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ABSTRACT

In order to develop a model mathematics preparation program for elementary teachers, faculty were drawn together from the Departments of Mathematics and Mathematics Education at The Ohio State University and representatives from the Columbus Public Schools. This two-year project involved 95 students, primarily freshman and sophomore women, who had demonstrated competency in high school mathematics. Following the completion of the program, several recommendations were presented. The project staff and consultants generally felt: (1) that study in instruction and math content should be coordinated; (2) that programs in preparing teachers to teach math should be upgraded by consulting mathematics faculties, education faculties, and public school personnel; (3) that school experience should be included in the coordinated content-methods instruction; and (4) programs in teacher training in mathematics should contain at least two mathematics courses. (SA)

A COMPARISON OF ALTERNATIVES AND AN IMPLEMENTATION OF A PROGRAM IN THE MATHEMATICS PREPARATION OF ELEMENTARY SCHOOL TEACHERS

(FINAL REPORT)

By

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of

The Ohio State University

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Abstract

Though virtually every child in this country is exposed to mathematics in the elementary schools, there has been little documentation comparing alternatives in the mathematics training of their teachers. The purpose of this project has been to unite faculty members in mathematics with faculty members in education and with school personnel in comparing several alternatives in order to formulate a model program in the mathematics preparation of elementary school teachers.

First Year

During the first year three approaches were identified and implemented over a ten week quarter: Pilot A, an integrated content-methods-field experience program using Indiana University Mathematics Methods Program materials; Pilot B, a coordinated content-methods-field experience program using traditional text materials; and Pilot C, a traditional program with separate instruction in content and methods and with a probability laboratory in lieu of field experience. Each group met ten hours per week. The time intended for content instruction ranged from approximately 50% in Pilot A to 75% in Pilot C and time devoted to field experience ranged from 0% in Pilot C to about 25% in Pilot A.

The subjects were primarily freshman and sophomore females who intended to teach at the elementary level and who had demonstrated competency in high school mathematics. Ninety-five students were randomly assigned to the three treatments. Twelve students either

failed to appear or dropped and another twelve with missing data were eliminated from the analysis. Of the remaining 71 subjects, 25 were in Pilot A, 27 were in Pilot B, and 19 were in Pilot C. The resulting groups were found to be equivalent on all background and pretest measures except for two attitude scales. The posttest attitude measures were adjusted to account for the initial differences on the two attitude scales.

Principal criteria under investigation were knowledge of mathematical concepts, knowledge of methods of teaching mathematics, problem solving ability, and attitudes toward mathematics, toward teaching mathematics, toward teaching elementary school children, and toward elementary school children. Data was collected via test instruments developed by the project staff and via interviewers, classroom observations, instructor logs, and student evaluations of the course. Possible limitations identified in the first year study included the teacher variable, instrumentation, and short duration of treatments.

The main results for the first year are summarized below:

1. A significant ($p \leq .01$) difference between Pilots A and B on Concepts was detected. This difference favored Pilot B.
2. A significant ($p \leq .1$) difference between Pilots A and C on Concepts was detected. This difference favored Pilot C.
3. A significant ($p \leq .1$) difference between Pilots A and C on Methods was detected. This difference favored Pilot A.
4. No significant ($p \leq .34$) differences among Pilots A, B, and C on Problem Solving were detected.

5. A significant ($p \leq .001$) positive change in student attitudes towards Mathematics was detected in all three Pilots combined.

6. A significant ($p \leq .001$) positive change in student attitudes towards Teaching Mathematics was detected in all three Pilots combined.

7. No significant ($p \leq .24$) differences in student attitude changes were detected among Pilots A, B, and C.

8. The students in Pilots A and B viewed the program more positively then did the students in Pilot C.

Second Year

The second year of the study was characterized by the need to provide instructional formats which were feasible for large numbers of students with typical constraints of staff and resources. In the interest of gleaning more information than the proposed case study of a single program could provide, the staff accepted the burden of developing, implementing, and evaluating two example programs.

The difference in the two versions occurred in the second quarter block when students were divided into two groups of roughly equal size: Group X, integrated content and methods with school experience using primarily MMP materials, and Group Y, coordinated content and methods with school experience using non-MMP materials.

Following a common mathematics content course given in the Winter Quarter, students were divided into Groups X and Y for the Spring Quarter Instruction in content, methods, and field experience. Students in

Group X met daily in small classes taught by either faculty or graduate students. Students in Group Y attended large content lectures given by a faculty member and small labs and problem sessions given by graduate students. They also attended small classes devoted to methods instruction which were taught by faculty and graduate students. Possible limitations identified in the second year study included lack of random assignment to treatments and in some cases less control of certain variables.

The main results for the second year are summarized below:

1. Significant differences were found on several background variables. Students in Group X had significantly ($p \leq .027$) lower gradepoint averages, had significantly ($p \leq .010$) less participation in the Freshman Early Experience Program, had significantly ($p \leq .044$) lower class standing, and were significantly ($p \leq .050$) younger than students in Group Y.

2. No significant ($p \leq .737$) difference between Group X and Y on Concepts was detected.

3. A significant ($p \leq .001$) difference between Groups X and Y on Methods was detected. This difference favored Group Y.

4. A significant ($p \leq .011$) difference between Groups X and Y on Problem Solving was detected. This difference favored Group Y.

5. A significant ($p \leq .039$) difference on the Teaching Elementary School Children scale was detected. This difference favored Group Y.

6. A significant ($p \leq .001$) positive change in student attitudes towards Mathematics was detected in both Groups combined.

7. A significant ($p \leq .01$) positive change in student attitudes towards Teaching Mathematics was detected in both Groups combined.

8. Significant differences were found on two of the post attitude scales adjusted for the initial differences in early experience participation and in the Attitude Toward Teaching Children premeasure. The Attitude of students in

Group X on the Teaching Mathematics and Teaching Elementary School Children scales was significantly ($p \leq .060$ and $p \leq .018$ respectively) lower than the Attitudes of students in Group Y.

The report offers the following recommendations:

Recommendation 1: Mathematics faculties, education faculties, and public school personnel should join together in upgrading the mathematics program for prospective elementary school teachers.

Recommendation 2: Instruction in mathematics content and methods should be strongly coordinated, but need not be combined.

Recommendation 3: School experience should be included in the coordinated content-methods instruction.

Recommendation 4: A mathematics content course should precede the combined content-methods-school experience package.

Recommendation 5: A significant portion of the content instruction (and of course the methods instruction) should be activity-oriented; moreover, the activity-based learning should lead the non-activity-based instruction whenever both are present.

Recommendation 6: Students should be required to display mastery of high school level algebra and geometry and give evidence of school participation as prerequisites to enrolling in the content and methods courses.

Recommendation 7: Lines of communication should be developed among institutions which train teachers to provide for cooperative efforts in the development of their programs.

Background

The Conference Board of the Mathematical Sciences National Advisory Committee on Mathematical Education (NACOME) in its 1975 report "Overview and Analysis of School Mathematics Grades K - 12" initiated its discussion of teacher education with the following remarks (page 81):

> The dominant feature of the mathematics teacher education picture is the absence of hard data concerning programs and practices, requirements, and characteristics of the products. Much of what is written, discussed in conferences, and used to justify recommended programs is based on sketchy impressionistic data, random cases of innovative activity and research, and opinion.

The report cited (page 88) several serious concerns of the National Council of Teachers of Mathematics (NCTM) Commission on Education of Teachers of Mathematics regarding current preservice teacher education: "One is the severe lack of research on and evaluation of teacher education programs and especially the need for shared information and coordination of research and evaluation efforts involving several teacher training institutions." In its recommendations for further research the NACOME report concluded the following (page 143):

> There should be continuing attempts to find a sound empirical basis for the recommendation of particular patterns, methods, and materials of instruction and of particular instructional and curricular organization. Needed are extensive evaluations of programs and comparative studies of alternative programs.

Groups such as the Committee on the Undergraduate Program in Mathematics (CUPM), the Cambridge Conference on Teacher Training and the NCTM Commission on Preservice Education of Teachers of Mathematics have provided helpful suggestions regarding the mathematical preparation of elementary school teachers, but have not provided hard data concerning various programs and practices.

One strategy for improving the preservice mathematics training of elementary teachers which has emerged is the blending of content and methods. For instance it was recommended by Roy Dubisch writing in the National Society for the Study of Education 1970 Yearbook, Mathematics Education, that

> 1. There is a great need to relate more closely what is being studied to problems of teaching elementary and secondary school mathematics.
>
> 2. Closely related to the need just described is the need to provide for a discussion of teaching methods along with content, rather than a study of content and methods in separate courses.

The major thrust for combined content-methods programs has been provided by the Indiana University Mathematics Methods Program (MMP). A brief summary of its impact was given in the NACOME report (page 87):

> There have been very few well-publicized programs experimenting with changes in the preservice mathematical education of teachers in recent years. A notable exception is the Indiana University Mathematics Methods Program which integrates content, methods, laboratory clinical and field experience within course modules, rather than separating them in the traditional manner. Teacher educators have expressed considerable favor for the concept of integrating content and methods. But there has been limited implementation in formal program structures. The difficulties of cutting across administrative and departmental lines in colleges and universities have blunted many attempts at organization change.

In its recommendations the report stated (page 139):

> Colleges of education, professional mathematics education organizations, accrediting agencies of teacher certification, and the mathematics community must cooperate to produce

> mathematics teachers knowledgeable in mathematics, aware of, oriented to, and practiced in a multitude of teaching styles and materials and philosophically prepared to make decisions about the best means to facilitate the contemporary, comprehensive mathematics education of their students. Further, the above bodies, together with local school boards and organizations representative of teachers must continually facilitate the maintenance of teachers' awareness of and input to current programs and issues.

In a detailed evaluation of the MMP (Archambault Final Report, 1974, pages 143-144) the Indiana University NSF project was described as "the first honest attempt at a set of materials which combine instruction in mathematics content and methods." The report continues

> However, there is also little doubt in the minds of the majority of the users of the MMP materials that a lack of content in the units will be a deterrent to their widespread use. Due to this weakness as well as to the vagueness of certain directions and activities, it is recommended that revisions in the units be made. To provide specific directions for these revisions an indepth analysis of the units orchestrated by an independent third party should be conducted. However, since there is conflicting speculation about how mathematics content affects teaching behavior, an empirical investigation of the effect of varying amounts of mathematics content on the later teaching performance of PSTs (preservice teachers) also should be undertaken.

Prior to the project discussed in this report, Ohio State University was representative of many institutions involved in the preparation of elementary school teachers; in spite of its commitment to the training of significant numbers of prospective teachers, there was a history of noncommunication between the Department of Mathematics, the College of Education, and local school personnel in developing the mathematics component of training programs for elementary teachers. However, in recent years several faculty members in the mathematics department have developed a strong interest in the content courses for preservice elementary teachers and have affected changes in that sequence. A Saturday morning on-campus mathematics program, primarily for inner city school children, founded by Arnold Ross gave unusual opportunity for faculty in the mathematics department to become involved with school mathematics and to work directly with children. In addition, the public schools in the Columbus area were requesting assistance in developing new programs and several faculty were working with groups of children in the schools. At the same time the College of Education was developing programs which permitted students to integrate methods courses with field experience in area schools. Thus, there was a strong impetus to nurture communication among the Department of Mathematics, the College of Education, and the local schools to seek improvement in the mathematics training of elementary teachers.

Project Goals

The overall purpose of the project was to unite the University's mathematics faculty with its education faculty and with public school personnel in providing meaningful preparation in mathematics for prospective elementary school teachers. The primary specific goals were the following:

1a. To define, using existing curriculum materials, three alternative content-methods programs with different formats and with varying amounts of mathematics content, methods, and school experience;

1b. To compare achievement in content, achievement in methods, and changes in attitude of prospective elementary school teachers enrolled in these three alternative content-methods programs;

2. To develop, implement, and evaluate example programs involving large numbers of preservice teachers.

Goals 1 and 2 reflect the principal thrust of the first and second years of project activity respectively.

Project Activity

In the sections which follow, project activity is discussed in light of the goals.

Goal 1a: To define, using existing curriculum materials, three alternative content-methods programs with different formats and with varying amounts of mathematics content, methods, and school experience.

The first six months of the project focused on the definition of three different programs in the mathematics preparation of prospective elementary school teachers.

The project staff used part of the first weeks of the project clarifying objectives so that the experimental instruction including varying amounts of mathematics content, methods, and school experience could be piloted during Spring Quarter 1977. Clearly a one quarter experience could not be expected to provide prospective teachers with all of the skills and understandings for teaching elementary school mathematics. The staff first identified those outcomes they considered desirable for teachers of elementary school mathematics. They then identified those characteristics which would be of high priority for the Spring Quarter experience rather than other aspects of the program such as the second mathematics course, student teaching, etc. The following objectives were identified for the spring instruction:

1. Students shall understand the mathematics topics of the elementary school curriculum contained in the spring course.
2. Students shall make appropriate selection of methods, materials, and learning activities for teaching a concept or topic.
3. Students shall use more than one model or explanation in teaching a concept or topic.

4. Students shall use appropriate sequencing of topics and activities in mathematics instruction.

5. Students shall recognize mathematics in the everyday life of a child.

6. Students shall formulate problems and solve them.

7. Students shall recognize and use valid reasoning and precise language.

8. Students shall identify the needs and learning difficulties of individual students in the study of mathematics.

Starting points for the exploration of various alternatives by the project staff were curriculum materials, journal articles, and presentations by visiting consultants. A study of MMP materials was aided by two visits to OSU by John LeBlanc, director of the MMP, and by a visit to Indiana University by the OSU faculty assigned to teach the MMP in this project. Experiences with combined programs at Michigan State University (including the MMP) were shared by consultant Lauren Woodby. Further insight was provided by Roy Dubisch and Shirley Hill, who have been active nationally in teacher training programs, and by Julius Goldberg, who has had extensive experience with Soviet teacher training programs.

The three alternative approaches were identified as follows:

Pilot A: Integrated content and methods with school experience using MMP materials.

Pilot B: Coordinated content and methods with school experience using non-MMP materials.

Pilot C: Separate content and methods with a probability laboratory and no school experience.

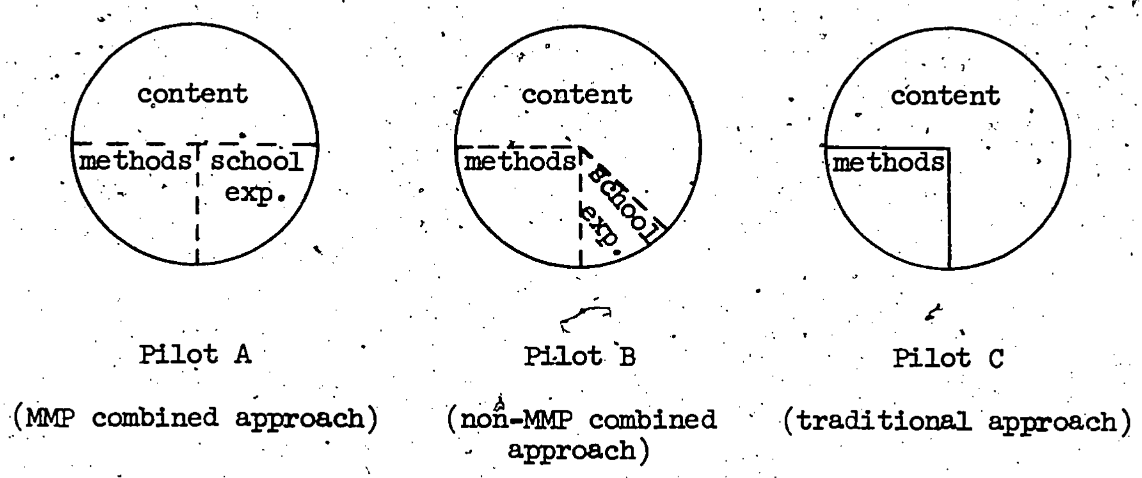

FIGURE 1. Time Alloted to Content, Methods, and School Experience by Pilot Groups

Figure 1 suggests the approximate emphasis on mathematics content, mathematics methods, and mathematics integrated school experience planned for each of the pilots. In each case there was a total of 10 quarter hours of instruction. Of course, the nature of the combined approaches often made it difficult to distinguish between the three components accurately.

The courses involved in the pilot instruction are described below. Each course met for ten weeks; credits refer to quarter hours. Each student enrolled in M105, E502, and either E289 or M294 depending on the pilot group.

M105: The first of two required five-hour mathematics content courses covering the arithmetic of the counting numbers, integers, and rational numbers, including applications to probability and statistics using a problem solving approach. Combinatorial questions and divisibility properties are included.

E502: A three-hour methods course focusing on the curriculum and instruction of mathematics in grades K - 6. Readiness, sequencing, materials selection and use, and evaluation are considered for the topics in M105.

E289: A two-hour field experience course in which university students teach small groups of children in the elementary schools.

M294: A two-hour mathematics lab developed to extend the study of probability and statistics in an activity-oriented setting.

The three pilots were constructed from the above courses and had the descriptive characteristics shown in Appendix A.

It should be noted that proficiency in high school level algebra either by placement examination or university coursework is a prerequisite to M105. In addition 90% of the students participate in the University's Freshman Early Experience Program (FEEP) in the schools prior to taking these courses. As a part of FEEP students spend about 12 hours per week for 9 weeks as observer/aides in elementary school classrooms.

Implementation occurred Spring Quarter 1977. There was a ten-week instruction period and three hours of testing during examination week. Each pilot met ten hours per week. Pilots A and B were team-taught with both instructors present in each class, while in Pilot C the mathematics instructors and the education instructor taught separately. A list of topics covered in the mathematics courses can be found in Appendix B. Results of the implementation are discussed under Goal 1b. A brief description of the three Pilots is given next.

Pilot A closely followed the MMP booklets. Content and methods instruction primarily involved having students do the activities in small groups under the supervision of the two instructors. Typically this involved the use of laboratory materials and elementary school textbooks. The MMP slide tapes were also used for the units covered. The pre-service teachers (PSTs) made two visits per week to a traditional elementary school classroom where they each taught a group of one to three children via an activity-oriented approach. PSTs made two or three visits with the same children and were given greater responsibility for planning the activities for each visit. During the quarter they sequentially encountered children in most grades one through six.

Pilot B covered the same topics using a conventional content textbook, Nuffield texts, a collection of resource materials, and conventional elementary school texts. The approach was more instructor-oriented, less activity-oriented than Pilot A. The content and methods instruction was highly coordinated, but fairly distinct, the emphasis was dependent largely on which instructor was leading the discussion. In all there was a greater content emphasis and lesser methods emphasis than in Pilot A. The PSTs made one visit per week to an elementary school featuring an informal learning approach where they instructed small groups of children via a learning center approach or tutored individual children. During the quarter they encountered children at various grade levels.

Pilot C used conventional texts in content and methods to cover the same topics as Pilots A and B and was supplemented by indepth laboratory experiences in informal probability and statistics. The instruction was divided into three parts taught by three separate instructors. Content instruction was presented in a highly problem-solving oriented lecture-discussion format. Methods instruction was

was conducted in a lecture discussion format supplemented by activities involving standard methods materials. The probability-statistics laboratory was activity-oriented primarily content instruction and was in lieu of school experience. A sample of the instructor-developed materials can be found in Appendix A.

Goal 1b: To compare achievement in content, achievement in methods, and changes in attitude of prespective elementary school teachers enrolled in three alternative content-methods programs.

The three alternatives described under Goal 1a were compared during Spring Quarter, 1977. Initial characteristics of the treatment groups were determined at the beginning of the quarter. Achievement in content, problem solving and methods and changes in attitude were measured according to the procedures described below. Student assessment of pilots was also obtained via course evaluations and interviews.

Description of Sample

The population of students for the project included those prospective elementary education majors who

1. were attending classes at the main campus of The Ohio State University during the Spring Quarter of 1977,
2. had not previously received credit for Mathematics 105 or Education 502, and
3. registered for the ten hour block of time reserved for this project in spring of 1977.

The 95 students in this population were randomly assigned to the three groups. A few students did not appear after assignment to groups and a few students dropped the courses. These and others with missing data on pre or post measures were eliminated from the analysis. The sample used for all group comparisons included those subjects remaining after dropouts and missing data subjects had been eliminated. This group was identified as the Final Sample. Table 1 is a summary of the samples for the pilot groups.

TABLE 1

Summary of Final Sample

	Pilot A	Pilot B	Pilot C	Total
Original Sample	31	32	32	95
Dropouts	3	2	7	12
Missing Data	3	3	6	12
Final Sample	25	27	19	71

Initial Characteristics

The initial characteristics of the subjects which were measured at the start of the quarter included certain background information, mathematics level, and attitudes. On the first day of the quarter each student completed a questionnaire, wrote a mathematics pretest, and responded to an attitude inventory. The questionnaire surveyed the following background characteristics:

1. Class standing or grade level
2. College grade point average (GPA)
3. Grade expected in Math 105 (first required content course)
4. Grade expected in Education 502 (methods)
5. College mathematics courses completed (college math rating). These courses were ranked as to level of mathematics.
6. Participation in a supervised school based field experience, FEEP (Freshman Early Experience Program)
7. Sex
8. Age

9. Intention of teaching in an elementary school
10. Years of high school mathematics
11. Average high school mathematics course grade

The students involved in the spring program were primarily freshmen and sophomores. They were average to above average students with college gradepoint averages around 2.50 to 3.00 on a four-point scale. Most of the students anticipated receiving grades of A or B for both the Math 105 and the Education 502 components of the program. Generally their expectations were high. Grades in the two courses usually average around C+ and B+ respectively. The college level mathematics background of these students varied considerably, but all students satisfied an elementary algebra prerequisite as demonstrated either by satisfactory performance on the university placement exam or by completion of OSU coursework. A small number of students indicated that they had taken pre-calculus or calculus courses beyond the prerequisite. Most of the students had been involved in the College of Education early field experience activities prior to registering for the Math 105 and Education 502 requirements.

Eighty-six percent of the students were women, most under the age of 21. The students reported having had three courses in high school mathematics at the algebra I level or higher. Their self-reported high school grade point average in high school mathematics was generally between 2.5 and 3.0 on a four-point scale. Percentage distributions for all background variables are given in Appendix C.

A Mathematics Pretest consisting of thirty multiple choice items was also given on the first day. The instrument was designed to obtain a measure of the student's understanding of concepts and problem solving skills before the course. Not all problems were chosen to reflect the course content. For example, several problems measured the ability of the student to translate a written problem into a mathematical representation; there was a consumer item and one or two items to test for basic vocabulary. Some items posed elementary problems in topics contained in the course: set operation, subsets, place-value, division of whole numbers, factors and multiples, integer arithmetic, order of fractions and decimals, division of fractions, rounding decimals, ratio and percent, and probability. The test statistics for the pretest are given in Appendix K. The means and standard deviations for background variables and the Mathematics Pretest are given for the final sample (n = 71) in Table 2. The same data for the missing data subjects and the dropouts are given in Appendices D and E.

The students with missing data ranged from low to high in mathematics ability over all three pilots. The equivalence of the three resulting pilot groups was tested using multivariate and univariate analyses of variance. The variables which indicate general academic ability or ability in mathematics were grouped together. The correlations calculated using all available data after dropouts are presented in Appendix F. These correlations indicate that these variables are generally related. Multivariate and univariate analysis of variance with these seven variables as the dependent variables were computed using the final sample. The results are presented in Appendix G. No significant (.05 level) differences were found between pilot groups on the academic background variables.

TABLE 2

Background Variables and Mathematics Skills Pretest Means and Standard Deviations by Pilot Group

Variable	Pilot A n = 25 $\bar{X}$	Pilot A SD	Pilot B n = 27 $\bar{X}$	Pilot B SD	Pilot C n = 19 $\bar{X}$	Pilot C SD
Math Pretest	20.68	4.00	21.18	3.08	22.05	2.55
Class Standing[a]	1.76	0.72	1.70	0.61	1.79	0.63
GPA[b]	2.83	0.49	2.88	0.49	2.91	0.59
Expected M105 Grade[b]	3.16	0.55	3.41	0.57	3.32	0.58
Expected E502 Grade[b]	3.48	0.59	3.56	0.51	3.47	0.61
College Math Rating[c]	3.56	1.36	3.52	1.37	3.60	1.22
Participation in FEEP[d]	1.12	0.33	1.04	0.19	1.16	0.38
Sex[e]	1.80	0.41	1.93	0.27	1.84	0.38
Age[f]	1.12	0.33	1.11	0.32	1.10	0.32
Intend Teaching in Elementary School[d]	1.08	0.28	1.00	0.00	1.21	0.71
Years of H.S. Math	2.80	0.91	3.00	0.83	2.90	0.57
Average H.S. Math Grade[b]	2.76	0.83	2.85	0.72	2.63	0.76

[a] 1 = Freshman, 2 = Sophomore, 3 = Junior, 4 = Senior, 5 = Other

[b] 4 = A, 3 = B, 2 = C, 1 = D, 0 = E

[c] Range 1 thru 9, 9 = most advanced

[d] 1 = Yes, 2 = No

[e] 1 = Male, 2 = Female

[f] 1 = Under 21, 2 = 21 - 24, 3 = 25 - 30, 4 = Above 30

The remaining five background variables

Class standing
Participation in Freshman Early Experience Program
Sex
Age
Intend Teaching in an Elementary School

were grouped together. The intracorrelations using all available data after dropouts for these variables are presented in Appendix H. These variables are descriptive of the non-academic background of the students in each pilot group. Multivariate and univariate analyses of variance with these five variables as the dependent variables were computed for the final sample and the results presented in Appendix I.

The results of the multivariate and univariate analyses of variance reveal no significant (.05 level) differences in non-academic abilities or backgrounds for Pilots A, B and C.

In addition to the background survey and the Mathematics Pretest the students completed an attitude instrument. The pretest attitude scores were obtained from the responses of the students to four semantic differentials each having nine pairs of bipolar adjectives. The students marked between these adjective pairs in the position which described their interest, perception and/or understanding of these four areas:

I. Mathematics
II. Teaching Mathematics
III. Teaching Elementary School Children
IV. Elementary School Children

The responses were scored relative to the project staff's perception of how the "ideal" elementary mathematics teacher would respond. The scales and the weights assigned to each response are presented in Appendix J.

The reliabilities for these scales are presented in Appendix K for the six times this instrument was administered throughout the quarter. The pretest attitude means and standard deviations for the final sample are presented in Table 5. Summary of attitude scores for missing data subjects is given in Appendix L.

The pretest attitude scale scores were subjected to one way multivariate and univariate analyses of variance to check for initial differences between Pilots A, B and C. The results are given in Appendix M. The multivariate analysis shows an overall significant ($p \leq 0.016$) difference in the attitudes which can be traced by inspection of the univariate tests to the attitudes involving children and teaching children. The results of the post-hoc analyses of the group means are reported in Appendix O.

Analysis of the pretest attitude responses indicate that students in Pilot B had interest, perceptions and understandings of teaching elementary school children more nearly ideal than did those in Pilot C. It is also evident that the Pilot A students had interests, perceptions and understandings of elementary school children which were more nearly ideal than did those in Pilot B. The correlation between attitudes toward elementary school children and toward teaching elementary school children was only 0.08 -- not significant (see Appendix P). Due to the differences on the pretest attitude scales and the lack of correlation reported above the Teaching Elementary School Children and Elementary School Children pretest attitude scales were selected as covariates for the analyses of the posttest attitude measures.

Content, Methods, and Problem Solving Achievement

The cognitive criterion measures for this project were as follows:

A. Mathematics Concepts Test (see Appendix Q)
B. Methods of Teaching Mathematics Test (see Appendix R)
C. Mathematics Problem Solving Test (see Appendix S)

Copies of the item statistics are included in Appendices Q, R, S. The summary statistics for these tests are given in Appendix T. The frequency distributions are given in Appendix U. Students had 55 minutes to write each test.

The intent in the preparation of these measures was to include items to assess the content of all three pilots. Some of the items were designed to assess elements of the project which all three pilots had in common while other items were relevant to only one or two of the pilots. As would be expected with such an instrument, the internal consistency reliabilities reported in Appendix T are low. The mathematics concepts were more consistent across the pilots than were the problem solving or methods components as is reflected by the reliability. Undoubtedly the quality of some of the items in the instruments also contributed to the low reliabilities.

The Mathematics Concepts Test was a 36 item test consisting of

A. Nineteen multiple choice items with only one correct choice
B. Five multiple choice (multiple correct answer) items
C. Four matching items
D. Eight items involving more extensive answers by students

The eight items in D called for listings, proofs, solutions of problems, and application of the mathematics. The Mathematics Concepts Test was constructed using the topic outline (see Appendix B) for the mathematics component of the project. The maximum possible score on this test was

44 and the mean across all pilots was 30.1. The average difficulty across all items was .28 which means the students overall experienced a 72 percent success rate on this test.

The Methods of Teaching Mathematics Test was a 20 item test consisting of

A. Twelve multiple choice items with only one correct response per item
B. Five true-false items
C. Two multiple choice items with multiple correct responses
D. One free response item.

The content of the methods instrument included

Selection and sequencing of activities (4 items)
Use and understanding of manipulatives (5 items)
Detection of error patterns (5 items)
Selection of models for instruction (1 item)
Equivalent representations of problem statements (5 items).

All items were scored as right or wrong except for the free response item which was scored from 0 to 2. The maximum possible total score was 21 and the mean across all pilots was 13.9. The reliability was very low (.42) due in part to the varied coverage of topics by the three pilots and in part to the quality of the items.

The Mathematics Problem Solving Test was a 10 item test made up of two parts:

Part I: Items 1-5 for which students were instructed to attempt all five.

Part II: Items 6-10 for which students were instructed to work as many as they could in the time available.

The problems were chosen to be in the spirit of the problems students had solved throughout the course but did not repeat any problem students had encountered in the course. Each of the ten problems was given a maximum possible score of ten. This resulted in a total maximum possible score of 100. The three M105 instructors jointly set the grading criteria for each problem. They then split up the problems for grading so that all students' responses to a given problem were read by the same grader. The mean score across pilot groups was 40.0. The reliability (.58) is effected by the testing procedure used. The students tended to work on different items out of the last five. Since all items were not attempted by all students, the internal consistency estimate is lower than if all items had been attempted by all students.

The means and standard deviations by Pilot Group for the post test measures of mathematics concepts, methods of teaching mathematics, and problem solving skills for mathematics are presented in Table 3.

TABLE 3

Means and Standard Deviations for Concepts, Methods, and Problem Solving Post Tests by Pilot Group (Year 1)

	Pilot A n = 25		Pilot B n = 27		Pilot C n = 19	
Variable	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
Concepts	26.56	6.09	33.00	5.30	30.32	3.97
Methods	14.84	2.32	14.04	2.52	13.26	2.16
Problem Solving	36.20	10.94	39.74	17.25	42.53	13.81

A one-way multivariate analysis of variance with three levels (Pilots A, B and C) was computed for the Concepts, Methods and Problem Solving tests. The results are given in Table 4.

TABLE 4

Multivariate Analysis of Variance for Concepts, Methods, and Problem Solving Criteria Differences Among Pilot Groups

Multivariate (3 dependent variables)	F(6,132)		$P \leq$
	7.1222		0.001
Univariate Variables	**F(2,68)**	**MS_B**	**$P \leq$**
Concepts	9.654	270.361	0.001
Methods	2.437	13.546	0.095
Problem Solving	1.072	221.510	0.348

A significant multivariate effect was found ($P \leq .001$). The univariate analyses of variance resulted in significant differences on the Mathematics Concepts Test ($P \leq .001$) and on the Methods Test ($P \leq .095$). To further isolate these effects, Scheffé comparisons are given in Appendix V.

The comparisons of Pilots A, B and C on the concepts measure indicate that Pilot A was significantly lower ($P \leq .01$) than Pilot B. Pilot A was significantly lower ($P \leq .1$) than Pilot C. Pilots B and C were not significantly different. (See Table 3 for means.)

The comparisons of Pilots A, B and C on the methods measure indicate a difference between the means for Pilot A and C ($P \leq .1$) with Pilot A greater. No other differences were found.

Due to apparent violations of homogeneity of variance for the problem solving variable a Kruskal-Wallis nonparametric test was made. A Chi Square Value of 3.86 ($P \leq .145$) resulted. The nonparametric tests produced results consistent with the analysis of variance.

Correlations of the criterion measures with the academic background variables and with the pre and post attitude measures are given in Appendices W and X.

Attitude Changes

Students responded to the attitude instrument every two weeks in addition to the pre and post measures. The means and standard deviations

TABLE 5

Means and Standard Deviations for Pre and Post Test Attitude Scales by Pilot Group

			Pre		Post	
Attitude Scale	Variable	n	$\bar{X}$	SD	$\bar{X}$	SD
Mathematics[a]	Pilot A	25	30.36	4.20	32.48	4.12
	Pilot B	27	30.96	4.47	33.15	3.99
	Pilot C	19	29.47	3.45	32.74	3.66
	Overall	71	30.35		32.80	
Teaching Mathematics[a]	Pilot A	25	34.20	2.65	35.16	2.21
	Pilot B	27	33.30	3.26	34.93	3.09
	Pilot C	19	33.47	3.26	34.74	2.28
	Overall	71	33.66		34.96	
Teaching Elementary School Children[b]	Pilot A	25	35.84	3.41	35.00	3.80
	Pilot B	27	37.81	3.14	36.59	2.27
	Pilot C	19	35.47	4.55	35.11	3.05
	Overall	71	36.49		35.63	
Elementary School Children[c]	Pilot A	25	32.92	2.38	32.32	2.66
	Pilot B	27	31.04	3.16	30.70	4.07
	Pilot C	19	32.00	3.43	32.42	3.55
	Overall	71	31.96		31.73	

[a]Maximum = 42

[b]Maximum = 43

[c]Maximum = 38

for pre and post test attitudes are given in Table 5. The effect of adjusting the post test means to account for initial differences is shown in Appendix Y. The results of the multivariate and univariate analysis of covariance for the attitude measures are given in Appendix Z. There were no significant differences in the effects of Pilots A, B and C on students attitudes toward mathematics, toward teaching mathematics, toward teaching elementary school children, or toward elementary school children.

In addition to comparing the three pilot groups on the four post attitude scales, an analysis of attitude changes for the combined groups was made. In Table 5 the pre and post test attitude scale means are given for each pilot group separately and for all three combined. Table 6 is a summary of the analysis of variance of change in attitude over time.

TABLE 6

Analysis of Variance of Change in Attitudes with Respect to Time

Attitude (Pre - Post)	Mean Squares	df	F	$P \leq$
Mathematics	220.870	1	27.003	.001
Teaching Mathematics	57.238	1	11.959	.001
Teaching Elementary School Children	22.781	1	2.534	.116
Elementary School Children	1.012	1	.258	.613

Significant positive changes ($P \leq .001$) were found over all groups for the Mathematics Attitude and for the Teaching Mathematics Attitude.

Because of the high incidence of missing data for the attitude scales during the 5th and 7th weeks, only data for the pre, 3rd, 9th, and post testing times were used for further investigation of attitude change during the quarter. Plots of the means of the Mathematics and Teaching Mathematics scales were made by pilot by testing time. Means were computed using only those students with complete data for all four reported testing times. The plots are given in Figures 2 and 3.

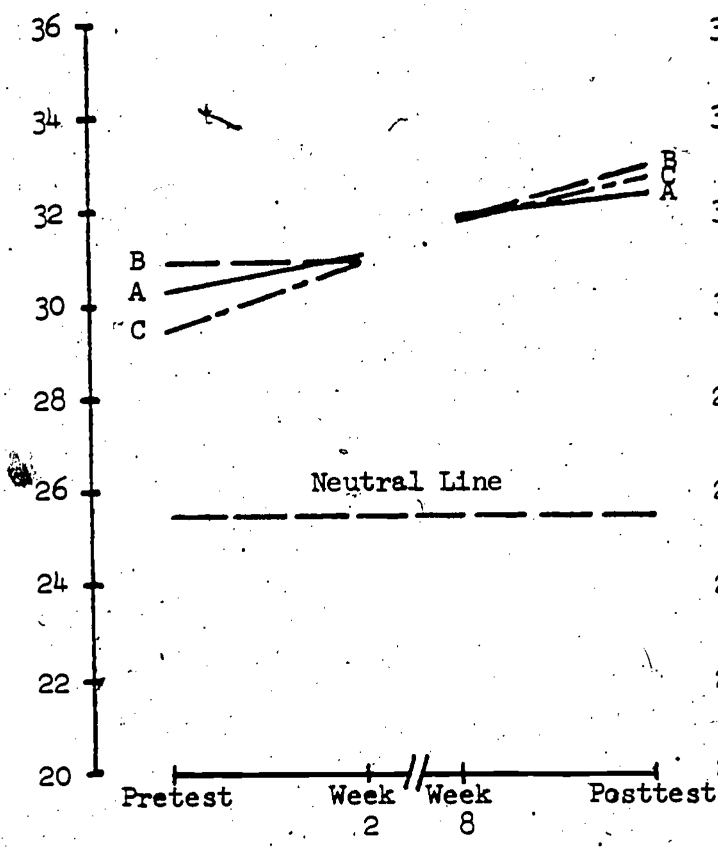

FIGURE 2. Attitude toward Mathematics with Respect to Time by Pilot Group

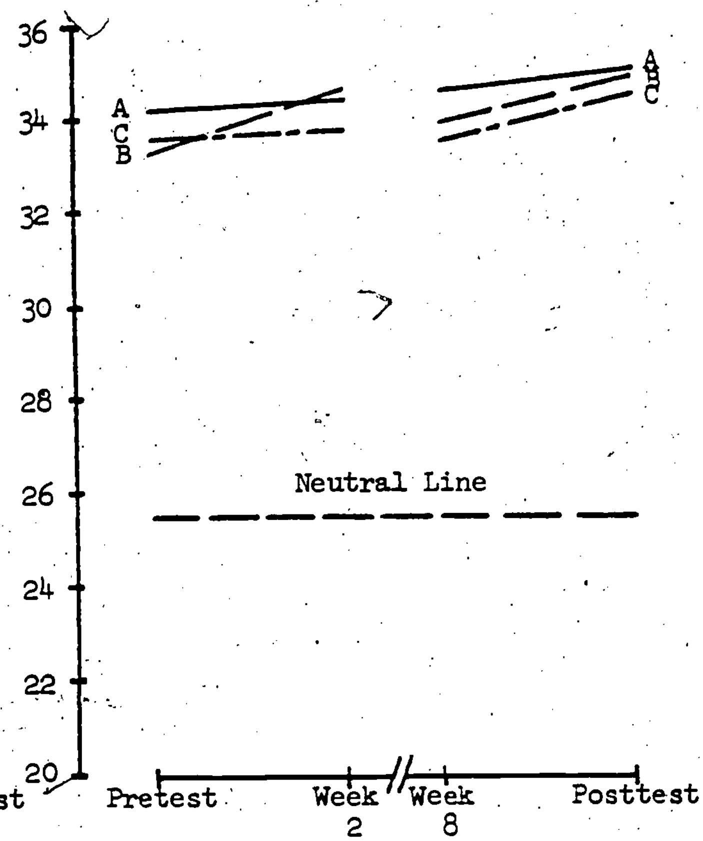

FIGURE 3. Attitude toward Teaching Mathematics with Respect to Time by Pilot Group

Student Assessments of Pilots

All students completed an evaluation form on which they reacted to statements designed to measure attitudes toward the courses in mathematics and in methods, the course assignments, the field experience or laboratory, the extent of integration in the instruction, grading procedures, and individual instructors.

All the evaluated areas in all courses (M105, EMC502, EMC289, M294) were average to high. The Pilot A instructors received the highest rating of the three on the items measuring the extent of integrated instruction. There was variation in the students' perceptions of the seven instructors involved.

In addition to the evaluation forms which were filled out by each student, five students were chosen at random from each Pilot and interviewed by members of the evaluation team. Analysis showed these students to be typical in terms of scores on the math pretest and all other pre-course variables. The students in each of the pilots felt that they had grown in their knowledge of concepts, methods, and problem solving and in their attitudes toward mathematics and teaching. Students in Pilots A and B felt they had gained in knowledge of and attitudes toward children. The students in Pilot C did not perceive these changes.

In the interviews students were asked to critique activities, materials, and organization for the areas of Mathematics Concepts Learning, Methods Skills, and Field Experience. The responses were in all cases positive but with Pilot C consistently lower. Pilot B students who were interviewed indicated that their field experience had not been as worthwhile as expected.

Students were asked to rank order the four following statements according to their own opinions of their importance and according to the importance assigned in their courses.

1. Selection and sequencing of learning activities based on the mathematics.
2. Selection and sequencing of learning activities based on the characteristics of the children.
3. Understanding of the mathematics concepts.
4. Interaction and communication with children.

Students in the Pilot A interview sample thought that understanding of mathematics concepts should have had the highest importance but that, in fact, selection and sequencing of learning activities based on the mathematics did. Pilot B students felt that interaction and communication with children should be most important but that understanding mathematics concepts had been. Pilot C students gave understanding of mathematics concepts as both their highest priority and also the emphasis of their courses. Pilot C and B students felt a higher importance should have been placed on interaction and communication with children. All three pilots felt that too much emphasis was placed on the selection and sequencing of learning activities based on the mathematics.

The 15 students in the interviews were asked two questions about the programs in which they participated.

1. Would you participate in a project like this one if the chance to do so came up again?
2. Would you recommend it to your friends?

The mean responses by pilot are given in Table 7.

TABLE 7

Mean Responses to Program Evaluation Interview Questions by Pilot Group (Year 1)

	Pilot A	Pilot B	Pilot C
Question 1	4.2	4.4	2.6
Question 2	4.4	4.2	2.8

(Scale: certainly not 1 2 3 4 5 very certainly)

The interview data suggests that students in Pilots A and B viewed the program more positively than did the students in Pilot C.

Summary of Results (Year 1 of Study)

1. A significant ($p \leq .001$) multivariate difference among Pilots on the post test criteria (Concepts, Methods, and Problem Solving) was detected. This effect primarily involved the Concepts ($p \leq .001$) and Methods ($p \leq .1$) criteria.
2. A significant ($p \leq .01$) univariate difference between Pilots A and B on Concepts was detected. This difference favored Pilot B.
3. A significant ($p \leq .1$) univariate difference between Pilots A and C on Concepts was detected. This difference favored Pilot C.
4. A significant ($p \leq .1$) univariate difference between Pilots A and C on Methods was detected. This difference favored Pilot A.
5. No significant ($p \leq .34$) univariate differences among Pilots A, B and C on Problem Solving were detected.

6. A significant ($p \leq .016$) multivariate difference among Pilots on the pretest attitude scales was detected. This effect primarily involved the Teaching Elementary School Children and Elementary School Children scales. (To adjust for these initial differences these two scales were selected as covariates for the analysis of the post test attitude measures.)

7. A significant ($p \leq .001$) positive change in student attitudes towards Mathematics was detected in all three Pilots combined.

8. A significant ($p \leq .001$) positive change in student attitudes towards Teaching Mathematics was detected in all three Pilots combined.

9. No significant ($p \leq .24$) differences in student attitude changes were detected among Pilots A, B and C.

10. The students in Pilots A and B viewed the program more positively than did the students in Pilot C.

Limitations

In reflecting on the project's first year, certain limitations deserve comment. Aside from the usual concern for satisfying the premises for the application of some of the statistical methods which have been employed in this study, there are some limitations which may bear on the findings.

Foremost among these is the teacher variable. A single team of two or three teachers taught the students in each pilot. Hence it is possible that the differences which occurred are in some measure attributable to the teaching teams rather than to the approaches themselves. Insofar as possible, assignment of instructors to treatments did, however, attempt to match the background and preferences of the teachers with the approaches in order to present each treatment most favorably.

The matching of instructors with treatments can be illustrated by the rationale for the particular assignment of teachers from the mathematics department to Pilots A, B and C. The Pilot A (MMP) instructor had participated in the development of the MMP materials and had taught using this approach previously. The instructors of Pilots B and C were instrumental in the development of current versions of the mathematics content course at the Ohio State University and were experienced in teaching this course. All of the instructors had experience in teaching mathematics to elementary school children as well as to prospective elementary school teachers.

Student course evaluations nevertheless suggested differences among the treatments in the perception of teachers by their students. It is difficult or impossible to identify the extent to which instructors and treatments affected each other. Further insight into the effects of the teacher variable was provided by the replication of the study being conducted during the project's second year.

A second possible limitation involves the instruments used in this study. The paucity of established instruments (particularly those which measure knowledge of the methods of teaching mathematics), the time constraints, and the limit on resources obviated thorough piloting of instruments during the first year.

It should be noted that emphasis was placed on providing a concepts instrument which measured the mathematics common to the three treatments. For example, though 20 percent of Pilot C instruction was devoted to the probability and statistics laboratory which was unique to that approach, the content instrument did little to probe this area. The authors feel that the instrument's concentration on content common to the three treatments may have introduced a bias disfavoring Pilot C, which covered the most topics, and favoring Pilot A, which covered the fewest topics.

Test statistics indicate that some of the greatest shortcomings were present in the instrument used to measure methods. It was a primary goal during the project's second year to refine all of the instruments, particularly the methods instrument.

A third possible limitation arises from the fact that there was a commitment by the project to render dichotomous approaches with regard to the MMP. Pilot A was to be "all MMP" and Pilots B and C were to be "no MMP." It may be that strict adherence to the existing form of the MMP materials in Pilot A (a philosophy not necessarily consistent with the materials themselves) resulted in introducing a bias against Pilot A, in that Pilots B and C were not similarly restricted. The second phase of this study replaced the "all MMP vs. no MMP" treatments by approaches which were "primarily MMP" and "non-MMP."

A fourth limitation of the first year study is the relatively short duration of the treatments. The university's quarter system imposed a constraint in that it was impossible to maintain treatment groups intact for more than eleven weeks. The conclusions reported herein are reported for a single one-quarter implementation; however, the second year of the project replicated the study by extending the experience to two quarters.

Possibly a bias was introduced by a Hawthorne effect. When students in Pilot C learned that they would not have a school experience -- a feature generally viewed with enthusiasm by students -- their attitudes may have been adversely affected. They may have held preconceived opinions regarding the relative merits of the three approaches which contributed to their viewing Pilot C less favorably. This may have contributed to the higher dropout rate for Pilot C.

Finally it must be noted that the schools used for the field experience were selected on the basis of proximity to campus and willingness of the principals to involve their schools in the project. Pilot A was assigned to a traditional elementary school and Pilot B to the Columbus designated alternative informal school. Thus there was a marked difference in these two experiences. In addition, the lack of experience of the Pilot B preservice teachers with nontraditional schools caused some to be uncomfortable in that environment.

Goal 2: To develop, implement, and evaluate a model program involving large numbers of preservice teachers.

Development

During the Spring and Summer Quarters of 1977 the project staff analyzed the data from the spring experimentation, shared their judgments on the various aspects of the instruction, took a realistic look at University resources, and made decisions for the instruction of several hundred students in combined programs during the Academic Year 1977-78.

The staff conjectured that in the first year the instruction by specially selected teachers with small groups of undergraduates contributed to a generally successful experience for most students during year 1. This may have accounted in some measure for the positive changes in attitude toward mathematics and toward teaching mathematics. One additional goal determined for the second year of experimentation was finding formats for instructing large numbers of students without sacrificing the positive attitude changes that were achieved in the small group instruction.

Among the features the staff wished to preserve from the first year's program were

1. coordination of content and methods instruction,
2. inclusion of a school experience component coordinated with the content/methods instruction, and
3. inclusion of a laboratory experience in the content component.

An additional consideration for the second year study was the sequencing of the course, this time to include two content courses (instead of one as in the first year experiment), one methods course, and one field experience course. The staff concluded that another feature of the second year study should be

4. completion of one mathematics content course before students enroll in a course in methods.

The advantages of offering one mathematics content course prior to the methods course together with the desirability of offering coordinated content-methods instruction led to the schedule for the second year study described below. Each course met for ten weeks; credits refer to quarter hours.

Quarter I (Winter 1978)

M105: The first of two required five-hour mathematics content courses covering (during winter 1978) the rational numbers and elementary probability with some number theory and constructive geometry.

Quarter II (Spring 1978)

M106: The second mathematics content course covering (during spring 1978) the whole numbers, measurement, and transformational geometry.

E502: A three-hour methods course focusing on the curriculum and instruction of mathematics of grades K - 8 including readiness, sequencing, materials selection and use, and evaluation.

E289: A two-hour field experience course in which university students teach small groups of children in the elementary schools.

The decision to do the rational number arithmetic before the whole number arithmetic was based mostly on the premise that the topics of M106 listed above were better suited for the coordinated content/methods courses. (In retrospect those who taught the courses agreed that the sequencing of these topics was unsatisfactory from the point of view of mathematics development.) A list of topics taught in the mathematics courses can be found in Appendix AA.

As was stated earlier in this report, proficiency in high school level algebra either by placement examination or university coursework is a prerequisite to M105. In addition 90% of the students participate in the Freshman Early Experience Program (FEEP) in the schools prior to taking these courses.

Though there was agreement on the four features for the second year study listed above, the staff recognized a need for further comparisons of materials and specific teaching formats. Limitations of the first year experiment, such as the involvement of only a few teachers per treatment and the commitment to render dichotomous approaches with regard to the MMP, prompted further experimentation. In the interest of gleaning more information than the proposed case study of a single program could provide, the staff accepted the burden of developing, implementing and evaluating two example programs.

The difference in the two versions occurred in the second quarter block, when the students were divided into two groups of roughly equal size:

Group X: Integrated content and methods with school experience using primarily MMP materials.

Group Y: Coordinated content and methods with school experience using non-MMP materials.

The schedule for the mathematics-related curriculum for the second year of the study is summarized in Figure 4. Further details are provided below.

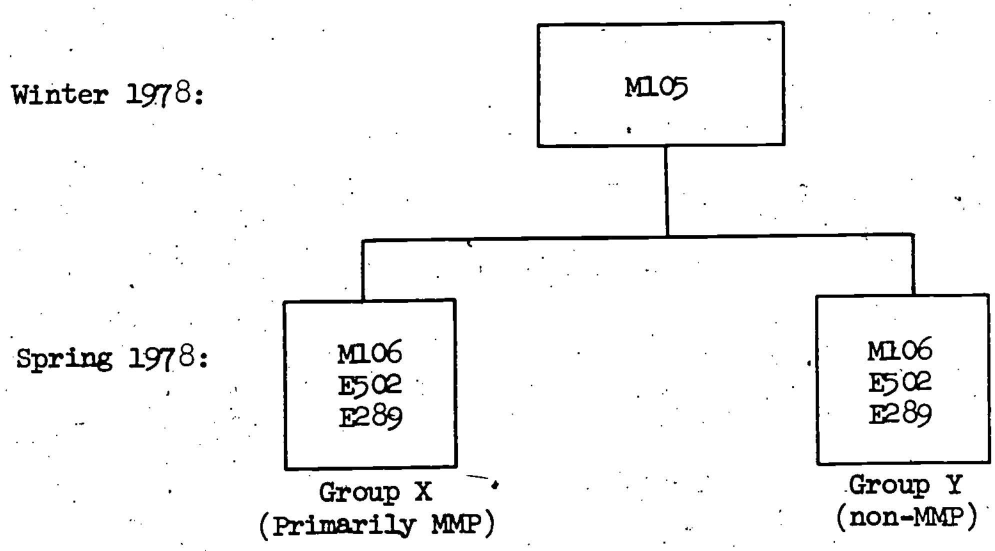

Figure 4. Schedule for the Mathematics-Related Curriculum for the Second Year Study.

Implementation

Over 300 students registered for the Winter Quarter M105 course. The course was taught using a conventional text (Begle) in the format of three lectures, one problem session, and one extended laboratory session each week. Experienced faculty members in the mathematics department coordinated the course and gave the lectures to groups of about one hundred. Teaching associates were utilized for problem sessions and laboratory sessions which averaged about 25 to 30 students each. This pattern of instruction is commensurate with departmental resources for most of its elementary courses.

Approximately 155 students enrolled in the Spring Quarter block. Demands on facilities required that classes meet at various times throughout the day. Group X classes met within the 8:30 A.M. - 12:30 P.M. time span. Group Y classes met within the 10 A.M. - 3 P.M. time span. An effort was made to assign students to treatments randomly, but time conflicts and certain quirks of the University's computerized scheduling process jeopardized the randomness somewhat.

The MMP units used by Group X were Numeration, Addition and Subtraction, Multiplication and Division, Rational Numbers, Measurement, and Transformational Geometry. The choice of the last two units added geometry to the project's experience with MMP material which was not part of the first phase. The materials were augmented by the mathematics text used during Winter Quarter to aid in providing the student with definitions, unifying topics which are fragmented in MMP, and focusing more attention on underlying concepts. In addition, the MMP materials were supplemented by instructor handouts in the direction of problem solving and certain methods topics such as learning theory. The students in Group X were divided among three classes, each taught three days per week for 108 minutes by an instructor from mathematics and two days per week for 108 minutes by an instructor from education. Sixty minutes of the time spent with the education instructor each week was given to school experience-about 30 minutes with children and 30 minutes for transportation. In order to get a comparison relating staffing patterns, one of the three sections was taught by two faculty members, one was taught by two teaching associates, and a third by a faculty member and a teaching associate.

Group Y was taught using conventional texts to cover the same material in a format close to that used in the Winter Quarter mathematics course. For the mathematics instruction the students in Group Y met each week together in three 48 minute lectures and were divided into four smaller groups, each which had one 48 minute problem session and one 90 minute (or less) laboratory per week. For the methods instruction students met in small groups with a faculty lecturer/coordinator for 108 minutes per week and with teaching associates 108 minutes per week. In addition, about 90 minutes per week were given to school experience-about 60 minutes with children and 30 minutes for transportation.

A summary of instructional time by Group is given in Figure 5. It should be observed that there was considerable imbalance in the appropriation of time between groups.

	Group X	Group Y
Content	324	282
Methods	156	216
School experience	30	60
Transportation	30	30
	540	588

Figure 5. Approximate Minutes per Week of Instructional Time to Program Components by Group (Year 2).

Evaluation

To provide a basis for comparison of the two alternatives described earlier, data was collected throughout Winter and Spring Quarters, 1978, according to the following schedule:

1. Initial background characteristics were determined at the beginning of Winter Quarter.
2. Scores on a common final exam administered at the end of Winter Quarter were used as a pretest of mathematics content for the Spring Quarter study.
3. Changes in attitude were measured at intervals throughout Winter and Spring Quarters.
4. Achievement in content, problem solving, and methods was measured at the end of Spring Quarter.
5. Student assessment of Groups was obtained via course evaluations and interviews at the close of Spring Quarter.

Assignment to treatments was finalized at the end of Winter Quarter. Of the 155 students enrolled at the beginning of Spring Quarter nine dropped out within the first two weeks of the quarter, one from Group X and eight from Group Y. An additional nine students were deleted from the study because of missing data, four from Group X and five from Group Y. The final sample for the second year study consisted of 137 students for whom data was complete, 68 from Group X and 69 from Group Y.

The initial characteristics of the subjects which had been measured at the start of Winter Quarter were the eleven background items listed on page 17. A summary of this data is provided in Appendix BB. It was found that students in Group X had significantly ($p \leq .027$) lower grade-point averages, had significantly ($p \leq .010$) less participation in the Freshman Early Experience Program, had significantly ($p \leq .044$) lower class standing, and were significantly ($p \leq .050$) younger than the students in Group Y.

Final exam scores for the common Winter Quarter mathematics course were used as a pre-measure of mathematics understanding for the Spring Quarter study. The test consisted of 50 objective type items and 12 subjective type items. The test and its statistics can be found in Appendix CC. The mean for Group X students was lower (no statistical significance, $p \leq .148$) than the mean for Group Y students.

Attitudinal measures were taken at intervals during Winter and Spring Quarters once again using the attitude scales listed on page 60 and detailed in Appendix J. For purposes of the study three of the applications of the attitude instrument were identified as the pretest, midtest, and posttest for the Spring Quarter study. These were administered at the conclusion of Winter Quarter, in the fifth week of Spring Quarter, and at the conclusion of Spring Quarter respectively. The results of the attitude measures and the test statistics for the attitude instrument are summarized in Appendix DD. An analysis of the pretest attitude scores showed an initial difference ($p \leq .038$) on the Teaching Elementary School Children Scale favoring Group Y. The initial difference on the Mathematics scaled approached significance ($p \leq .135$), also favoring Group Y. (See Appendix EE.)

Examination of the background information and the results of the premeasures for mathematics content and attitude cast doubt on the premise that the groups involved in the second year of the study were equivalent. Evidently, the schedule changes resulting from time conflicts led to nonequivalent groups in terms of these variables. In an effort to correct this deficiency, certain variables were used as covariates in the subsequent analysis.

The cognitive criterion measures were redone for the second year study. Items from the first year's instruments were screened using the item analysis results and relevance for the second year study. The satisfactory items formed a core which was supplemented by new items resulting in the Content, Methods, and Problem Solving instruments found in Appendices FF-HH. The item statistics are included.

The Mathematics Concepts Test was a 30 item instrument covering the content of the Spring Quarter 1978 mathematics course. All test items tested material common to the instruction given to Groups X and Y. Students had 55 minutes to write the test.

The Mathematics Methods Test was a 39 item instrument covering the content of the Spring Quarter 1978 methods course. Most test items tested material common to the instruction given to Groups X and Y. The test for the second year was of a considerably broader scope than the first year instrument. Students had 90 minutes to write the methods test.

The Problem Solving Test was a 10 item instrument consisting of problems which were designed to be unfamiliar yet solvable by the students in both Groups. Half of the items were taken from the test administered during the first year study. Students had 55 minutes to write the test.

Means and standard deviations by Group for the three post test measures are given in Table 8. A one-way multivariate analysis of variance was computed for the Concepts, Methods, and Problem Solving Tests. The results are given in Table 9.

Table 8

Means and Standard Deviations for Concepts, Methods, and Problem Solving Tests by Group (Year 2)

	Group X			Group Y		
Variable	$\overline{X}$	$\overline{X}$ adj.	SD	$\overline{X}$	$\overline{X}$ adj.	SD
Concepts	69.19	69.90	9.10	70.94	70.40	9.58
Methods	51.10	51.84	8.28	59.29	58.57	9.45
Problem Solving	47.48	48.46	13.68	55.83	54.87	15.29

*Means are adjusted for background differences in Class Standing and Grade Point Average

A significant multivariate effect was found ($p \leq .001$). The univariate analyses of variance resulted in significant differences on the Methods Test and on the Problem Solving Test, but no significant difference on the Concepts Test.

The comparisons of Groups on the Methods measure indicate that Group X was significantly lower ($p \leq .001$) than Group Y. The comparison of Groups on the Problem Solving Test indicate that Group X was significantly lower ($p \leq .011$) than Group Y.

Table 9

Multivariate Analysis of Covariance for Concepts, Methods, and Problem Solving Criteria Differences Between Groups Using Class Standing and Grade Point Average as Covariates (Year 2)

Test of Program Differences

Multivariate	F(3,131)		p ≤
	8.93		0.001

Univariate	F(1,133)	MS_B	p ≤
Concepts	0.20	14.22	0.657
Methods	22.97	1445.01	0.001
Problem Solving	6.66	1310.94	0.011

Correlations of the criterion measures with the academic background variables and with pre and post attitude measures are given in Appendices II and JJ.

An analysis of the attitude measures can be found in Appendices DD - EE and KK - MM. A comparison of means for Post Test Attitudes by Group adjusted for initial differences in early experience participation and in the Attitude Toward Teaching Children premeasure reveals that the Attitude of Group X on the Teaching Mathematics and Teaching Elementary School Children scales were significantly ($p \leq .060$ and $p \leq .018$ respectively) lower than the Attitudes of Group Y.

In addition to comparing the two Groups on the four post attitude scales, an analysis of attitude changes for the combined groups was made. (See Appendix OO.) Significant positive changes were found over all groups for the Mathematics Attitude ($p \leq .001$) and for the Teaching Mathematics Attitude ($p \leq .01$).

In order to measure student reaction to the programs, 24 students were interviewed by members of the evaluation team, four students from each of the three Group X classes and twelve students from Group Y. The results of these interviews are summarized in Appendix NN. There were no significant differences in the perceptions of those interviewed with regard to goals and objectives of the programs nor with regard to outcomes or evaluations of the programs.

Students from both programs felt that

1. understanding of the mathematics concepts received the greatest emphasis,
2. interaction and communication with children received the least emphasis, and
3. selection and sequencing of learning activities was intermediate in terms of emphasis.

They generally felt that neither of the limited field experience programs associated with this project was as valuable as the more extensive Freshman Early Experience Program they had participated in.

During the interviews students were asked to rank both the influence of and the time spent with textbooks, teachers, manipulative materials, pupils in the schools, and fellow students during the Spring Quarter. The sharpest contrast occurred with respect to "fellow students," which Group X students ranked first and Group Y students ranked last in terms of influence. Students in both groups ranked time spent with textbooks higher in terms of time spent than in terms of influence. The ranking of time spent with teachers was significantly lower ($p \leq .001$) in Group X than in Group Y, which is consistent with the data of Figure 5 on page and with the intended formats of the approaches.

Students in both Groups indicated a preference for participating in a similar program again if given the opportunity and recommending the experience to friends as reported in Appendix PP, Part III. A comparison of this information with the interview results of the previous year (Table 7, page 32) reveals that the mean responses for the second year program were lower than those of Pilots A and B and higher than that of Pilot C during the first year program.

In summary, the statistical analysis indicates that the performance of students in Group Y was higher in terms of the Methods, Problem Solving, Attitude toward Mathematics, and Attitude toward Teaching Mathematics as measured.

Summary of Results (Year 2 of Study)

1. Significant differences were found on several background variables. Students in Group X had significantly ($p \leq .027$) lower gradepoint averages, had significantly ($p \leq .010$) less participation in the Freshman Early Experience Program, had significantly ($p \leq .044$) lower class standing, and were significantly ($p \leq .050$) younger than students in Group Y.

2. A significant ($p \leq .001$) multivariate difference among Groups on the post test criteria (Concepts, Methods, and Problem Solving) was detected. This effect primarily involved the Methods and Problem Solving criteria.

3. No significant ($p \leq .737$) univariate difference between Group X and Y on Concepts was detected.

4. A significant ($p \leq .001$) univariate difference between Groups X and Y on Methods was detected. This difference favored Group Y.

5. A significant ($p \leq .011$) univariate difference between Groups X and Y on Problem Solving was detected. This difference favored Group Y.

6. A significant ($p \leq .039$) univariate difference on the Teaching Elementary School Children scale was detected. This difference favored Group Y.

7. A significant ($p \leq .001$) positive change in student attitudes towards Mathematics was detected in both Groups combined.

8. A significant ($p \leq .01$) positive change in student attitudes towards Teaching Mathematics was detected in both Groups combined.

9. Significant differences were found on two of the post attitude scales adjusted for the initial differences in early experience participation and in the Attitude Toward Teaching Children premeasure. The Attitude of students in Group X on the Teaching Mathematics and Teaching Elementary School Children scales was significantly ($p \leq .060$ and $p \leq .018$ respectively) lower than the Attitudes of students in Group Y.

Limitations

While the second year study enjoyed the advantages of having more subjects, more teachers involved, and more time to develop and refine instruments, it also had several disadvantages. The imbalance in the allotment of time between groups was one outgrowth of the priority given to model program development over strict experimental control. Another example of the relaxed controls was the deliberate freedom in choice of materials (primarily MMP vs. non-MMP) rather than the rendering of strictly dichotomous approaches. Also as mentioned earlier, assignment to treatments was not random.

Though a formal comparison of the two treatments was conducted, the subordination of experimentation to achieving the optimal program suggests that the data generated during the second year study should be regarded with caution. Apropos to this discussion is the viewing of the second year effort as two case studies supported by somewhat informal experimentation.

Interpretations and Recommendations

Because of inherent limitations and other constraints within a project of this scope, this report does not attempt to offer absolute conclusions; yet, it would be neglectful if it failed to share the opinions which evolved from the study. The remarks which follow are offered in this spirit. The opinions expressed are based in part on the data and in part on informal observations.

An encouraging aspect of this project was the ability of the University's mathematics faculty, its education faculty, and the school personnel to work together in upgrading the mathematics program for prospective elementary school teachers. Despite a history of non-communication at this university, the two faculties were able to unite effectively. Moreover ties with school personnel were substantially strengthened. The impact of this cooperation was felt beyond the pre-service teacher training program and is likely to have a continuing positive effect. In particular the inservice program has grown dramatically, with an increasing number of well-received course offerings on- and off-campus. From this emerges the rather general but strong recommendation for similar institutions that mathematics faculties, education faculties, and public school personnel should join together in upgrading the mathematics program for prospective elementary school teachers.

A primary program recommendation from the study is that instruction in mathematics content and methods be strongly coordinated. There is considerable support for this conclusion in the findings, especially in terms of student reactions and attrition rates, but perhaps more significant is the widespread support of this precept from staff members of both the mathematics and education departments.

The use of the word "coordinated" rather than "combined" is intentional in that the experiences during the two years suggest that for a large audience a combined program (i.e., one in which instruction in content and methods is highly integrated) is more demanding of staff and facilities without producing better results than a coordinated program (i.e., one in which instruction in content and methods is strongly related but not integrated). The greater demands of the MMP type combined program arise from the need to have instructors qualified in both content and methods teaching as well as in providing small classes in a mostly activity-oriented format. For large audiences the task of finding an adequate number of qualified teachers and of providing ample laboratory facilities needed for most of the instruction can be prohibitive. Justification for the added burden of doing a combined program is lacking, for when compared to the coordinated versions (Pilot B of the first year study and Program Y of the second year study), criteria measures for the combined programs were in no case significantly higher, while those of the coordinated program were significantly higher in several instances.

A third recommendation is that school experience be included in the coordinated content-methods instruction. This recommendation is based on the results of the formal student interviews together with informal feedback from students and instructors. Chief among student criticism of the school experience is the complaint that the 30 to 60 minute sessions (at most three with the same child or group of children) were insufficient to provide depth and continuity. While recognizing that the area of school experience is itself worthy of considerable investigation, the authors offer on the basis of informal and limited observations the following specific suggestions for the school experience component:

1. Preservice teachers (PSTs) should work with small groups of children, beginning with no more than two children.

2. PSTs should work with children at a variety of grade levels.

3. PSTs should have at least one opportunity to work with the same child (or small group of children) over an extended period of time.

4. PSTs should be given increasingly more responsibility in the planning of lessons for successive sessions with the same group of children.

A fourth recommendation assumes that the program to train prospective elementary shcool teachers in mathematics contains at least two mathematics courses. (The thought of less is inconceivable in light of the importance of mathematics in the elementary school curriculum!) It is recommended that a mathematics content course precede the combined content-methods-school experience package referred to above. This suggestion is offered to avert the possibility of placing the prospective teachers in the position (during the school experience component) where they are asked to teach mathematics which they do not yet understand. Prospective teachers also need a certain level of mathematics competence before beginning their methods study.

Reaction of consultants to two long standing features of the program at this university are supportive of a fifth recommendation, that students be required to display mastery of high school level algebra and geometry and give evidence of school participation as prerequisites to enrolling in the content and methods courses. Proficiency in high school level algebra and geometry is hardly a lofty goal since graduates of such programs in many states are certified to teach grades K - 8, and these are not uncommon topics in the middle school grades. Early school participation of a general nature, which may range from observing to aiding experienced classroom teachers, provides a perspective helpful in making the experience in a specific area, in this case mathematics, more meaningful.

Consensus of the staff leads to a sixth recommendation, that a significant portion of both the content instruction and the methods instruction be activity-oriented and that whenever possible, the activity-based learning precede and motivate other instruction. The laboratory setting should serve as a vehicle for discovery rather than for verification.

The last recommendation is again more general. The beneficial interchanges fostered by the visitations of consultants to this project suggest that institutions which train teachers develop lines of intercommunication to provide for cooperative efforts in the development of their programs. The preparation of teachers is a vital and challenging task which can best be met by cooperation within institutions and among institutions.

Summary of Recommendations

Recommendation 1: Mathematics faculties, education faculties, and public school personnel should join together in upgrading the mathematics program for prospective elementary school teachers.

Recommendation 2: Instruction in mathematics content and methods should be strongly coordinated, but need not be combined.

Recommendation 3: School experience should be included in the coordinated content-methods instruction.

Recommendation 4: A mathematics content course should precede the combined content-methods-school experience package.

Recommendation 5: Students should be required to display mastery of high school level algebra and geometry and give evidence of school participation as prerequisites to enrolling in the content and methods courses.

Recommendation 6: A significant portion of both the content instruction and the methods instruction should be activity-oriented and whenever possible, the activity-based learning should precede and motivate other instruction.

Recommendation 7: Lines of communication should be developed among institutions which train teachers to provide for cooperative efforts in the development of their programs.

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Appendix A
Course Descriptions by Pilot
(Year 1)

Pilot A

Format: Integrated content and methods with field experience, using MMP materials.

Coursework: M105, E502, [illegible] (school experience).

Instructors: J. Schultz, L. Stull

Texts: Selected activities from the following MMP units: Numeration, Addition and Subtraction, Multiplication and Division, Number Theory, Rational Numbers, and Probability and Statistics.

Classes: Both instructors participating in each class, 8 hours per week of instruction in M105 and E502.

School Experience: 2 hours/week at Fifth Avenue Elementary School, grades 1, 2, 3, 5. School experience modeled MMP format.

Pilot B

Format: Integrated content and methods with school experience, using materials other than MMP.

Coursework: M105, E502, E269 (school experience).

Instructors: D. Haefele, Joan Leitzel

Texts: Begle: The Mathematics of the Elementary School, and Nuffield: Mathematics in the Later Primary Years, and a collection of resource books and school materials assembled in a reading area.

Classes: Both instructors participating in each class, 9 hours/week of instruction in M105 and E502.

School Experience: 1 hour/week at Indianola Informal School, both primary and intermediate levels. Student teachers worked one-on-one with school pupils.

Pilot C

Format: Separate content and methods instruction with probability laboratory.

Coursework: M105, E502, M294 (probability and statistics laboratory).

Instructors: J. Riedl (M105), A. Osborne (E502), S. Demarin (M294).

Texts: Kelley-Richert, Elementary Mathematics for Teachers (for M105); NCTM 37th Yearbook: Mathematics Learning in Early Childhood (for E502). Text materials for M294 were developed by the instructor. Sample materials are provided on the next few pages.

Classes: M105 met 4 hours/week.
E502 met 3 hours/week.
M294 met 3 hours/week.

Laboratory Experience: Students participated in a probability and statistics laboratory.

Math 294 Sample Materials

Name ______________________

Laboratory Report 8 : And still more on brown bag experiments

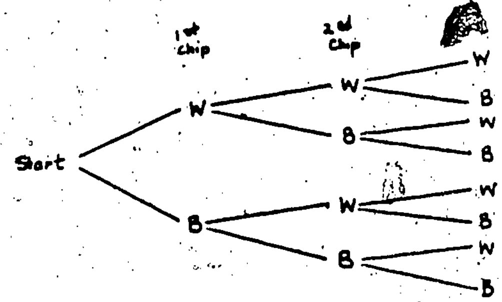
A. Place 5 white and 3 blue chips in a bag. Play "blue chips" 10 times using these 8 chips. Record your outcome below; then record them on the board.

Trial	Outcome
1	
2	
3	
4	
5	

Trial	Outcome
6	
7	
8	
9	
10	

When everyone has recorded data on the board make a graph illustrating the distribution of outcomes for the class

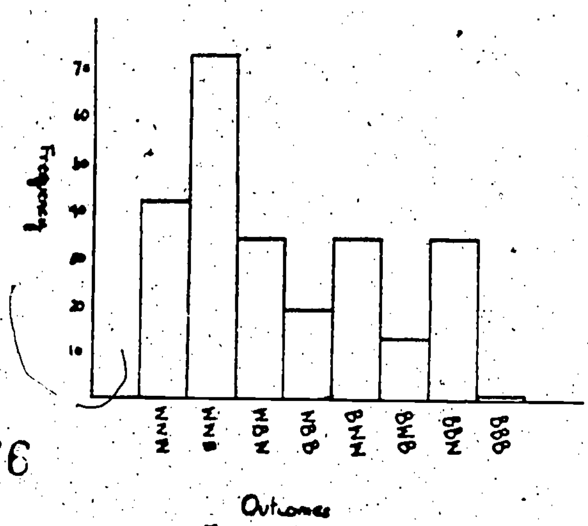
Use the "tree diagram" below to find the theoretical distribution of outcomes



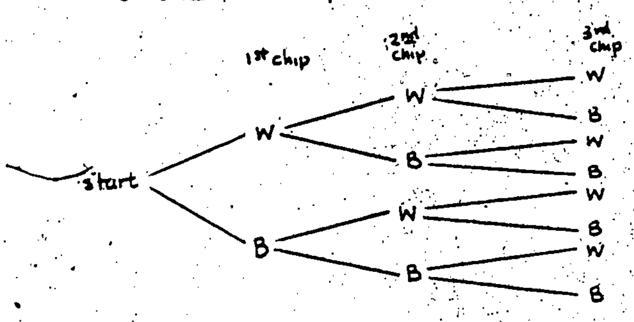
Make a graph illustrating the number of times you would expect to achieve each outcome in ~~every~~ 512 tries (on the average

Compare this graph with the one you made from the class data.

B. In laboratory 3 you performed a similar experiment. 5 whit[e] and 3 blue chips were placed in each bag. Chips were draw[n] one at a time but not replaced before the next chip was drawn. The graph below shows the distribution of outcomes for 26 people.



Use the tree diagram below to find the theoretical distribution of outcomes for this experiment



Make a graph illustrating the theoretical distribution of outcomes.

Compare the four graphs you have made

Find the "chances" of obtaining 3 W, 2W, 1W, 0 W for each game

Result	"With Replacement"	"Without Replacement"
3 White		
2 White		
1 White		
0 White		

C. Make a similar analysis of the "blue chips" game we are playing. In laboratory 5, we found the chances of obtaining each number (0, 1, 2, 3) of blue chips and used these to decide that the game should cost \$2.40. Suppose we played "blue chips without replacement;" how much should we charge for it? Complete the table of chances

	Regular "Blue Chips"	Blue Chips Without Replacement
3 blue	216/1000	
2 blue	432/1000	
1 blue	288/1000	
0 blue	64/1000	
Cost for fair game	\$2.40	

D. Using the bag of 5 white and 3 blue chips play this game. Draw chips, one at a time, without replacement, until you have drawn all 3 blue chips. Repeat 10 times

Trial	Outcome
1	
2	
3	
4	
5	

Trial	Outcome
6	
7	
8	
9	
10	

How many outcomes are possible for this game?

DON'T FORGET TO PLAY BLUE CHIPS

Appendix B

Syllabus for M105 (Mathematics Concepts Course)
(Year 1)

Set Theory and Number Concepts
- 1-1 correspondence
- matching sets
- equal sets
- subsets
- # of n-subsets of an m-set (Pascal's triangle)
- set operations
  - union
  - intersection
  - cartesian product
  - difference
- finite vs infinite sets
- counting numbers

Fundamental Operations with Whole Numbers
- addition: definition, models, properties, and algorithms
- subtraction: definition, models, properties, and algorithms
- multiplication: definition, models, properties, and algorithms
- division: definition, models, properties, and algorithms
- relationship between operations (inverse operations, etc.)
- counting arguments

Numeration Systems
- place value concept
- non-decimal systems and operations within these
- modular number systems and operations within these
- properties of number systems

Number Theory
- prime and composite numbers
- divisibility and divisibility rules
- prime decomposition and unique factorization into primes
- GCD (including the Euclidean algorithm) and LCM

Integers: Definition, models, properties and operations

Rational Numbers
- models for fractions
- operations with fractions and rational decimals
- ordering rational numbers
- place value concepts and expanded notation
- rounding decimals
- equivalent forms (fractions, decimals, ratios, and %)
- algebraic definitions of operations and properties

Probability

Elementary statistics

Appendix C

Frequency Distribution for Background Variables
(Year 1)

Mathematics Pretest Totals	
15	4%
16	6
17	6
18	10
19	8
20	4
21	10
22	18
23	4
24	6
25	15
26	6
27	3

Class Standing	
Freshman	37%
Sophomore	52
Junior	11

GPA	
Below 2.00	3%
2.00-2.49	23
2.50-2.99	[illegible]
3.00-3.49	27
3.50-4.00	13

Expected M105 Grade	
C	6%
B	59
A	35

Expected E502 Grade	
C	3%
B	44
A	54

College Math Rating	
2	3%
3	77
4	1
5	7
6	6
7	3
8	3

Participation In Freshman Early Experience Program	
Yes	90%
No	10

Sex	
Male	14%
Female	86

Age	
Under 21	89%
21-24	11

Intend Teaching Elementary School	
Yes	94%
No	6

Years of H.S. Math	
1	[illegible]
2	32
3	41
4	25

Average H.S. Math Grade	
D	3%
C	35
B	45
A	17

Appendix D

Background Variables and Mathematics Pretest Means
for Missing Data Subjects by Pilot Group
(Year 1)

Variable	Pilot A	Pilot B	Pilot C
Math Pretest	14.67 (3)	23.67 (3)	2[illegible].17 (6)
Class Standing[a]	2.00 (3)	2.00 (3)	2.00 (6)
GPA[b]	2.99 (3)	3.99 (3)	3.33 (6)
Expected M105 Grade[b]	3.33 (3)	4.00 (3)	3.60 (5)
Expected E502 Grade[b]	3.33 (3)	4.00 (3)	3.60 (5)
College Math Rating[c]	2.67 (3)	5.00 (3)	4.00 (6)
Participation in FEEP[d]	1.67 (3)	1.00 (3)	1.17 (6)
Sex[e]	2.00 (3)	1.67 (3)	[illegible] ([illegible])
Age[f]	1.33 (3)	1.00 (3)	1.00 (5)
Intend Teaching in Elementary School[d]	1.33 (3)	1.00 (3)	1.00 (4)
Years of H.S. Math	2.00 (3)	3.33 (3)	3.00 (5)
Average H.S. Math Grade[b]	3.00 (3)	3.00 (3)	3.00 (5)

[a] 1 = Freshman, 2 = Sophomore, 3 = Junior, 4 = Senior

[b] 4 = A, 3 = B, 2 = C, 1 = D, 0 = E

[c] Range 1 thru 9, 9 = most advanced courses

[d] 1 = Yes, 2 = No

[e] 1 = Male, 2 = Female

[f] 1 = Under 21, 2 = 21-24, 3 = 25-30, 4 = Above 30

Appendix E

Background Variables and Mathematics Pretest Means
for Dropouts by Pilot Group
(Year 1)

Variable	Pilot A $n_a = 3$	Pilot B $n_b = 2$	Pilot C $n_c = 7$
Math Pretest	21.67	9.50	21.71
Class Standing[a]	2.33	1.50	1.[illegible]
GPA[b]	3.24	2.50	2.60
Expected M105 Grade[b]	3.00	3.50	3.43
Expected E502 Grade[b]	3.67	4.00	3.57
College Math Rating[c]	3.67	2.00	3.29
Participation in FEEP[d]	1.00	1.00	1.14
Sex[e]	1.67	1.00	1.86
Age[f]	1.33	1.00	1.14
Intend Teaching in Elementary School[d]	1.00	1.00	1.17
Years of H.S. Math	2.67	2.50	2.86
Average H.S. Math Grade[b]	2.00	3.00	3.43

[a] 1 = Freshman, 2 = Sophomore, 3 = Junior, 4 = Senior

[b] 4 = A, 3 = B, 2 = C, 1 = D, 0 = E

[c] Range 1 thru 9, 9 = most advanced courses

[d] 1 = Yes, 2 = No

[e] 1 = Male, 2 = Female

[f] 1 = Under 21, 2 = 21-24, 3 = 25-30, 4 = Above 30

Appendix F
Intracorrelations of Academic Ability Background Variables
(Year 1)

	Pre-Math Total	GPA	Expected M105 Grade	Expected E302 Grade	College Math Rating	Years of H.S. Math
GPA	.27** (83)					
Expected M105 Grade	.21* (82)	.24* (82)				
Expected E302 Grade	.17 (82)	.24* (82)	.68*** (82)			
College Math Rating	.27** (83)	.17 (83)	.39*** (82)	.21* (82)		
Years of H.S. Math	.31** (82)	.17 (82)	.17 (81)	.11 (81)	.23* (82)	
Avg. H.S. Math Grade	.33*** (82)	.25** (82)	.41*** (81)	.18 (81)	.20* (82)	.47*** (82)

*$p \leq .05$ **$p \leq .01$ ***$p \leq .001$

Numbers In Parentheses Indicate Sample Sizes

Appendix G
Multivariate Analyses of Variance for Academic Ability Background Differences Between Pilot Groups
(Year 1)

Multivariate (7 dependent variables)

$F(14, 124)$ .697 $P <$ .774

Univariate Variable	$F(2,68)$	MS_B	$P\leq$
Math Pretest	.932	10.234	.399
GPA	.127	.140	.881
Expected M105 Grade	1.245	.402	.294
Expected E302 Grade	.162	.051	.851
College Math Rating	.013	.023	.987
Years of H.S. Math (Algebra and higher)	.404	.260	.669
Average H.S. Math Grade	.456	.271	.636

Appendix H
Intracorrelations of Non-Academic Background Variables
(Year 1)

	Class Standing	Participation In FEEP	Sex	Age
Participation In FEEP	-.05 (83)			
Sex	-.26** (82)	-.16 (82)		
Age	.37*** (82)	.11 (82)	-.19 (82)	
Intend Teaching Elem. School	.03 (81)	.01 (81)	.09 (81)	.32** (81)

*$p \leq .05$ **$p \leq .01$ ***$p \leq .001$

Numbers In Parentheses Indicate Sample Sizes

Appendix I
Multivariate Analysis of Variance for Non-Academic Background Differences Between Pilot Groups
(Year 1)

Multivariate (5 dependent variables)

$F(10, 128)$ .883 $P<$ .551

Univariate Variable	$F(2,68)$	MS_B	$P\leq$
Class Standing	.103	.045	.902
Participation in FEEP	1.002	.090	.373
Sex	.866	.107	.425
Age	.012	.001	.988
Intend Teaching Elementary	1.530	.248	.224

Appendix J
Weights Assigned by Project Staff to Responses on Attitude Scales[a]

I. MATHEMATICS

confident	5	4	3	2	1	insecure
thinking	5	4	3	2	1	memorizing
symbols	1	2	3	4	3	ideas
uncomfortable	1	2	3	4	5	at ease
easy	1	3	4	4	3	hard
exciting	5	4	3	2	1	dull
practical	5	4	3	2	1	useless
boring	1	2	3	4	5	challenging
rigid	1	2	3	4	3	flexible

II. TEACHING MATHEMATICS

rigid	1	2	3	4	4	flexible
exciting	5	4	3	2	1	dull
lecture	1	3	5	4	2	laboratory
confident	5	4	3	2	1	insecure
easy	1	3	4	4	2	hard
uncomfortable	1	2	3	4	5	at ease
problem	1	2	3	4	5	challenge
tell	1	3	4	[illegible]	1	discuss
one way	1	2	3	4	[illegible]	many ways

III. TEACHING ELEMENTARY SCHOOL CHILDREN

confident	5	4	3	2	1	insecure
talking	1	1	[illegible]	1	1	listening
play	1	3	3	4	2	work
reluctant	1	2	3	4	5	eager
organized	5	4	3	2	1	chaos
uncomfortable	1	2	3	4	5	at ease
problem	1	2	3	4	5	challenge
easy	1	1	5	3	1	hard
exciting	5	4	3	2	1	dull

IV. ELEMENTARY SCHOOL CHILDREN

passive	1	2	3	4	5	active
excited	3	4	3	2	1	bored
good	3	4	3	2	1	bad
curious	4	4	3	2	1	uninterested
listening	1	2	4	3	1	talking
work	1	3	4	2	1	play
eager	2	4	3	2	1	reluctant
chaos	1	3	4	3	1	organized
timid	1	3	2	3	1	bold

[a]Semantic differential scoring weights determined by Project staff, summer 1977.

Appendix K
Mathematics Pretest and Attitude Instrument Test Statistics
(Year 1)

Mathematics Pretest Test Statistics

Number of items	30
Mean score	21.2
Standard deviation	3.72
Average item difficulty (Proportion missing item)	.29
Average item score to total score correlation	.24
Reliability (Internal Consistency α)	.69

Attitude Instrument
Internal Consistency Reliabilities (Cronbach's Alpha)[a]

Attitude Scale	Testing Time: Pre-Test	Week 3	Week 5	Week 7	Week 9	Post-Test
Mathematics	0.66	0.90	0.97	0.97	0.95	0.96
Teaching Mathematics	0.10	0.88	0.97	0.97	0.95	0.96
Teaching Elementary School Children	0.86	0.92	0.98	0.97	0.94	0.96
Elementary School Children	0.86	0.93	0.97	0.96	0.93	0.95

Appendix L
Means for Pre and Posttest Attitudes, and Concepts, Methods, and Problem Solving Postests for Missing Data Subjects by Pilot Groups
(Year 1)

	Pilot A n = 3	Pilot B n = 3	Pilot C n = 6
Pretest Attitudes:			
Mathematics	29.0	31.5**	31.0
Teaching Mathematics	32.7	35.3	34.0
Teaching Elementary School Children	36.0	41.0*	35.0
Elementary School Children	32.3	34.0*	29.5
Posttest Attitudes:			
Mathematics	--	36.3	35.0*
Teaching Mathematics	--	37.7	32.0*
Teaching Elementary School Children	--	36.3	33.0*
Elementary School Children	--	32.7	23.0*
Concepts	25.3	33.3	31.8
Methods	14.0	11.7	12.3
Problem Solving	33.0	48.0	48.5

*n = 1 **n = 2

Appendix M
Multivariate Analysis of Variance for Pretest Attitude Scores Among Pilot Groups
(Year 1)

Multivariate (4 dependent variables)

	$F(8, 130)$		$P<$
	2.463		0.016

Univariate Variable	$F(2,68)$	MS_B	$P \le$
Attitude:			
Mathematics	0.727	12.369	0.487
Teaching Mathematics	0.617	5.760	0.542
Teaching Elementary School Children	2.904	38.788	0.062
Elementary School Children	2.581	23.035	0.083

Appendix N
Notation for Group Comparisons

Information with respect to significant differences between groups has been summarized schematically throughout this report in the manner described in Winer (1971):

A C B

The treatments are ordered from low to high according to their group means. Treatment groups underlined by a common line do not differ significantly from each other; treatments not underlined by a common line do differ significantly. In the diagram above Treatment A does not differ significantly from Treatment C, Treatment C does not differ significantly from Treatment B, but Treatment A and Treatment B do differ significantly.

Appendix O
Scheffé Multiple Comparisons of Univariate Differences Between Pilot Groups on Pretest Attitude Scales
(Year 1)

Pretest Attitude: Teaching Elementary School Children ($P \le 0.062$ on univariate test)

Comparison	Mean Difference	$\hat{\psi}/\hat{\sigma}_{\hat{\psi}}$ [a]	Result
Pilot A - Pilot B	-1.98	1.960	NSD
Pilot A - Pilot C	0.37	0.333	NSD
Pilot B - Pilot C	2.35	2.147*	$P \le 0.11$

C A B (See Appendix N)

Pretest Attitude: Elementary School Children ($P \le 0.083$ on univariate test)

Comparison	Mean Difference	$\hat{\psi}/\hat{\sigma}_{\hat{\psi}}$ [a]	Result
Pilot A - Pilot B	1.88	2.268	$P \le 0.10$
Pilot A - Pilot C	0.92	1.012	NSD
Pilot B - Pilot C	-0.96	1.073	NSD

B C A

*Critical value of $\hat{\psi}/\hat{\sigma}_{\hat{\psi}}$ for $P \le 0.10$ is 2.18.

a. Where $\hat{\psi}$ is the Scheffé contrast for comparing means and $\hat{\sigma}_{\hat{\psi}}$ is the square root of the variance of the contrast (standard error of the contrast as in Glass pp. 388-389, 1970).

Appendix P
Intercorrelations of Attitude Pretests
(Year 1)

	Mathematics	Teaching Mathematics	Teaching Elem. Sch. Children
Teaching Math	.49*** (82)		
Teaching Elem. S. Children	.11 (80)	.32** (81)	
Elem. S. Children	.19* (80)	.08 (81)	.08 (81)

Appendix Q
Mathematics Concepts Test
Items and Item Statistics
(Year 1)

Item	Difficulty[a]	Correlation[b]	Group Comparisons[c]
1	.05	.02	CAB
2	.19	.13	CAB
3	.27	.25	ACB
4	.18	.36	CAB
5	.08	-.03	CBA
6	.07	.18	ACB
7	.20	.31	ACB
8	.33	.44	ACB
9	.29	.21	ACB

Item

1. $\frac{2}{3} \div \frac{5}{7}$ equals

   (1)A. $\frac{9}{8}$ (95)B. $\frac{14}{15}$

   (1)C. $\frac{15}{14}$ (1)D. $\frac{10}{21}$

   (1)E. None of the above

2. The least common multiple of 36 and 14 is

   (4)A. 504 (81)B. 252

   (0)C. 42 (10)D. 2

   (6)E. None of the above

3. Which of the following are prime numbers?

   7, 9, 57, 91

   (11)A. only 7 and 91
   (10)B. only 7, 57 and 91
   (1) C. only 7, 9 and 91
   (73)D. only 7
   (5) E. All of the numbers are prime

4. For what bases is 7811 a meaningful numeral?

   (1) A. 1,2,...,9,10
   (82)B. 9,10,11,...
   (2) C. 2,3,4,...
   (6) D. 9 only
   (6) E. 1,2,3,...
   (2) No response

a. Difficulty is percentage of subjects who got this item wrong.
b. Correlation between Item and (Test - Item).
c. See Appendix N.

5. Which numeral names the number represented in the chart?

Thousands	Hundreds	Tens	Ones
2	12	8	23

   (1)A. 212,823 (1)B. 21,231

   (1)C. 32,803 (92)D. 3,303

   (5)E. None of the above

6. The greatest common factor of 30 and 84 is

   (1)A. 2 (93)B. 6

   (1)C. 420 (4)D. 2520

   (1)E. None of the above

7. Round the decimal 231.7638 to the nearest hundredth.

   (1)A. 200 (1)B. 231

   (18)C. 231. (0)D. 231.763

   (80)E. None of the above

8. The following set has ______ non-empty subsets.

   {m, n, o, p}

   (0)A. 4 (2)B. 12

   (67)C. 15 (18)D. 24

   (12)E. None of the above

9. Which arrangement of the following decimals is in order from smallest to largest?

   (1) A. .6, .61, .611, .601
   (4) B. .601, .6, .61, .611
   (0) C. .611, .61, .6, .601
   (71)D. .6, .601, .61, .611
   (23)E. None of the above
   (1) No response

Item	Difficulty	Correlation	Group Comparisons
10. A bag contains 3 blue marbles, 2 green ones, and 5 red ones. A marble is selected at random. What is the probability it is green? (80)A. 1/5 (1) B. 1/3 (4) C. 1/4 (10)D. 1/10 (6) E. None of the above	.20	.25	ACB
11. If a coin is tossed twice, what is the probability of getting a head once and a tail once? (7) A. 1/3 (61)B. 1/2 (30)C. 1/4 (0) D. 2/3 (1) E. None of the above	.39	-.09	CBA
12. For all whole numbers a, b, c, and d ($b \neq 0$, $d \neq 0$), the sum $\frac{a}{b} + \frac{c}{d}$ is equivalent to which of the following? (6) A. $\frac{a+c}{b+d}$ (2) B. $\frac{a+c}{bd}$ (2) C. $\frac{ab+cd}{bd}$ (87)D. $\frac{ad+bc}{bd}$ (2) E. None of the above	.13	.16	BAC
13. Which of the following statements can always be used to describe the least common multiple of two counting numbers? The least common multiple is -- (1) A. the product of the two numbers (10)B. the product of the factors of both numbers (0) C. the sum of the factors of the two numbers (7) D. the largest counting number divisible by each of the two numbers (??)E. the smallest counting number divisible by each of the two numbers	.18	.22	BAC
14. Which of the following is always a correct statement about division? (a, b, and c are all non-zero) (10)A. $a \div (b+c) = (a \div b) + (a \div c)$ (2) B. $a \div (b-c) = (a \div b) - (a \div c)$ (14)C. $(a \div b) \div c = (a \div a) \div (b \div c)$ (43)D. $(a + b) \div c = (a \div c) + (b \div c)$ (28)E. More than one of these (2) No Response	.57	.18	CBA
15. How many positive divisors does the number $2^2 \times 3^3 \times 5^2$ have? (10)A. 7 (4) B. 12 (4) C. 23 (72)D. 36 (11)E. None of the above	.28	.10	CAB
16. Which response best describes the statement "For all whole numbers a, b, c, $a \times (b \times c) = (a \times c) \times (b \times c)$"? (17)A. This is a true statement known as the distributive law of multiplication. (1) B. This is a true statement related to the associative law. (10)C. This is a true statement combining the associative and distributive laws. (0) D. This is a true statement illustrating the cancellation property. (72)E. This is not a true statement.	.28	.33	BAC
17. Which response best describes the expression "$0 \div 0$"? (2) A. It is defined to be 1 (18)B. It is defined to be 0 (64)C. It is undefined because $0 \times Y = 0$ has many solutions. (16)D. It is undefined because $0 \times Y = 0$ has no solution. (0) E. It is defined differently in different number systems.	.36	.49	ACB
18. Which arrangement of the following fractions is in order from smallest to largest? (0) A. 4/13, -4/13, 4/17, -4/17 (75)B. -4/13, -4/17, 4/17, 4/13 (1) C. 4/13, 4/17, -4/17, -4/13 (2) D. -4/17, 4/17, -4/13, 4/13 (20)E. -4/17, -4/13, 4/13, 4/17 (1) No Response	.25	.20	ACB

Item	Difficulty	Correlation	Group Comparisons
19	.19	.40	ABC
20	.31	.41	ACB
21	.13	-.02	BCA
22	.14	.27	ACB
23	.76	.07	BCA
24	.43	.18	BAC
25	.07	-.09	CAB
26	.16	.33	BAC

19. Which of the following rational numbers can be written as terminating decimals?

$\frac{7}{8}, \frac{9}{13}, \frac{17}{25}, \frac{11}{40}, \frac{6}{41}$

(81) A. $\frac{7}{8}, \frac{17}{25}, \frac{11}{40}$ (0) B. $\frac{9}{13}$ and $\frac{11}{40}$

(5) C. $\frac{7}{8}$ and $\frac{17}{25}$ (4) D. $\frac{9}{13}$ and $\frac{6}{41}$

(10) E. All of the numbers satisfy the condition

Directions: In 20-24 there may be <u>more than one</u> correct answer. Read each question carefully and blacken the appropriate space or spaces to signify the correct answer or answers.

20. The set or sets that can be put in one-to-one correspondence with $\{1,2,3,...\}$ would be

A. the set of people in China
B. $\{2,4,6,8,...\}$
C. the set of positive odd numbers less than 500
D. $\{...,-3,-2,-1\}$

21. The set $\{0,1,2,...\}$ is closed under

A. Addition
B. Subtraction
C. Multiplication
D. Division

22. For which of the following sets, K, is $N(K) = 0$

A. $K = \{\ \}$
B. $K = \{0\}$
C. $K = \{0,1\}$

23. In which of the following pairs of counting numbers is the first entry prime and the second entry composite?

A. 7, 101 B. 31, 87
C. 1, 20 D. 41, 51

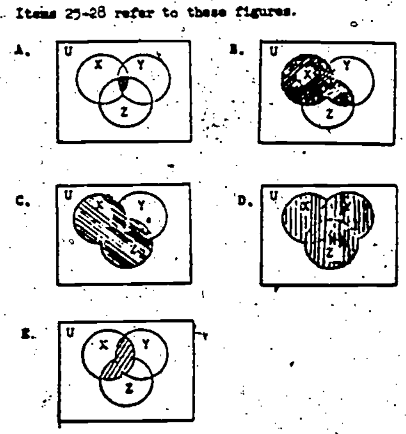
24. Which of the following are perfect squares?

A. 6! B. $2^3 \times 5^4$

C. $2^3 \times 3^3$ D. $9^2 \times 2$

E. $3^2 \times 5^4$

Items 25-28 refer to these figures.



25. $(X \cup Y) \cup Z$

(2) A. (0) B. (1) C.

(93) D. (2) E.

(1) No Response

26. $X \cap (Y \cap Z)$

(84) A. (2) B. (0) C.

(0) D. (1) E. (1) No Response

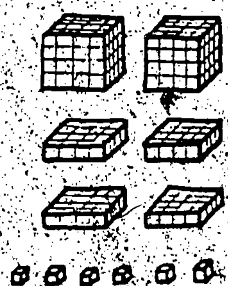
Item	Difficulty	Correlation	Group Comparisons
27. $X \cup (Y \cap Z)$ (0) A. (98) B. (0) C. (1) E. (1) No Response	.02	.08	CBA
28. $X \cup Z$ (0) A. (0) B. (98) C. (0) E. (1) No Response	.02	.06	BAC
29. Find the product in base 5 . $\begin{array}{r} 43_{five} \\ \times\ 32_{five} \\ \hline \end{array}$	.31	.25	ACB
30. Given the seventh row of Pascal's triangle 1 7 21 35 35 21 7 1 A. Write the 8th row B. Compute the number of 4 element subsets of a 9 element set.	.16	.36	ACB
31. Prove or disprove the following statement. If a is a factor of m , then $2a$ is a factor of $6m$.	.58	.37	ABC
32. Prove or disprove the following statement. If a is a factor of $b + c$, then a is a factor of b and a is a factor of c .	.37	.32	CAB
33. List all of the elements which have multiplicative inverses in the arithmetic of integers modulo 13 .	.80	.41	ACB
34. A. Compute the greatest common factor of 938 and 1162 . B. Is it possible to find integers X , Y so that $1162X + 938Y = 42$? Yes ________ No ________	.33	.31	ABC

Item	Difficulty	Correlation	Group Comparisons
35. A test was scored on a scale from 0 to 4 . The table below shows the number of students obtaining each score. What was the average score?	.37	.40	ACB

Score	Tally	Number of Students
4	𝍸 I	6
3	𝍸 II	7
2	𝍸 IIII	9
1	IIII	4
0	III	3

Item	Difficulty	Correlation	Group Comparisons
36. Discuss the main similarities and the main differences between the integers and the rational numbers.	.73	.32	ACB

Appendix E
Mathematics Methods Test Items and Item Statistics
(Year 1)

Item	Difficulty[a]	Correlation[b]	Group Comparisons[c]
1. Which sequence of problems would be best for teaching addition to first graders? A. 3 +5, 7 +4, 12 +6, 26 +5, 31 +25, 14 +18 B. 3 +5, 12 +6, 7 +4, 31 +25, 26 +5, 14 +18 C. 3 +5, 7 +4, 12 +6, 14 +18, 26 +5, 31 +25 D. 3 +5, 7 +4, 12 +6, 31 +25, 26 +5, 14 +18 E. 3 +5, 7 +4, 12 +6, 31 +25, 14 +18, 26 +5	.51	.35	CAB
2. Which numeral best describes the set of Dienes's blocks shown on the right after all possible trades are completed?  A. 2226_{four} B. 2606_{four} C. 312_{four} D. 3012_{four} E. 26_{four}	.43	.10	BCA

a = Difficulty is percentage of subjects who got this item wrong.
b = Correlation is between Item and (Test - Item).
c = See Appendix H.

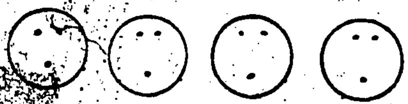
Item	Difficulty	Correlation	Group Comparisons
3. Which of the following would be suitable examples of sets for first and second grade students? A. {Peter, Paul, Mary} B. {I, II, III} C. {2 x 3, 3 x 3, 4 x 3} D. A and B only E. A, B, and C	.40	.13	CBA
4. What answer would you expect Judy to get for $\frac{2}{7} + \frac{1}{21}$ = if the error pattern of her previous work looked like this? I. $\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12} = 1\frac{7}{12}$ II. $\frac{1}{9} + \frac{2}{3} = \frac{7}{9} + \frac{6}{9} = 1\frac{1}{9}$ A. $1\frac{6}{147}$ B. $\frac{10}{21}$ C. $1\frac{10}{21}$ D. $1\frac{8}{147}$ E. $1\frac{8}{21}$	.10	.28	CAB
5. Which of the following sequences represents the order in which the following should be taught? (a) Written '4' (b) Spoken 'four' (c) Written 'IV' (d) Written 'four' A. (a), (b), (c), (d) B. (b), (c), (a), (d) C. (c), (d), (a), (b) D. (b), (a), (d), (c) E. (b), (d), (a), (c)	.12	.16	CAB

Item	Difficulty	Correlation	Group Comparison
6. What answer would you expect Jerry to get for 26 x37 if the error pattern of his previous work looked like this? I. 43 x 27 = 13201 II. 57 x 23 = 11571 III. 33 x 78 = 23364 A. 6003 D. 7622 B. 11362 E. 1362 C. 7262	.05	.10	ACB
7. Here are sample problems of George's work with long division: 5)1645 → 329; 6)3642 → 607; 7)2877 → 411; 4)1624 → 46; 3)1269 → 423 Which of the following best describe George's achievement on these five problems? A. George got all five problems correct. B. George makes errors and will probably miss most long division exercises. C. George makes errors but will probably only miss certain long division exercises. D. George makes errors with long division problems that seemingly have no pattern. E. George got the majority of the problems correct and his errors were due to faulty recall of basic multiplication facts.	.10	.12	BCA

Item	Difficulty	Correlation	Group Comparisons

8. Which one of the following models for multiplication leads most easily to the algorithms for multiplication of the common fractions and mixed numbers?

A.

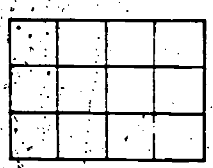


Question: How many dots are here?

B.

Question: How many points of intersection are here?

C.



Question: How many squares cover this figure? 

D.

Question: How many dots in this array? .30 .08 CAB

9. Shawn adds decimals in the following manner

.1 + .21 = .022 .42 + .237 = .00657

After interviewing her, it is found that her rule for decimal point placement is to count the number of decimal places in the addends and put this many places in the sum from right to left.

Which remediation practice seems likely to best help?

A. Have her change her decimals to common fractions, proceed by adding the fractions, and then convert her answer back to decimals.

B. Make a chart of place value, (i.e. "ones" column, "tenths" column, etc.) and have her do the exercises on it.

C. Have her estimate her answer before she begins an exercise.

D. Tell her that she is using the rule for multiplication of decimals and demonstrate the correct procedure, and give her 10 more exercises to work.

E. None of the above activities is likely to help Shawn. .34 -.17 ACB

10. Mary does the following problem as indicated.

$$\begin{array}{r} 56 \\ -\ 39 \\ \hline 17 \end{array}$$

Which choice below most nearly represents Mary's reasoning?

A. 56 - 39 = 50 + 6 - 30 - 9
= 40 + 10 + 6 - 30 - 9
= (40 - 30) + (10 - 9) + 6
= 10 + (1 + 6)
= 10 + 7
= 17

B. 56 - 39 = 50 + 6 - 30 - 9
= 40 + 16 - 30 - 9
= (40 - 30) + (16 - 9)
= 10 + 7
= 17

C. 56 - 39 = 50 + 6 - 30 - 9
= 50 + 6 + 10 - 10 + 30 - 9
= 50 + 16 - 40 - 9
= (50 - 40) + (16 - 9)
= 10 + 7
= 17 .14 .26 CBA

11. In what order should the following materials generally be used in teaching place value?

(a) Multibase blocks (c) popsicle sticks
(b) abacus (d) graph paper

A. (c), (a), (b), (d) B. (b), (c), (a), (d)
C. (d), (b), (a), (c) D. (a), (b), (c), (d)
E. (c), (b), (a), (d) .47 .11 BCA

12. If a blue is worth 7 yellows, a yellow is worth 7 greens, a green is worth 7 reds, and a red is worth 7 whites, then which of the following is worth the same as 5 blues?

A. 4 blues, 7 yellows, 7 greens, 7 reds, and 7 whites

B. 4 blues and 49 whites

C. 1715 whites

D. 35 whites

E. none of the above .35 .29 BAC

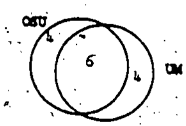
Directions:

In questions 13-17 you will be given a situation. You must decide if the listed expressions (13-17) are appropriate representations of the situation. If you feel the expression is appropriate, indicate this by marking A on your answer sheet. If the expression is not appropriate, indicate this by marking B on your answer sheet. Remember <u>more than one expression may be appropriate</u>. Do NOT concern yourself with working the problem.

Situation:

Ohio State University (OSU) and University of Michigan (UM) each play 10 football games not counting the game they play against each other. Six of the games each plays are against the same Big Ten teams. What is the total number of different teams which the 2 schools play?

13.

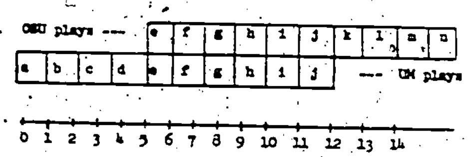


A. appropriate B. not appropriate .14 .12 CBA

14. $N(OSU) + N(UM) - N(OSU \cap UM) = 10 + 10 - 6$

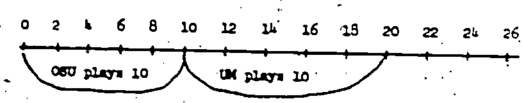
A. appropriate B. not appropriate .48 .04 BAC

15.



A. appropriate B. Not appropriate .18 .16 CBA

16.

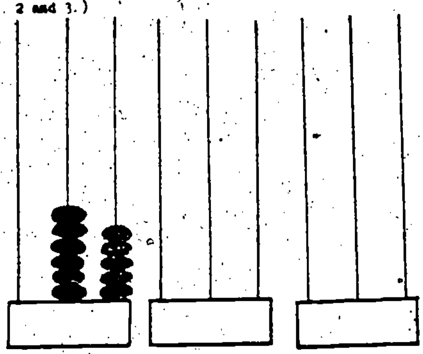


A. appropriate B. not appropriate .02 .09 CBA

17. $N(OSU \cup UM) + N(OSU \cap UM) = 8 + 6$

A. appropriate B. not appropriate .17 .13 ABC

Item	Difficulty	Correlation	Group Comparisons
Directions: In this section there may be <u>more than one</u> correct answer. Read each question and blacken the appropriate space or spaces to signify the correct answer or answers. †*			
18. Which of the following numeration concepts should a person know before he can understand regrouping in addition and subtraction? A. counting up to three place numbers B. place value C. digits D. addends and subtrahends E. counting by tens	.88	.03	<u>BCA</u>
19. A sixth-grade teacher would have her class develop a sieve of Eratosthenes primarily in order to help the pupils to A. understand the evolution of place value B. grasp the significance of number characteristics C. develop an easier way to factor numbers D. determine which numbers are prime numbers E. realize the value of tens as a base of our number system	.80	-.06	<u>BAC</u>*
20. Show a step-by-step solution on a twenty-bead abacus for the problem 65 + 5. (Step one is done for you. Draw in the beads for steps 2 and 3.)	.13	.23	<u>CBA</u>



step 1 step 2 step 3

† .05 ≤ p ≤ .10 * p ≤ .05, but no pairwise differences

Appendix 8
Mathematics Problem Solving Test
Items and Item Statistics
(Year 1)

Item	Difficulty[a]	Correlation[b]	Group Comparison[c]
1. A car wash charges \$1.75 for a wash but reduces prices in relation to the amount of gasoline purchased. amount: 0-7.9 gal, 8-9.9 gal, 10-11.9 gal, 12-14.9 gal, 15 gal to fill cost: \$1.75, \$1.38, \$1.28, \$1.18, \$.98 If you buy 10.7 gallons of gas at the car wash at a price of 65.9 cents per gallon and you also have your car washed, what is the total cost?	.12	.41	<u>BAC</u>
2. A 17-element set has 131072 subsets. How many of these subsets contain 9 or more elements?	.64	.31	<u>ACB</u>
3. How many distinct three-digit numbers can be represented with only the numerals 1, 2, and 3? (A numeral may be used more than once in any given representation.)	.30	.33	<u>BAC</u>
4. Find the smallest (a) square (b) cube which has 180 as a factor. Explain your reasoning. Do the same using 1440 as a factor.	.72	.23	<u>ABC</u>
5. A <u>palindrome</u> is a whole number which reads the same forwards as backwards. For example, 131, 2552, 34,743 are palindromes. A. How many 2-digit palindromes are there? 3 digits? B. What whole number greater than 1 is a factor of every 4-digit palindrome? Prove your answer.	.60	.24	<u>BCA</u>
6. Find the smallest rational number which when divided by $\frac{6}{35}$, $\frac{10}{21}$, and $\frac{15}{49}$ yields a whole number as the quotient in each case.	.8[illegible]	.1[illegible]	<u>CBA</u>

a = Difficulty is percentage of subjects who got this item wrong.
b = Correlation between Item and (Test - Item).
c = See Appendix N.

Item	Difficulty	Correlation	Group Comparison
7. In your class of 11 students, on a recent 20 point quiz, the average was 13. Unfortunately you misplaced Joan's paper. The remaining 10 scores were 12, 18, 3, 17, 11, 19, 10, 15, 14, 6. What was Joan's score?	.32	.17	<u>ACB</u>
8. A. How many whole numbers less than or equal to 30 have either 2 or 3 as a factor? Explain your reasoning. B. How many whole numbers less than or equal to 900 have either 3 or 5 as a factor? Explain your reasoning.	.69	.32	<u>BAC</u>
9. It takes 867 digits to number the pages of a book. (Page 9 uses one digit, page 37 uses two digits, etc.) A. How many pages are there in the book? B. How many times does the digit 5 appear?	.89	.26	<u>ACB</u>
10. What is the smallest integer greater than 1 which is simultaneously -- A. A square and a cube? B.* A square, a cube, and a fifth power? * It is sufficient to find the prime factorization (using exponential notation) for this integer.	.84	.29	<u>BAC</u>

Appendix T
Concepts, Methods, and Problem Solving Test Statistics
(Year 1)

	Concepts	Methods	Problem Solving
Number of Items	36	20	10
Maximum Score	44	21	100
Mean Score	30.1	13.9	40.0
Standard Deviation	5.90	2.44	15.0
Average Difficulty (percentage wrong)	.28	.31	.60
Average Item To (total minus item) Correlation	.23	.14	.27
Internal Consistency Reliability (Cronbach's α)	.75	.42	.58

Appendix V
Scheffé Multiple Comparisons of Univariate Differences
Between Pilot Groups on Post Test Criteria
(Year 1)

Concepts ($P \leq 0.001$ on univariate test)

Comparison	Mean Difference	$\hat{\psi}/\hat{\sigma}_{\hat{\psi}}$ [a]	Result
Pilot A - Pilot B	-6.44	4.38	$p \leq 0.01$
Pilot A - Pilot C	-3.76	2.33	$p \leq 0.1$
Pilot B - Pilot C	2.68	1.69	NSD

A   C ——— B   (See Appendix X)

Methods ($P \leq 0.095$ on univariate test)

Comparison	Mean Difference	$\hat{\psi}/\hat{\sigma}_{\hat{\psi}}$	Result
Pilot A - Pilot B	0.80	1.22	NSD
Pilot A - Pilot C	1.58	2.20	$P \leq 0.1$
Pilot B - Pilot C	0.78	1.10	NSD

C ——— B ——— A

[a] $\hat{\psi}$ is the Scheffe contrast for comparing means and $\hat{\sigma}_{\hat{\psi}}$ is the square root of the variance of the contrast (standard error of the contrast as in Glass pp. 388-389, 1970).

Appendix W
Intercorrelations of Academic Ability Background Variables
with Criterion Variables
(Year 1)

	Concepts Criterion	Methods Criterion	Prob. Solv. Criterion
Math Pretest	.39*** (83)	.27* (83)	.43*** (83)
GPA	.42*** (83)	.11 (83)	.21 (83)
Expected M105 Grade	.31** (82)	.02 (82)	.23* (82)
Expected E502 Grade	.29** (82)	.10 (82)	.14 (82)
College Math Rating	.38*** (83)	.10 (83)	.24* (83)
Years of H.S. Math	.38*** (82)	.07 (82)	.14 (82)
Avg. H.S. Math Grade	.34** (82)	.25* (82)	.18 (82)

$^{*}p \leq .05$ $^{**}p \leq .01$ $^{***}p \leq .001$

Number in Parenthesis Indicate Sample Sizes

Appendix U
Frequency Distributions by Pilots on Concepts,
Methods, and Problem Solving Tests
(Year 1)

Frequency Distribution by Pilots on Mathematics Concepts Test

	Pilot A	Pilot B	Pilot C	Total
Below 20	1	1	0	2
20-21	3	0	2	5
22-23	3	0	0	3
24-25	7	0	0	7
26-27	3	1	0	4
28-29	0	5	4	9
30-31	1	4	5	10
32-33	4	2	6	12
34-35	1	4	1	6
36-37	1	4	0	5
38-39	0	4	1	5
40-41	1	2	0	3

Frequency Distributions by Pilots on the Methods of Teaching Mathematics Test

	Pilot A	Pilot B	Pilot C	Total
9	1	1	0	2
10	0	1	3	4
11	1	3	1	5
12	2	4	3	9
13	1	2	3	6
14	6	3	4	13
15	5	3	1	9
16	2	6	3	11
17	3	3	1	7
18	4	0	0	4
19	0	1	0	1

Frequency Distribution by Pilots on the Mathematics Problem Solving Test

	Pilot A	Pilot B	Pilot C	Total
Below 10	0	2	1	3
10-14	2	0	0	2
15-19	0	1	2	3
20-24	1	0	0	1
25-29	2	4	0	6
30-34	5	0	0	5
35-39	2	8	3	13
40-44	6	3	2	11
45-49	6	1	5	12
50-54	1	2	3	6
55-59	0	3	2	5
60-64	0	2	1	3
Above 64	0	1*	0	1

*Actual score was 82

Appendix X
Intercorrelations of Attitude Pretests, Attitude Posttests and Criterion Measures.
(Year 1)

	Attitude Pretests: Mathematics	Attitude Pretests: Teaching Mathematics	Attitude Pretests: Teaching Elem. Sch. Children	Attitude Pretests: Elementary School Children	Attitude Posttests: Mathematics	Attitude Posttests: Teaching Mathematics	Attitude Posttests: Teaching Elem. Sch. Children	Attitude Posttests: Elementary School Children	Criterion Measures: Concepts	Criterion Measures: Methods
Attitude Pretests: Teaching Mathematics	.49*** (82)									
Attitude Pretests: Teaching Elem. Sch. Children	.11 (80)	.32** (81)								
Attitude Pretests: Elem. Sch. Children	.19* (80)	.08 (81)	.08 (81)							
Attitude Posttests: Mathematics	.50*** (74)	.29** (75)	.01 (73)	-.17 (73)						
Attitude Posttests: Teaching Mathematics	.29* (74)	.41*** (75)	.36*** (73)	-.13 (73)	.42*** (75)					
Attitude Posttests: Teaching Elem. Sch. Children	.11 (74)	-.01 (75)	.30** (73)	.09 (73)	.20* (75)	.25* (75)				
Attitude Posttests: Elem. Sch. Children	.10 (74)	.09 (75)	-.13 (73)	.66*** (73)	-.26* (75)	.08 (75)	.22* (75)			
Criterion Measures: Concepts	.34*** (82)	.06 (83)	.12 (81)	.01 (81)	.19 (75)	.01 (75)	.07 (75)	.04 (75)		
Criterion Measures: Methods	.18* (82)	.14 (83)	-.08 (81)	.09 (81)	.15 (75)	.03 (75)	.00 (75)	.06 (75)	.34*** (83)	
Criterion Measures: Problem Solving	.27** (82)	.19* (83)	.05 (81)	.14 (81)	.29** (75)	.11 (75)	.02 (75)	.25* (75)	.45*** (83)	.34*** (83)

n

$*p \leq .05$ $**p < .01$ $***p < .005$

Appendix Y
Raw and Adjusted Means for Attitude Post Test Scores by Pilot Group
(Year 1)

	Pilot A $n = 29$		Pilot B $n = 27$		Pilot C $n = 19$	
Attitude:	$\bar{X}_{RAW}$	$\bar{X}_{ADJ}$[a]	$\bar{X}_{RAW}$	$\bar{X}_{ADJ}$	$\bar{X}_{RAW}$	$\bar{X}_{ADJ}$
Mathematics	32.48	32.69	33.15	32.94	32.74	32.76
Teaching Mathematics	35.16	35.54	34.93	34.37	34.74	35.03
Teaching Elementary School Mathematics	35.00	35.00	36.59	36.47	35.10	35.28
Elementary School Children	32.32	31.72	30.70	31.17	32.42	32.55

[a]The adjusted means are the post attitude scale scores with the variance due to initial differences on the two pre test attitude scales removed.

Appendix Z
Multivariate Analysis of Covariance for Attitude Score Differences Among Pilot Groups
(Year 1)

Multivariate (4 dependent variables).

	$F(8, 126)$:		$P <$
	1.315		0.242
Univariate Variables	$F(2, 66)$	MS_B	$p \leq$
Attitude Post Tests:			
Mathematics	0.023	0.356	0.975
Teaching Mathematics	1.381	7.759	0.258
Teaching Elementary School Children	1.452	13.413	0.241
Elementary School Children	1.369	9.781	0.261

Appendix AA

Syllabus for Mathematics Concepts Courses (Year 2)

M105

Counting techniques including Pascal's Triangle

Primes, factoring, divisibility

Rational numbers and their arithmetic

Probability

Decimals

Ratios, rate, percent

Geometric figures

Congruence

Similarity

M106

One-to-one correspondence

Numeration

Whole number arithmetic

Measurement of lengths, angles, areas, volumes

Geometric transformations

Appendix BB

Background Variable Means and Standard Deviations by Group

(Year 2)

	X $n = 68$		Y $n = 69$	
Variable	$\bar{X}$	SD	$\bar{X}$	SD
Class Standing[a]	1.74	0.68	1.99	0.76
GPA[b]	2.87	1.00	3.05	1.03
Expected M105 Grade[b]	3.35	.66	3.39	0.62
College Math Rating[c]	2.72	1.34	3.07	1.48
Participation in FEEP[d]	1.21	0.41	1.06	0.24
Sex[e]	1.93	0.26	1.91	0.28
Age[f]	1.12	0.32	1.32	0.78
Years of H.S. Math	2.84	0.96	2.88	0.96
Average H.S. Math Grade[b]	2.97	0.69	3.00	0.75
M105 Final Exam Score	103.93	14.89	107.31	16.36

a 1=Freshman, 2=Sophomore, 3=Junior, 4=Senior
b 4=A, 3=B, 2=C, 1=D, 0=E
c Range 1 thru 9. 9=most advanced courses
d 1 Yes, 2 No
e 1 Male, 2 Female
f 1 Under 21, 2=21-24, 3=25-30, 4=Above 30

Appendix CC
First Mathematics Content Course Final Examination and Test Statistics (Year 2)

Math 10? Final Exam Name ______________________

150 points Rec. Instructor ______________________

Winter 1978 Recitation Day Tue Thur

Questions 1-25 3 points each

Questions 26-40 5 points each

In this section choose one answer for each question.

1. $4\frac{1}{4} \div 2\frac{1}{3}$ equals
   - A. $\frac{20}{11}$
   - B. $\frac{51}{28}$
   - C. $\frac{28}{51}$
   - D. $\frac{119}{12}$
   - E. None of the above

2. The least common multiple of 36 and 14 is
   - A. 504
   - B. 252
   - C. 42
   - D. 2
   - E. None of the above

3. Which of the following are prime numbers? 7, 9, 57, 91
   - A. only 7 and 91
   - B. only 7, 57 and 91
   - C. only 7, 9, 91
   - D. only 7
   - E. All of the numbers are prime

4. Which numeral names the number represented in the chart?

ones	tenths	hundredths	thousandths
2	12	8	23

   - A. 2.12823
   - B. 2.1231
   - C. 3.303
   - D. 3.2103
   - E. None of the above

5. Round the decimal 231.7638 to the nearest hundredth.
   - A. 200
   - B. 231
   - C. 231.77
   - D. 231.763
   - E. None of the above

6. $GCF(3 \times 5^2 \times 7 \times 11, 2^2 \times 3 \times 5 \times 7^2) =$
   - A. $3 \times 5 \times 7$
   - B. $2 \times 3 \times 5 \times 7 \times 11$
   - C. $2^2 \times 3 \times 5^2 \times 7^2 \times 11$
   - D. 1
   - E. $2^2 \times 3^2 \times 5^3 \times 7^3 \times 11$

7. Which arrangement of the following decimals is in order from smallest to largest?
   - A. .6, .61, .611, .601
   - B. .601, .6, .61, .611
   - C. .611, .61, .6, .601
   - D. .6, .601, .61, .611
   - E. None of the above

8. A bag contains 3 blue marbles, 2 green ones, and 5 red ones. A marble is selected at random. What is the probability it is green?
   - A. 1/5
   - B. 1/3
   - C. 1/4
   - D. 1/10
   - E. None of the above

9. For all whole numbers a, b, c, and d ($b \neq 0$, $d \neq 0$), the sum $\frac{a}{b} + \frac{c}{d}$ is
   - A. $\frac{a + c}{b + d}$
   - B. $\frac{a + c}{bd}$
   - C. $\frac{ab + cd}{bd}$
   - D. $\frac{ad + bc}{bd}$
   - E. None of the above

10. Which of the following statements can always be used to describe the least common multiple of two counting numbers?

    The least common multiple is --
    - A. the product of the two numbers.
    - B. the product of the factors of both numbers.
    - C. the sum of the factors of the two numbers.
    - D. the largest counting number divisible by each of the two numbers.
    - E. the smallest counting number divisible by each of the two numbers.

11. Which of the following is always a correct statement about division? (a, b, c are all $\neq 0$)
    - A. $a \div (b + c) = (a \div b) + (a \div c)$
    - B. $a \div (b - c) = (a \div b) - (a \div c)$
    - C. $(a + b) \div c = (a \div c) + (b \div c)$
    - D. $(a \div b) \div c = (a \div c) \div (b \div c)$
    - E. More than one of these

12. How many factors does the number $2^2 \times 3^3 \times 5^2$ have?
    - A. 7
    - B. 12
    - C. 23
    - D. 36
    - E. None of the above

13. Which response best describes the statement "For all rational numbers a, b, c, $a \times (b \times c) = (a \times c) \times (b \times c)$"?
    - A. This is a true statement known as the distributive law of multiplication.
    - B. This is a true statement related to the associative law.
    - C. This is a true statement combining the associative and distributive laws.
    - D. This is a true statement illustrating the cancellation property.
    - E. This is not a true statement.

14. Which response best describes the expression "0 ÷ 0"?

A. It is defined to be 1.
B. It is defined to be 0.
C. It is undefined because $0 \times Y = 0$ has many solutions.
D. It is undefined because $0 \times X = 0$ has no solution.
E. It is defined differently in different number systems.

15. Which of the following rational numbers can be written as terminating (finite) decimals?

$\frac{7}{8}, \frac{9}{13}, \frac{17}{25}, \frac{11}{40}, \frac{6}{41}$

A. only $\frac{7}{8}, \frac{17}{25}, \frac{11}{40}$
B. only $\frac{9}{13}$ and $\frac{11}{40}$
C. only $\frac{7}{8}$ and $\frac{17}{25}$
D. only $\frac{9}{13}$ and $\frac{6}{41}$
E. All of the numbers satisfy the condition.

16. In a horse race, I decide to bet on two horses, horse A and horse B. If horse A has probability 1/6 of winning and horse B has probability 1/4 of winning, what is the probability that either horse A or horse B wins? (No ties allowed)

A. 1/24
B. 4/6
C. 2/10
D. 7/24
E. 5/12

17. A child decides that the best way to add fractions would be $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$. Which of the following best describes the reason he should be dissuaded.

A. That is not the way it is done.
B. This sort of operation doesn't apply to any real situation.
C. This method would be too easy.
D. Different representation of the rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ by fractions would lead to different sums.
E. You have to get a common denominator before you can add.

18. A rectangle has how many lines of symmetry?

A. always only 1
B. always only 2
C. always only 4
D. sometimes 2, sometimes 4
E. sometimes 1, sometimes 2

19. Which arrangement of the following fractions is in order from smallest to largest?

A. $\frac{8}{9}, \frac{9}{10}, \frac{9}{11}, \frac{11}{12}$
B. $\frac{11}{12}, \frac{9}{10}, \frac{8}{9}, \frac{9}{11}$
C. $\frac{11}{12}, \frac{9}{11}, \frac{9}{10}, \frac{8}{9}$
D. $\frac{9}{10}, \frac{9}{11}, \frac{11}{12}, \frac{8}{9}$
E. $\frac{9}{11}, \frac{8}{9}, \frac{9}{10}, \frac{11}{12}$

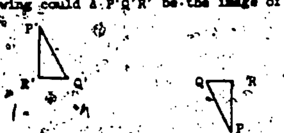
20. A bag contains 6 red chips and 5 blue chips. Two chips are drawn in succession from the bag (without replacement). What is the probability that both chips are blue?

A. $\frac{9}{11}$
B. $\frac{20}{121}$
C. $\frac{2}{11}$
D. $\frac{25}{121}$
E. None of these

21. 1.08 ÷ .144 =

A. .75
B. .705
C. 7.5
D. 75
E. 70.5

22. Δ PQR → Δ P'Q'R'. For which of the following could Δ P'Q'R' be the image of Δ PQR? (R', Q', Q and R lie along the same line)



A. a slide
B. two slides
C. a turn
D. a flip and a turn
E. a slide followed by a flip

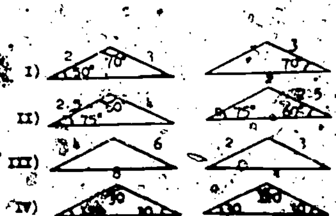
23. Find x.

A. 7/3
B. 3/2
C. 14/3
D. 21/2
E. cannot be determined

24. Given ΔABC and ΔDEF with AB = 8, AC = 7.5, BC = 4.7, DE = 8, DF = 7.5, EF = 4.7 and measure of angle B equal to 56°. What is the measure of angle F?

A. 56°
B. 124°
C. 72°
D. 90°
E. Cannot be determined

25. Which of the following pairs of Δ's are congruent?



A. I
B. II
C. III
D. IV
E. none of the above

In questions 26-30 there may be more than one correct answer. Mark all appropriate responses.

26. In which of the following pairs of counting numbers is the first entry prime and the second entry composite?

A. 7, 101
B. 31, 87
C. 1, 20
D. 41, 51
E. 12, 61

27. Which of the following are perfect squares?

A. $6 \times 5 \times 4 \times 3 \times 2 \times 1$
B. $2^3 \times 5^4$
C. $2^3 \times 3^3$
D. $9^2 \times 2$
E. $3^2 \times 5^4$

28. Which of the following represent facts which are true for the arithmetic of rational numbers but not for whole numbers?

A. There is an additive identity, i.e., a zero.
B. Every nonzero number has a multiplicative inverse (reciprocal).
C. Every number has an additive inverse (opposite).
D. The distributive property holds.
E. Division can be carried out whenever the divisor is ≠ 0.

29. 9 is a factor of N and 12 is a factor of N. Which of the following must also be factors of N?

A. 3
B. 36
C. 108
D. 18
E. 6

30. A certain number, written base 10, has a sum of digits 36 and a last digit of '0'. One knows that which of the following are factors?

A. 6
B. 15
C. 36
D. 4
E. 90

Directions: Answer the following questions. Show your work in the space provided below each question.

31. Given the seventh row of Pascal's triangle 1 7 21 35 35 21 7 1

A. Write the 8th row

B. Compute the number of ways of choosing a subcommittee of 4 persons from a group of 9 people.

32. Prove or disprove the following statement. If a is a factor of m then 2a is a factor of 6m.

33. Prove or disprove the following statement. If p is a prime factor of b + c then p is a factor of b or p is a factor of c.

34. List all of the elements which have multiplicative inverses on the clock with 18 numbers.

35. A. Compute the greatest common factor of 938 and 1162.

B. Reduce $\frac{1162}{938}$ to lowest terms.

36. A merchant buys a product and then sells it for $[illegible]. This selling price provides exactly a 20% profit on his original investment. What was the original cost of the product?

37. On a clock with 34 numbers, find the numbers representing

A. 118 A. ______
B. -13 B. ______
C. the multiplicative inverse (reciprocal) of 23 C. ______

38. Does the set {1,3,4,5,6,7,8,9,...} with the usual multiplication of counting numbers possess unique factorization? Justify you response.

39. Find a pair of numbers whose LCM is 300 and whose GCF is 18 (or argue that this is not possible.)

40. A school district with 6000 students has a teacher to pupil ratio of 1:30. How many teachers need to be hired to achieve a teacher to pupil ratio of 1:27?

First Mathematics Content Course Final Exam

Test Statistics (Year 2)

Number of Items	62
Possible Score	150
Number of Subjects	146
Mean	104.30
Standard Deviation	19.80
Average Item - Total Correlation	0.28
Average Difficulty	0.29
Internal Consistency Reliability	0.83

Appendix DD

Means and Standard Deviations of Attitude Scales by Group (Year 2)

Attitude Scale		Group X n = 68 $\bar{X}$	Group X SD	Group Y n = 69 $\bar{X}$	Group Y SD
Mathematics	Pre[a]	30.16	4.96	31.46	4.37
	Mid[b]	31.43	4.23	32.57	3.39
	Post[c]	31.52	4.21	32.48	3.53
Teaching Mathematics	Pre	34.01	2.81	34.25	3.44
	Mid	34.48	2.43	34.88	2.74
	Post	34.38	2.49	35.44	2.26
Teaching El. School Children	Pre	35.65	3.73	36.81	2.66
	Mid	35.32	3.15	36.03	3.24
	Post	35.00	3.38	36.65	2.82
Elementary School	Pre	31.56	3.21	31.56	3.08
	Mid	31.02	3.16	31.68	3.09
	Post	31.61	3.47	32.19	2.78

a Pre. = Pretest, given at the conclusion of Winter Quarter

b Mid = Midtest, given during the 5th week of Spring Quarter

c Post = Posttest, given during at the conclusion of Spring Quarter

Appendix EE

Multivariate Analysis of Variance for Pretest Attitude Scores Between Groups (Year 2)

Multivariate	$F(4,132)$	$p \le$
	1.64	0.167

Univariate	$F(1,135)$	MS_B	$p \le$
Mathematics	2.66	58.06	0.105
Teaching Math.	0.21	2.08	0.647
Teaching El. School Children	4.36	46.45	0.039
El. School Children	0.00	0.00	0.990

Appendix FF

Mathematics Concepts Test Items, Item Statistics and Test Statistics (Year 2)

Math 106 | Final Exam | Name ______________________

Spring 1978 | Concepts | M106 Instructor(s) ______________________

You have 55 minutes for this 30 item test. Work neatly. The exam is worth 100 points as indicated.

Part I (63 points) Choose the one best answer. — Difficulty[a] / Correlation[b] / Group Comparison[c]

1. Which numeral names the number represented in the chart?

Thousands	Hundreds	Tens	Ones
2	12	8	23

A. 212,823
B. 21,231
C. 32,103
D. 3,303
E.* None of the above

Difficulty .08 — Correlation .10 — Group Comparison XY

2. The following set has ______ non-empty subsets: {m, n, o, p}

A. 4
B. 12
C. 15
D. 24
E. None of these

Difficulty .26 — Correlation .23 — Group Comparison XY

3. Which response best describes the expression "0 ÷ 0"?

A. It is defined to be 1.
B. It is defined to be 0.
C. It is undefined because 0 x Y = 0 has many solutions.
D. It is undefined because 0 x X = 0 has no solution.
E. It is defined differently in different systems.

Difficulty .33 — Correlation .39 — Group Comparison XY

4. The associative property for addition of counting numbers is a consequence of which of the following for any choice of sets S and T (and R)?

A. (S x T) x R = S x (T x R)
B. S ∩ T = T ∩ S
C. S x T = T x S
D. (S ∪ T) ∪ R = S ∪ (T ∪ R)
E. (S x T) x R matches S x (T x R)

Difficulty .21 — Correlation .16 — Group Comparison XY

5. In the problem shown, one must "carry" 2. What is the actual value of this 2?

```
   25
  126
 x 29
 1134
 252
 3654
```

A. 10²
B. 2
C. 250
D. 20
E. None of the above.

Difficulty .28 — Correlation .44 — Group Comparison YX

a Difficulty is percentage of subjects who got this item wrong.
b Correlation between Item and (Test - Item).
c See Appendix H.

6. In using the usual subtraction algorithm to compute 496 - 128 we regroup and consider 496 as

A. 4 tens 9 hundreds and 6 ones
B. 3 hundreds 19 tens and 6 ones
C. 4 hundreds 8 tens and 16 ones
D. 4 hundreds 19 tens and 5 ones
E. None of the above.

Difficulty .02 — Correlation .17 — Group Comparison XY

7. The base four numeral for the number of letters in the English alphabet is

A. 130_{four}
B. 32_{four}
C. 26_{four}
D. 122_{four}
E. None of the above.

Difficulty .29 — Correlation .22 — Group Comparison YX ($p \le .021$)

8. Which of the following is listed in order smallest to largest?

A. 6 + 2, 6 + 3, 6 + -1, 6 + -2, -6 + -3
B. 6 + -1, 6 + -2, -6 + -3, 6 + 2, 6 + 3
C. -6 + -1, 6 + -1, 6 + -2, 6 + 3, 6 + 2
D. 6 + -1, 6 + [illegible] 6 [illegible] + 2, -6 + -1
E. None of the above.

Difficulty .05 — Correlation .14 — Group Comparison YX

9. -3 x (6 + (-8)) =

A. -26
B. 30
C. 6
D. -24
E. None of the above.

10. In what base is the addition problem

```
  132
+ 352
  504
```

being worked?

A. 3
B. 8
C. 6
D. [illegible]
E. None of the above.

Difficulty .06 — Correlation .04 — Group Comparison YX ($p \le .004$)

11. If P is a set with 6 elements and Q is a set with 7 elements, then N(P ∪ Q)

A. equals 1
B. equals 6
C. equals 7
D. equals 13
E. Cannot be determined.

Difficulty .15 — Correlation .39 — Group Comparison YX

	Difficulty	Correlation	Group Comparisons

12. The figure shown has the following symmetries:

A. 2 turn symmetries only
B. 2 turn symmetries and only 1 flip-symmetry
C. 2 flip symmetries and at least 1 turn symmetry
D. only 1 turn symmetry, only 1 flip symmetry, and only 1 glide symmetry
E. [illegible] above

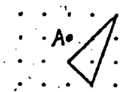
.37 .08 XY

13. Let [illegible] and [illegible] the transformations of the coordinate plane where f is a 90° [illegible] turn (rotation) and g is a flip (reflection) in the line [illegible] and $(0,5)$. Then $g \circ f\ (1,2) =$

A. [illegible]
B. [illegible]
C. $(-1,-2)$
D. $(-2,-1)$
E. None of the above.

.74 .20 XY

14. A triangle is placed on a geoboard as shown. Which of the following shows the geoboard after it has been rotated 90° counter-clockwise about point A ?



.25 .28 YX

None of these.

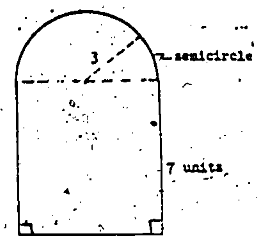
A. B. C. D. E.

15. If the image of the point $(1,2)$ under a slide is $(3,1)$, what is the image of $(1,1)$ under that slide?

.07 .16 XY

A. $(3,3)$
B. $(3,2)$
C. $(3,0)$
D. $(3,1)$
E. $(1,3)$

16. The perimeter of the figure to the right is exactly



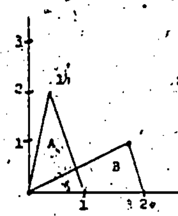
A. $26 + 3\pi$
B. $13 + \frac{9\pi}{2}$
C. 29
D. 26
E. $20 + 3\pi$

17. The area of the figure in the previous problem is exactly

A. $42 + 3\pi$
B. $49 + 9\pi^2$
C. $42 + \frac{9\pi}{2}$
D. 46.5π
E. 56.13

	Difficulty	Correlation	Group Comparisons
Item 16.	.38	.28	XY ($p \leq .031$)
Item 17.	.25	.47	XY

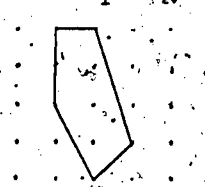
18. Compare the areas of these triangles:



.14 .21 YX

A. the area of A is greater than the area of B.
B. the area of B is greater than the area of A.
C. the areas of A and B are equal.
D. the areas of A and B are unknown.
E. None of the above.

19. Find the area of the displayed pentagon :



A. 5
B. 6
C. $7\frac{1}{2}$
D. $8\frac{1}{2}$
E. 10

.07 .18 XY

20. Four persons have volunteered for president of the Garden Club and three have volunteered for secretary. The nominating committee will choose 2 for each office to put on a ballot. In how many ways may the committee select the four candidates? (The ordering of names on the ballot is immaterial.)

.41 .13 YX

A. 9
B. 72
C. 12
D. 144
E. 18

Math 106-Final Exam - Page 5

	Difficulty	Correlation	Group Comparisons
21. If an operation $a * b$ is defined on the whole numbers by $a * b = b$ (for example, $3 * 5 = 5$) then the operation *			
A. is both commutative and associative	.88	.26	XY
B. is commutative but not associative			
C. is associative but not commutative			
D. is neither commutative nor associative			
E. None of the above.			

Part II (22 points) There may be more than one correct answer. Choose all correct answers.

	Difficulty	Correlation	Group Comparisons
22. The set or sets that can be put in one-to-one correspondence with {1, 2, 3, . . .} would be			
A. {2, 4, 6, 8, . . .}	.10	.48	YX
B. the set of positive odd numbers less than 500	.03	.14	XY
C. {. . ., -3, -2, -1}	.29	.37	XY
D. {0, 1, 2, 3, . . .}	.09	.35	XY
23. For the segments $\overline{AB}$ and $\overline{CD}$ which are true			
A. The set of points in $\overline{AB}$ matches the set of points in $\overline{CD}$.	.47	.55	YX
B. The set of points in $\overline{AB}$ matches a proper subset of the points in $\overline{CD}$.			
C. The set of points in $\overline{CD}$ matches a proper subset of the points in $\overline{AB}$.	.69	.46	YX
D. $\overline{AB}$ is congruent to $\overline{CD}$.	.03	.07	XY
E. The length of $\overline{AB}$ is greater than the length of $\overline{CD}$.	.04	.12	YX
24. Which of the following describe $n-m$ if n and m are counting (whole) numbers with $n \geq m$?			
A. $n - m = N(A \sim B)$ where $N(A) = n$, $N(B) = m$, and $B \subset A$.	.13	.30	XY
B. $n - m = N(C \cap D)$ where $N(C) = n$, $N(D) = m$, and $C \cap D \neq \{\}$.	.16	.05	YX
C. $n - m$ is the counting (whole) number t satisfying $t + m = n$.	.11	.16	XY
D. $n - m = N(R \cup S \cup T)$ where R, S, T are m disjoint sets and $N(R) = N(S) = N(T) = n$.	.04	.23	XY ($p \leq .044$)
E. None of the above.			
25. On the set of even integers {. . . , -6, -4, -2, 0, 2, 4, 6, . . .} the operation x (multiplication)			
A. is associative	.08	.45	YX
B. has a multiplicative identity	.49	.33	YX
C. has multiplicative inverses for each non-zero member	.45	.46	YX
D. satisfies the cancellation property			

26. When working with only counting (whole) numbers, which of the following describe $p \div q$?	Difficulty	Correlation	Group Comp.
A. $p \div q$ is the counting (whole) number t satisfying $q \times t = p$.	.10	.03	YX
B. $p \div q$ is the counting (whole) number t satisfying $p \times t = q$.	.01	.11	XY
C. $p \div q$ is the number of members in each set occurring when a set with p members is subdivided into q matching subsets.	.25	.39	XY ($p \leq .022$)
D. $p \div q$ is the number of disjoint sets each with q members whose union is a set with p members.	.57	.11	YX
E. $p \div q$ is the exact number of times q must be subtracted from p to obtain 0.	.18	.20	XY ($p \leq .001$)

Part III (15 points)

	Difficulty	Correlation	Group Comparisons
27. Find the product in base five: $43_{five} \times 32_{five}$	[illegible]	[illegible]	YX
28. a. List three properties which hold for both the integers and the rational numbers.	.25	.31	XY
b. State one property which holds for one of these (integers or rational numbers), but not the other. Be specific.	.60	.34	XY
29. a. Tell whether the following statement is true or false: "If s, t, and r are integers with $s < t$ and $t < r$, then $s < r$."	.03	.14	YX
b. Defend your answer to part a.	.70	.43	YX ($p \leq .001$)
30. Using basic definitions show that $3 + 4 = 7$.	.22	.27	XY ($p \leq .001$)

Concepts Test Statistics

(Year 2)

Number of Items	46
Possible Score	95
Number of Subjects	146
Mean	69.92
Standard Deviations	9.38
Average Item - Total Correlation	0.17
Average Difficulty	0.25
Internal Consistency Reliability	0.63

Appendix GG
Mathematics Methods Test Items, Item Statistics, and Test Statistics.
(Year 2)

OHIO STATE UNIVERSITY

EMC 502
Spring, 1978
Final Exam

Your Name: ______________ (first) ______________ (last) (please print)

Instructor's Name: ______________

This exam may include some questions related to topics which you may have had little or no experience with in EMC 502 this quarter. Do not worry about these questions. Answer them if you can and go on to the next question.

(Circle the letter which represents the best response to each item.)

	Difficulty[a]	Correlation[b]	Group Comparisons[c]

1. Which of the following problems would be the best one to use in developing the array idea of multiplication?
   - A. How many 4-digit license plates can be made using the numerals 0 thru 9?
   - B. A cricket jumps 3 feet each time it jumps. If it jumps 6 times, how far does it travel?
   - C. Tim has 4 different colored shirts and 3 different colored ties. How many different outfits are possible?
   - D. How much will it cost Sherri to buy three oranges that cost 4¢ each?
   - E. How many coins will Jim need to fill a page that has three rows across and four columns?

   .13 .21 XY ($p \leq .016$)

2. In teaching addition of one-digit numbers we use the types of activities listed below:
   - (a) Activities using manipulative materials
   - (b) Written drill on basic facts
   - (c) Worksheets on which examples are illustrated

   Which of the following sequences indicates the order in which each type of activity should be emphasized?
   - A. c, b, a
   - B. a, c, b
   - C. b, a, c
   - D. c, a, b
   - E. a, b, c

   .12 .32 XY

3. In what order should the following materials generally be used in teaching place value?
   - (a) Dienes multibase blocks
   - (b) Abacus
   - (c) Popsicle sticks

   - A. a, b, c
   - B. a, c, b
   - C. b, a, c
   - D. c, a, b
   - E. c, b, a

   .37 .27 YZ

a Difficulty is percentage of subjects who got this item wrong.
b Correlation between Item and (Test - Item).
c See Appendix N.

4. A sequence for teaching operations (+ and -) on single digit numbers is given below. Indicate the point in this sequence where problems like $3 + \square = 7$ would first appear.

   Sequence for Teaching — Difficulty / Correlation / Group Comparison

   $2 + 1 = \square$, $3 + 1 = \square$

   A. (here) ⟶

   $2 + 5 = 5 + \square$, $3 + 4 = \square + 3$

   B. (here) ⟶

   $2 - 1 = \square$, $3 - 1 = \square$

   C. (here) ⟶

   $6 - 4 = \square$, $7 - 3 = \square$

   D. (here) ⟶

   $6 + 5 - 3 = \square$, $7 + 3 - 4 = \square$

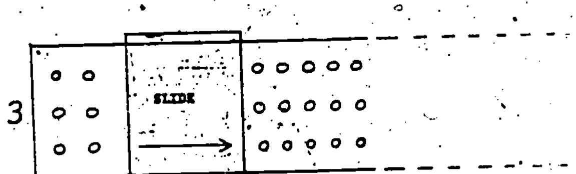
   E. (here) ⟶

   .54 .10 XY

5. Which of the following models for multiplication affords the most natural illustration of the associative property of multiplication?
   - A. Repeated addition
   - B. Number line
   - C. Stacks of blocks
   - D. Area model (graph paper)
   - E. Linear measurement

   .64 .25 XY ($p \leq .001$)

6.

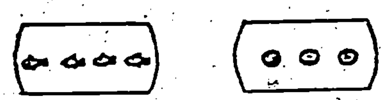


$3 \times \square = 12$

.64 .21 XY

The above diagram suggests an appropriate way to introduce which of the following:
   - A. The "missing factor" approach to division
   - B. The "repeated addition" approach to multiplication
   - C. That multiplication is commutative
   - D. That multiplication is distributive over addition
   - E. None of the above

7.



The quality that makes card 1 a better choice than card 2 for use in illustrating ordinal number concepts is: — Difficulty / Correlation / Group Comparisons
   - A. There are more fish than balls.
   - B. The fish have a clear direction while the balls do not.
   - C. The fish are clearly fish, but the balls could be perceived as marbles, peanuts, or other objects.
   - D. Children respond better to pictures of animals than to pictures of inanimate objects.
   - E. The fish are less abstract than the balls.

   .57 .38 XY ($p \leq .001$)

8. Which sequence of problems would be best for teaching addition to first graders?

   A. $\frac{3}{+5}$, $\frac{7}{+4}$, $\frac{12}{+6}$, $\frac{26}{+5}$, $\frac{31}{+25}$, $\frac{14}{+18}$

   B. $\frac{3}{+5}$, $\frac{12}{+6}$, $\frac{7}{+4}$, $\frac{31}{+25}$, $\frac{26}{+5}$, $\frac{14}{+18}$

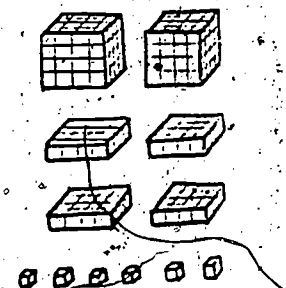
   C. $\frac{3}{+5}$, $\frac{7}{+4}$, $\frac{12}{+6}$, $\frac{14}{+18}$, $\frac{26}{+5}$, $\frac{31}{+25}$

   D. $\frac{3}{+5}$, $\frac{7}{+4}$, $\frac{12}{+6}$, $\frac{31}{+25}$, $\frac{26}{+5}$, $\frac{14}{+18}$

   E. $\frac{3}{+5}$, $\frac{7}{+4}$, $\frac{12}{+6}$, $\frac{31}{+25}$, $\frac{14}{+18}$, $\frac{26}{+5}$

   .57 .08 YX

9. Which numeral best describes the set of Dienes's blocks shown on the right after all possible trades are completed?



   - A. 2226_{four}
   - B. 2406_{four}
   - C. 3312_{four}
   - D. 3012_{four}
   - E. 246_{four}

   .29 .26 YX ($p \leq .044$)

Questions 10 to 12 each state an objective of the elementary mathematics program. For each objective indicate the grade level for which it is most appropriate. — Difficulty / Correlation / Group Comparisons

10. Ability to multiply 2- and 3-digit numbers by 2-digit numbers.
   - A. 2
   - B. 3
   - C. 4
   - D. 5
   - E. 6

   .47 .13 XY

11. Compute with decimal fractions.
   - A. 2
   - B. 3
   - C. 4
   - D. 5
   - E. 6

12. Group objects to illustrate $2 + (3+4) = (2+3) + 4$.
   - A. 2
   - B. 3
   - C. 4
   - D. 5
   - E. 6

   .22 .13 XY

13. Several children in your class (grade 1) have problems with numeration and addition. However, you feel you must move on to subtraction of one-digit numbers. Which of the following children do you expect will have the most basic trouble understanding subtraction?
   - A. Amy who confuses 3 and Σ.
   - B. Billy who doesn't conserve number.
   - C. Carol who is weak on her basic addition facts.
   - D. Debra who goes back to 1 when asked to "count on."
   - E. Eddie who doesn't concentrate and makes all kinds of errors.

   .47 .28 XY

14. What answer would you expect Judy to get for $\frac{3}{7} \div \frac{1}{21} = \square$ if the error pattern of her previous work looked like this:

   I. $\frac{3}{4} + \frac{1}{3} = \frac{9}{12} + \frac{4}{12} = 1\frac{5}{12}$ II. $\frac{7}{9} + \frac{2}{3} = \frac{7}{9} + \frac{6}{9} = 1\frac{1}{9}$

   - A. $1\frac{16}{147}$
   - B. $\frac{10}{21}$
   - C. $1\frac{10}{21}$
   - D. $1\frac{8}{147}$
   - E. $1\frac{8}{21}$

   .06 .11 XY

	Difficulty	Correlation	Group Comparisons

15. When asked to reduce $\frac{16}{64}$, Frank says "Cancel the sixes $\frac{16}{64}$ to get $\frac{1}{4}$."

Of the following, which is the best first response you could make to Frank?

A. "Frank, you forgot to factor the numerator and denominator."
B. "Frank, I would have to count that wrong, although the answer is right."
C. "Frank, can you reduce $\frac{12}{48}$ that way?"
D. "Frank, can you reduce $\frac{12}{24}$ that way?" .49 .18 XY
E. "Frank, it is an accident that you got the right answer."

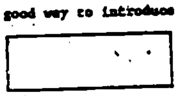
16. Here are sample problems of George's work with long division:

329	67	411	46	423
5/1645	6/3642	7/2877	4/1624	3/1269
15	36	28	16	12
14	42	7	24	6
10	42	7	24	6
45		7		9
45		7		9

Which of the following best describes George's achievement on these five problems? .08 .16 XY

A. George got all five problems correct.
B. George makes errors and will probably miss most long division exercises.
C. George makes errors but will probably only miss certain long division exercises.
D. George makes errors with long division problems that seemingly have no pattern.
E. George got the majority of the problems correct and his errors were due to faulty recall of basic multiplication facts.

17. Which is a good way to introduce finding the area of a rectangular region?



.12 .30 XY ($p \leq .006$)

A. Fit square tiles on the region.
B. Use a ruler to measure the sides.
C. Draw a diagonal to make two triangles.
D. Start with the formula $A = \ell \times w$.
E. None of the above.

18. Which of the following is a listing of Bruner's three levels of knowing in order from lowest level to highest?

A. Iconic, Symbolic, Enactive
B. Concrete, Pre-operational, Formal .14 .29 XY ($p \leq .001$)
C. Enactive, Iconic, Symbolic
D. Enactive, Symbolic, Iconic
E. Pre-operational, Concrete, Formal

19. In Gagne's terms, the learning of an algorithm for subtraction of 2-digit numbers is an example of

A. Principle learning
B. Chaining .64 .26 XY ($p \leq .001$)
C. Verbal association
D. Multiple discrimination
E. Concept learning

20. There are a lot of superstitions about teaching problem solving to children. Which of the following common beliefs is most nearly true?

A. Children are always turned off by story problems unless the problems are translated for them.
B. Children should not be allowed to work on a problem unless the teacher knows the answer.
C. Children should be given problems only after they have mastered the algorithms which yield rapid solutions.
D. Children should be encouraged to solve problems in their own ways.
E. Most of problem solving in mathematics is performing the right computation. .30 .29 YX

21. Research indicates that all of the following approaches are useful in teaching children to solve problems. Which of these is most important?

A. Teach children the "wanted-given analysis" approach as a method for solving problems.
B. Teach children a sequence of questions to ask themselves as they solve problems.
C. Give children a large number of varied problems.
D. Give children an opportunity to act out problems.
E. Give children some problems orally and others in written form. .75 .33 XY

22. Which of the following is the most difficult for elementary school children to understand?

A. $0 \div 0$ .23 .20 XY ($p \leq .023$)
B. $0 \div 1$
C. 0×0
D. 1×0
E. 1×1

	Difficulty	Correlation	Group Comparison

23. Which of the following best describes the mathematics content of standardized achievement tests?

A. Most questions are straight computation; there are a few questions on understanding of concepts.
B. Most questions are straight computation; there are a few word problems.
C. There are a few computation problems; most of the test consists of problems and concepts.
D. Questions on computation, concepts and word problems are all given emphasis.
E. The tests vary so much among grade levels and test publishers that none of the above applies. .62 .21 XY ($p \leq .013$)

24. The major asset of the chip trading game in teaching place value is

A. Children can learn lots of different bases.
B. Children can experience place value before formal instruction.
C. Children can learn to handle much larger numbers than was once thought.
D. The colorful chips are more interesting to children than regular drill activities.
E. Children can perform computations without having memorized the basic facts. .36 XY ($p \leq .011$)

25. The position of the National Council of Supervisors of Mathematics is that the core of the mathematics curriculum should be

A. Abstract reasoning.
B. Basic facts. .38 .22 XY
C. Computation.
D. Measurement.
E. Problem solving.

26. The single most important idea in developing student understanding of multiplication of numbers with more than one digit is probably

A. that 1 is the multiplicative identity.
B. the commutative law. .76 .31 XY
C. the distributive law.
D. the need for a place holder in the second partial product.
E. the need to organize and keep track of "carry digits".

27. Which of the following types of readiness activities is not considered a necessary prerequisite to learning number?

A. Comparing
B. Measuring .40 .23 YX
C. Classifying
D. Ordering
E. None of the above

28. Which of the following is least useful in helping children understand and use the algorithm for multiplying larger numbers?

A. Place value
B. Multiplication is repeated addition .43 .23 YX
C. Distributive principle
D. Multiplication facts
E. Addition facts

29. Which of the following would not be considered as part of the development of the idea of cardinal number, such as the concept of fiveness?

A. Pairing members of sets to determine if the sets are equivalent.
B. Showing a set with 5 members as 4 and 1 more.
C. Practicing forming the numeral "5". .45 .21 XY
D. Drawing pictures of sets with 5 members.
E. Identifying sets with 5 members.

30. According to Piagetian studies, what is the order of learning conservation of length as compared to conservation of numerousness?

A. Conservation of length learned first. .68 .24 YX
B. Conservation of numerousness learned first.
C. Both types of conservation learned simultaneously.
D. No predictable order in learning the two types of conservation.

31. The area of this sheet of paper (one side) is about

A. 7 square meters. .34 .45 XY ($p \leq .001$)
B. 7 square millimeters.
C. 700 square centimeters
D. 700 square millimeters
E. 7 hectares

Difficulty | Correlation | Group Comparisons

32. What would be the probable order for learning the following geometric concepts?

(a) betweenness
(b) inside-outside
(c) congruence

.34 .36 XY

A. (a) and (b) together followed by (c).

B. (a) followed by (b) and (c) together.

C. (a) and (c) together followed by (b).

D. (c), (b), (a).

33. Your second grade class is conducting a survey. Which of the concepts (mean, median, mode) can you introduce meaningfully at this grade level?

A. All three.
B. Mean and median.
C. Median and mode only.
D. Mode only.
E. None.

.77 .16 YX

34. Two types of acceleration for children gifted in mathematics have been identified. In vertical acceleration the children are introduced to the topics and text materials from higher grade levels. In horizontal acceleration they are

A. involved in peer teaching activities.

.30 .37 XY

B. actually placed in higher grades for mathematics.

C. given problems and activities which require insight and sophisticated application of the math at their own grade.

D. given responsibility for classroom activities using math such as collecting money.

E. given activities similar to those of the rest of the class but involving bigger numbers and more complicated computations.

35. The process of counting objects is essentially

A. a memory process.
B. a matching process.
C. an addition process.
D. a theoretical process.

.38 .25 YX

36. In what order would the following be considered in the development of addition, subtraction, multiplication or division of whole numbers?

(a) The algorithm for computing
(b) The concept of the operation
(c) The basic facts

A. a, b, c

.40 .31 YX ($p \leq .015$)

B. b, a, c

C. b, c, a

D. c, a, b

E. c, b, a

37. Here are 5 subtraction problems worked by John:

75	43	239	684	793
-32	-27	- 63	-368	-261
43	24	234	324	532

What answer would you expect John to give to the problem 5627 -2853

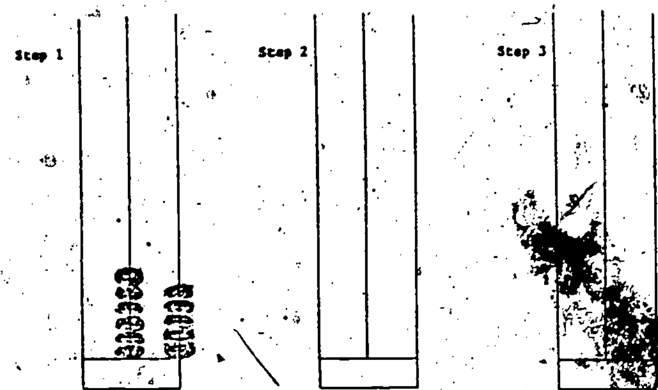
A. 2774
B. 2834
C. 3234
D. 3374
E. None of the above

.03 -.03 YX ($p \leq .030$)

What would you do to help John correct his error? (Write your explanation below.)

.58 .05 YX

38. Show a step-by-step solution on a twenty-bead abacus for the problem 65 + 5. (Step one is done for you. Draw in the beads for steps 2 and 3.)



Difficulty | Correlation | Group Comparison

.45 .10 XY

39. On the next two pages you will find two activities, A and B, described. You are a first grade teacher and are considering using each of these in your classroom. Evaluate each activity and decide whether you will use it or not. Your evaluation should include discussion of:

(a) the probable objective(s) of the activity
(b) the validity and usefulness of the activity as a means toward attaining the objective(s)
(c) the appropriateness of the objective(s) and the activity to the first grade level.

Record your evaluations of Activity A (yellow) and Activity B (pink) on the sheet below the description of the activity. You may use the back of the sheet if you need more space.

Go on to next page.

Activity A

Materials: A pair of dice, one blue and one red.

Description: Two children play the game in pairs. Each starts with a marker at 0 on a number line. One child rolls the dice and subtracts the value showing on the blue die from the value showing on the red die. He then moves his marker the number of units represented by the difference. If the difference is positive the marker is moved to the right, if negative, to the left. Play continues with the children taking turns. The first player to reach 30 on the number line wins the game.

Difficulty | Correlation | Group Comparisons

.48 .32 XY ($p \leq .001$)

Activity B

Materials: Plastic boxes of Cuisenaire rods.

Description: Children play the game in pairs. The first child chooses a non-white rod and places a shorter rod below it. The second player must choose the rod which when placed with the shorter rod would make a "train" as long as the first rod and tell the "minus story" using numbers. Then the second child chooses a pair of rods and the first child must choose the missing rod and tell the "minus story". A player gets one point for a correct match and the appropriate "minus story". If the correct match and "minus story" are not given, the player who chose the rods gets the point. Play continues until one player scores ten points.

.62 .30 XY

Appendix HH

Problem Solving Test Items, Item Statistics, and Test Statistics (Year 2)

Math 106
Spring 1978

Final Exam

Problem Solving

Name ______________________

M106 Instructor(s) __________

You have 55 minutes for this 10 item test. Work neatly in the space provided. Each problem is worth 10 points.

	Difficulty[a]	Correlation[b]	Group Comparisons[c]
1. A car wash charges $1.75 for a wash but reduces prices in relation to the amount of gasoline purchased.			

amount	0-7.9 gal	8-9.9 gal	10-11.9 gal	12-14.9 gal	15 gal to fill
cost	$1.75	$1.38	$1.28	$1.18	$.98

If you buy 10.7 gallons of gas at the car wash at a price of 65.9 cents per gallon and you also have your car washed, what is the total cost?

	Difficulty[a]	Correlation[b]	Group Comparisons[c]
Item 1	.13	.16	XY

Methods Test Statistics

(Year 2)

Number of Items	40
Possible Score	98
Number of Subjects	146
Mean	55.12
Standard Deviation	9.68
Average Item - Total Correlation	0.15
Average Difficulty	0.41
Internal Consistency Reliability	0.61

2. If you select at random one of the subsets of a 17 element set, what is the probability it contains at least 9 elements?

	Difficulty	Correlation	Group Comparisons
Item 2	.90	.19	XY

3. A palindrome is a whole number which reads the same forwards as backwards. For example, 131, 2552, and 34743 are palindromes.

How many 2-digit palindromes are there? 3 digit palindromes?

	Difficulty	Correlation	Group Comparisons
Item 3	.39	.24	YX

a Difficulty is percentage of subjects who got this item wrong.
b Correlation between Item and (Test - Item).
c See Appendix N.

In your class of 11 students, on a recent 20 point quiz, the average was 11. Unfortunately, you misplaced Joan's paper. The remaining 10 scores were 14, 18, 3, 17, 11, 19, 10, 15, 14, 6. What was Joan's score?

	Difficulty	Correlation	Group Comparisons
Item 4	.22	.18	YX

5. It takes 867 digits to number the pages of a book. (Page 9 uses one digit, page 37 uses two digits, etc.)

A. How many pages are there in the book?

B. How many times does the digit 5 appear?

	Difficulty	Correlation	Group Comparisons
Item 5	.80	[illegible]	YY

6. A train that is one mile long is traveling 1 mile each 3 minutes. How long does it take this train to pass through a 2-mile tunnel?

	Difficulty	Correlation	Group Comparisons
Item 6	.40	.28	XY ($p \leq .001$)

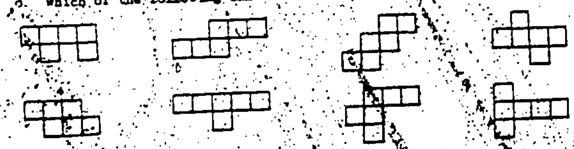
7. A car travels 20,000 miles on five tires which are rotated so that each is in use for the same distance. How many miles is each tire in use?

	Difficulty	Correlation	Group Comparisons
Item 7	.70	.41	YX

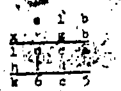
Math 106
Final - Page 3

	Difficulty	Correlation	Group Comparisons
8. Which of the following can be folded to make a cube (with no overlapping)?	.19	.12	XY (p ≤ .001)
9. Find whole number values of a, b, c, d, e, f, g, and h so that the following multiplication problem is correctly worked out: (Please answer by writing out the complete problem.)	.33	.21	XY
10. a) How much will it cost to tile the floor if you use only triangular tiles?	.56	[illegible]	.16	XY (p ≤ .031)
10. b) What is the least you could spend to tile the floor if both square and triangular tiles were used?	.96	.24	XY (p ≤ .020)

8\. Which of the following can be folded to make a cube (with no overlapping)?



9\. Find whole number values of a, b, c, d, e, f, g, and h so that the following multiplication problem is correctly worked out: (Please answer by writing out the complete problem.)



10\. A 10 foot by 20 foot rectangular room is to be tiled in brown and beige in such a way that on one side of the diagonal line all tiles are brown, on the other side all are beige. Available are tiles in both brown and beige in two shapes: 2 foot by 2 foot squares and right triangles with one leg of length 1 foot, one leg of length 2 feet. Tiles cost $1 apiece, regardless of shape.

a) How much will it cost to tile the floor if you use only triangular tiles?

b) What is the least you could spend to tile the floor if both square and triangular tiles were used?

Problem Solving Test Statistics

Number of Items	11
Possible Score	100
Number of Subjects	146
Mean	51.05
Standard Deviation	15.37
Average Item - Total Correlation	0.23
Average Difficulty	0.51
Internal Consistency Reliability	0.53

Appendix II

Intercorrelations of Academic Background Variables with Criterion Variables (Year 2)

	Concepts Criterion	Methods Criterion	Prob. Solving Criterion
GPA	.43 ***	.49 ***	.31 ***
Expected M105 Grade	.17 *	.22 **	.22 **
College Math Rating	.13 *	.01	.28 ***
Years of H.S. Math	.20 **	.04	.25 ***
Average H.S. Math Grade	.36 ***	.22 **	.27 ***

n = 137

* p ≤ .05

** p ≤ .01

*** p ≤ .001

Appendix JJ

Intercorrelations of Attitude Pretests, Attitude Posttests and Criterion Measures (Year 2)

		Attitude Pretests: Mathematics	Attitude Pretests: Teaching Mathematics	Attitude Pretests: Teaching Elem. School Children	Attitude Pretests: Elem. School Children	Attitude Posttests: Mathematics	Attitude Posttests: Teaching Mathematics	Attitude Posttests: Teaching Elem. School Children	Attitude Posttests: Elem. School Children	Criterion Measures: Concepts	Criterion Measures: Methods
Attitude Pretests	Teaching Mathematics	.58 ***									
	Teaching Elem. School Children	.24 **	.27 **								
	Elementary School Children	.00	.10	.17 *							
Attitude Posttests	Mathematics	.65 ***	.36 ***	.17 *	-.05						
	Teaching Mathematics	.27 ***	.46 ***	.24 **	.00	.37 ***					
	Teaching Elem. School Children	.15	.21 *	.49 ***	.02	.16	.43 ***				
	Elementary School Children	-.12	-.12	.17 *	.47 ***	-.11	-.07	.08			
Criterion Measures	Concepts	.18 *	.12	.00	-.19 **	.22 **	.07	-.05	-.14		
	Methods	.08	.05	.12	-.01	.15	.09	.16	.08	.41 ***	
	Problem Solving	.28 ***	.05	.12	-.06	.29 ***	-.00	.10	-.05	.51 ***	.43 ***

n = 137 for all cells

* p ≤ .05 ** p ≤ .01 *** p ≤ .001

Appendix KK

Means for Post Test Attitudes by Group Adjusted for Initial Differences in Freshman Early Experience Program and Attitude Toward Teaching Children Pretest (Year 2)

Attitude Scale	Group X	Group Y
Mathematics	31.60	32.40
Teaching Mathematics	34.52	35.30
Teaching Elementary School Children	35.24	36.24
Elementary School Children	31.77	32.07

Appendix LL

Multivariate Analysis of Covariance of Post-test Attitudes by Group Using Freshman Early Experience Program and Attitude Toward Teaching Children Pretest as Covariates (Year 2)

Test of Homogeneity of Regression

Multivariate	$F(8,256)$		$p \leq$
	0.78		0.620

Univariate	$F(2,131)$	MS_B	$p \leq$
Mathematics	0.87	13.00	0.421
Teaching Mathematics	0.08	0.47	0.919
Teaching Elementary School Children	0.25	1.95	0.778
Elementary School Children	1.46	14.10	0.237

Test of Program Differences

Multivariate	$F(4,130)$		$p \leq$
	1.98		0.101

Univariate	$F(1,133)$	MS_B	$p \leq$
Mathematics	1.37	20.44	0.244
Teaching Mathematics	3.61	19.67	0.060
Teaching Elementary School Children	5.78	44.48	0.018
Elementary School Children	0.29	2.84	0.591

Appendix MM

Analysis of Variance of Attitudes by Group (Year 2)

Attitude toward Mathematics

Source	df	SS	MS	F	$p \leq$
Between Subjects					
Program	1	128.89	128.89	3.12	NS
Testing	136	5609.34	41.25		
Within Subjects					
Testing	2	116.12	58.06	10.46	.001
Program x Testing	2	1.32	0.66	0.12	NS
Subjects x Testing/Program	272	1509.88	5.55		
TOTAL	413	7365.55			

Attitude toward Teaching Mathematics

Source	df	SS	MS	F	$p \leq$
Between Subjects					
Program	1	32.50	32.50	2.26	NS
Testing	136	1957.97	14.40		
Within Subjects					
Testing	2	44.50	22.25	5.70	.01
Program x Testing	2	14.53	7.26	1.86	NS
Subjects x Testing/Program	272	1061.59	3.90		
TOTAL	413	3111.09			

Attitude toward Teaching Elementary School Children

Source	df	SS	MS	F	$p \leq$
Between Subjects					
Program	1	149.76	149.76	7.78	.01
Testing	136	2618.00	19.25		
Within Subjects					
Testing	2	26.44	13.22	2.33	NS
Program x Testing	2	17.52	8.76	1.55	NS
Subjects x Testing/Program	272	1540.60	5.66		
TOTAL	413	4352.32			

Appendix NN

Means, Standard Deviations, and T-Tests on Interview Data by Group (Year 2)

I. Goals and Objectives

How would you rank the following goals in terms of importance to an elementary school teacher? In terms of the emphasis given to them during the last two quarters?

	Importance							Emphasis						
	Program X		Program Y					Program X		Program Y				
	$\bar{x}$	SD	$\bar{x}$	SD	t	df	$p \leq$	$\bar{x}$	SD	$\bar{x}$	SD	t	df	$p \leq$
Selection and sequencing of learning activities based on the mathematics.	2.00	1.10	2.25	0.96	-0.58	21	.567	2.75	1.06	3.00	0.85	[illegible]	22	.530
Selection and sequencing of learning activities based on the characteristics of the children.	2.09	0.83	2.42	1.16	-0.77	21	.452	2.08	0.90	[illegible]	[illegible]	[illegible]	22	[illegible]
Understanding of the mathematics concepts.	3.09	1.14	2.33	1.23	1.53	21	.141	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22	[illegible]
Interaction and communication with children	2.82	1.17	3.00	1.13	-0.38	21	.708	1.50	0.67	1.42	0.79	0.28	22	[illegible]

II. Resources and Activities

A. How would you rank order the following in terms of influence on your learning this quarter? In terms of amount of time you spent with each this quarter?

(5 = most
1 = least)

	Influence							Time Spent						
	Program X		Program Y					Program X		Program Y				
	$\bar{x}$	SD	$\bar{x}$	SD	t	df	$p \leq$	$\bar{x}$	SD	$\bar{x}$	SD	t	df	$p \leq$
Textbook(s)	3.17	1.34	2.25	1.14	1.81	22	.084	4.33	1.37	3.33	1.61	1.64	22	.116
Teachers	3.25	1.60	4.25	1.06	-1.81	22	.085	3.17	0.72	4.58	0.52	-5.56	22	.000
Manipulative Materials	2.75	1.60	3.50	1.09	-1.34	22	.193	2.92	1.38	2.75	1.06	0.33	22	.743
Pupils in Public Schools	2.42	0.90	3.33	1.30	-2.01	22	.057	1.42	0.52	1.75	0.75	-1.26	22	.219
Fellow Students	3.42	1.56	1.67	0.98	3.28	22	.003	3.17	1.27	2.58	1.24	1.14	22	.267

Attitude Toward Elementary School Children

Source	df	SS	MS	F	$p \leq$
Between Subjects					
Program	1	24.64	24.64	1.24	NS
Testing	136	2703.53	19.88		
Within Subjects					
Testing	2	24.58	12.29	2.45	NS
Program x Testing	2	5.95	2.97	0.59	NS
Subjects x Testing/Program	272	1364.80	5.02		
TOTAL	433	4123.50			

II (cont.)

B. How did the mathematics preparation over the last two quarters compare in time and quality with content preparation in other areas?

(3: Math > Other Areas; 2: Math = Other Areas; 1: Math < Other Areas)

	Program X $\bar{X}$	Program X SD	Program Y $\bar{X}$	Program Y SD	t	df	$p \leq$
Time	2.56	0.88	2.20	0.92	0.86	17	.401
Quality	2.44	0.53	2.40	0.52	0.19	17	.85

C. Did you have FEEP? (1 = YES 0 = NO)

How did this school experience compare with the FEEP?

(Comparison of this program with FEEP: 3: this program > FEEP; 2: this program = FEEP; 1: this program < FEEP)

	Program X $\bar{X}$	Program X SD	Program Y $\bar{X}$	Program Y SD	t	df	$p \leq$
Have FEEP	1.00	0.00	1.00	0.00	0.00	22	1.000
Compare	1.50	0.80	1.08	0.29	1.70	22	.111

III. Outcomes and Evaluation

Where would you mark each of the following so as to represent the area in which you have learned the most?

	Program X $\bar{X}$	Program X SD	Program Y $\bar{X}$	Program Y SD	t	df	$p \leq$
Problem Solving	7.83	2.25	6.75	1.71	1.33	22	.198
Concepts	9.25	1.82	9.83	1.34	-0.90	22	.380
Methods	6.92	2.64	7.42	2.02	-0.52	22	.608

Would you participate in a project like this one if the chance to do so came up again?

Would you recommend this experience to your friends?

(1: definitely not; 5: definitely yes)

Program X $\bar{X}$	Program X SD	Program Y $\bar{X}$	Program Y SD	t	df	$p \leq$
3.83	1.27	3.58	1.00	0.54	22	.596
3.42	1.50	3.58	1.41	-0.29	22	.775

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