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ABSTRACT

This guide presents lesson plans for the teaching of mathematics within the biomedical curriculum. It is designed to accompany the student text. Lessons concentrate on biomedical applications of the subject mathematical techniques, and are designed to be taught in close cooperation with the science lessons of the curriculum. Each lesson plan presents objectives, class periods required, overview and remarks, and a key to problems presented in the student text. (RE)

B I O M E D I C A L   M A T H E M A T I C S

UNIT I

MEASUREMENT,
LINEAR FUNCTIONS
AND DIMENSIONAL ALGEBRA

INSTRUCTOR'S MANUAL
REVISED VERSION, 1975

THE BIOMEDICAL INTERDISCIPLINARY CURRICULUM PROJECT

SUPPORTED BY THE NATIONAL SCIENCE FOUNDATION

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INTRODUCTION TO UNIT I

The course you are about to teach is unlike most other mathematics courses in at least two important ways. First, this course is a part of a larger interdisciplinary program. We wish to stress from the start the importance of maintaining close contact with the other members of the Biomedical teaching team. The unity which you can achieve will largely dictate the quality of the interdisciplinary experience for your students. At the most basic level, your pacing and the pacing of the other instructors must be closely coordinated at times. There are specific instances in which the intended sequence of events is essential. For example, Science Lesson 15 must be presented prior to mathematics Lesson 19 (see Interdisciplinary Ties). Situations will arise in which you will need to adjust the pacing of your material to accomodate the needs of other team members. Flexibility will be an asset in making a success of this course.

The other unusual aspect of this course is its biomedically applied orientation. We have adopted the ideal of applying the mathematics developed to the complete solution of biomedically relevant problems whenever possible. For example, skills associated with the solution of algebraic equations are applied to predicting the length of time that a scuba diver can stay under water. Following this sequence, knowledge associated with the study of linear functions is used to predict the blood-alcohol concentration for given times after drinking. This information is related to legislation of blood-alcohol concentration and driving.

Because of the respiration theme of this first unit, the science course begins with a consideration of the behavior of gases. For this reason, the mathematics course begins with a review of the concepts of length, area and volume and the distinctions between them. This is followed by the introduction of measurement, a topic basic to all of applied mathematics. We have found that students have difficulty handling the uncertainty inherent in their measurements. Therefore, explicit attention to uncertainty begins early. Again and again throughout the course the idea of uncertainty in measurement is returned to and built upon. Note, however, that arithmetic operations on ranges of uncertainty are deferred until the next unit. It is for this reason that almost all of the answers to problems in this unit are "nice," i.e., whole numbers or mixed numbers with small denominators.

Our advice to you, the teacher, is to sidestep the issues of rounding off and significant digits until they are taken up in the next unit. Our treatment of these topics will also differ somewhat from the typical. It is our contention that the significant digit conventions in common use are simplistic and inaccurate. We thought it better to defer the treatment of the arithmetic operations with approximate numbers until sufficient mathematical skills had been developed to handle a more accurate treatment. Along the same vein you will notice the use of the "=" and "≃" signs. When an equation becomes approximate the = sign is replaced by the ≃ sign.

Another theme that is begun early is an emphasis on the metric system of measurement. All graphs that appear in both the Science and the Mathematics Text are metric based. Recommendations for the scaling of graphs are based on the assumption that there will be metric based graph paper available for the students. In particular, 2 mm grid graph paper is heavily used. You may find such paper unavailable locally, necessitating arrangements for producing it. We also assume that you will have at least one meter stick in the classroom, and that every student has a metric ruler.

Because this is a math course the curriculum moves on to a review of functions. The strictly mathematical content of the course receives its inspiration from SMSG-type treatments. Many of the ideals of the SMSG will be encountered in this course. For example, the attention to precise mathematical language is retained. Another ideal that we are in complete agreement with is that students should not be taught principles that later have to be unlearned.

A review of linear functions follows. Knowledge of linear functions is then applied to the analysis of data arising from Charles Law [Volume = m(Kelvin temperature)] investigation in science class. A similar analysis of Boyle's Law [$\frac{1}{\text{volume}}$ = m(pressure)] data follows. Then linear functions are applied to a study of correlations between height and a quantity called vital capacity. Vital capacity is the measure of the maximum amount of air produceable between maximum inspiration and maximum expiration. These applications lead into the previously mentioned scuba diving problems, which are followed in turn by the blood-alcohol concentration problems.

Also included in Unit I is an examination of the technique called "dimensional algebra." An example of this is the following unit conversion problem.

$$4 \text{ yards} \times \frac{3 \text{ ft}}{\text{yd}} \times \frac{12 \text{ in}}{\text{ft}} \times \frac{2.54 \text{ cm}}{\text{in}} = 365.76 \text{ cm}$$

This is a topic that is typically treated only in science courses. Many times the reason why this technique works is glossed over by science texts. That repeated multiplication by one is what is happening is explained and used to develop the technique. This is another instance of a topic that should be sidestepped until it is taken up in due course. Prior to that point, help is provided whenever a unit conversion is needed.

There are a few points about the teaching of these materials that are not stated explicitly in the lesson plans. It is assumed that when you have completed the presentation of new material each day, the students will use any remaining time to begin working on the associated problem set. Thus the students should make a general rule of bringing the student text to class. Any remaining problems (whether you have assigned some or all of the problems in a set) are to be completed outside class. It is further assumed that the beginning of the next class will be devoted to a review of the preceding day's work, demonstration of the solution or specific problems, etc.

If you have a transparency machine available you may want to make use of the ones listed under the category of Supplementary Transparencies. We have provided printed copies suitable for use in such a machine.

INTERDISCIPLINARY TIES
UNIT I

It is essential that you have in your possession the following data on the days indicated.

<u>Lesson</u>

19	Data from Science Laboratory Activity 15
20	Data from Science Laboratory Activity 16
22	Data from Science Laboratory Activity 18

INTRODUCTORY LESSON: ORIENTATION TO THE BIOMEDICAL CURRICULUM

SUGGESTED TEACHING PROCEDURES:

The two purposes of this lesson are to inform students of the basic structure and intent of the Biomedical Curriculum, and to demonstrate to students that their three instructors are working as an interdisciplinary team. To accomplish this, you and your two colleagues should meet with the students for the first class period of the semester. If Science is the first of the three classes scheduled, all three instructors may still have a full class period to dispose of administrative details. If this is not the case, you may be able to arrange schedules so that each of you will have at least some of your own class time with the students. Although arranging schedules so that three teachers are free during the same period may be difficult, starting the course with this joint orientation will go far toward encouraging you and your students to approach curricular topics with an interdisciplinary perspective.

This first meeting can be organized in a number of ways. An essential ingredient is the provision for student questions. This will allow students to see how you and your colleagues interact as a team. Each of you may wish to react to the entire program from your own perspective, thus allowing students the opportunity to see how different perspectives can and should be applied to a central topic such as health.

Specifically, your team may be able to locate an article in the local newspaper which will allow each of you to indicate how your teaching area can provide a perspective on the topic. For example, a drought or famine in some area of the world, accidents, coronary heart disease, or organ transplants are appropriate for analysis by the mathematician, the natural scientist, and the social scientist. A statement about changes in the health characteristics of a population, such as an increase in the frequency of an illness, or an increase in the proportion of persons over sixty-five, is also appropriate for analysis from all three perspectives. A health care delivery topic, such as a shortage of equipment or a financial dilemma, can be used to illustrate how all three perspectives may be applied.

Other examples will occur to you and your colleagues as you plan for this meeting, or you may want to use another approach. The organization of this initial lesson is not as important as the results you are able to obtain in orienting students to the curriculum in an interdisciplinary manner.

In the next class session each of the three teachers may have time to pursue in greater depth the perspective each course can supply for a health topic. If students come to understand that the three courses will seldom be totally related to each other on specifics, but will often be related to each other on general topics, they will begin the course of study with the appropriate interdisciplinary approach.

LESSON 1: WHAT KIND OF COURSE IS THIS?

OBJECTIVES:

The student will perform arithmetic operations with signed numbers.

PERIODS RECOMMENDED:

One

SUPPLEMENTARY TRANSPARENCIES:

Transparencies I-M-1a, b

OVERVIEW AND REMARKS:

The bulk of this lesson is devoted to operations with signed numbers. Before beginning that material, however, we suggest some introductory remarks along the following lines.

The Biomath course is primarily a mathematics course. The sequence and contents of the lessons were created with an eye to the gradual development of mathematical skills. Every attempt has been made to prevent this course from becoming a vocational one. To do otherwise would tend to prohibit students from taking additional math courses following high school graduation. The following is a list of mathematical topics to be covered during the biomath course. Transparency I-M-1a can be used to display this list if you wish.

1. Linear Equations
2. Vectors
3. Elementary Statistics
4. Quadratic Equations
5. Symbolic Logic
6. Elementary Trigonometry
7. Binomial Theorem
8. Elementary Differential Calculus
9. Exponentials and Logarithms.

The students may want to know which of these subjects are taught in other math classes in their high school. Such information would be helpful to them, along with a very brief notion of what each topic is about.

Most current math programs beyond Algebra I are concerned with the development of increasingly complicated abstract mathematical concepts. The emphasis of this course will be on the application of mathematical concepts and skills to the solution of problems arising mainly in the realm of biomedicine and frequently from experiments in the Bioscience Lab.

The students should understand that the math program that they are about to embark upon is not a complete "about face" from their prior experience. The emphasis on precise mathematical language is retained. The ideal of not teaching something that has to be unlearned later is retained. (An example of this is that students used to be taught that a larger number could not be subtracted from a smaller number. Later on they were told that this was not true.) Some interest in the derivation of mathematical relationships is retained. However, more emphasis on computational skills is built into the course than they would encounter in the conventional Algebra II course.

The developers of the curriculum recognize the existence of a certain amount of tedious and potentially boring activity connected with the development of computational skills. In order to alleviate this aspect of the process the developers have adopted the ideal of created problems that have "something extra" whenever possible. Note that math texts typically concentrate solely on the development of a particular mathematical skill (e.g., the "age" problems of Algebra I). In the development of appropriate problems, as much energy was directed to the "reality factor" as to the mathematical concerns. When the students have finished working a problem they often will have simultaneously learned a fact completely outside the academic field of math. Below is a sampling of the kinds of computations that the student should be able to perform upon completion of the Biomedical Curriculum (also available on Transparency I-M-1b).

1. How much air is left in the lungs after one exhales to the fullest?

2. How long a scuba diver may stay at a given depth when the tank pressure, volume and temperature are known.

3. Approximate maximum blood-alcohol concentration when the proof and volume of beverage and the mass of the drinker are known.

4. How long is it necessary to wait after drinking a given amount of liquor before it is legal to drive.

5. How to interpret and alter stated drug concentrations as a nurse or pharmacist would have to do on the job.

6. How to compute and state the accuracy of an experimental result as a lab technician must do.

7. The number of calories in a food from the quantities of protein, fat and carbohydrate in it.

8. The least expensive combination of two varieties of nuts or beans to meet or exceed the protein content of lean beef.

9. The magnification factor involved in interpreting X-rays as an X-ray technician must be able to do.

10. Determine whether an elementary school class of children has a standard level of vision correction.

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LESSON 2: APPLIED MATH PROBLEMS

OBJECTIVE:

The student will solve word problems involving signed numbers.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

In Lesson 1 the students encountered arithmetic problems involving signed numbers. Such problems are exceptional in the applied setting; typically an applied problem requires considerable interpretation before it reduces to a strictly arithmetic or algebraic form. Therefore, "word" problems will be a routine occurance in this course. Therefore, the problem set will follow a pattern which is designed to ease the difficulty inherent in word problems. The first problem of a given type will be broken down into a number of "bite size pieces" facilitating the translation from verbal to numeric. Subsequent problems of the same type will provide only some of the intermediate steps, challenging the student to provide the others. Often, the student will finally be faced with a problem which he or she must solve without the benefit of intermediate instructions.

All the above remarks are amply illustrated in the text and problem set of this lesson. Indeed, the basic intention of the lesson is to familiarize the students with the approach to word problems in the biomath course.

KEY--PROBLEM SET 2:

1. a. -2 1 = -3

   b. Elmo's temperature fell 3° C

2. a. The kidney removes waste products from the body for excretion in the urine.

   b. Very little or very much extra body water.

   c. The objective is to have the intake match the output.

   d. decreased

   e. 400 ml

   f. 400 + 400 + 250 = 1050 ml

   g. 150 ml

   h. (-350) + (-200) + (-375) + (-150) + (-100) = -1175

   i. 1050 - 1175 = -125

   j. Output exceeded intake by 125 ml.

   k. Intake should be increased by 125 ml.

3. a. 600 + 500 + 200 + 14 = 1315

   b. (-600) + (-500) + (-66) + (-150) + (-100) = -1950

   c. 1315 - 1950 = -635

   d. The patient's intake should be increased by 635 ml during the next time period.

4. a. 2 hr

   b. 3 hr

   c. 200 - (-20) = 220

   d. t = -40 hr

5. 6 meters

6. a. debts and expenditures

   b. cash on hand or taken in

   c. increase

   d. -53.14 + 16.33 + 24.53 + 47.19 - (-117.74) - 96.14 = +56.51

e. Probably

7. a. Darius

b. -519 - (-558) = 39 years

8. a. 14,495 - (-282)

b. 14,495 - (-282) = 14,777 ft.

LESSON 3: PATHS AND SURFACES

OBJECTIVES:

The student will distinguish between situations involving one space and those involving two space.

The student will be able to use the formulas for the areas for rectangles, triangles, trapezoids and circles to find the areas of given plane figures.

The student will use subscript notation correctly in algebraic manipulations.

PERIODS RECOMMENDED:

One

SUPPLEMENTARY REFERENCE:

Farrell, Margaret A. Area from a Triangular Point of View. The Mathematics Teacher, LXIII: 1 January 1970, pp. 18-21

OVERVIEW AND REMARKS:

In Lessons 3 through 6, the notion of the dimension of a space is considered. The appropriate units (cm, cm^2 or com^3) are introduced and basic formulas for areas and volumes of geometric figures are presented. The supplementary reference explores an alternate approach to area, namely the use of triangular units of area instead of square units of area.

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KEY--PROBLEM SET 3:

1. a. 1
   b. 2
   c. 1
   d. 2
   e. 1
   f. 1
   g. 2
   h. 1
   i. 1
   j. 2
   k. 2
   l. 1
   m. 1
   n. 2
   o. 2
   p. 1
   q. 2
   r. 1

2. a. b_1
   b. A_Δ
   c. x_i
   d. m_s
   e. L_{mo}
   f. P_0

3. a. A sub trapezoid
   b. x sub two
   c. m sub three
   d. a sub i
   e. yellow sub marine
   f. a sub set
   g. Commie subversive
   h. hard subject
   i. fat sub contract
   j. very sub tle
   k. add sub tract
   l. I'm sub normal

4. a. $b_1 + b_2 = b_1 + b_2$
   b. $a_1 + a_1 = 2a_1$

5. a. 10
   b. 15
   c. 35
   d. $\frac{15}{2}$
   e. 25
   f. 10

20

6. a. cm
   b. cm^2
   c. cm^2
   d. cm^2
   e. cm^2
   f. cm^2

7. a. 10 cm
   b. 50 cm^2
   c. 6 cm
   d. 15 cm^2
   e. 65 cm^2
   f. 65 cm^2
   g. yes

8. a. 10 cm^2
   b. 5 cm^2
   c. 3600 cm^2
   d. 0.36 m^2
   e. 154 cm^2

9. a. 434 cm^2
   b. ~154 cm^2
   c. ~588 cm^2

10. a. 48 cm^2
    b. 12 cm^2
    c. 161 cm^2
    d. 24 cm^2
    e. 6 cm^2
    f. 30 cm^2
    g. 221 cm^2

11. a. 470 cm^2
    b. 470 mm^2

12. ~411.5 cm^2

21

LESSON 4: GROUPING AND DISTRIBUTIVITY

OBJECTIVE:

The student will group like terms.

The student will apply the distributive principle to the solution of problems.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

Area problems are used as a means of reviewing the distributive law and the grouping of like terms.

KEY--PROBLEM SET 4:

1. $A_{\square} + 2A_{\triangle} + 2A_{\triangle}$

2. $R + 5S + 3T + 3C + P$

3. $6\frac{1}{2}a + 20b$

4. $\frac{3}{2}b_1h_1 + b_2h_2$

5. a. $R_1 = xz$

   b. $R_2 = xy$

   c. $xy + xz = x(y + z)$

6. $\frac{h}{2}b_1 + \frac{h}{2}b_2$

7. $-xy + 2x$

8. $2z - 12$

9. $8b + 32$

10. $-mn - 4m$

11. $\frac{h}{2}(b_1 + b_2)$

12. $4(3z + y)$

13. $x(-a + b)$ or $-x(a - b)$

14. $-2(a + 4b)$ or $2(-a - 4b)$

15. $32(2a - b)$

16. $A_{total} = 2A_{\triangle} + A_{\triangle}'$

17. $A_{total} = \frac{1}{2}A_{\square} + 2A_{\triangle} + \frac{1}{2}A_{\square}$

18. $5a + b$

19. $5a + b$

20. $\frac{3}{2}a + 3b + \frac{1}{2}c$

21. $-3a - 4b + c$

22. $4a$

23. $3a + 2b$

24. $-7y - 2x$

25. $4a + 4x$

26. $a_{total} = \frac{3}{2}b_1h + \frac{1}{2}b_2h$

27. $-2ab + bc - 2c$

28. $a - 4bc - c$

29. $-ab - 2c + d$

30. $3ab - 3ac + c$

31. $x^2 - 2x - 8$

32. $2a^2 + 3ab + b^2$

33. $80^2 - 16 = 6384$

34. $MN - 10N + 2M - 20$

35. $x^3 + y^3$

36. $x^3 - y^3$

LESSON 5: VOLUME

OBJECTIVE:

The student will manipulate and substitute into algebraic formulas connected with volume.

PERIODS RECOMMENDED:

One

2.;

KEY--PROBLEM SET 5:

1. a. $V = \ell wh$

   $V = 10 \cdot (1.5) \cdot 2$

   $V = 30 \text{ cm}^3$

   b. rectangular solid

2. a. $V = \frac{1}{2} \cdot \overset{10}{20} \cdot (1.3) \cdot (9.1)$

   $V = 13 \cdot (9.1)$

   $V = 118.3 \text{ m}^3$

   b. triangular prism

3. a. $V = \frac{1}{2}(b_1 + b_2)ah$

   $V = \frac{1}{2}(6 + 5) \cdot 5 \cdot \frac{1}{10}$

   $V = \frac{1}{2} \; 11 \cdot 5 \cdot \frac{1}{\underset{2}{10}}$

   $V = \frac{11}{4}$

   $V = 2\frac{3}{4} \text{ cm}^3$

   b. solid with trapezoidal base

4. a. $V = \pi r^2 h$

   $V \simeq (3.14)(9^2)(\frac{1}{3})$

   $V \simeq (3.14)(81 \cdot \frac{1}{3})$

   $V \simeq 3.14 \cdot 27$

   $V \simeq 84.78 \text{ m}^3$

   b. (right) cylinder

5. a. 320 cm^3

   b. 560 cm^3

   c. 350 cm^3

   d. 350 cm^3

5. e. 550 cm^3

   f. 150 cm^3

   g. 850 cm^3

6. a. 300 cm^3

   b. 75 in^3

   c. 9 cm^2

   d. 2 mm

   e. 15 m

   f. 30 in^2

   g. 5.25 m^3

   h. 1.8 cm^3

7. a. 10 cm

   b. 6 cm

   c. 40 cm

   d. 12 cm

   e. 12 cm

8. a. $37,500 \text{ cm}^3$

   b. $30,000 \text{ cm}^3$

   c. rises

   d. 4.5 cm

LESSON 6: A TWO DIMENSIONAL UNIVERSE

OBJECTIVE:

The student will answer basic questions involving the geometries of one, two and three space.

The student will compute the density of a substance when given the mass and volume.

PERIODS RECOMMENDED:

One

SUPPLEMENTARY REFERENCE:

Abbot, Edwin A. Flatland. Sixth Edition, Dover Publications, New York, 1952

SUPPLEMENTARY FILM:

16-mm film: Flatland. 12 min., color, 1965. McGraw-Hill Films, 330 West 42nd Street, New York, New York 10036

OVERVIEW AND REMARKS:

This lesson begins with an exploration of one, two and three space and the projection of higher dimension figures on lower dimension spaces. Spaces of dimension higher than three are briefly discussed. The film Flatland provides a worthwhile jumping off point for discussion if you find it available. If you do obtain it be sure to tell the class to listen carefully since the narrator has an unfamiliar accent.

The latter part of this lesson treats the concept of density and the formula for computing it. Science Lessons 4 and 5 also discuss density, so the concept should be familiar to the students to some extent.

KEY--PROBLEM SET 6:

1. line

2. Momentarily, a sphere would look like a point when the sphere first came into contact with Flatland. Following this, the sphere would look like a line segment which would at first grow and then shrink and finally disappear.

3. 2

4. tug of war

5. 3

The units for the answers for Problems 6 through 14 are grams per cm^3.

6. $\rho = 2.702$ (by book)

   $\rho = 2.7$ (by calculation)

7. $\rho = 10.5$ (by either method)

8. $\rho = 10.5$ (by either method)

9. $\rho = 19.3$ (by book)

   $\rho \simeq 19.306$ (by calculation)

10. $\rho \simeq 8.933$ (by book)

    $\rho \simeq 8.9379$ (by calculation)

11. $\rho = 8.933$ (by book)

    $\rho \simeq 8.9385$ (by calculation)

12. $\rho \simeq 7.14$ (by book)

    $\rho \simeq 7.139$ (by calculation)

13. $\rho \simeq 7.14$ (by book)

    $\rho \simeq 7.1392$ (by calculation)

14. $\rho \simeq 4.50$ (by book)

    $\rho \simeq 4.50$ (by calculation)

15. $\rho \simeq 11.3$ g per ml

16. $\rho \simeq .8$ g per ml

LESSON 7: DIMENSIONAL ALGEBRA I

OBJECTIVES:

The student will find outside sources of unit conversion information.

The student will use conversion factors to convert units.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

This is the first in a series of three lessons convering the important topic of dimensional algebra. The concepts contained herein will be used throughout both the math and science courses.

Note that the second and third lessons in this sequence have been postponed until the end of Unit I (Lessons 35 and 36). Both of these later lessons use scientific notation and the laws of exponents which have not been covered at this point. Lesson 35 involves conversions of square and cubic units, so you need not be concerned with such conversions at this time.

Problem 1 in the problem set is intended to acquaint the student with references where conversion factors can be obtained. Sources which contain such information on unit conversion include:

1. dictionaries and encyclopedias under such headings as "measure," "metric system" and "weights and measures."

2. physics textbooks

3. chemistry textbooks

4. Handbook of Chemistry and Physics (under "Conversion Factors").

PREPARATION FOR FUTURE LESSON:

At this point you should persue Lesson 9. The activity in that lesson requires that the class have available a large number of containers (cans, bottles, etc.) with the labels still on them. The labels on the containers will be checked to see if they meet legal requirements.

In order to have a large number of containers available, you should instruct the class to bring empty ones from home within the next two days. It would also be wise to collect a few of your own for distribution to forgetful students.

KEY--PROBLEM SET 7:

1a. 1 furlong ≈ .2012 km

b. 1 rod ≈ 5.029 m

1 rod (Brit. volume) ≈ 28.317 m^3

c. 1 link (Gunter's) ≈ 20.12 cm

(Ramden's) ≈ 30.48 cm

d. 1 chain (Gunter's) ≈ 20.12 m

(Ramden's) ≈ 30.48 m

e. 1 pole ≈ 5.029 m

f. 1 cord-foot ≈ .4531 m^3

g. 1 cord ≈ 3.625 m^3

h. 1 fortnight ≈ 14 days

i. 1 astronomical year ≈ 365 days, 5 hr, 48 min, 45.51 sec

j. 1 fathom ≈ 1.829 m

k. 1 cable (U.S. Navy) ≈ 219.46 m

(Brit.Navy) ≈ 185.32 m

l. 1 hand ≈ 10.16 cm

m. 1 nautical mile (Brit.) ≈ 1.8532 km

(Int.) ≈ 1.852 km

n. 1 league (Naut. Brit.) ≈ 5.5596 km

(Naut. Int.) = 5.556 km

(Statute) ≈ 4.828 km

o. 1 gill (Brit.) ≈ 142 ml

(U.S.) ≈ 118 ml

p. 1 palm ≈ 7.62 cm

q. 1 imperial gallon ≈ 4.546 liters

r. 1 minim (Brit.) ≈ .059 ml

(U.S.) ≈ .062 ml

s. 1 drop (U.S) ≃ .062 ml

t. 1 fluid dram (U.S.) ≃ 3.697 ml

u. 1 centare = 1 m^2

v. 1 hectare = 10,000 m^2

w. 1 quintal (metric) = 100 kg

x. 1 stere = 1 m^3

y. 1 are = 100 m^2

z. 1 myriameter = 10 km

aa. 1 micron = .0001 cm

ab. 1 grain ≃ .064799 g

ac. 1 pennyweight ≃ 1.5552 g

ad. 1 scruple (apoth.) ≃ 1.2960 g

(Brit. fluid) ≃ 1.183877 m^3

ae. 1 troy ounce ≃ 31.103 g

af. 1 stone ≃ 6.35 kg

ag. 1 barn = 10^{-24} m^2

ah. 1 pottle ≃ 2.27 liters

ai. 1 skein ≃ 109.728 m

aj. 1 span ≃ 22.86 cm

ak. 1 ell ≃ 114.3 cm

al. 1 noggin ≃ 142 ml

am. 1 butt (Brit.) ≃ .47696919 m^3

2. correct

3. $\frac{\text{24 hours}}{\text{day}}$

4. $\frac{\text{12 cobs}}{\text{plant}}$

5. $\frac{\text{24 hr}}{\text{day}} \cdot \frac{\text{365 day}}{\text{yr}}$

6. correct

7. correct

8. $\frac{\text{fl oz}}{29.6\text{ ml}}$

9. correct

10. $\frac{\text{year}}{365\text{ days}} \cdot \frac{\text{century}}{100\text{ years}}$

11. $\frac{60\text{ sec}}{\text{min}}$

12. $\frac{3600\text{ sec}}{\text{hr}}$

13. $\frac{\text{g}}{1000\text{ mg}}$

14. $\frac{\text{lb}}{\text{gallon}}$

15. $\frac{1000\text{ ml}}{\text{liter}}$

16. $\frac{\text{ml}}{\text{breath}}$

17. $\frac{\text{girls}}{\text{student}}$

18. $3\text{ miles} \cdot \frac{5280\text{ ft}}{\text{mile}} \cdot \frac{.305\text{ m}}{\text{ft}} \cdot \frac{1\text{ km}}{1000\text{ m}} \simeq 4.8312\text{ km}$

19. $2\text{ lb} \cdot \frac{16\text{ oz}}{\text{lb}} \cdot \frac{28.35\text{ g}}{\text{oz}} \cdot \frac{1\text{ kg}}{10^3\text{ g}} \simeq .9072\text{ kg}$

20. $720\ \frac{\text{mi}}{\text{day}} \cdot \frac{1\text{ day}}{24\text{ hr}} \cdot \frac{1.61\text{ km}}{\text{mi}} \simeq 48.3\ \frac{\text{km}}{\text{hr}}$

21. $960\text{ minims} \cdot \frac{1\text{ fl dm}}{60\text{ minims}} \cdot \frac{1\text{ fl oz}}{8\text{ fl dm}} \cdot \frac{1\text{ pt}}{16\text{ fl oz}} = \frac{1}{8}\text{ pt}$

22. $8.36\ \frac{\text{lb}}{\text{gallon}} \cdot \frac{1\text{ kg}}{2.2\text{ lb}} \cdot \frac{1\text{ gal}}{3.8\text{ liters}} \simeq 1\ \frac{\text{kg}}{\text{liter}}$

23. $1\ \frac{\text{pack}}{\text{day}} \cdot \frac{365\text{ days}}{\text{year}} \cdot \frac{20\text{ cigarettes}}{\text{pack}} \simeq 7300\ \frac{\text{cigarettes}}{\text{year}}$

24. $300\ \frac{\text{gallons}}{\text{hr}} \cdot \frac{3.79\text{ liters}}{\text{gallon}} \cdot \frac{\text{hr}}{60\text{ min}} \simeq 18.95\ \frac{\text{liters}}{\text{min}}$

25. $73\ \frac{\text{tons}}{\text{day}}$

31

LESSON 8: MEASUREMENT

OBJECTIVES:

The student will express measurements in terms of ranges of imprecision.

The student will write the range of imprecision corresponding to a given graph.

The student will explain why the number of pulse beats counted in a given time interval is somewhat dependent on when the interval starts.

PERIODS RECOMMENDED:

Two

SUPPLEMENTARY TRANSPARENCIES:

Transparencies I-M-8a, b

OVERVIEW AND REMARKS:

This is an important lesson since measurements will arise routinely throughout the science course. The beginning of the lesson discusses the three basic types of measurements, yes or no, counting and continuous. You may want to use Transparency I-M-8a at this point.

The middle of the lesson treats the continuous measurement case. This material will be used extensively at the beginning of Unit II in a discussion of error propagation under various arithmetic operations.

TEACHER ACTIVITIES:

In Science Lesson 14 the students will have occasion to take pulse measurements in connection with the step test.

The end of this math lesson is intended to point out that the pulse count sometimes depends on when the counting interval begins. The following activity will help convey this idea.

1. Have the students choose partners and practice locating each other pulse. Explain that the pulse is taken by placing the index and middle fingers of one hand about $1\frac{1}{2}$ inches from the wrist joint on the inner side of the wrist nearest the thumb (see Transparency I-M-8b).

2. When all have located their partners' pulses, tell them that they are to count each other's pulse beats during a 15-second interval. Start and finish the interval for the whole class yourself. Instruct them to keep their findings secret, but to be sure to write them down somewhere.

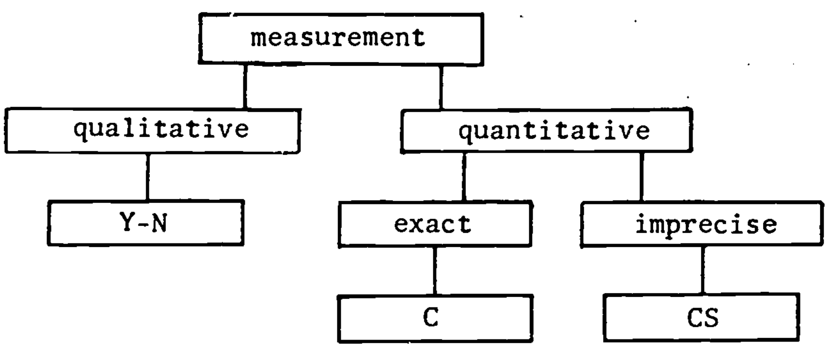
3. Ask the students to repeat the same count on themsleves. As before start and stop the counting interval for the entire class.

4. Ask the students to compare the number of beats they counted to the number obtained by the other person. Doubtless the two counts will differ in many cases.

5. Initiate a discussion on possible reasons for the discrepancy. There exist many valid explanations. Section 8-5 presents one possible reason which is inherent in the technique of measurement used.

KEY--PROBLEM SET 8:

1. C
2. CS
3. Y-N
4. Y-N
5. CS
6. CS
7. C
8. Y-N
9. Y-N
10. C
11. CS
12.



13. a. 3.5 = L

    b. 4.6 = H

    c. 4.05 = m

    d. .55 = Δm

    e. 4.05 $\pm$ .55 = m $\pm$ Δm

14. a. .4

    b. .7

    c. .55

    d. .55 $\pm$ .15

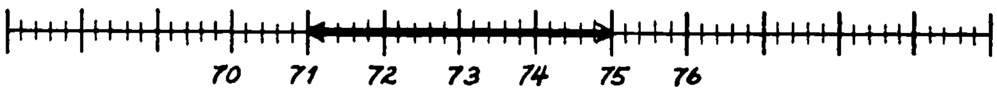
15. 4.55 .35

16. 2.75 .15

17. 39 2

18. 52.25 1.25

19. .425 .015

20.


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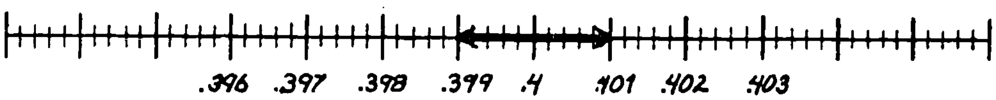

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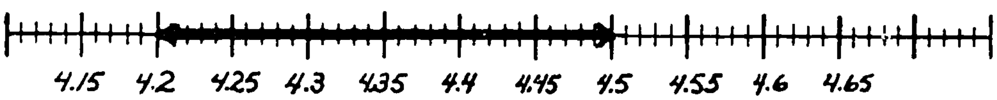

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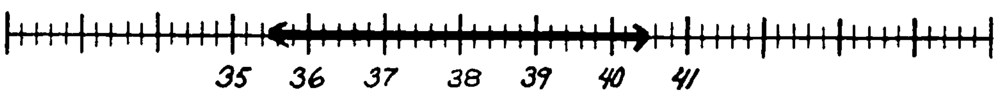

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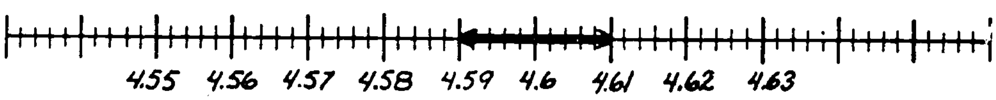


25. d = .5

26. d = .1

27. d = .25

28. d = 1

29. d = .5

30. d = .05

31. d = .01

32. d = 30° C

33. d = 10

34. a. 1

b. 3.2 and 3.3

c. count $\frac{1}{3}$ (answers may vary a little)

d. .033... ≈ .03

e. 3.23

f. .01

g. 3.23 .01

For the answers to Problems 35 through 40 the midpoints of the students answers should fall within the given range of imprecision.

The imprecision of the student's answers should agree with the imprecision of the answer given.

35. 1.85 ± .02

36. 10.58 ± .025

37. 4.0 ± .2

38. 3.85 ± .025

39. 38.3 ± .5

40. 315 ± 5

41. a. 12.5 cm

b. equal

c. 6

d. 7

e. 1 pulse

f. no

g. 1

LESSON 9: FAIR PACKAGING AND LABELLING ACT

OBJECTIVE:

The student will determine whether a manufacturer's statement of contents on a container meets legal requirements.

PERIODS RECOMMENDED:

One

REFERENCE:

"Rules, Regulations, Statement of General Policy or Interpretation and Exemptions Under the Fair Packaging and Labeling Act." Federal Trade Commission, October 1, 1971. Available from

Federal Trade Commission
6th St. and Pennsylvania Ave., N.W.
Washington, D.C. 20580

You might wish to ask your school librarian to obtain a copy of this pamplet.

SUPPLIES:

foot rulers (at least one per two students)

OVERVIEW AND REMARKS:

In this lesson the students will read the actual statutes of the Federal Trade Commission regarding requirements for minimum type size in statements of quantity of contents.

The lesson as planned attempts to mirror a real life, outside-of-the-classroom situation as closely as possible. The FTC statutes are difficult reading and could have been simplified. They are not simplified because generally no one is available to simplify things for us. It is felt that a little exposure to "bureaucratese" is a good preparation for today's world. As you read the regulations, watch for the following.

1. The word "circumference" is misused in the definition of the "principle display panel."

2. The regulations for type size reduce to the following set of statements.

Let A = area of principal display panel in in^2.

If $A \leq 5$ then the minimum type size is $\frac{1}{16}$ in.

If $5 < a \leq 25$ then the minimum type size is $\frac{1}{8}$ in.

If $25 < A \leq 100$ then the minimum type size is $\frac{3}{16}$ in.

If $100 < A \leq 400$ then the minimum type size is $\frac{1}{4}$ in.

If $400 < A$ then the minimum type size is $\frac{1}{2}$ in.

If the statement of net quantity is blown, embossed, etc., directly on the container, then $\frac{1}{16}$ in. should be added to the minimum type size.

TEACHER ACTIVITIES:

When the class has read and discussed the statutes, distribute the containers which you and the students have collected for the activity. Depending on the number of containers, the students can work individually or in small groups to determine whether the labels meet legal requirements. The problem set is also useful in testing comprehension of the statutes.

KEY--PROBLEM SET 9:

1. F
2. T
3. F
4. T
5. T
6. F
7. T
8. T
9. F
10. T
11. F
12. F
13. a. T

    b. T
14. a. ~9.42 in

    b. ~47.1 in^2

    c. ~18.84 in^2

    d. FTC regulations 500.18 (b) (2)

    e. no, no
15. a. ~12.57 in

    b. 40.22 in^2

    c. $\frac{3}{16}$ in
16. a. 84 in^2

    b. $\frac{3}{16}$ in
17. a. ~40 in^2

    b. $\frac{3}{16}$ in

    c. $\frac{1}{4}$ in
18. a. ~12.56 in^2

    b. $\frac{1}{8}$ in

39

LESSON 10: GRAPHS

OBJECTIVES:

The student will graph ordered pairs.

The student will find the ordered pair corresponding to a point on a graph.

The student will graph regions of imprecision.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The majority of this lesson is a review of the cartesian coordinate system. The only unfamiliar material involves regions of imprecision and their graphical depiction.

KEY--PROBLEM SET 10:

1. a. x-coordinate, horizontal coordinate, abscissa

   b. y-coordinate, vertical coordinate, ordinate

   c. 1st blank any one of the answers to part a

      2nd blank any one of the answers to part b

      3rd blank "comma"

      4th blank "parentheses"

2. It can be done by putting the y-coordinate first.

3. Cartesian

4. .2

5. .1

6. .05

7. .04

8. 1

9. .01

10. .005

11. a. (9, 1)

    b. (9, 2.4)

    c. (8.2, 4.4)

    d. (7.2, 2.5)

    e. (5.4, 5.6)

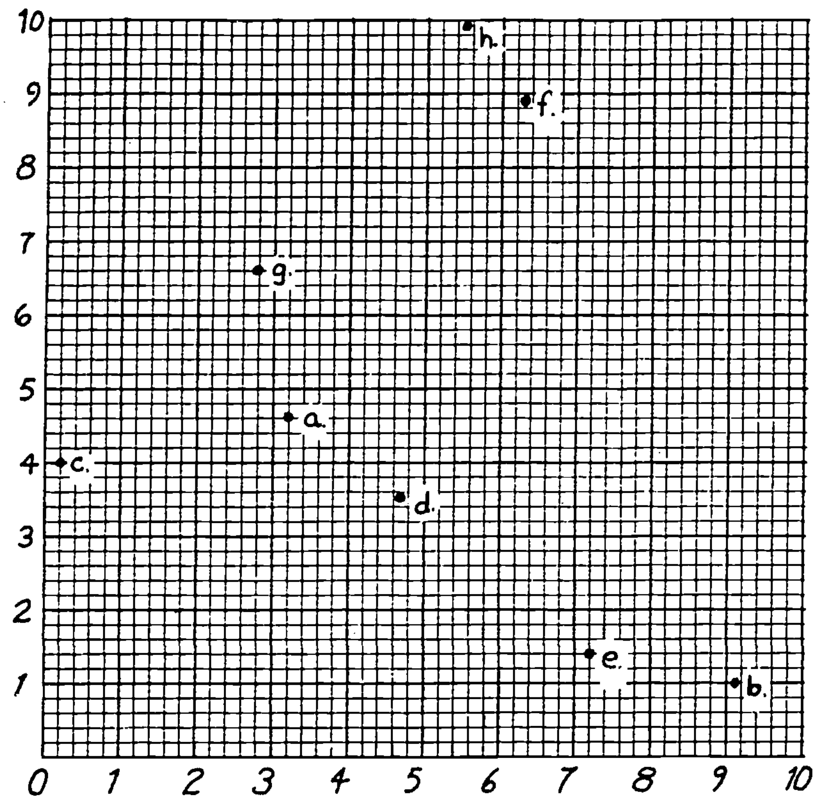
    f. (4.3, 1.1)

    g. (3, 7.2)

    h. (2.7, 9.4)

41

12.



13. a. (2, .25)

b. (1.75, .55)

c. (1.55, 1.85)

d. (1.20, 2.05)

e. (.85, .40)

f. (.35, 1.15)

g. (-.5, .25)

h. (-1, 1)

i. (-1.4, 1.35)

j. (-1.85, .15)

k. (-1.5, -1.5)

l. (-.75, -.75)

m. (-.15, -.10)

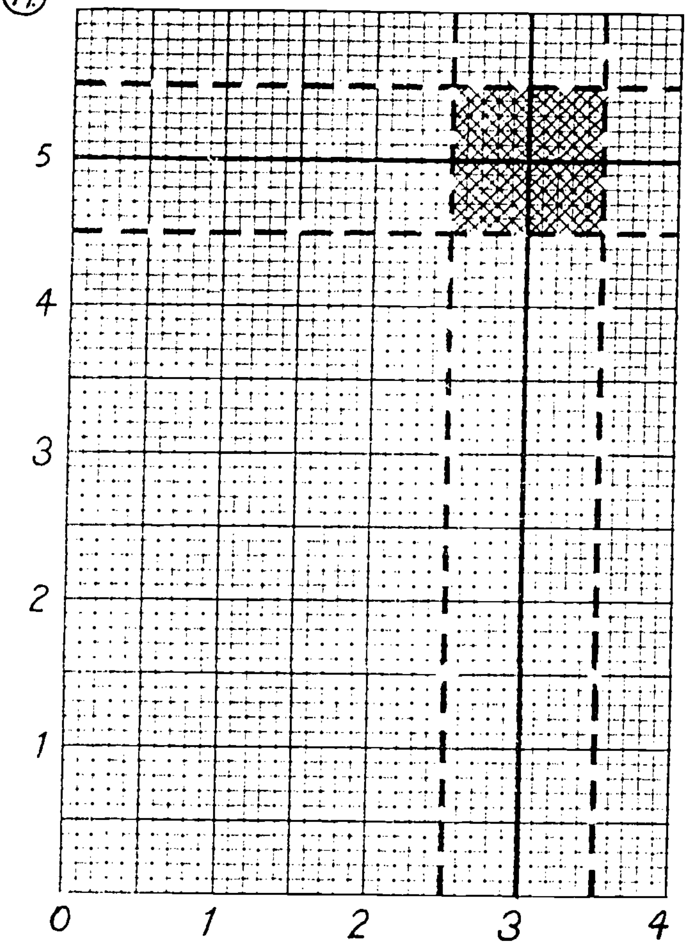
n. (.40, -.45)

o. (1, -1)

p. (2.4, -2)

4?

14.



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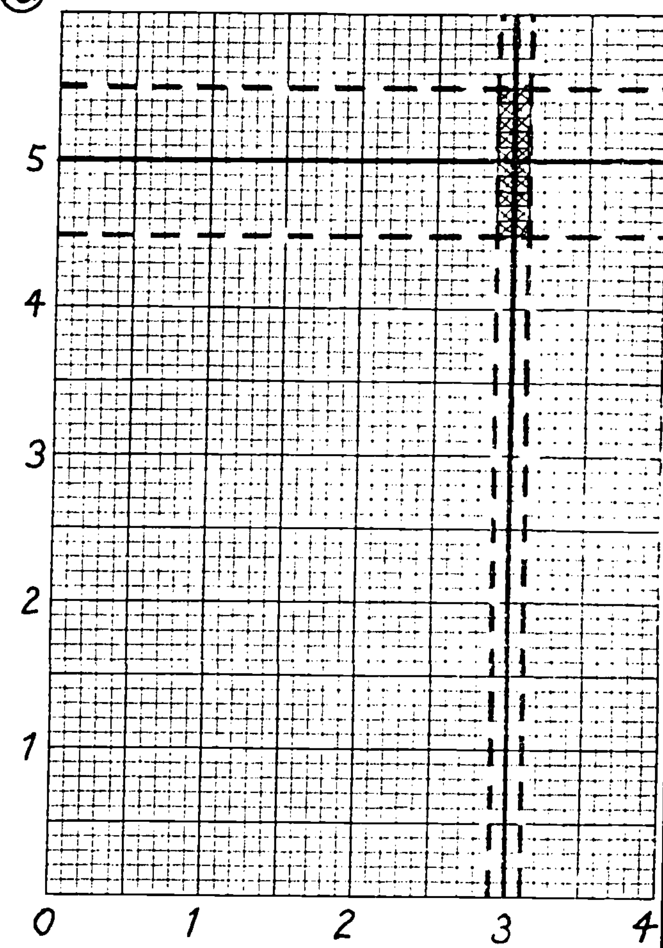

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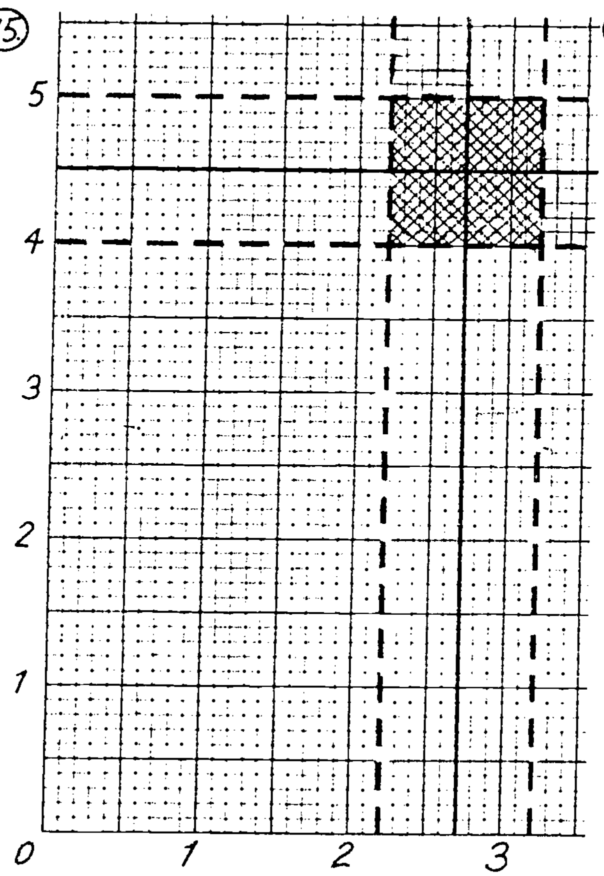

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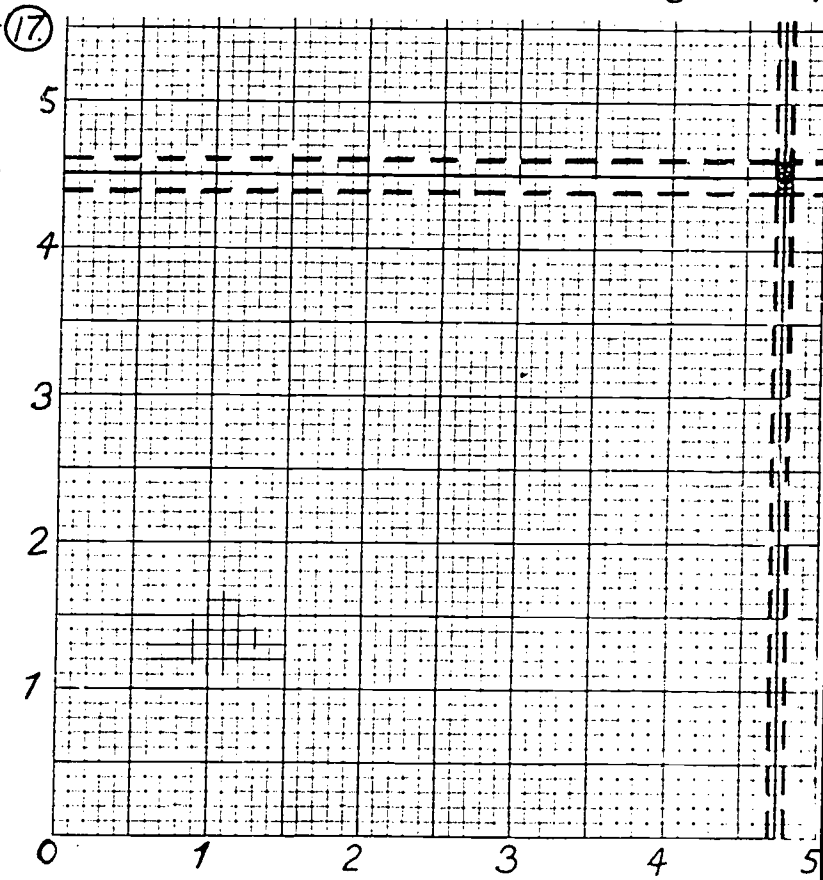


LESSON 11: THE REQUIREMENTS FOR A GOOD GRAPH

OBJECTIVES:

When given a set of ordered pairs to be graphed, the student will determine a scale which makes favorable use of available space and which lends itself to simple computations.

PERIODS RECOMMENDED:

Two

OVERVIEW AND REMARKS:

Graphing is a skill which will receive considerable attention throughout the biomath course. This particular lesson has many features which will be encountered repeatedly. First, there is an activity which involves measurements leading to a set of ordered pairs. The pairs are then graphed and a line drawn which seems to best represent the resulting scattered points. This line can then be used in making predictions and the degree of scattering suggests a range of imprecision associated with a particular prediction.

Expertise in the above techniques will be developed over a period of time. In this lesson the principal concern is that of choosing a proper scale to be used in plotting a set of ordered pairs. Later on, in Lessons 19 through 23, graphing techniques in data analysis will be explored in more detail.

The graphical techniques outlined above parallel the typical statistical treatment in which linear regression is used to find the equation of a "best approximating straight line." The range of imprecision would then be expressed in terms of standard diviations.

TEACHER ACTIVITIES:

There are a number of bodily measurements which can be conducted in class and which will generate sets of ordered pairs for the purpose of gaphing. Section 11-1 outlines one possibility namely the measurement of reach versus height for several individuals.

One approach which could be used to generate numerous graphing problems is the following. Make several measurements on each individual, such as height, reach, foot length, forum length, knee to bottom of foot, and so on. The class can then be divided into graphs and each group assigned the plotting of one pair of measurements. Each group will then be faced with a unique scaling problem, and several examples will be available for general discussion.

KEY--PROBLEM SET 11:

1. a. The scales should be easy to read and interpret.

   b. The plotted points should fill the available space as completely as possible while maintaining the first objective.

2. The graph is unacceptable because the points do not fill the available space well.

3. The grid is 18 cm x 24 cm with five divisions to the centimeter.

4. false

5. false

6. a. 4 cm height per big unit, .8 cm height per little unit.

   b. 3 cm height per big unit, .6 cm height per little unit.

7. a. 2 cm reach per big unit, .4 cm reach per little unit.

   b. 1.5 cm reach per big unit, .3 cm reach per little unit.

8. a. 2 cm for the big units, .4 cm for the little unit.

   b. 1.5 cm for the big units, .3 cm for the little units.

9. a. 10 mm for the big units, .2 mm for the little units.

   b. 5 mm for the big units, 1 mm for the little units.

10. a. .2 cm^3 for the big units, .04 cm^3 for the little units.

    b. .15 cm^3 for the big units, .03 cm^3 for the little units.

11. a. 10 mm for the big units, 2 mm for the little units.

    b. same as a

12. a. .25 cm^3 for the big units, .05 cm^3 for the little units.

    b. .2 cm^3 for the big units, 104 cm^3 for the little guys.

13. a. The thing being measured.

    b. The units used.

14. The first number on an axis should be a whole number multiple of the scale for the big units.

15. 66.8 cm

16. 80.4 cm

17. 183 cm

18. 159 cm

19. People who make clothes for sure. There are probably other answers which are just as good.

LESSON 12: FUNCTIONS

OBJECTIVES:

The student will recognize sets of ordered pairs which are functions.

The student will recognize graphs which represent functions.

The student will solve problems involving function notation, domain, range, finite functions and infinite functions.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

This lesson initiates the study of functions in the math course. Basic terminology is introduced and the students are given some notions on distinguishing functions from nonfunctions. The actual formula definition of a function is introduced in Lesson 14.

Lessons 15 through 18 are concerned with linear functions. After this development there are several biomedical applications in the latter part of Unit I.

KEY--PROBLEM SET 12:

2. a. infinite

   b. finite

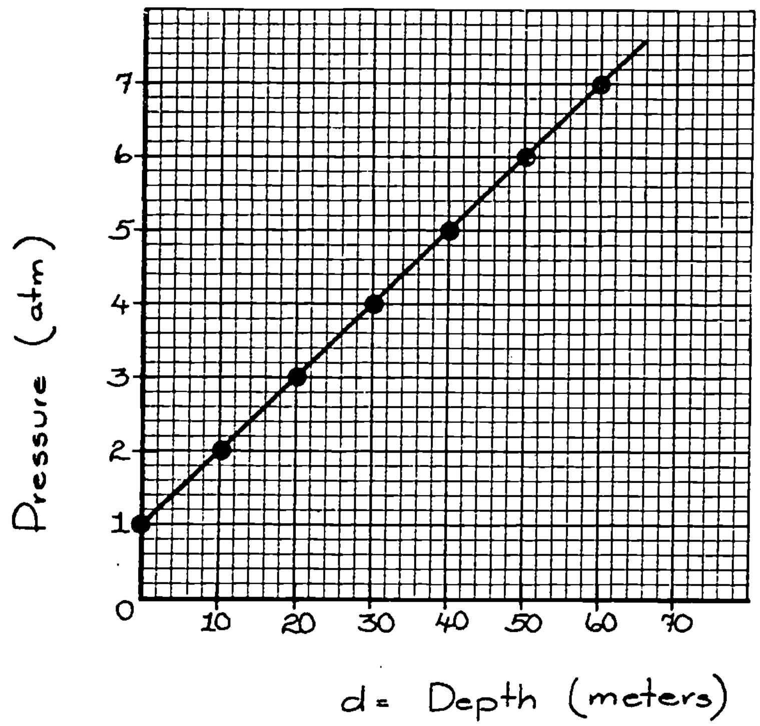
3. a. $f(x) \simeq 24$ c. $f(x) \simeq 41$

   b. $f(x) \simeq 72$ d. $f(x) \simeq 84$

4. a. $x \simeq 1.7$ c. $x \simeq 3.5$

   b. $x \simeq 4.15$ d. $x \simeq 2.25$

5.



6. The left column: d = depth (meters)

7. a. f = {(0, 1), (10, 2), (20, 3), (30, 4), (40, 5), (50, 6)}

   b. finite

   c. infinite

8. a. $g(d) \simeq 2.5$ c. $g(d) \simeq 3.8$

   b. $g(d) \simeq 5.5$ d. $g(d) \simeq 4.6$

9. a. $d \simeq 15$ c. $d \simeq 60$

   b. $d \simeq 48$ d. $d \simeq 33$

10. b, c, e, f

11. a, d, e

48

LESSON 13: FUNCTION MACHINES

OBJECTIVES:

The student will "guess the rule" which a function machine is using.

The student will supply the range when the domain and rule are known.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

This is a straightforward lesson which is intended to build intuition on the properties of functions so that the formal definition of Lesson 14 will appear plausible. No particular problems are anticipated.

49

KEY--PROBLEM SET 13:

1. a. (3) f. (1)
   b. (7) g. (4)
   c. (2) h. (10)
   d. (5) i. (9)
   e. (6) j. (8)

2. a. $\{1, 9, -9\}$
   b. $\{2, 17, -46\}$
   c. $\{29\}$
   d. $\{8, 4, 2, 16\}$

3. a. $f(x) = 4x$
   b. $f(x) = 8 - x$
   c. $f(x) = 7$
   d. $f(x) = \frac{1}{3}x - 1$
   e. $f(x) = 4x + 4$
   f. $f(x) = x + 3$

4. b, d, and e are sick

50

LESSON 14: A DEFINITION FOR FUNCTION

OBJECTIVE:

The student will differentiate between functional and non-functional relationship.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The new element in this lesson is the following formal definition of a function.

Let D and R be any two sets. A function from D to R is a set of ordered pairs with first coordinates from D and second coordinates from R with the function property that if (x, y) and (x, y') belong to the function, then y = y'.

51

KEY--PROBLEM SET 14:

1. b. (142, 48), (142, 49)

   c. (19, 49), (19, 51)

   f. (A, 23), (A, 29)

   or (A, 23), (A, 37)

   or (A, 29), (A, 37)

   or (B, 14), (B, 17)

   g. (x_2, y_2), (x_3, y_3)

   i. (dozen eggs, 53¢), (dozen eggs, 55¢)

2. b. children to mothers

   d. sons to fathers

3. a. f = {(0, 0), (2, 2), (4, 4)}

   b. f = {(-1, 2), (0, 4), (1, 6)}

   c. f = {(-3, 3), (-1, 3), (0, 3), (1, 3), (3, 3)}

   d. f = {(2, 6), (3, 5), (7, 1), (9, -1)}

   e. f = {(-4, -2), (2, 1), (8, 4)}

   f. f = {(-4, 10), (2, 7), (8, 4)}

   g. f = {(0, 7), (1, 8), (2, 9), (3, 10)}

   h. f = {(-2, 0), (-1, 2), (0, 4), (1, 6), (2, 8)}

4. a. Any value for x, except the last one shown, that has more than 11 digits to the right of the decimal point and the corresponding value for f(x) are not included in his list.

   b. No

   c. Yes, to the first part, and no, to the second. He should retire.

5. a. finite
   b. finite
   c. infinite
   d. infinite
   e. finite
   f. infinite
   g. infinite
   h. finite
   i. finite
   j. finite

6. a, b, c, f, g, h, j

7. a. function, infinite
   b. function, finite
   c. function, infinite
   d. function, infinite
   e. function, infinite
   f. non-function
   g. non-function
   h. non-function
   i. function, infinite
   j. non-function
   k. function, infinite
   l. non-function

LESSON 15: SLOPE

OBJECTIVE:

The student will calculate the slopes of straight lines.

PERIODS RECOMMENDED

one

OVERVIEW AND REMARKS:

This is a standard sort of lesson on slope determination. Slope is defined as rise divided by run and you will need to emphasize that either or both can be negative numbers.

KEY--PROBLEM SET 15:

1. a. -4

   b. 4

   c. -4

   d. 4

   e. -4

   f. 4

   g. 4

   h. -4

   i. -1

   j. -1

   k. -1

2. -2

3. 0

4. undefined

5. 7

6. $\frac{1}{4}$

7. $\frac{26}{3}$

8. a. + d. -

   b. 0 e. undefined

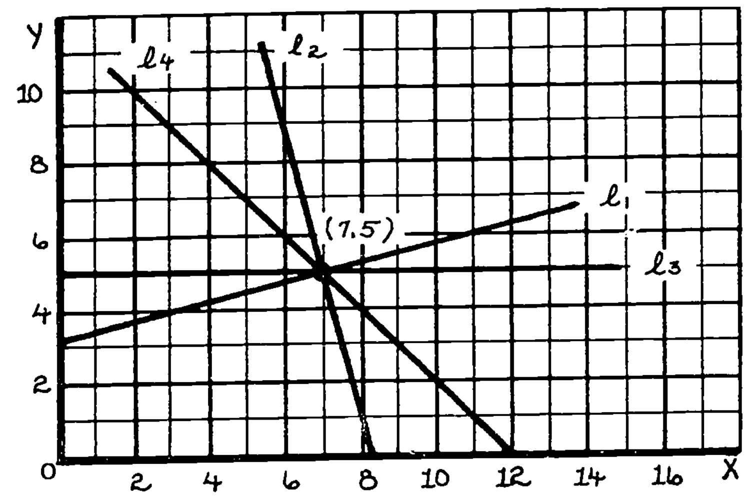
   c. + f. -

9. a. -3

10. $\frac{-3}{10}$

5.

10.



$m_1 = \frac{1}{4}$

$m_2 = -4$

$m_3 = 0$

$m_4 = -1$

a. -4

b. it gets steeper

c. they appear to be perpendicular

11. $\overline{AB}$: m = 1

$\overline{AC}$: m is undefined

$\overline{BC}$: m = -1

12. $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ all have a slope of $\frac{6}{5}$

13. a. AB: m = $\frac{3}{2}$

AC: m = $\frac{3}{2}$

BC: m = $\frac{3}{2}$

b. m = $\frac{3}{2}$

14. slope = 7.6

15. slope = 4.5

LESSON 16: LINEAR FUNCTIONS OF A SPECIAL TYPE

OBJECTIVES:

The student will graph functions of the form $f(x) = mx$.

The student will derive equations of lines passing through the origin.

The student will convert graphs and statements into the language of direct proportion.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

This lesson initiates the study of equations representing straight lines. Linear functions will play an important role throughout the math course and occur frequently in biomedical and scientific problems. The following are examples of quantities which are linearly related.

1. Respiration rate is a linear function of the concentration of CO_2 in the blood over a limited domain.

2. From the end of the second month to about the end of the eighth month the length of the human fetus is a linear function of time spent in the womb.

3. The concentration of alcohol in the blood is a linear function of the length of time that has passed since the blood-alcohol level reached its maximum.

57

KEY--PROBLEM SET 16:

1. a. The graph will be a line.

   b. The slope of the line will be the coefficient of x.

   c. The line will go through the origin.

2. a. $m = \frac{3}{2}$

   b. $m = 1$

   c. $m = \frac{-21}{2}$

   d. $m = -5$

   e. $m = \frac{103}{62}$

   f. $m = 103$

3. a. $y = \frac{11}{4}x$

   $m = \frac{11}{4}$

   b. $y = \frac{19}{4}x$

   $m = \frac{19}{4}$

   c. $y = -\frac{23}{4}x$

   $m = -\frac{23}{4}$

   d. $y = 2x$

   $m = 2$

   e. $y = -\frac{34}{7}x$

   $m = -\frac{34}{7}$

   f. $y = 10x$

   $m = 10$

   g. $y = \frac{1}{2}x$

   $m = \frac{1}{2}$

   h. $y = -\frac{9}{2}x$

   $m = -\frac{9}{2}$

   i. $y = 32x$

   $m = 32$

   j. $y = 47x$

   $m = 47$

4. a. $y = x$

   b. $y = -16x$

   c. $y = \frac{7}{2}x$

   d. $y = \frac{10}{3}x$

   e. $y = \frac{3}{7}x$

   f. $y = -\frac{3}{8}x$

   g. $y = 0$

   h. $x = 0$

   i. $y = -8x$

   j. $y = \frac{9}{64}x$

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8. $V = kT$

9. $V = km$

10. $B = kH$

11. $g = kd$

12. $f = km$

13. $L = km$

14. $M = kK$

15. $F = ka$

16. $F = kP$

17. $I = kA$

18. $f = kd$

19. $V = kS^3$

20. $m = kV$

21. a. The mass of water is directly proportional to its volume.

 b. slope = $\frac{1}{1000}$

 c. $\frac{1}{1000}$

 d. $m = \frac{1}{1000}V$

22. a. The mass of mercury is directly proportional to its volume.

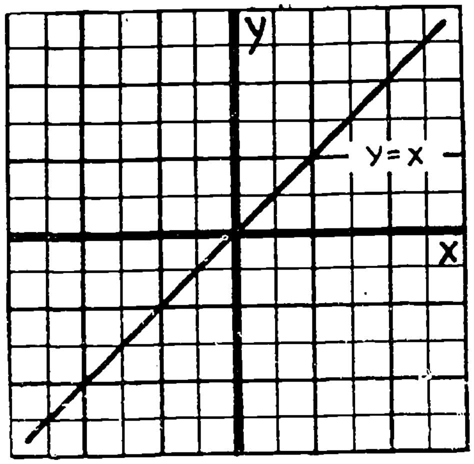
 b. slope = $\frac{14}{1000}$ or any equivalent expression

 c. $\frac{14}{1000}$

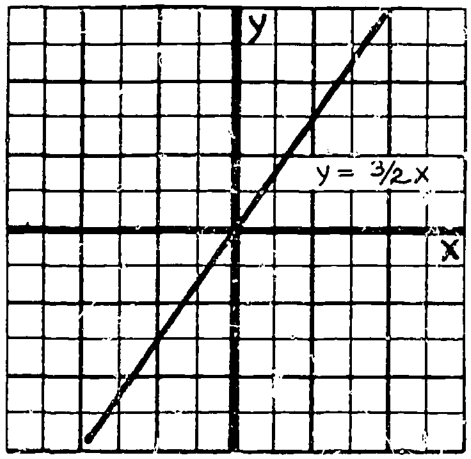
 d. $m = \frac{14}{1000}V$

59

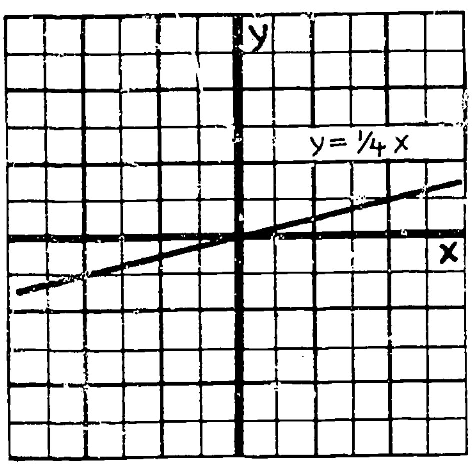
5. a.



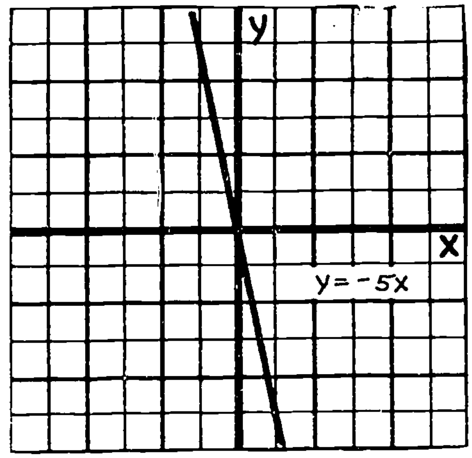
b.



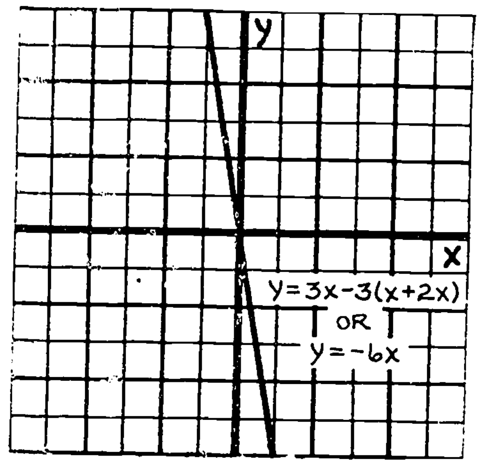
c.



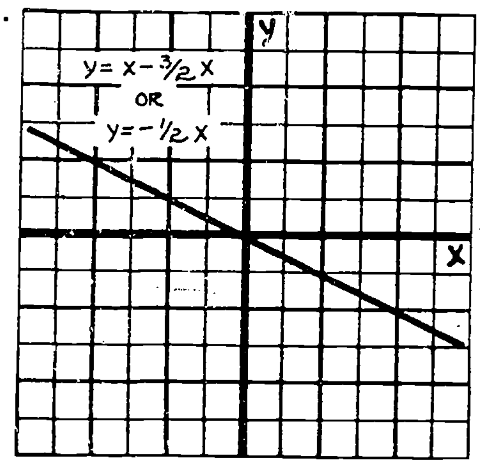
d.



e.



f.



6. a. y

b. m

c. x

7. a. $m = kN$

b. m

c. k

d. N

LESSON 17: MORE LINEAR FUNCTIONS

OBJECTIVES:

The student will graph functions of the form $y = mx + b$.

The student will derive equations of lines when given their graphs.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

The previous lesson considered lines passing through the origin. This one treats the more general case when the y-intercept is not necessarily zero.

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KEY--PROBLEM SET 17:

1. a. $m = -\frac{3}{2}$; $b = 3$

   b. $m = 4$; $b = -7$

   c. $m = 7$; $b = \frac{8}{3}$

   d. $m = -\frac{12}{7}$; $b = -\frac{12}{7}$

   e. $m = 3$; $b = 0$

   f. $m = 0$; $b = \frac{18}{5}$

2. a. $y = 3x - 21$

   $m = 3$; $b = -21$

   b. $y = 3x + 4$

   $m = 3$; $b = 4$

   c. $y = 17x - 8$

   $m = 17$; $b = -8$

   d. $y = \frac{8}{3}x$

   $m = \frac{8}{3}$; $b = 0$

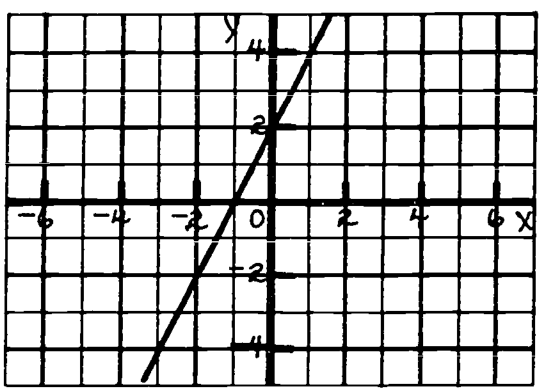
   e. $y = \frac{7}{2}$

   $m = 0$; $b = \frac{7}{2}$

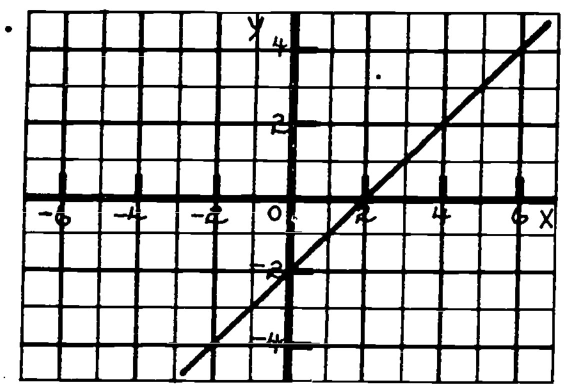
   f. $y = 19x - 6$

   $m = 19$; $b = -6$

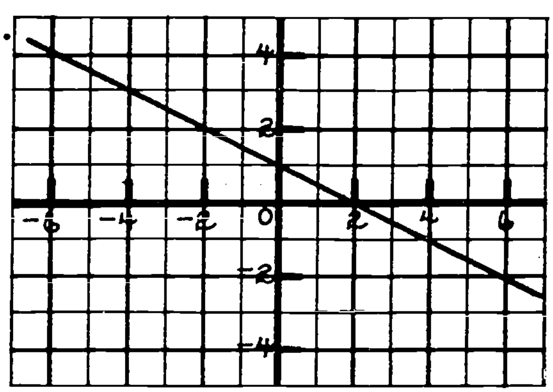
3. a.



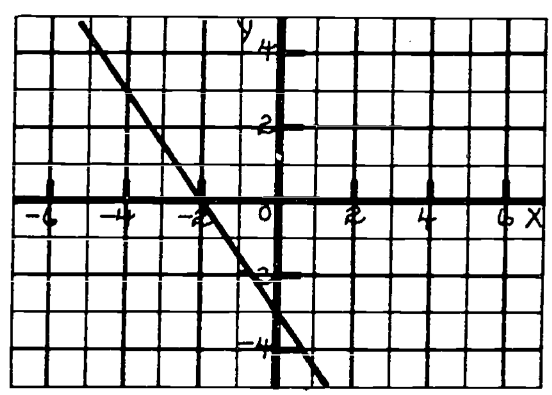
c.



b.



d.



4. a. $y = 2x + 2$

   b. $y = 8x - 6$

   c. $y = 4$

   d. $y = -\frac{1}{2}x - 4$

   e. $y = -5x + 6$

   f. $x = 4$

5. a. $A = (4, 14)$

   $B = (16, 21)$

   b. + ; because it's rising to the right.

   c. $m = \frac{7}{12}$

   d. 12

   e. $y = \frac{7}{12}x + 12$

   f. $(6, 15\frac{1}{2})$

   $(10, 17\frac{5}{6})$

   $(12, 19)$

   $(15, 20\frac{3}{4})$

   g. Approximately the values from 4 liters/min to 16 liters/min, inclusive.

   h. 14 mm Hg to 21 mm Hg, inclusive.

LESSON 18: FINDING THE EQUATION FOR A LINE FROM TWO POINTS ON THE LINE

OBJECTIVE:

The student will derive the equation of a line from knowledge of the coordinates of two points on the line.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

This straightforward lesson completes the theoretical treatment of linear functions. Applications begin in the next lesson on Charles' Law.

6.:

KEY--PROBLEM SET 18:

1. a. true

   b. false

   c. false

2. a. $\frac{2}{3}$

   b. $\frac{2}{3}$

   c. 3

   d. −2

3. a. false

   b. true

   c. false

4. a. $\frac{1}{4}$

   b. $-\frac{3}{2}$

   c. 3

   d. 3

5. $y = \frac{3}{2} \cdot x$

6. $y = 8$

7. $y = x + 4$

8. $y = 3x - 5$

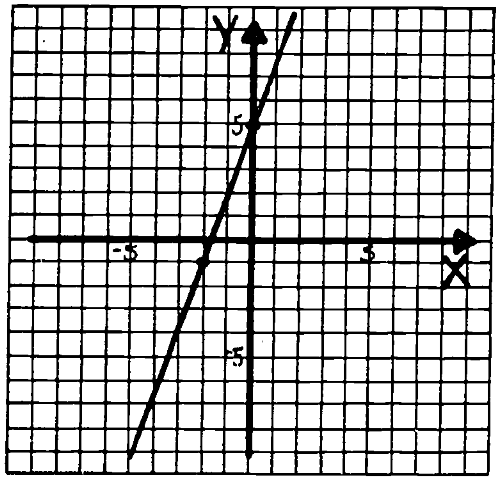
9. $y = \frac{3}{2}x + \frac{21}{2}$

10. $y = 6x + 3$

11. $y = -3x + 19$

12. $y = 2x - 4$

13. $y = 2x + 7$

14. a. 

b. 5

c. slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are any two distinct points on the line.

d. slope = 3

e. $y = 3x + 5$

LESSON 19: CHARLES' LAW

OBJECTIVES:

The student will construct a graph of the results of the Charles' Law activity (Science Laboratory Activity 15).

The student will interconvert Celcius and Kelvin temperatures.

PERIODS RECOMMENDED:

One

SUPPLIES:

Each student's data from Science Laboratory Activity 15 (The Effect of Temperature Change on the Volume of a Confined Gas). Obtain from the science teacher prior to class and distribute to the students at the beginning of the period.

OVERVIEW AND REMARKS:

This lesson is devoted to an analysis of student data from Science Laboratory Activity 15. In the activity the students measured the volume of a quantity of confined air as a function of Celcius temperature. The air was confined in a column so that volume could be measured in terms of length.

The principal new concept in this math lesson is that of absolute zero and the way in which Kelvin temperatures relate to the experimental results. The ordered pair (-273, 0) is added to each students' data set and the new point set is graphed (see Problem 7). This leads naturally to a review of graph scaling. Finally each student draws a best line through the point set. It is anticipated that the points will show a reasonably strong linear pattern. Introducing the Kelvin temperature scale, we finally arrive at a mathematical statement of Charles' Law.

$$V = mT$$

where

V = volume

T = Kelvin temperature

m = proportionality constant

If possible, the graphing should be done in class so that you can provide needed assistance.

KEY--PROBLEM SET 19:

1. 273° K

2. -273° C

3. 283° K

4. -23° C

5. 233° K

6. 127° C

7. Results will depend in student data.

LESSON 20: A GRAPH OF $\frac{1}{V}$ AS A FUNCTION OF P

OBJECTIVES:

The student will construct a graph of $\frac{1}{V}$ as a function of P, using the data from Science Laboratory Activity 16.

PERIODS RECOMMENDED:

One

SUPPLIES:

Each student's data from Science Laboratory Activity 16 (The Effect of Pressure Change on The Volume of a Confined Gas). Obtain from the science teacher prior to class and distribute to the students at the beginning of the period.

OVERVIEW AND REMARKS:

This lesson and Lesson 21 are devoted to the analysis of data from Science Laboratory Activity 16. This activity measured the relationship between pressure and volume (Boyle's Law) for a confined gas. Two experiments were available, the plunger/syringe activity and the liquid filled tube activity. The science class will have performed only one but the analysis of this lesson applies to either.

The data as you receive it should look like the sample data in Section 20-1 of the Student Text. Note that in the case of the plunger/syringe activity the volumes will be expressed as ranges of imprecision.

The task in this lesson is the preparation of a suitable graph displaying the data. Since Boyle's Law is not a linear relationship, the (pressure, volume) ordered pairs will not lie on a straight line (see sample graphs in text). However, the resulting curve can be straightened out if the reciprocal of volume is graphed as as a function of pressure. This is true because the usual statement of Boyle's Law

$$PV = k$$

can also be written as

$$\frac{1}{V} = \left(\frac{1}{k}\right)P$$

which is a linear relationship between $\frac{1}{V}$ and P.

See Problem Set 20 for the instructions on the graphing task. The graphs should be saved for the following period (Lesson 20). In Lesson 21 the graphs will be used to obtain an estimate of atmospheric pressure.

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KEY--PROBLEM SET 20:

1. g/cm^2
2. ml
3. cm of tubing
4. He uses a library.
5. He reserves the right to try to reproduce anybody else's results.
6. False
7. nonlinear
8. linear
9. Boyle's Law
10. Chemistry and physics textbooks and many other places.

11. and 12. Individual answers

LESSON 21: BOYLE'S LAW

OBJECTIVE:

The student will analytically determine the horizontal intercept of the linear function obtained in Lesson 20.

PERIODS RECOMMENDED:

One

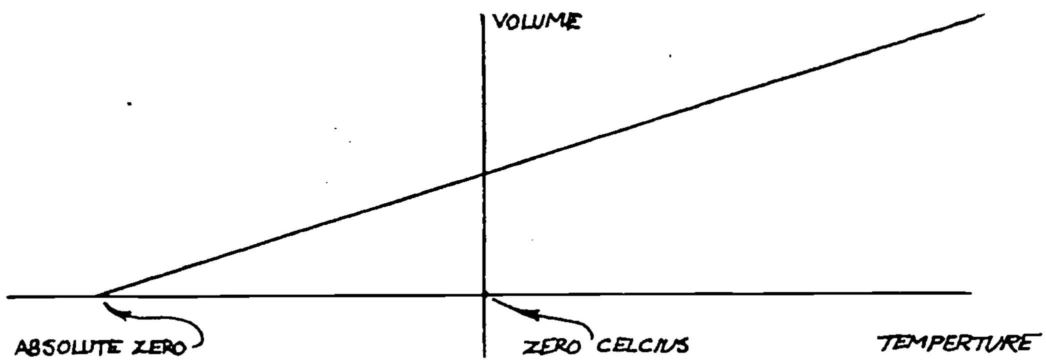
SUPPLIES:

Each students graph from Lesson 20

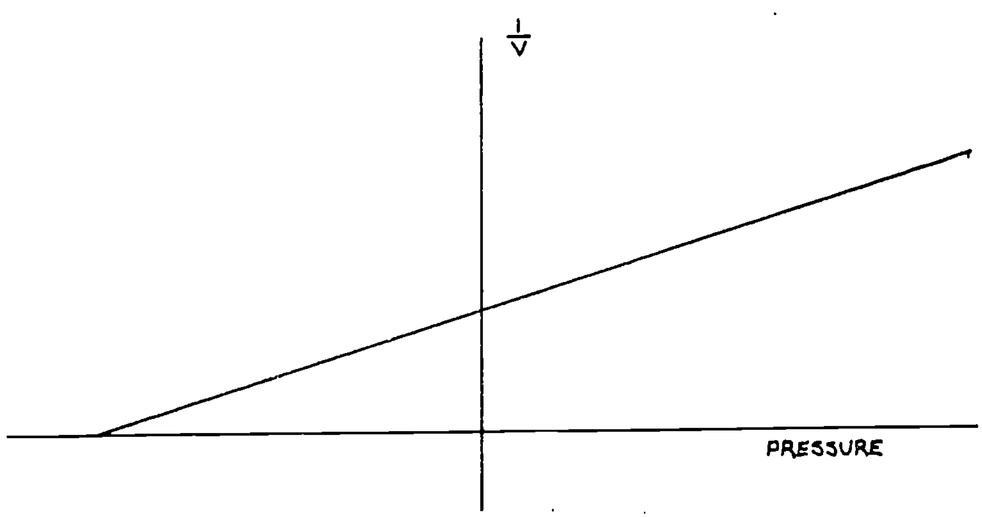
Atmospheric pressure (in g/cm^2) on day Science Laboratory Activity 16 was performed (obtained from science teacher)

OVERVIEW AND REMARKS:

The approach in this lesson parallels that of Lesson 19, in which the Charles' Law data was analyzed. In that lesson it turned out that 0° C was not the same as "absolute" zero. Absolute zero corresponds to the horizontal intercept of the linear function.



Compare the graph above with a typical one relating $\frac{1}{V}$ and P (with an approximating line drawn in).



Here zero pressure is not the same as "absolute" zero pressure. This is true simply because guage pressure reads zero at a pressure of the linear function should be approximately equal to the negative of atmospheric pressure.

TEACHER ACTIVITIES:

The task in this lesson is the determination of the horizontal intercept of the line approximating the plotted points. The result is then compared with atmospheric pressure.

1. Instruct each student to draw the "best" approximating line through his or her points.

2. In theory, the horizontal intercept of the resulting linear function could be found graphically (see Section 21-2). However, the method of choice is computational. The technique is explained in Section 21-3.

2. When the students have computed their intercepts, compare the results with the actual atmospheric pressure on the day of the science activity.

72

KEY--PROBLEM SET 21:

1. Any two of these three

   a. The graph will be linear

   b. The graph will go through the origin

   c. The slope will be m.

2. 0° C is not 0° absolute.

3. 273° had to be added.

4. °K or the units for any other absolute temperature scale.

5. 273° C

6. a. $\frac{1}{V}$

   b. $\frac{1}{k}$

   c. P

7. Our zero pressure is not zero absolute pressure.

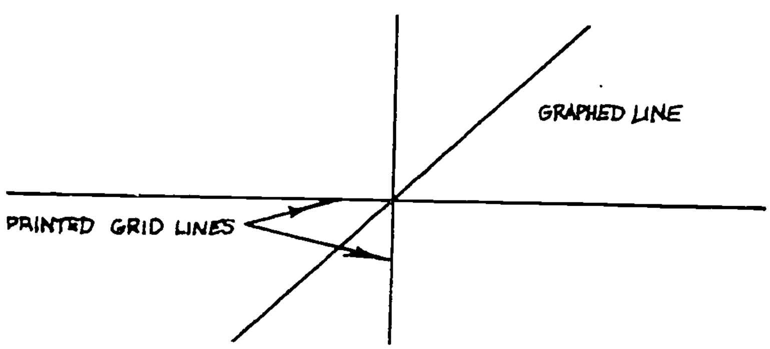
8. The distance should be atmospheric pressure for the day that the activity was performed.

9. Atmospheric pressure

10. True

11. It minimizes the effect of scale reading error on the slope calculation.

12.



13. It greatly simplifies a necessary division.

14. Either point which corresponds to a rise of .030 would lie off of the graph paper.

15, 16 and 17. Individual answers.

7.

LESSON 22: A GRAPH OF VITAL CAPACITIES

OBJECTIVE:

The student will compare plotted sets of data both for linearity and strength of relationship.

The student will prepare a graph of vital capacity as a function of height cubed.

PERIODS RECOMMENDED:

One

SUPPLIES:

A list of heights and vital capacities for students in the class (results of Science Laboratory Activity 18 on Forced Expiratory Volume). Obtain from science teacher.

SUPPLEMENTARY TRANSPARENCIES:

Transparencies I-M-22a, b, c, d, e

REFERENCES:

1. Bjure, Jan. Spirometric Studies in Normal Subjects. Act Pediatrics. 52: May 1963, pp. 232-40.

2. Ferris, B.G., and C.W. Smith. Maximum Breathing Capacity and Vital Capacity in Female Children and Adolescents. Pediatrics, 12: Oct. 1953, pp. 341-52.

3. Ferris, B.G., J. L. Whittenberger and J.R. Gallagher. Maximum Breathing Capacity and Vital Capacity of Male Children and Adolescents. Pediatrics, 9: June 1952, 659-69.

OVERVIEW AND REMARKS:

This lesson and Lesson 23 are devoted to an analysis of the vital capacity data collected during Science Laboratory Activity 18, The data is graphed and the results are examined in the light of statistical studies taken from the medical literature (see references). This lesson is concerned only with the graphing part of the task. There are two aspects involved.

1. The determination of a variable which bears an approximately linear relationship to vital capacity.

2. The graphing of vital capacity as a function of the appropriate variable when it has been determined.

TEACHER ACTIVITIES:

A. You will find in the Student Text a number of graphs which display the relationship between vital capacity and other variables. These graphs should be discussed in detail and judged on the following two criteria.

1. The degree to which the points appear to be linear.

2. The "tightness" with which the points cluster around the line.

Some of the graphs are clearly preferable to others according to these criteria. These may be disagreement on the final choice, but it can be verified by a statistical analysis that vital capacity vs. height^3 is most favorable for both sexes.

You may find the transparencies useful in discussing the notion of linear approximations. A grease pencil can be used to sketch in lines which seem to best represent the data points.

B. The final task in this lesson is the preparation of vital capacity vs height^3 graphs for each sex based on the data from Science Laboratory Activity 18. Each student can be assigned on of the two graphing problems. The table on page 204 can be used to compute values of height^3. The students should not try to draw approximating lines through the data points at this time. That activity will come up in the next lesson. Be sure to tell the students to save their graphs.

7.;

LESSON 23: ANALYSIS OF VITAL CAPACITY DATA

OBJECTIVE:

The student will plot linear equations which have been found by statistical methods to describe vital capacity data.

When given the height of a male or female, the student will state the edicted vital capacity as a range of imprecision.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

At this point the students have graphed the vital capacity data for the biomed class. It is natural to ask how one can decide whether any of the values are cause for medical concern. In order to answer this question it is necessary to have available a statistical analysis based on a large sample. The class results can then be viewed as part of a larger distribution of values. In this lesson, some results from a statistical analysis are summarized. A region of the vital capacity vs height[3] graph is defined in terms of an upper and lower limiting line. It is expected that 95% of all 17-year-old people will be found within the strip bounded by the two lines. Naturally, the equations of the bounding lines differ for males and females. In each case the equation of a "best" or average line is also presented.

TEACHER ACTIVITIES:

It is suggested that the students work on the problem set in class so that you can provide needed asssistance.

KEY--PROBLEM SET 23:

2. 2.29 ± .7 liters

3. 2.50 ± .7 liters

4. 3.25 ± .7 liters

5. 4.15 ± .7 liters

6. 7.475 ± .725

7. 4.625 ± .725

8. 5.1475 ± .725

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LESSON 24: SOLVING EQUATIONS

OBJECTIVES:

The student will correctly use the terms open sentence and equivalent equation.

The student will solve first degree equations.

PERIODS RECOMMENDED:

one

SUPPLEMENTARY TRANSPARENCIES:

Transparencies I-M-24a, b

OVERVIEW AND REMARKS:

Problems involving the gas laws often require the solution of first degree equations. This lesson provides a review of equation solving. The treatment is traditional and no particular problems are anticipated.

KEY--PROBLEM SET 24:

1. a. $2 \cdot 7 = 10 + 4$

   b. $9 + 6 = 96$

   c. Any equation but $2 \cdot 7 = 10 + 4$ or $9 + 6 = 96$

   d. $2x = 5$

   e. $2x + 3 = 2x - 3$

   f. $12x = 6$ and $4x = 2$

2. $\{2\}$
3. $\{83\}$
4. $\{21\}$
5. $\{39\}$
6. $\{16\}$
7. $\{79\}$
8. $\{-68\}$
9. $\{13\}$
10. $\{-97\}$
11. $\{3\frac{1}{3}\}$
12. $x = \frac{5}{6}$
13. $x = 2$
14. $T = 44.8$
15. $N = -2$
16. $m = -24$
17. $N = 0$
18. $x = -6$
19. $x = 40$
20. $x = 44$
21. $z = 69$
22. 20.16 liters
23. 3000 mm Hg
24. 7,969.5 mm Hg • liters
25. 444 ml
26. $10\frac{1}{3}$ liters
27. 370 ml
28. $159\frac{1}{4}$ ml
29. 48 liters
30. 27.3 ml

79

LESSON 25: THE COMBINED GAS LAW

OBJECTIVE:

The student will use the combined gas law to solve applied problems.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

In this lesson the combined gas law is derived and then used in solving problems. The applications of the combined gas law will continue in Lesson 26 which discusses scuba diving problems.

KEY--PROBLEM SET 25:

1. $V = 80$

2. $T' = 240$

3. $N = 285$

4. $r = 141$

5. $t = 42{,}174$

6. $m = -2862$

7. $z = -1$

8. $A = -7\frac{1}{2}$

9. $T = -\frac{1}{9}$

10. $x = 4$

11. a. $V_i = 5$ liters

    b. $V' = 6$ liters

12. a. $V_i = 7.2$ liters

    b. $V' = 6$ liters

13. $T' = 168.3° K$

14. $V' = 600\ cm^3$

15. $T' = 2800° K$

16. $V' = \frac{1}{3}$ liter

17. $V = 3.6$ liters

8 i

LESSON 26: PRESSURE, DEPTH AND AQUALUNGS

OBJECTIVE:

Same as Lesson 25.

PERIODS RECOMMENDED:

three

OVERVIEW AND REMARKS:

There are several situations in which a knowledge of the gas laws might be important for life support. For example,

astronauts in space
divers in the ocean
a patient under anesthesia
mountain climbers
passengers in airplanes flying at high altitudes

Any situation in which people are separated from the atmosphere and are forced to carry their air in bottles

In this lesson the concern will be diving problems. However, the technqiues involved are transferrable to other types of respiration life-support situtations such as those encountered in biomedical careers.

The students may be interested in the origin of the term "scuba." It stands for "self-contained underwater breathing apparatus."

KEY--PROBLEM SET 26:

1. 2400 liters

2. 125 atm

3. a. 1200 liters

   b. 304° K

   c. 1 atm

   d. 12 liters

   e. 285° K

   f. 93.75 atm

4. a. $1\frac{1}{2}$ atm

   b. $3\frac{1}{2}$ atm

5. a. 50 atm

   b. 12 liters

   c. 4 atm

   d. 150 liters

   e. 5 minutes

   f. 12 liters, i.e., the capacity of the tank

6. a. PV = P'V'

   b. 169 atm

   c. 15 liters

   d. 1.3 atm

   e. 1950 liters

   f. 50 minutes

   g. 15 liters

7. 280 liters

8. 20 minutes

9. a. 200 atm

   b. 306° K

   c. 14 liters

   d. 1.4 atm

   e. 289° K

   f. $1888\frac{8}{9}$ liters

   g. $55\frac{5}{9}$ minutes

10. 80 minutes = 1 hour, 20 minutes

11. a. 203 atm

    b. 308° K

    c. 297° K

    d. yes

    e. $\frac{P}{T} = \frac{P'}{T'}$

    f. $\frac{203}{308} = \frac{P'}{297}$

    g. 195.75 atm

12. 231 atm

13. a. 1800 liters

    b. 180 minutes = 3 hours

14. 225 minutes = 3 hours, 45 minutes

15. 240 minutes = 4 hours

8.

LESSON 27: SCIENTIFIC NOTATION I

OBJECTIVE:

The student will convert numbers stated in decimal notation into scientific notation and the converse.

The student will be able to determine the value of n in conversions similar to $96.9 \times 10^{12} = .969 \times 10^{n}$.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

Scientific notation will be used throughout the Biomed course, and in particular it will be used in the upcoming science lessons dealing with pH. In this lesson, and in Lesson 28, the students will be exposed to basic skills in working with scientific notation. This lesson concentrates on intercoversion between scientific and decimal notation and Lesson 28 treats multiplication and division of numbers in scientific notation.

Please note that the definition of scientific notation in this program is more general than that in most treatments. We do not require the decimal part of a number in scientific notation to have one digit to the left of the decimal point. This allows more flexibility; the expression of a number can reflect what is convenient in a particular calculation.

KEY--PROBLEM SET 27:

1. 2

2. 2

3. 0

4. -2

5. -1

6. -3

7. 1

8. -2

9. 4

10. 3

11. 9.648×10^3

12. 4.3×10^1

13. 5.867 or 5.867×10^0

14. 1×10^3

15. 6.0708×10^1

16. 2×10^{-3}

17. 5.76×10^{-2}

18. 1.8×10^{-5}

19. -3.572×10^2

20. -9.09×10^{-3}

21. -2×10^{-1}

22. 6800

23. 7.6006

24. .0576

25. -30,010

26. -.47

27. .000771

28. 98

$$\left.\begin{array}{ll} 29. & \text{b. } 1 \times 10^{100} \\ 30. & 9.3 \times 10^{7} \text{ miles} \\ 31. & 1 \times 10^{-6} \text{ meter} \end{array}\right\} \text{or any equivalent expression}$$

32. a. 1 parsec ≃ 4.3 [illegible] years

≃ 3.08×10^{18} cm

b. 1 barn = 1×10^{-24}

c. 1 nanosecond - 1×10^{-9} sec

80

LESSON 28: SCIENTIFIC NOTATION II

OBJECTIVES:

The student will multiply and divide numbers stated in scientific notation.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

This is a straightforward lesson. Multiplication and division of numbers in scientific notation are based on the following exponential properties, which should be stressed.

$$10^{p} \times 10^{q} = 10^{p+q}$$

$$\frac{10^{p}}{10^{q}} = 10^{p-q}$$

87

KEY--PROBLEM SET 28:

1. 10^5

2. 10^3

3. 10^3

4. 10^{-5}

5. 10^{-10}

6. 10^2

7. x^5

8. x^3

9. x^3

10. x^{-3}

11. 6×10^5

12. 2.25×10^4

13. 1.8×10^{-5}

14. 5×10^{-4}

15. 1.0807×10^{15}

16. 2.266×10^{12}

17. 2×10^3

18. 3.05×10^{-4}

19. 6×10^{-12}

20. 2.08×10^5

21. 20×10^0

22. 1.5×10^6

23. 1.75×10^7

24. $.125 \times 10^{-3}$

25. a. 1×10^{100}

    b. 1×10^{200}

26. 5×10^{-3} liter

27. 9.4608×10^{15} meters

(or any equivalent expressions)

28. 1.89216×10^{22} meters

29. a. 2.83824×10^{25} meters

    b. about 2 billion years old

LESSON 29: DECIMAL POWERS OF TEN

OBJECTIVE:

The student will find the approximate values of expressions involving fractional powers of ten.

The student will convert expressions of the form 10^r, where r is a decimal, into scientific notation.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

In the study of pH in science class, the students will find it necessary to work with fractional powers of ten. This lesson is intended to lay the necessary groundwork. Mathematically, this lesson closely resembles Lesson 28. The only new aspect is the introduction of fractional exponents. The entire subject of fractional exponents will be covered in greater detail in the second year math course.

KEY--PROBLEM SET 29:

1. 10

2. tenth

3. 5

4. fifth

5. $x = 10^{.5}$

6. second or square

7. We will use $\frac{5}{4}$ instead of 1.2589 because $\frac{5}{4}$ is easier to work with.

8. $(10^{.1})(10^{.1}) \simeq \frac{5}{4} \cdot \frac{5}{4}$

$\simeq \frac{25}{16}$

$\simeq 1 \frac{9}{16}$

$\simeq$ 1.5125 which is close to 1.6

9. $(10^{.3})(10^{.1}) \simeq 2 \quad \frac{5}{4}$

$\simeq$ 2.5 which agrees exactly

10. $(10^{.4})(10^{.1}) \simeq (2\ 5)(\frac{5}{4})$

$\simeq \frac{12.5}{4}$

$\simeq$ 3.125 which is close to 3.2

11. $(10^{.5})(10^{.1}) \simeq (3.2)(\frac{5}{4})$

$\simeq (.8)(5)$

$\simeq$ 4.0 which agrees exactly

12. $(10^{.9})(10^{.1}) \simeq (7.9)(\frac{5}{4})$

$\simeq \frac{39.5}{4}$

$\simeq$ 9.85 which is close to the exact value of 10.

90

13. 1.6

14. 3.2

15. 2.5

16. 7.9

17. 4.0

18. 4.0

19. 2.5

20. 1.25

21. 1.6×10^{1}

22. 2.5×10^{3}

23. 2.0×10^{7}

24. 7.9×10^{9}

25. 5×10^{1}

26. 4×10^{2}

27. 1.25×10^{8}

28. 3.2×10^{101}

29. 6.3×10^{24}

30. 2.5×10^{15}

31. 3.2×10^{-2}

32. 1.25×10^{-8}

33. 4×10^{-9}

34. 2.0×10^{-22}

35. 6.3×10^{-18}

36. 5×10^{-5}

37. 2.5×10^{-8}

38. 1.6×10^{-25}

39. 1.25×10^{-1}

40. 6.3×10^{-8}

LESSON 30: REVIEW

OBJECTIVE:

The student will review the concepts of Lessons 24 through 29.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

Some or all of the problems in the problem set can be assigned as a review. A breakdown of the problem set by section is included as an aid in choosing appropriate problems.

KEY--PROBLEM SET 30:

1. a. 10^3

   b. 10^{-15}

   c. 10^7

2. 1.8×10^2 °K

3. a. s = 8

   b. t = 16

   c. x = 80

   d. h = 6

4. a. 1.88×10^2ml

   b. 7.5 atm

5. a. 200 atm

   b. ~2.5 atm

   c. ~12 liters

   d. ~960 liters

   e. ~32 minutes

6. a. $\{10^{-11}\}$

   b. $\{10^{15}\}$

   c. $\{10^2\}$

7. a. ~50 atm

   b. ~12 liters

   c. ~120 minutes = 2 hours

8. a. $10^{7.9}$

   b. fifth

   c. second

   d. $10^{-1.4}$

9. a. 7.9×10^4

   b. 2.0×10^7

   c. 2.0×10^{-2}

   d. 7.9×10^{-100}

LESSON 31: ALCOHOL INGESTED ON AN EMPTY STOMACH

OBJECTIVES:

The student will be able to interpret graphs relating blood-alcohol concentration to time after consumption.

The student will be able to determine the amount of alcohol present in an amount of a beverage when the concentration is stated as a percent.

PERIODS RECOMMENDED:

two (This allows time for a discussion of the legal aspects of alcohol consumption. See table later in lesson plan.)

OVERVIEW AND REMARKS:

Lessons 31 through 33 are devoted to the study of alcohol concentration in the bloodstream. When alcohol is ingested there is a distributive phase during which the blood concentration rises, eventually reaching a peak value. At this point, if no more alcohol is introduced to the system, the concentration in the blood will drop as the alcohol is metabolized. This latter phase can be well described by a linear function. It is for this reason that the alcohol material has been included at this point in the math course.

Similar studies of metabolism and elimination rates have been carried out for numerous drugs. The resulting information is essential in deciding the dosage interval necessary to maintain a desired concentration in a patient's bloodstream. As it turns out, the vast majority of drugs are not metabolized according to a linear relationship. Instead, blood concentration obeys a pattern of exponential decay. This subject will be treated in more detail in the second year biomath course. For the present, it is well to point out that the behavior of alcohol in the human body is exceptional.

Lesson 31 and 32 are concerned with alcohol ingested on an empty stomach while Lesson 33 treats the case of ingestion with or after a meal. Both cases involve a linear function but there are differences in peak blood concentrations and elmination rates. The causes of these differences are explained in Section 33 of the Student Text.

An integral part of this lesson is a treatment of the legal aspects of alcohol consumption. A brief introduction to this subject is included in Section 31-4. We have also included some supplementary material here.

SUPPLEMENTARY MATERIAL: STATE DRUNK-DRIVING STATUTES

Most states have enacted leglislation defining intoxication while driving in terms of specific concentrations of alcohol in the blood. This is because a blood test can be easily administered and provides the court with objective evidence.

The table that follows gives information on the drunk-driving statutes of the different states as of December 1971. The states are listed alphabetically and fall under one of two headings: X - Y states or X = Y states. For the X - Y states the statutes are of the following form.

If the blood-alcohol content is below X, it is presumptive evidence of non-intoxication.

If the blood-alcohol content is between X and Y, it is evidence of non-intoxication, but not presumptive.

If the blood-alcohol content is above Y, it is presumptive evidence of intoxication.

When there is presumptive evidence of intoxication, the court assumes that the defendant was intoxicated unless the defense can persuade him that this was not the case, from other evidence.

When there is presumptive evidence of non-intoxication, the court will assume the defendant was not intoxicated unless the prosecution can prove otherwise from other evidence.

For the X = Y states the statutes are of the following form.

If the blood-alcohol content is above Y it is presumptive evidence of intoxication. The statute says nothing about the concentration for presumption of non-intoxication.

(1), (2), (3), (4) and * refer to notes at the end of the table.

State	X - Y States	X = Y States
Alabama	No statutory guidelines	
Alaska	.05 - .10	
Arizona	.05 - .15	
Arkansas	.05 - .10	
California	.05 - .10	
Colorado	.05 - .15	
Connecticut	.05 - .10	
Delaware		(1) If level >.10 driver is guilty.

State	X - Y States	X = Y States
District of Columbia	.05 - .15	
Florida	No statutory guidelines	
Georgia	.05 - .10	
Hawaii	.05 - .10	
Idaho		.08
Illinois	.05 - .10	
Indiana		
Iowa		.10
Kansas		(2) .10
Kentucky	.05 - .10	
Louisiana	No statutory guidelines	
Maine	.05 - .10	
Maryland	No statutory guidelines	
Massachusetts	.05 - .15	
Michigan	.07 - .10	
Minnesota	.05 - .10 (.10 is conclusive.)	(3)
Mississippi	.10 - .15	
Missouri	.05 - .15	
Montana	.05 - .10	
Nebraska		(4) If level >.10 driver is arrested.
Nevada	.05 - .10	
New Hampshire	.05 - .10	
New Jersey	.05 - .10	
New Mexico	.05 - .10	
New York	*	
North Carolina		.10
North Dakota	.05 - .10	
Ohio	.05 - .10	
Oklahoma	No statutory guidelines	
Oregon	.05 - .10	
Pennsylvania	No statutory guidelines	
Rhode Island	.05 - .10	

State	X - Y States	X = Y States
South Carolina	.05 - .10	
South Dakota	.05 - .10	
Tennessee		.15
Texas	No statutory guidelines	
Utah	.05 - .08	
Vermont	.05 - .10	
Virginia	.05 - .15	
Washington	.05 - .10	
West Virginia	.05 - .10	
Wisconsin	.05 - .15	
Wyoming	.05 - .15	

Notes:

(1) The Delaware statute states that anyone with a blood-alcohol concentration greater than .10% is guilty.

(2) The Kansas statute is slightly different from the others in its category. Below .10%, the defendant is presumed to be not intoxicated; about 10% the defedant is presumed to be intoxicated.

(3) The Minnesota statute is slightly different from the others in its category. It states that .10% or more is conclusive, not presumptive, evidence of intoxication. If the defendant has a concentration greater than .10% he is guilty.

(4) Nebraska's statute doesn't talk about presumptive evidence. It merely states that those individuals with a concentration greater than .10% will be put under arrest.

*New York has two separate offenses:

(a) driving while intoxicated; and

(b) driving while one's senses are impaired by alcohol.

A blood-alcohol percentage of less than .05% is presumptive evidence that the subject is neither intoxicated nor impaired. If there is no other strong indication of drunkenness or impairment, he cannot be prosecuted.

A blood-alcohol percentage of between .05 and .08% is presumptive evidence that the subject is not intoxicated, as above. On the other hand, it is evidence that the subject is impaired, and the subject might be prosecuted for impairment.

A blood-alcohol percentage of .08% or more is presumptive evidence that the subject is guilty of driving while impaired. It is evidence that the subject is intoxicated, but not necessarily as strongly presumptive of guilt of intoxication as it is of guilt of impairment.

If time allows, you may want to call your local city hall to check on the current status of drunk-driving laws in your state. They can also provide information on how blood alcohol concentration is tested and so on.

KEY--PROBLEM SET 31:

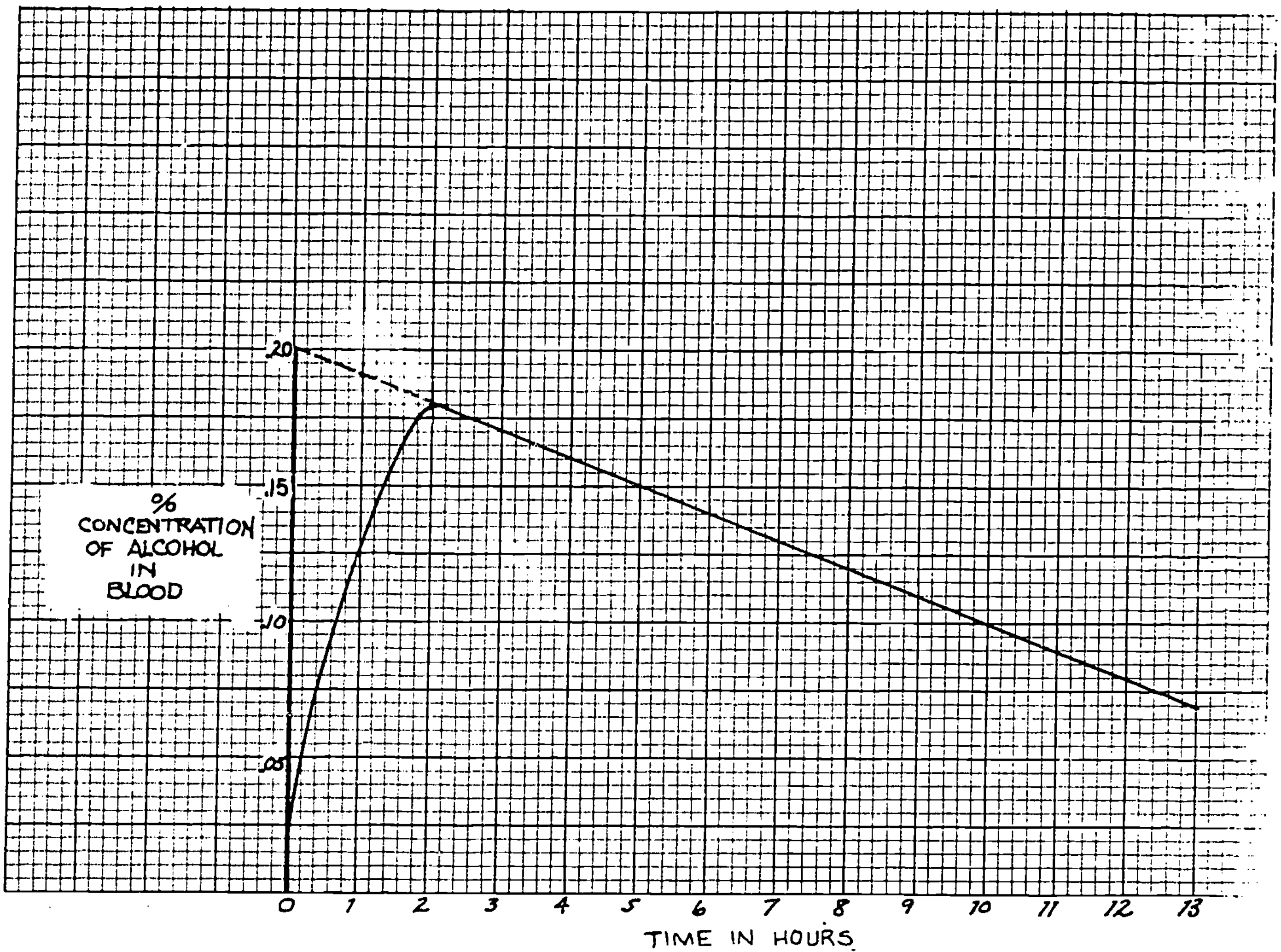
1. 350 ml
2. 90 ml
3. 4.65 liters
4. $11\frac{1}{9}\%$
5. 5%
6. a. 375 ml
   b. 37.5%
7. a. 10 ml
   b. 4 ml
8. a. 90 ml
   b. 100 ml
   c. 190 ml
   d. 47.5%
9. true
10. false
11. true
12. true
13. false
14. false
15. true
16. true
17. true
18. a. 2 hours
    b. .12%
    c. -.01
    d. $c \approx -.01t + .12$
    e. no
    f. no, because the concentration would be negative

g. 12 hours

h. 2 hr $\leq$ t $\leq$ 12 hr

i. The line will coincide with the x-axis for t > 12, because the concentration of alcohol cannot be less than zero.

19. a.

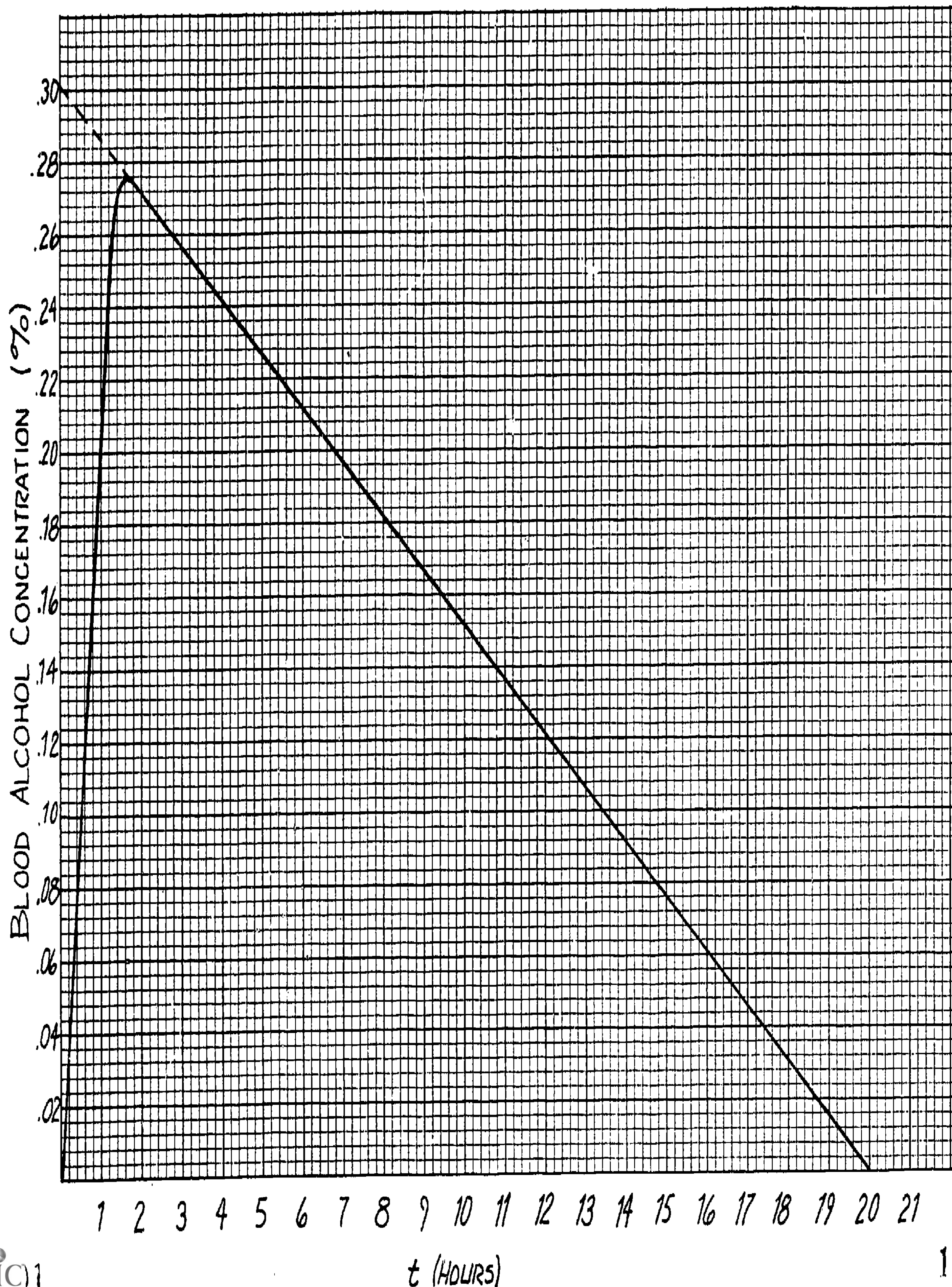


b. 10 hours

c. 20 hours

d. For t = 1, c $\simeq$ .19 by the equation. This is incorrect according to the graph. The formula gives the correct values for c <u>only</u> for t $\geq$ t_m; t = 1 which is less than t_m.

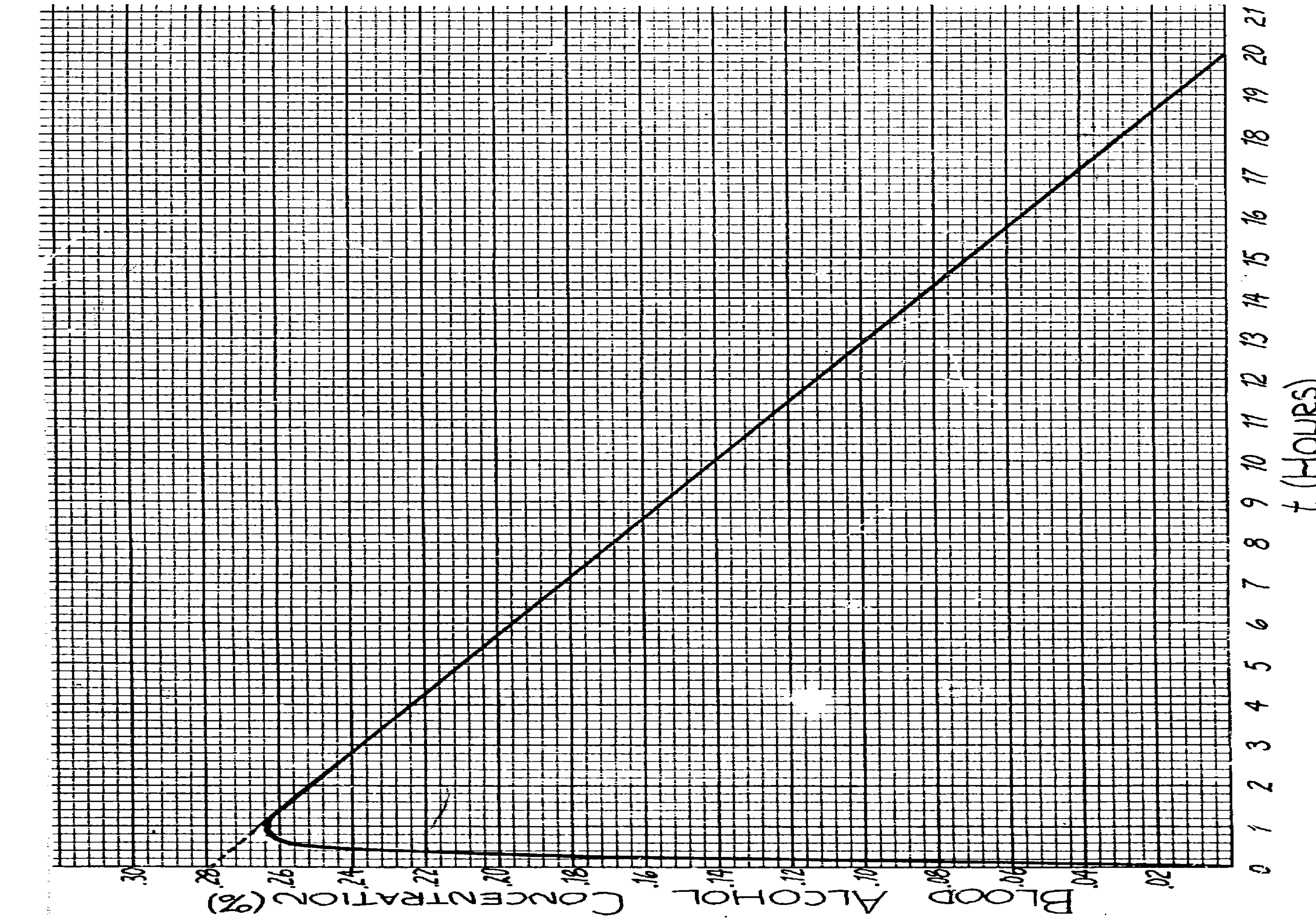
20. a.


BLOOD ALCOHOL CONCENTRATION (%)
.30
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
t (HOURS)

b. About 13.3 hr from the graph. Exactly $13\frac{1}{3}$ hr by calculation.

c. 20 hours

21. a.



b. 20 hours

c. About .265 from the graph. Exactly .266 by calculation.

1:06

LESSON 32: ALCOHOL INGESTED ON AN EMPTY STOMACH (CONT'D)

OBJECTIVE:

The student will solve problems involving the relations $c \simeq \beta t + c_o$ and $c_o \simeq \frac{A}{Mr}$.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

This lesson continues the treatment of alcohol ingestion on an empty stomach. The following questions are treated.

1. Given the mass of the individual and the amount of alcohol consumed, how can one find the "initial" blood alcohol concentration?

2. How can one find the maximum blood alcohol concentration actually attained?

3. How can one predict the blood alcohol concentration at any subsequent time?

If you have students in your class with a limited command of the English language, you may find it necessary to give explicit directions for Problem 6 of the problem set.

KEY--PROBLEM SET 32:

1. a. 126 ml

   b. .2%

   c. .1775%

   d. $3\frac{1}{3}$ hours

   e. 10 hours

2. a. 90.4 ml

   b. .2%

   c. .1805%

   d. 10 hours

   e. $15\frac{5}{13}$ hours

3. a. .2%

   b. 10 hours

   c. $13\frac{1}{3}$ hours

4. $t = -\frac{c_o}{\beta}$

5. a. .2%

   b. .234%

   c. .1712%

   d. .2199%

   e. no

f. if r, t_m and β were the same for both Dave and Jack

LESSON 33: ALCOHOL INGESTED WITH A MEAL

OBJECTIVE:

The student will apply knowledge of linear functions to the solution of problems relating to blood-alcohol concentrations resulting from the ingestion of alcohol on a full stomach.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

There are no new mathematical concepts in this lesson. The only change is in the values of the constants used in computations. The new values are

$$r \simeq 14$$
$$t_m \simeq 2.5 \text{ hours}$$
$$\beta \simeq -.01\% \text{ per hour}$$

The Student Text explains how food in the stomach influences the absorption and elimination of alcohol. Notice that the explanation is consistent with the new values of the constants. r is now 14 instead of 9 which means that the "initial" concentration c_o will be lower. t_m is 2.5 hours instead of 1.5 hours which means that the peak blood alcohol concentration takes longer to occur. Finally, β is -.01% instead of -.015% which implies a slower rate of elimination from the body. If the students can gain an intuitive feeling for these relationships, they will find it easier to decide which constants to use in a given problem.

Be on the lookout for confusion over Problem 13, which is a long word problem.

KEY--PROBLEM SET 33:

1. higher
2. slower
3. lowering c_o
4. slowing the rate of elimination
5. higher, steeper
6. a. $\beta = -.015\%$ per hour

   b. faster
7. empty, full
8. a
9. a. roughly reproducible

   b. differs significantly
10. F
11. a. yes

    $A \simeq 63$ ml

    $c_o \simeq \frac{7}{45}\%$

    at $t = 2$ hr, $c \simeq \frac{113}{900}\% > .10\%$

    b. no

    $c_o \simeq .1\%$

    The maximum concentration attained at $t_m \simeq 2.5$ hr is

    $c \simeq .075\% < .10\%$
12. a. .216%

    b. 6.6 hr

    c. 16.5 hr

    d. 21.6 hr

    e. .336%

    12.4 hr

    19 hr

    22.4 hr
13. 5,134.4 ml = 5.1345 liters
14. 700 ml
15. 18.375 ml
16. 100 kg

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LESSON 34: REVIEW

OBJECTIVE:

The student will review the concepts of Lessons 1 through 33.

PERIODS RECOMMENDED:

two

OVERVIEW AND REMARKS:

Some or all of the problems in the problem set can be assigned as a review. A breakdown of the problem set by section is included as an aid in choosing appropriate problems.

KEY---PROBLEM SET 34:

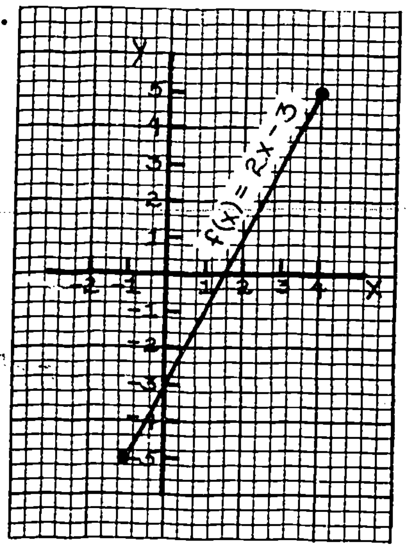
1. a. (-2, 4), (0, -2), (1, 1)

   b. 4, 6

   c. 12

2. a. -3

   b.



   c. 2

   d. -3

3. a. 3

   b. -3

   c. 0

   d. -2

   e. -2

   f. 4

   g. y = 3x - 2

   h. y = -3x - 2

   i. y = 4

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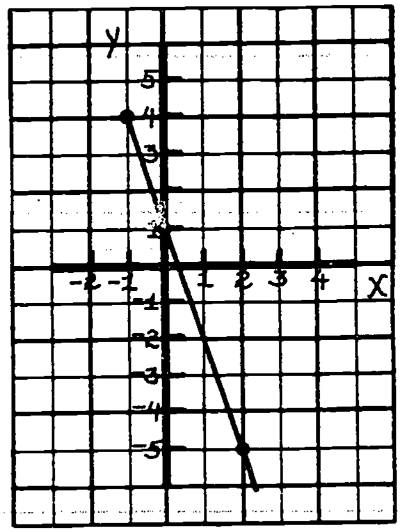
4. a.



b. $y = -3x + 1$

c. -3

5. a. 2

b. -2

c. $y = 2x - 2$

d. 5

6. 5 kg per sq cm

7. a. 50 kg

b. $\frac{1}{2}$

c. 2 sq cm

8. a. $h = 13$

b. $p = 4$

c. $s = 10$

d. $x = 220$

9. a. 4.3×10^{-1}

b. 1.2×10^{7}

c. 2.5×10 or 2.5×10^{1}

10. 1.55 ml

11. a. 1.8×10^2 °K

b. $3\frac{1}{3}$ atm

12. a. 4 atm

b. 200 liters

c. 20 minutes

13. a. .054 liters

b. 54 ml

c. .08%

d. 2.5 hours

14. a. 3.2×10^7

b. 3.2×10^{-8}

c. 5×10^{-14}

15. a. $10^{-6.3}$

b. $10^{-10.2}$

c. $10^{-2.7}$

LESSON 35: DIMENSIONAL ALGEBRA II

OBJECTIVES:

The student will apply dimensional algebra to problems involving square units and cubic units.

PERIODS RECOMMENDED:

one

OVERVIEW AND REMARKS:

This lesson and the following lesson continue the study of dimensional algebra which began in Lesson 7. In this lesson the emphasis is on units of area and volume. Several problems in the problem set require the scientific notation skills developed in Lessons 27 and 28.

11.

KEY--PROBLEM SET 35:

1. Correct

2. Correct

3. Correct

4. $.022 \frac{\text{moles}}{\text{cm}^3} \cdot \left(\frac{1 \text{ cm}}{10 \text{ mm}}\right)^3 \cdot 6 \times 10^{23} \frac{\text{molecules}}{\text{mole}} = 1.32 \times 10^{19} \frac{\text{molecules}}{\text{mm}^3}$

5. $.91 \frac{\text{g}}{\text{liter}} \cdot \frac{1 \text{ liter}}{1000 \text{ ml}} \cdot 1 \frac{\text{ml}}{\text{cm}^3} \cdot \left(\frac{100 \text{ cm}}{\text{m}}\right)^3 \simeq 9.1 \times 10^2 \frac{\text{g}}{\text{m}^3}$

6. Correct

7. $18 \frac{\text{ft}^3}{\text{hr}} \cdot \left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right)^3 \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \left(\frac{100 \text{ cm}}{\text{m}}\right)^3 \approx 8502 \frac{\text{cm}^3}{\text{min}}$

8. $32 \frac{\text{ft}}{\text{sec}^2} \cdot \left(\frac{3600 \text{ sec}}{\text{hr}}\right)^2 \cdot \frac{1 \text{ m}}{3.28 \text{ ft}} \approx 1.26 \times 10^8 \frac{\text{m}}{\text{hr}^2}$

9. $4.7 \frac{\text{m}^3}{\text{hr}} \cdot \left(\frac{100 \text{ cm}}{\text{m}}\right)^3 \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{4.7 \times 100^3}{60} \frac{\text{cm}^3}{\text{min}}$

10. $\left(\frac{\text{m}}{\text{ft}}\right)^2$

11. $\left(\frac{100 \text{ cm}}{\text{m}}\right)^3$

12. $\left(\frac{12 \text{ in}}{\text{ft}}\right)^2$

13. $\left(\frac{100 \text{ cm}}{\text{m}}\right)^3$

14. $\frac{1 \text{ ml}}{\text{cm}^3}$

15. $\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3$

16. $\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3$

17. $\frac{\text{grams}}{\text{mole}}$

18. $\frac{ft^3}{gallons}$

19. $\left(\frac{3600\ sec}{hr}\right)^2$

20. $\sim \frac{1}{3.6^3}\ m^3 \approx .27\ m^3$

21. $\sim 2.4 \times 10^{-5}\ m^3$

22. $\sim 1.21\ m^2$

23. $\sim \frac{2}{15}\ m^3 \approx .13\ m^3$

24. $\sim 1.07\ \frac{kg}{cm^2}$

25. $\sim 10^{-2}\ \frac{moles\ NaCl}{liter\ solution}$

26. $\sim 504\ \frac{liters}{hr}$

27. $\sim 1\ \frac{g}{m^3}$

28. $\sim 10^{-1}\ \frac{mole\ HCl}{liter\ solution}$

29. $\sim .0029\ \frac{ft^3 - atm}{°K}$

30. a. $\left(\frac{1.0936\ yd}{1\ m}\right)$

b. ~ 109.36 yd

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LESSON 36: DIMENSIONAL ALGEBRA III

OBJECTIVES:

The student will apply dimensional algebra to problems involving rates.

The student will solve air pollution problems involving the units of $\mu g/m^3$ and ppm.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

This is the last of the dimensional algebra lessons. The air pollution units, $\mu g/m^3$ and ppm, are potentially confusing and the two pertinant examples in the text should be covered carefully. A comparison of percentage (parts per hundred) and ppm (parts per million) may help clarify the latter units.

KEY--PROBLEM SET 36:

1. $80 \frac{\text{beats}}{\text{minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot 3 \text{ hours} = 14{,}400 \text{ beats}$

2. $2.3 \times 10^6 \cdot \frac{\text{cells}}{\text{second}} \cdot \frac{60 \text{ seconds}}{\text{minute}} \cdot \frac{60 \text{ minutes}}{\text{hour}} \cdot 2 \text{ hours} = 1.656 \times 10^{10} \text{ cells}$

3. $\frac{16 \text{ g}}{100 \text{ ml}} \cdot \frac{1000 \text{ ml}}{1 \text{ liter}} \cdot 5.5 \text{ liters} = 880 \text{ g}$

4. $\frac{10 \text{ cm}^3 \text{ CO}}{10^6 \text{ cm}^3 \text{ air}} \cdot 10^3 \text{ cm}^3 \text{ air} = 10^{-2} \text{ cm}^3 \text{ CO}$

5. $\frac{.3 \text{ cm}^3 \text{ SO}_2}{10^6 \text{ cm}^3 \text{ air}} \cdot \frac{100^3 \text{ cm}^3 \text{ air}}{1 \text{ m}^3 \text{ air}} \cdot 10 \text{ m}^3 \text{ air} = 3 \text{ cm}^3 \text{ SO}_2$

6. $1 \text{ cm}^3 \text{ O}_3$

7. $.83 \text{ m}^3$ air

8. 1.66 m^3 air

9. a. $\frac{1 \ \mu^3}{10^3 \ \mu\text{g}} \cdot \frac{10^6 \ \mu\text{g}}{1 \text{ g}} \cdot 1 \text{ g} = 10^3 \text{m}^3$

   b. $5 \times 10^3 \text{ m}^3$

10. a. $\frac{600 \text{ ml}}{\text{breath}} \cdot \frac{12 \text{ breaths}}{\text{min}} \cdot \frac{1 \text{ cm}^3}{10^6 \text{ cm}^3} \cdot \frac{1 \ \mu\text{g}}{\text{m}^3} \cdot \frac{60 \text{ min}}{\text{hr}} \cdot \frac{24 \text{ hr}}{\text{day}} \simeq 10.368 \frac{\mu\text{g}}{\text{day}}$

    b. $\frac{10.368 \ \mu\text{g}}{\text{day}} \cdot \frac{1\text{g}}{10^6 \ \mu\text{g}} \cdot \frac{365 \text{ days}}{\text{yr}} \simeq 3.78432 \times 10^{-3} \frac{\text{g}}{\text{yr}}$

11. a. $\frac{1 \text{ ml}}{5 \text{ mg}} \cdot \frac{10^3 \text{ mg}}{\text{g}} \cdot .02 \text{ g} = 4 \text{ ml}$

12. $\frac{1 \text{ cm}^3}{500 \text{ mg}} \cdot \frac{10^3 \text{ mg}}{\text{g}} \cdot .4 \text{ g} = .8 \text{ cm}^3$

13. $\frac{1 \text{ ml}}{5 \times 10^5 \text{ units}} \cdot 4 \times 10^5 \text{ units} = .8 \text{ ml}$

14. In the following solution, A = alcohol, B = blood and W = body weight.

$$\frac{2 \text{ g A}}{100 \text{ g B}} \cdot \frac{1 \text{ g B}}{\text{ml B}} \cdot \frac{1 \text{ ml A}}{.8 \text{ g A}} \cdot \frac{8 \text{ ml B}}{100 \text{ g W}} \cdot \frac{1000 \text{ g W}}{\text{kg W}} \cdot 70 \text{ kg W} \simeq 14 \text{ ml alcohol}$$

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15. $\frac{9 \times 10^3 \text{ cells}}{\text{mm}^3} \cdot \frac{10^3 \text{ mm}^3}{\text{cm}^3} \cdot \frac{1 \text{ cm}^3}{\text{ml}} \cdot \frac{8 \text{ ml}}{100 \text{ g}} \cdot \frac{10^3 \text{ g}}{\text{kg}} \cdot 50 \text{ kg} \simeq 3.6 \times 10^{10} \text{ cells}$

16. $\frac{1 \text{ min}}{5 \text{ liters}} \cdot \frac{1 \text{ liter}}{10^3 \text{ ml}} \cdot \frac{1 \text{ ml}}{\text{cm}^3} \cdot \frac{10^6 \text{ cm}^3}{\text{m}^3} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ day}}{24 \text{ hr}} \cdot 750 \text{ m}^3 \simeq 104.16 \text{ days}$

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