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ABSTRACT

This is volume three of a three-volume set for teachers using SMSG junior high school text materials. Each unit contains a commentary on the text, answers to all the exercises, a copy of the questionnaire used for evaluating the material, and a summary of comments by the teachers using the text. Unit topics include: (1) what is mathematics; (2) the scientific seesaw; (3) statistics; and (4) chance. (MP)

JUNIOR HIGH SCHOOL MATHEMATICS UNITS

VOLUME III, APPLICATIONS

Commentary for Teachers

SMSG

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VOLUME III, APPLICATIONS

Commentary for Teachers

Prepared by the SCHOOL MATHEMATICS STUDY GROUP

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These units were prepared in the summer of 1958 at Yale University by a School Mathematics Study Group writing team. The units were taught in a number of classes during the academic year 1958-59. No major editing of them has been attempted, but typographical and other errors have been corrected.

When the units were used, the teachers were invited to submit comments concerning their experiences. A questionnaire was filled out by teachers for each unit taught. A copy of this questionnaire and a summary of comments by the teachers is included for each unit.

In addition, in those cases where unit tests were prepared during the year, a collection of suggested test items appears at the end of the commentary on the unit.

This volume includes the units concerned with applications of mathematics. These are units I, XI, XII, XIII in the numbering system originally used.

TABLE OF CONTENTS

UNIT I

WHAT IS MATHEMATICS

TO THE TEACHER:

This unit is designed to give the pupil an appreciation for the importance of mathematics. Its objectives are:

I. To develop an understanding of what mathematics is as opposed to simple computation.

II. To motivate pupils by pointing out the need for mathematicians.

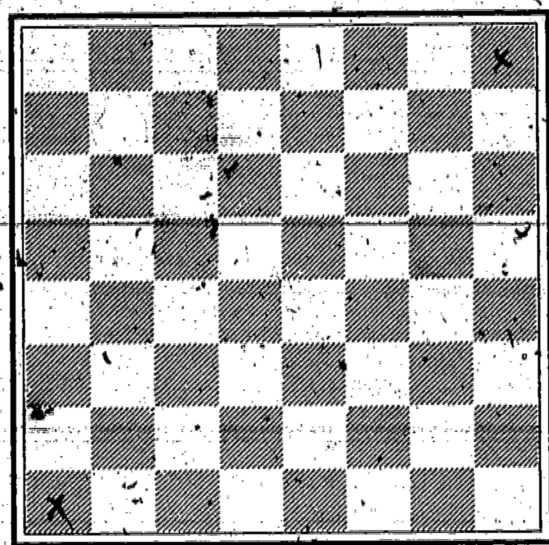
Good attitudes will be built if you use imagination and enthusiasm in getting these objectives across to the children.

Since this unit is so different from a textbook, it will need a different treatment. One way of presenting this unit will be to let the students take the unit home and read it. Follow this with daily class discussions on each part and work the exercises. Encourage the more able students to solve the "BRAINBUSTERS" but be ready to help if they have difficulty. The daily class discussions will help the slower learners digest the information and hold their interest.

The first section introduces deductive thinking. It attempts to show that logical reasoning can be used to solve difficult problems more easily than they can be solved by computation. It should be pointed out that most mathematics is a method of deductive reasoning. Where inductive reasoning is used in the early stages of a problem the final answer is almost always obtained by deductive reasoning. Also, mathematics has many aspects but its most powerful tool is the method of reasoning.

This concept of deductive reasoning is not an easy one to follow. You may want to include problems of your own selection to increase the student's understanding. You may want to include the problem of the checkerboard.

Suppose you have a checkerboard and dominoes just large enough to cover two squares on the checkerboard. Can you place the dominoes on the board in such a way as to cover all the board except two opposite corner squares?



In solving the problem is it practical to try out all the possible ways the dominoes may be placed on the board? This would be difficult because there are more than 65,536 ways to cover the whole board. The solution may be found in another way:

How many squares are there altogether on the board? (64)
How many squares must be covered? (62)
What is special about the two squares next to each other? (They are of different colors)
What is special about the two opposite corners? (They are the same color)

If you place any number of dominoes on the board can you say anything about the kinds of squares which will be covered? How does this compare with the kinds of squares which you are supposed to cover? Do you have to make even one experiment in order to get the answer to the original problem? Can two squares of the same color be covered with one domino? The answer to this question should help some of the students reason why the solution is impossible.

After introducing deductive reasoning, it is still necessary to give the students an understanding of its importance. Examples of scientific advancements in which deductive reasoning was used might be the Salk Vaccine, man-made satelites, atomic submarines, and digital computers. Students should have little difficulty finding examples of this sort for discussion in class.

Answers to exercise on page 2

1. 367. Birthdate indicates the same day. We must keep in ~~mind the possibility of February 29 as a birthdate.~~ Hence the smallest number for which we can prove two students have the same birthdate is 367.

2. 1. If the number is greater than 1 there is the possibility (however remote) that the two pupils were born on the same date. This should be carefully pointed out to the pupils so they may appreciate more fully the need for proving their conclusions.

3. 13. Most pupils should be able to reason this out, but don't rush the discussion so that the pupils miss out on the idea of deductive reasoning used in the solution.

4. 6. Again, most pupils should be able to reason this out.

5. 1. As in problem 2 this is the only acceptable conclusion if we are to be sure that no two people see the same show.

6-10. The "if-then" idea may be used here to show the pupils what may happen if they make certain choices. It might be a good idea to let the pupils study the problem and then have one pupil attempt the correct solution. If an error is made another pupil may take over. Problems 6 and 7 are not difficult. Many pupils will solve them without the need of a diagram, but they should be given permission to use paper and pencil if necessary. As they are discussed, a diagram on the board may be helpful for slower learners.

6. Only one boy may receive 3.

7. A = 200 pound man
B = 100 pound boy
C = 100 pound boy

B and C cross river.
B returns. A crosses
C returns for B

8. Man takes goose and returns alone. He takes fox and returns with goose. He takes corn across river and returns alone to pick up goose. (Students will enjoy working out solutions.)

9. Most pupils will need paper and pencil for this one. As ~~they attempt the solution in front of class it is a good~~ idea to minimize the help from others in the class. If ~~an error is made, another student should be selected to~~ present his solution. It should be pointed out that a crossing necessitates a landing, i.e. jumping from boat onto boat is not considered ethical.

$M_1\ M_2\ M_3$ — $C_1\ C_2\ C_3$

$M_1\ C_1$ cross river. M_1 returns.
$C_2\ C_3$ cross river. C_1 returns.
$M_1\ M_2$ cross river. $M_1\ C_2$ return.
$M_1\ M_3$ cross river. C_3 returns.
$C_1\ C_2$ cross river. C_1 returns.
$C_1\ C_3$ cross river.

10. Balance the two groups of 3 marbles each. If they balance then it is only necessary to balance the remaining two marbles to find the heavy one. If the two groups of 3 marbles do not balance, take the heavier group. Of the 3 marbles in the heavier group balance any 2 marbles. If they balance the remaining marble is the heaviest one. If the 2 marbles do not balance the heaviest will be 1 of the 2 on the balance.

The second section deals with Gauss' method of addition for a group of numbers written in an arithmetic series. This may be used to dramatize how some pupils apply insight to finding a solution to a problem. That Gauss was only 10 years old at the time might be of interest to the class. Your better pupils should be told that there are other than Gauss' method of quickly finding the sum of a series of numbers. Some students might be encouraged to discover methods of their own for adding number series quickly.

The "middle number" method is one that may be used. This method can be used for an even or odd number of integers. The following examples may be used to explain this method to the students who have tried to discover other methods:

Example A. $1 + 2 + 3 + 4 + 5 + 6 + 7 = ?$

In this series the middle number (in this case "4") is the average of the series. The sum is the product of the middle number (4) and the number of integers (7) in the series. $4 \cdot 7 = 28$.

Example B. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = ?$

In this series the middle number lies between 4 and 5 and may be thought of as 4.5 or 4 1/2. Again the product of this "middle number" and the number of integers in the series will produce the correct sum. $4\ 1/2 \cdot 8 = 36$.

You may want to give the students several more problems on which they may apply this method.

Answers to Exercise Page 4

1. $1 + 5 = 6$ $5 \times 6 = 30$ $\frac{30}{2} = 15$

2. $\frac{(0 + 8)\,5}{2} = 20$

3. $\frac{(1 + 3)\,3}{2} = 6$

4. $\frac{(2 + 6)\,3}{2} = 12$

5. $\frac{(0 + 8)\,9}{2} = 36$

6. $\frac{(1 + 15)\,8}{2} = 64$

7. $\frac{(0 + 20)\,11}{2} = 110$

8. $\frac{(1 + 25)\,25}{2} = 325$

9. $\frac{(7 + 15)\,5}{2} = 55$

10. $\frac{(22 + 30)\,5}{2} = 130$

11. $\frac{(0 + 50)\,51}{2} = 1275$

12. $\frac{(0 + 200)\,201}{2} = 20100$

Yes
Yes
Yes

If we start with 0 there are 201 integers in the series giving us $\frac{(0 + 200)\,201}{2}$. If we start with 1 there are 200 integers in our series and we have $\frac{(1 + 200)\,200}{2}$. The products of like factors are equal. The method also may be used in a series if we select a number other than 1 or 0 as starting points. Some of the better students may investigate whether the method works in other number series.

John Friedrich Karl Gauss was born in Brunswick, Germany in 1777. He died in 1855 at the age of 78. He signed his masterpieces Karl Friedrich Gauss. Many of the references use this name.

Together with Newton and Archimedes, Gauss is labeled as one of the three greatest mathematicians of all time. He is frequently referred to as the "Prince of Mathematicians."

One day, prior to his third birthday, Gauss was watching his father make out the weekly payroll for the laborers under his charge. Suddenly the lad said, "Father, the reckoning is wrong, it should be . . . ." After carefully checking the figures, Gauss' father made the necessary corrections.

After his seventh birthday Gauss started school. At the age of twelve he questioned Euclid's geometry. At fifteen he entered Caroline College and mastered the classics. By seventeen he had studied the works of Euler, Lagrange and Newton. Newton was his hero. He never lowered his esteem for him.

Gauss had no financial worries during his twenties and thirties. He was supported by a pension from the Duke of Brunswick. These years were his most prolific, largely due to the Duke's generosity, as this relieved Gauss of financial problems.

When he was twenty Gauss decided to be a mathematician instead of a philologist (study of languages, classics and their historic relationship.)

His first masterpiece was published in 1801 entitled "Disquisitions." In 1805 while still being supported by his pension, he married. His wife died in 1809 after the birth of their son, Louis. Gauss remarried a year later.

When the Duke of Brunswick died in 1806 Gauss had to find a way to earn a living. In 1807 he was appointed Director of the Göttingen Observatory and Lecturer of Mathematics at Göttingen University.

Gauss made startling discoveries in the field of astronomical mathematics. He calculated the orbits of planets and the comets.

Financially he was not successful. Most of his works were not published until after his death. The royalties from his texts published prior to his death were small.

This section is only a brief introduction to probability. Although students may become interested at this point and attempt more complex problems it would be better if they waited. The unit on probability comes later in the text.

Exercise, page 6-

1. (1 out of 52) This simple problem should guide the students to realize that there are 52 cards in all and only one ace of spades. Therefore, probability is only one out of 52.

2. (4 out of 52 or 1 out of 13) Probability is a ratio of possible correct selections out of total possibilities. The student should be reminded that this does not mean that he will draw an ace with the first 13 draws, but merely that his chances are one out of 13.

3. (1 out of 6) Because a die has six sides and only one side has two dots.

4. BRAINBUSTER (1 out of 36) The more advanced students should reason that there are 36 possible combinations. A table may be constructed to show the 36 possibilities.

Show the 36 possibilities:

	1	2	3	4	5	6	7	8	etc.
1st die	1	1	1	1	1	1	2	2	
2nd die	1	2	3	4	5	6	1	2	

Since there is only one way of making 2 ones, the probability is one out of 36.

5. 1/8

Why you need to know mathematics

This section should be used primarily for reading by the students and class discussion. Some words like deduction and induction will need to be discussed beyond the reading in the text. One way to improve and extend one's vocabulary is to introduce a new word (or several new words) each day. The teacher may not wish to adapt this whole section to a formal class assignment. It may be wise merely to assign the reading for homework and then let students voice any questions raised by the reading.

Supplement to Teachers Guide, Unit I

Supplementary Problems and Answers
(Answers are in the square brackets)

I. Gauss' method for the addition of a group of numbers.

1. Add the first five odd numbers, beginning with 1, [25]; the first six, [36]; the first seven, [49]; the first 10, [100]. Is there a general law? [The sum is the square of the number of addends]. Add the first 1000 odd numbers. [1000^2 or 1,000,000].

2. What is the 5th even number beginning with 2, [10]; the 7th, [14]; the 10th, [20]; the 1000th, [2000]. How is the 5th odd number related to the 5th even number? [The 5th odd number is one less than the 5th even number]. What is the 1000th odd number? [The 1000th odd number is one less than the 1000th even number]. Reverse this process. Fill in the blanks:

10 is the [5]th even number
28 is the [14]th even number
9 is the [5]th odd number
99 is the [50]th odd number

14

II. Probability

1. What is the probability that a head appears when one coin is tossed? [1/2, one out of 2].

2. If two coins are tossed what is the probability that both coins show tails? [1/4] that exactly one head shows [1/2].

3. There are four aces (from a playing card deck) to be dealt to four people. What is the probability that the first person who receives a card gets the ace of hearts? [1/4 , there are 4 possibilities, one of which is favorable].

UNIT I

Sample Test Questions

PART I. TRUE - FALSE

T 1. A mathematician works more like a poet than a bookkeeper.

T 2. Mathematics is an art as well as a science.

F 3. Computing machines are taking the place of mathematicians in creating new mathematics.

T 4. Mathematics can be a hobby for some people.

T 5. Mathematics is used by the government in making important decisions.

F 6. The arithmetic textbook is the best source of mathematics.

F 7. A mathematician prefers to solve problems by trying all possibilities.

F 8. Since science is more important in our society today, mathematics is less important.

PART II. MULTIPLE CHOICE.

1. The most important part of a mathematician's work is:
   - A. measuring and drawing.
   - B. recording the data systematically.
   - C. counting and computing.
   - *D. thinking and reasoning.
   - E. none of these.
2. Deductive reasoning can best be defined as:
   - A. reasoning by trying all possibilities.
   - B. reasoning by experimentation.
   - *C. reasoning by logical thinking.
   - D. reasoning by trial and error.
   - E. reasoning by popular vote.

3. If 4 boys receive a total of 13 candy bars, a conclusion that is always true if each boy receives at least one (without cutting any bar) is:

A. one boy receives 10 candy bars.

B. each boy receives at least 3 candy bars.

*C. one boy receives at least 4 candy bars.

D. one boy receives 5 candy bars.

E. none of these.

4. If you wish to be <u>sure</u> that <u>no</u> two students have birthdays in the same month, the greatest number of students you could choose is:

A. 2

B. 12

C. 13

D. 364

*E. none of these.

5. The vowels are a, e, i, o, u, and y. The probability of choosing a vowel from the letters of the word "unable" would be:

*A. one out of two.

B. one out of three.

C. one out of four.

D. one out of six.

E. none of these.

6. The sum of all the whole numbers from 4 to 41 can be quickly found by:

A. multiplying 45 by 41 and dividing the product by 2.

B. multiplying $22\frac{1}{2}$ by 37

*C. multiplying 45 by 38 and dividing the product by 2.

D. multiplying $22\frac{1}{2}$ by 41.

E. none of these.

7. The product of two consecutive numbers such as 6 × 7 will always be even because:

A. it will always have a 2 in its one's place.

B. none of these are true since the statement is not true.

*C. one of the two numbers is even.

D. when multiplying any two numbers the product is even.

E. all of the above are correct.

8. If you pick 7 different numbers from 1 to 10, what is the least number of them which must be even?

A. 0

B. 1

*C. 2

D. 3

E. None of these.

9. What is the least number you must pick out of a group of 20 children in order to be sure that at least two must be of the same sex?

A. 2

*B. 3

C. 11

D. 19

E. None of these.

10. If you multiply a two-digit number by a three-digit number, the greatest possible answer you can get is:

*A. 98,901

18

B. 100,000.

C. 1,000,000

D. 998,901

E. none of these.

11. Here is a string.

Suppose you cut it a few times, only one piece at a time. Each cut changes the total number of pieces. How many peices will you have if you make 17 cuts?

A. 16

B. 17

*C. 18

D. 19

E. None of these.

12. Ann multiplied a number by itself. The answer was one of the following. Which one was it?

*A. 22801

B. 22802

C. 22803

D. 22807

E. 22,808

13. A club of 18 boys had a baseball team (9 players) and a football team (11 players). Five boys were on neither team. How many were on both?

A. 2

*B. 7

C. You can't tell from this information.

D. None of these answers.

PART III. PROBLEMS.

1. Add the odd integers from 1 to 99. (2500)
2. Add 9 + 13 + 17 + 21 + 25 + 29 + 33 + 34. (172)
3. What is the least number of pupils you must have to be sure that at least 4 pupils have their birthday in the same month? (37)
4. If you had a die with eight faces, numbered 1 to 8, what would be the probability of throwing a 1? ($\frac{1}{8}$)
5. If you had a pair of dice, each with 8 faces numbered 1 to 8, what would be the probability of tossing a two? ($\frac{1}{64}$)
6. What is the probability of selecting an eight from a regular deck of playing cards? ($\frac{1}{13}$)
7. What is the probability of drawing a heart from a regular deck of playing cards? ($\frac{1}{4}$)

Date ____________

UNIT: ____

Name of Teacher: ____________________________________

Name of School: _____________________________________

City: ______________________ State: ________________

Number of days given to the teaching
(including testing) of this unit: ____________________

Approximate dates: __________________________________

USE THE BACK OF THIS SHEET IF YOU NEED EXTRA SPACE TO
ANSWER ANY OF THE QUESTIONS BELOW

1. Make a statement about the ability level of the pupils in the class and state whether your school uses some plan of homogeneous grouping.

2. What parts of the unit proved to be the most teachable?

3. What parts of the unit proved to be the most difficult to teach?

   Did you omit any part?

4. Did you use any supplementary developmental materials? ________

   If so, what were they, and at what points were they used?

5. Did you find it necessary to provide the pupils with additional practice material: ____________

   If so, was it from textbooks or did you write your own?

6. Do you think that a unit on this topic should be included in regular textbooks for 7th and 8th grades? ____________

7. Please make ANY additional comments about your teaching experience with this unit which you think would be helpful to the Panel responsible for preparing and experimenting with textbook materials for grades 7 and 8.

Summary of Teachers' Comments

Reports on Unit 1 were received from 75 teachers who taught it in 110 classes. Of these classes 50 were grouped according to ability (35 high, 9 medium, 6 low) and 60 were heterogeneous. Three-fourths of the classes were seventh grade and most of the rest were eighth grade classes. There were a few classes at the sixth, ninth, or tenth grade levels. Most of the teachers agreed that the unit should be included in the seventh grade.

The time spent on the unit varied from 2 to 17 days. Some 70% of the teachers taught it for from 3 to 8 days. It is suggested that probably 5 or 6 days is a reasonable time for most classes.

The teachers regarded the sections as most easily taught or most difficult in the following order of frequency:

Teachability:	(1) Gauss	(2) Deduction	(3) Probability
Most difficult:	(1) Probability	(2) Deduction	(3) Gauss

Note that there were differences of opinion with respect to teachability and difficulty.

A few representative comments made by teachers were:

> The interest of students was very high. Also there was participation in discussion by students who normally are reluctant to contribute.
>
> There was very good reaction to this material as an opening unit. There was evidence that the family at home had been brought into some discussions and their interest was whetted for more.
>
> This type of material should be spaced strategically throughout a text.
>
> I recognized that my teaching of the unit to my second participating class was better.
>
> The interest span of the lower group is longer than it was when they were studying traditional work.

UNIT XI

THE SCIENTIFIC SEESAW

This unit can be used in a mathematics class or a science class or it can be planned as a joint project. It might well follow Unit I, "What Mathematics Is and Why You Need to Know It."

Much of the science that is taught in the schools today is descriptive science, and it omits or neglects the quantitative aspect of the subject. Yet only by a study of quantitative data can one learn to carry out the four steps of a scientific experiment, namely:

1. The scientist observes what happens and collects and studies the quantitative data.
2. To explain the facts he has observed, he states his hunch as a hypothesis which expresses the pattern he sees in the data.
3. He makes predictions, telling what will be seen if certain other observations in the experiment are made.
4. He goes back to the equipment and tests his prediction. If it works in a reasonable number of cases, he can state his discovery as a theory or principle. No amount of tests will "prove" the principle, but a rather large number of tests will show that the principle is very likely to be true. The principle is usually stated with mathematical terms and symbols.

The purposes of this unit on "The Scientific Seesaw" are:

(a) to illustrate by a simple experiment the typical inductive method of science;

(b) to give pupils experience in collecting mathematical data in an experiment in science;

(c) to help pupils see the importance of the quantitative aspect of the physical experiment;

(d) to furnish applications of some mathematical skills.

The equipment is very simple. A meter stick should always be available in a science or a mathematics classroom. If there is no set of weights, a batch of pennies can be used, but newer ones will be more similar in weight than old dirty ones.

The Law of the Lever is one that can be discovered rather easily if measurements are carefully made, and yet it is very fundamental in science and in its use in much equipment that we see around us such as crowbars, tongs, scissors, etc.

It is recommended that each pupil take part in the experiment. The teacher may find it profitable to divide the class into groups of five pupils, so that each group may carry out its own experiment. In the group, one pupil may hold the string by which the bar is suspended, one may adjust the weights and distances, a third may read the scale and the remaining two act as recorders. As the experiment proceeds, the pupils may rotate these responsibilities so that each one has a chance to perform them all, and to experience the complete experiment.

When pupils are doing the experiment and recording measurements in Table I, help them discover two relationships: if the weight is increased, the distance from the fulcrum is decreased; also, if a weight is doubled, the distance from the fulcrum is only half as much, if the weight is made three times as much, the distance is only one-third as much, etc. The products of weight and distance in Table I should all be approximately 120.

Tables II, III, and IV suggest ways of beginning the measurements in order to get a large number of readings and also to help bring out certain relationships. Also, reading the meter stick should increase pupils' familiarity with this kind of measurement.

After recording all the measurements in their tables, pupils should be able to see and formulate in their own words the Law of the Lever, namely that the product of one weight and its distance from the fulcrum is equal to the product of the other weight and its distance from the fulcrum. Briefly, $wd = WD$.

The purpose of Table V is to see whether pupils can use arithmetic and apply the law to specific cases. The last four figures in the table show that if a sufficiently long lever is used, a tremendous weight can be lifted.

Graphing is a skill that needs to be used both in mathematics and science. This experiment gives a set of figures which can be graphed easily and yet they form a curve which these pupils probably have not studied. The teacher may wish to follow up this graphing with several other graphs for practice which would yield similar curves, as $WD = 48$ or $WD = 96$. The hyperbola is such an interesting curve that the better students should be encouraged to investigate it and its uses further.

In all this work, help pupils to become aware of the importance of mathematics in science and of the necessity of mathematical data to carry out experiments.

ANSWERS

P. 2. Table I

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
w	10	10	10	10	10	10	10
d	12	12	12	12	12	12	12
W	10	20	5	8	15	24	12
D	12	6	24	15	8	5	10

P. 3. Table II

w	16	16	16	16	16	16	16
d	6	6	6	6	6	6	6
W	16	24	6	12	4	8	48
D	6	4	16	8	24	12	2

P. 4. Table III

w	20	40	10	15	30	$7\frac{1}{2}$	50
d	15	$7\frac{1}{2}$	30	20	10	40	6
W	20	20	20	20	20	20	20
D	15	15	15	15	15	15	15

P. 4. Table IV

w	18	18	18	18	18	18	18
d	5	5	5	5	5	5	5
W	15	10	30	45	9	6	5
D	6	9	3	2	10	15	18

P. 4. Exercises.

1. (a) They are placed the same distance from the fulcrum.
   (b) The new distance is $\frac{1}{2}$ of the original distance.
   (c) The new D is twice as large as the original D.
   (d) wd = WD.
   The products of the distance times the weight on each side are equal.

2. Table V.

w	25	35	12	45	21	15	23	11	$14\frac{6}{7}$	14	10	100	100	100
d	3	4	5	3	5	6	4	8	7	5.5	50	50	500	5000
W	15	20	7	15	14	20	12	$8\frac{4}{5}$	13	10	100	1000	10000	100000
D	5	7	$8\frac{4}{7}$	9	$7\frac{1}{2}$	4.5	$7\frac{2}{3}$	10	8	7.7	5	5	5	5

3. Suggest checking cases with the smallest numbers.

P. 6. Exercises.

2. (a) As the weight increases the distance decreases.
   (b) As the distance increases the weight decreases.
   Note: The teacher may wish to point out that this is an inverse relationship.

3. (a) $W = 6\frac{2}{3}$ (b) $W = 8$ (c) $W =$ about 13.

4. All are "No."

5.

If	W	6	7	8	9	16	17	21	23	25
Then	D	20	$17\frac{1}{7}$	15	$13\frac{1}{3}$	$7\frac{1}{2}$	$7\frac{1}{17}$	$5\frac{5}{7}$	$5\frac{5}{23}$	$4\frac{4}{5}$

ANSWERS

2. Table V.

w	25	35	12	45	21	15	23	11	$14\frac{6}{7}$	14	10	100	100	100
d	3	4	5	3	5	6	4	8	7	5.5	50	50	500	5000
W	15	20	7	15	14	20	12	$8\frac{4}{5}$	13	10	100	1000	10000	100000
D	5	7	$8\frac{4}{7}$	9	$7\frac{1}{2}$	4.5	$7\frac{2}{3}$	10	8	7.7	5	5	5	5

3. Suggest checking cases with the smallest numbers.

P. 6. Exercises.

2. (a) As the weight increases the distance decreases.
   (b) As the distance increases the weight decreases.
   Note: The teacher may wish to point out that this is an inverse relationship.

3. (a) $w = 4$ (b) $w = 8$ (c) $w =$ about 13.

4. All are "No."

5.

If	W	6	7	8	9	16	17	21	23	25
Then	D	20	$17\frac{1}{7}$	15	$13\frac{1}{3}$	$7\frac{1}{2}$	$7\frac{1}{17}$	$5\frac{5}{7}$	$5\frac{5}{23}$	$4\frac{4}{5}$

Date_______________

UNIT:_______________

Name of Teacher:_______________

Name of School:_______________

City:_______________ State:_______________

Number of days given to the teaching (including testing) of this unit: _______________

Approximate dates:_______________

USE THE BACK OF THIS SHEET IF YOU NEED EXTRA SPACE TO ANSWER ANY OF THE QUESTIONS BELOW

1. Make a statement about the ability level of the pupils in the class and state whether your school uses some plan of homogeneous grouping.

2. What parts of the unit proved to be the most teachable?

3. What parts of the unit proved to be the most difficult to teach?

   Did you omit any part?__________

4. Did you use any supplementary developmental materials:______

   If so, what were they, and at what points were they used:

5. Did you find it necessary to provide the pupils with additional practice material:__________

   If so, was it from textbooks or did you write your own?

6. Do you think that a unit on this topic should be included in regular textbooks for 7th and 8th grades?__________

7. Please make ANY additional comments about your teaching experience with this unit which you think would be helpful to the Panel responsible for preparing and experimenting with textbook materials for grades 7 and 8.

UNIT XI

Summary of Teachers' Comments

The chapter on the scientific seesaw was used by 12 teachers in 18 sections, largely with heterogeneous grouping. Those who taught it liked it and invariably found it well received by the students.

Teachers reported spending from 3 to 10 days on the material. At least 5 days were devoted to the chapter in most classes. In several schools the material was taught in cooperation with the science teacher and in at least one instance the unit was taught in the science classroom.

Supplementary material was seldom used although a few teachers introduced other types of levers and a number suggested this as a desirable option. Graphing was hard for some classes and for a few others the law of the lever caused difficulty. In one or two cases the teachers were dissatisfied with the experimental equipment at their disposal. There was general agreement that the chapter was especially valuable as a tie in with science and as an introduction to applications of mathematics.

"UNCLE SAM AS A STATISTICIAN"

One of the interesting applications of elementary mathematics is in statistics, the fundamental ideas of which often come up in grades 7 and 8 and even earlier. One application of these ideas is in the statistics needed by the federal government to carry out the mandates of the Constitution. This use is very extensive, and many millions of dollars are spent each year to gather statistics for the government. For instance, in 1954 the federal government spent almost $50,000,000 just on collecting statistics. The breakdown was as follows:

Department of Agriculture	$ 5,985,0[illegible]
Department of Commerce (includes Bureau of Census)	21,782,[illegible]
Dept. of Health, Education, Welfare	1,286,[illegible]
Dept. of Labor, Bureau of Labor Statistics	5,345,[illegible]
Dept. of Treasury, including Internal Revenue Service	1,633,000
Federal Trade Commission	234,000
Securities and Exchange Commission	155,000
Other statistics for Federal Civilian agencies	16,000,000

Since most junior high school students study the Constitution, the relating of their mathematics to this study should strengthen both subjects.

The purpose of this unit are three-fold:

(1) To develop the elementary concepts of statistics through a study of some of the data that our government must collect.

(2) To develop an appreciation of the important use of mathematics in the social studies, thereby motivating and strengthening both.

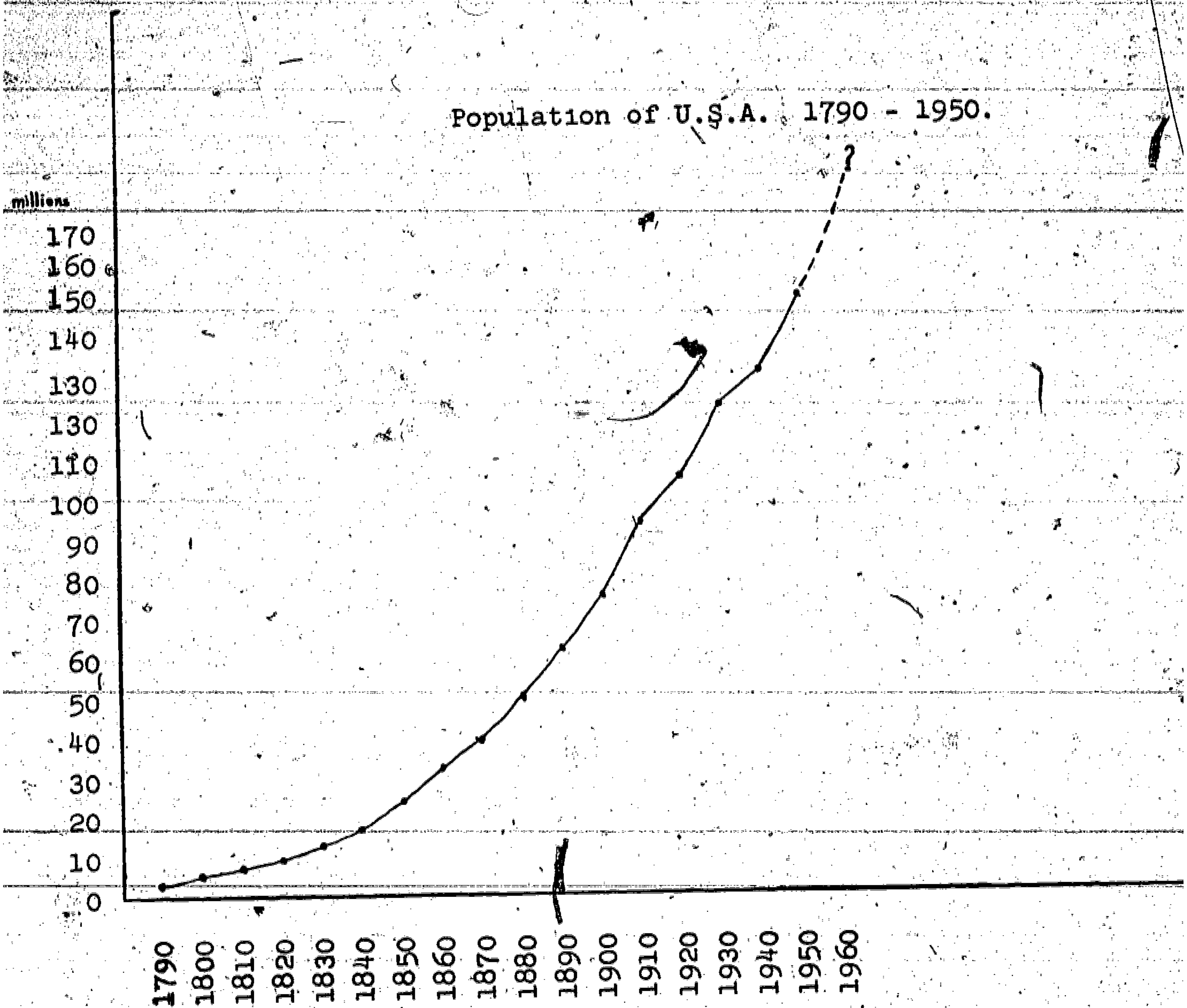
(3) To show how a mathematical interpretation of statistics is very important in making necessary planning and in predicting by the government.

The questions on pages 1-3 of the unit should serve to stimulate pupils to think about the problems and to find some of the answers.

The list of agencies on page 4 whose main purpose is to gather statistics gives some notion about the great extent of the task.

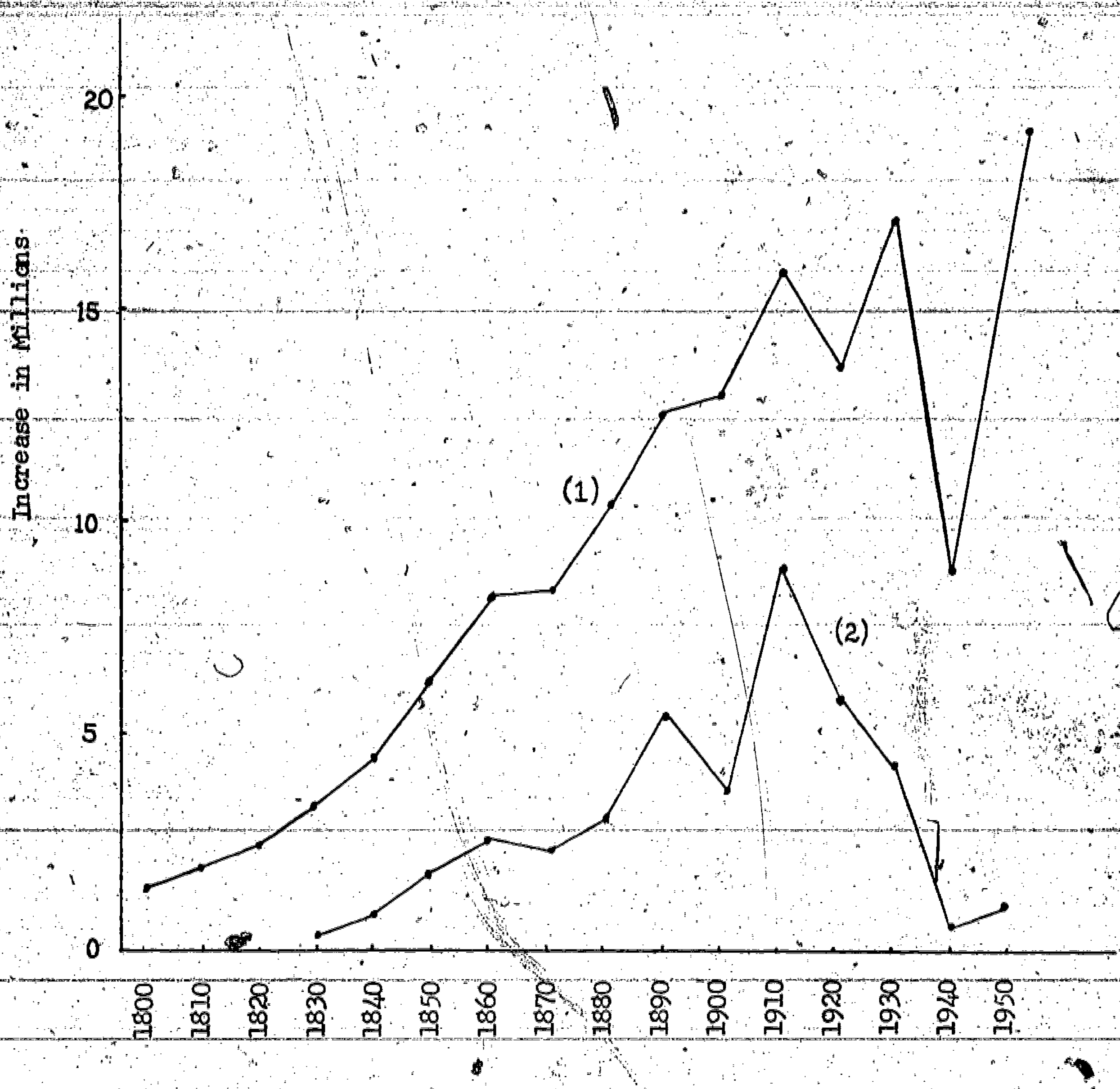
An excellent source book for this unit is the STATISTICAL ABSTRACT OF THE UNITED STATES which has been published annually since 1878. It is the standard summary of statistics on the industrial, social, political, and economic organization of the United States, and is prepared under the direction of the Bureau of the Census.

The following graph is the type that pupils are asked to do on page 6.



The two following line-graphs, asked for on pages 7 and 8, show significant similarities and also reflect certain changes in national policy, such as the "quota laws" of 1923. This pair of graphs help to answer the questions on pages 7 and 8.

Increase in population of U.S.A. by decades 1790 - 1950 (1)
Immigration by decades 1820 - 1950 (2)



Decade ending:

From page 8 on, is found the development of the meanings of:

RANGE - the difference between the lowest and highest numbers in the table

ARITHMETIC MEAN - the sum of the observations divided by the number of them

MEDIAN - the middle number in the set of data

MODE - the number that occurs most often

MEAN DEVIATION - the average of the deviations from the mean

Pupils' work with the tables that are given should bring out the following general concepts:

(a) The range is a simple but important figure, since it shows whether there is a large or small spread in the observations.

(b) The mode is an interesting figure but does not tell anything about the distribution of the data.

(c) The median is rather easy to find, but only tells that half the numbers are below and half are above. A few inordinately large or small numbers have no effect on the median. (See Exercises 6 and 7, pages 20 and 21)

(d) The mean is the number that the layman usually refers to as the "average". It is affected by every number, and so a few very large or very small numbers might change a mean considerably.

(e) Deviation (or difference) from the mean is one way of studying the spread of a set of numbers. For instance, one set of figures may be very concentrated and group themselves closely around the arithmetic mean, while another set may have a very wide spread on one or both sides of the mean. Only the average of the deviations is developed here. The negative numbers should not trouble the pupils, but rather should give another intuitive introduction to signed numbers. The sign is not taken into consideration in averaging the absolute deviations. Further study of deviations, such as finding the so-called standard deviation, is a next step in analyzing sets of data.

Let pupils know that they have had merely a glimpse into this interesting study of statistics which is playing such an important role in life today.

The following list suggests some topics which could be used as special reports or special projects:

1. The number of automobiles manufactured each year for the last 10 years.
2. The number of airplanes produced each year for the last 10 years. (Can you find out how many airplanes there were in the U. S. in 1914?)
3. Urban population compared with farm population in recent years.
4. Amount of air-conditioning.
5. Air travel vs. railroad travel.
6. Farm machinery, such as number of tractors and motor trucks.
7. The television industry.
8. Percentage of population in school each year.

It may be desirable that the teacher have the pupils collect data (such as temperatures) and analyze the data as a classroom project.

ANSWERS

P. 6

1. Population has been increasing.

2. 1940 is out of line.

3. 1810; High immigration rate and beginning of westward movement.

4. 1940; The depression.

5. 1950; A slight depression.

6. 1846-47 was the height of the famine. Migration to U. S. increased.

7. (a) $1.9 + 2.4 + 3.3 + 4.2 + 6.1 = 17.9$ (million

   or $23.2 - 5.3 = 17.9$

   (b) $8.2 + 8.4 + 10.4 + 12.7 + 13.1 = 52.8$

   or $76.0 - 23.2 = 52.8$

   (c) $16.0 + 13.7 + 17.1 + 8.9 + 19.0 = 74.7$

   or $150.7 - 76 = 74.7$

8. (a) $\frac{17.9}{5.3}$ or 340% (nearest 10%)

   (b) $\frac{52.8}{23.2}$ or 230% (nearest 10%)

   (c) $\frac{74.7}{76.0}$ or 98% (Nearest 1%)

9. (a) Increase in population.

   (b) According to 8(a) 1800-50 contained the greatest rate of increase.

   (c) Over 200,000,000.

   (d) Population figures are used to solve problems in:

   1. Taxation.
   2. Budgets.
   3. Social welfare.
   4. School enrollment.
   5. Transportation.
   6. Food supplies.

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P. 7

1. Immigration has decreased.

2. Immigration in the 1931-1940 period. These were the depression years.

3. General trends are the same.

6. 1830-1860 1900-1910

   1870-1890 1940-1950

7. (a) Increased. Western movement and expansion of industry.

   (b) Decreases. Panic years and war of 1898.

   (c) Both increase and decrease. Economic expansion and World War I.

   (d) Decrease. Overpopulated and depression.

8. Seven, five.

9. Decreased, but not as much as the 10 years before and after.

10. There has been less and less immigration. It is extremely difficult to predict the immigration during 1950-1960 since the situation in Hungary. The graph, however, indicates that if the trend continues the number may be as low as 100,000.

P. 10

1. 86.6

2. 10,188

3. 2,318

4. 60°

5. The depression was at its highest point during 1932-1934.

6. World War II reduced unemployment.

7. (a) Auto sales drop during a depression.

   (b) Unemployment compensation paid increases.

   (c) Food and clothing sales drop.

P. 12

1. The median score is 87.

   The mean score is 84.

2. See graph.

3. The median is 62°.

P. 15

1. See graph.

2. The highest bar is the mode.

3. The median by a rapid examination is about 4000. There are about as many below this figure as there are above it.

4. The mode is 85.

5. The median is 85.

6. The median is about 72.

7. See graph.

8. See graph.

9. The rate of patents issued decreased from 1935-1950, and then increased during the 1951-55 period.

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P. 17

1. Deviation from mean was greatest in 1953 and 1954. Reason: These were years of an upturn in business.
2. Deviation from mean was least in 1951.
3. Average deviation = 12.2
4. Mean = 83.4

   Average deviation = 2.7 (to nearest 10th.)
5. Mean = 84

   Average deviation = 9.6
6. (a) Mean of Exercise 5 is .6 greater.

   (b) Average deviation of Exercise 5 is 6.9 greater.

   (c) Average deviation indicates spread (variability) (dispersion) of the test grades.

P. 19

1. (a) Mode = 1

   (b) No.
2. (a) Median = 2

   (b) No.
3. Mean = 49
4. Deviations from mean are -46, -47, -47, -47, -47, -48, -48, -48, -48, -48, -48, -25, -28, -33, -20, -6, +210, +208, and +225 respectively.

   Average deviation = 67.
6. (a) Mean = $6,000

   (b) Three are above.

   (c) Eight are below.

(d) No.

(e) Median is 5,000

(f) Yes.

7. (a) Mean = 57

(b) Six are above

(c) Seven are below

(d) Yes

(e) Median is 48

(f) No

NUMBER OF FAMILIES AS A FUNCTION OF INCOME

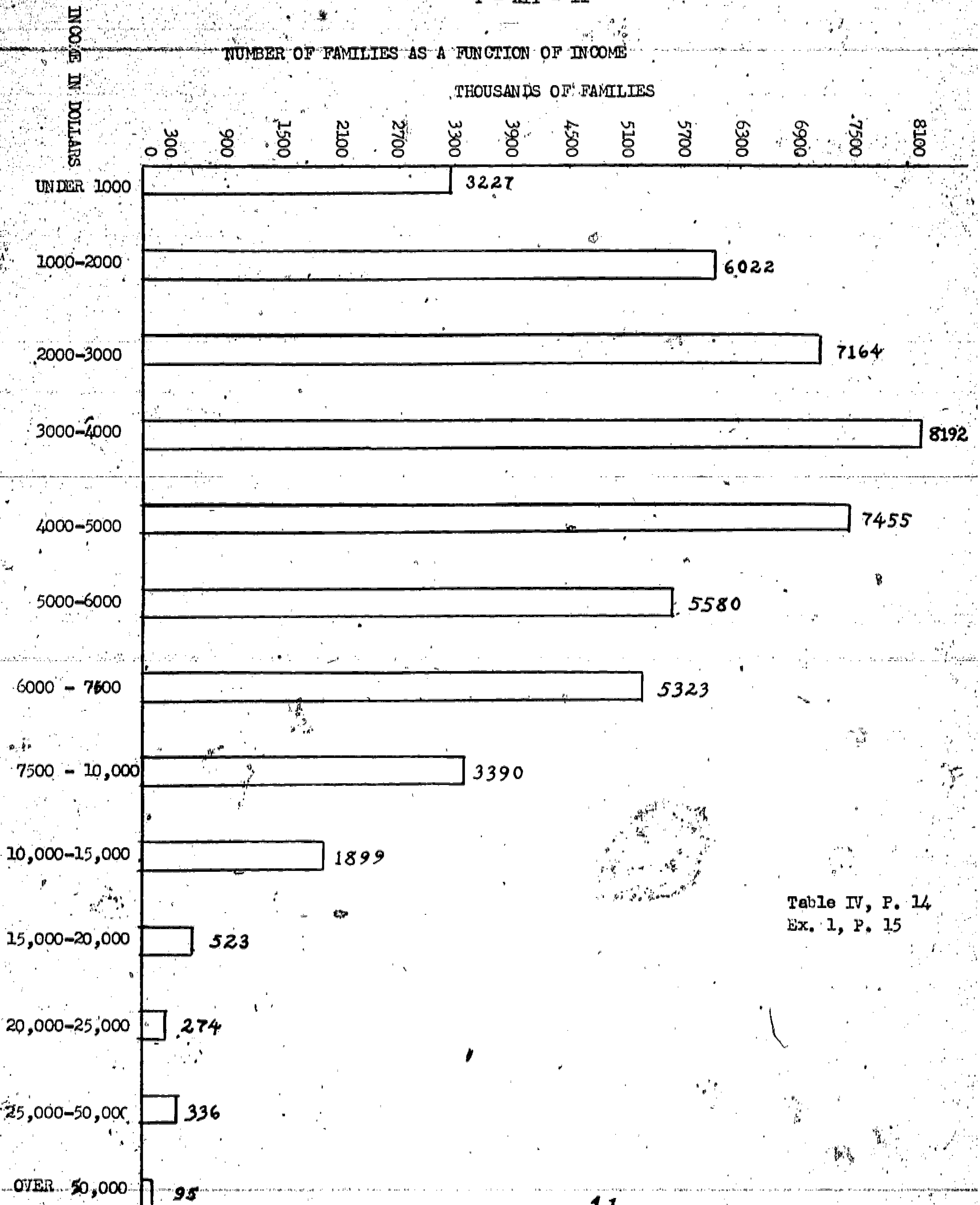
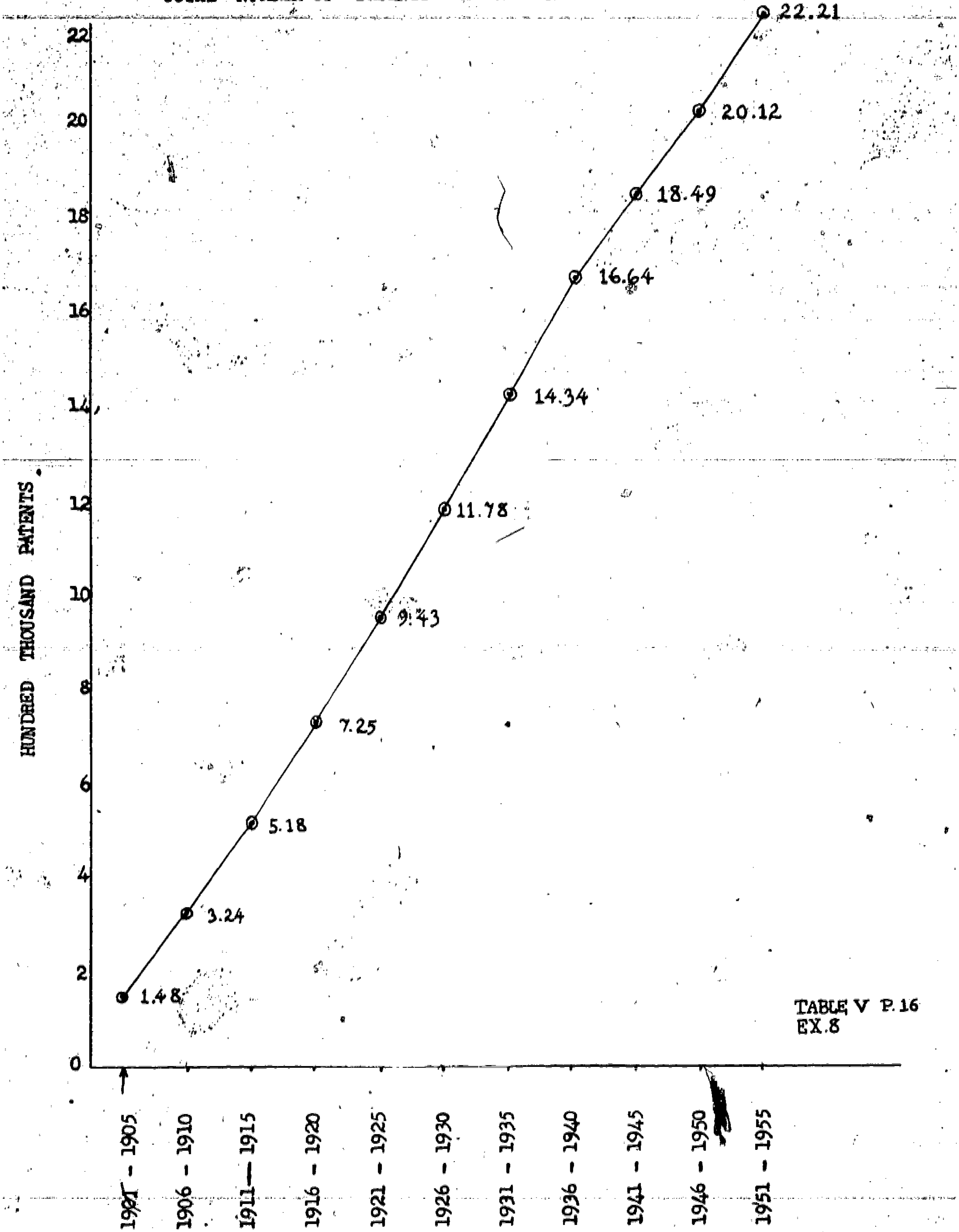


Table IV, P. 14
Ex. 1, P. 15

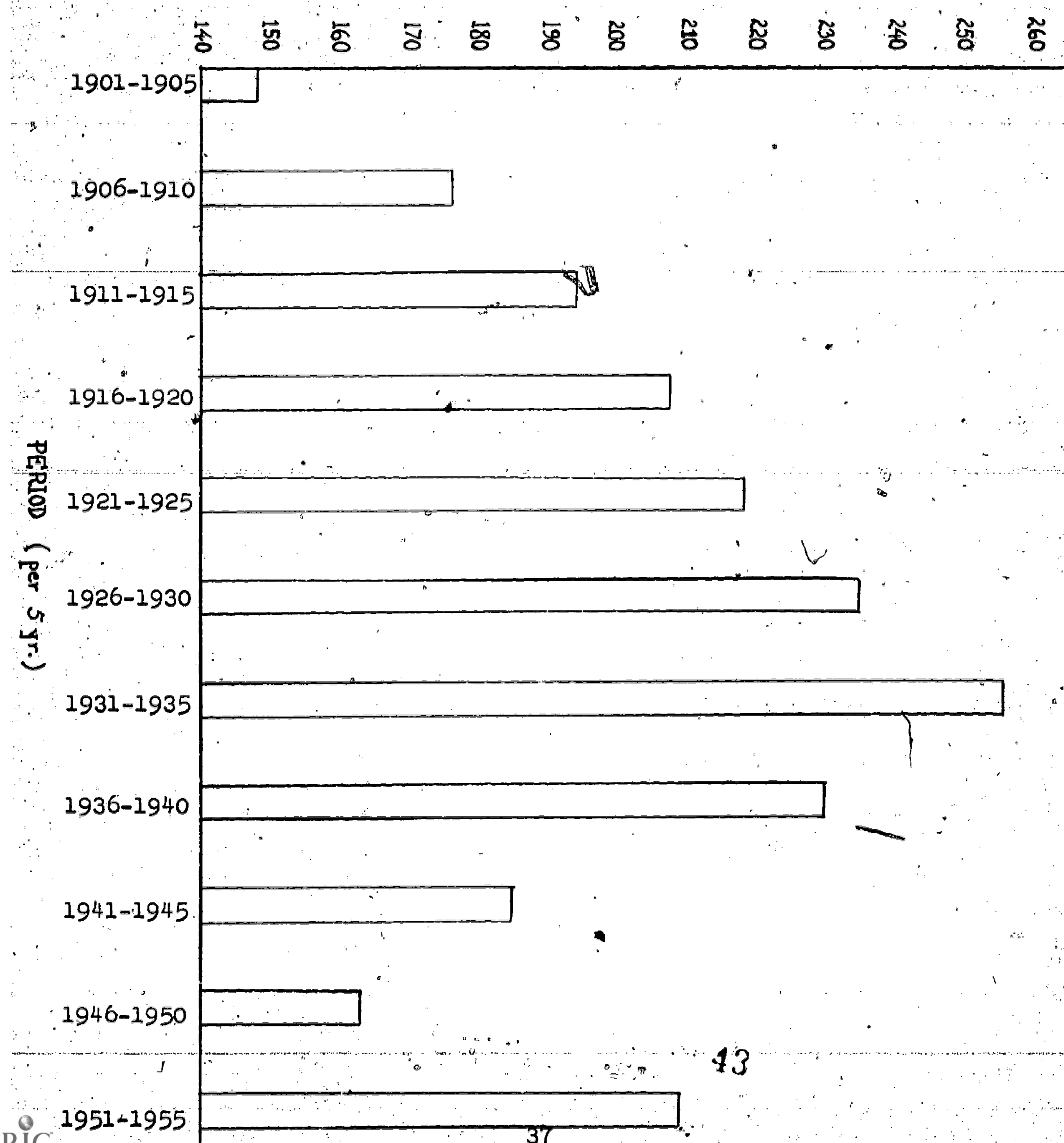
TOTAL NUMBER OF PATENTS AS A FUNCTION OF PERIOD.



NUMBER OF PATENTS ISSUED AS A FUNCTION OF PERIOD

PATENTS ISSUED (THOUSANDS)

Table V, P. 16
Ex. 7.



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Sample Test Questions

PART I. MATCHING.

Match the following by placing the letter corresponding to the best description before each of the numbered words or phrases.

Problem 1.

(G) 1. Arithmetic mean	A. Average of the differences from the mean
(B) 2. Deviation	B. Difference of each value from the mean
(D) 3. Graph	C. Difference of each value from the median
(A) 4. Mean Deviation	D. A picture of statistical data
(E) 5. Median	E. Middle value in a set of data
	F. Number of observations cited
	G. Sum of the observations divided by the number of them

Problem 2.

(H) 1. Mode	A. Collection and arrangement of facts, in a condensed form, for ready reference
(B). 2. Range	B. Difference between the highest and lowest values
(E) 3. Statistical inference	C. General direction or prevailing tendency
(A) 4. Table	D. Largest value that occurs
(C) 5. Trend	E. Method of drawing conclusions or making decisions on the basis of samples of data

F. Number of times a value occurs

G. Reasoning from the general to the particular

H. Value occurring most often

PART II. MULTIPLE CHOICE.

1. The median and the mean of a set of data will be:

   *A. sometimes the same.

   B. always the same.

   C. never the same.

   D. none of the above.

2. In the table to the right, the median age is nearest to:

   A. five.

   B. four.

   *C. ten.

   D. fourteen.

Age	Number in age group
0 - 4	2
5 - 9	5
10 - 14	12

3. From the table on the right, the mean of the data can not be computed because:

   A. no mean exists.

   B. values are large.

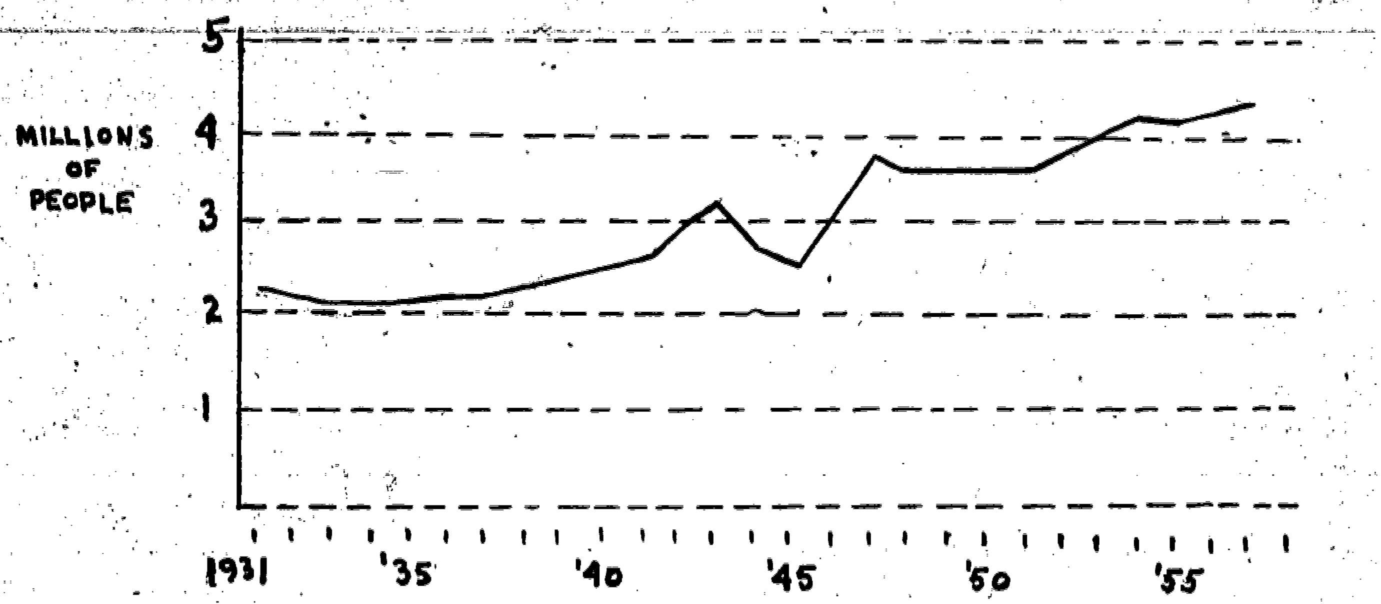
   C. grouping is wide.

   *D. data are open-ended.

Age	Number in age group
0 - 10	53
11 - 20	62
Over 20	73

The next four questions refer to the following graph.

BIRTHS IN THE U.S.



4. Which of the following years is out of line with the general trend?

   A. 1940

   B. 1952

   *C. 1945

   D. 1956

5. In which period did the number of births increase the most?

   A. 1935-40

   *B. 1945-50

   C. 1955 on

   D. before 1935

6. The number of births in 1957 was 4.3 million. The number "4.3" has been rounded off to what place value?

   A. ten-thousands

   B. ten-millions

   C. millions

   *D. hundred-thousands

7. A prediction of the number of births in 1960 would be:

*A. 4.6.

B. 5.4.

C. 4.0.

D. 3.9.

The next two questions are based on this data:

14, 15, 18, 28, 29, 30, 31, 31, 40

8. If the 15 were changed to 19, the median would:

A. increase.

B. decrease.

*C. remain the same.

D. disappear.

9. If one more item, 43, were added, the median would:

*A. increase.

B. decrease.

C. remain the same.

D. disappear.

The next two questions are based on this set of items:

85, 85, 80, 80, 80, 77, 73, 72, 70

The mean of this set is 78, the median is 80, the mode 80.

10. The deviation from the mean of the score "85" is:

A. 3

*B. 7

C. 0

D. 6

11. The average deviation from the mean is:

A. 4.44.

B. 23

*C. 5.86.

D. 40.

A factory owner checks the time of arrival of each worker, labeling the number of minutes early by a numeral with a minus sign. Here are the data: -12, 3, -10, -1, 15, 4, -3, 0, -2, -3, -15, 12, -10, -5, 7

He made a table, the first two groups of which are given here:

Time	Number of Workers
-19 to -16	0
-15 to -12	2

12. The next group should be:

A. -11 to -9.

*B. -11 to -8.

C. -12 to -9.

D. -10 to -7.

13. How many workers would fall into the group "1 to 4"?

A. 5

B. 4

*C. 2

D. 3

16. Mark the statements which are always true with a plus sign, and those which are not always true with a minus sign.

(-) 1. The average deviation is the same as the arithmetic mean.

(+) 2. The range is the difference between the smallest value and the largest value in a given set of data.

(+) 3. Extreme values have no effect on the median.

(+) 4. The arithmetic average and arithmetic mean both have the same meaning.

(+) 5. If seventy-nine test scores are arranged in order of sign the 40th score will be the median.

(+) 6. The value which occurs most frequently in a given set of data is called the mode.

Statistia

Statistia is a country with a population of nine people. Although it is small, it likes to do things bigger countries do; it has pride. When it takes its ten-year census, it puts the information in the form of a table. Here is some information about the population, and a table based on it.

The youngest citizen is a girl two years old.
There are three twelve-year-old boys.
There is one eighteen year old girl.
There is one twenty-seven year old woman.
There are two men whose ages are thirty-one and thirty-two.
The oldest citizen is ninety-three.

Population Table.

0 - 9	1	50 - 59	0
10 - 19	4	60 - 69	0
20 - 29	1	70 - 79	0
30 - 39	2	80 - 89	0
40 - 49	0	90 - 99	1

The old man likes to think that the population of Statistia is middle-aged; he quotes the average age to show that this is so. The boys say that the population is very young; they give the mode to prove their point. The twenty-seven year old woman says that she herself is near the median age. Answer the following questions, giving figures to prove your point.

1. Is the old man right? Why? (Yes, from his point of view, since the mean age is 26.5 years.)

2. Is the woman right? Why? (No, since the median age is 18 years.)

3. Are the boys right? Why? (Yes, but only in the sense that the mode is 12 years.)

Some of the citizens assert that the mean of the population does not really represent the age-group of the population correctly. They quote the mean deviation to prove this. Give your opinion, quoting numerical data to support your opinion.

4. (Yes, for the mean deviation is 17.0 years.)

One twelve year old boy suddenly discovers that he is actually only three, since he was born on February 29th and has had only three birthdays. However, he says that even if this is true, it doesn't make any difference in the mode. One of the others says that the median will change, while another says that the mean is constant. Tell who is right and why.

(The mode remains unchanged and so also does the median. However, the mean will be reduced.)

PART III. COMPLETION.

The scores for the baseball team were: 4, 7, 11, 5, 7, 8, 13, 9, 7, 7, 10, Complete the range, mean, median, mode, and average deviation.

1. Range = (9)

2. Mean = (8)

3. Median = (7)

4. Mode = (7)

5. Average deviation = (2)

Date ______________

UNIT: ____

Name of Teacher: ________________________________

Name of School: ________________________________

City ____________________ State: ______________

Number of days given to the teaching (including testing) of this unit: ______________

Approximate dates: ________________________________

USE THE BACK OF THIS SHEET IF YOU NEED EXTRA SPACE TO ANSWER ANY OF THE QUESTIONS BELOW

1. Make a statement about the ability level of the pupils in the class and state whether your school uses some plan of homogeneous grouping.

2. What parts of the unit proved to be the most teachable?

3. What parts of the unit proved to be the most difficult to teach?

   Did you omit any part? _______

4. Did you use any supplementary developmental materials: ________

   If so, what were they, and at what points were they used:

5. Did you find it necessary to provide the pupils with additional practice material: ___________

   If so, was it from textbooks or did you write your own?

6. Do you think that a unit on this topic should be included in regular textbooks for 7th and 8th grades? _____________

7. Please make ANY additional comments about your teaching experience with this unit which you think would be helpful to the Panel responsible for preparing and experimenting with textbook materials for grades 7 and 8.

Summary of Teachers' Comments

Unit XII, "Uncle Sam as a Statistician" was taught to nine classes by nine different teachers. Six of the classes contained students of various abilities and in three of the classes the students were above average in ability. The number of days spent on teaching the unit ranged from 6 days to 17 days. The average was about 12 $\frac{1}{2}$ days.

In general the teachers found all the unit easy to teach. Some teachers indicated that the materials on central tendency and graphs were especially liked by the students. It was reported that some students needed assistance in finding trends from data and the average of deviations. About half of the teachers found it helpful to provide additional supplementary developmental and practice material. Some teachers planned the unit with the social studies teacher and highly recommended such a cooperative arrangement.

With minor suggestions, the teachers who taught the unit recommended its inclusion in the mathematics program for either the seventh grade or eighth grade.

CHANCE

The purpose of this unit is to introduce a few of the most elemental properties of probability. It was decided that the word "chance" would be used in place of "probability". If you prefer the term "probability," by all means use it.

The unit attempts to illustrate that the estimation of a chance or probability statement is often a matter of counting. Thus, there are exercises in which the pupil must count events. It is also illustrated that in order to count, one must be able to classify events or identify classes. We chose to use the classification of "True" and "False." Through the unit it would be desirable to stress this important condition for the counting process. Other examples that a teacher could use to illustrate a dichotomy of classes are: black and white, on and off, right and left, win and lose, and so on.

The term "chance statement" is used instead of "statement" to apply to a statement involving an element of chance, like those on the first page and in exercises throughout the unit. When we associate a number with a chance statement we call this number the measure of the chance that the statement is true. We sometimes shorten this expression to merely "the measure of the chance statement." While this is a use of the term "measure" somewhat different from the way we use this term in measuring lengths or in measuring time it does, in common with all measurements, associate a number (an estimation in a sense) with an object or situation or quality in the physical world, - in this case with a chance statement. This is useful in many ways.

This measure is especially useful in making decisions based on chance. As mathematics is applied, in the social sciences even more than in the physical sciences, it becomes necessary to agree upon a basis or scale of measurement which can be expressed by number, but which is neither determined directly nor exactly.

Exercises pp. 3-7

The first set of exercises is designed to introduce the concepts of classifying, counting, ratio, estimation, and the meaning and measure of chance.

We must keep in mind that a probability statement is based on a model or upon looking at past experience, but is used to look ahead and as the basis of statements about future events. The looking ahead is, therefore, concerned with a great many events like the ones investigated in the problem. The best we can do with the information we have in any problem is to set up some procedure for estimating the probability. We would recommend that this point be discussed many times with the pupils. It is proper to say "The chance that A will occur is 7/10." This means that if the event would occur many times, say 1,000,000, the number of times (in which A occurs) would be very close to 700,000 or as a decimal the chance is about .70 and more accurately (by mathematics beyond this unit) it would be almost certain that the chance is between .69 and .71. A statement like this was not included in the text and is only recommended for discussion with the pupils.

Questions may be raised, and should be, about the two methods of finding a measure of a chance statement discussed on pages 7 and 8. In the example on page 1 about the weather, chance is estimated or measured by experimental evidence or empirical evidence. Generally we associate this procedure with scientific method; at least, this is the scientific method we first study. It is a matter of drawing conclusions from a set of observations. Some people call this the "inductive process." Here we are using this method to estimate the truth value of the chance statement.

In contrast to this method we may also construct a theoretical model of our problem. As mentioned in the text, tossing coins provides an easy and convenient model to talk about. The possibilities are in general easy to list and each possibility is easy to classify as T and F. Once again when we say that the chance that exactly 2 heads appear when 3 coins are tossed is 3/8 we mean that over many trials the measure of chance would approximate (or get close to) 3/8. Chance looks ahead to many cases.

Exercises pp. 10-11

Encourage the pupil actually to set up a model by listing all possibilities and then to write the truth values of each possibility.

You may wish to illustrate a pattern for listing heads and tails.

Pattern for 2 coins

(1)	(2)
H	H
H	T
T	H
T	T

The first column is grouped in twos, the second alternately H and T. There are 2^2 possibilities. The two classes, heads and tails, determine the base, 2. The exponent 2 is the number of coins.

Extended to 3 coins we would expect 2^3 possibilities. This checks with our list. Arrange them as follows:

Pattern for 3 coins

(1)	(2)	(3)
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

The first column is in groups of four, the second column is in groups of two, and the third column alternately H and T.

For four coins we would expect 2^4 possibilities. Arrange the first row in groups of 8, the second in groups of 4, the third in groups of 2, and the last alternately H and T.

The schemes above are certainly not necessary but they are convenient. Truth tables of symbolic logic make use of a similar arrangement just for convenience.

You may wish to ask questions like: How many possibilities would we have if we tossed 5 coins and how could they be arranged conveniently in a table?

Decisions and Chance

This particular section provides review exercises in another setting. We are introducing a use of probability which is extremely important to science, industry, and business. You may emphasize that we use chance in this sense when it is the best way to help us make a choice. We make use of information and models to arrive at some measure by which we can select one or the other of two choices. More than two choices could of course also be involved.

The question will probably come up about the size of the difference between the measures of chance and whether this is a significant difference. This issue should not be entirely dodged but mathematical analysis at this time would be beyond the purpose and scope of this unit. However, we should not glide over this question without an intuitive discussion of it.

Experiments 7 and 8 would provide the best background for the discussion about the significance of the differences between the measures of the chance statements.

You may point out that mathematically this difference can be established according to the "chance" one wishes to take. If it is an important decision (say that there is a great deal of money involved) one would demand a greater difference between the two measures than if there were little at stake. In the case of our experiment we would settle for a small difference, say .05. The difference should be agreed upon before the counting is done. The final decision then rests on whether or not the difference was greater than .05.

Certainly if the measures are about equal we would be satisfied that neither garden was better than the other.

The problem of "testing hypothesis" and discussion of "confidence intervals" may be found in any mathematical statistics book.

These ideas of probability are discussed in an elementary fashion in the following books:

John G. Kemeny, J. Laurie Snell, Gerald L. Thompson, <u>Introduction to Finite Mathematics</u>, Englewood Cliffs, N.J., Prentice-Hall, Inc., 1957.

<u>Probability and Statistical Inference</u>, Commission on Mathematics, College Entrance Examination Board, 425 West 117th Street, New York 27, New York, 1957.

Throughout the unit you may wish to discuss why the exercises stipulate such things as "without a plan," "you are not to look inside the box," "plant the seeds the same depth," and "add the same amount of water to each box." Chance becomes meaningless if influence is exerted over the problem. These are examples of "bias." Point out that minimizing bias (it can never be completely eliminated in an experimental design) is one of the most difficult problems in estimating a measure of chance which proves to be worthwhile information.

Experiment 6 is called a "π board." The average found in the experiment should be a close approximation to π.

You can expect answers of about 3.1 to 3.3. Notice that we did not ask the pupil to write a chance statement or to find the measure of a chance statement in this experiment. You may wish to do this.

ANSWERS

Exercise pp. 3-7

1) a) We must first assume we know who "I" refers to. If we assume this, then we have a declarative sentence which can be determined to be either true or false.

   b) (1) T (2) F (3) F (4) T (5) F (6) F (7) F (8) T (9) T (10) F (11) F (12) T (13) F (14) F (15) T (16) F

   c) 3/8

2) 3/8

3) C = 3/10 (1) T (2) T (3) F (4) F (5) F (6) F (7) T (8) F (9) F (10) F

4)

Draws	Truth Value
B_1B_2	F
B_1W	T
B_2W	T

C = 2/3

5) C = 1/3

6) C = 2/3

7) C = 1/2

8) C = 1/6

Exercise pp. 10-11

1) C = 1/4

2) C = 3/4

3) C = 1/2

4) C = 3/8

5) C = 1/8

6) C = 7/8

7) a) HH 1/4; HT 1/4; TH 1/4; TT 1/4

   b) 1

8) a) HHH 1/8; HHT 1/8; HTH 1/8; HTT 1/8; THH 1/8; THT 1/8; TTH 1/8; TTT 1/8

   b) 1

9) 1/4; 1/4; 3/8; 1/16

10) C = 2/3

11) 2/3 (If you include the possibility of standing in between them)

1/2 (If this is not one of the possibilities)

Exercise pp. 12-13

1) Statement A - 3/8 Statement B - 1/2 Statement B is more likely.

2) Statement A - 1/2 Statement B - 1/2 Both are equally likely.

3) Statement A - 1/4 Statement B - 3/8 Statement B is more likely.

4) Sunshine seems more likely - 11/20 to 7/20 in favor of sunshine.

Experiments pp. 13-16

1) a) 1/2 b) 25 c) ----

2) -----

3) -----

4) Toss three coins for, say 100 trials and record the number of trials in which exactly two heads turn up. (problem 5 would be similar - use ten coins instead)

5) -----

6) -----

7) -----

8) -----

9) -----

UNIT XIII

Sample Test Questions

PART I. TRUE - FALSE

T 1. If the measure of our chance of winning the basketball game next Friday is $\frac{2}{5}$, we would forecast a loss.

F 2. The number that measures the chance that a statement is true is the ratio of true statements to false statements.

F 3. If a box contains 2 red marbles and 3 green marbles, the liklihood of drawing a red marble is the same as drawing a green marble.

F 4. If a penny is tossed 100 times, we will always get exactly 50 heads.

T 5. If two pennies are tossed 100 times, we expect to get 2 heads about 25 times.

T 6. The chance that your friend has the same birthday as yours is about $\frac{1}{365}$.

F 7. Truth value may be measured with a ratio such as $\frac{2}{3}$.

F 8. "Truth value" and "measure of chance" mean the same thing.

T 9. A forecast of rain with a probability (measure of chance) of 0.7 means that it is more likely to rain than not to rain.

T 10. Decision making can be based on a knowledge of chance.

T 11. A forecast of rain is a chance statement.

PART II. MULTIPLE CHOICE.

1. Suppose you toss a coin five times and each time it comes up heads. The chance of tossing a head on the sixth toss is:

A. $\frac{5}{6}$

64

B. $\frac{1}{6}$

C. $\frac{1}{5}$

*D. $\frac{1}{2}$

E. none of the above.

2. If the weatherman forecasts the weather correctly 20 times in one month and incorrectly 10 times, the measure of the chance that his forecast was correct is

A. $\frac{1}{2}$

B. $\frac{2}{1}$

*C. $\frac{2}{3}$

D. $\frac{3}{2}$

E. none of the above is correct.

3. A drawer contains 10 blue stockings and 10 red stockings. What is the smallest number of stockings you need to take to be sure you have a pair of the same color stockings?

A. 2

*B. 3

C. 5

D. 10

E. 11

4. There are two roads from Milltown to Luck and three roads from Luck to Turtle Lake. How many different ways can John travel from Milltown to Turtle Lake?

*A. 6

B. 5

C. 3

D. 2

E. None of these.

PART III. MEASURES OF CHANCE.

FIND THE MEASURE OF THE CHANCE THAT THESE STATEMENTS ARE TRUE.

($\frac{1}{4}$) 1. If two pennies are tossed, exactly two tails show.

($\frac{3}{4}$) 2. If two pennies are tossed, at least one tail shows.

($\frac{1}{8}$) 3. If three pennies are tossed, exactly three tails show.

($\frac{1}{2}$) 4. If three pennies are tossed, at least two heads show.

FOR EACH OF THE FOLLOWING EXERCISES, CHOOSE THE EVENT MOST LIKELY TO HAPPEN BY COMPARING THEIR MEASURES OF CHANCE. SHOW THE MEASURE OF CHANCE FOR EACH STATEMENT.

($\frac{1}{2}$) 5. A. If two coins are tossed, exactly 1 head shows.

($\frac{3}{4}$) B. If two coins are tossed, at least 1 tail shows.

($\frac{1}{2}$) 6. A. If three coins are tossed, at least two tails show.

($\frac{7}{8}$) B. If three coins are tossed, at least 1 head shows.

($\frac{2}{3}$) 7. A. If two cards out of three numbered 2, 3, 4 are chosen without looking, the sum will be odd.

($\frac{1}{3}$) B. If two cards out of three numbered 3, 4, 5 are chosen without looking, the sum will be even.

PART IV. COMPUTATION.

COMPUTE THE MEASURES OF CHANCE FOR THESE EVENTS.

($\frac{1}{3}$) 1. A box contains a red marble, green marble and yellow marble. If you draw two marbles without looking, what are the chances that you draw the red and green marble in any order.

($\frac{2}{3}$) 2. Three jackets are in a dark closet. One jacket belongs to you, one to Bill, and one to Bob. If you reach into the closet and take out two jackets, what is the chance that you get your own jacket?

3. A box contains the letters A, B, and C. For each of the following draws indicate the truth value of the chance statement that <u>one of the letters is B if any two letters are drawn.</u>

DRAWS	TRUTH VALUE
A B	(T)
B A	(T)
A C	(F)
C A	(F)
B C	(T)
C B	(T)

($\frac{1}{6}$) 4. What is the chance of drawing A B in that order from the box in the example above?

($\frac{1}{2}$) 5. Two letters are to be selected at the same time from these letters, A, B, M, T, in a box. If the letters are picked without looking, what is the chance that B will be one of the letters picked? Make a truth table for the chance statement that one of the letters picked is B.

A B	T
A M	F
A T	F
B M	T
B T	T
M T	F

($\frac{1}{6}$) 6. ~~In problem 27 above, what is the chance that A and M~~ will be picked?

($\frac{1}{6}$) 7. A die has six faces with 1, 2, 3, 4, 5, or 6 dots on each face. What is the measure of chance that a 6 will turn up when the die is tossed?

($\frac{1}{6}$) 8. What is the measure of chance of tossing a total of 7 with two dice?

($\frac{1}{12}$) 9. A die has 12 faces with 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12 dots on each face. What is the measure of chance that a 7 will turn up when the die is tossed?

PART V. DEFINITION AND ESSAY.

1. In the formula $C = \frac{T}{S}$, what is the meaning of each letter?

   C = (Measure of chance)

   T = (Number of times a statement is true)

   S = (Sum of the number of times the statement is true or false.)

2. Describe an experiment in which you compare the measure of chance with the actual events in the experiment.

Date ______________

UNIT: ____

Name of Teacher: ______________________________________

Name of School: _______________________________________

City: ______________________ State: ____________

Number of days given to the teaching
(including testing) of this unit: ______________________

Approximate dates: ___________________________________

USE THE BACK OF THIS SHEET IF YOU NEED EXTRA SPACE TO
ANSWER ANY OF THE QUESTIONS BELOW

1. Make a statement about the ability level of the pupils in the class and state whether your school uses some plan of homogeneous grouping.

2. What parts of the unit proved to be the most teachable?

3. What parts of the unit proved to be the most difficult to teach:

   Did you omit any part? ______

4. Did you use any supplementary developmental materials: ______

   If so, what were they, and at what points were they used:

5. Did you find it necessary to provide the pupils with additional practice material: ______

   If so, was it from textbooks or did you write your own?

6. Do you think that a unit on this topic should be included in regular textbooks for 7th and 8th grades?

7. Please make ANY additional comments about your teaching experience with this unit which you think would be helpful to the Panel responsible for preparing and experimenting with textbook materials for grades 7 and 8.

Summary of Teachers' Comments

Most of the nine teachers who reported using the unit spent 6 to 10 days on it, with 7 days being the most representative. One teacher spent as much as 20 days on the unit. The classes were largely of the heterogeneous type. Only two teachers had high ability groups.

All teachers seemed to feel that the entire unit was very teachable. In one case, where the material was used with a high ability group, the unit was said to be too easy. It was the feeling of most teachers that no part of the unit was too difficult for the seventh or eighth grade level. Only in one case was vocabulary mentioned as a difficulty. If there were any omissions it was only because of a lack of time rather than for any other reason. In such a case the class omitted an experiment or two.

Six of the nine teachers used supplementary practice material. Materials used were:

- World of Science - The Golden Library
- Introduction to Finite Mathematics - Kemeny, Snell, Thompson Prentice Hall, 1957.
- Original materials by teachers
- Newspaper data
- Baseball data
- Models for chance written by pupils for others to solve

General comments written by teachers centered around the fact that the unit was interesting to the pupils, fairly easy, and in some cases even too easy. There were some reactions from the pupils regarding the sameness of the exercises which at times made the work monotonous. Some wanted more application to everyday life and others to science.

Some pupils indicated they had to think more than when they used the textbook and preferred the material to that of the textbook. All the teachers agreed that the unit was suitable for seventh and eighth grade levels. One teacher commented that the material would be more suitable if it were broadened.