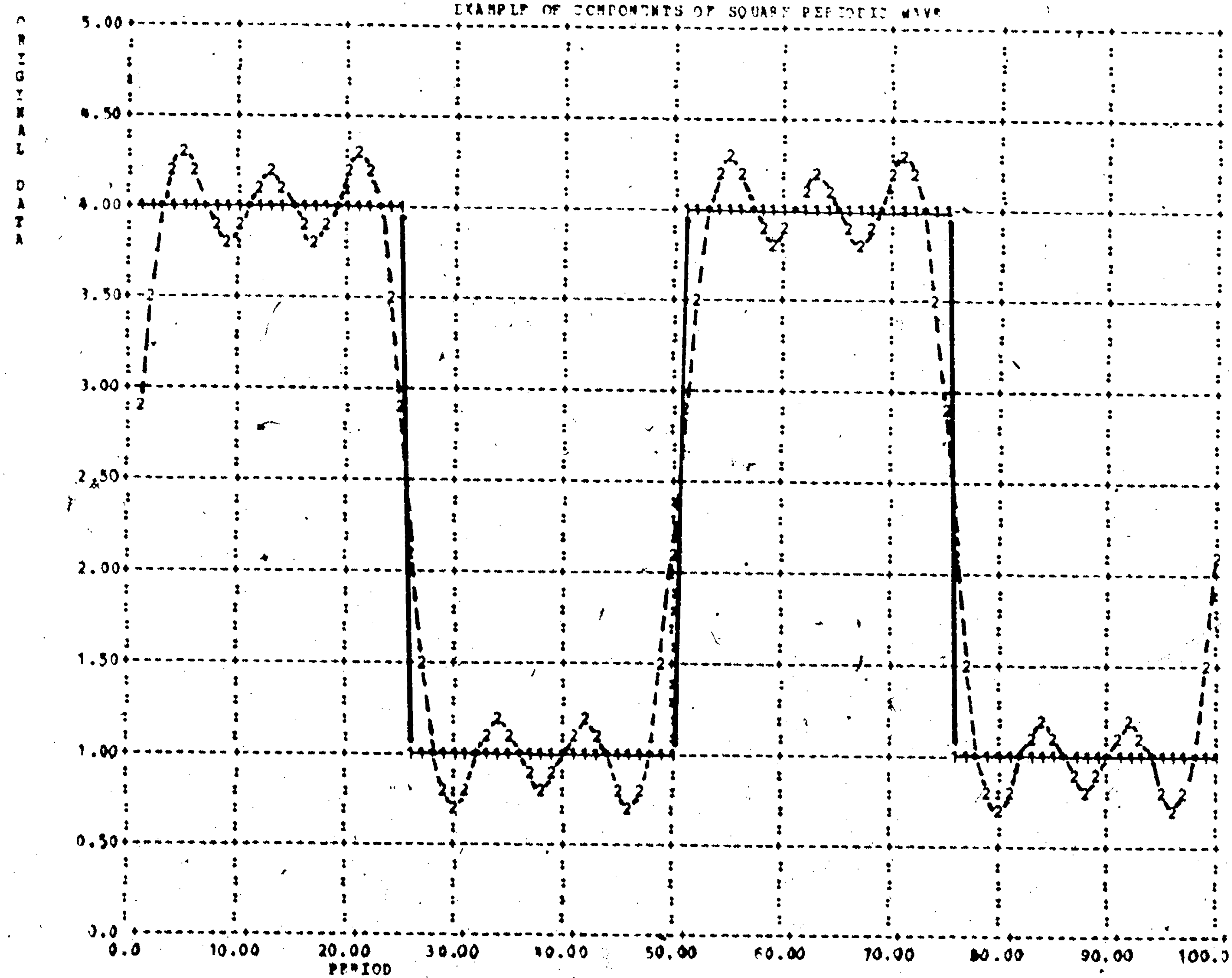
EXAMPLE OF COMPONENTS OF SQUARE PERIODIC WAVE
ORIGINAL DATA
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.0
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
PERIOD

FIGURE 8

FIGURE 9

TYPICAL PERIODOGRAM WITH SIDE-LOBES

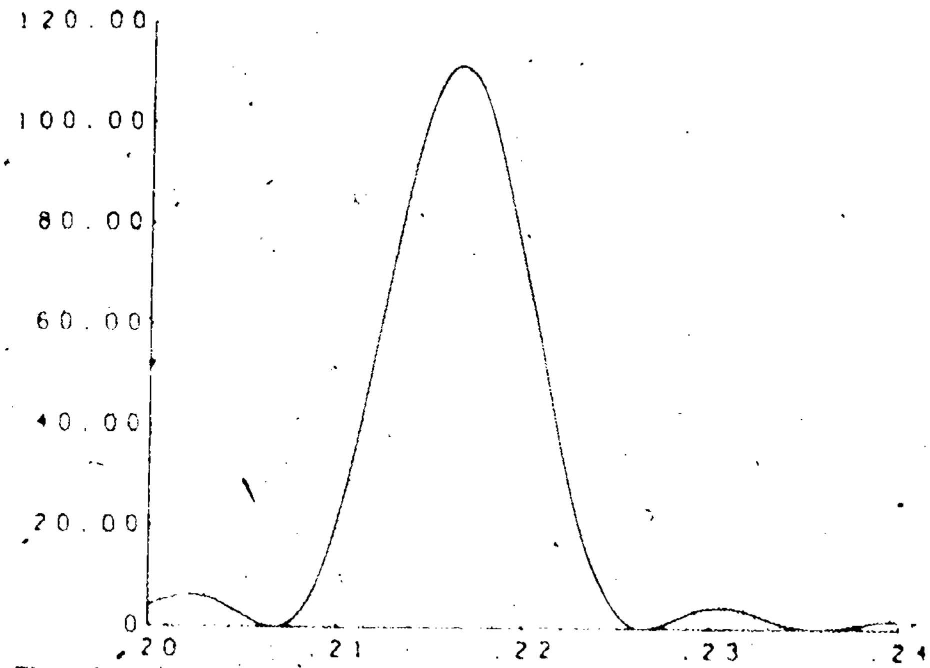

Figure 2.1 Periodogram of the variable-star data for frequencies ω, $0.20 < \omega < 0.24$.

From Bloomfield (1976)

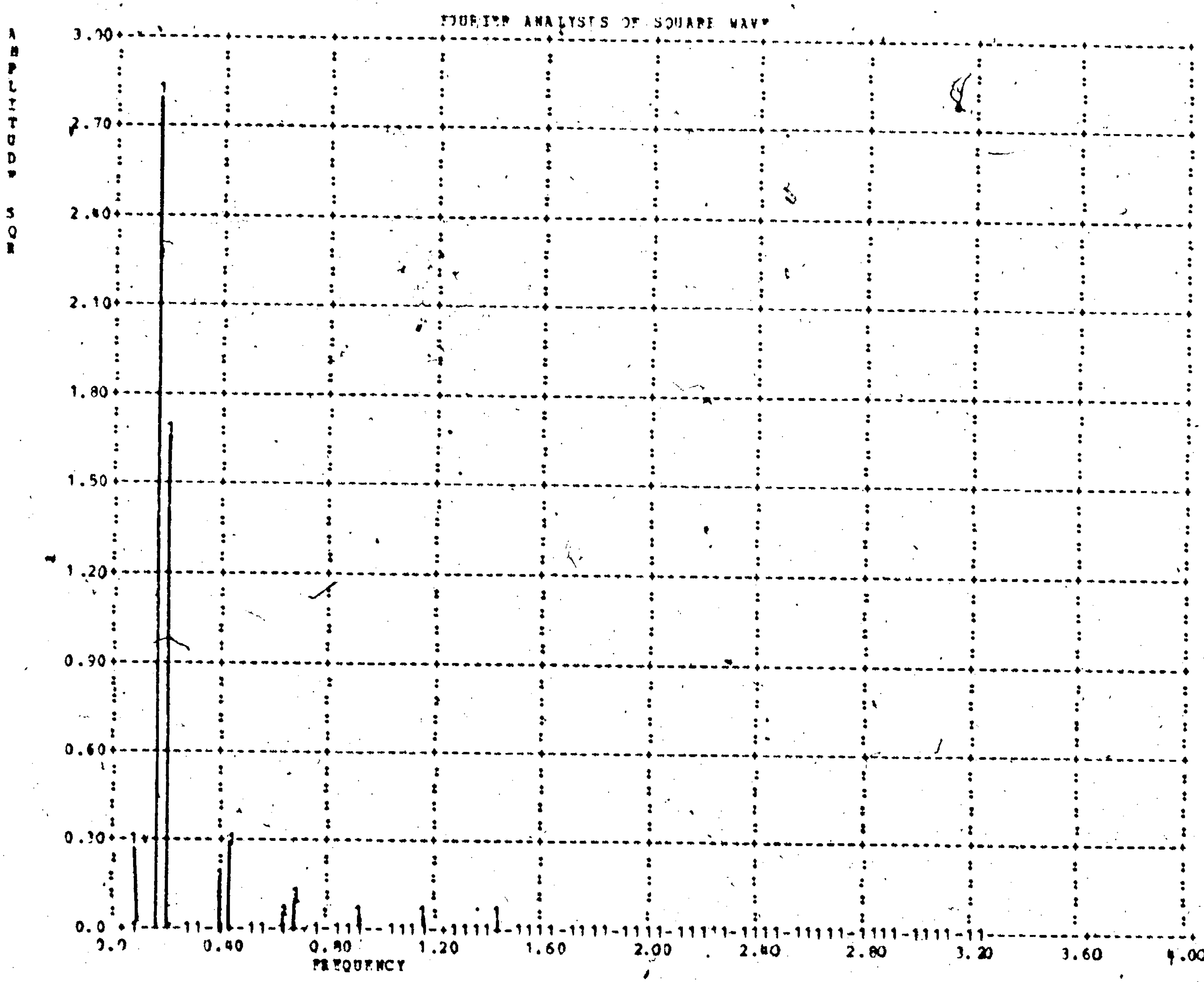
FOURIER ANALYSIS OF SQUARE WAVE
AMPLITUDE SQR
3.00
2.70
2.40
2.10
1.80
1.50
1.20
0.90
0.60
0.30
0.0
0.0
0.40
0.80
1.20
1.60
2.00
2.40
2.80
3.20
3.60
4.00
FREQUENCY

FIGURE 10

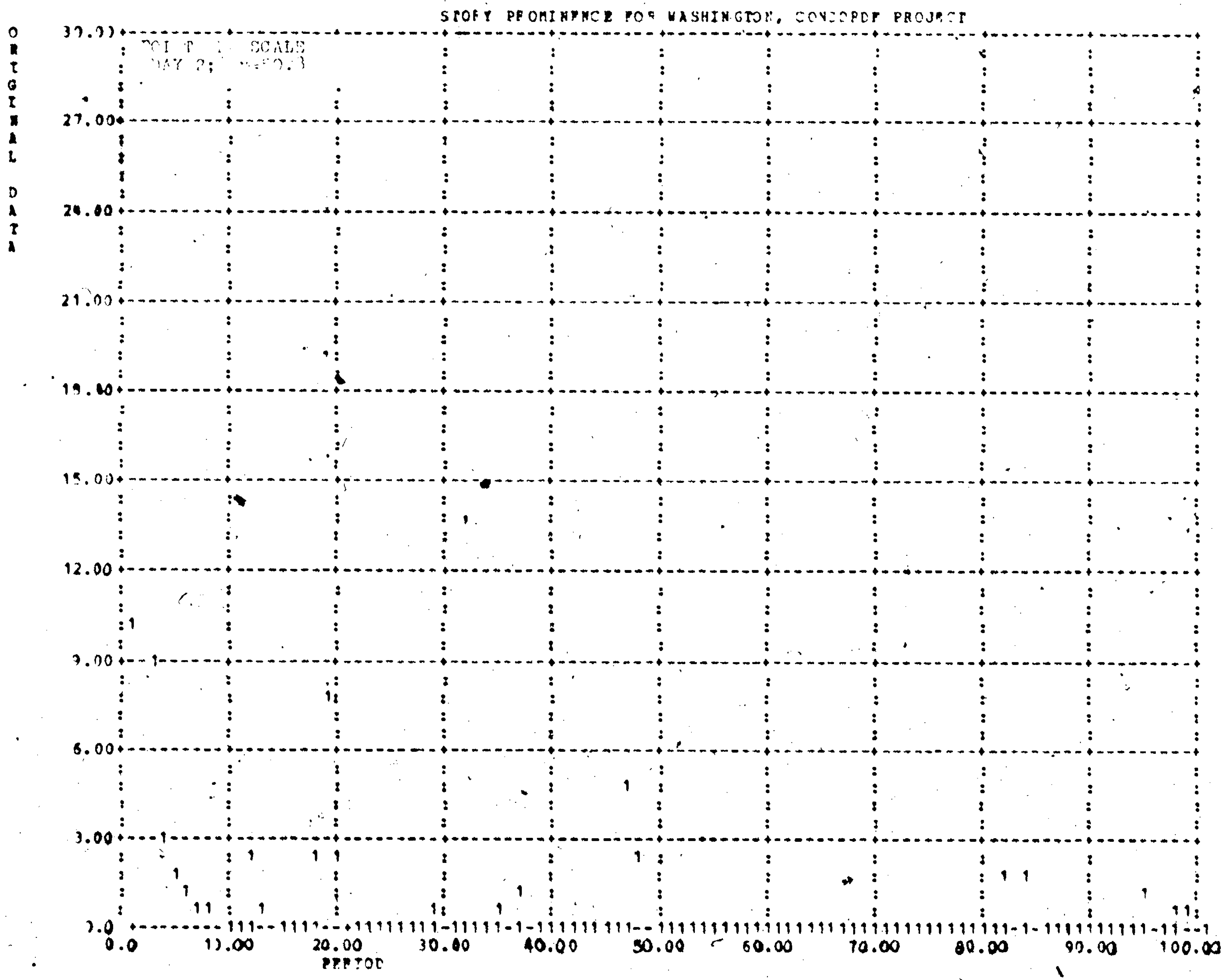
STORY PROMINENCE FOR WASHINGTON, CONCORDE PROJECT
ORIGINAL DATA
PERIOD
30.00
27.00
24.00
21.00
18.00
15.00
12.00
9.00
6.00
3.00
0.0
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00

FIGURE 11a

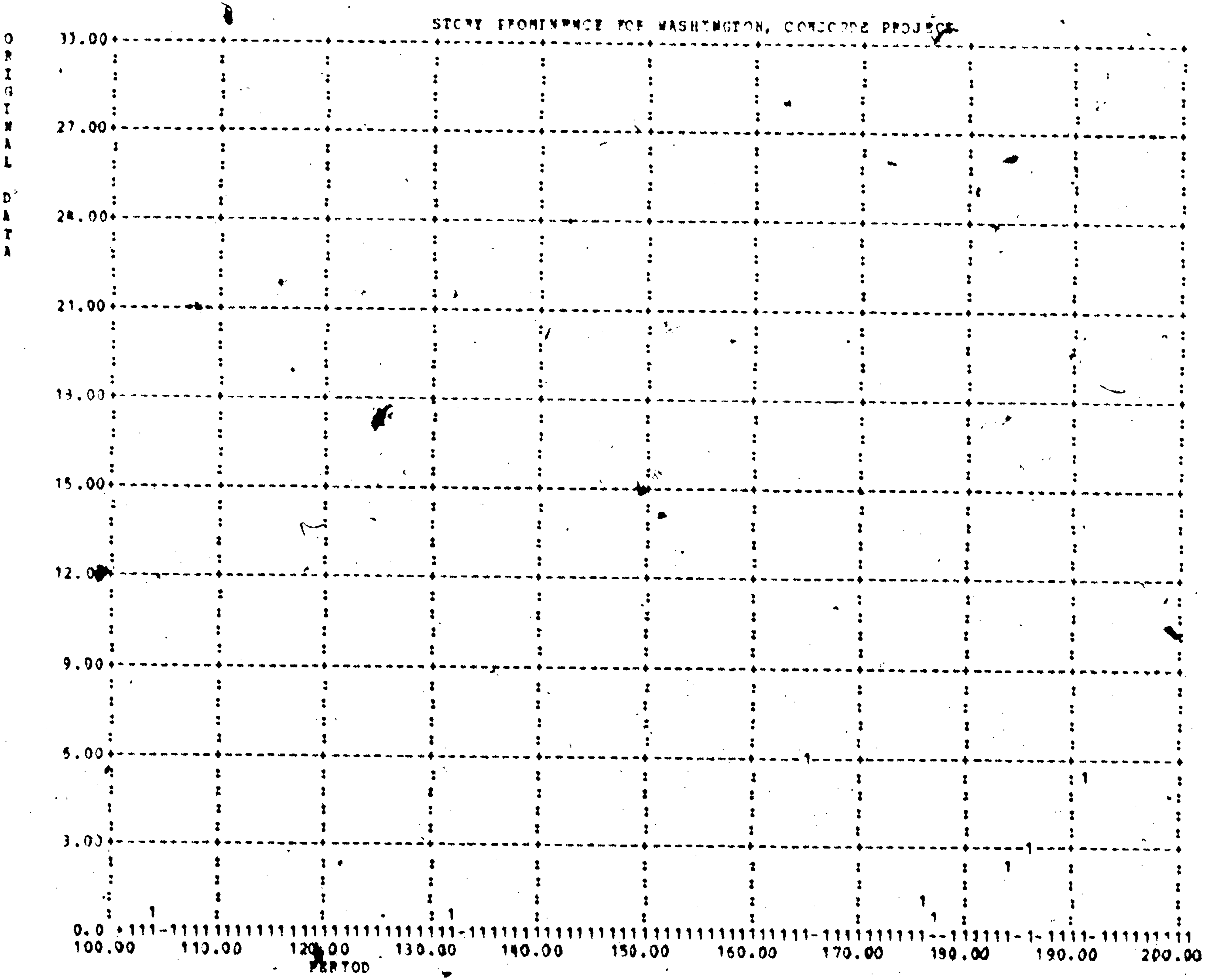
STORY PROMINENCE FOR WASHINGTON, CONCORDE PROJECT
ORIGINAL DATA
33.00
27.00
24.00
21.00
18.00
15.00
12.00
9.00
6.00
3.00
0.0
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
PERIOD

FIGURE 11b

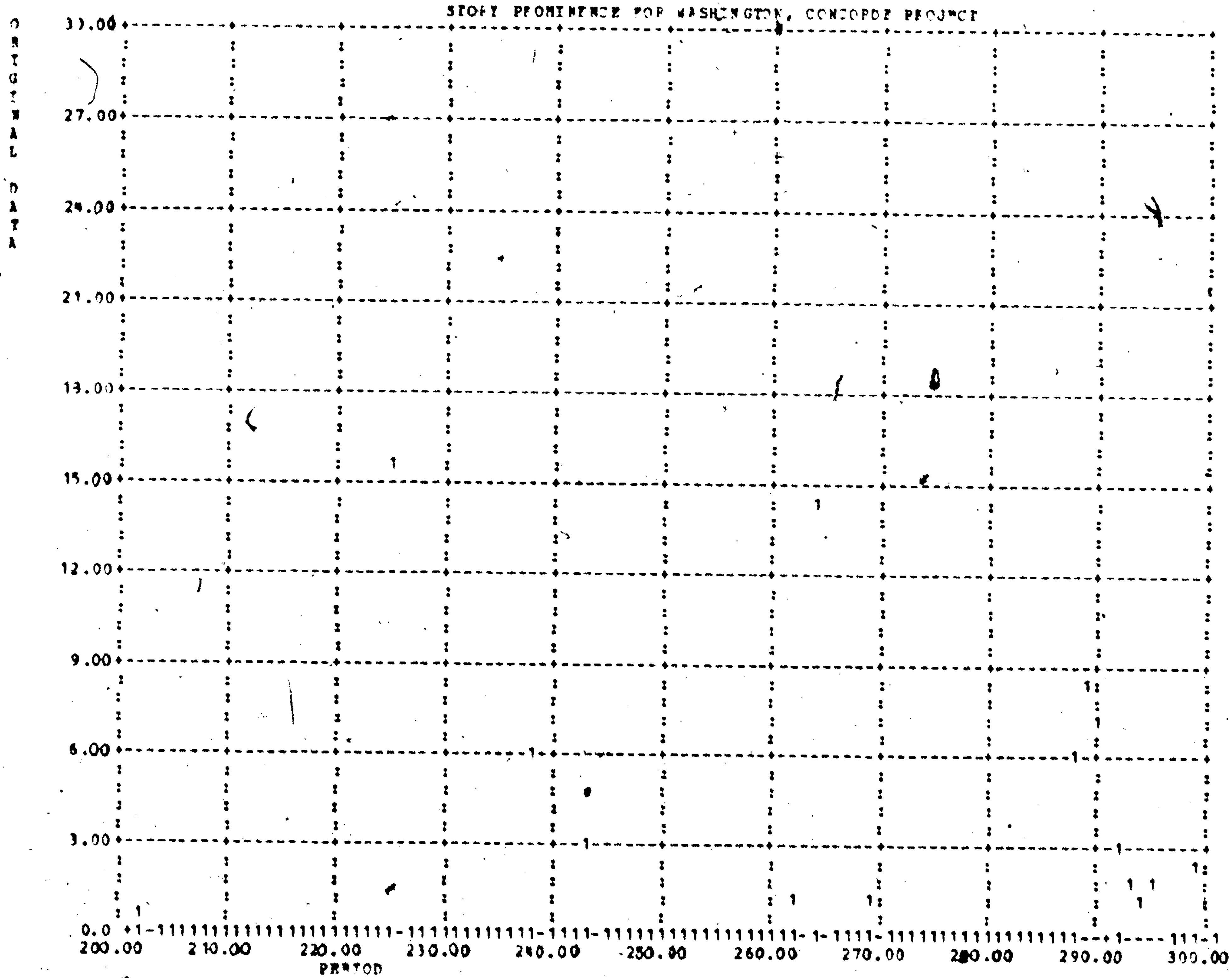
STORY PROMINENCE FOR WASHINGTON, CONCORDE PROJECT
ORIGINAL DATA
30.00
27.00
24.00
21.00
18.00
15.00
12.00
9.00
6.00
3.00
0.0
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00
280.00
290.00
300.00
PERIOD

FIGURE 11c

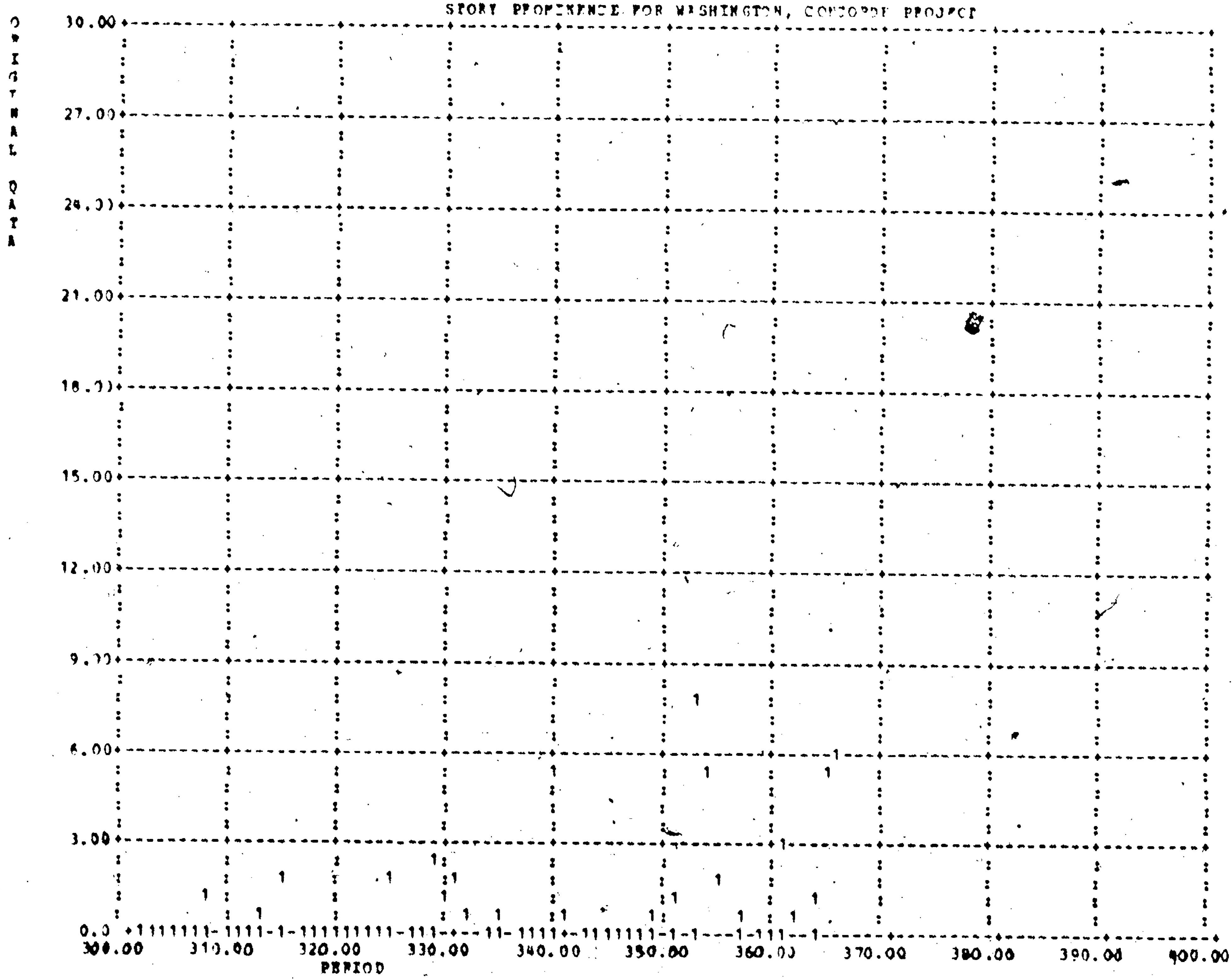
STORY PROMINENCE FOR WASHINGTON, CONCORDE PROJECT
ORIGINAL DATA
PERIOD
30.00
27.00
24.00
21.00
18.00
15.00
12.00
9.00
6.00
3.00
0.0
300.00
310.00
320.00
330.00
340.00
350.00
360.00
370.00
380.00
390.00
400.00

FIGURE 11c

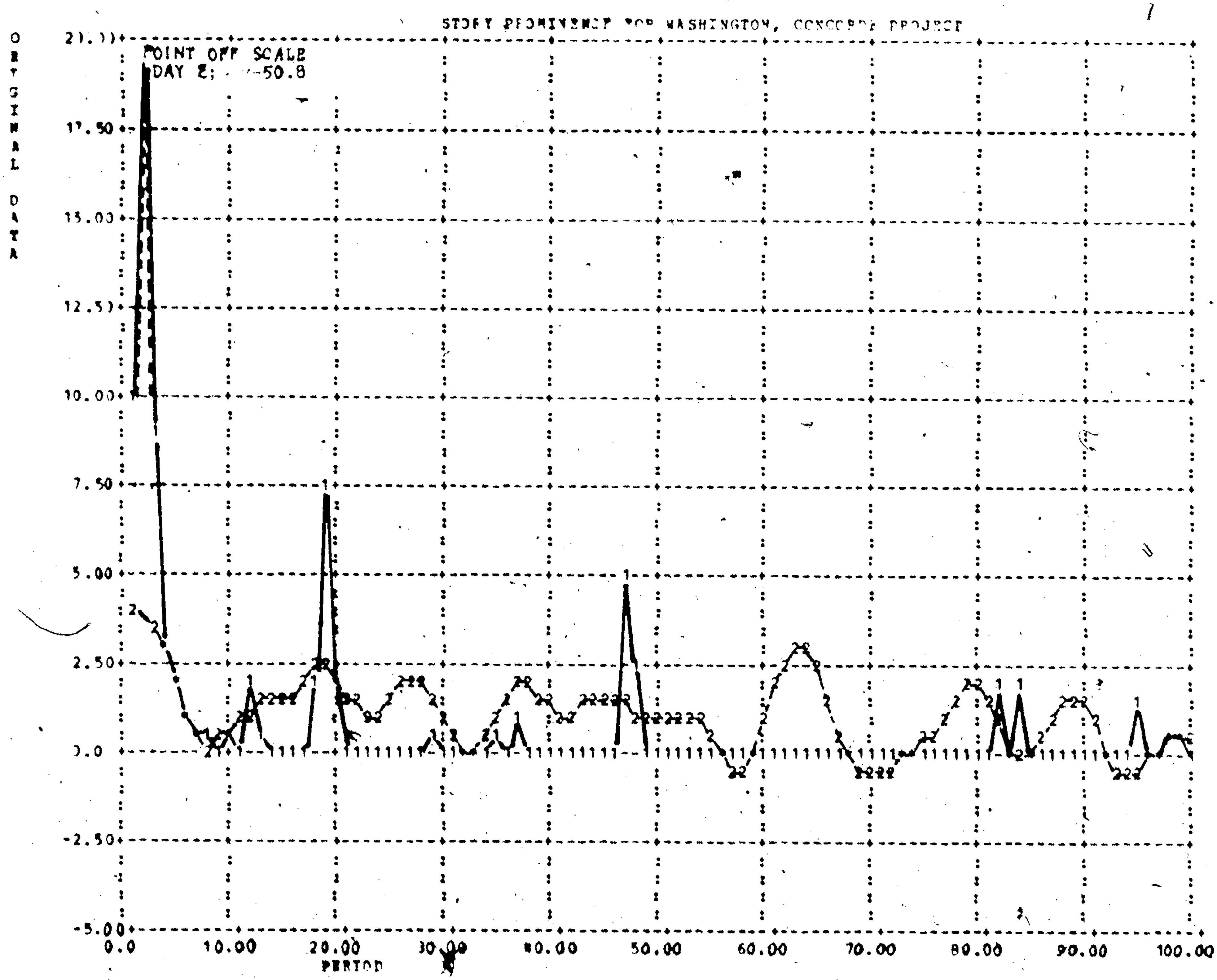
STORY PROMINENCE FOR WASHINGTON, CONCORDE PROJECT
ORIGINAL DATA
POINT OFF SCALE
DAY 2: -50.8
20.00
17.50
15.00
12.50
10.00
7.50
5.00
2.50
0.0
-2.50
-5.00
0.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
PERIOD

FIGURE 12a

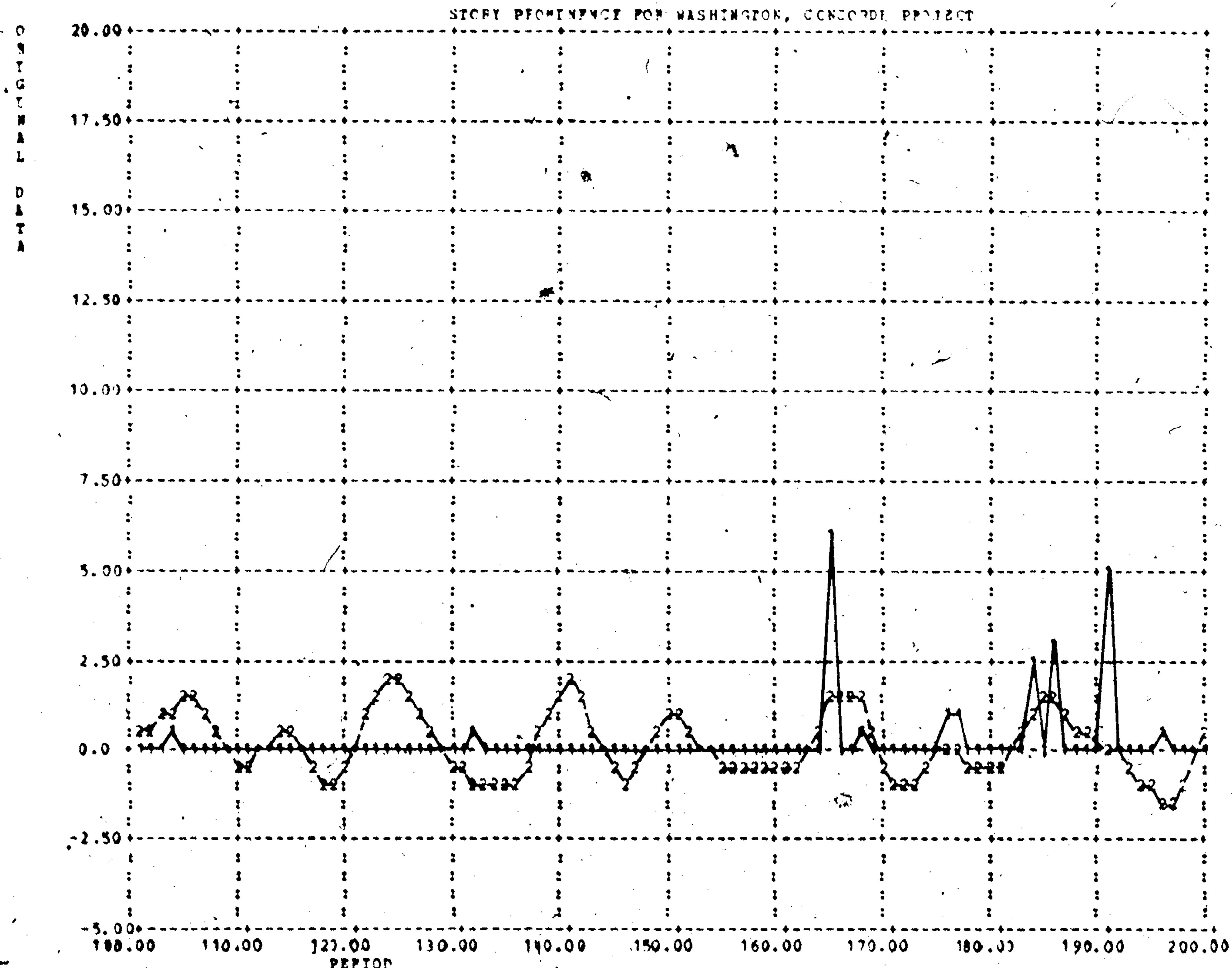
STORY PROMINENCE FOR WASHINGTON, CONCORDE PROJECT
ORIGINAL DATA
20.00
17.50
15.00
12.50
10.00
7.50
5.00
2.50
0.0
-2.50
-5.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
PERIOD

FIGURE 12b

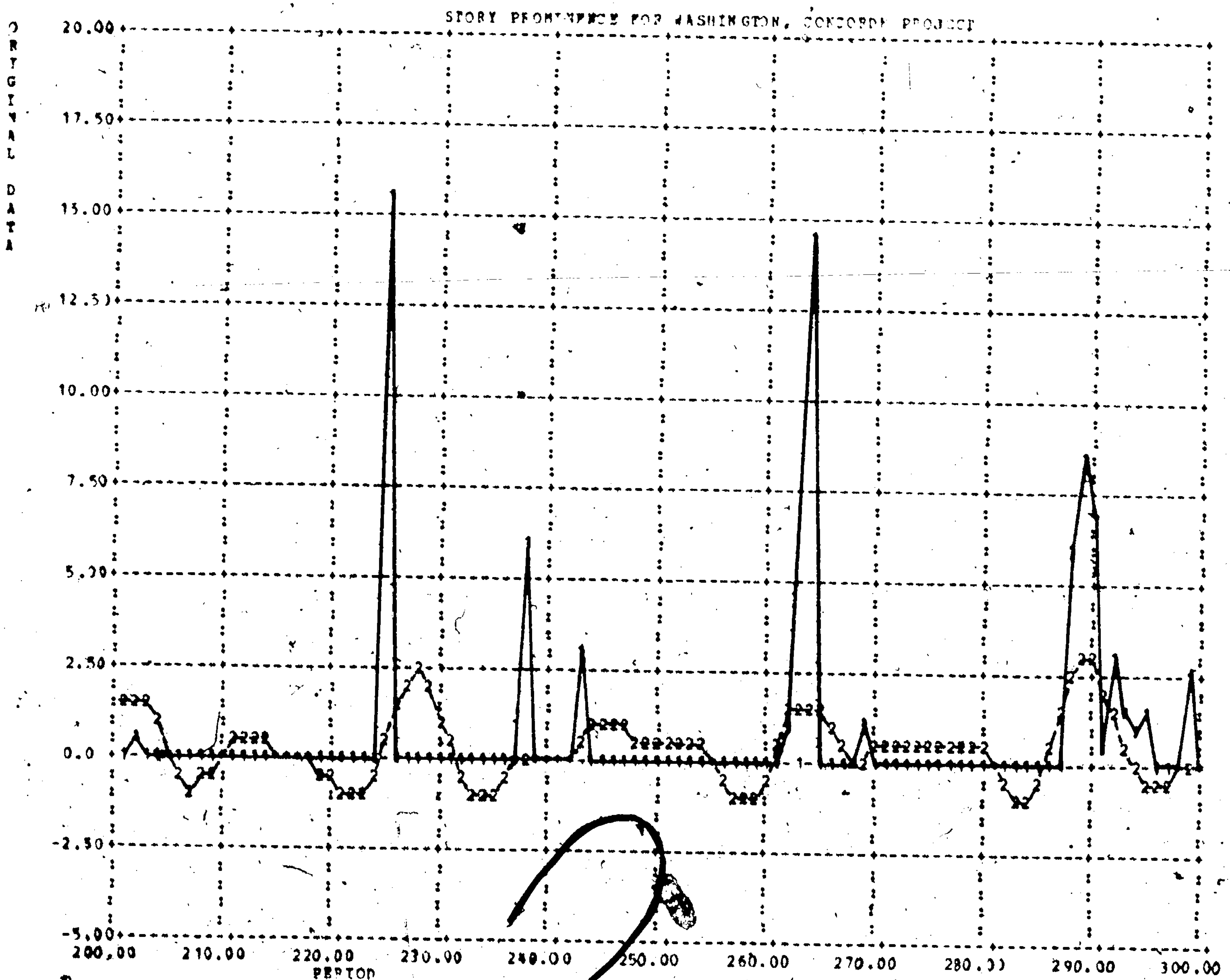
STORY PROMINENCE FOR WASHINGTON, CONCORDE PROJECT
ORIGINAL DATA
20.00
17.50
15.00
12.50
10.00
7.50
5.00
2.50
0.0
-2.50
-5.00
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00
280.00
290.00
300.00
PERIOD

FIGURE 12c

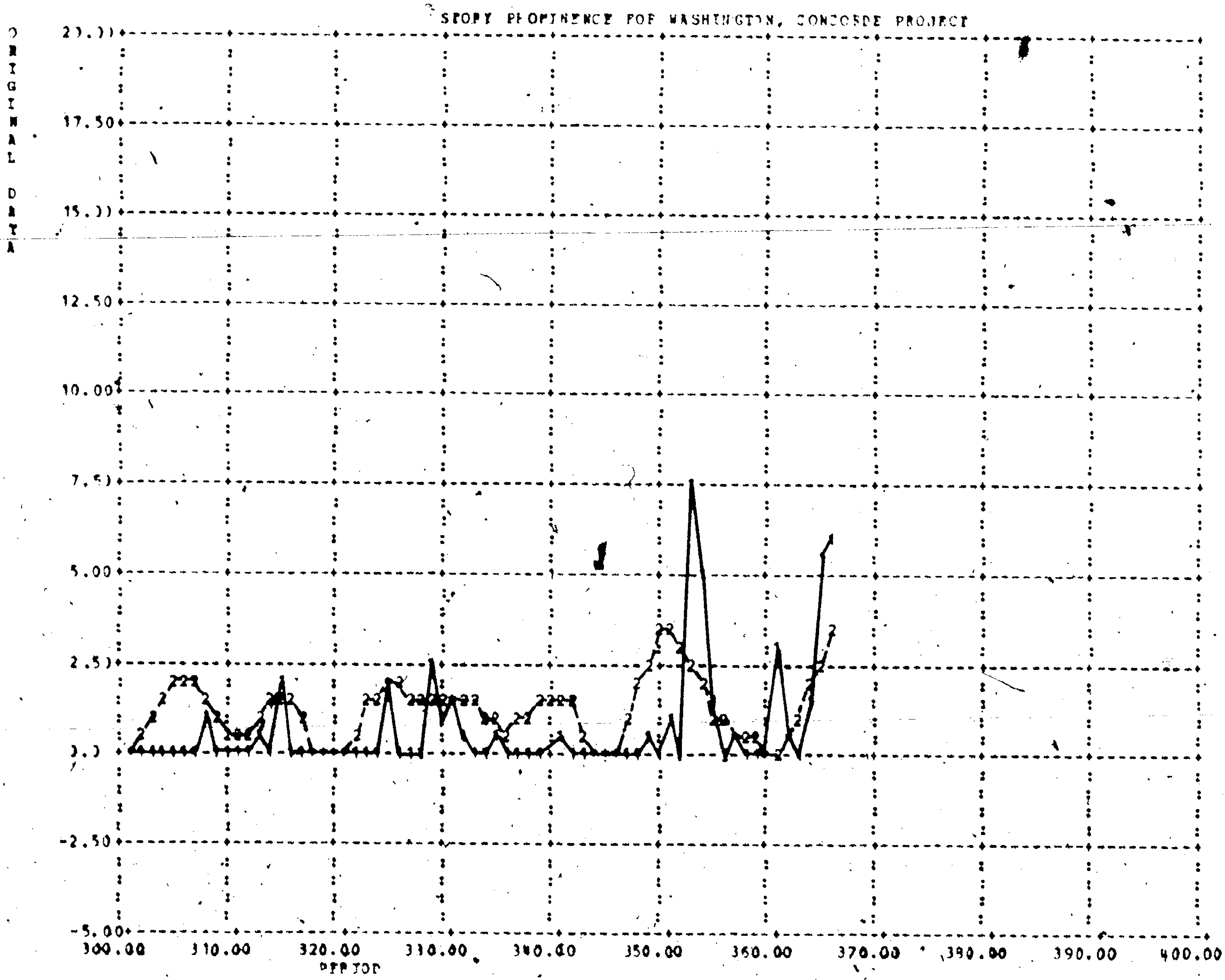
STORY PROMINENCE FOR WASHINGTON, CONCORDE PROJECT
ORIGINAL DATA
20.00
17.50
15.00
12.50
10.00
7.50
5.00
2.50
0.0
-2.50
-5.00
300.00
310.00
320.00
330.00
340.00
350.00
360.00
370.00
380.00
390.00
400.00
PERIOD

FIGURE 12d

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